CONFINEMENT AND YANG–MILLS THERMODYNAMICS FROM CORRELATION FUNCTIONS

Leonard Fister
NUI Maynooth

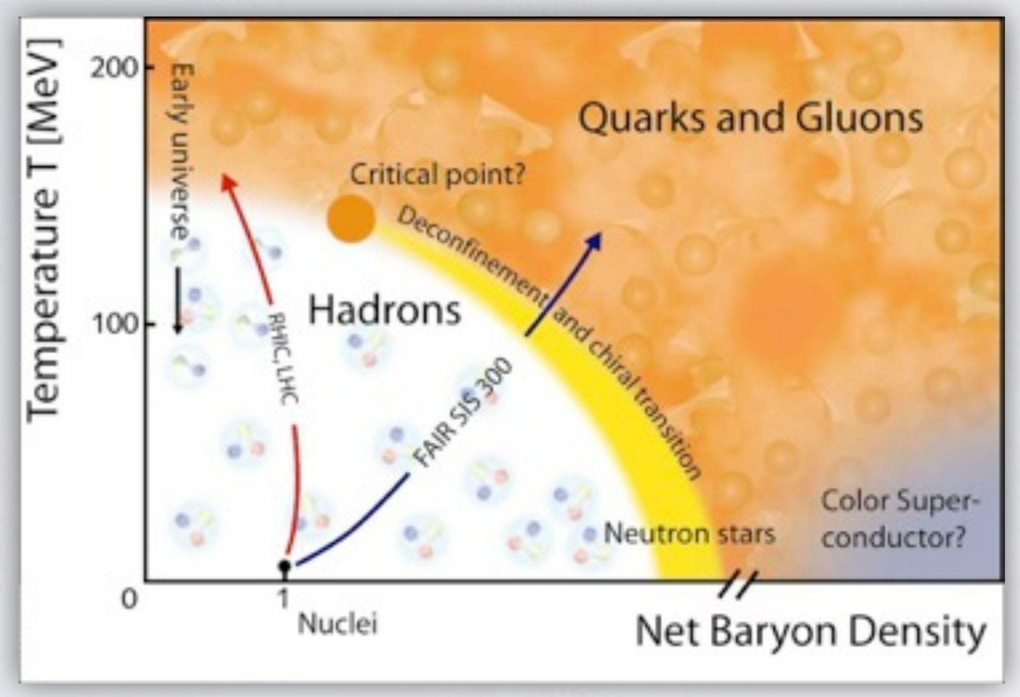
LF, J.M. Pawlowski, arXiv: 1301.####[hep-ph].

LF, J.M. Pawlowski, arXiv: 1112.5440 [hep-ph].

LF, J.M. Pawlowski, PoS QCD-TNT-II2011 (2011) 021 [arXiv: 1112.5429 [hep-ph]].

Delta13,
Heidelberg, January 11-12, 2013

Motivation: QCD Phase Diagram



GSI Darmstadt

characteristic features at low energies

- dynamical chiral symmetry breaking
- confinement

non-perturbative computation of physical observables from microscopic dynamics

here: study aspects of the phase diagram with non-perturbative **functional continuum methods**

- functional renormalisation group,
- Dyson–Schwinger equations,
- *n*PI-techniques

→ static quark confinement via the Polyakov loop potential

phase transition order, phase transition temperature,
confinement criterion via infrared behaviour of propagators

→ thermodynamics of pure gluodynamics (Yang–Mills theory)

pressure at temperatures around the phase transition

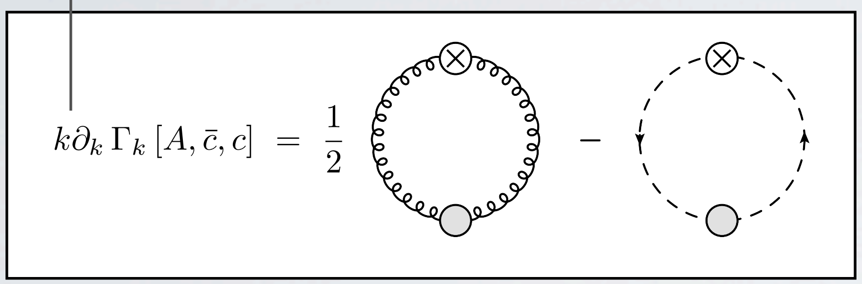
OUTLOOK

- Motivation
- (Thermal) Yang–Mills Propagators
- Quark Confinement
- Thermodynamics of Yang–Mills Theory

FUNCTIONAL METHODS FOR YANG–MILLS

- Functional Renormalisation Group (FRG)

k ... energy scale: integrate fluctuations energy-shell-wise from UV to IR

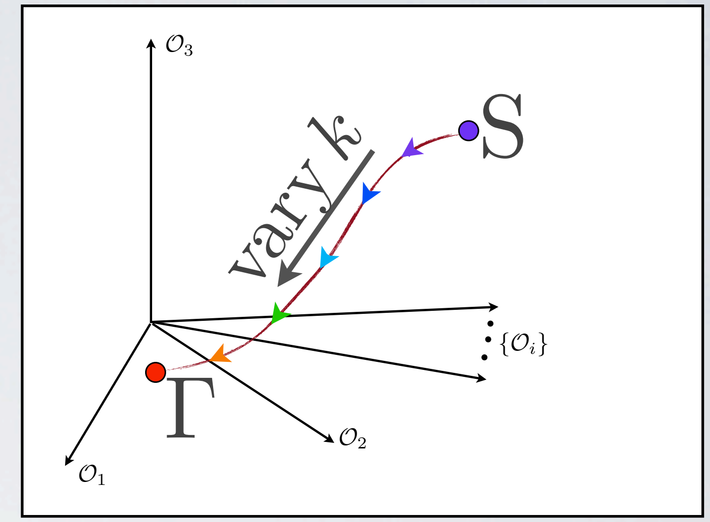


flow along in theory space:

... spanned by *all* operators

... start with classical action S

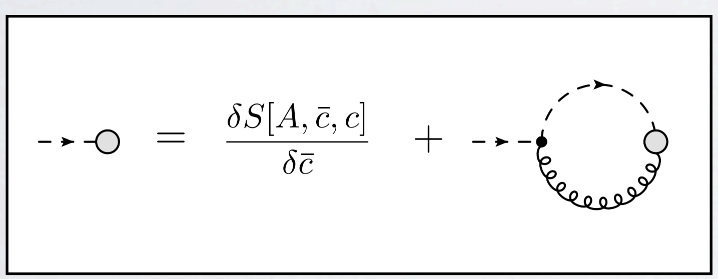
... quantum theory Γ at $k \to 0$

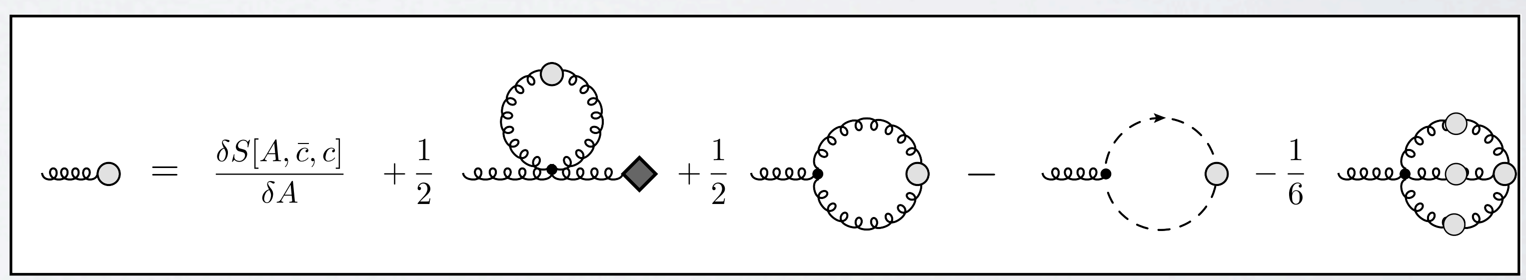


- Dyson–Schwinger Equations (DSEs)

originate from

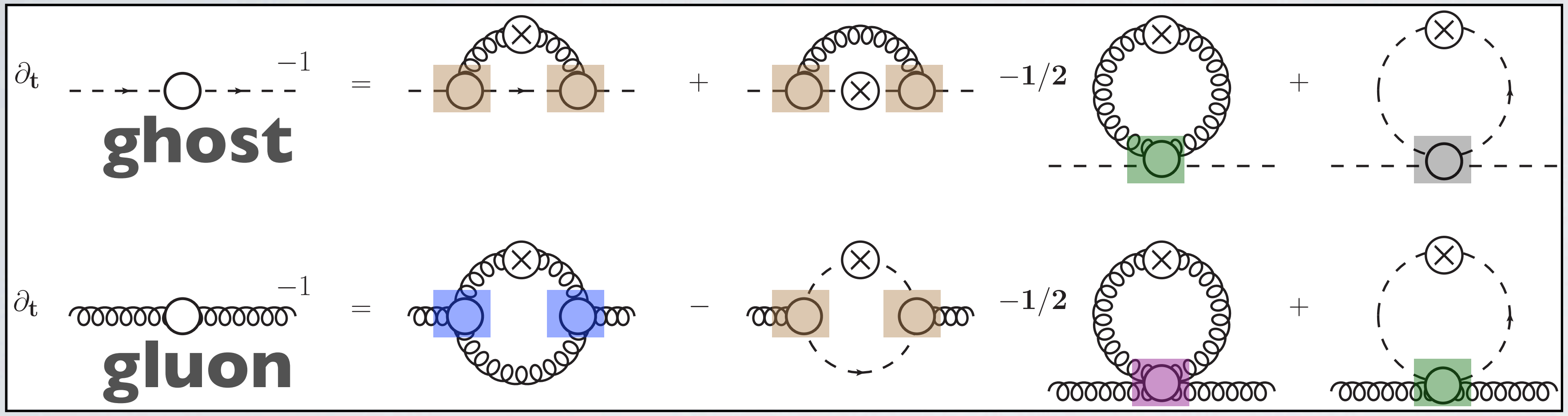
$$\int \mathcal{D}\phi \frac{\delta}{\delta\phi} e^{-S[\phi]+J\cdot\phi} = 0$$





... Both FRG and DSEs provide **exact descriptions** of the full theory in terms of **correlation functions.**

(Landau gauge) YANG–MILLS PROPAGATORS

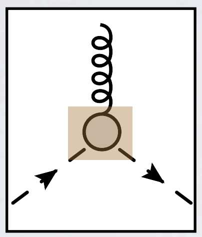


.... FRG equations,

DSE equations see e.g.

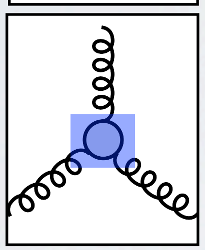
R. Alkofer, L. von Smekal, Phys.Rept. 353, 281 (2001).
C.S. Fischer, J.Phys. G32, R253 (2006).

truncation based on

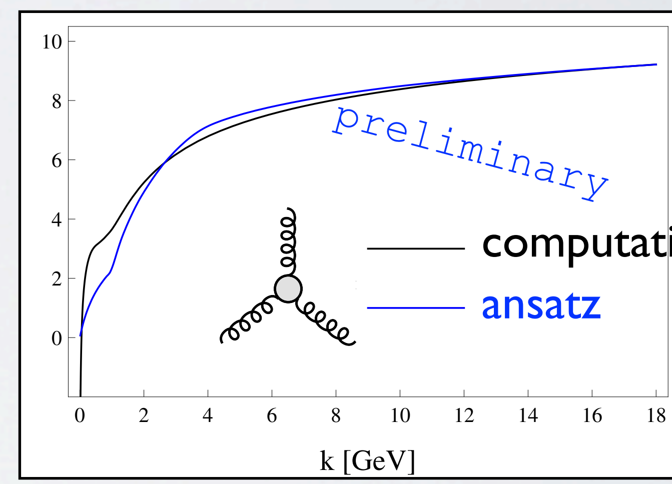


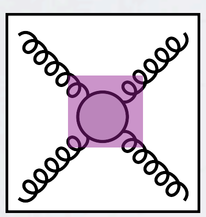
FRG LF, J.M. Pawlowski, arXiv: 1112.5440 [hep-ph].

DSE M.Q. Huber, L. von Smekal, arXiv: 1211.6092 [hep-th].

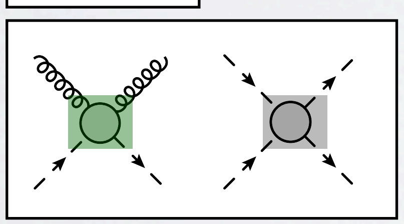


FRG LF, J.M. Pawlowski, in preparation.





