Thermal crossover with $N_f = 2$ twisted mass quarks : fixing the transition temperature(s) and measuring the gluon and ghost propagators at the crossover

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for the tmfT Collaboration

Δ-Meeting on QCD, Nonequilibrium Dynamics,
Complex Systems and Simulation Methods

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Outline:

1. Motivation
2. Introduction: Twisted Mass
3. Recalling the Global Phase Structure
4. Simulation Setup for the Crossover Temperature(s)
5. Towards the Chiral Limit
6. Gluon and Ghost Propagators:
   Unquenched with Twisted Mass Quarks
7. Conclusions and Outlook

1. Motivation

- **Thermodynamic simulations mostly with staggered fermions (computationally least demanding !)**
- **Unsolved problems: rooting, locality, flavor symmetry ?**
- **Necessary to test/compare various fermion discretization schemes.**
- **Wilson fermions have a clear flavor interpretation !**
- **Wilson fermions break chiral symmetry explicitly.**
- **Wilson fermions have a subtle chiral behavior and a complicated phase structure, both at $T = 0$ and at finite teperature.**
- **Wilson fermions are very sensitive to finite a effects.**

2. Introduction: twisted mass

Improvement necessary. Two methods for Wilson fermions:

1. **Clover improvement (for finite T, see DIK Collaboration)**
2. **Twisted mass improvement**

What is twisted mass ?

Wilson twisted mass action for the fermion sector in the twisted basis :

$$S_F[\psi\,,\bar{\psi}\,,U] = a^4 \sum_x \left[\bar{\psi}(x)\left(D_W + m_0 + i\mu_0\gamma_5\tau^3\right)\psi(x)\right]$$

where $D_W = \gamma_\mu \frac{1}{2a}\left(\nabla_\mu + \nabla^*_\mu\right) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu$ **is the Wilson-Dirac operator.**

Can be rewritten as

$$S_F[\Psi\,,\bar{\Psi}\,,U] = a^4 \sum_x \Big[\bar{\Psi}(x)\big(\gamma_\mu \frac{1}{2a}\left(\nabla_\mu + \nabla^*_\mu\right) - \frac{ar}{2}\mathrm{e}^{-i\omega\gamma_5\tau^3}\nabla^*_\mu\nabla_\mu + M_0\big)\Psi(x)\Big]$$

in the physical basis $\bar{\Psi}$, Ψ, which is related to the twisted basis $\bar{\psi}$, ψ by the non-anomalous chiral rotation

$$\Psi = \exp\left(i\frac{\omega}{2}\gamma_5\tau^3\right)\psi\ ,$$
$$\bar{\Psi} = \bar{\psi}\exp\left(i\frac{\omega}{2}\gamma_5\tau^3\right)$$

with

$$M_0 = \sqrt{m_0^2 + \mu_0^2}\,, \qquad \tan(\omega) = \mu_0/m_0\ ,$$

the bare polar quark mass and bare twist angle.

For maximal twist related through :

$$\Psi = \frac{1}{\sqrt{2}}(1 + i\gamma_5\tau^3)\psi \quad \textbf{and} \quad \bar{\Psi} = \bar{\psi}\frac{1}{\sqrt{2}}(1 + i\gamma_5\tau^3)$$

Introducing Wilson's hopping parameter $\kappa = 1/(2am_0 + 8r)$ **and rescaling as usual** $\psi \rightarrow \sqrt{a^3/(2\kappa)}\psi$ **leads to the standard form of twisted fermion action appearing in the simulation code :**

$$S_F[\chi\,,\bar{\chi}\,,U] = \sum_x \Big[\bar{\chi}(x)\left(1 + i2\kappa a\mu_0\gamma_5\tau^3\right)\chi(x)$$
$$- \kappa \sum_\mu \bar{\chi}(x)\Big((r-\gamma_\mu)U_{x,\mu}\chi(x+\hat{\mu}) + (r+\gamma_\mu)U^\dagger_{x-\hat{\mu},\mu}\chi(x-\hat{\mu})\Big)\Big]$$

Due to the spin-flavour structure of the twisted mass term, $ia\mu_0\gamma_5\tau^3$**, parity is a symmetry only combined with a discrete flavour rotation, while flavour symmetry is broken explicitly according to the pattern** $SU_V(2) \rightarrow U_{(3)}(1)$**.**

$U_{(3)}(1)$ **is the subgroup of flavour rotations generated by** τ^3**.**

Standard usage in conjunction with tree-level Symanzik improved gauge action

$$S_G[U] = \beta \left[c_0 \sum_P \left(1 - \frac{1}{3} \mathbf{Re}\ (\mathrm{tr}\, [U(P)]) \right) + c_1 \sum_R \left(1 - \frac{1}{3} \mathbf{Re}\ (\mathrm{tr}\, [U(R)]) \right) \right]$$

where $\beta = 6/g_0^2$ (with g_0 the bare gauge coupling) and $U(P)$, $U(R)$ are plaquette and rectangle loops.

