QCD thermodynamics in magnetic fields: HRG (and lattice)

[arXiv:1301.1307]

Gergely Endrődi

University of Regensburg

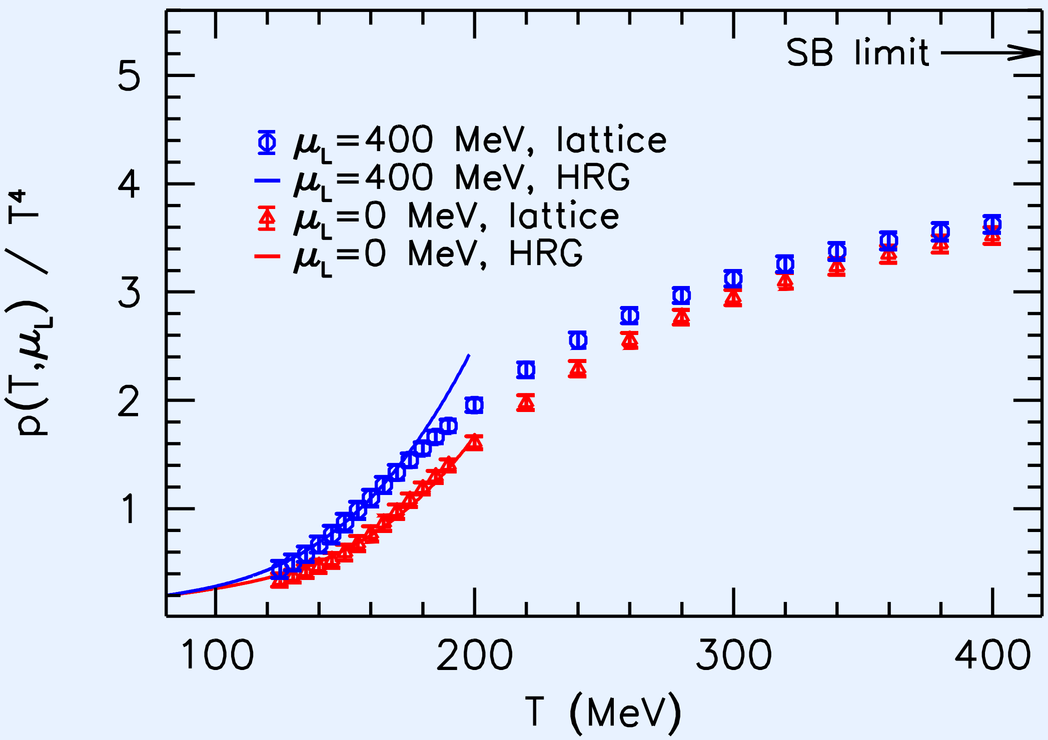
Delta 2013

Heidelberg, 12. January 2013

Introduction

- QCD deconfined phase reproduced in high energy colliders
- experimental results well described by near-ideal relativistic hydrodynamics
- equilibrium description of the system is given by the equation of state (EoS)
- EoS relevant for heavy ion collisions, early Universe and neutron stars
- high T: perturbation theory describes q, g degrees of freedom
- around T_c: lattice simulation of the EoS is computationally very demanding + conceptual problems with $\mu \neq 0$
- low T: surprisingly well described by HRG

