Inverse magnetic catalysis at the QCD transition

Falk Bruckmann
(Univ. Regensburg)

Δ, Heidelberg, Jan. 2013

with G. Bali, G. Endrődi, Z. Fodor, S. Katz, T. Kovács,
S. Krieg, A. Schäfer, K. Szabó

JHEP 1202 (2012) 044, PRD 86 (2012) 071502, in prep.^{2}

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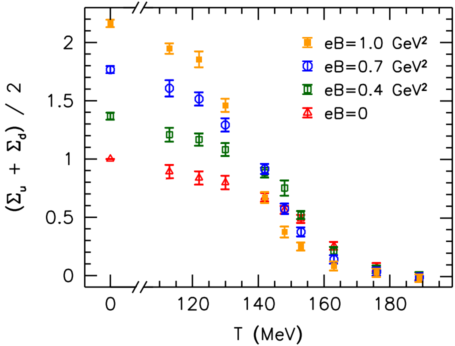
Main results and outline

- (light) quark condensate condensate as a function of T at fixed B's:



$T = 0$: Magnetic Catalysis

- Model Conjecture
- Monte Carlo ✓ D'Elia et al. 10

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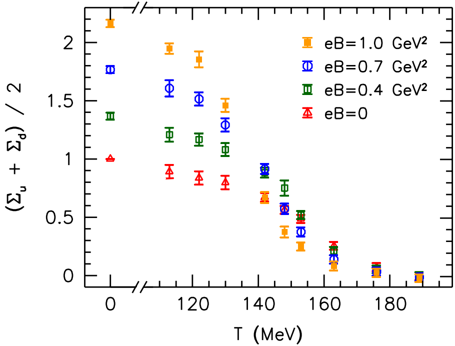

$T = 0$: Magnetic Catalysis

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Inverse Magnetic Catalysis: $T_c(B) \searrow$

- Important: Masses of Current quarks
- Investigate Monte Carlo configurations!

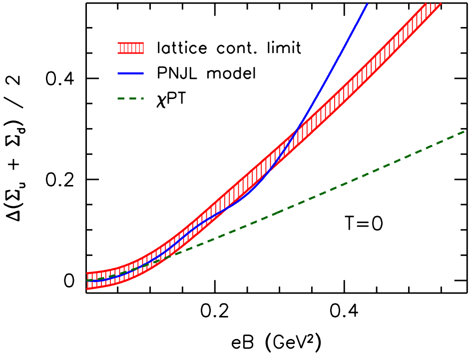
  $\Rightarrow$ sea and valence effects

Magnetic catalysis

- change of condensate $\left[\langle\bar\psi\psi_{u,d}\rangle(B) - \langle\bar\psi\psi_{u,d}\rangle(0)\right] m_{u=d} \sim \Delta\Sigma_{u,d}$

  cont. limit of $N_f = 1+1+1$ (staggered) quarks at physical masses

Bali, FB, Endrődi et al. 11



chiral perturbation theory — Cohen, McGady, Werbos 07, Andersen 12

& NJL model — Gatto, Ruggieri 10

$\Rightarrow$ well approximated unless $eB > \begin{matrix}0.1\\0.3\end{matrix}$ GeV^2 (approaches valid there?)

Magnetic Catalysis: picture

NJL model

Müller, Schramm[2] 92, Gusynin, Miransky, Shovkovy 96

$$\mathcal{L} = \bar\psi \not{D}[B]\psi + G\, O(\psi^4)$$

- chiral condensate in mean field:

$$\langle\bar\psi\psi\rangle \equiv \mathrm{tr}\frac{1}{\not{D}[B]+m} \qquad \ldots \text{just } B\text{, otherwise free}$$

$$= m_0\, qB \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} e^{-m_0^2 s} \coth(sqB) \qquad \ldots \text{proper time (Mellin trafo)}$$

↑ degeneracy

Schwinger 51

$$= m_0 \left[\Lambda^2 + qB \log\frac{qB}{m_0^2}\right] \qquad \ldots \text{for small } m_0\text{, large } B$$

- dynamical mass m from gap equation:

$$m = G\langle\bar\psi\psi\rangle = mG\left[\Lambda^2 + qB\log\frac{qB}{m^2}\right]$$

