# For a Few Zero Modes More

$$\not{D}\,\psi = 0$$

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Zero Modes - solutions of Dirac-Weyl Equation $\not{D}\psi = 0$ specialise to self-dual backgrounds

i) Nahm transform

ii) B. Cheng and CF, zero modes for abelian BPS monopoles

iii) Jackiw-Rebbi mechanism

Self duality 4 d Euclidean

Space coords $x_0, x_1, x_2, x_3$

Freeze one coord, say $x_0$

SD → BPS equations defining monopoles

Freeze two coords, $x_0$ and $x_3$

SD → Hitchin equations

$$[\bar{D}, \Phi] = 0$$

$$[D, \bar{D}] = [\Phi, \bar{\Phi}]$$

$\Phi = A_0 + i A_3$ complex Higgs

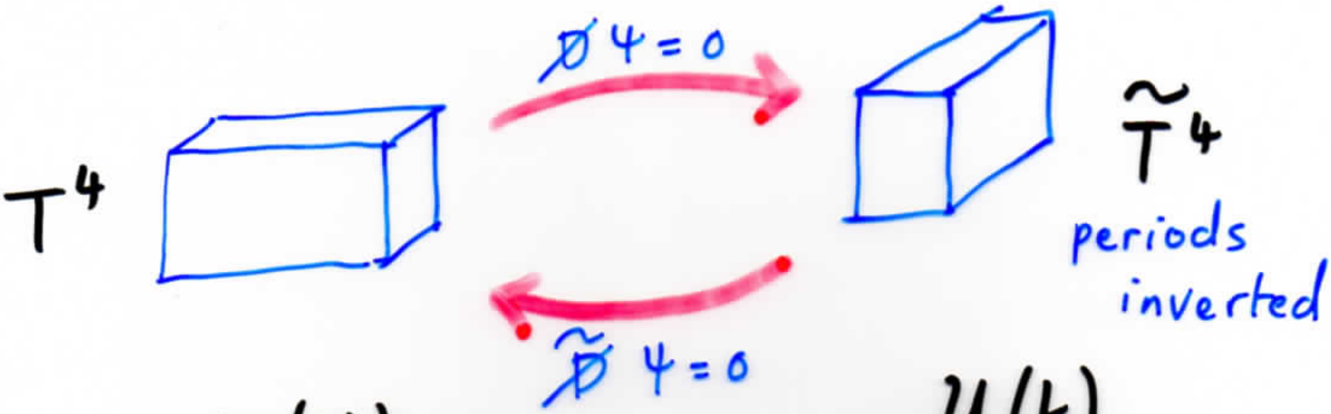
$D = D_1 + i D_2$

Freeze 3 coords

SD → Nahm equations

# $T^4$ Nahm Transform

(Braam + van Baal)




| | |
|---|---|
| $U(N)$ | $U(k)$ |
| top charge $k$ | top charge $N$ |
| $A_\mu(x)$ | $\hat{A}_\mu(z)$ |
| $x \in T^4$ | $z \in \tilde{T}^4$ |

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If $k=1$ dual potential abelian ?

Taking Periods $\to \infty$ Dual

Periods $\to 0$

$T^4 \longleftrightarrow \tilde{T}^4$

$T^3 \times R \longleftrightarrow \tilde{T}^3$ BPS

$T^2 \times R^2 \longleftrightarrow \tilde{T}^2$ Hitchin

doubly-periodic

$S^1 \times R^3 \longleftrightarrow \tilde{S}^1$ Nahm

calorons

$R^4 \longleftrightarrow \bullet$

For charge $k=1$

solutions dual potential

$\hat{A}$ abelian

Freeze a coordinate $\equiv$ set a period to zero dual period $\to \infty$

$T^3 \longleftrightarrow \tilde{T}^3 \times R$

$T^2 \times R \longleftrightarrow \tilde{T}^2 \times R$

doubly periodic monopole | doubly periodic monopole

$S^1 \times R^2 \longleftrightarrow \tilde{S}^1 \times R$

periodic monopole | Hitchin

$R^3 \longleftrightarrow R$

BPS monopole | Nahm

Again for charge $k=1$ $\hat{A}$ abelian

Freeze two coordinates $\equiv$ set two periods zero

$$T^2 \longleftrightarrow \tilde{T}^2 \times R^2$$

$$S^1 \times R \longleftrightarrow \tilde{S}^1 \times R^2$$

$$R^2 \longleftrightarrow R^2$$

Hitchin Hitchin

Again for $k=1$

$\hat{A}$ abelian

Cases where dual theory gives Hitchin equations:

$$T^2 \times R^2 \longleftrightarrow \tilde{T}^2 \qquad S^1 \times R^2 \longleftrightarrow \tilde{S}^1 \times R$$

$$R^2 \longleftrightarrow R^2$$

Hitchin Equations

$$[\bar{D}, \Phi] = 0 \qquad [D, \bar{D}] = [\Phi, \bar{\Phi}]$$

In abelian case they collapse to

$$\partial_{\bar{z}} \Phi = 0 \qquad [D, \bar{D}] = 0$$

$\Phi$ analytic (or meromorphic)

DP instantons CF and J.M. Pawlowski

$$T^2 \times R^2 \longleftrightarrow \tilde{T}^2$$

Periodic Monopoles Cherkis and Kapustin

$$A_1 = A_2 = 0 \qquad \Phi(z) \overset{!}{=} \lambda \sinh z$$

$\lambda$ complex constant

$$\Big\downarrow \text{NT}$$

smooth periodic monopole
charge 1 !!

Need Nahm zero modes ...

## A Periodic Monopole via the Nahm Transform

Weyl operators:

$$-\frac{i}{2}D = \begin{pmatrix} -\frac{1}{2}(x_0 + ix_3) - \Phi(z) & \partial_z - \frac{1}{2}x_1 \\ \partial_{\bar{z}} + \frac{1}{2}x_1 & -\overline{\Phi(z)} - \frac{1}{2}(x_0 - ix_3) \end{pmatrix}$$

$$\frac{i}{2}D^\dagger = \begin{pmatrix} -\frac{1}{2}(x_0 - ix_3) - \overline{\Phi(z)} & -\partial_z + \frac{1}{2}x_1 \\ -\partial_{\bar{z}} - \frac{1}{2}x_1 & -\Phi(z) - \frac{1}{2}(x_0 + ix_3) \end{pmatrix}.$$

Constants $x_1$, $x_0$ and $x_3$ are the coordinates of $S^1 \times R^2$

$x_1$ is the periodic coordinate : $x_1 \equiv x_1 + 1$.

$\Phi(z) = \lambda \sinh z$ (Cherkis and Kapustin).

To construct periodic monopole find two normalisable zero modes for $D^\dagger$ for each point $(x_1, x_0, x_3) \in S^1 \times R^2$

Easier case. Take $\Phi(z) = \frac{1}{2}e^z$. Here $D^\dagger$ has *one* zero mode. This gives a $U(1)$ periodic monopole of unit charge.