DSE C. Kellermann, C.S. Fischer, Phys.Rev.D78, 025015 (2008).



via resummations

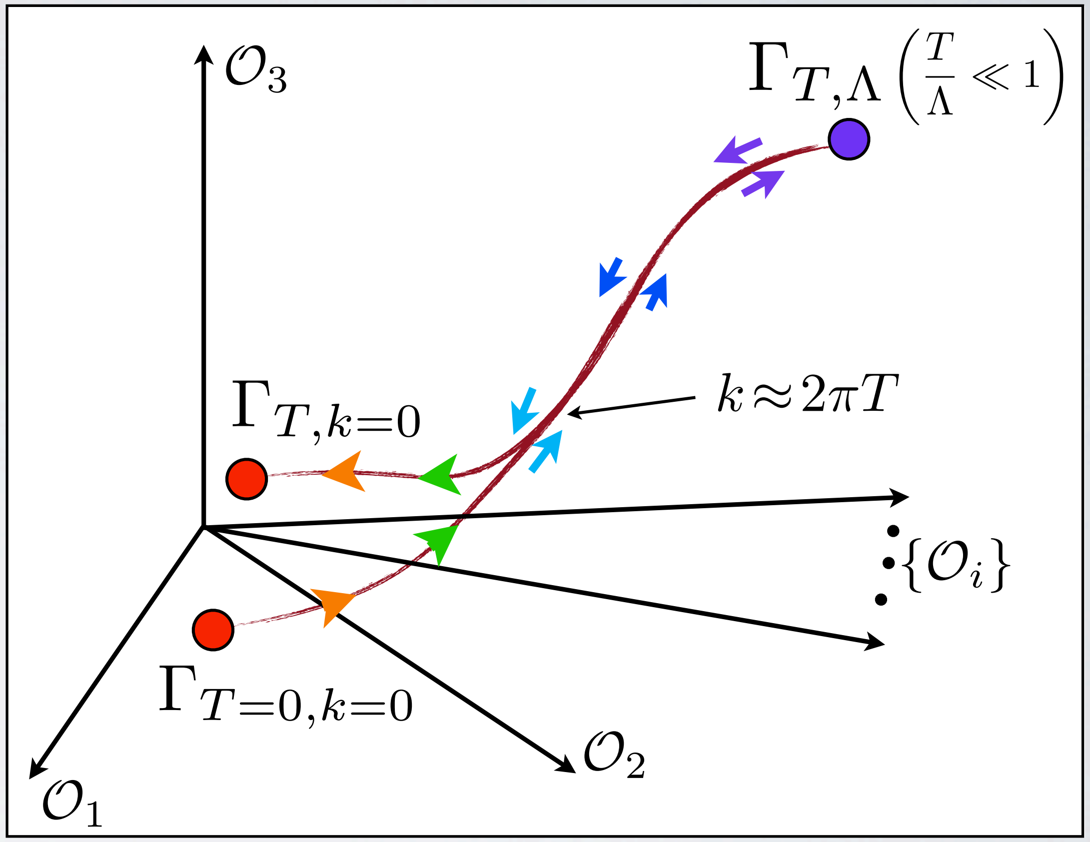
Propagators have non-trivial **temperature** and **momentum dependence**, **both** are **indispensable**(, in particular **for thermodynamics**).

THERMAL FRG

purely thermal fluctuations

$$\Delta\Gamma_{k,T} = \Gamma_{k,T} - \Gamma_{k,T=0}$$

D. Litim, J.M. Pawlowski, arXiv: hep-th/9901063.
D. Litim, J.M. Pawlowski, JHEP 11 (2006) 026.



temperature effects restricted to infrared $k \lesssim 2\pi T$

two-step procedure:

1. computation of quantum effects
2. add thermal fluctuations to quantum theory

practical advantages:

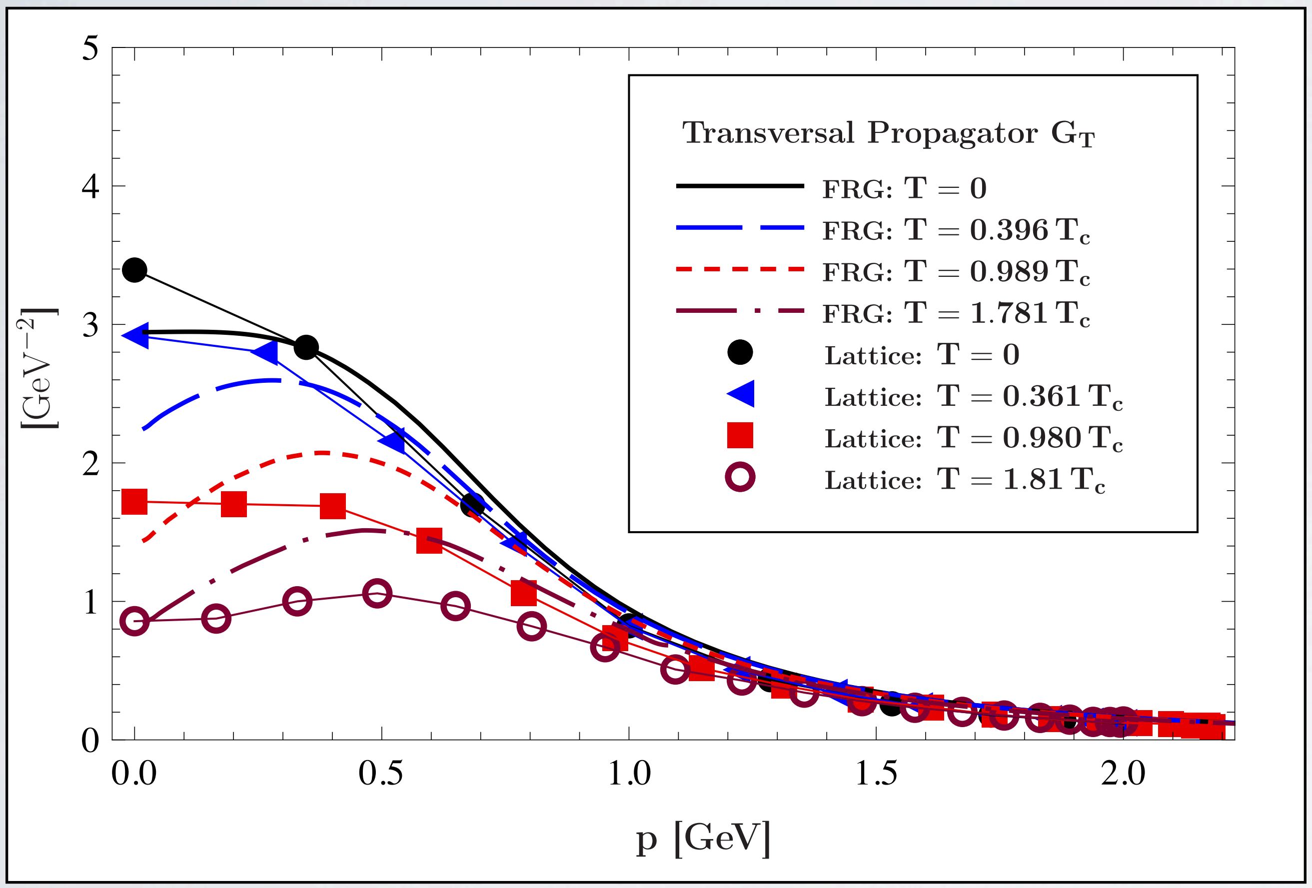
- quantum theory can be taken from any method, i.e. also from lattice gauge theory
- truncation errors affect only the infrared

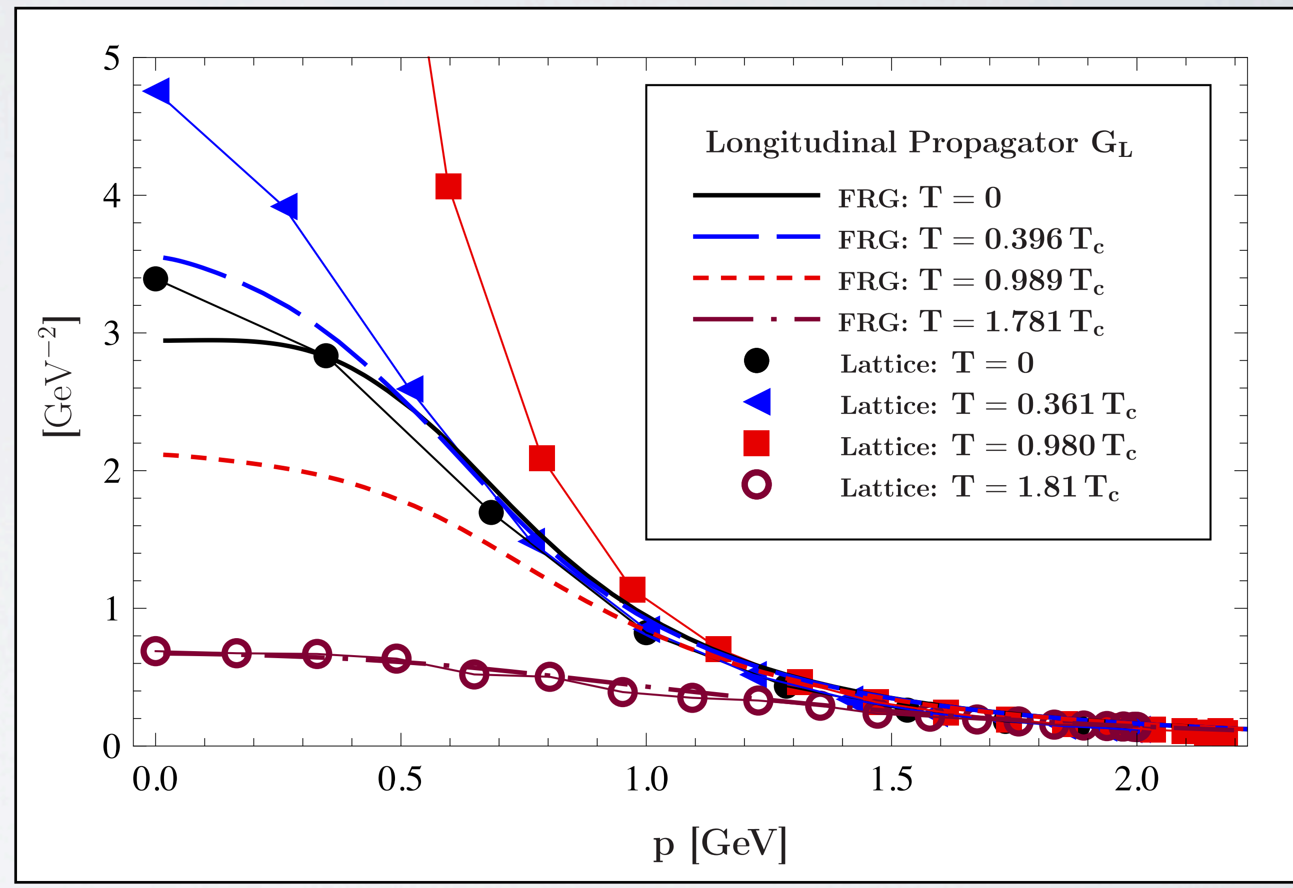
YANG–MILLS PROPAGATORS

momentum dependence at **non-zero** temperature

at finite temperature: the gluon propagator has two projections wrt the heatbath

transversal gluon propagator | **longitudinal gluon propagator**





FRG results from:

LF, J.M. Pawlowski, arXiv: 1112.5440 [hep-ph].

LF, J.M. Pawlowski, PoS QCD-TNT-II2011 (2011) 021 [arXiv: 1112.5429 [hep-ph]].

lattice data taken from:

A. Maas, J.M. Pawlowski, L. von Smekal, D. Spielmann, Phys. Rev. D85 (2011) 034037.

Quark Confinement

POLYAKOV LOOP POTENTIAL

The expectation value of the **Polyakov loop**, $\langle L[A_0]\rangle$, relates to the free energy F_q of a single quark.

→ order parameter for static quark confinement

$$e^{-F_q/T} \sim \langle L[A_0]\rangle = \left\langle \frac{1}{N_c}\mathcal{P}\ e^{ig\int_0^{1/T} dt A_0} \right\rangle \qquad \begin{cases} = 0 \ \ldots \text{ confinement} \\ > 0 \ \ldots \text{ deconfinement} \end{cases}$$

Also $$L[\langle A_0\rangle] \begin{cases} = 0 & \text{if } \langle L\rangle = 0 \\ \geq \langle L\rangle & \text{if } \langle L\rangle > 0 \end{cases}$$ is an order parameter.

J. Braun, H. Gies, J.M. Pawlowski, Phys. Lett. B684, 262 (2010).
F. Marhauser, J.M. Pawlowski, arXiv: 0812.1144 [hep-ph].

$L[\langle A_0\rangle]$ easily accessible in **background field formalism:**

$\langle A_0\rangle$ minimum of **effective potential** $V[A_0]$ of **constant background field** A_0.

→ in FRG, DSE, 2PI, ...