- always trivial solution $m = 0$ preserving chiral symmetry
- nontrivial solution $m > 0$ breaking chiral symmetry

$$m = \sqrt{qB}\, \exp\left(-\frac{1}{G\, qB}\right) \exp\left(\frac{\Lambda^2}{qB}\right)$$

for arbitrarily small magnetic fields & small couplings G

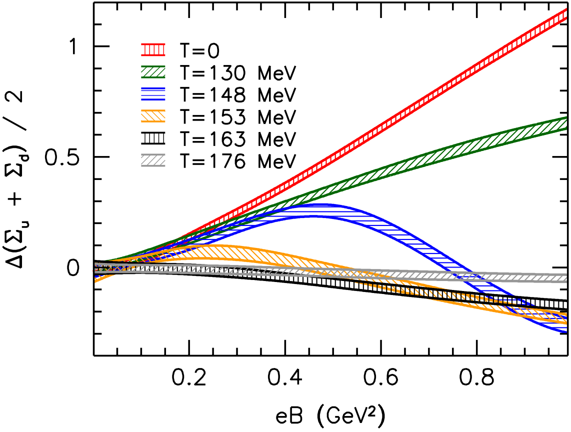
nonperturbative

magn. field catalyzes condensate and dyn. mass

- in contrast vanishing magnetic field:
  nontrivial solution only for strong couplings G

Inverse magnetic catalysis

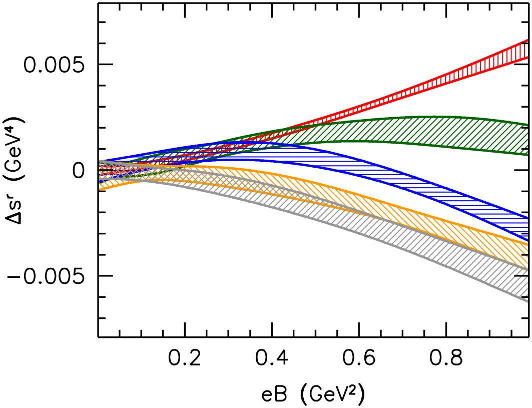
- again change of condensate, for finite T: Bali, FB, Endrődi et al. 12



non-monotonic behaviour: MC turns into IMC around T_c

Inverse magnetic catalysis for gluons

- change of gluonic action, for finite T: Bali, FB, Endrődi et al. in prep.



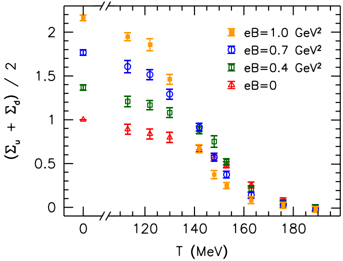
non-monotonic behaviour: MC turns into IMC around T_c

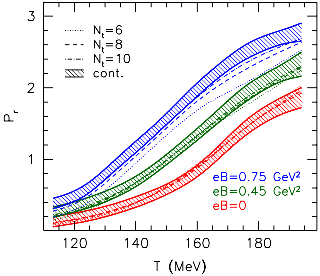
related to condensates via trace anomaly!

details

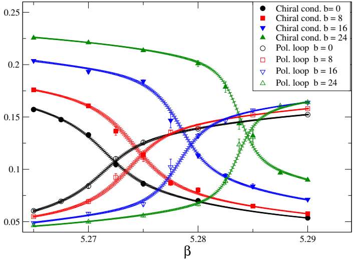
Inverse magnetic catalysis: mass dependence

$1+1+1$ (staggered, smeared) us
at phys. masses





$1+1$ (staggered) D'Elia et al. 10
at higher-than-phys. masses



similar in $SU(2)$ Ilgenfritz et al. 12

! $T_c(B)$ different

! monotonicities different

rationale: continuum limit,

light quark masses!