Example: HRG vs lattice

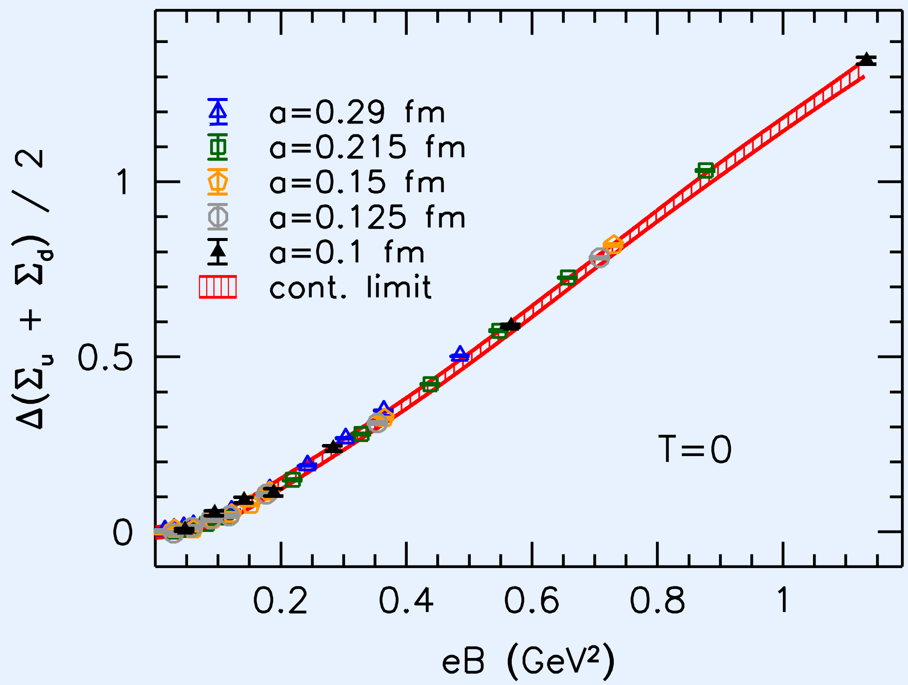


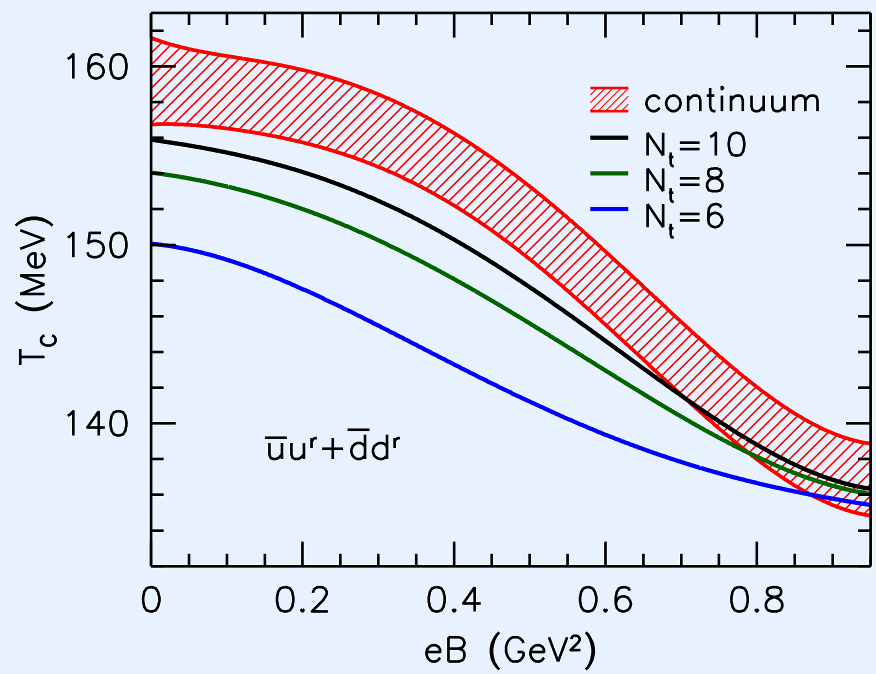
- HRG approximation: take a gas of non-interacting hadrons, and sum up the individual contributions to the free energy
- agreement with lattice $T \lesssim 130 - 150$ MeV, both at $\mu = 0$ and $\mu > 0$ [Borsányi, GE et al '11, '12]

Role of magnetic fields

- relevant state parameters: T and μ, and also B
- systems with strongly interacting matter and magnetic fields
  - dense neutron stars, magnetars
  - non-central heavy ion collisions
  - early universe cosmology
- magnitudes: reaching up to $eB \sim \mathcal{O}(\Lambda^2_{\text{QCD}})$
- B acts as a probe of the QCD vacuum:
  - enhances chiral symmetry breaking at $T = 0$
    [Gusynin et al '96]
  - $B - T$ phase diagram structure
    [Bali, Bruckmann, GE et al '11]

Magnetic field and CSB





- $\bar{\psi}\psi = T/V \cdot \partial \log \mathcal{Z} / \partial m_q$
- $T = 0$: magnetic catalysis $\bar{\psi}\psi(B) - \bar{\psi}\psi(0) > 0$
- $T > 0$: modified by gluonic back-reaction
  [talk by Bruckmann]
- result: $T_c(B)$ decreases [Bali, Bruckmann, GE et al '11, '12]