POLYAKOV POTENTIAL - REPRESENTATIONS

eff. potential

$$V[A_0] = \frac{T}{\text{volume}} \Gamma[A_0; a = 0]$$

background field method:
gluon $A = A_0 + a$
(temporal) background ↑ ↑ fluctutation about background

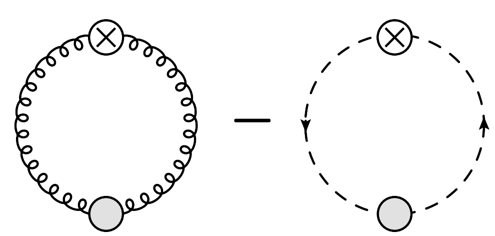
↓

Confinement is immanent, if minima of $V[A_0]$ equal confining values,

i.e. at these $\langle A_0 \rangle_{\text{conf}}$: $L[\langle A_0 \rangle_{\text{conf}}] = 0$.

FRG:

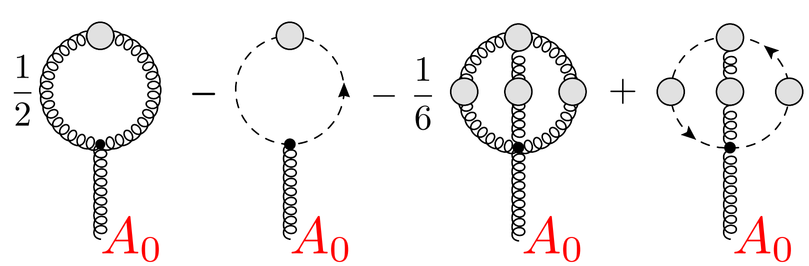
$$k\partial_k \Gamma_k [A_0; a, \bar{c}, c] = \frac{1}{2}$$



J. Braun, H. Gies, J.M. Pawlowski, Phys. Lett. B684, 262 (2010).
J. Braun, A. Eichhorn, H. Gies, J.M. Pawlowski, Eur. Phys. J. C70, 689 (2010).
LF, J.M. Pawlowski, arXiv: 1301.#### [hep-ph].

DSEs:

$$\frac{\delta(\Gamma - S)}{\delta A_0} =$$



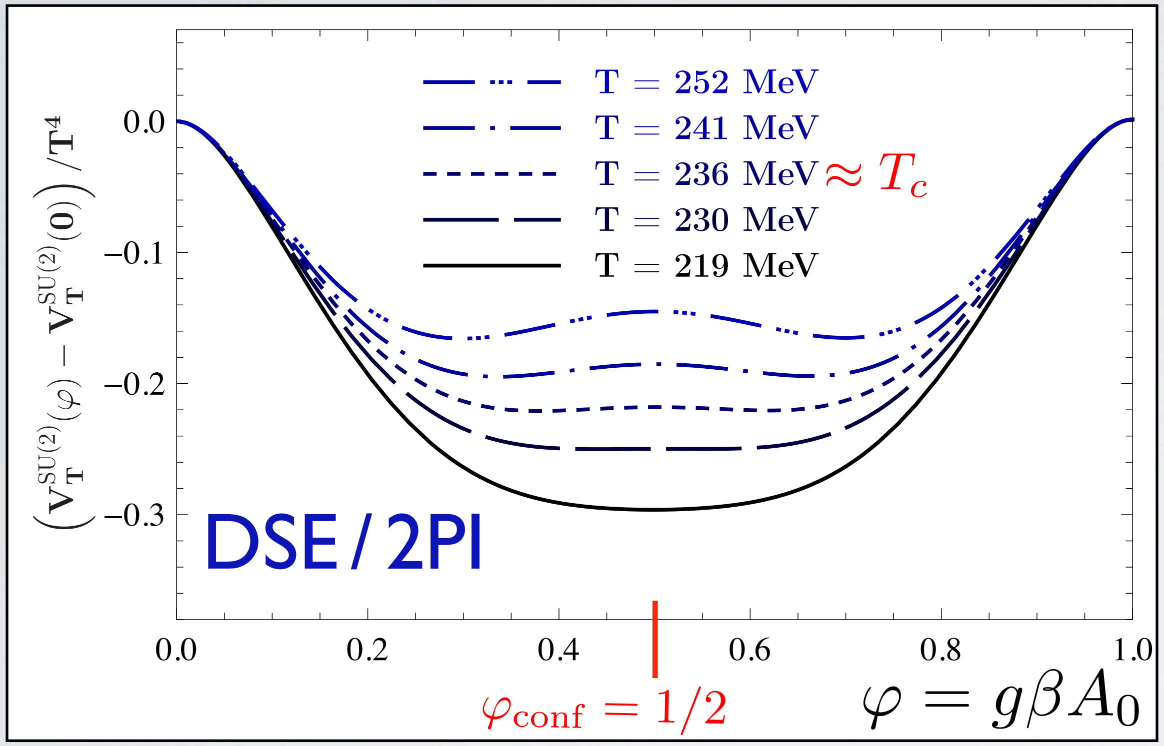
LF, J.M. Pawlowski, arXiv: 1301.#### [hep-ph].

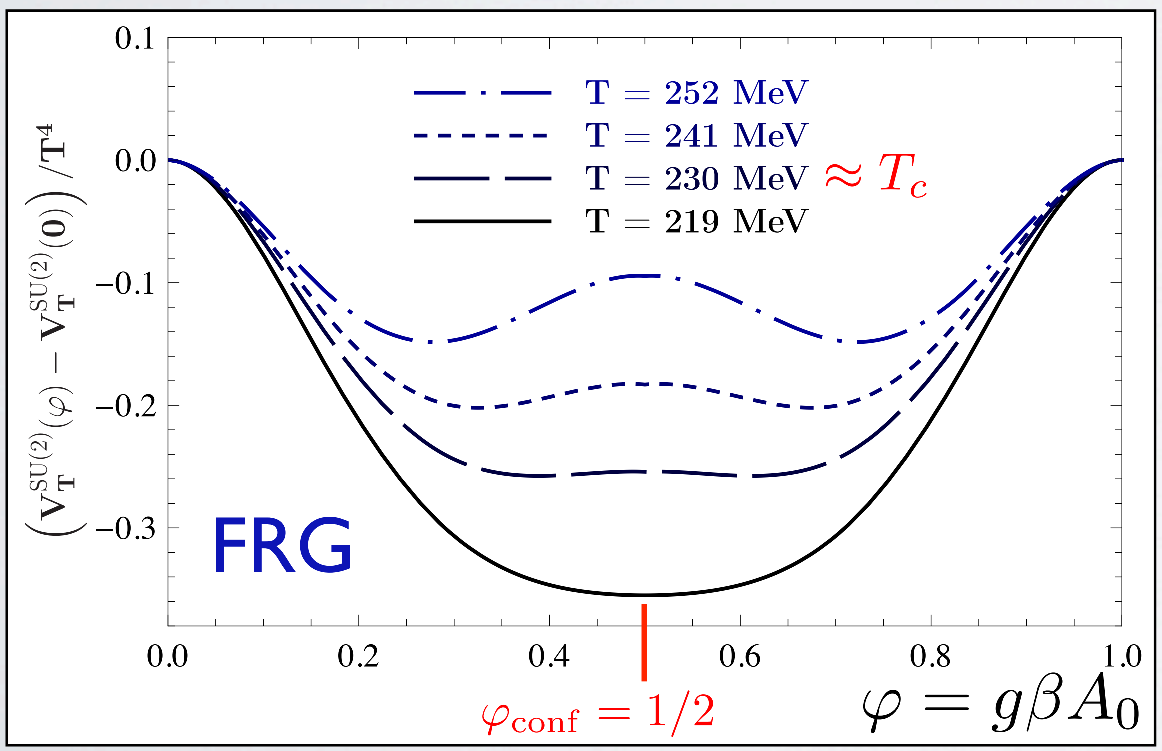
2PI:

for the purpose presented here it is equivalent to the DSE

LF, J.M. Pawlowski, arXiv: 1301.#### [hep-ph].

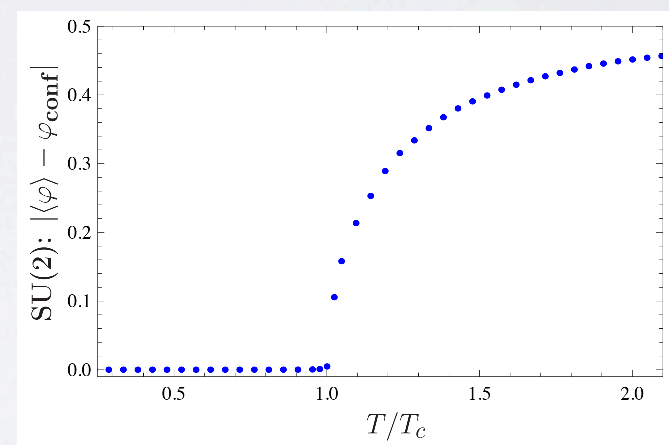
POLYAKOV POTENTIAL - SU(2)





DSE / 2PI	$T_c^{DSE}/\sqrt{\sigma} \approx 0.56$
FRG	$T_c^{FRG}/\sqrt{\sigma} \approx 0.548$
lattice gauge theory*	$T_c^{latt.}/\sqrt{\sigma} \approx 0.709$

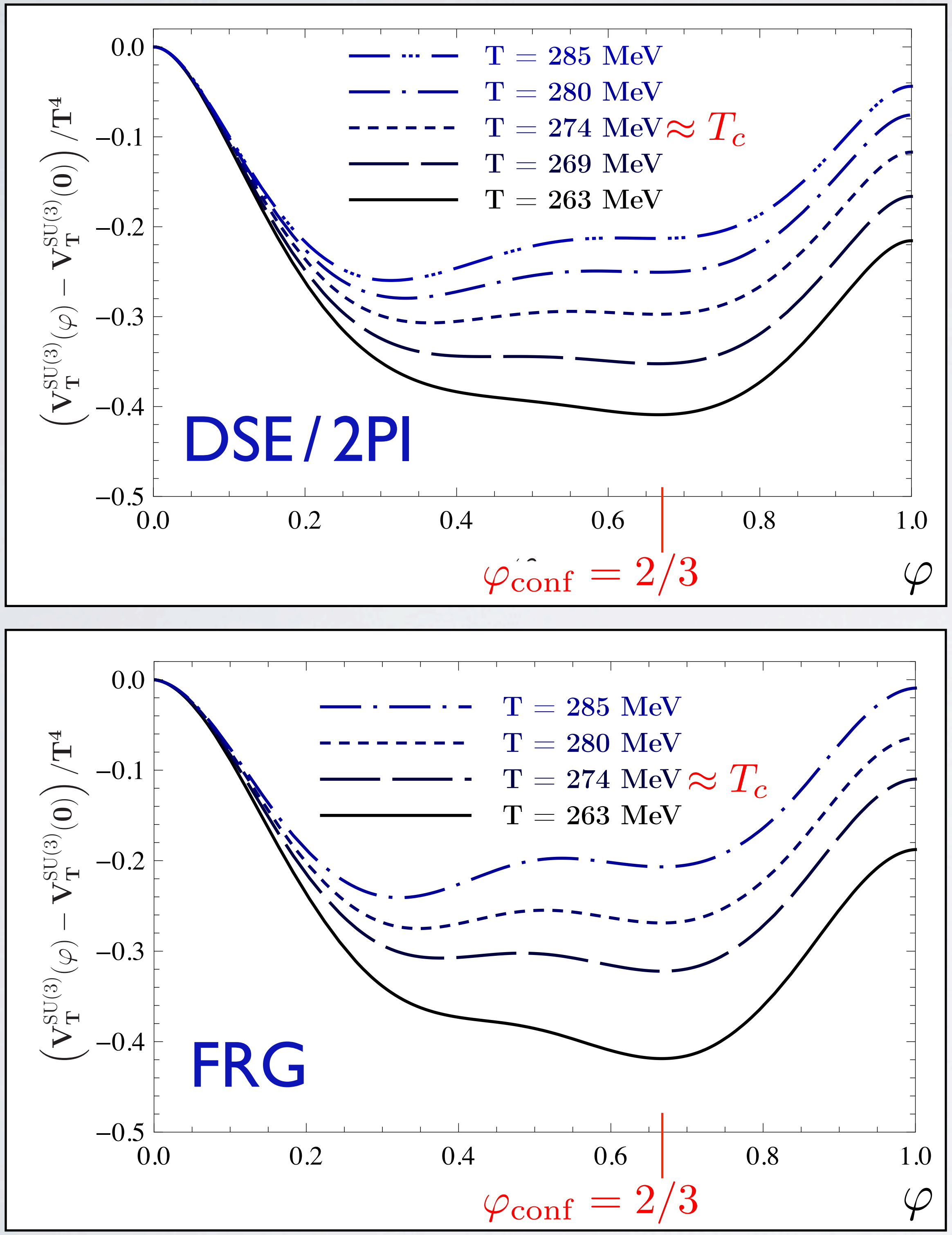
- **minimum moves smoothly** away from $\varphi = 1/2$
  $\longleftrightarrow$ second order phase transition for SU(**2**)



- implicit temperature dependence of propagators has a 10% effect
- not sensitive to scaling/decoupling

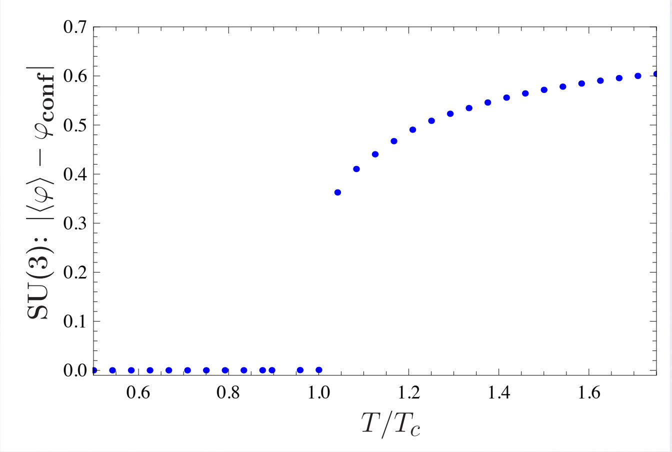
*Lattice data taken from B. Lucini, M. Teper, U. Wenger, JHEP 01, 061 (2004).