▸ cross-checked

$$\bar\psi\psi^{\rm full} = \frac{\int DA e^{-S_g} \det(\not{D}[B]+m)\,{\rm tr}(\not{D}[B]+m)^{-1}}{\int DA e^{-S_g} \det(\not{D}[B]+m)}$$

D'Elia, Negro 11

$\simeq$

$$\bar\psi\psi^{\rm val} = \frac{\int DA e^{-S_g} \det(\not{D}[0]+m)\,{\rm tr}(\not{D}[B]+m)^{-1}}{\int DA e^{-S_g} \det(\not{D}[0]+m)}$$

B in observable

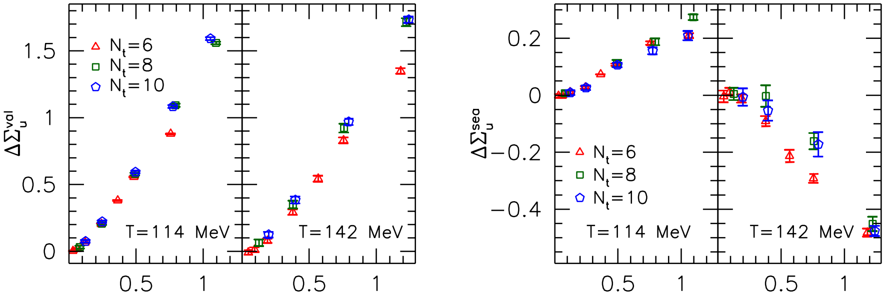
$+$

$$\bar\psi\psi^{\rm sea} = \frac{\int DA e^{-S_g} \det(\not{D}[B]+m)\,{\rm tr}(\not{D}[0]+m)^{-1}}{\int DA e^{-S_g} \det(\not{D}[B]+m)}$$

B in config. generation

- at low T and around T_c:

FB, Endrődi, Kovács (in prep.)



Inverse magnetic catalysis: mechanism

$$\bar{\psi}\psi^{\text{full}} = \frac{\int DA e^{-S_g} \det(\not{D}[B]+m)\,\mathrm{tr}(\not{D}[B]+m)^{-1}}{\int DA e^{-S_g} \det(\not{D}[B]+m)}$$

D'Elia, Negro 11

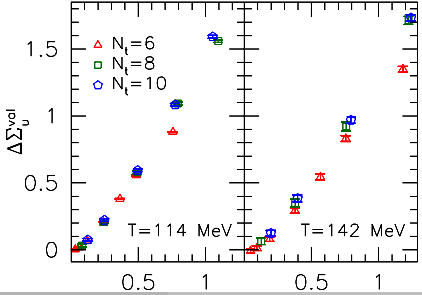
$$\simeq$$

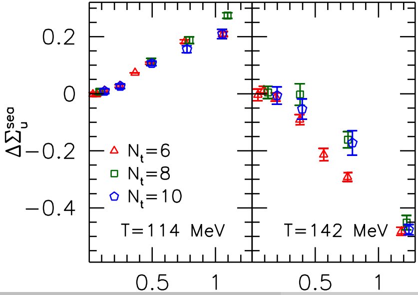
$$\bar{\psi}\psi^{\text{val}} = \frac{\int DA e^{-S_g} \det(\not{D}[0]+m)\,\mathrm{tr}(\not{D}[B]+m)^{-1}}{\int DA e^{-S_g} \det(\not{D}[0]+m)}$$ B in observable

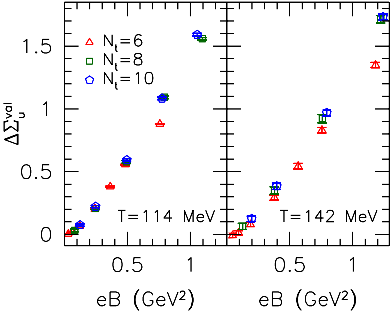
$$+$$

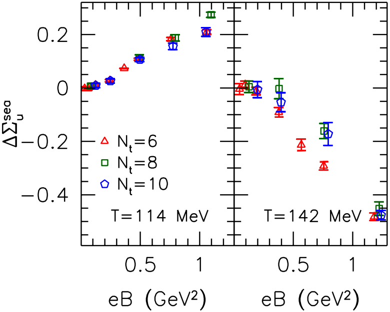
$$\bar{\psi}\psi^{\text{sea}} = \frac{\int DA e^{-S_g} \det(\not{D}[B]+m)\,\mathrm{tr}(\not{D}[0]+m)^{-1}}{\int DA e^{-S_g} \det(\not{D}[B]+m)}$$ B in config. generation

