QCD phase diagram with functional methods

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HIC for FAIR
Helmholtz International Center

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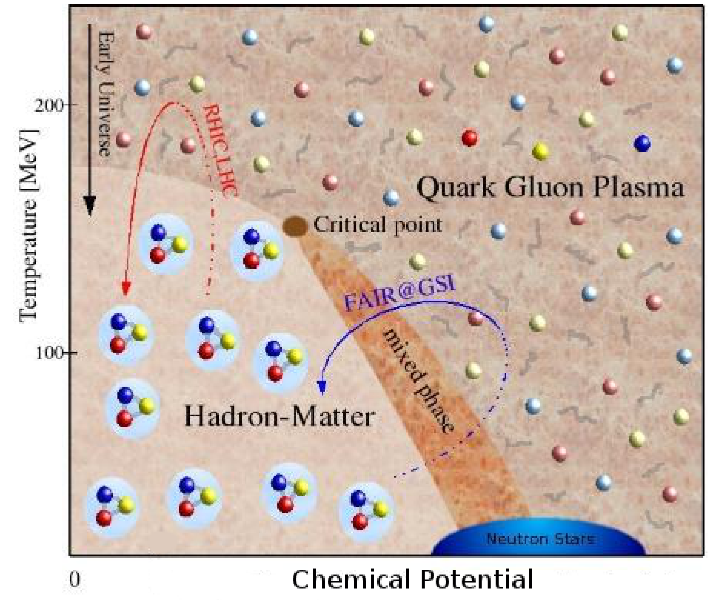
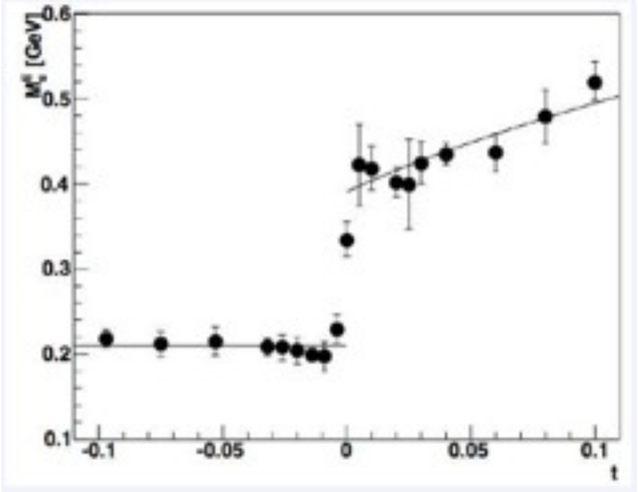
HELMHOLTZ GEMEINSCHAFT

Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001
CF, Luecker, Phys. Lett. B 718 (2013) 1036-1043

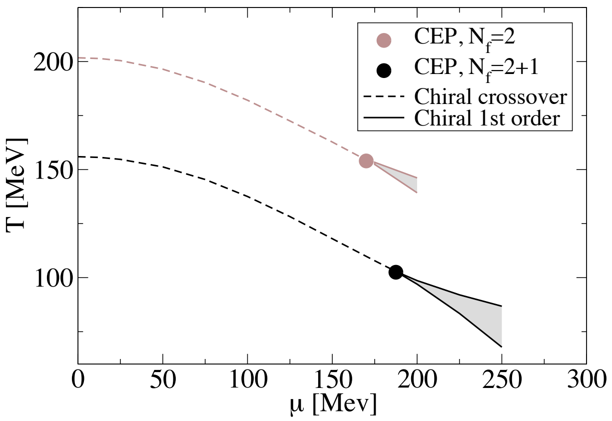


1. Introduction

2. Gluons at zero and finite temperature

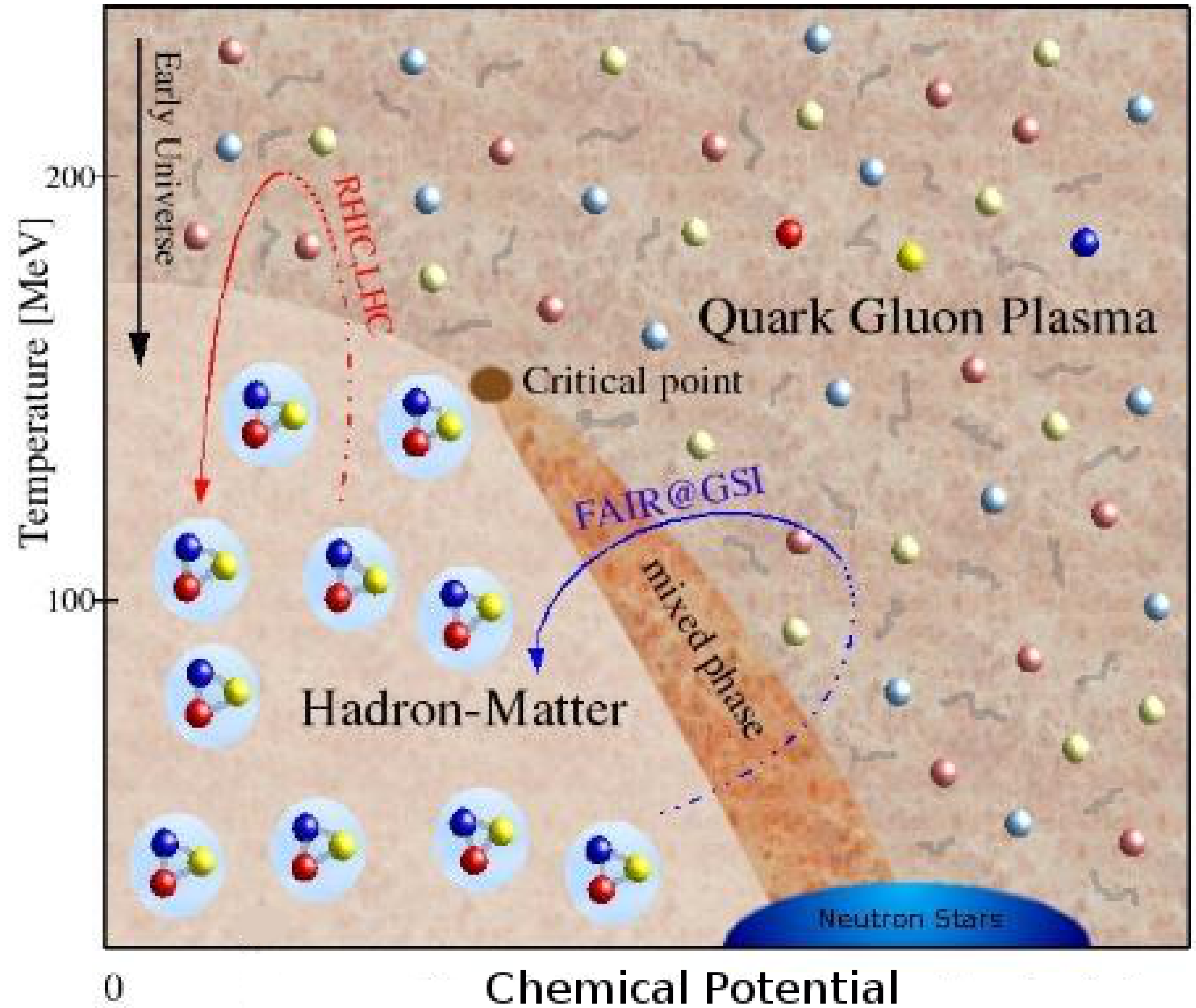


3. Quarks and the QCD phase diagram



Saturday, January 12, 2013

QCD phase diagram

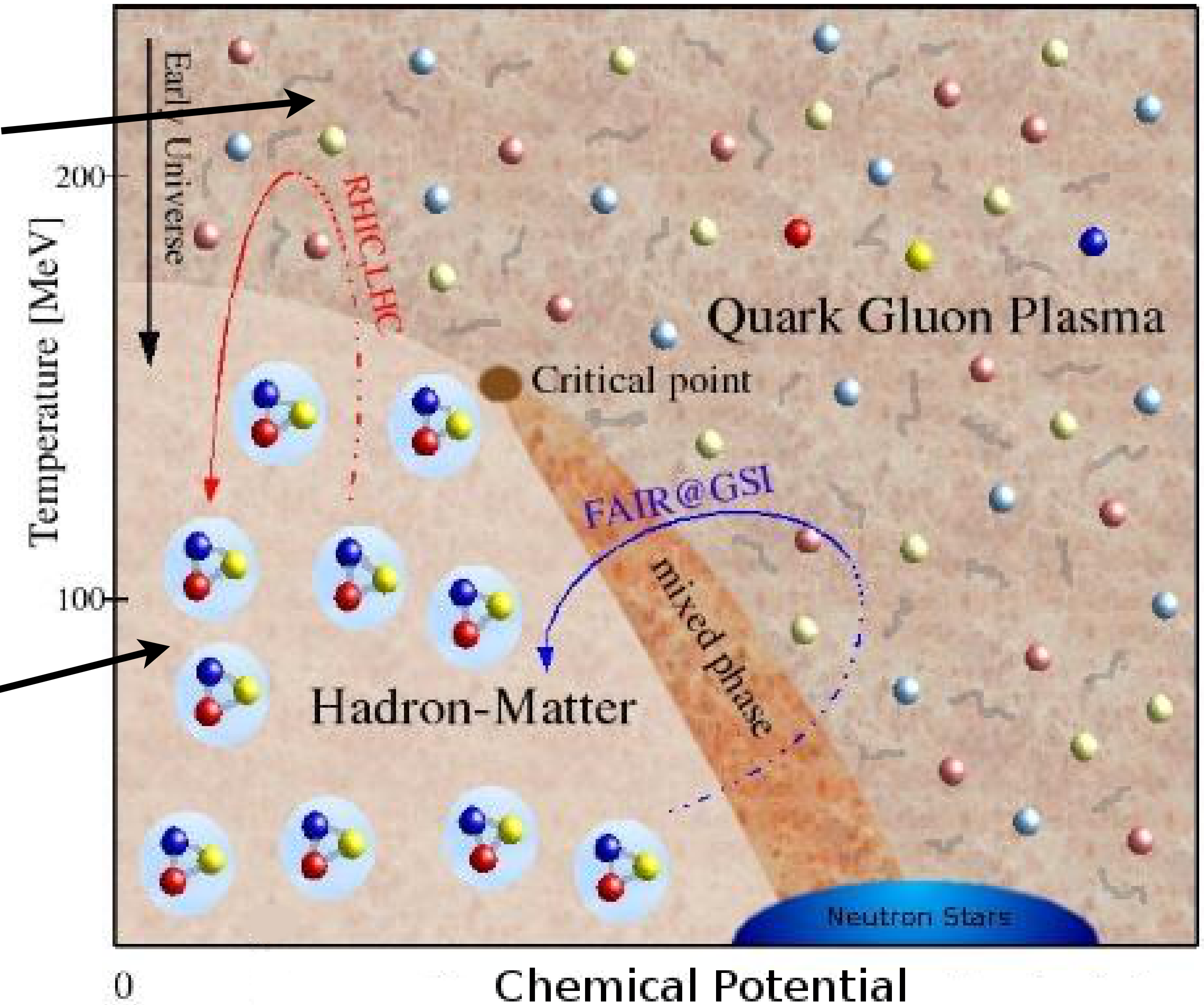


Interesting open questions:

- Details of phase transitions
- Existence and location of critical point
- Properties of quarks and gluons in different phases
- Consequences for astrophysics

Quarks de-confined and (almost) massless

Quarks confined and massive



Interesting open questions:

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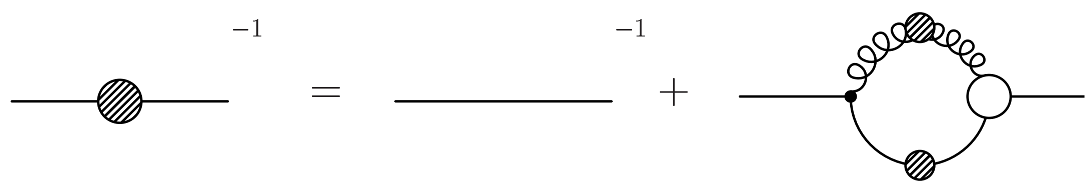
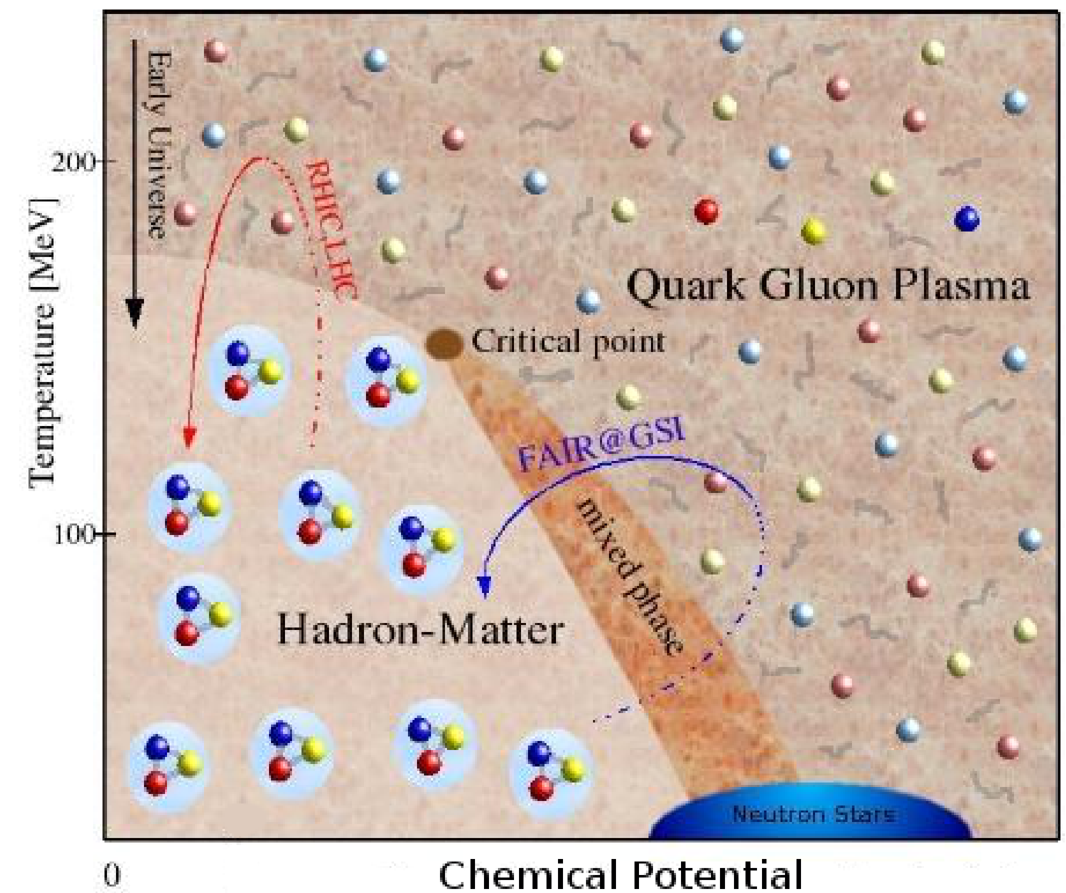
QCD phase transitions I



$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$



Phase transitions:

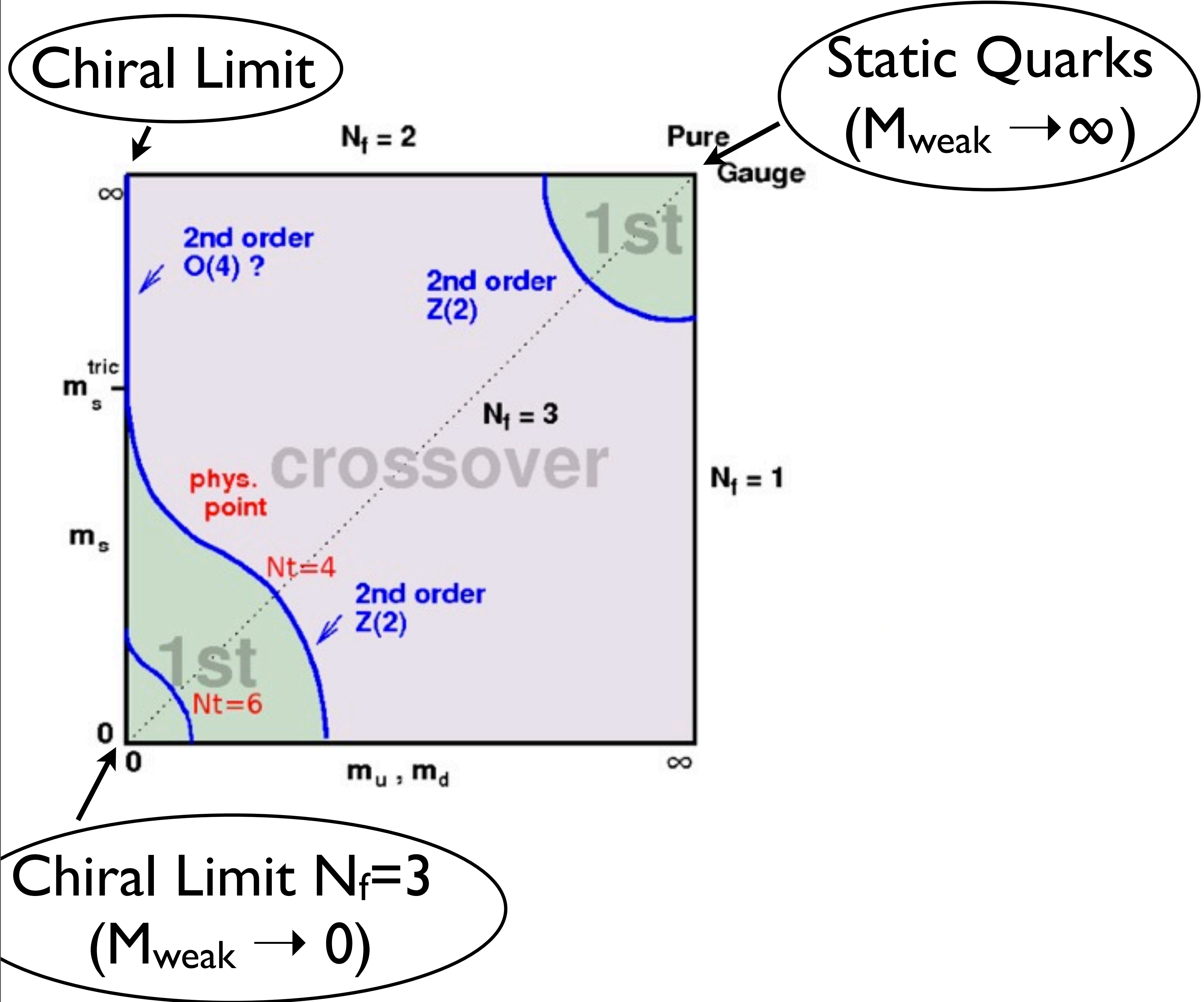
- Chiral limit ($M_{weak} \to 0$): order parameter chiral condensate

$$\langle \bar{\psi}\psi \rangle = Z_2 N_c Tr_D \int \frac{d^4p}{(2\pi)^4} S(p)$$

- Static quarks ($M_{weak} \to \infty$): order parameter Polyakov-loop

$$\Phi \sim e^{-F_q/T}$$

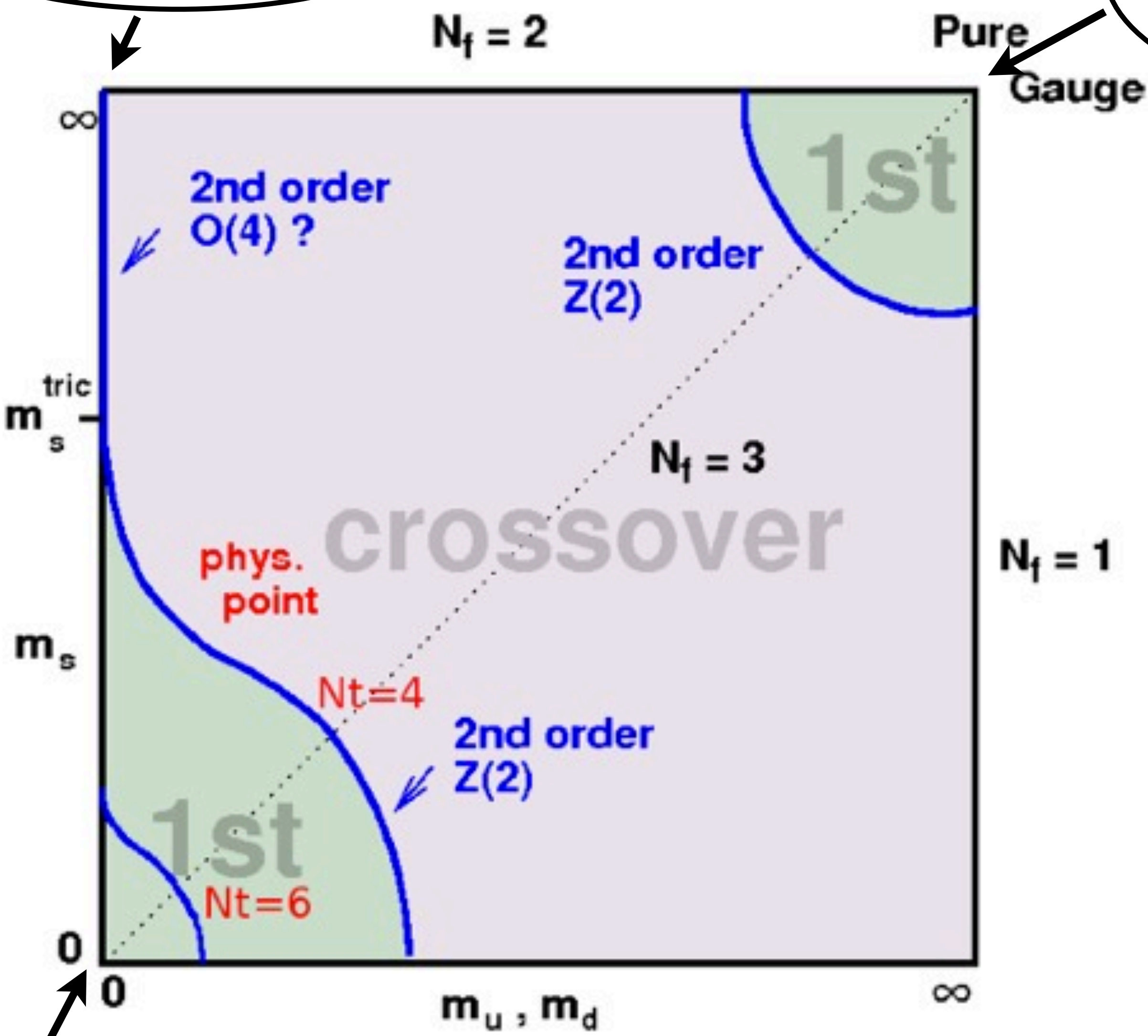
QCD phase transitions II

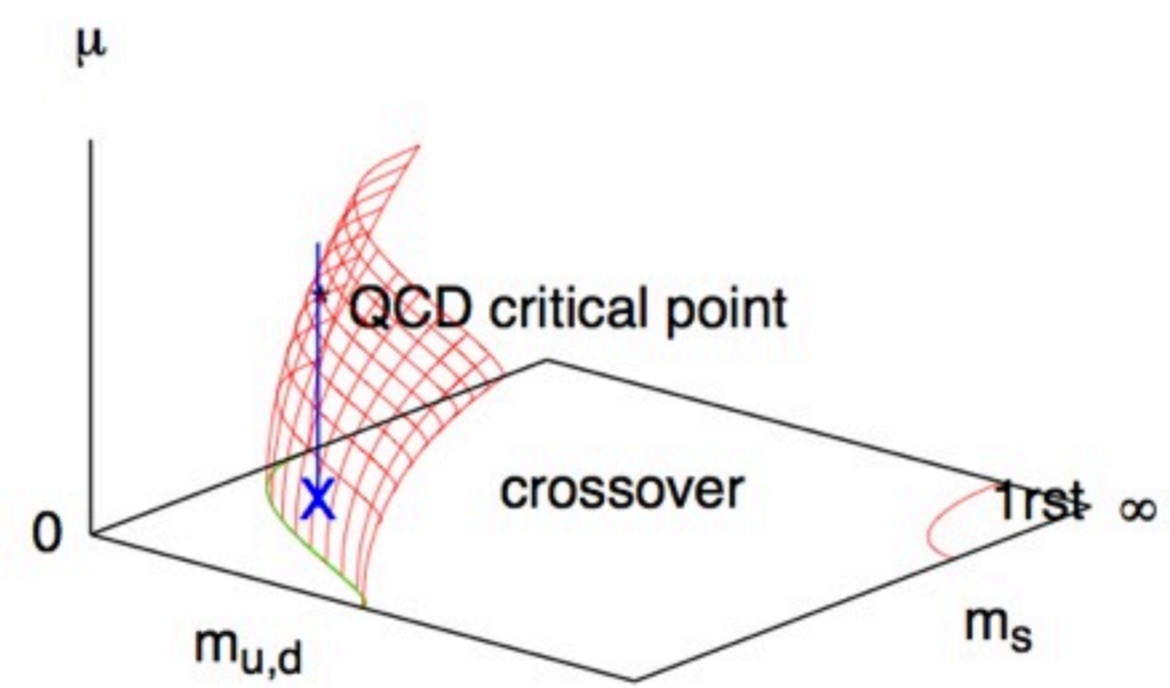


QCD phase transitions II

Chiral Limit

Static Quarks ($M_{weak} \to \infty$)



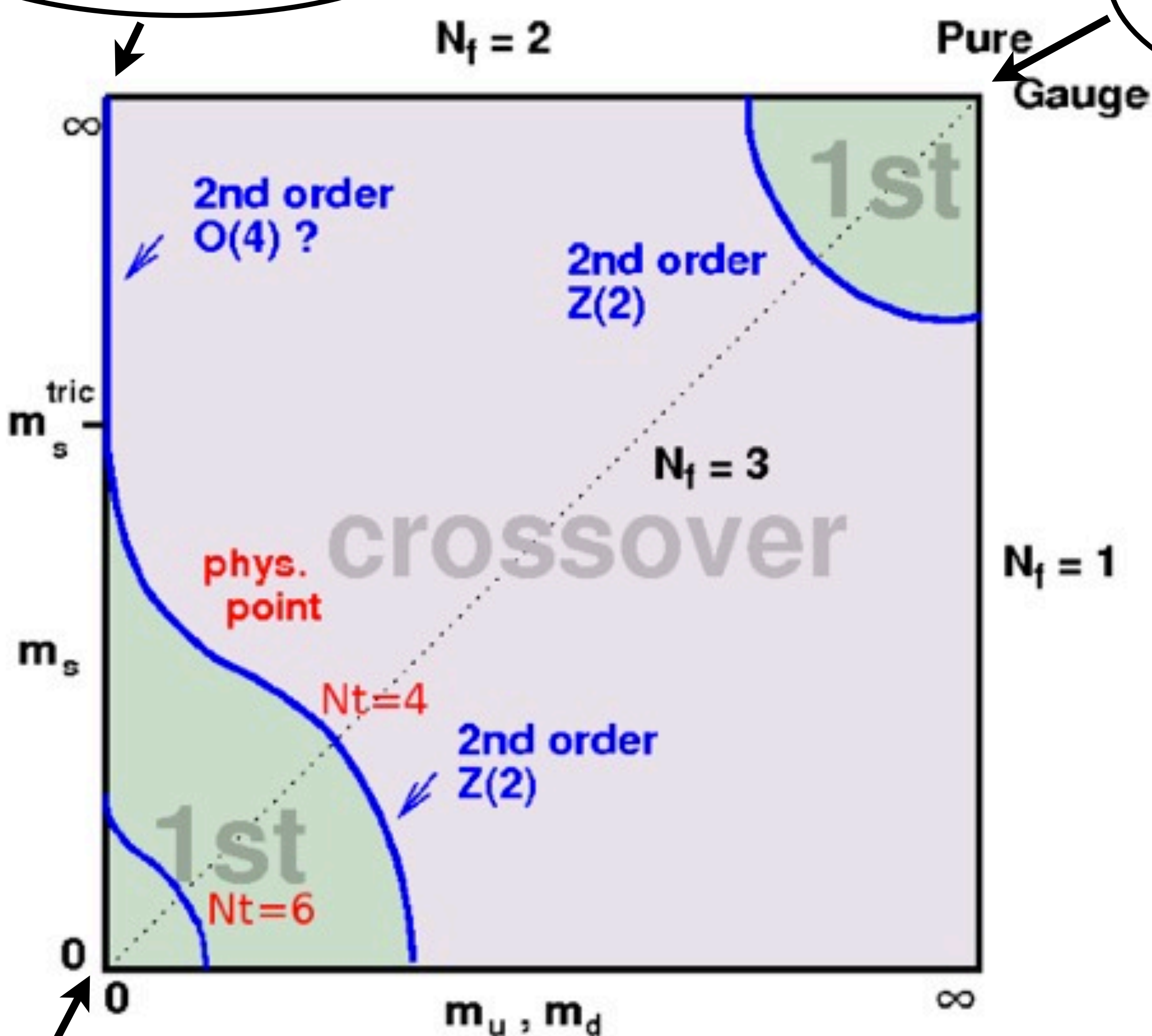


Chiral Limit N_f=3 ($M_{weak} \to 0$)

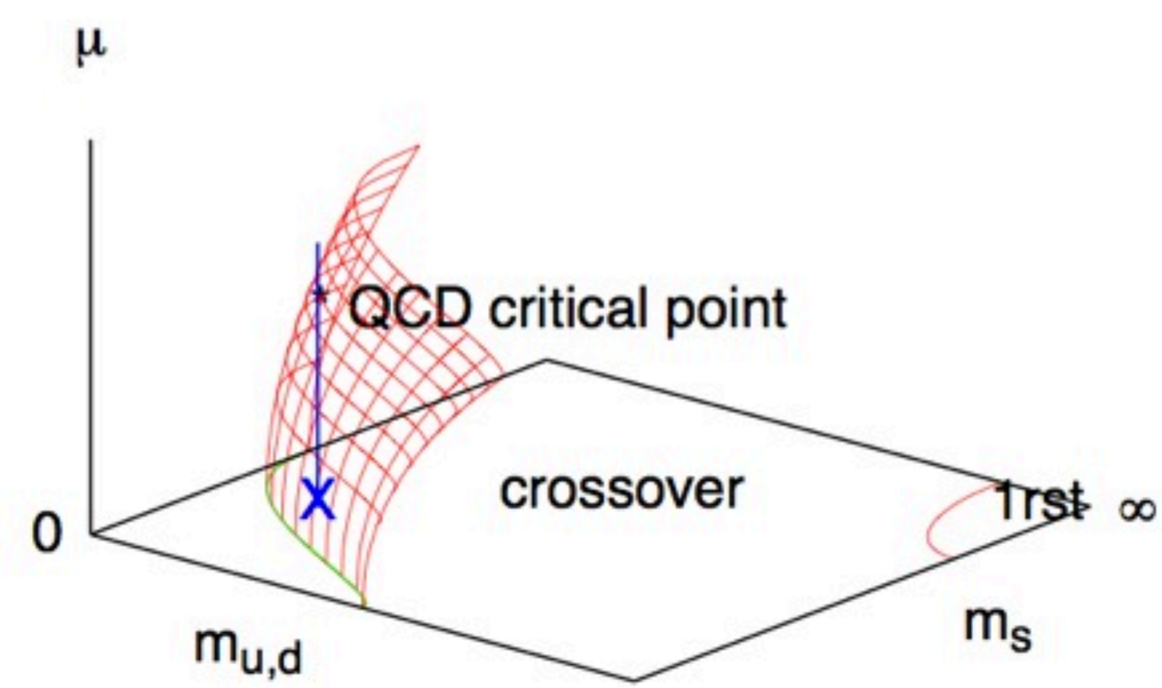
QCD phase transitions II

Chiral Limit

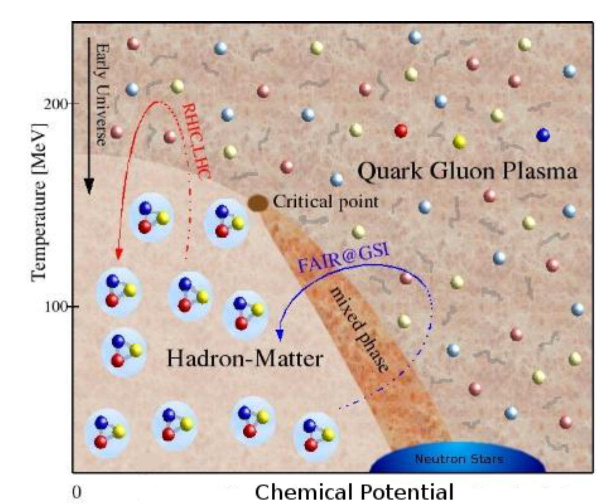
Static Quarks ($M_{weak} \to \infty$)



Chiral Limit N_f=3 ($M_{weak} \to 0$)



Is this happening ??