POLYAKOV POTENTIAL - SU(3)



DSE / 2PI	$T_c^{DSE}/\sqrt{\sigma} \approx 0.651$
FRG	$T_c^{FRG}/\sqrt{\sigma} \approx 0.655$
lattice gauge theory*	$T_c^{latt.}/\sqrt{\sigma} \approx 0.643$

- **minimum jumps** away from conf. value $\varphi = 2/3$
  $\longleftrightarrow$ first order phase transition for SU(**3**)

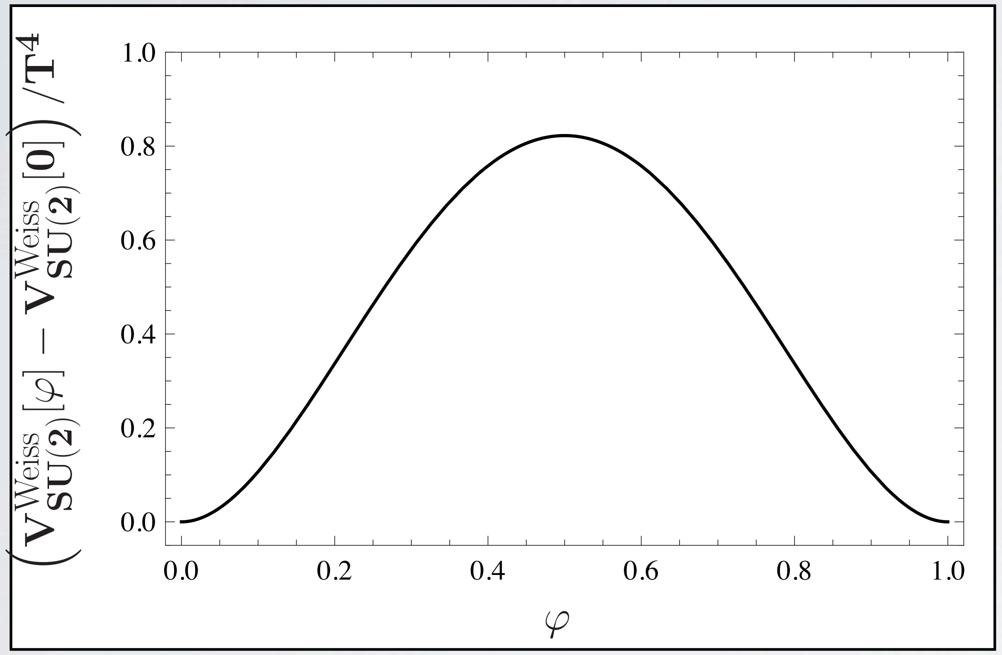


*Lattice data taken from B. Lucini, M. Teper, U. Wenger, JHEP 01, 061 (2004).

CONFINEMENT CRITERION

perturbation theory → **Weiss potential**

N. Weiss, Phys. Rev. D24, 475 (1981).
D.J. Gross, R.D. Pisarski, L.G. Yaffe, Rev. Mod. Phys. 53, 43 (1981).

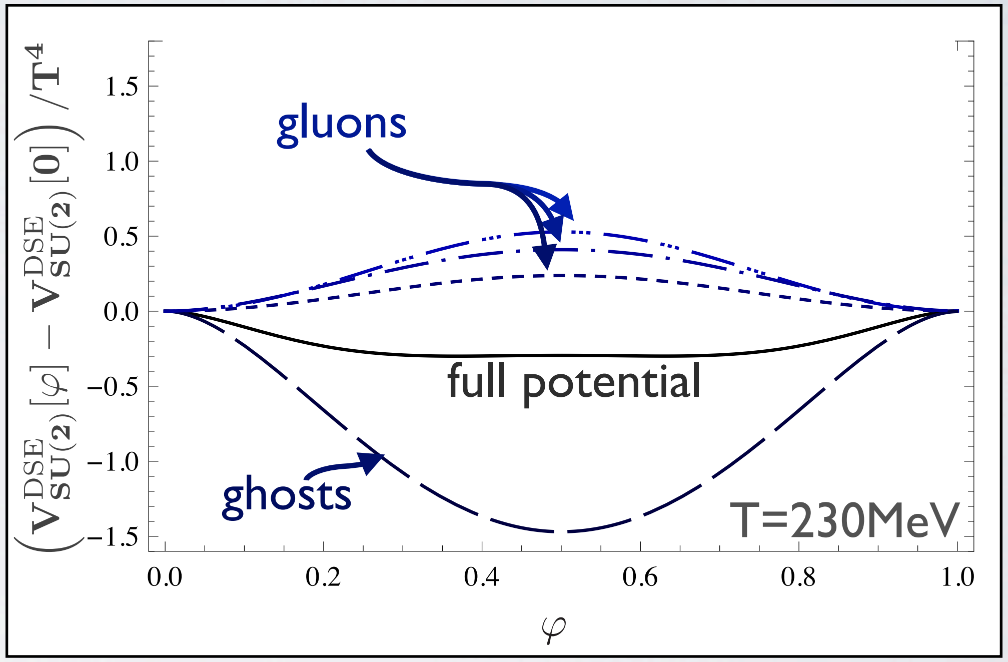


two (transversal) gluonic modes, others cancel exactly

minima at **integer** values of $\varphi = g\beta A_0$
confining value of φ at 1/2,
$\rightsquigarrow$ **no confinement in perturbation theory**

non-perturbatively

gluon suppression
ghost enhancement



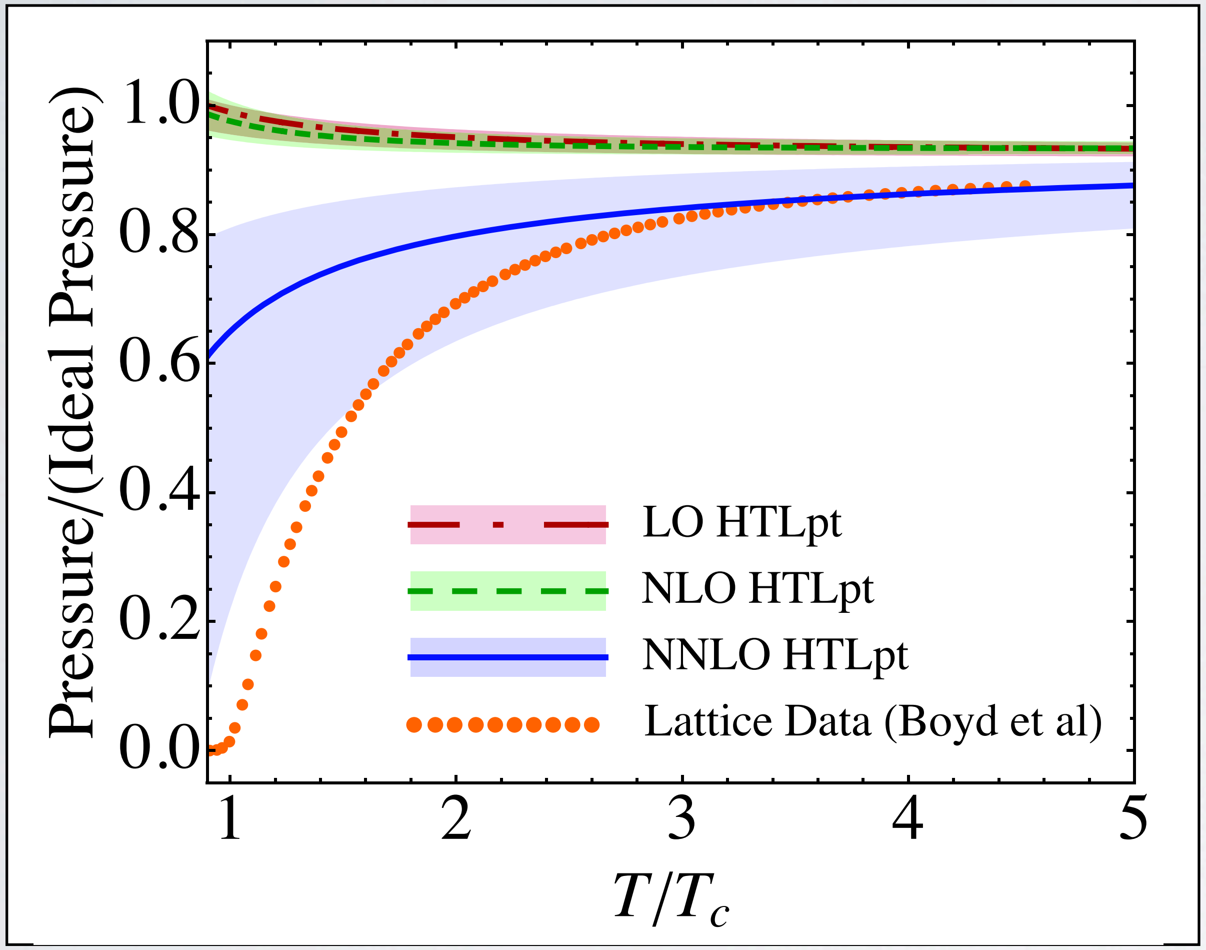
gluon modes have **positive** contributions,
ghost modes have **negative** contributions,

no exact cancellation of modes,
ghosts dominate at small temperatures
$\rightsquigarrow$ **confinement at small temperatures**

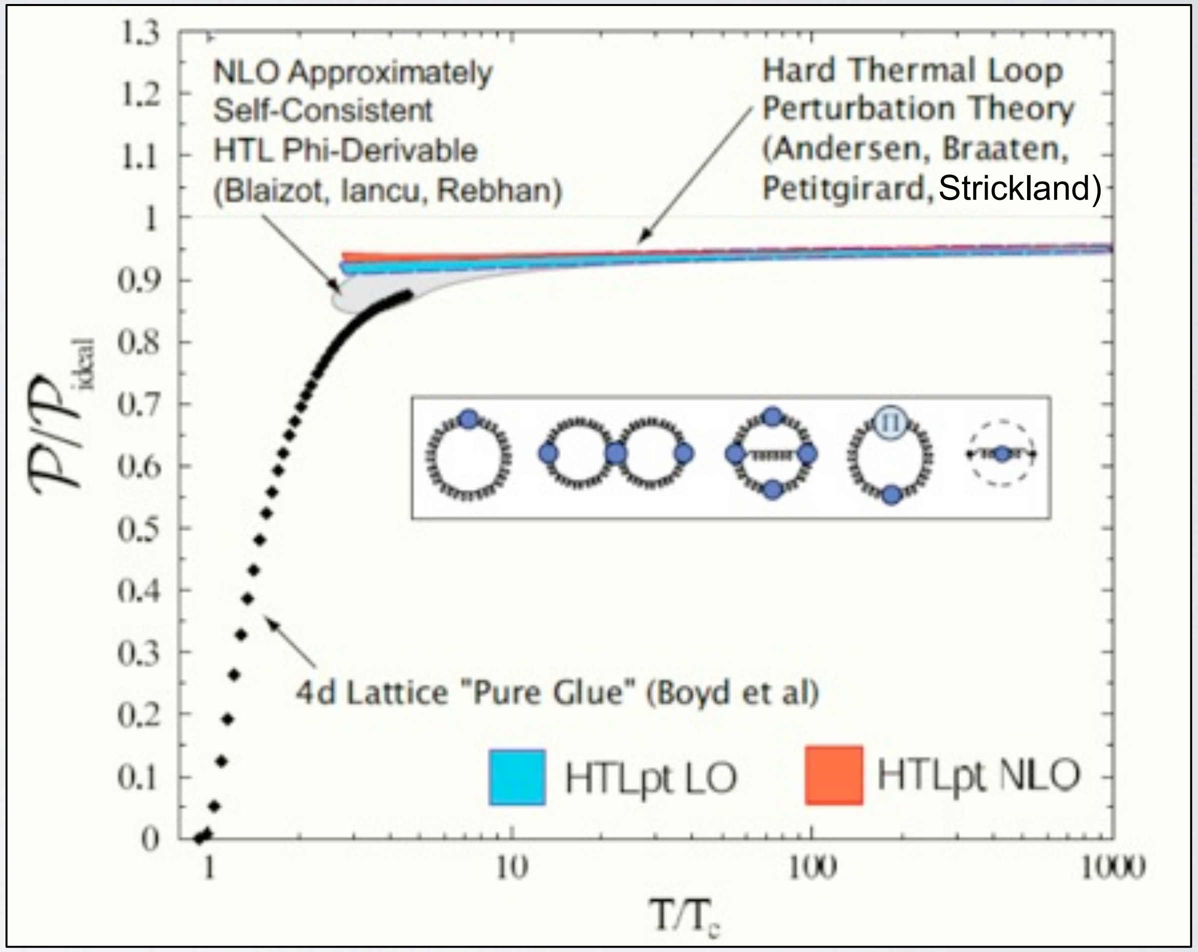
confinement criterion:
infrared **suppressed gluons** and **non-suppressed ghosts** → **confinement**