Thermodynamics for $B > 0$

- free energy $\mathcal{F} = -T \log \mathcal{Z}$

$$\mathcal{S} = -\frac{\partial \mathcal{F}}{\partial T}, \qquad \mathcal{M}_B = -\frac{\partial \mathcal{F}}{\partial B}, \qquad p = -\frac{\partial \mathcal{F}}{\partial V} = -\frac{\mathcal{F}}{V}.$$

- fundamental relation

$$\mathcal{F} = \mathcal{E} - T\mathcal{S} - B\mathcal{M}_B.$$

- intensive quantities

$$s = \frac{\mathcal{S}}{V}, \qquad \epsilon = \frac{\mathcal{E}}{V}, \qquad f = \frac{\mathcal{F}}{V}, \qquad m_B = \frac{\mathcal{M}_B}{V}.$$

- energy density given as

$$\epsilon = Ts + Bm_B - p.$$

- speed of sound

$$c_s^2 = \left.\frac{\partial p}{\partial \epsilon}\right|_B = \left.\frac{\partial p}{\partial T}\right|_B \bigg/ \left.\frac{\partial \epsilon}{\partial T}\right|_B .$$

Free energy

- free particle (m, s, q) in a magnetic field $B \parallel z$ has energies

$$E(p_z, k, s_z) = \sqrt{p_z^2 + m^2 + 2qB\,(k + 1/2 - s_z)},$$

- free energy density in terms of energy levels

$$f(s) = \mp \sum_{s_z} \sum_{k=0}^{\infty} \frac{qB}{2\pi} \int \frac{\mathrm{d}p_z}{2\pi} \Bigg[\underbrace{\frac{E(p_z, k, s_z)}{2}}_{\text{vacuum}} + \underbrace{T \log(1 \pm e^{-E(p_z,k,s_z)/T})}_{\text{thermal}} \Bigg],$$

- thermal part is finite, calculated numerically
- vacuum part

$$f^{\text{vac}}(s) = f(s)|_{T=0}$$

  is divergent due to charge renormalization
- background field method [Abbott '81]
  β-function using fermion propagator in B-field

Free energy, for spin-zero particle

- dimensional regularization (ϵ, μ) gives:

$$\Delta f^{\text{vac}} = \frac{(qB)^2}{192\pi^2}\left[\left(\frac{2}{\epsilon} - \gamma - \log\left(\frac{m^2}{\mu^2}\right)\right) + f(m^2/qB)\right].$$

- include energy of the field itself and redefine B

$$\Delta f^{\text{vac,r}} = \Delta f^{\text{vac}} + \frac{B^2}{2}, \qquad B^2 = Z_q B_r^2, \qquad q^2 = Z_q^{-1} q_r^2,$$

note $qB = q_r B_r$

- with the renormalization constant

$$Z_q^{\text{scalar}} = 1 + \frac{1}{2}\beta_1^{\text{scalar}}\, q_r^2 \left(-\frac{2}{\epsilon} + \gamma + \log\left(\frac{m_\star^2}{\mu^2}\right)\right), \qquad \beta_1^{\text{scalar}} = \frac{1}{48\pi^2},$$

$m_\star$ is a scale fixed to the physical mass

- the renormalized free energy is

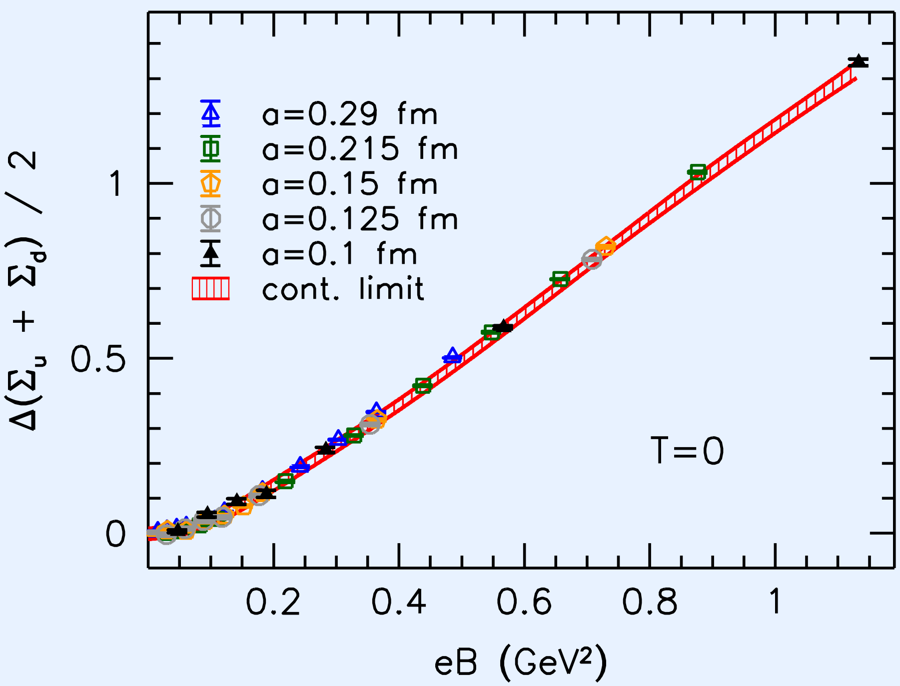
$$\Delta f^{\text{vac,r}}\Big|_{m=m_\star} = \frac{B_r^2}{2} + \underbrace{\frac{(qB)^2}{192\pi^2}\cdot f(m^2/qB)}_{\mathcal{O}((qB)^4)}$$