QCD in covariant gauge

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \exp\left\{ - \int_0^{1/T} dt \int d^3x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\,(F^a_{\mu\nu})^2 + \text{gauge fixing} \right) \right\}$$

Landau gauge propagators in momentum space, $p = (\vec{p}, \omega_p)$:

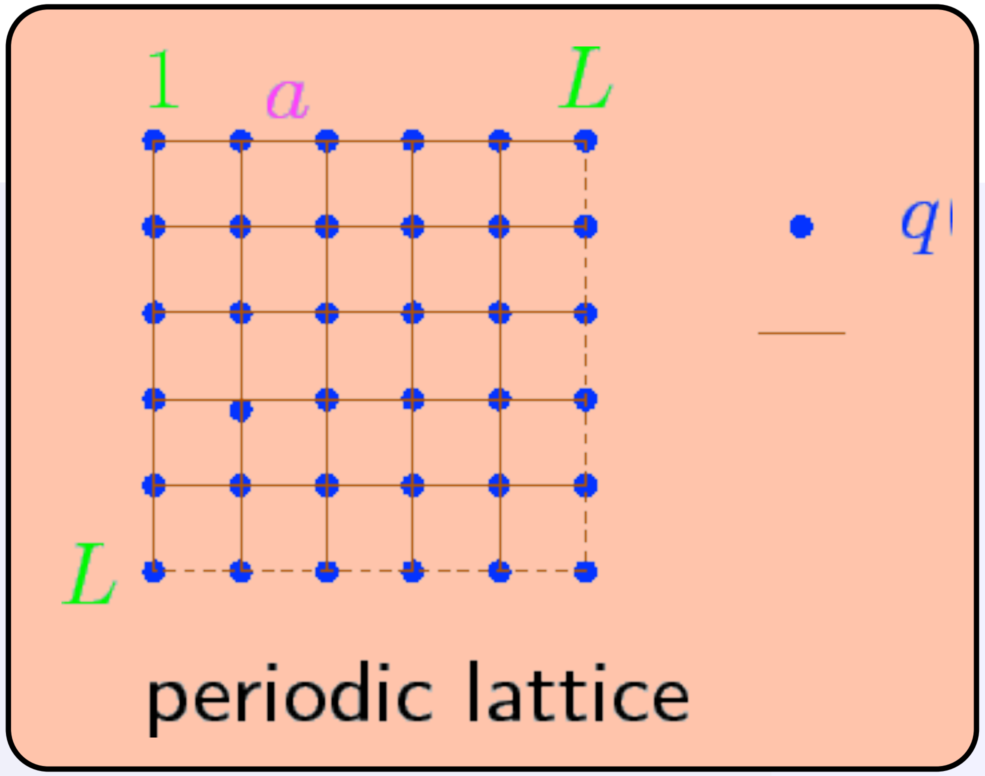
$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{Z_T(p)}{p^2} P^T_{\mu\nu}(p) + \frac{Z_L(p)}{p^2} P^L_{\mu\nu}(p)$$

$$S^{\text{Quark}}(p) = Z_f(p) \left[-i\,\vec{\gamma}\vec{p} - i\,\gamma_4 \omega_n\, Z_c(p) + M(p) \right]^{-1}$$

The Goal:

Gauge invariant information from gauge fixed functional approach

Lattice QCD vs. DSE/FRG: Complementary!



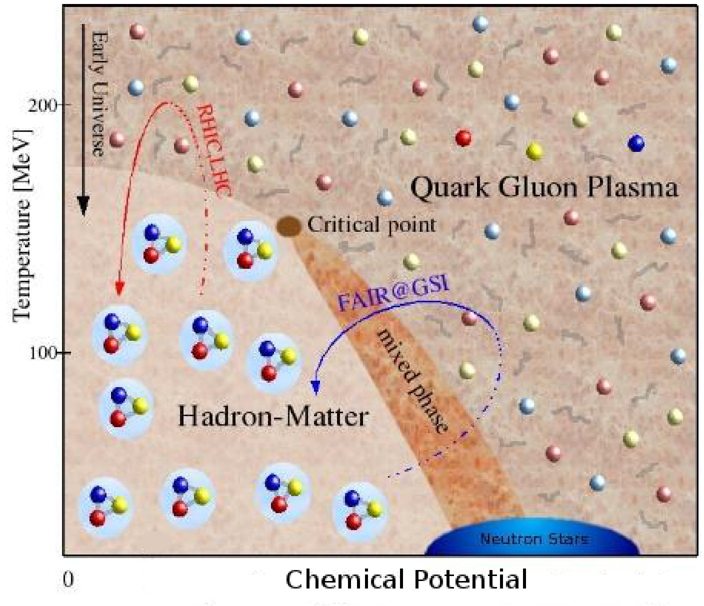
- Lattice simulations
  - Ab initio
  - Gauge invariant

Fodor, Karsch, Phillipsen...

- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Analytic solutions at small momenta
    CF, J. Pawlowski, PRD 80 (2009) 025023
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible: e.g. $(g\text{-}2)_\mu$
    T. Goecke, C.F., R. Williams, PLB 704 (2011); PRD 83 (2011)
  - Chemical potential: no sign problem

- Models: PNJL, PQM
  - Technically easier
  - Exploratory

Weise, Schaefer,...



1.Introduction

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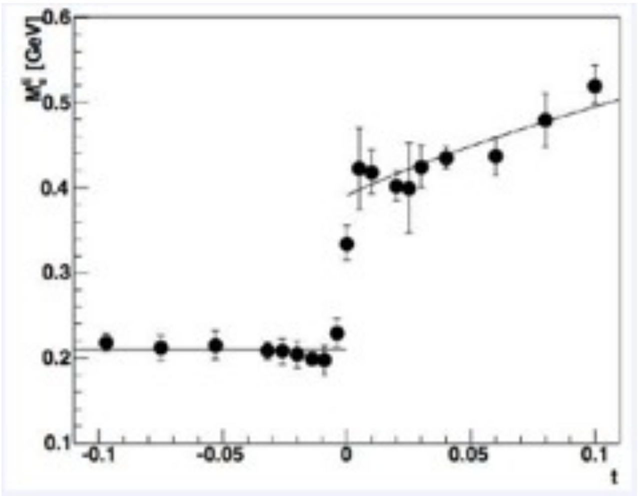

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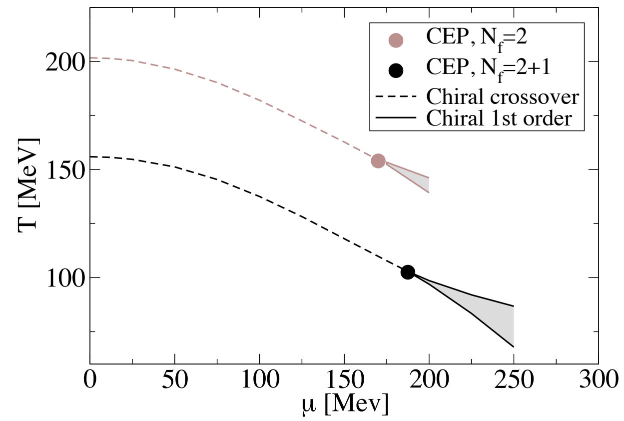
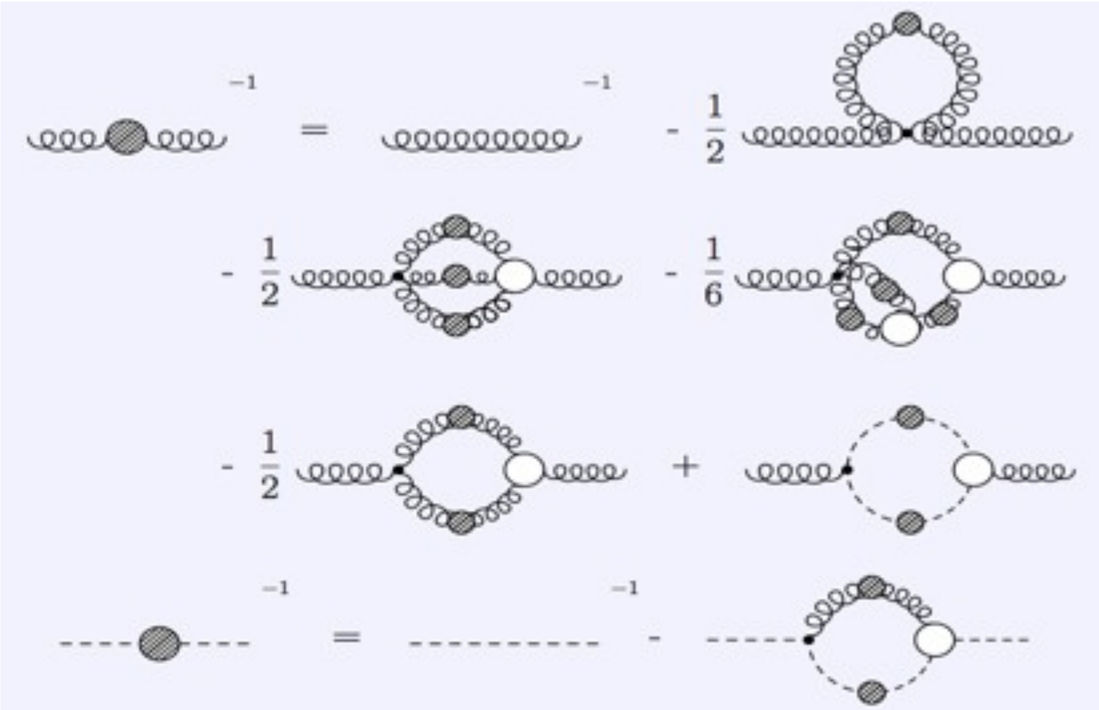
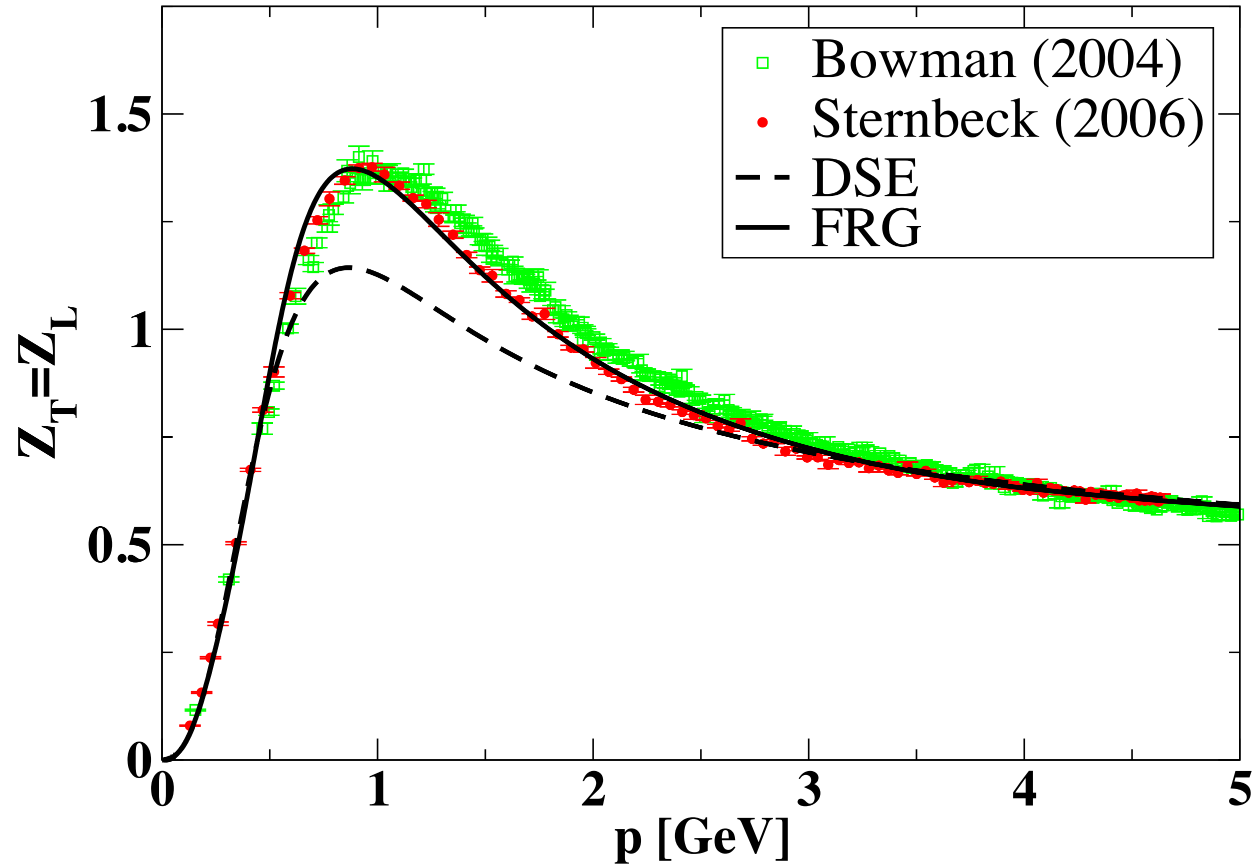






CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.

- Small momenta: $Z(p^2) \sim p^2$, i.e. gluon mass generation

  Cornwall PRD **26** (1982) 1453; Cucchieri, Mendes, PoS **LAT2007** (2007) 297.
  Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008); Boucaud, et al. JHEP **0806** (2008) 099

- Deep infrared: subtle questions related to gauge fixing...