The weight coefficients satisfy $c_0 + 8\, c_1 = 1$, and $c_1 = -1/12$ (this is the tree-level improvement condition).

This choice corresponds to the conventions used for ETMC's data at $T = 0$ (needed for calibration of the lattices).

Algorithm:

- generalised hybrid Monte-Carlo algorithm,
- with even-odd preconditioning,
- with Hasenbusch trick,
- with multiple time scale integration according to the Sexton-Weingarten scheme.

see: C. Urbach and K. Jansen, twisted mass programming suite.

Advantages of the twisted mass approach :

- prevents exceptional configurations
- twisted mass provides a natural infrared cutoff
- it should be easier to reach light PS masses
- at maximal twist, with κ set to $\kappa_c(\beta)$,
  the twisted mass term takes the role of the mass term,
  while automatic $O(a)$ improvement is guaranteed;
  was observed first by R. Frezzotti, G. C. Rossi (2004)

Disadvantage :

- explicit flavor symmetry breaking (small effect on m_{π^0},
  has to be checked, calibration by means of $m_{\pi^\pm}$ measurements)

Chiral limit ?

- For natural quark masses: crossover instead of phase transition.
- Nature of the phase transition with $N_f = 2$ flavors in the chiral limit (not physical !) is however of principal interest; not settled up to now :
  Does it belong to the $Z(2)$ or $O(4)$ equivalence classes ?
  Majority of previous results compatible with $O(4)$.
  Or is it even a first order transition (Pisa group) ?
- How is that realized for twisted mass fermions ?

How many flavors can be simulated ?

- Rotation in flavor space involves two flavors : $N_f = 2$
- Two (degenerate or non-degenerate) doublets : $N_f = 2 + 1 + 1$

First steps

Our activity (at Humboldt University) comes from two roots:

1. **non-quenched study of the Aoki phase, mainly with respect to its limits in β, as a lattice artefact (2003/2005): an external source $\mu_0 \to 0$ induces symmetry breaking with an order parameter**

$$\lim_{\mu_0 \to +0} \lim_{V \to \infty} \langle \bar{\psi} i \gamma_5 \tau^3 \psi \rangle \neq 0$$

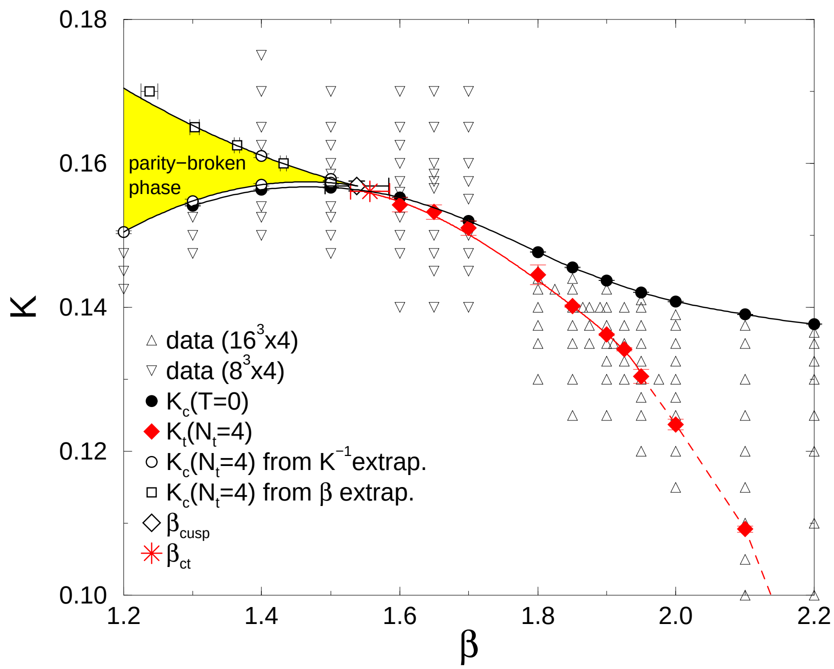
   that leads to spontaneous parity-flavor symmetry breaking (remnant "magnetization" in the limit $\mu_0 \to +0$)

2. **entering finite-temperature studies in the twisted mass approach, forming the tmfT ("twisted mass, finite T") collaboration, the "little brother" of the European Twisted Mass Collaboration**

Now we are working much closer to the continuum limit, at $N_\tau = 10$ and 12.

Aoki phase in unquenched simulations was first discovered with $N_\tau = 4$! (cf. Ali Khan et al., PRD 63 (2001) 034502)

Entangled with the physical finite-temperature transition !