Free energy - mass dependence

- so the renormalized expression is

$$\Delta f^{\mathrm{vac,r}}\Big|_{m=m_\star} = \frac{B_r^2}{2} + \mathcal{O}(B^4)$$

how can this give a condensate of $\mathcal{O}(B^2)$?



Free energy - mass dependence

- so the renormalized expression is

$$\Delta f^{\text{vac,r}}\Big|_{m=m_\star} = \frac{B_r^2}{2} + \mathcal{O}(B^4)$$

how can this give a condensate of $\mathcal{O}(B^2)$?

- renormalization does not change with m!

$$\Delta f^{\text{vac,r}}(m, m_\star) = \Delta f^{\text{vac}}(m) + \frac{B_r^2}{2} Z_q^{\text{scalar}}(m_\star),$$

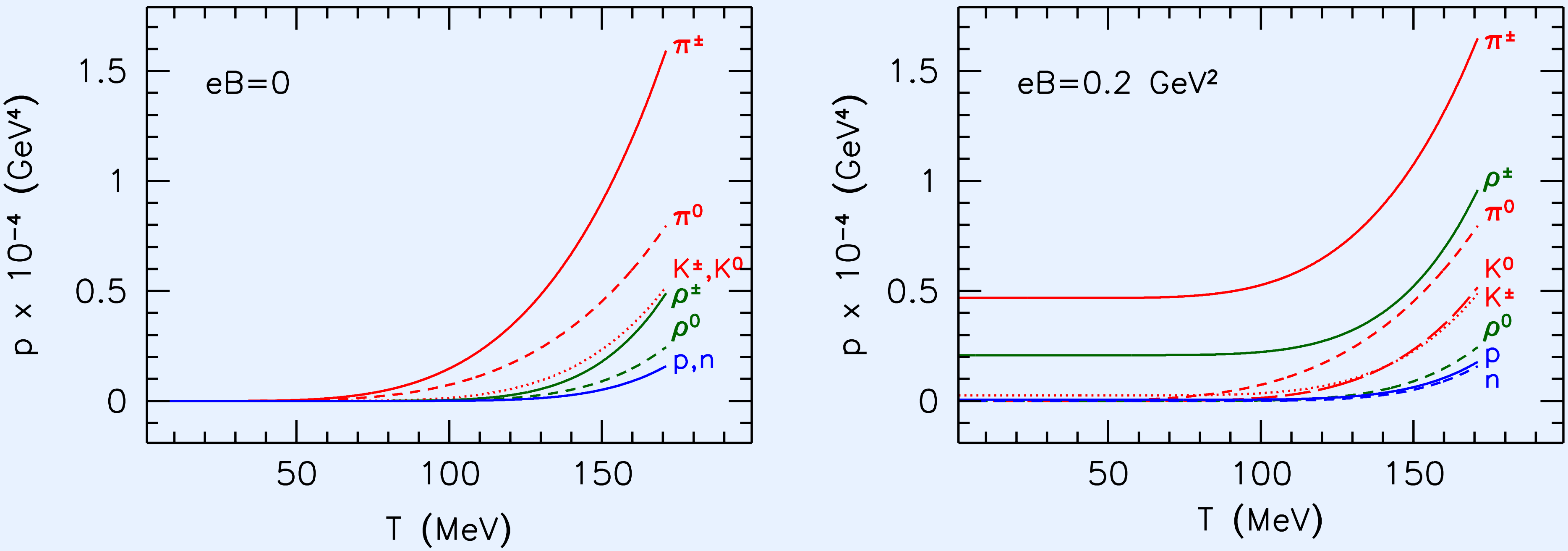
$$\frac{m_q \Delta\bar{\psi}\psi}{m^2} = -\frac{\partial}{\partial m^2}\Delta f^{\text{vac,r}}(m, m_\star) = -\frac{\partial}{\partial m}\Delta f^{\text{vac}}(m)$$