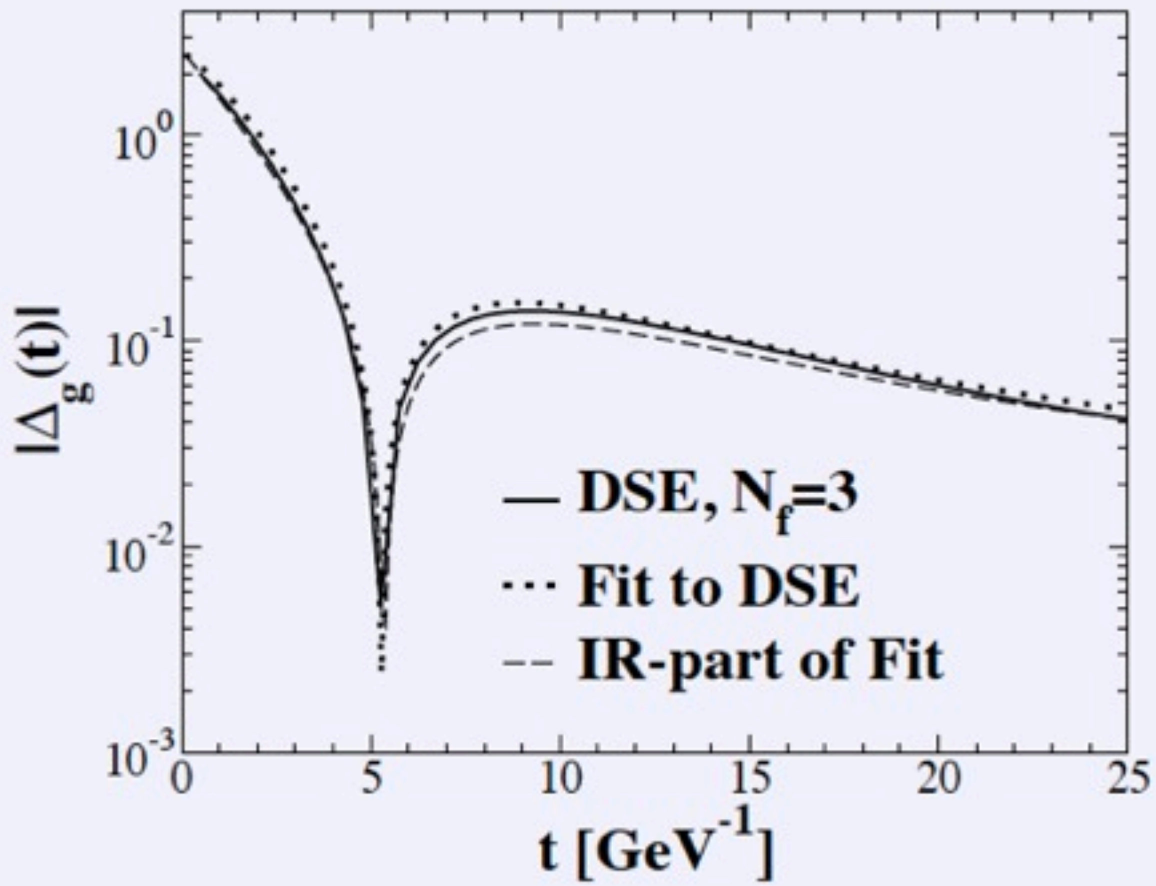
  Maas, PLB 689 (2010) 107; Sternbeck, Smekal, EPJC 68 (2010) 487

- Timelike momenta: Positivity violations $\rightarrow$ gluon screening

  Alkofer, Detmold, C.F. and Maris, PRD 70 (2004) 014014

Gluon: positivity violation

$$\Delta_g(t) := \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4+\vec{p}\vec{x})} \frac{Z(p^2)}{p^2}$$



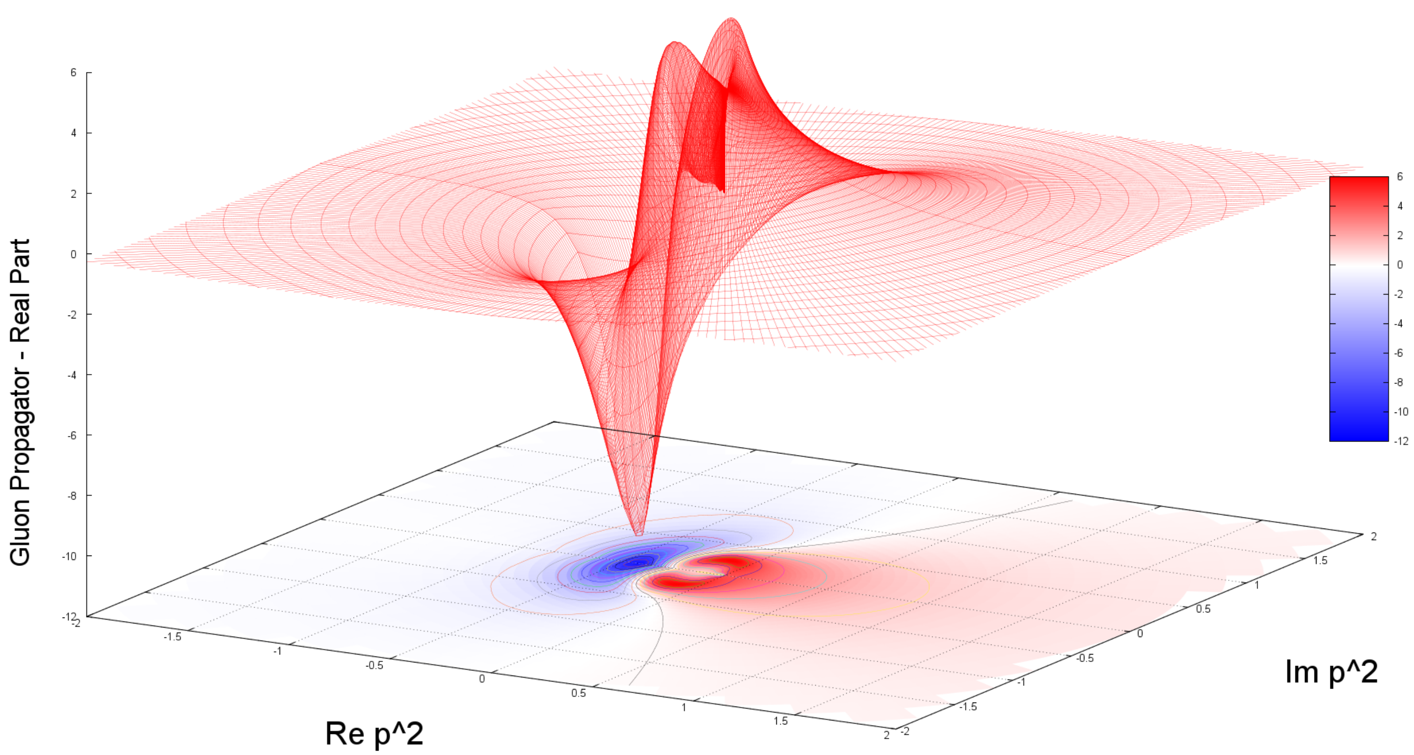
- Violation of positivity ⇒ **no physical asymptotic gluons**
- Cut on the timelike momentum axis ?

R. Alkofer, W. Detmold, C. F., P. Maris, Phys. Rev. D **70** (2004) 014014

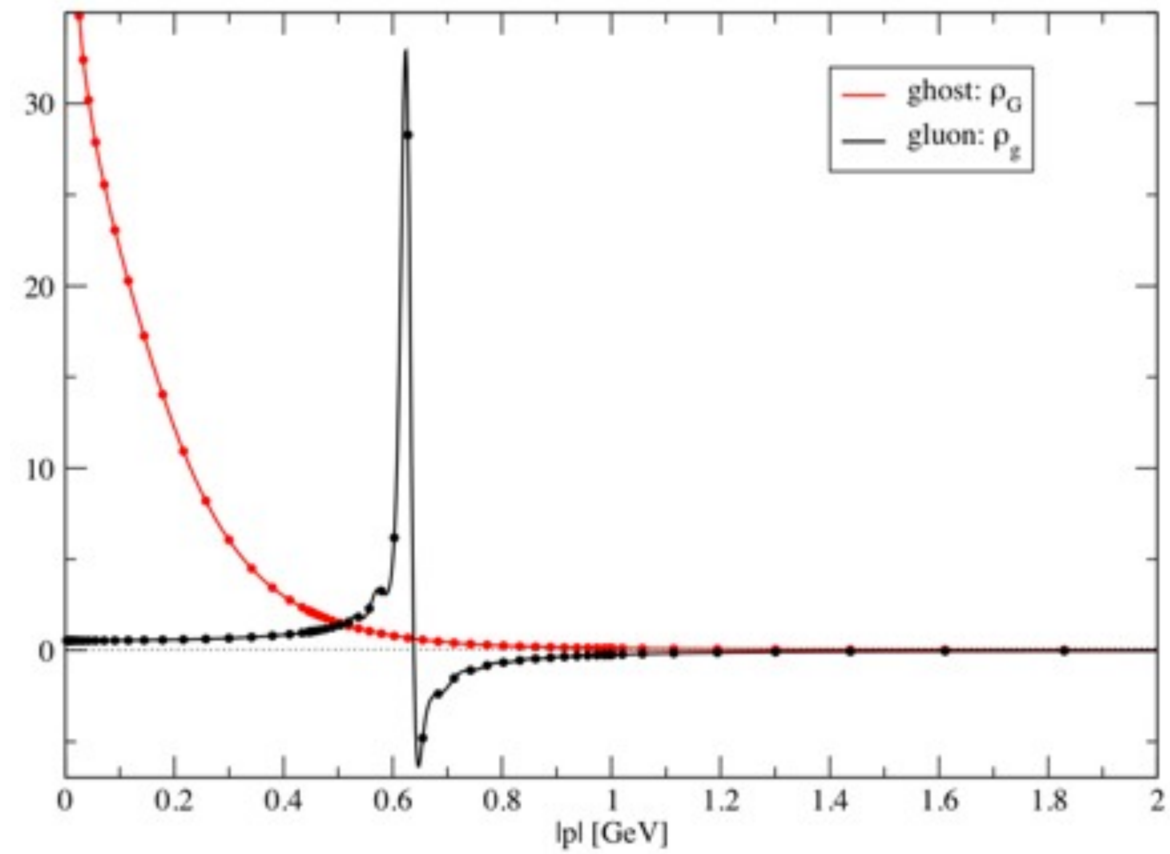
C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408-2437.

Gluon: analytic structure

Gluon: real part



Gluon: spectral function

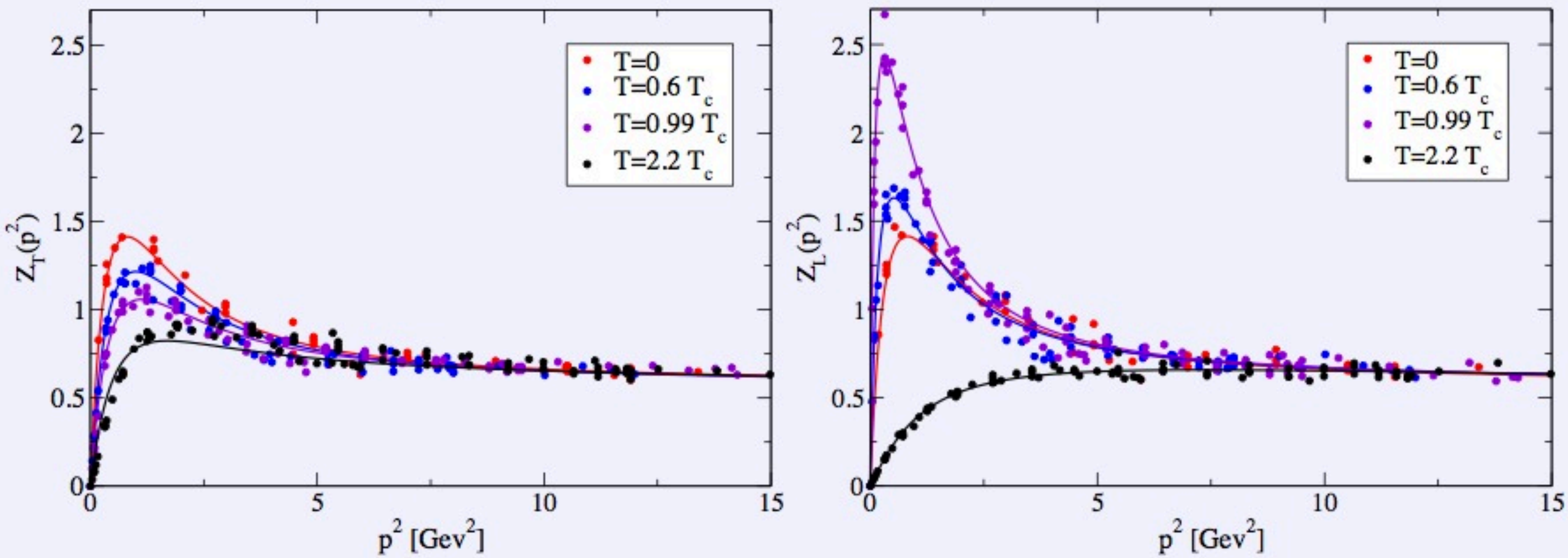


Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001

- Ghost and Gluon DSE solved in the complex p^2-plane
- No non-analytic structure outside real axis
- Cut for timelike real momenta $p^2 < 0$
- Spectral function: Oehme-Zimmermann relation satisfied

Glue at finite temperature ($T \neq 0$)

T-dependent gluon propagator from lattice simulations:



- Difference between electric and magnetic gluon
- Maximum of electric gluon around T_c

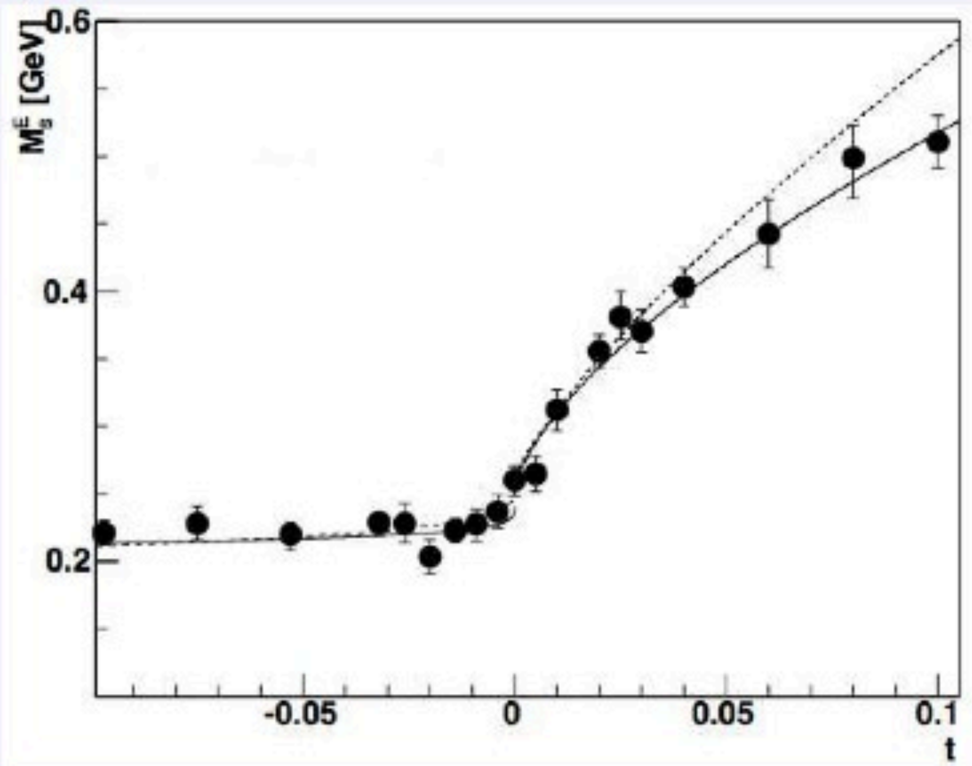
Cucchieri, Maas, Mendes, PRD 75 (2007)

C.F., Maas and Mueller, EPJC 68 (2010)

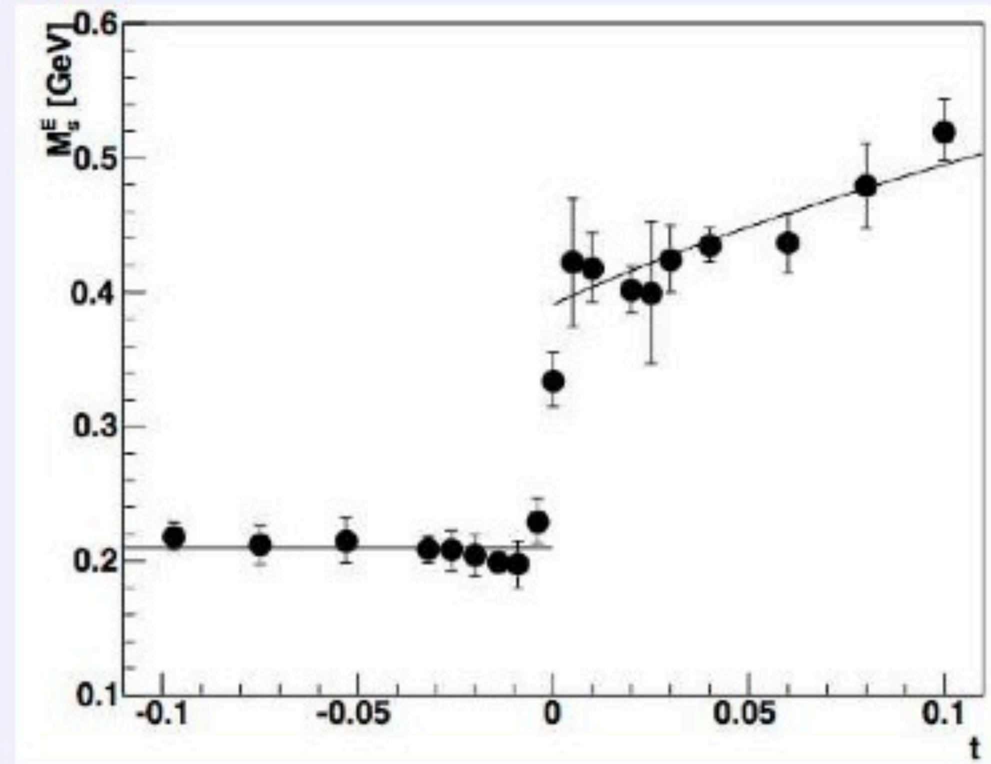
Cucchieri, Mendes, PoS FACESQCD (2010) 007.

Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker, Sternbeck, [arXiv:1108.1735 [hep-lat]].

SU(2)



SU(3)

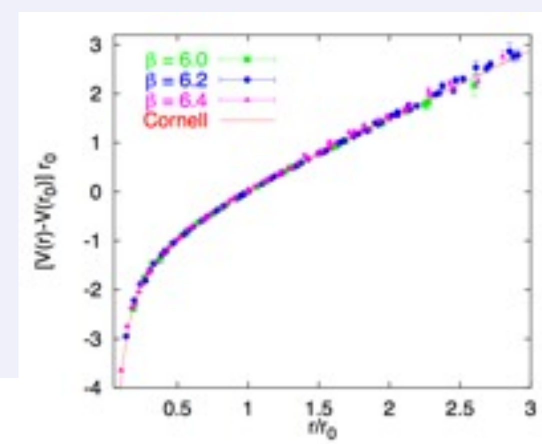


$t = (T - T_c)/T_c$

Maas, Pawlowski, Smekal, Spielmann, arXiv:1110.6340.

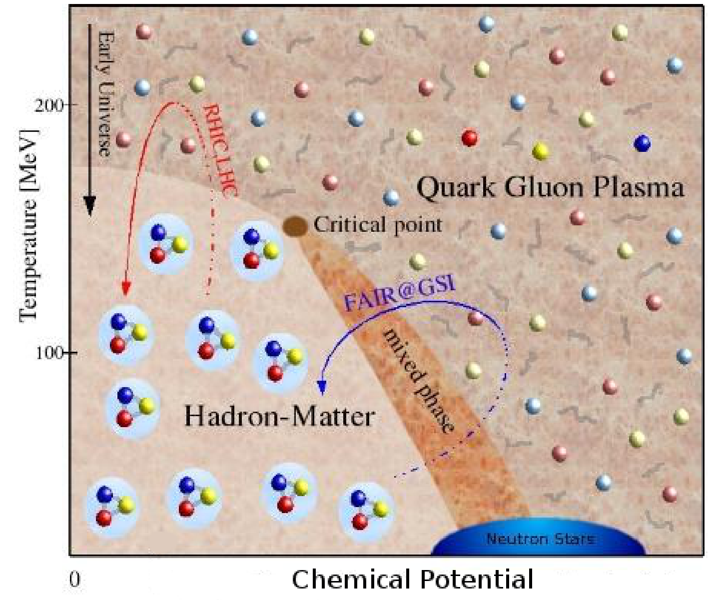
C.F., Maas and Mueller, EPJC 68 (2010)

- phase transition of second and first order clearly visible in electric screening mass



Bali, Phys. Rept 343 (2001)

Overview



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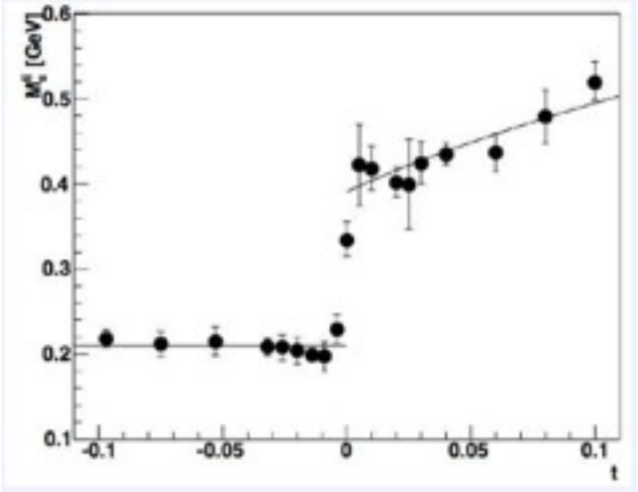

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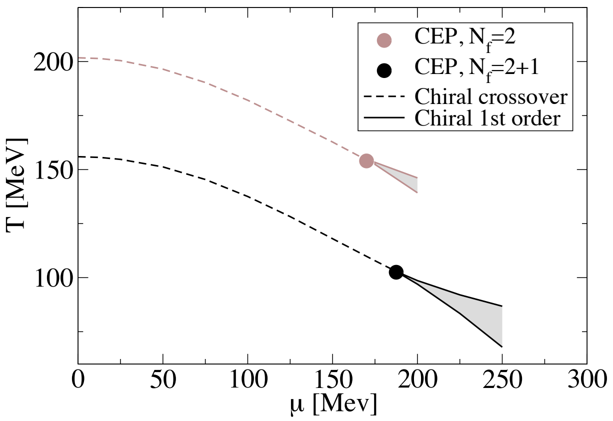
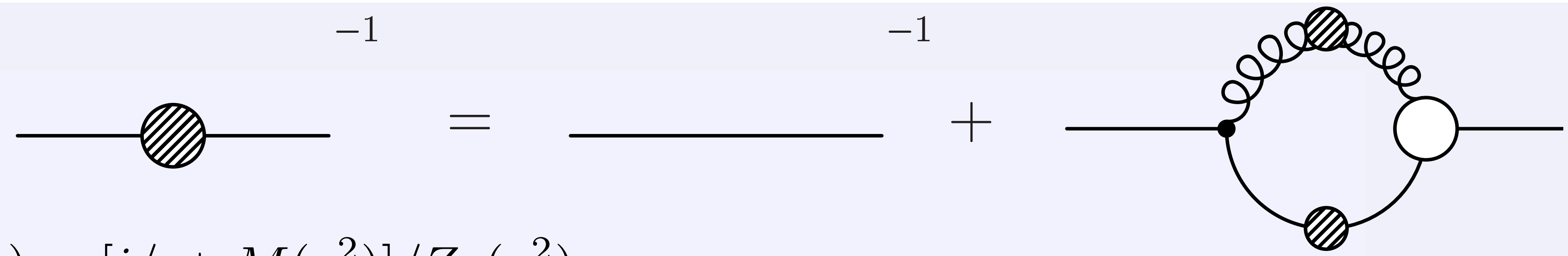


Properties of QCD: Dynamical mass generation



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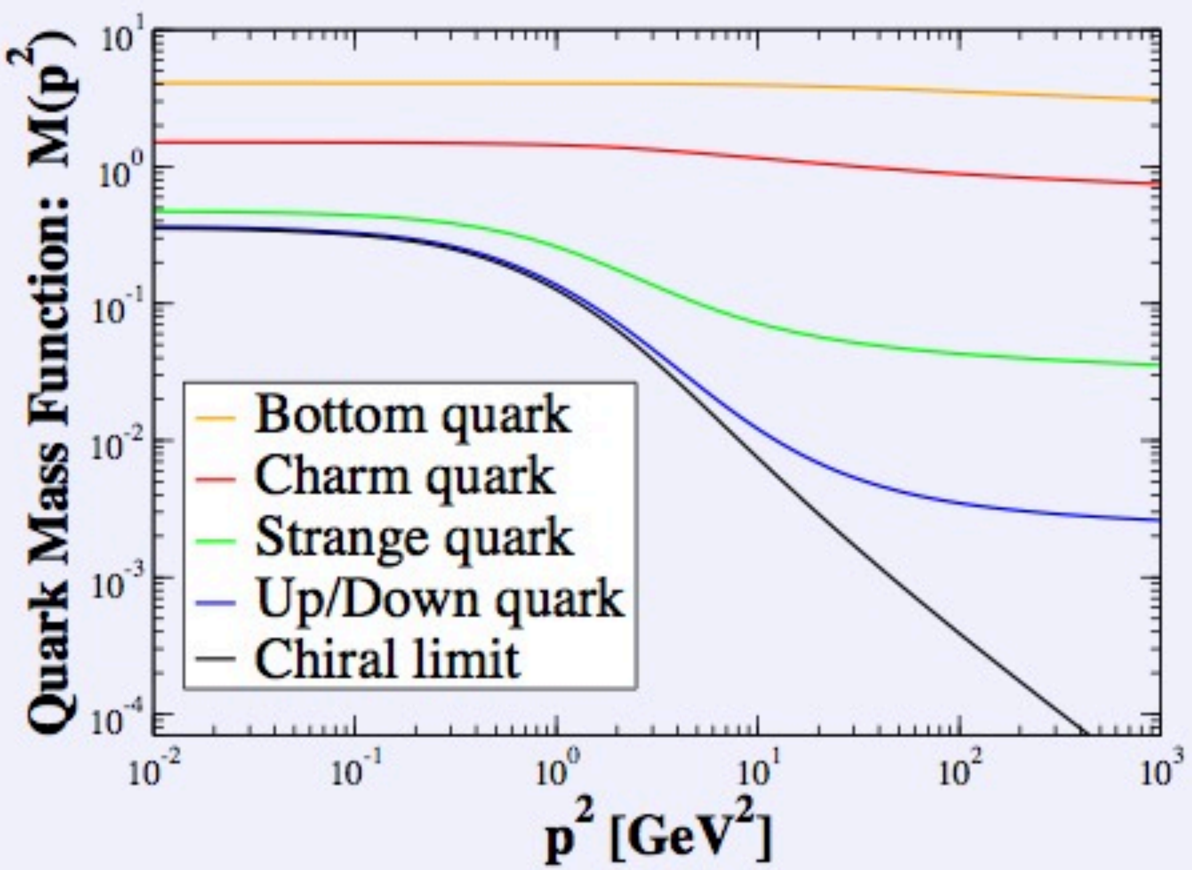

C.F. J.Phys.G G32 (2006) R253-R291

- $M(p^2)$ momentum dependent!
- Dynamical masses $M_{strong}(0) \approx 350$ MeV
- Flavour dependence because of M_{weak}
- $\langle\bar{\psi}\psi\rangle \approx (250\text{MeV})^3$

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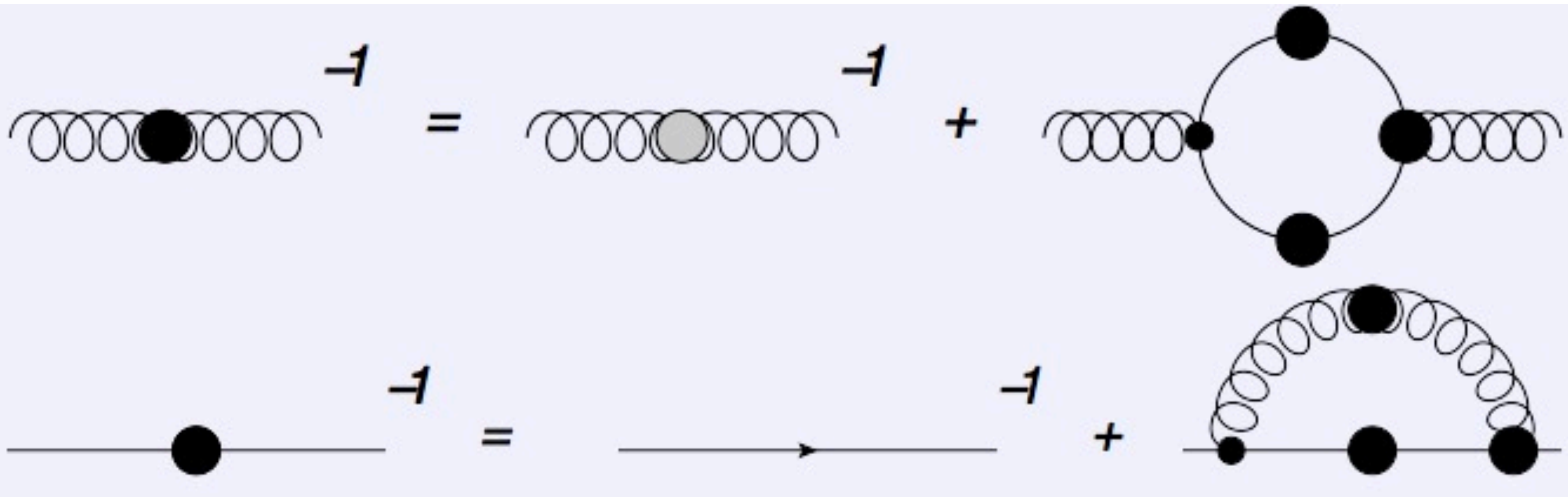
The ordinary chiral condensate



- quenched lattice gluon propagator + DSE-quark-loop
- $T = 0$: quark-gluon vertex studied via DSEs

  Alkofer, C.F., Llanes-Estrada, Schwenzer, Annals Phys.324:106-172,2009.