In fact, a whole sequence of discontinuities (phase transitions) since then became discernible closer to the continuum limit :

- **the Aoki phase (a discontinuity in μ_0),**
- **a metastability region (a discontinuity in κ),**
- **an unphysical but finite-temperature transition became visible at $\kappa_T >> \kappa_c(\beta)$: of deconfining nature, originating from the doubler structure.**

Finally, the physically relevant finite-temperature region (crossover line) has been identified :

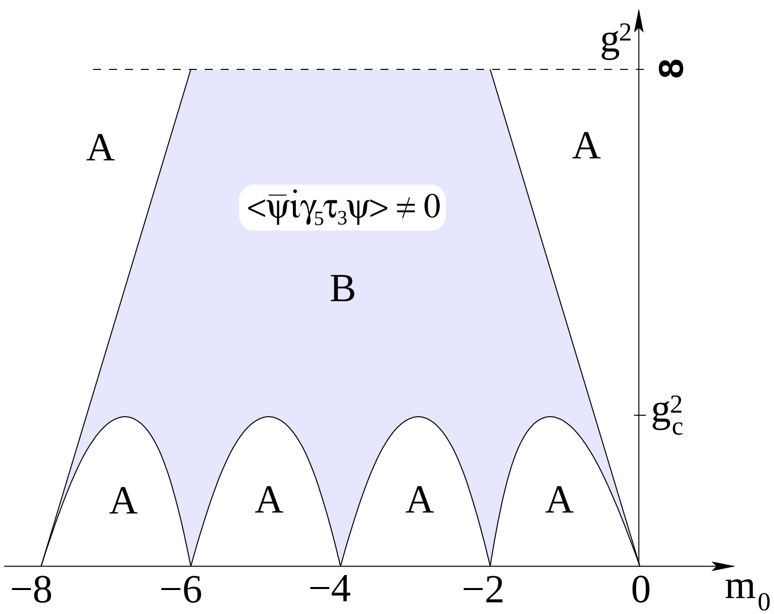
- **κ_T decouples (with $\mu_0 = 0$) from $\kappa_c(\beta, T = 0)$ (chiral limit $m_\pi \to 0$);**
- **is connected via a critical endpoint and a first order line to the quenched endpoint at $\kappa = 0$.**