- this also implies that

$$\frac{m_q \Delta\bar{\psi}\psi}{m^2} = \frac{\partial}{\partial m}\frac{B_r^2}{2} Z_q^{\text{scalar}}(m) = \frac{1}{4}\beta_1^{\text{scalar}}\frac{(qB)^2}{m^2} + \mathcal{O}(B^4)$$

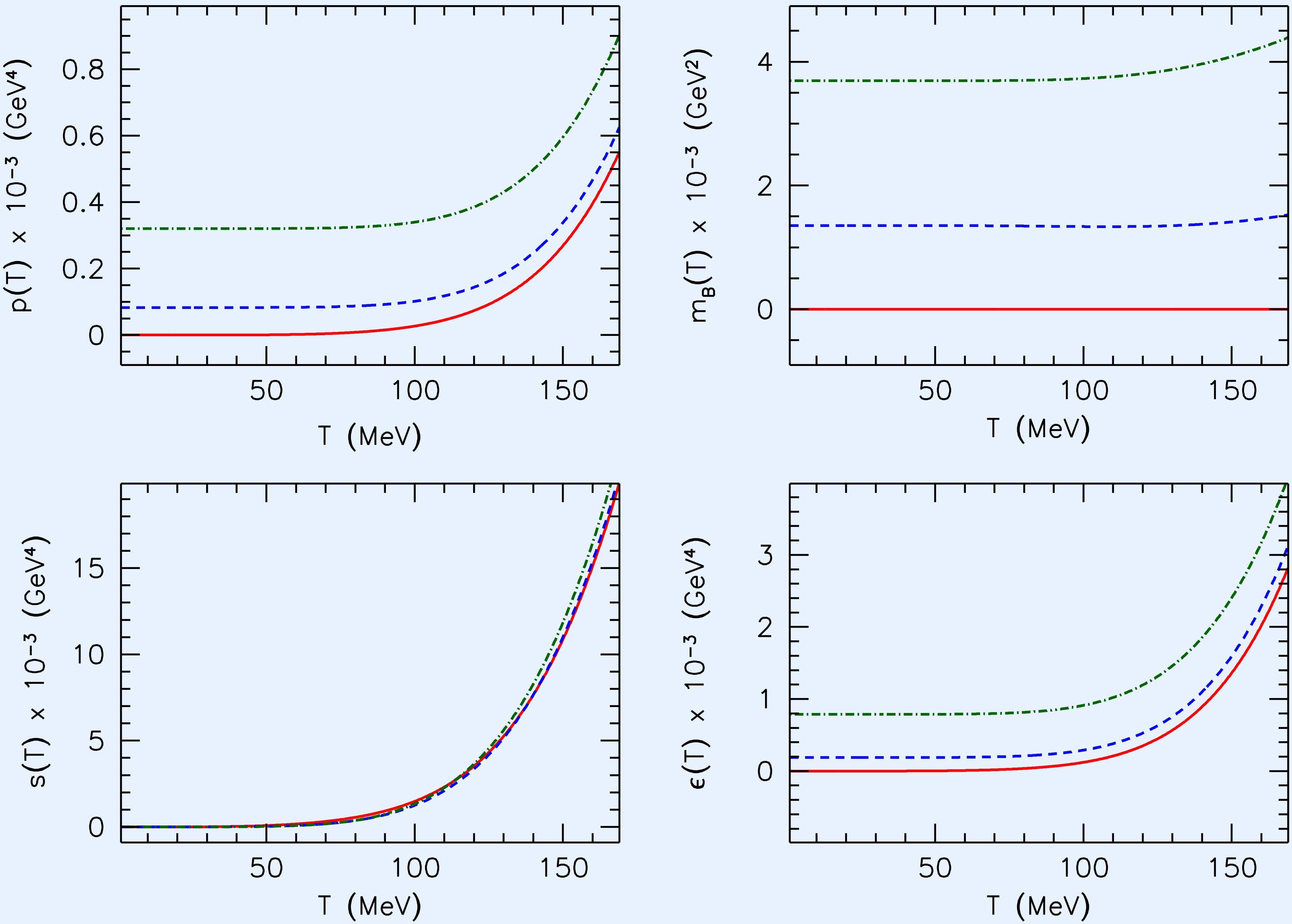
magnetic catalysis $\Leftrightarrow$ scalar QED is not asympt. free