Yang–Mills Thermodynamics

PRESSURE FROM OTHER METHODS



J.O. Andersen, M. Strickland, N. Su, Phys. Rev. Lett. 104 (2010).



from a talk by M. Strickland

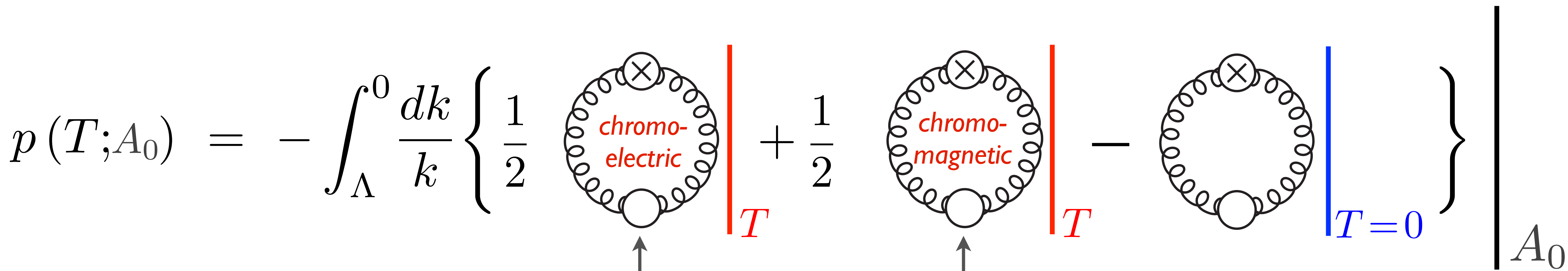
PRESSURE FROM THE FRG

The thermal pressure p is the effective action evaluated on the EoM, normalised in the vacuum.

$$p_k\,(T;A_0) = -\Delta\Gamma_{k,T}(A_0) \quad = -\left(\Gamma_{k,T} - \Gamma_{k,T=0}\right)$$

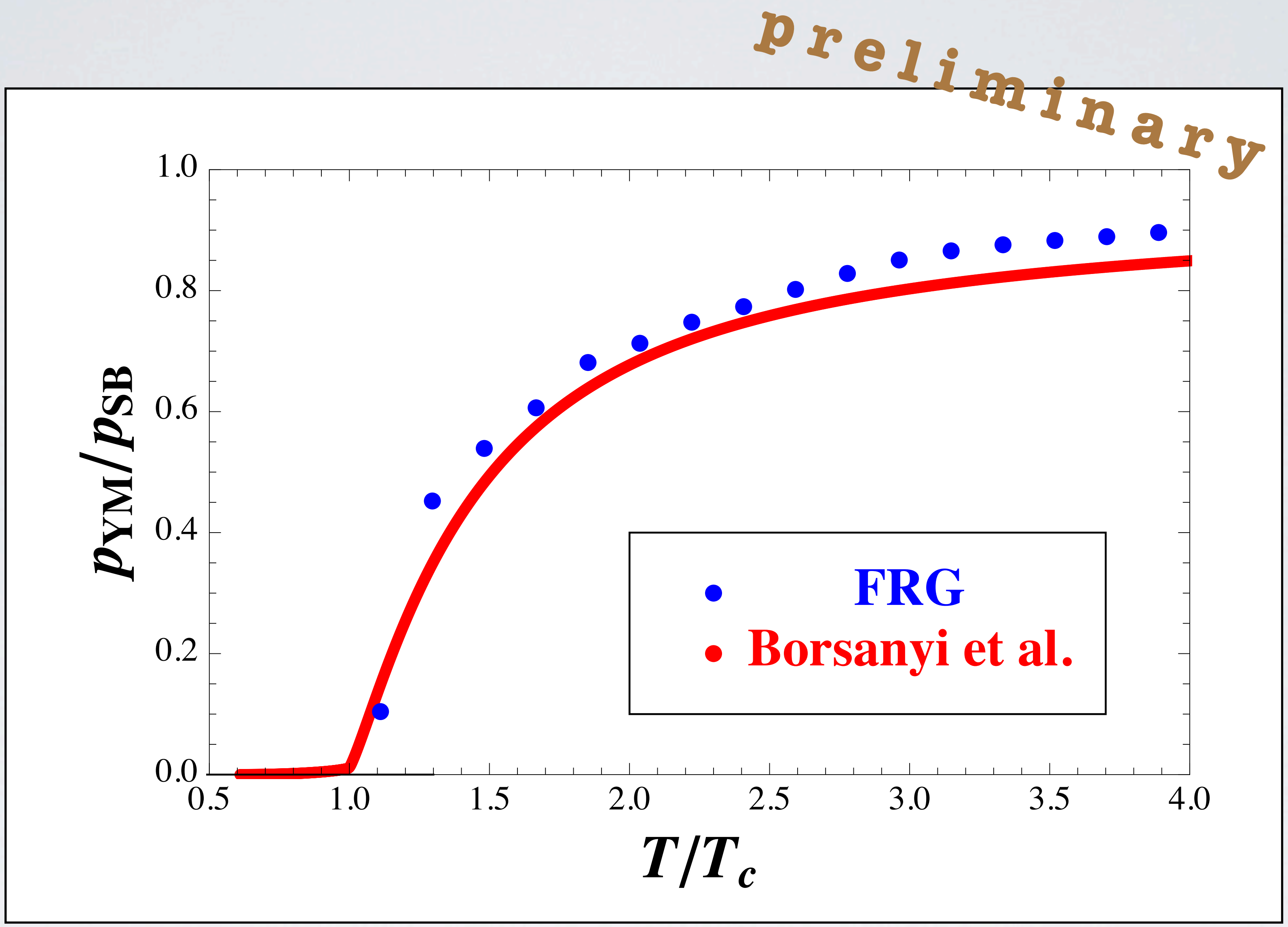
projection onto physical subspace

- one chromoelectric +
- one chromomagnetic mode



Polyakov loop potential is crucial for the critical physics.
Implicit temperature dependence of the propagators for quantitative accuracy.

PRESSURE



lattice data from
S. Borsanyi, G. Endrodi, Z. Fodor, S. Katz and K. Szabo, JHEP 1207, 056 (2012).

Polyakov loop potential is crucial for the critical physics.
Implicit temperature dependence of the propagators for quantitative accuracy.

CONCLUSIONS

Functional methods allow to study the **QCD phase diagram.**

functional renormalisation group, Dyson–Schwinger equations (, 2PI-techniques)

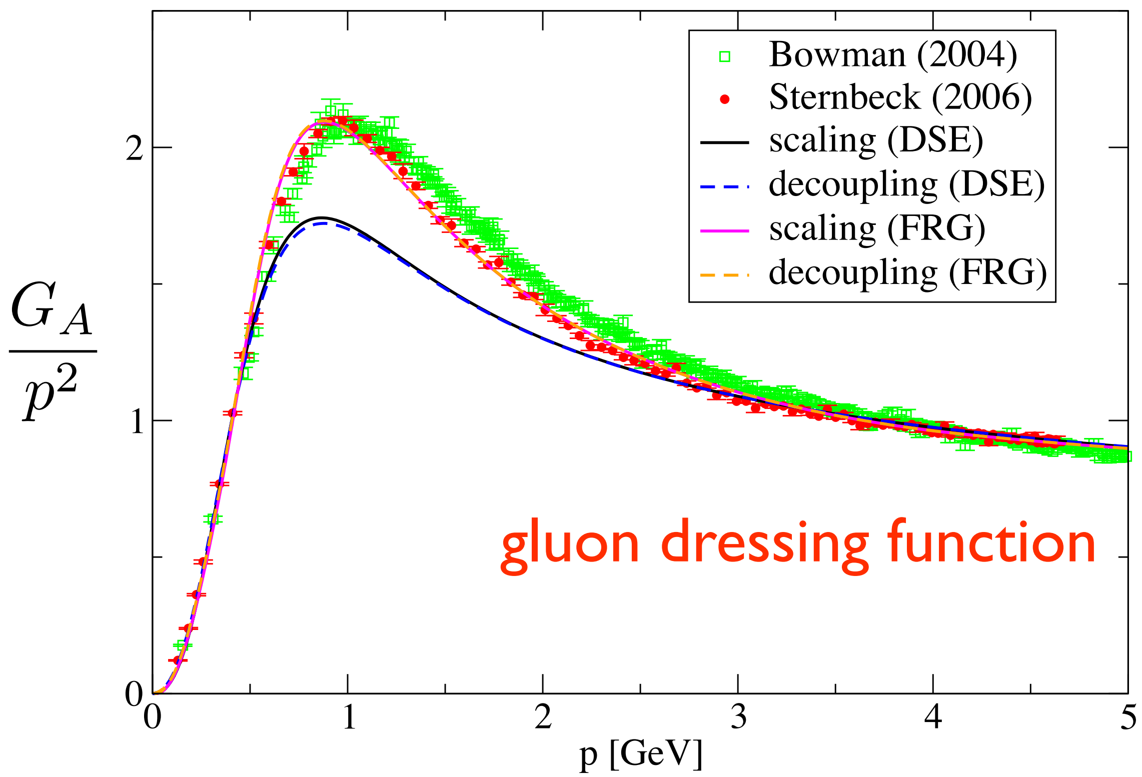
- Thermal Propagators:
  - **Temperature dependence** is **crucial** for confinement and **thermodynamics.**
  - Chromo**magnetic** gluon matches lattice data at all temperatures.
  - Chromo**electric** —— " —— , except around $T \approx T_c$.

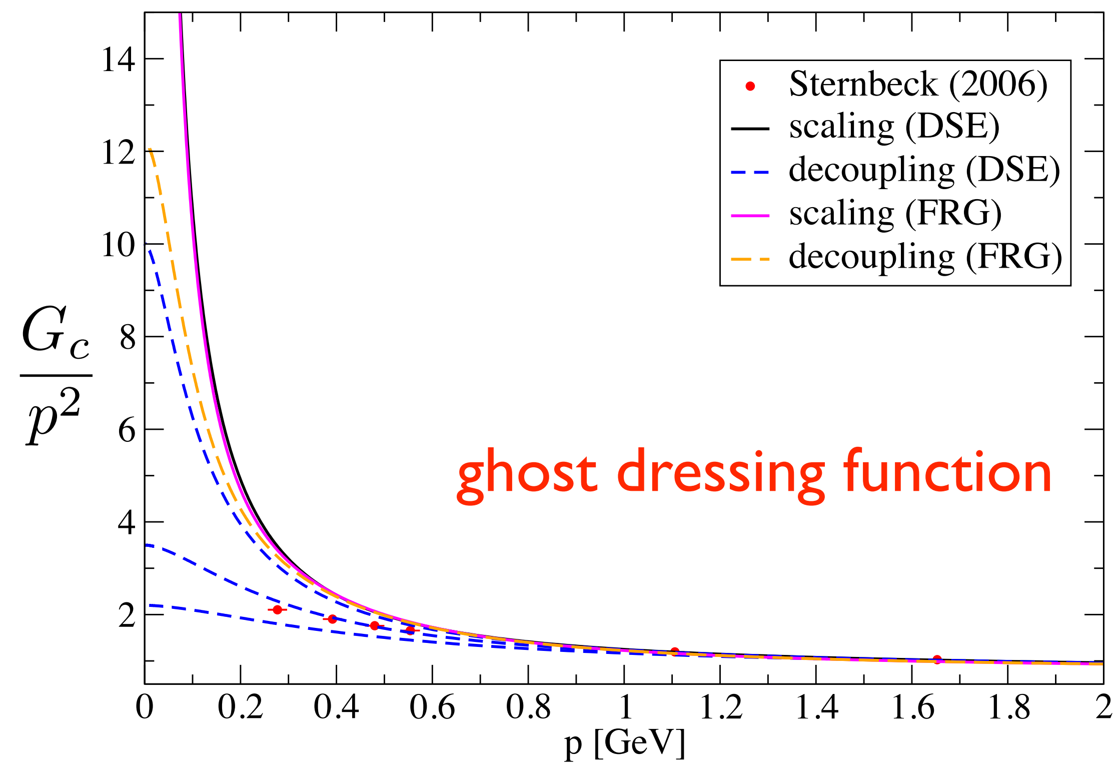
- Confinement:
  - second order phase transition for SU(2), first order for SU(3)
  - critical temperatures $\frac{T_c^{\mathrm{SU(2)|SU(3)}}}{\sqrt{\sigma}} \approx \begin{cases} .55 \;|\; .65 & \text{functional methods} \\ .709 \;|\; .646 & \text{lattice gauge theory} \end{cases}$
  - **Criterion** for confinement: gluons must be IR suppressed, while ghosts must *not.*

- Thermodynamics:
  - **Pressure** at all temperatures, **even for** $T \lesssim 3\,T_c$.