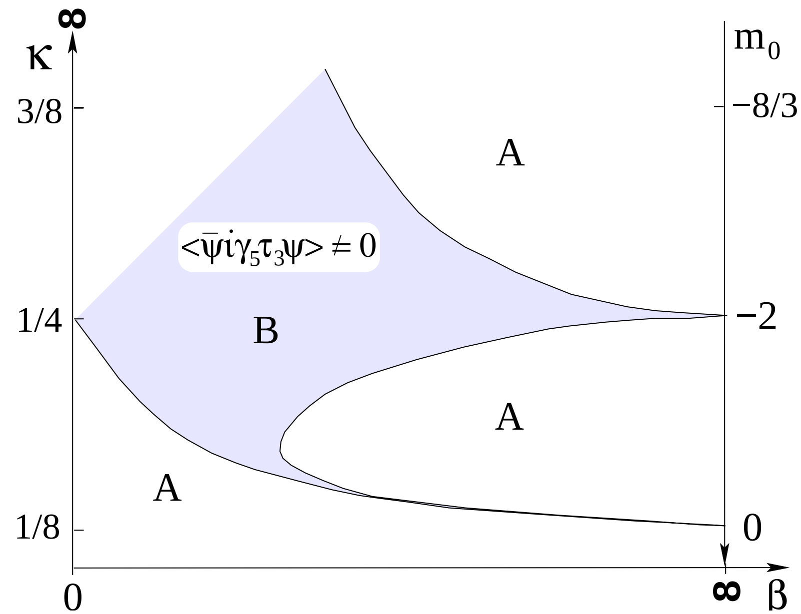
3. Recalling the global phase structure

Hypothetical Aoki phase in the m_0-g^2 and in the β-κ diagram

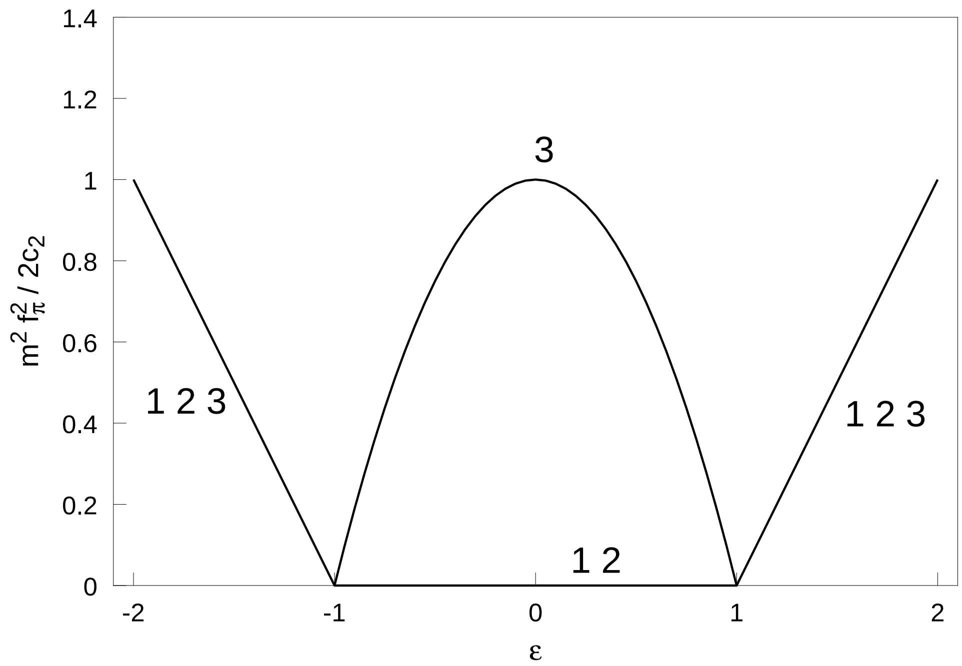
"A numerical reinvestigation of the Aoki phase with $N_f = 2$ Wilson fermions at zero temperature", PRD 69 (2004) 074511

E.-M. I., W. Kerler, M. Müller-Preussker, A. Sternbeck, H. Stüben



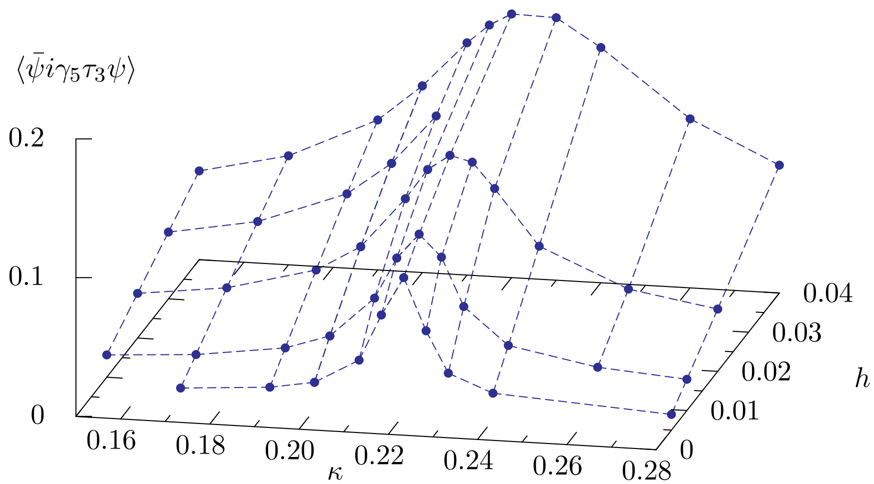


characteristic signal: degeneracy and splitting of π^0 and $\pi^\pm$ masses



"Predicting the Aoki phase using the chiral Lagrangian",
Stephen R. Sharpe, Robert L. Singleton, Jr.,
Nucl. Phys. Proc. Suppl. 73 (1999) 234-236

First order signal (jump) along the $h = \mu_0$ direction



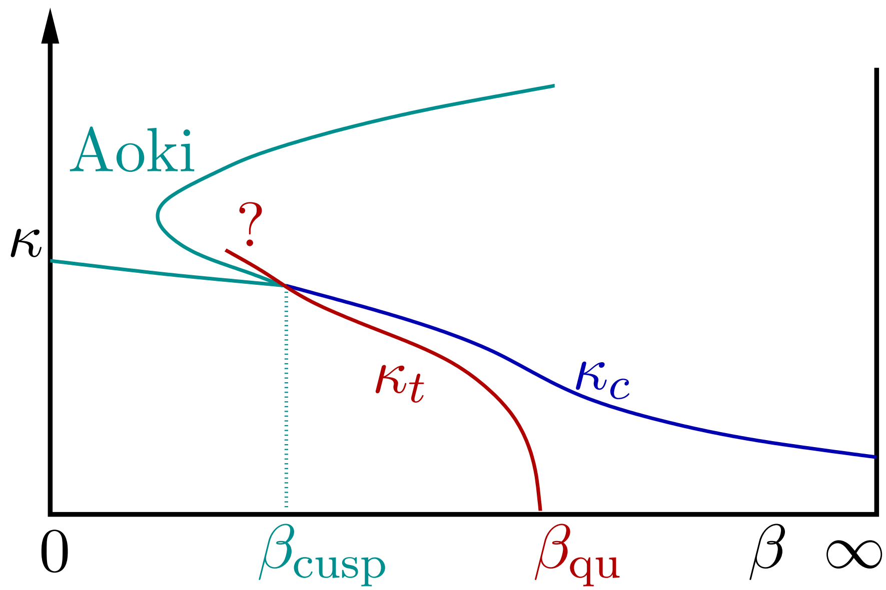
cf. Ising model at $T < T_{\mathbf{Curie}}$ with magnetic field $H \to \pm 0$