Contributions to pressure

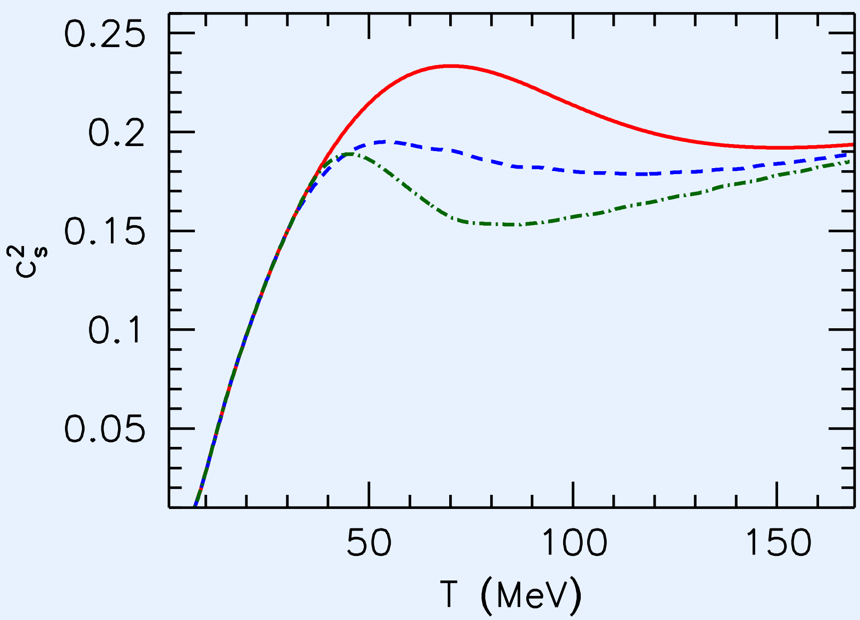


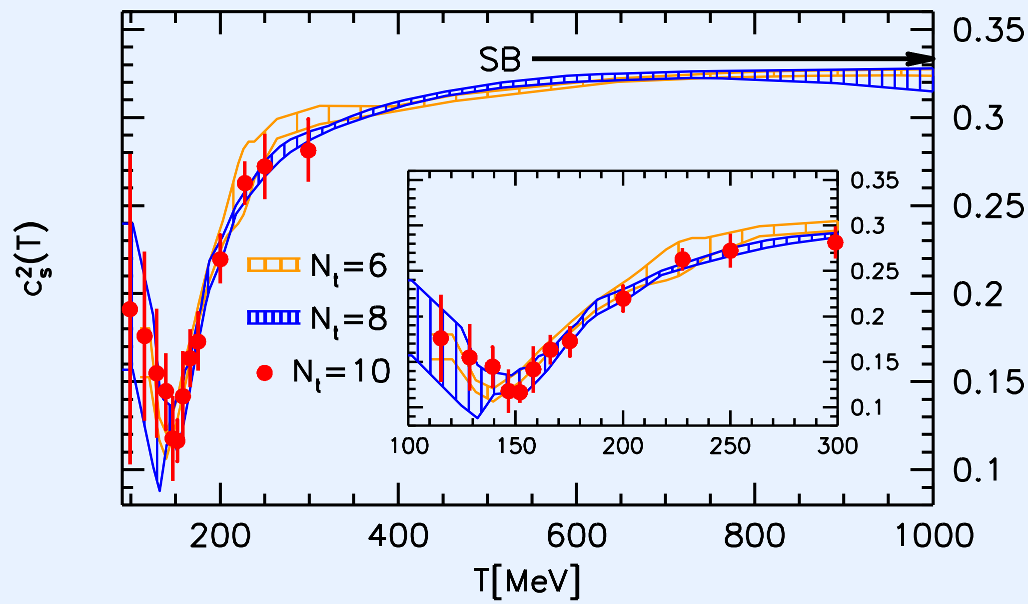
- pion-dominance at $T = 0$ is lost as B grows
- thermal contribution $\exp(-m_{\text{eff}}/T)$
  - $m_{\text{eff}}^2 \sim m^2 + qB(1-2s)$
  - grows for $\rho^\pm$ and decreases for $\pi^\pm$
- now sum up the individual contributions