. . . supplementary material

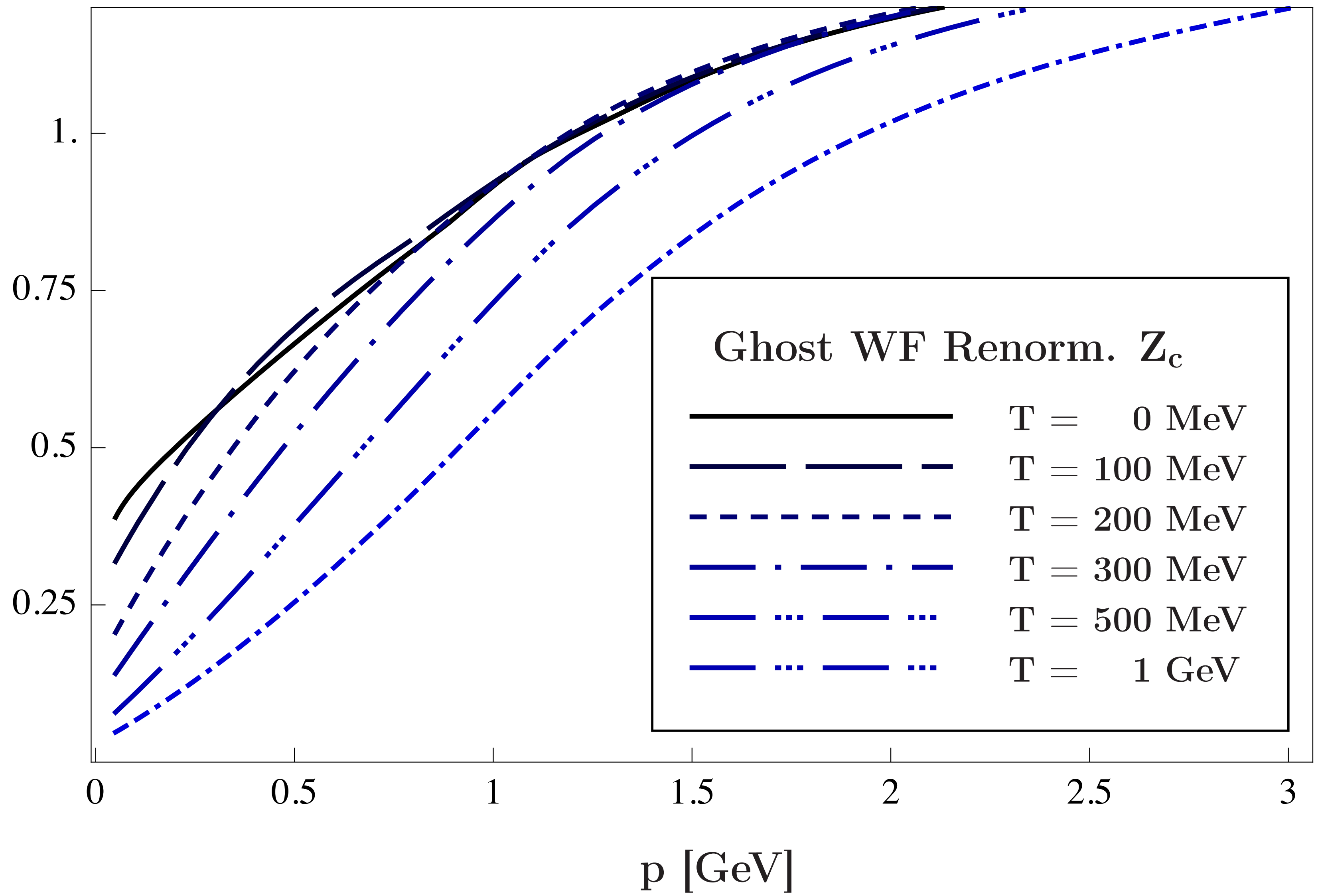
T=0 Propagators





taken from: C.S. Fischer, A. Maas, J.M. Pawlowski, Annals Phys. 324 (2009).

Ghost Wave-Function Renormalisation

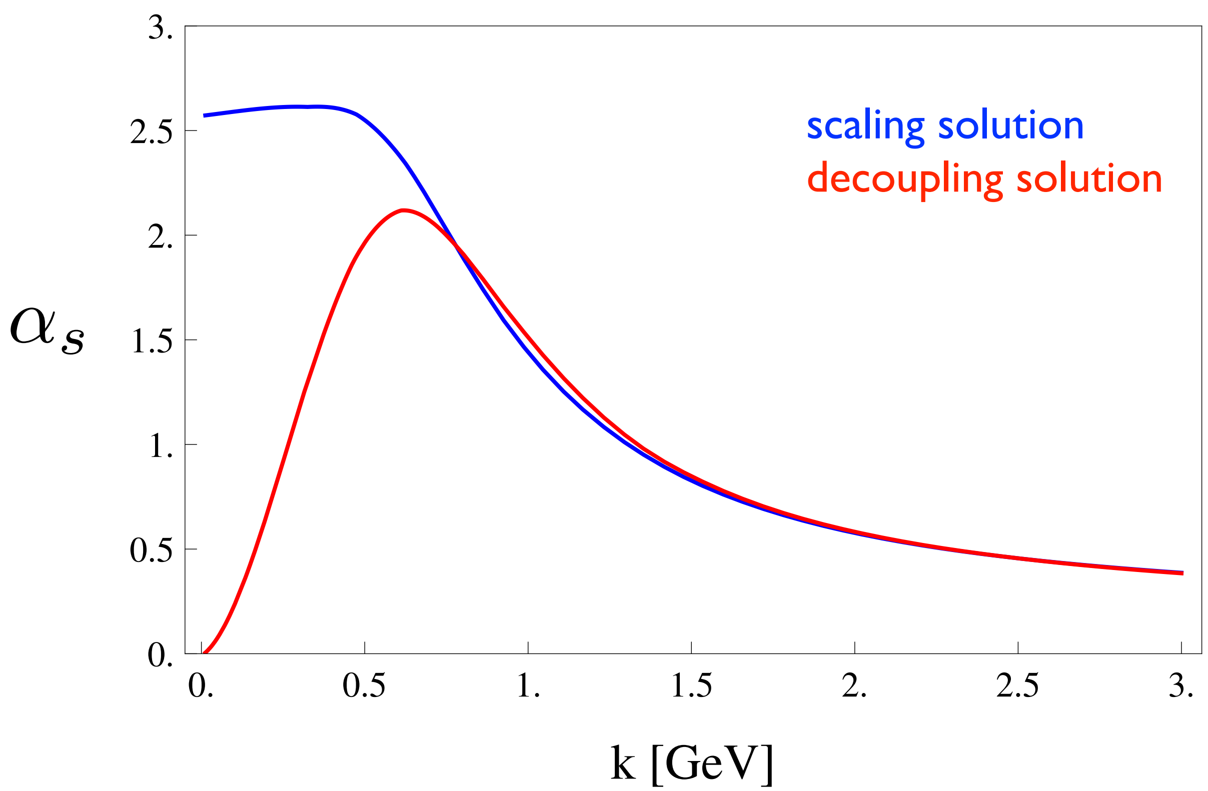


$$Z_c = \frac{1}{p^2\, G_c}$$

Coupling at Vanishing Temperature

L. von Smekal, A. Hauck, R. Alkofer, Ann. Phys. **267** (1998) 1.
C. Lerche, L. von Smekal, Phys. Rev. **D65** (2002) 125006.

J.M. Pawlowski, D.F. Litim, S. Nedelko, L. von Smekal, Phys. Rev. Lett. **93** (2004) 152002.
C.S. Fischer, H. Gies, JHEP **10** (2004) 048.

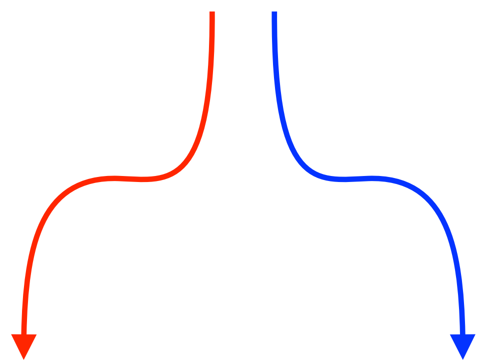


definition of the running coupling from the ghost-gluon vertex:

$$\alpha_s(k) = \frac{g_0^2\, Z^2_{\bar{c}Ac}(k)}{4\pi}\, k^6 G_c^2(k^2)\, G_A(k^2)$$

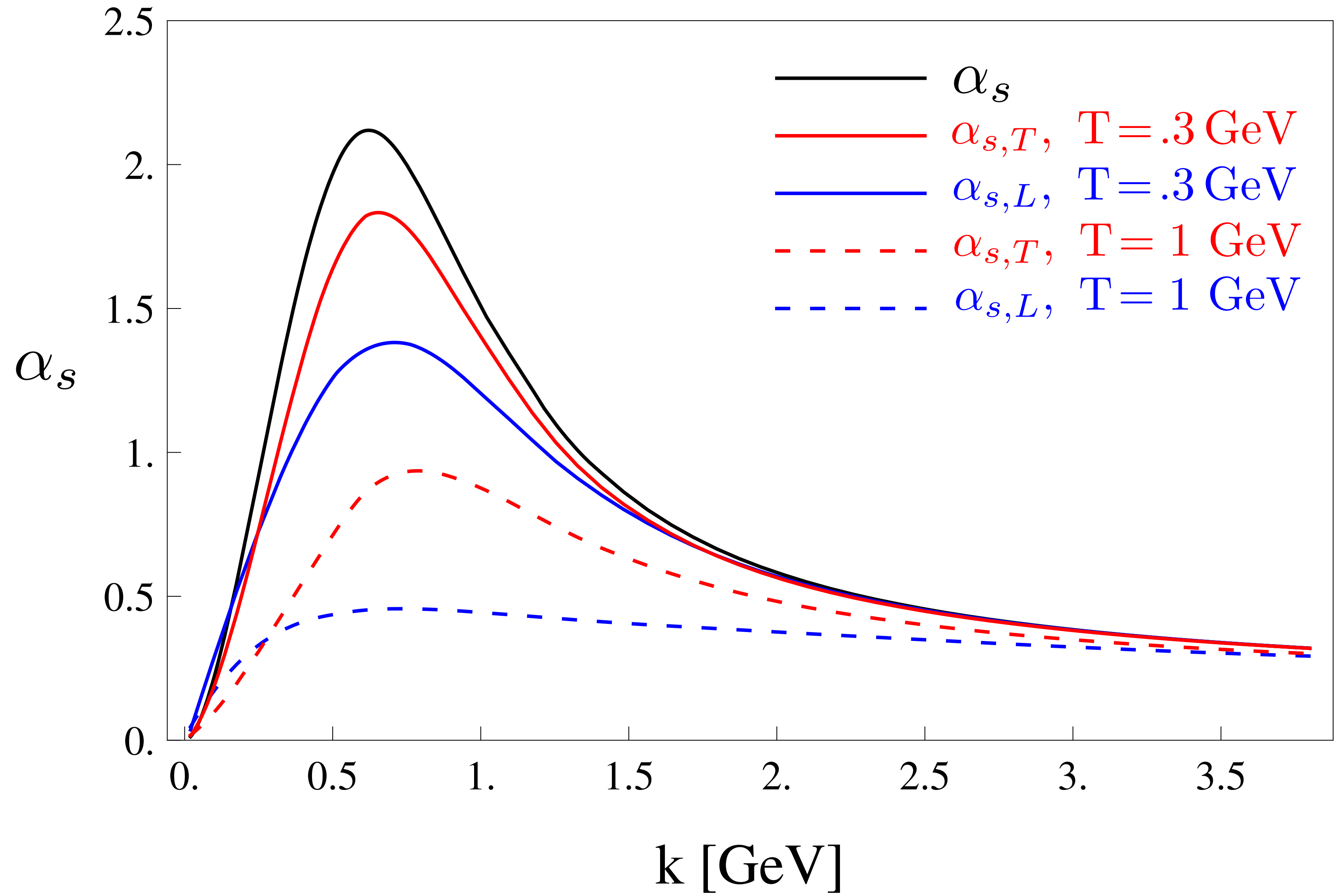
Coupling at Non-Vanishing Temperature

The coupling depends on the gluon propagator.