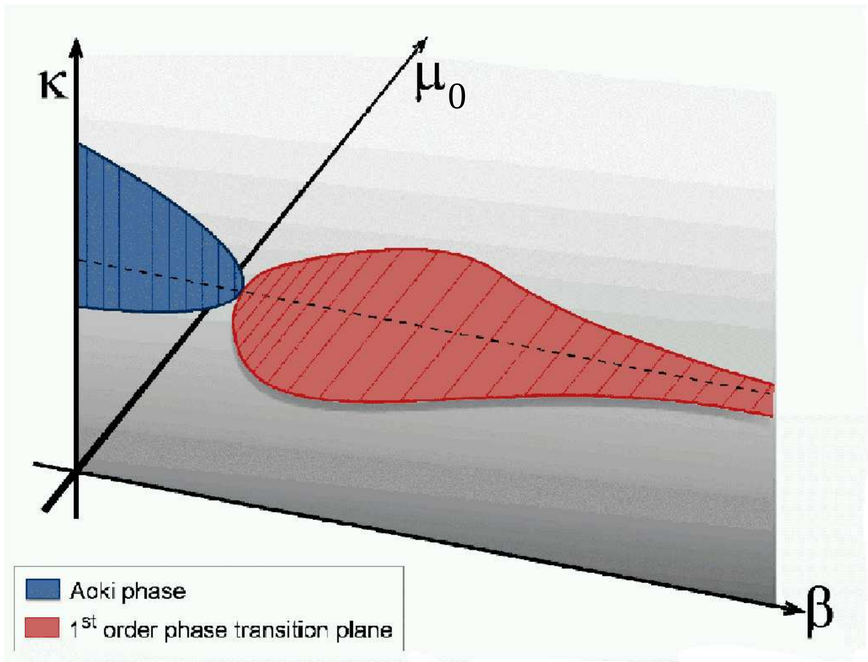
- **Aoki phase has a finite range in κ**
- **and has a finite range also in β !**

Over a broad β range a complicated phase stucture is known by now, w
"unwanted" phase transitions, as **non-physical artefacts of Wilson acti**

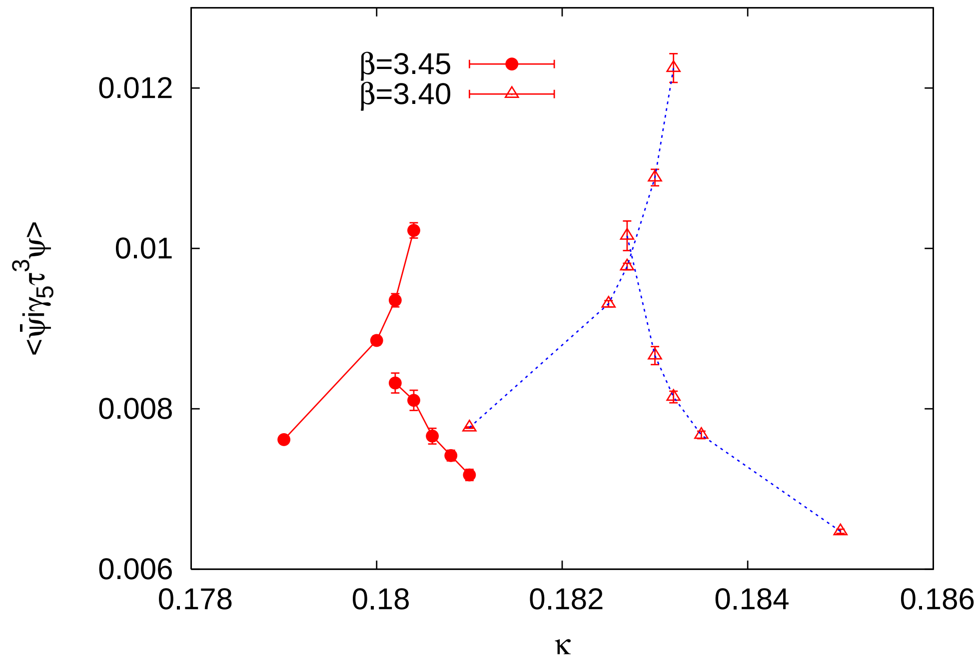
Improved gauge action becomes important here !