Equation of state at 0, 0.2, 0.3 GeV^2



Equation of state





- $m_B > 0$: suggests decreasing $T_c(B)$ [Fraga et al '12]
- speed of sound suggests decreasing $T_c(B)$

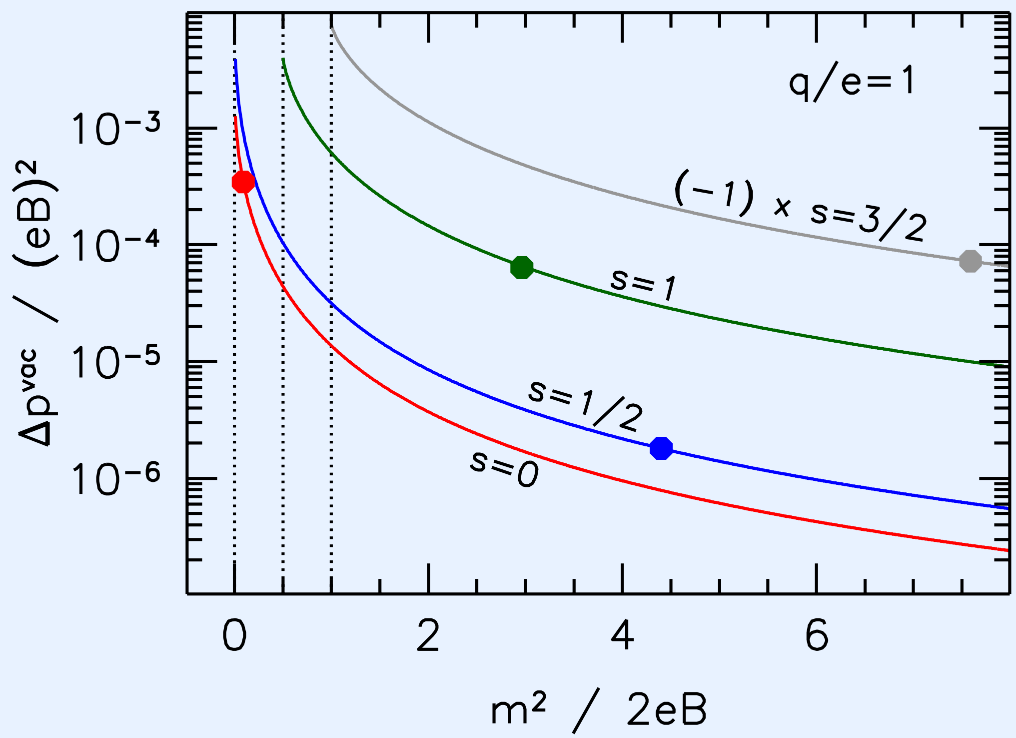
Conclusions

- HRG model for $B > 0$
- pion dominance at $T = 0$ is lost if $eB \gtrsim 0.2$ GeV^2
- $m_B > 0 \rightarrow$ paramagnetic QCD vacuum at $T < T_c$
- $c_s^2(B,T) \rightarrow$ decreasing $T_c(B)$, cf. lattice
  [Bali, Bruckmann, GE et al '11]
- relation between $\mathcal{O}(B^2)$ magnetic catalysis and scalar QED β-function!

$$m_q \Delta \bar{\psi}\psi = \frac{1}{4}\beta_1^{\text{scalar}}(qB)^2 + \mathcal{O}(B^4), \qquad \beta_1^{\text{scalar}} > 0.$$

- details in [arXiv:1301.1307]

Backup - Spin channels



- model only works if $m^2/2qB + 1/2 - s > 0$
  - $\rho^{\pm}$: $eB < m_\rho^2$, Δ^{++}: $eB < m_\Delta^2/4$
- $s = 3/2$ channel gives negative pressure at $T = 0$
  - inconsistencies of $s = 3/2$ theory [Zwanziger et al '69]

  $\rightarrow$ exclude Δ