Black hole thermodynamics under the microscope

Kevin Falls, University of Sussex

DELTA 2013

January 11, 2013

Outline

Main Ideas [1]:

- Understanding black hole (BH) thermodynamics as arising from an averaging of degrees of freedom via the renormalisation group.
- Go beyond the semi-classical approximation using a systematic coarse graining idea, interpolating from largest to smallest BH masses.

Outline:

- Black hole thermodynamics.
- Coarse grained model.
- Implications for asymptotic safety.
- Conclusion.

[1] based on KF and D Litim arXiv:1212.1821

Black holes in general relativity

- Solutions to Einsteins equations
- Stationary solutions are parametrized by just M, J and q.
- End point of gravitational collapse
- Uniqueness $M, J, q \to$ "No hair"

$$A = A(M, J, q) \tag{1}$$

- First law [2]

$$\frac{\kappa}{8\pi G_N}\delta A = \delta M - \Omega\delta J - \Phi\delta q \tag{2}$$

[2] Bardeen, Carter and Hawking '73

Black holes and thermodynamics

- Four laws of black hole mechanics
- Analogous to the laws of thermodynamics

Thermodynamics	Black holes
$T\delta S = \delta Q$	$\frac{\kappa}{8\pi G}\delta A = \delta Q$
T	κ
$\delta S \geq 0$	$\delta A \geq 0$

- Generalised second law [3] $\delta S + \delta S_{BH} \geq 0$

[3] Bekenstein '73

Beyond the analogy

- Hawking radiation ('75) QFT on curved space-time

$$T = \hbar \frac{\kappa}{2\pi}, \qquad S_{BH} = \frac{A}{\hbar 4G}. \tag{3}$$

- Jacobson(95')

$$\frac{\delta Q}{T} = \delta S \iff G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi G \, T_{\mu\nu} \tag{4}$$

for all local Rindler causal horizons.

- Suggests a deep connection between classical gravity, thermodynamics and quantum mechanics due to the presence of causal horizons.
- Is gravity/space-time fundamental or emergent?

Problems

- Black hole thermodynamics appears when quantum fields propagate on a background black hole spacetime.
- A thermal bath of particles, as seen by observers far from the horizon, seems to contain no information of the matter that initially collapsed to form the black hole
- What happens when the black hole evaporates away completely? Information loss?
- Black hole themodynamics seems to suggest that there exists an underlying microstructure of space-time
  $\Longrightarrow$ What are the fundamental degrees of freedom?
- Strings, branes ? Spin foam?...

Quantum gravity

- Saddle point approximation to the Euclidean path integral

$$e^{-\Gamma} = \int \mathcal{D}g_{\mu\nu}\, e^{-I_{EH}[g_{\mu\nu}]}, \qquad \frac{\delta I_{EH}}{\delta g_{\mu\nu}}[\bar{g}_{\mu\nu}] = 0 \tag{5}$$

$$\approx e^{-I_{EH}[\bar{g}_{\mu\nu}]} \tag{6}$$

- Hawking and Gibbons found that in this approximation

$$\Gamma = \frac{F}{T} \equiv \frac{M}{T} - S \tag{7}$$

- Here we utilise the coarse grained effective action Γ_k which provides a set of scale dependent models where fluctuations $p^2 > k^2$ have been included.

Wilsonian black hole thermodynamics

- Here we utilise the coarse grained effective action Γ_k which provides a set of scale dependent models where fluctuations $p^2 > k^2$ have been included.
- Jacobson and Satz arXiv:1212.6824 [hep-th]. Can the long wavelength modes $p^2 < k^2$ be interpreted as accounting for the entanglement entropy?
- Becker and Reuter arXiv:1205.3583 [hep-th]. Running boundary terms.

Wilsonian black hole thermodynamics

- Implement quantum corrections to the physics of black hole thermodynamics using the renormalisation group.
- We consider $4d$ gravity coupled to $U(1)$ gauge fields.
- The action

$$\Gamma_k[g_{\mu\nu}, A_\mu] = \int d^4x \sqrt{-\det g_{\mu\nu}} \left[\frac{1}{8\pi G_k} \mathcal{R} + \frac{1}{4\alpha_k} F^{\mu\nu} F_{\mu\nu} \right] + S_m \,.$$

- Scale dependent couplings G_k and $\alpha_k \equiv \frac{e_k}{4\pi}$ which run under the renormalisation group flow of the theory

Scale dependent spacetime

- At fixed k, and by varying Γ_k with respect to the $g_{\mu\nu}$ and A_μ we recover Einstein-Maxwell theory.
- Solutions include a family of Kerr–Newman-type black holes additionally parameterised by the RG scale k via the running couplings.

$$A = A(M, J, q; k) \tag{8}$$

- Semi-classical limit ($k \to 0$) we have $G \approx 6.674 \times 10^{-11}\ \mathrm{N}\,(\mathrm{m}/\mathrm{kg})^2$ and $\alpha \approx \frac{1}{137}$.