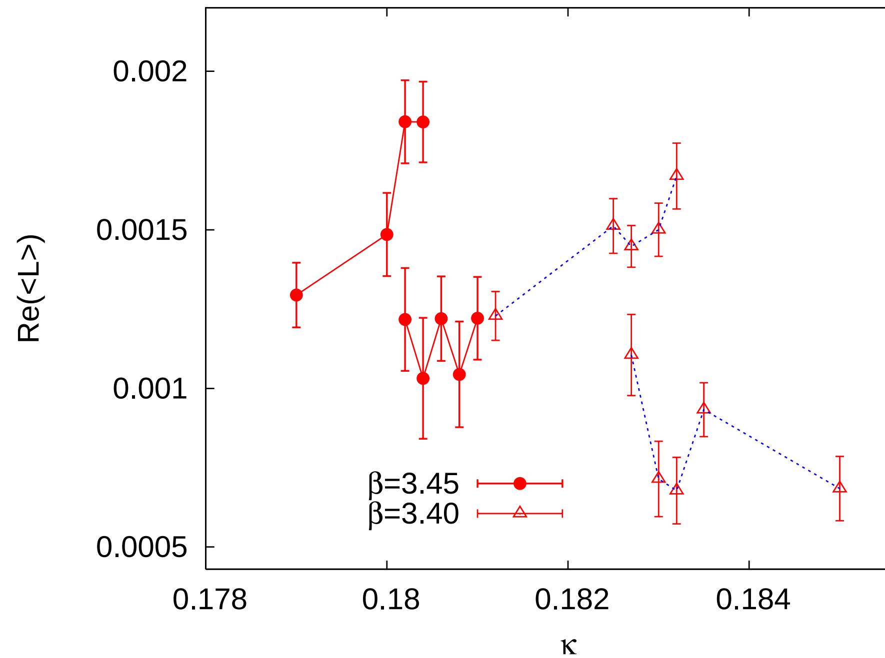
- **First example: Aoki phase (see above), not existing for $\beta > \beta_{\text{cusp}}$.**
- **Next, a first order transition extending in μ_0 direction was found.**