  C.F, R. Williams, PRL **103** (2009) 122001

  $T \neq 0$: ansatz, T, μ and mass dependent (STI)
- Order parameter for chiral symmetry breaking:

$$\langle \bar{\psi}\psi \rangle = Z_2 N_c T \sum_{n_p} \int \frac{d^3p}{(2\pi)^3} \, Tr_D \, S(\vec{p}, \omega_p)$$

The dual condensate/dressed Polyakov loop

Then define dual condensate Σ_n:

$$\Sigma_n \;=\; -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle\overline{\psi}\psi\rangle_\varphi$$

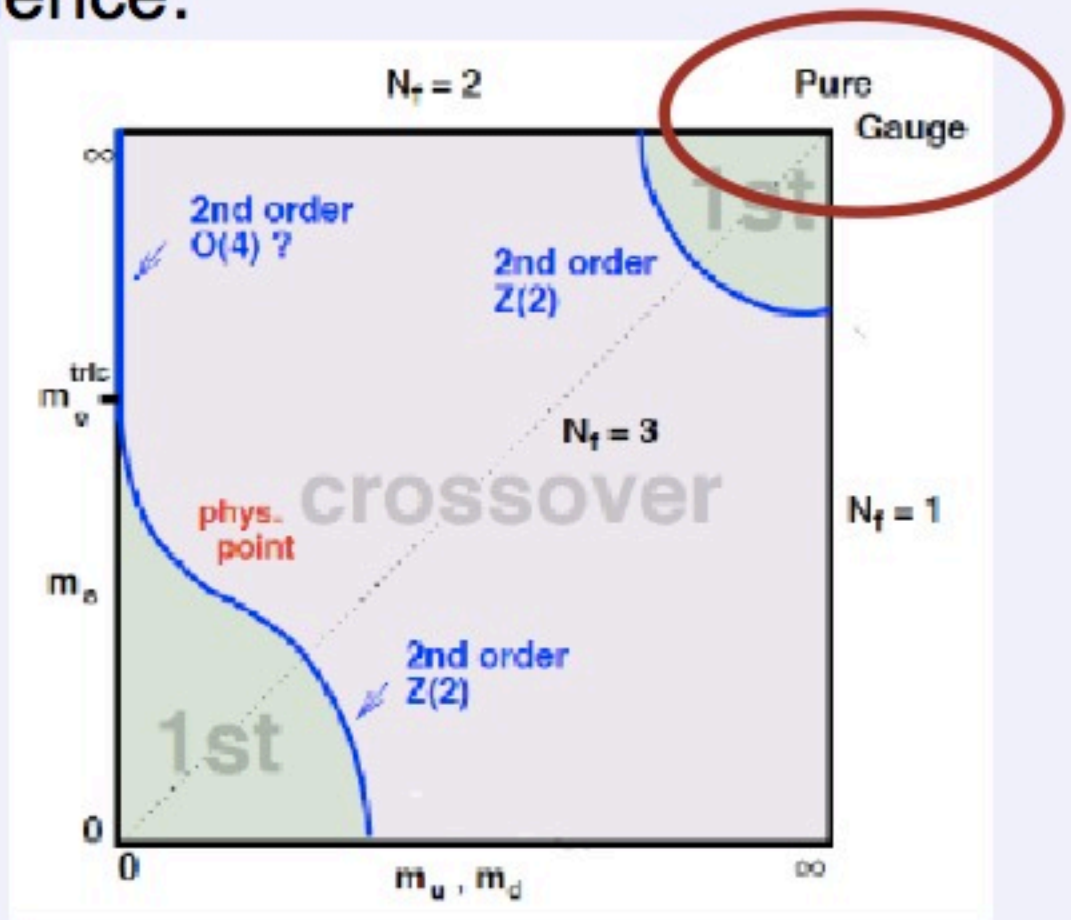
- $n = 1$ projects out loops with $n(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop: order parameter for center symmetry breaking
- Σ_1 is accessible with functional methods

  C.F., PRL **103** (2009) 052003

C. Gattringer, PRL **97**, 032003 (2006)
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** 094007 (2008).
F. Synatschke, A. Wipf and K. Langfeld, PRD **77**, 114018 (2008).
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, PRL 106 (2011)

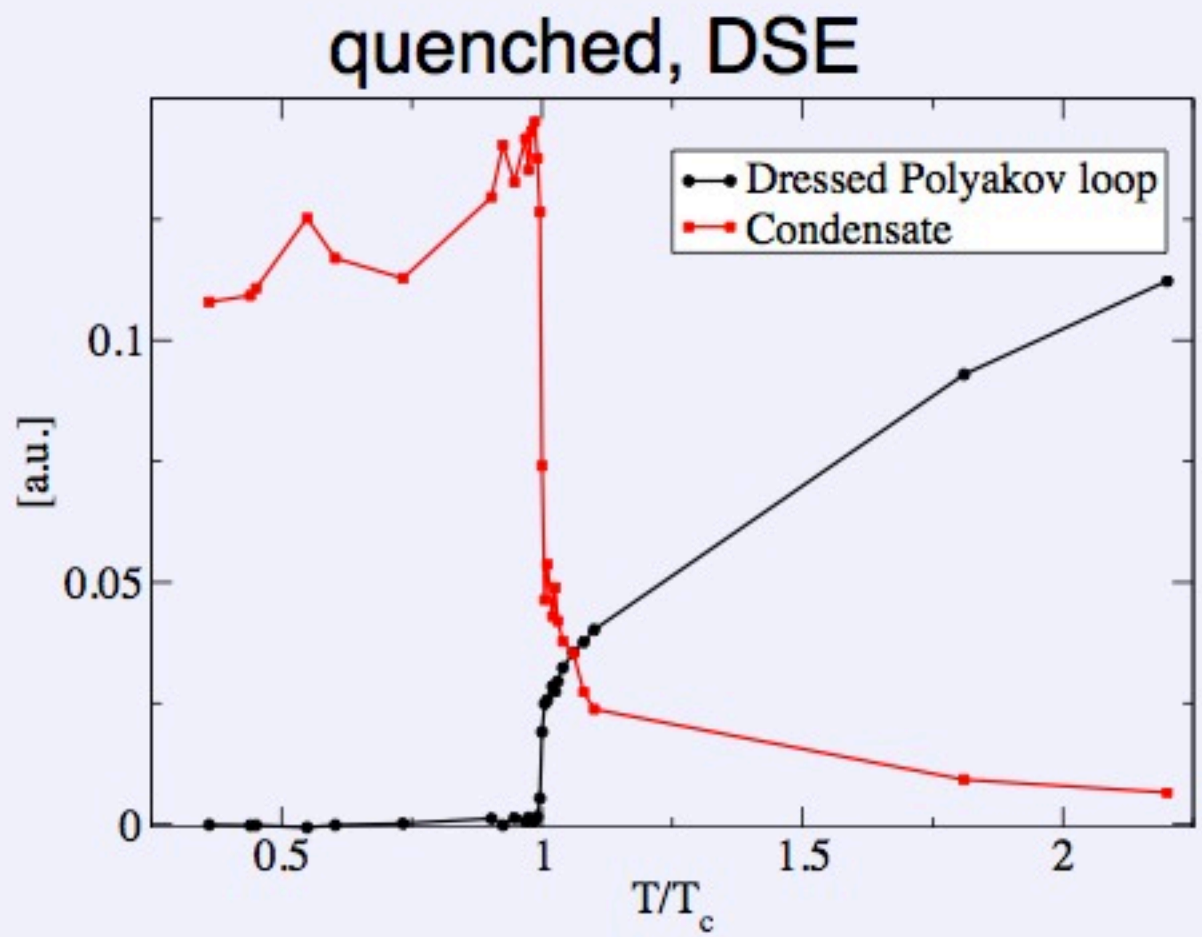
QCD phase transition: heavy quark limit/quenched

Quark mass dependence:



- Expect: Transitions controlled by deconfinement
- SU(2) second order, SU(3) first order

Transition temperatures, quenched



Luecker, C.F., arXiv:1111.0180; C.F., Maas, Mueller, EPJC 68 (2010).

- SU(2): $T_c \approx 305$ MeV
  SU(3): $T_c \approx 270$ MeV
- $T \leq T_c$: increasing condensate due to electric part of gluon

cf. Buividovich, Luschevskaya, Polikarpov, PRD 78 (2008) 074505.
cf. Braun, Gies, Pawlowski, PLB 684 (2010) 262-267.

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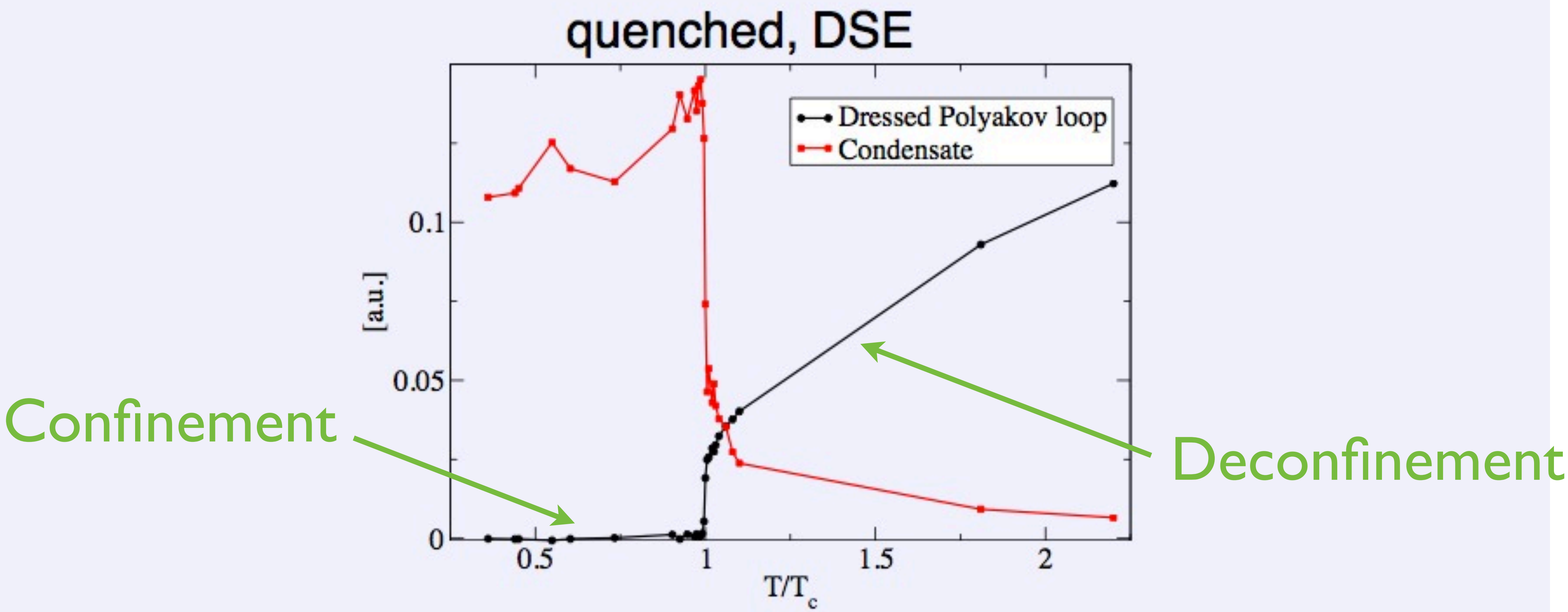
Transition temperatures, quenched



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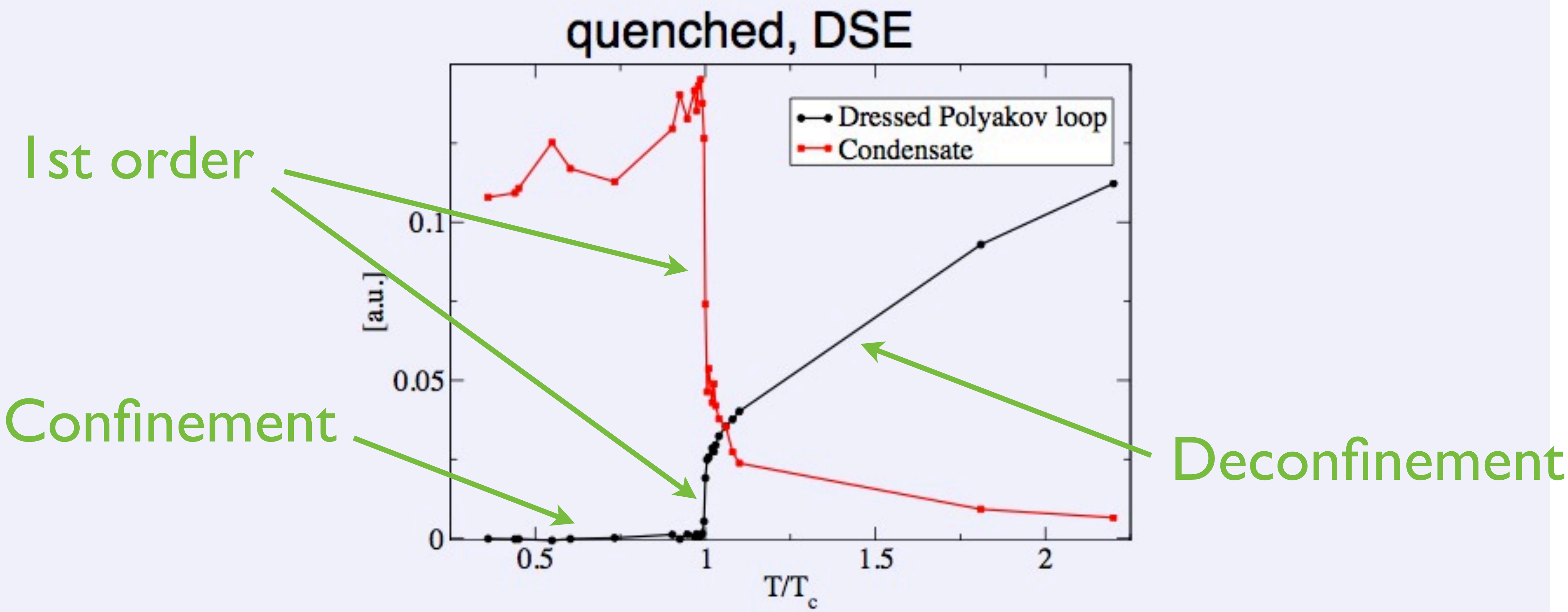

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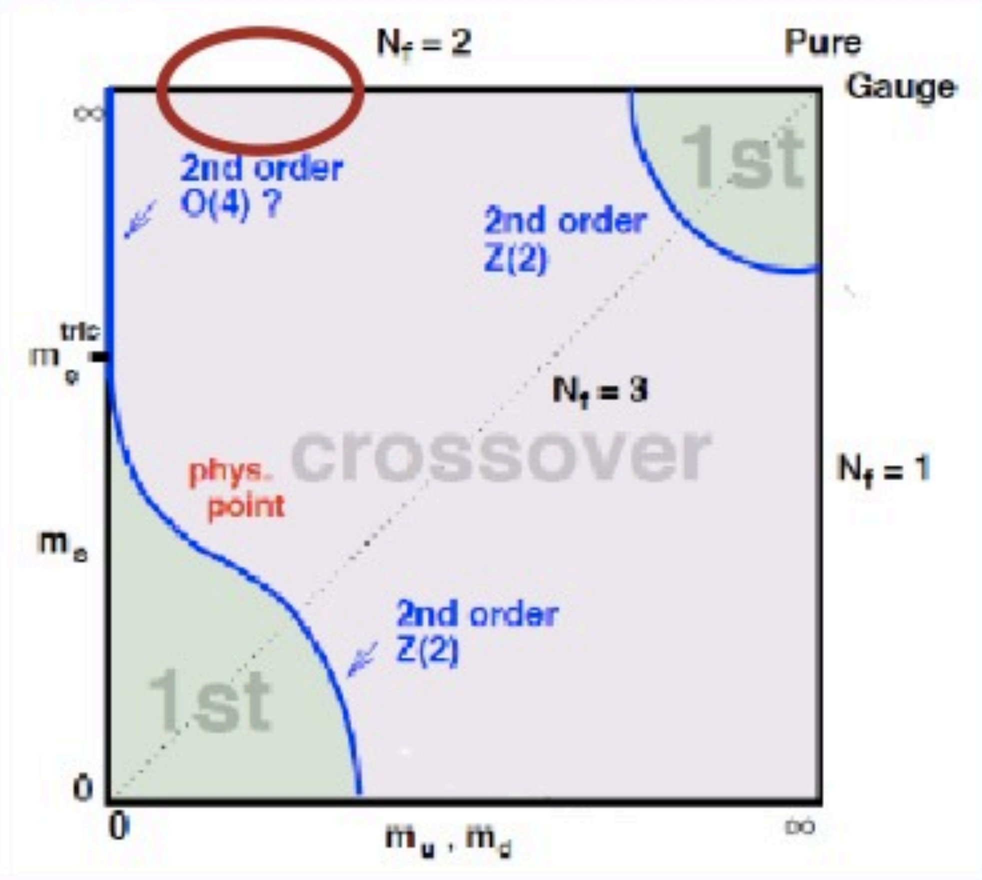
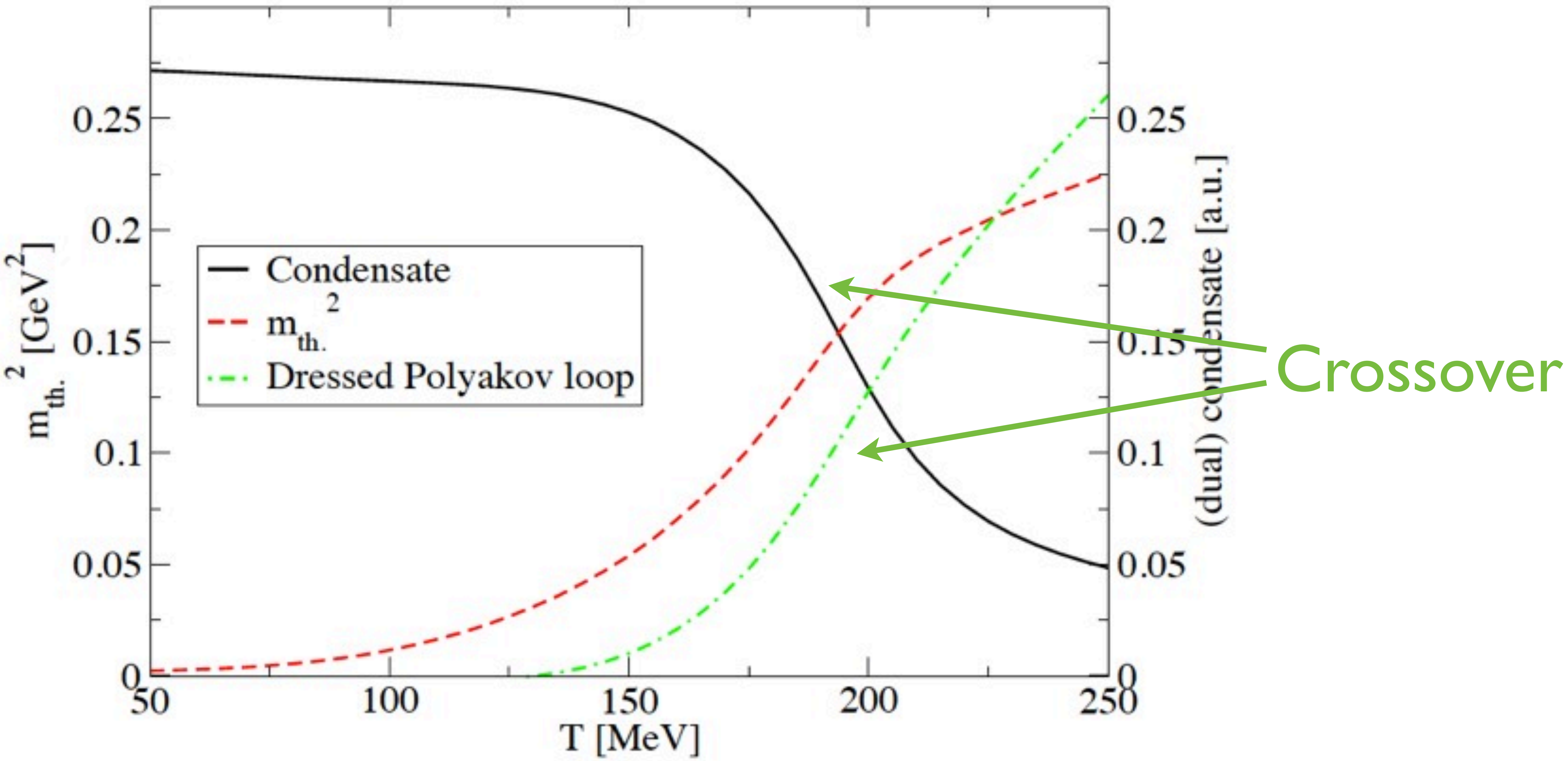
QCD phase transitions: N_f=2