transversal coupling $\alpha_{s,T}$

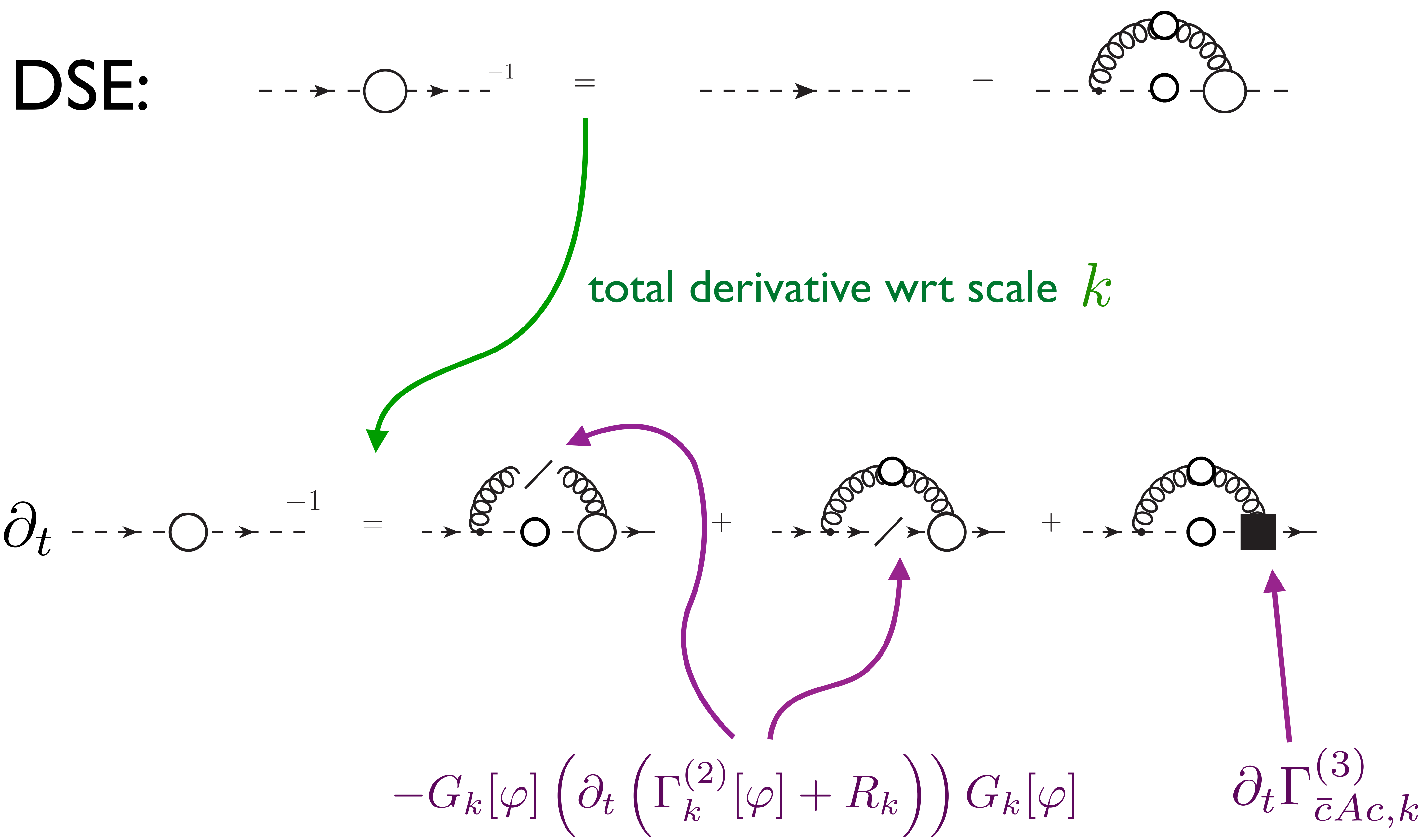
longitudinal coupling $\alpha_{s,L}$



Dyson–Schwinger Approximation for the Ghost

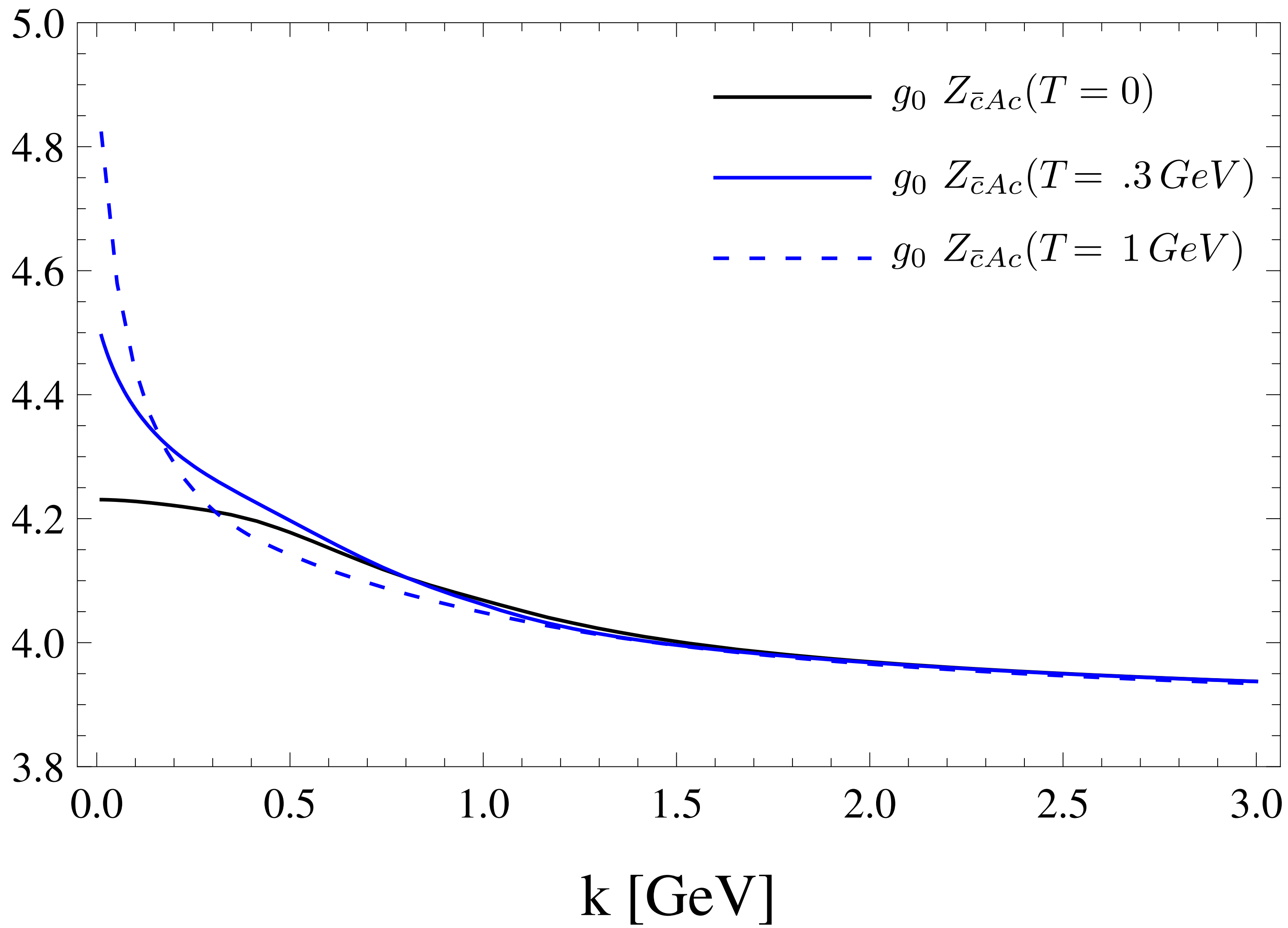
Use:

The flow equation is the differential form of the Dyson–Schwinger equation.



Temperature Effects on the Ghost-Gluon Vertex

Mild temperature effect on the vertex below the scale $2\pi T$.



Gluonic Vertices — Ansätze vs. Computation

ansatz

$$\Gamma^{(3)}_{A^3} \sim S^{(3)}_{A^3}\Big|_{g=1} \sqrt{4\pi\alpha_s}\, (\mathbf{Z_A})^{\frac{3}{2}}$$

$$\Gamma^{(4)}_{A^4} \sim S^{(4)}_{A^4}\Big|_{g=1} 4\pi\alpha_s\, (\mathbf{Z_A})^{\mathbf{2}}$$

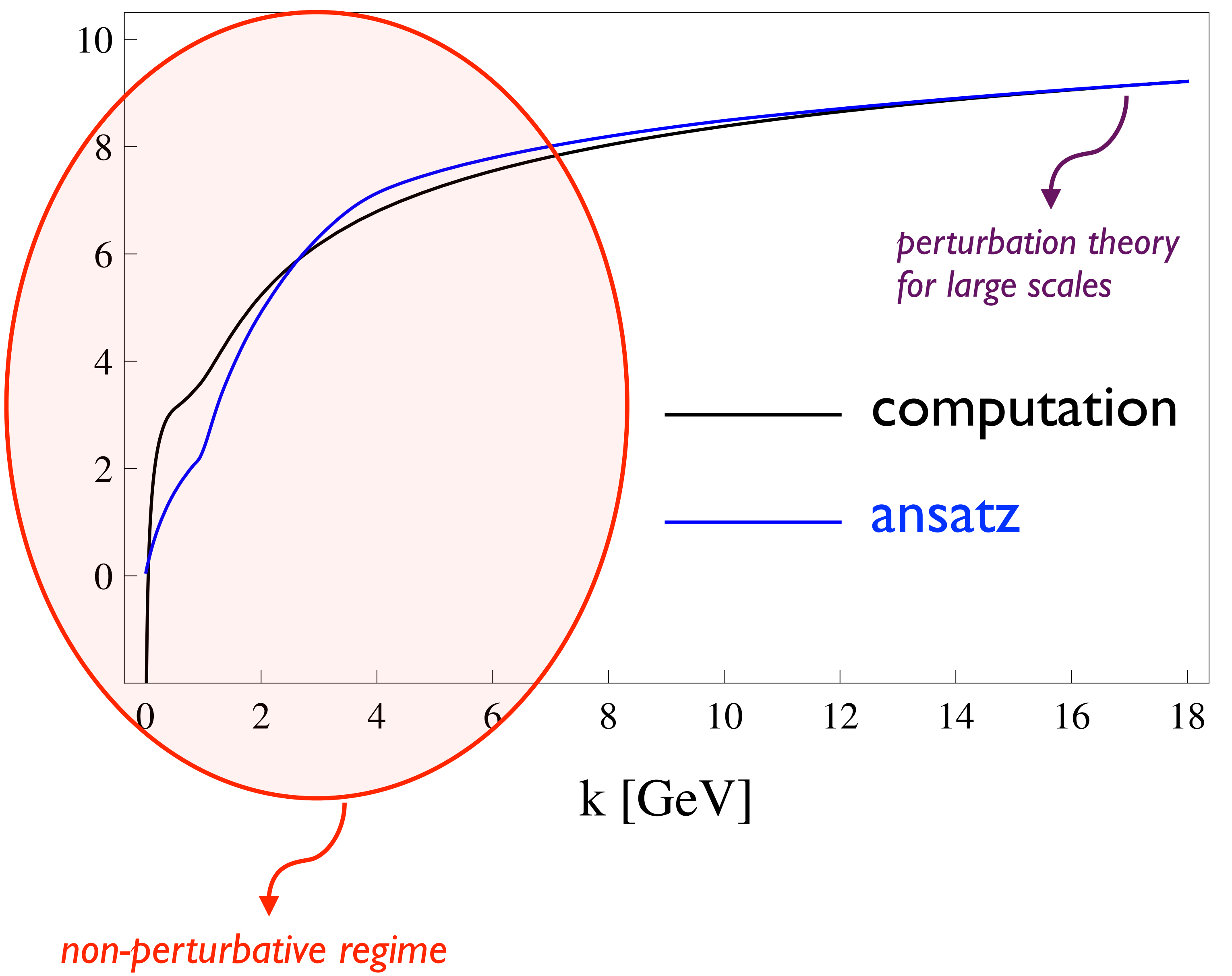
vertex $\sim$ class. tensor–struct.