- around T_c: $\bar{\psi}\psi^{\text{sea}} < 0$ and competing with $\bar{\psi}\psi^{\text{disfavorval}} > 0 \Rightarrow$ IMC









$\not{D}[B]$ has small eigenvalues with degeneracy B

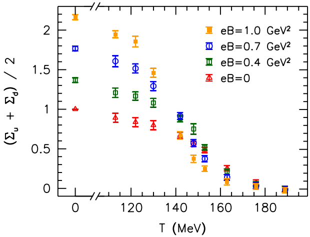
$\Downarrow$ in <u>valence</u> trace	$\Downarrow$ in <u>sea</u> determinant
generates condensate	avoided: low probability
= statement about the change of the spectrum	= statement about the typical gauge field
(even quenched)	= feedback of quarks

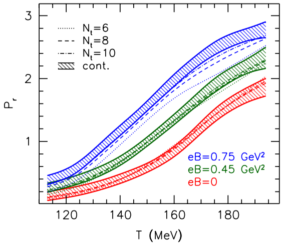
sea effect is particularly effective near T_c! why increasing at low T?

washed out for heavy quarks

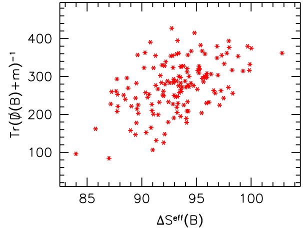
Inverse magnetic catalysis: role of Polyakov loop

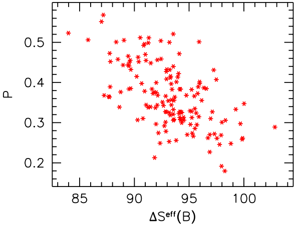
again: condensate Polyakov loop





- reweighting from $B = 0$ with eff. ferm. action $-\Delta S^{\mathrm{eff}} = \log \frac{\det \not{D}[B]+m}{\det \not{D}[0]+m}$





near T_c

$\Rightarrow$ small action prefers smaller condensate and larger Polyakov loop

Inverse magnetic catalysis: picture

Polyakov loop = effectively part of fermionic boundary conditions

large Polyakov loop $\rightleftharpoons$ few small eigenvalues

- high T: large eff. Matsubara frequency
- small T: tr $P \simeq 0 \rightleftharpoons$ many small eigenvalues, condensate

with B: larger Polyakov loop $\rightleftharpoons$ smaller sea condensate

particularly sensitive at the transition and for small quark masses

Free case

free quarks with magn. field B and constant Polyakov loops

$$P = \begin{pmatrix} e^{2\pi i\varphi} & & \\ & e^{-2\pi i\varphi} & \\ & & 1 \end{pmatrix} \begin{cases} \varphi = 1/3: & \mathrm{tr}P = 0 \quad \text{'conf. phase'} \\ \varphi \to 0: & \mathrm{tr}P \to 1 \quad \text{'deconf. phase'} \end{cases}$$

'confined' minus 'deconfined' free energy (eff. action):

$$f(B,\varphi) - f(B,0) \sim -\log\det(\not{D}[B,\varphi] + m) + \log\det(\not{D}[B,0] + m) \quad \text{sea!}$$
$$= \frac{qB}{\pi^2}\int_0^\infty \frac{ds}{s^2}\, e^{-m^2 s}\coth(qBs)\big[\theta_3((\varphi + \frac{1}{2})\pi, e^{-1/4sT^2}) - \theta_3(..\varphi = 0..)\big]$$

integrand positive and

✓grows with smaller $m \Rightarrow$ light quarks prefer deconfinement, $T_c(m) \searrow$

(✓would decrease with imag. $\mu \Rightarrow$ prefers confinement, $T_c(i\mu) \nearrow$)

! grows with $B \Rightarrow$ prefers deconfinement, $T_c(B) \searrow$

Summary

- magnetic catalysis: $\langle\bar{\psi}\psi\rangle(B) \nearrow$ at $T = 0$
- inv. magnetic catalysis: $\langle\bar{\psi}\psi\rangle(B) \searrow$ at $T \simeq T_c$
  - only for light (phys.) quark masses
  - sea quark back reaction, in other approaches!?
  - role of Polyakov loop: in line with fewer small eigenvalues
- QCD crossover: $T_c(B)$ decreases slightly