First order signal now by hysteresis scans along the κ-direction

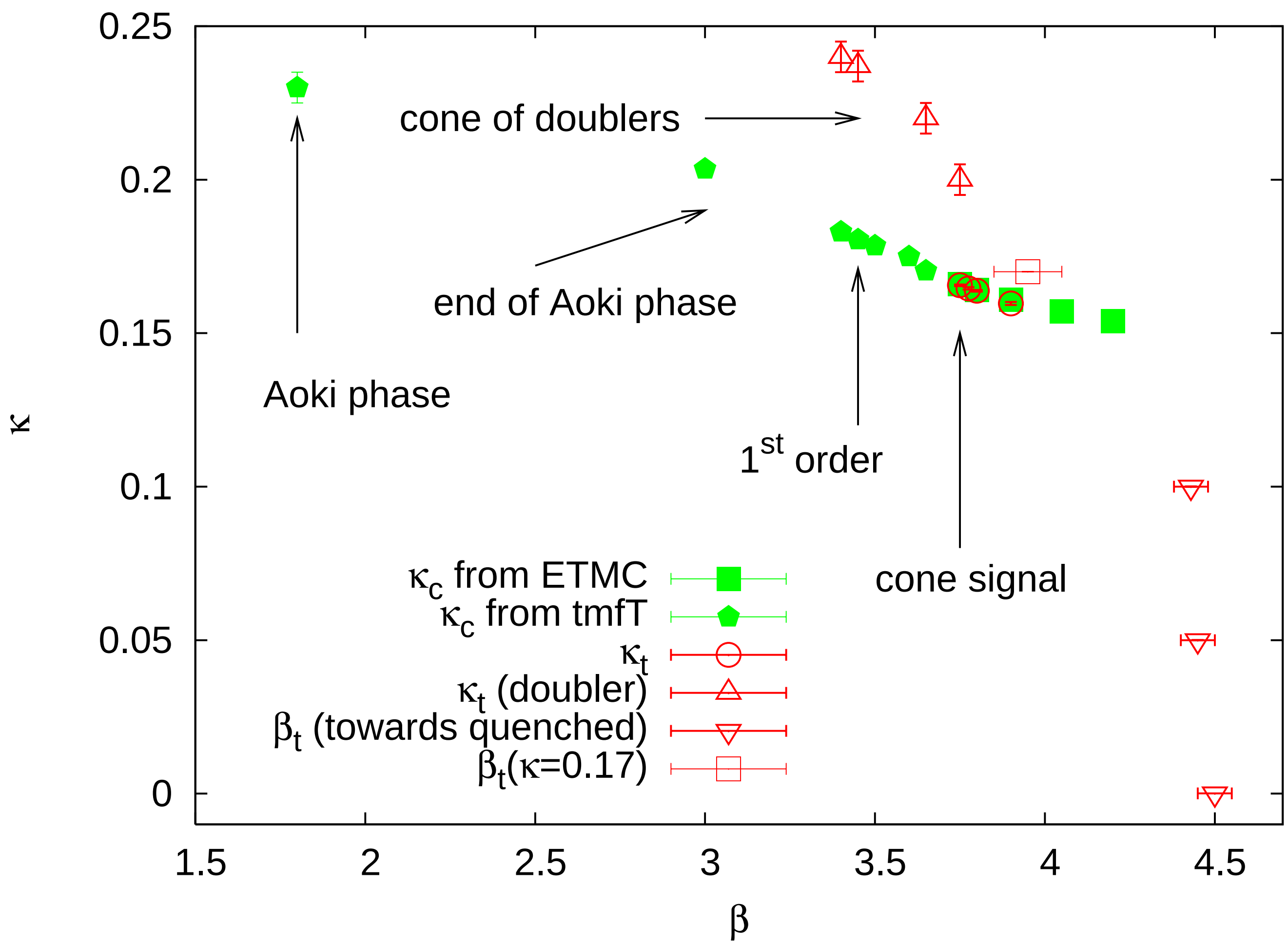




A gross overview has been given in PRD 80 (2009) 094502.

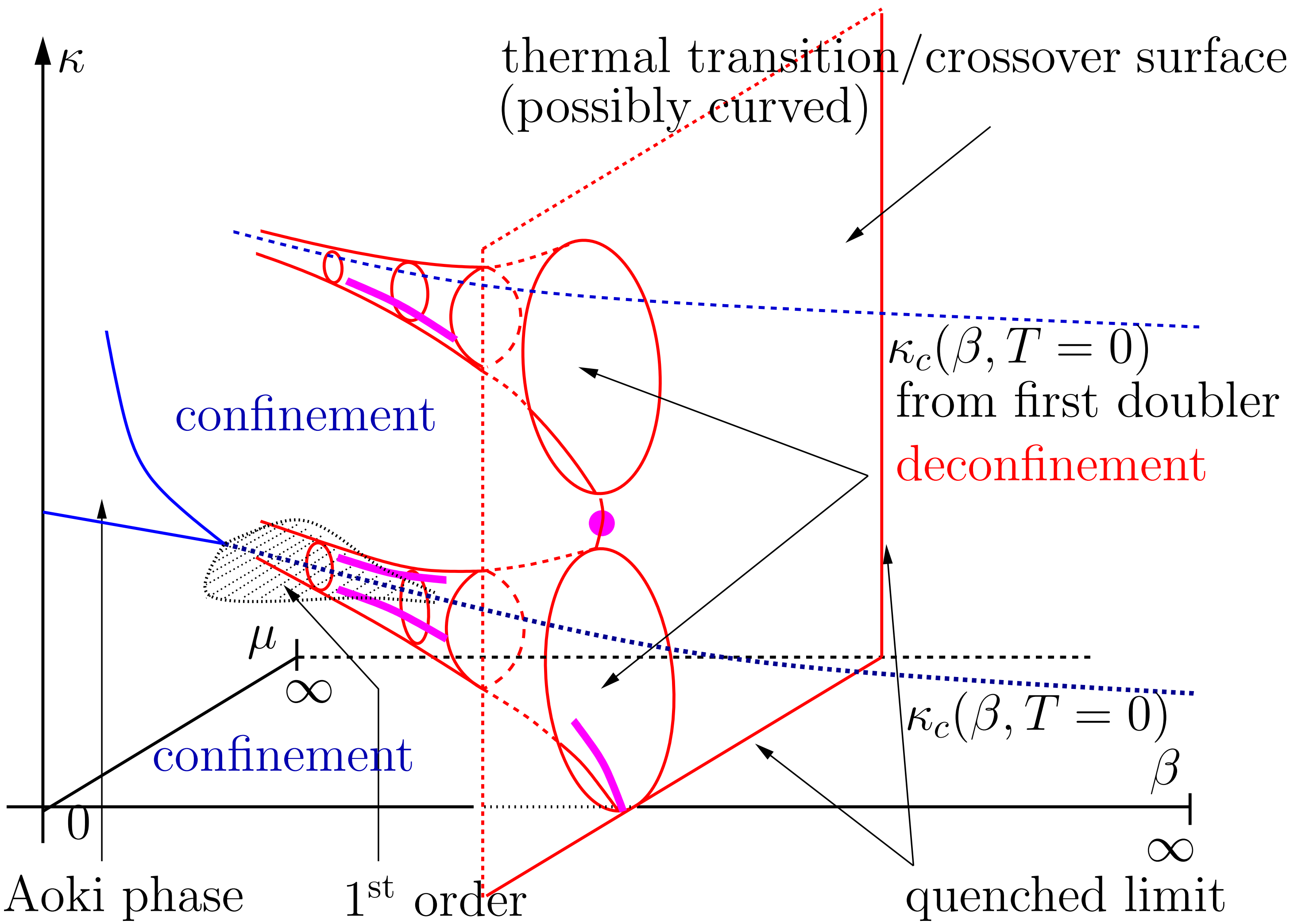
“Phase structure of thermal lattice QCD with $N_f = 2$ twisted mass Wilson fermions”, PRD 80 (2009) 094502

E.-M. I., K. Jansen, M.P. Lombardo, M. Müller-Preussker, M. Petschlies, O. Philipsen, L. Zeidlewicz



All this is summarized in

The full 3-dimensional phase diagram (Creutz' conjecture)



Some κ-scans at fixed μ_0

have been used to localize the cone and estimate its width.