$\times$ non–pert. runn. coupl.

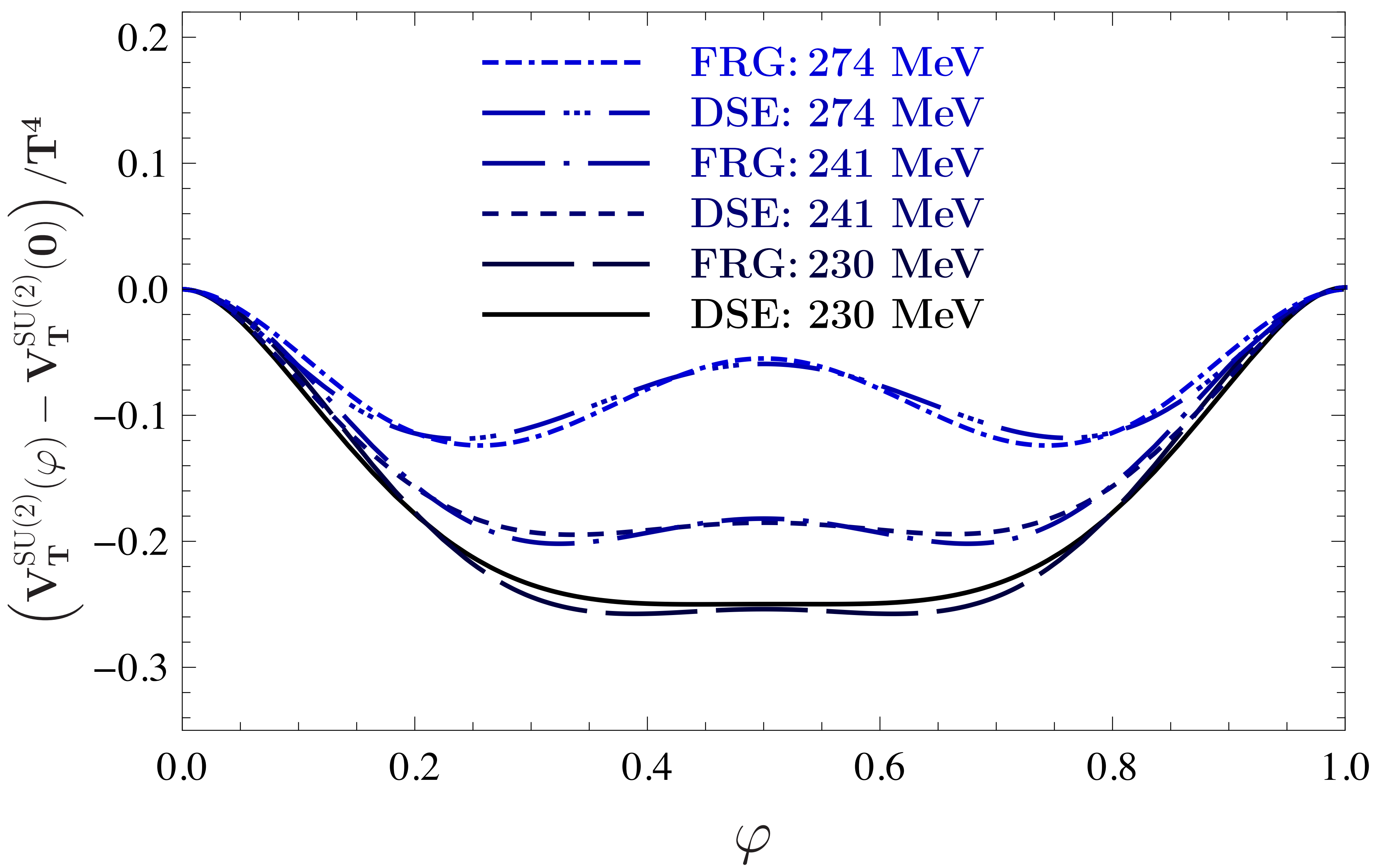
$\times$ **RG running**

trigluon vertex $\Gamma^{(3)}_{A^3}$

(Ansatz vs. Computation)

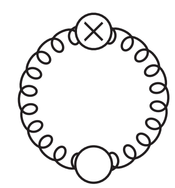
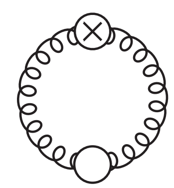


Polyakov Loop Potential — Amplitude DSE/2PI/FRG

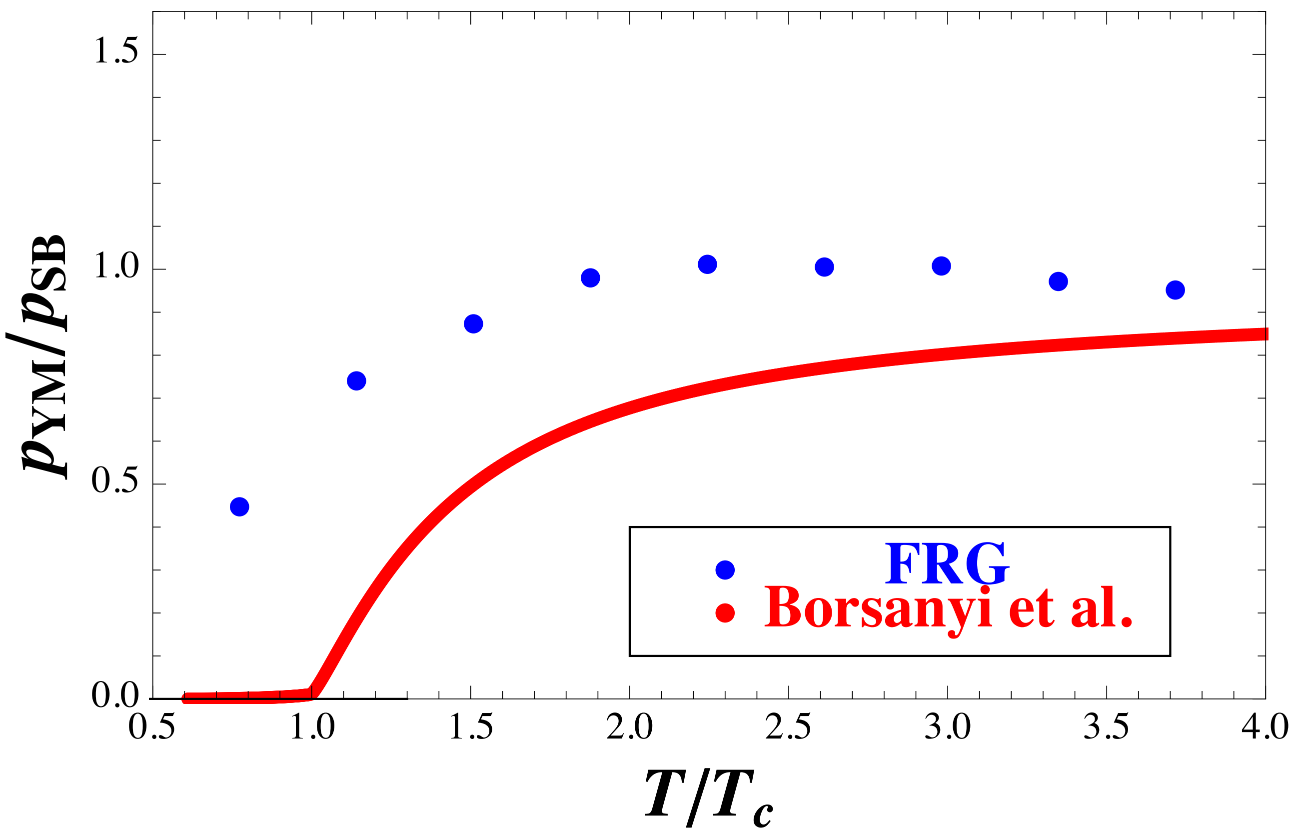


Pressure with T=0 props

preliminary

$$-p\,(T) \;=\; \int_\Lambda^0 \frac{dk}{k}\Bigg\{$$  $$\Big|_T \;-\;$$  $$\Big|_{T=0}\Bigg\}$$

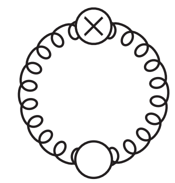
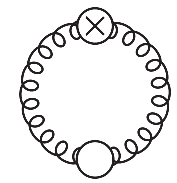
$$\sum\!\!\!\!\!\!\int_p G_{\mathbf{T=0},k}\;\partial_t R_k \qquad\qquad \int_p G_{T=0,k}\;\partial_t R_k$$



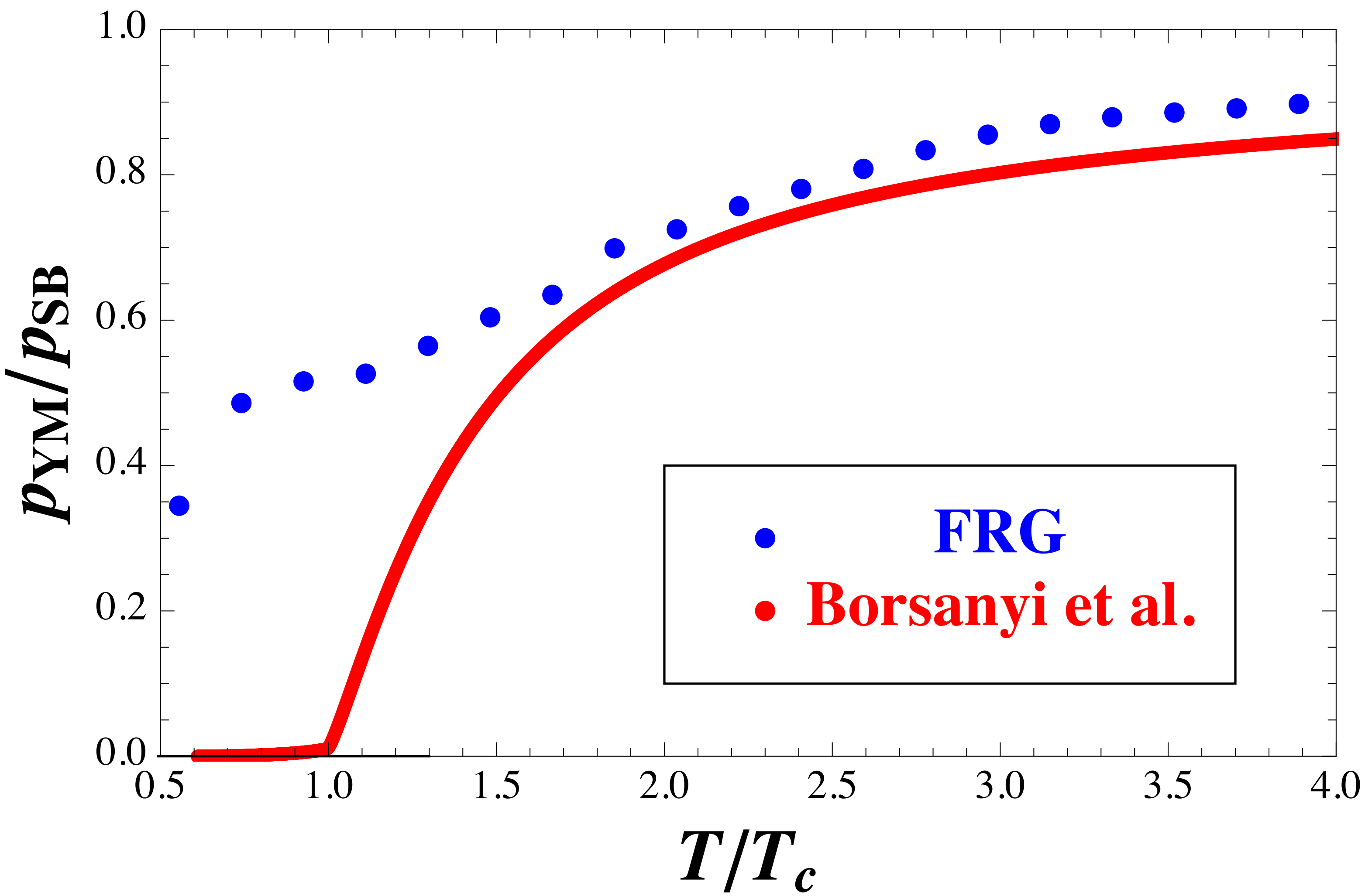
Temperature dependence of the propagators is important!

Pressure **without** Polyakov Loop

preliminary

$$-p(T) = \int_\Lambda^0 \frac{dk}{k}\Bigg\{$$  $$\Big|_T -$$  $$\Big|_{T=0}\Bigg\}$$

$$\sum\!\!\!\!\!\!\!\int_p G_{T,k}\, \partial_t R_k \qquad\qquad \int_p G_{T=0,k}\, \partial_t R_k$$



Polyakov loop potential crucial for critical physics.