Flowing entropy

- Even off-shell the entropy we will consider the entropy to be given by

$$S_k = \frac{A}{4G_k}, \qquad \delta S_k = \frac{\delta A}{4G_k} \tag{9}$$

Which can be obtained directly from the running Einstein-Hilbert action using Wald's definition of the entropy or by relating the Euclidean action to the free energy.

- The RG flow for the off-shell entropy taken at constant area is then given by

$$\frac{\partial}{\partial \ln k} S_k = -S_k \frac{\partial \ln G_k}{\partial \ln k} \tag{10}$$

Scale identification

- We will assume that there exists a scale $k = k_{\rm opt}$, associated to the macroscopic spacetime geometry, such that $\Gamma_{k_{\rm opt}}$ gives a good saddle point approximation to the full functional integral.

$$k_{\rm opt} = k_{\rm opt}(M, J, q) \tag{11}$$

- Under this identification we have a new set of RG improved Kerr-Newman-type black holes defined by a new state function

$$A(M, J, q) = A(M, J, q; k_{\rm opt}) \tag{12}$$

Thermal equilibrium

- We now imagine that a small amount of matter to falls into a black hole of mass M, angular momentum J and charge q.
- While assuming the relation

$$\frac{\delta Q}{T} = \delta S_{k_{\rm opt}} \tag{13}$$

holds with $\delta S_{k_{\rm opt}} = \frac{\delta A}{4 G_{k_{\rm opt}}}$.

- Heat crossing the horizon of a black hole

$$\delta Q = \delta M - \Omega \delta J - \Phi e_{k_{\rm opt}} \delta q \,, \tag{14}$$

Thermal equilibrium

- After the matter has fallen in the parameters M, J and q are shifted which implies that the scale k_{opt} is also changed.
- Under these assumptions one finds

$$\delta k_{\text{opt}} \propto \delta A, \qquad \text{thus} \quad k_{\text{opt}}(M,J,q) \equiv k_{\text{opt}}(A(M,J,q))$$

- Dimensional analysis $\Longrightarrow k_{\text{opt}}^2 = \frac{4\pi}{A}\xi^2$
- ξ depends on the RG-scheme used $\xi = \xi(R_k)$.
- Will set $\xi = 1$ for simplicity.

Mass function

- Modified relation between the area A and the parameters M, J and q.

$$M^2 \equiv \frac{4\pi}{A}\left[\left(\frac{A + 4\pi G_{\mathrm{opt}}(A) e_{\mathrm{opt}}^2(A) q^2}{8\pi G_{\mathrm{opt}}(A)}\right)^2 + J^2\right]$$

- Temperature

$$T = 4 G_{\mathrm{opt}}(A) \frac{\partial M}{\partial A}$$

- Semi-classical limit $A \to \infty$

Asymptotic safety

- Our reasoning so far has been independent of the actual form of the RG running couplings and therefore the UV completion of gravity
- Asymptotic safety is a possible UV completion of gravity
- Dimensionless coupling constants reach a non-Gaussian fixed point at high energies.
  $\longrightarrow$ Ensures that the continuum limit may be taken.
- Finite number of relevant directions flowing away from the fixed point to the IR.
  $\longrightarrow$ Theory is predictive.

Asymptotic safety and black hole thermodynamics

- Go beyond the semiclassical approximation assuming

$$\frac{1}{G_k} = \frac{1}{G_N} + \frac{k^2}{g^*} \tag{15}$$

- Cross over between classical $G \approx G_N$ and fixed point $G \propto k^{-2}$ scaling governed by the characteristic energy scale

$$E_c^2 = g^* M_P^2 = \frac{g^*}{G_N} \tag{16}$$

Asymptotic safety and black hole thermodynamics

- Inserting the running Newton's constant into the mass function with $q = 0$ and resolve for A

$$A_{\pm} = 4\pi G_N \left(2G_N M^2 - G_N M_c^2 \pm 2\sqrt{G_N^2 M^4 - J^2 - G_N^2 M_c^2 M^2} \right) .$$

- Characteristic mass scale

$$M_c^2 = \frac{1}{g^*} M_P^2 .$$

  - Smallest black hole mass.