Quark mass dependence:



- $N_f = 2$, physical up/down quark masses
- Transition controlled by chiral dynamics

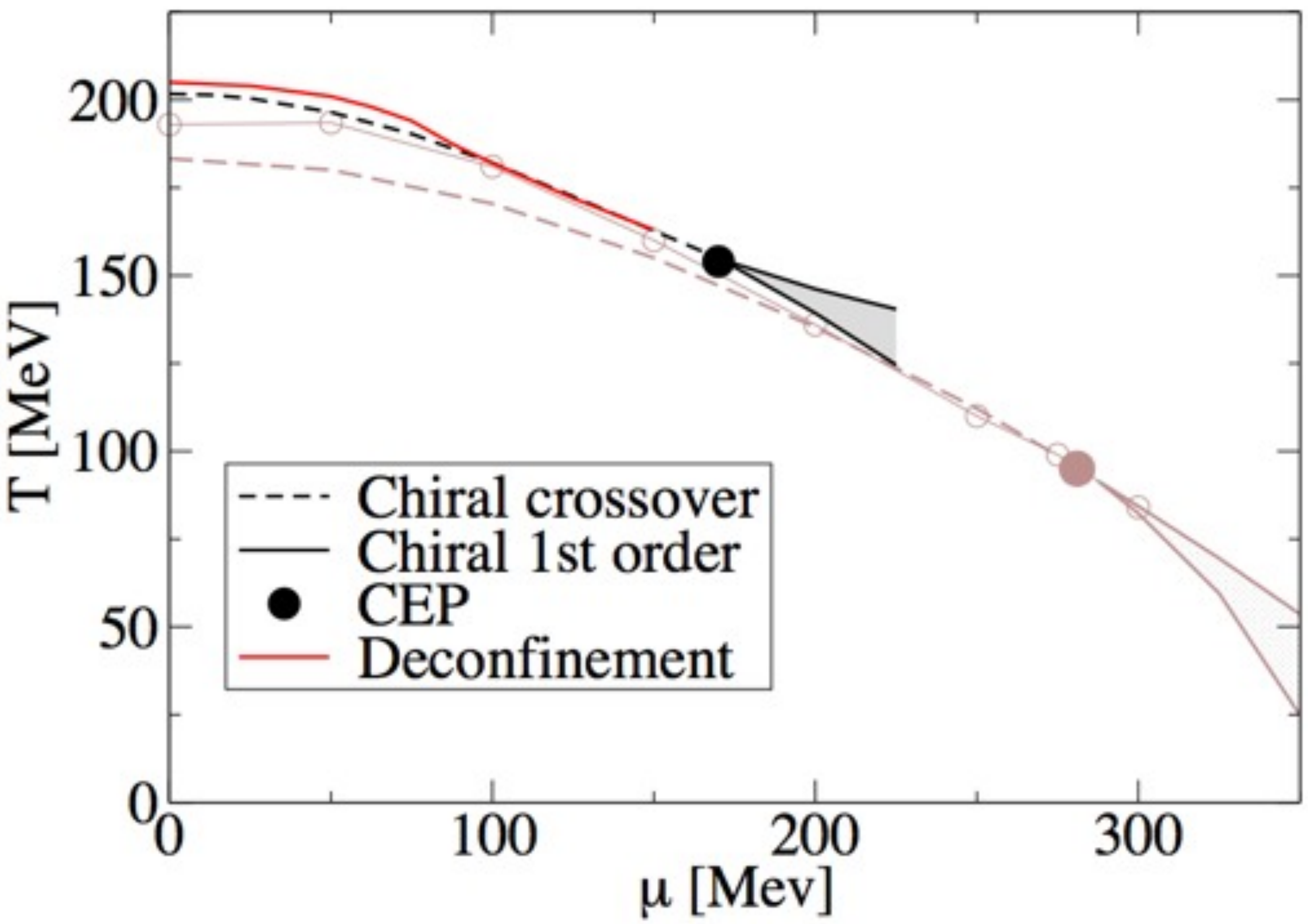
N_f=2: Transition temperatures at μ=0

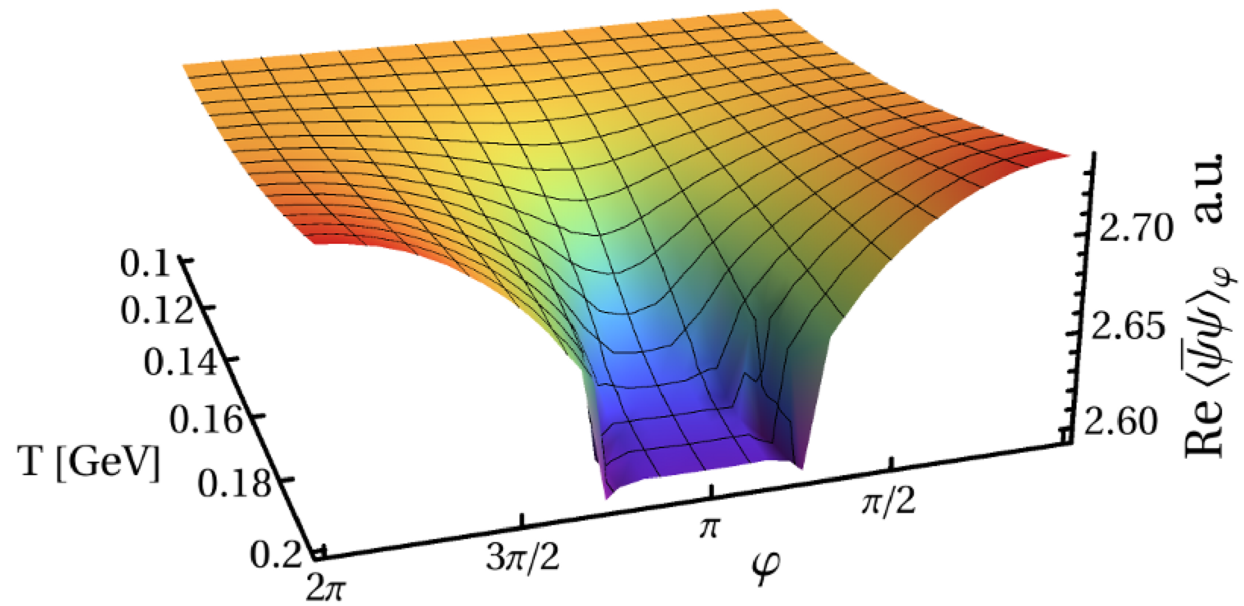


CF, Luecker, Mueller, PLB 702 (2011) 438-441
CF, Luecker, PLB 718 (2013) 1036.

- T_χ ≈203 MeV
- T_{conf} ≈205 MeV
- similar results in FRG-approach

Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011) 022002



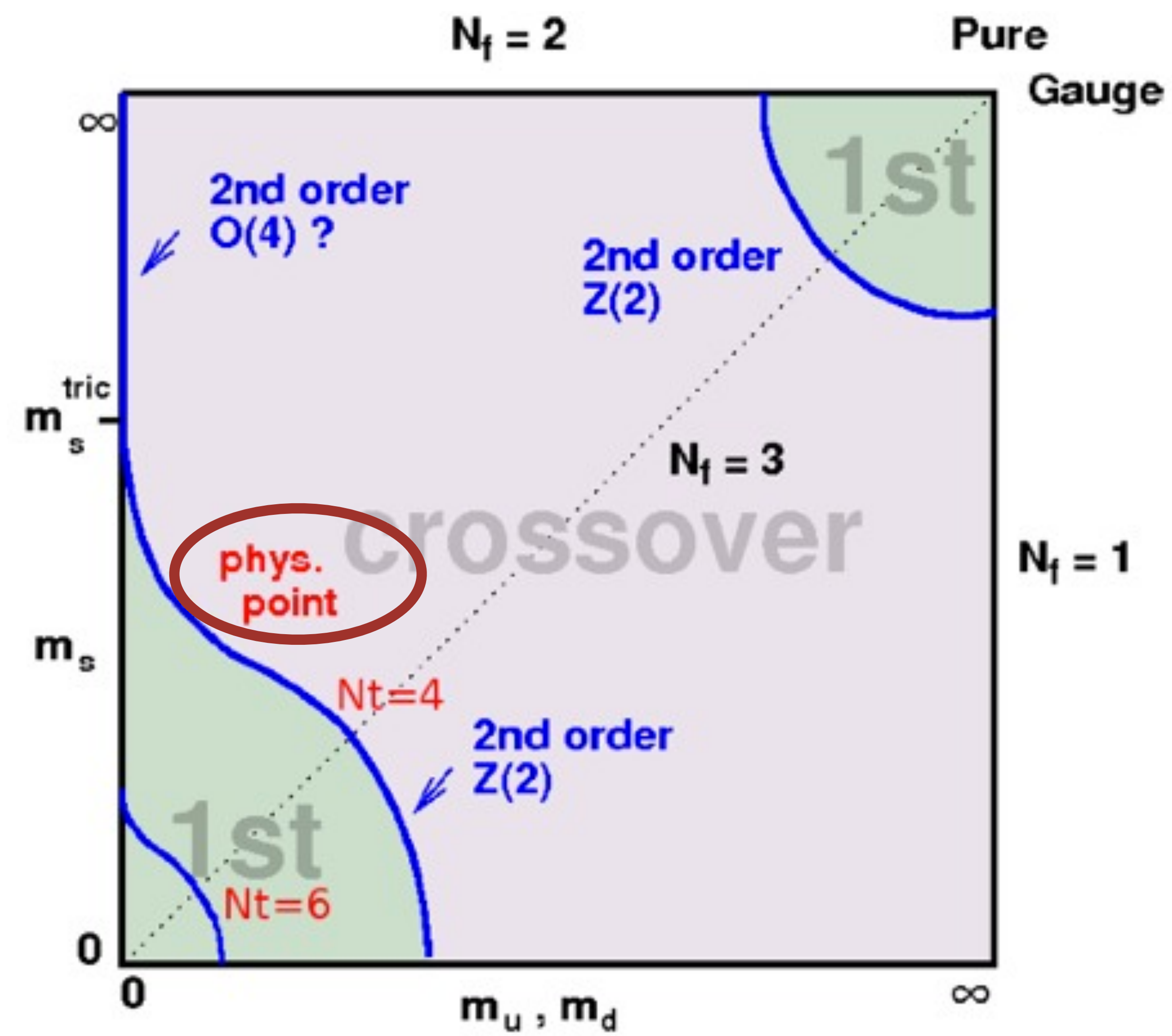


CF., Luecker, Mueller, PLB 702 (2011) 438-441
CF., Luecker, PLB 718 (2013) 1036

- chiral CEP
- crucial: backreaction of quark onto gluon
- qualitative agreement with RG-improved PQM model

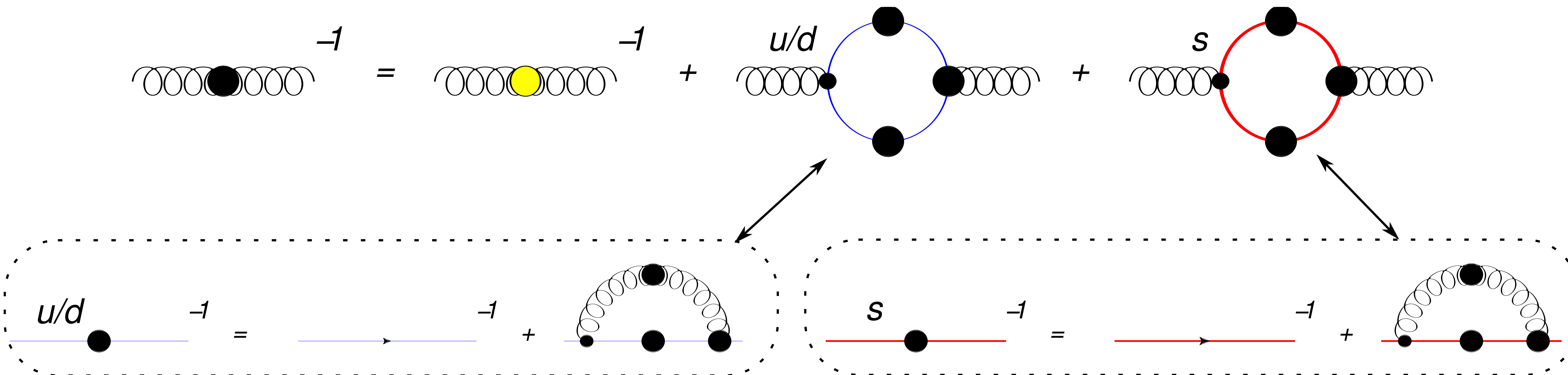
Herbst, Pawlowski, Schaefer, PLB 696 (2011)