Backup: Strong Magnetic fields

early universe	$\sqrt{eB} \simeq 2$ GeV	
RHIC/LHC non-central collisions charged spectators B perp. to reaction plane	0.1..0.5 GeV	QCD scale!
neutron stars, magnetars	1 MeV	$B \simeq 10^{14}$ G

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cf. strongest field in lab		10^5 G (10^7 G unstable)
refrigerator magnet		100 G
earths magn. field		0.6 G

Backup: Simulation details

as for transition studies at $B = 0$ Budapest-Wuppertal

- tree-level improved gauge action
- stout smeared staggered fermions (rooting trick)
- 2 light quarks + strange quark, charges $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$
- lattice spacing set at $T = 0, B = 0$
  physical pion masses
  set by f_K, f_K/m_π and f_K/m_K
- $T = 0$: $24^3 \times 32$, $32^3 \times 48$ and $40^3 \times 48$ lattices
- $T > 0$: $N_t = 6, 8, 10$ meaning $a = 0.2, 0.15, 0.12$ fm
  $N_s = 16, 24, 32$ for finite volumes
- condensates from stochastic estimator method with 40 vectors
- magn. flux quanta: $N_B \leq 70 < \frac{N_x N_y}{4} = 144$

Backup: Trace anomaly/interaction measure

definition:

$$I = \epsilon - 3p = -T^5 \frac{\partial}{\partial T} \frac{\log f}{T^4}$$

$$\stackrel{\text{lattice}}{=} -\frac{1}{V^4} \frac{\partial \log Z}{\partial \log a} \qquad V_4 = V/T$$

free/conformal system: $f \sim T^4$ [SB], $\log Z$ is scale-indep. $\Rightarrow I = 0$

lattice:

$$-I = \frac{1}{V_4} \left(\frac{-\partial \log Z}{\partial \beta} \frac{-\partial \beta}{\partial \log a} + \sum_f \frac{\partial \log Z}{\partial m_f} \frac{\partial m_f}{\partial \log a} \right)$$

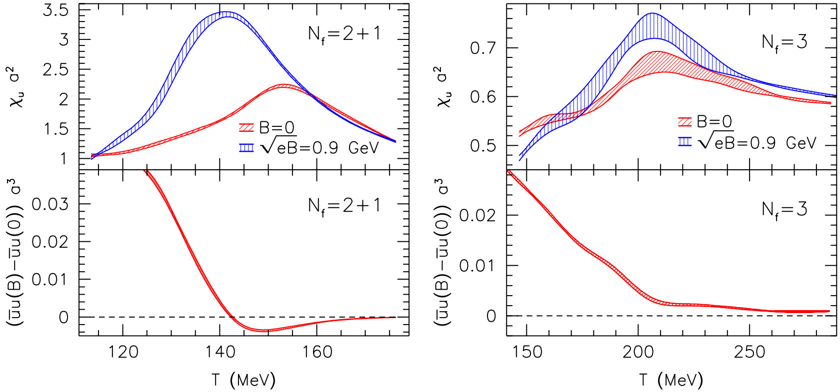
$$= \langle s \rangle \overbrace{\frac{-\partial \beta}{\partial \log a}}^{>0} + \sum_f \underbrace{\langle \bar{\psi}_f \psi_f \rangle m_f}_{\text{mult. renorm.}} \overbrace{\frac{\partial \log m_f}{\partial \log a}}^{\text{from LCP}}$$

prescription for mult. renorm. of action density

(all add. renorms. removed via differences in B)

Backup: Mass sensitivity

- what if we put $(m_{\text{light}}, m_{\text{light}}, m_{\text{strange}}) \rightarrow (m_{\text{strange}}, m_{\text{strange}}, m_{\text{strange}})$?



T-dep. of *u*-susceptibility (top) and change of *u*-condensate (bottom)

$\Rightarrow$ effects of decreasing T_c & inverse magn. catalysis disappear

light quark masses are important