- Semi-classical limit $1/g^* \to 0$ leads to $M_c \to 0$ and $E_c \to \infty$.

Temperature

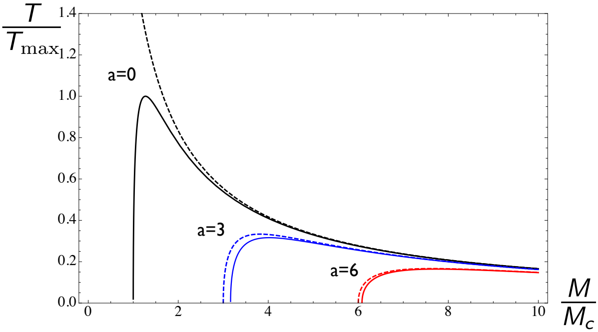

Figure: Horizon temperature as a function of the black hole mass, comparing classical gravity with asymptotically safe gravity with $g_* = 1$ for several angular momenta $a = J/M$, with a given in units of $1/M_c$.

Specific heat

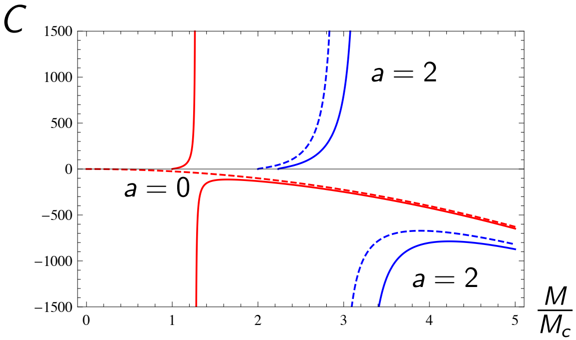

Figure: Specific heat as a function of the black hole mass, comparing classical gravity with asymptotically safe gravity ($g_* = 1$, solid lines) for several angular momenta a, given in units of $1/M_c$.

Conformal scaling

- Suggested by O. Aharony and T. Banks and by A. Shomer that gravity cannot exist as a local QFT
- Consider the case for general d with $J = q = 0$
- At a UV fixed point we expect that a theory behaves as a CFT where by the entropy and energy should scale as $S \sim (RT)^{d-1}$ and $E \sim R^{d-1}T^d$.
- For black holes the radius R depends on the energy $E = M$.
- Therefore we consider the relation

$$\frac{S}{R^{d-1}} \sim \left(\frac{E}{R^{d-1}}\right)^{\nu} . \tag{17}$$

With $\nu_{\rm CFT} = \frac{d-1}{d}$. However for classical black holes $\nu_{\rm BH} = \frac{1}{2}$

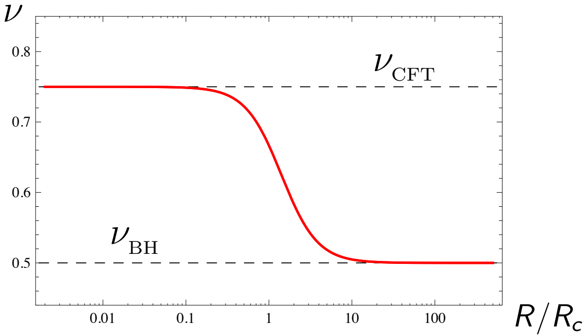

Figure: Scaling index for an asymptotically safe Schwarzschild black hole in four dimensions interpolating between the classical value $\nu_{\rm BH}$ for large horizon radii and the conformal limit $\nu_{\rm CFT}$ for small radii.

RG improved Schwarzschild metric and statistical entropy

- Reuter and Bonnano 00': Put the RG improved Schwarzschild metric into the classical Euclidean action to obtain the free energy

$$F = \frac{r_+}{2G_N} - \frac{A}{4G_N} T \tag{18}$$

- If instead the action with $G_N \to G_{\rm opt}(A)$ is used and identify k^2 with the inverse area one obtains

$$F = M - ST \tag{19}$$

where $S = \frac{A}{4G_{\rm opt}(A)}$

Conclusions

Black hole thermodynamics + RG

- is compatible with RG running of couplings assuming a scale dependent entropy
- requires that RG cutoff is set by BH area
- has physical interpretation that sub-horizon modes are integrated out - consistent picture.
- Implies thermodynamics is consistent even away from the semi-classical limit.

Conclusions

Asymptotic Safety + Black hole thermodynamics

- Existence of smallest black hole mass M_c.
- Maximum temperature T_{max}.
- Generic existence of inner horizon.
- Potential existence of BH remnant.
- Conformal scaling.