QCD phase transitions: N_f=2+1



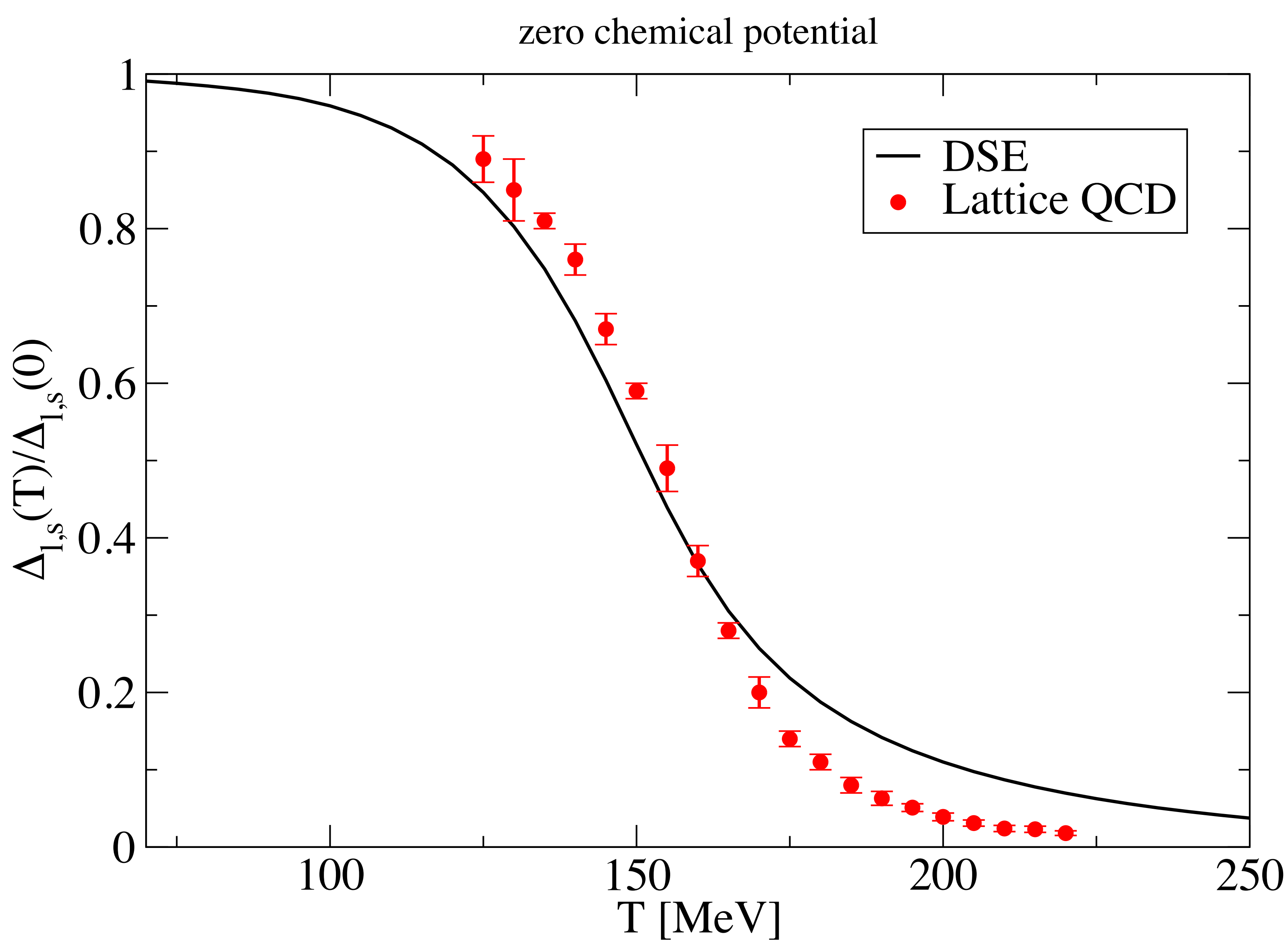
- Physical up/down and strange quark masses
- Transition controlled by chiral dynamics
- at μ=0: compare to available lattice results

DSEs with N_f=2+1



- solve coupled system of three equations

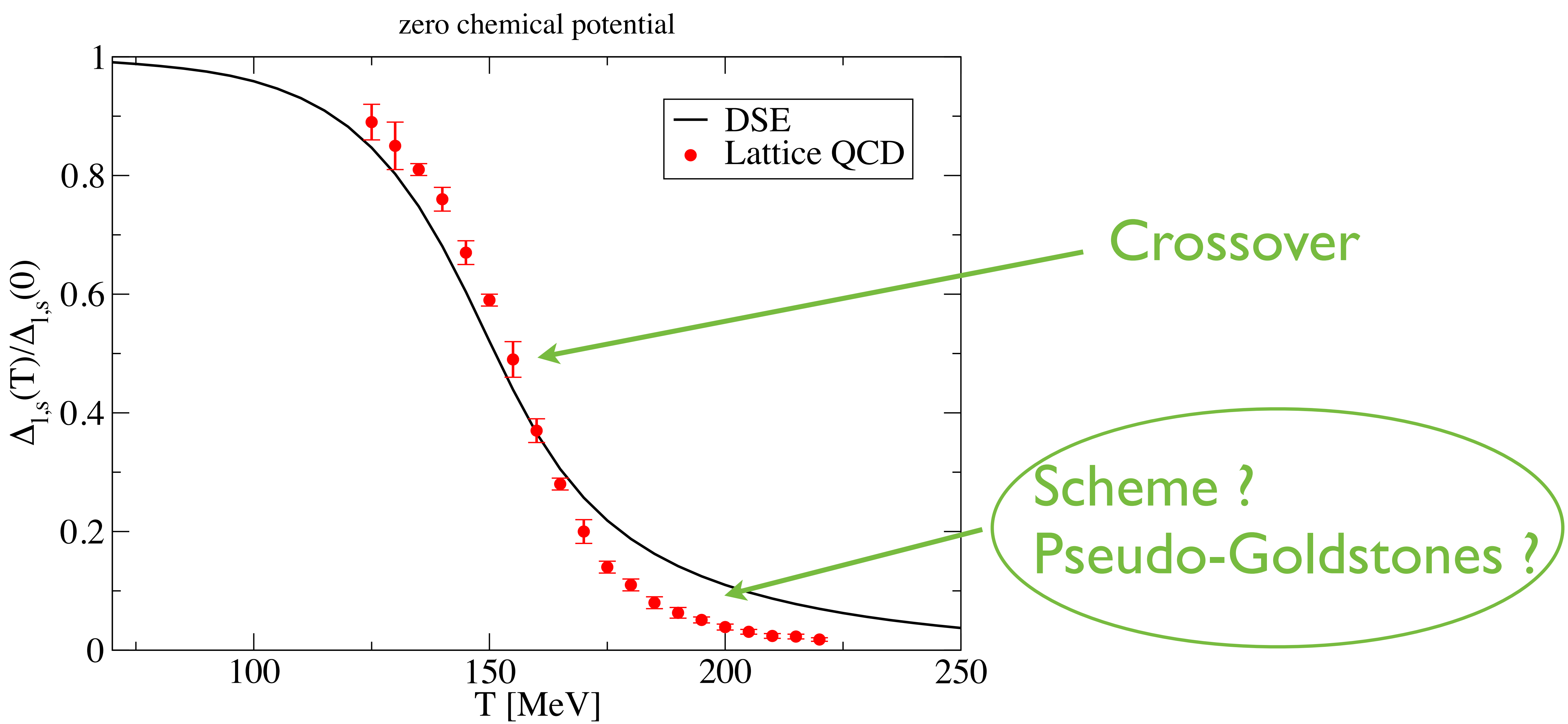
N_f=2+1, zero chemical potential



Lattice: Borsanyi *et al.* [Wuppertal-Budapest Collaboration], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036

- semi-quantitative agreement

N_f=2+1, zero chemical potential



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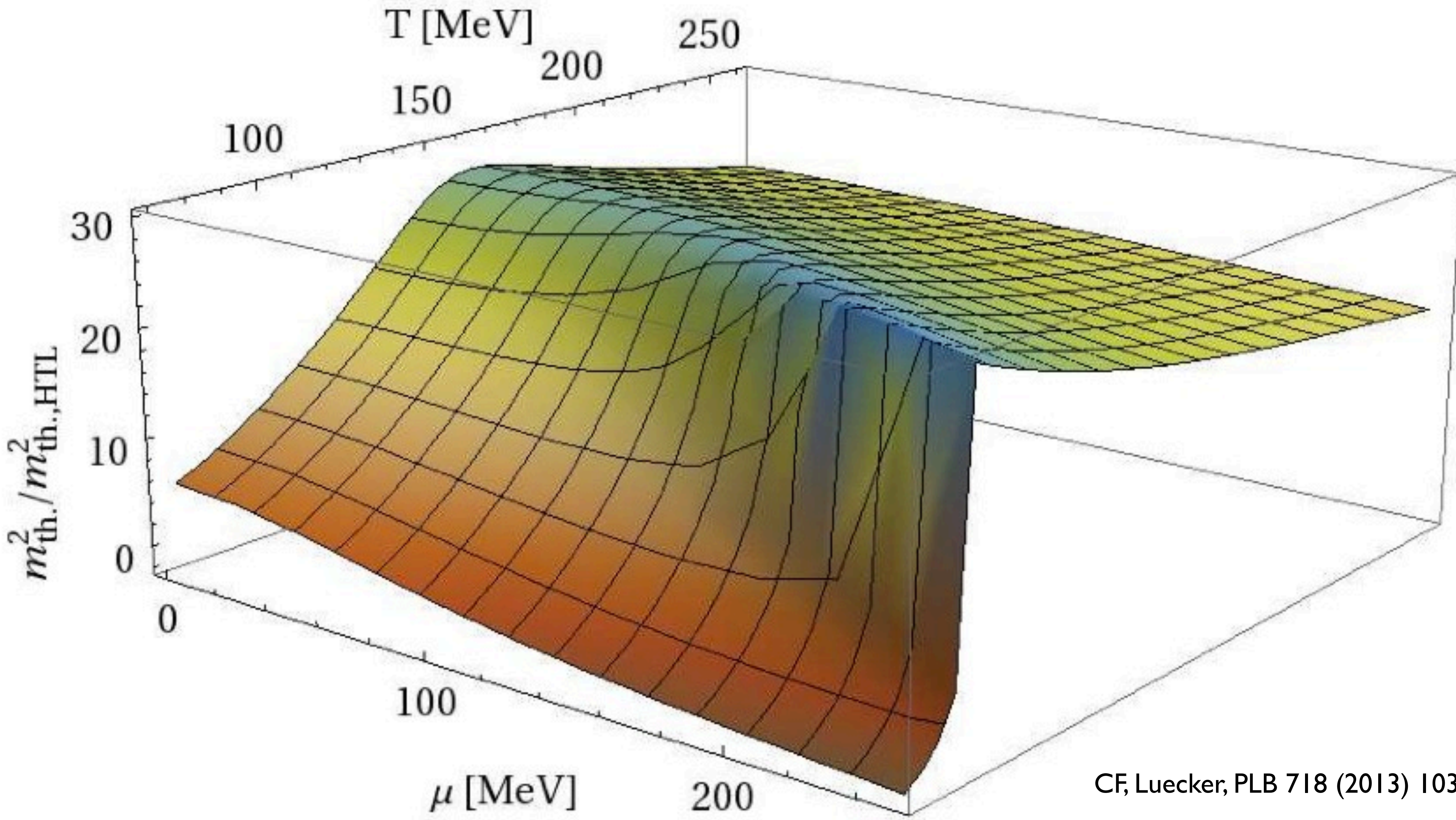
DSE: CF, Luecker, PLB 718 (2013) 1036

- semi-quantitative agreement

Nf=2+1: thermal electric gluon mass

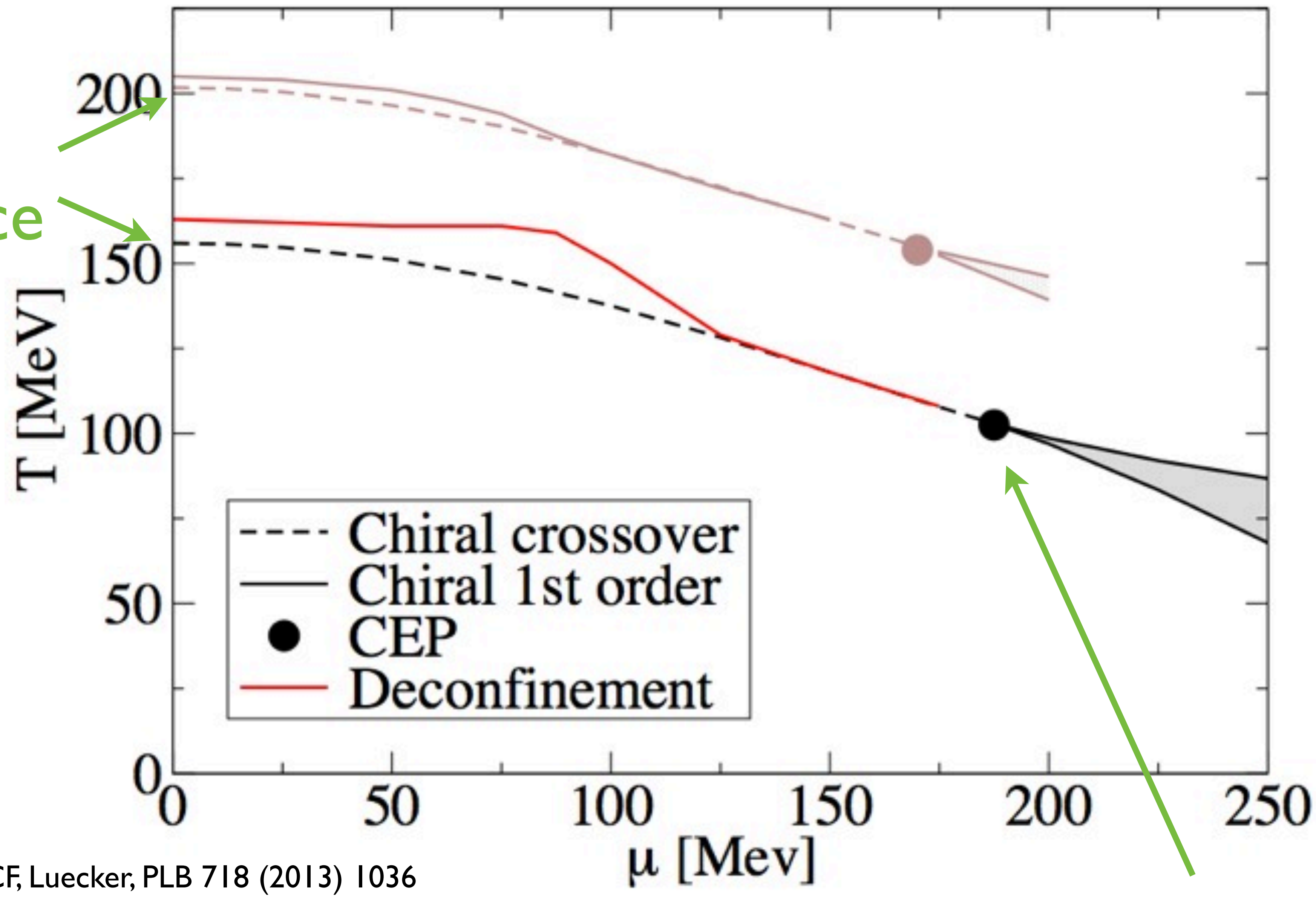


CF, Luecker, PLB 718 (2013) 1036

- large temperatures: behavior as expected from HTL
- first order transition at large chemical potential

N_f=2+1: phase diagram

T_c agree with lattice



CF, Luecker, PLB 718 (2013) 1036

CEP at large μ

- no quarkyonic region
- baryon effects missing...

Nc=2, PQM: Strodthoff, Schaefer and Smekal, PRD 85 (2012) 074007

- curvature too large... ?!

N_f	CEP	κ
2 (HTL)	(280,90)	0.23
2	(171,154)	0.37
2+1	(190,100)	0.28

Summary

- Gluon spectral functions at T=0: positivity violation
- Temperature dependent gluon propagator
  - characteristic behavior of electric gluon
  - 'melting' of magnetic gluon with temperature
- Deconfinement T_c from dressed Polyakov-loop via DSEs
- QCD with finite chemical potential (beyond mean field)
  - backreaction of quarks onto gluons important
  - N_f=2+1: CEP at $\mu_c/T_c > 1$