Confirmation of Creutz' conjecture !

What is physically relevant here ?

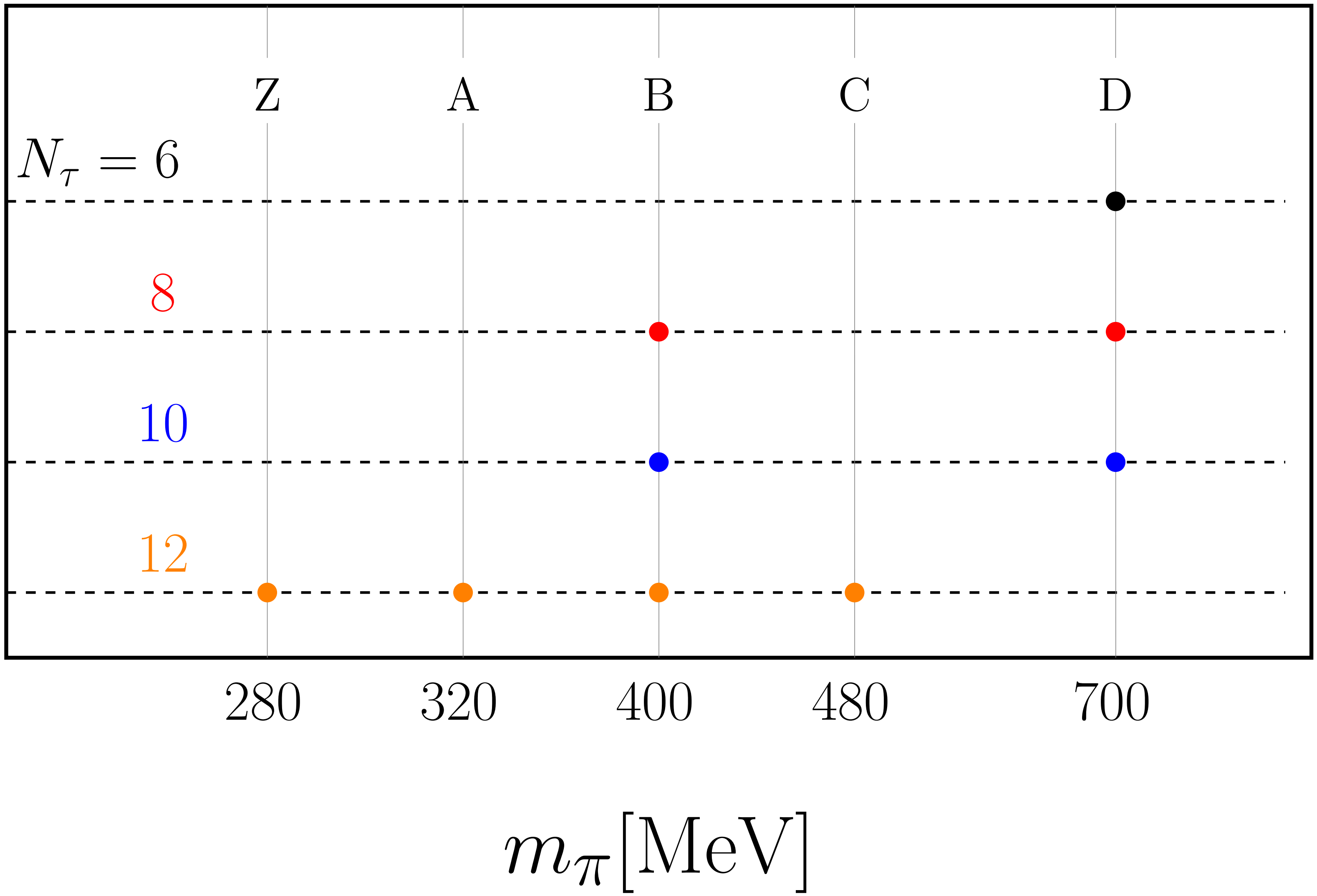
- **Only the lower cone connects with the quenched limit !** **(varying the quark mass $\rightarrow \infty$, a critical endpoint is passed, connecting the crossover to a first order line ending in the quenched endpoint at $\kappa = 0$.)**
- **How does a line of constant physics intersect the cone !**
- **The LCP should run at maximal twist !**
- **The LCP should not run at $\mu_0 = \text{ const}$!**

The analysis rests heavily on calibration simulations at $T = 0$

- **by the ETM Collaboration**
- **and partly by the tmfT Collaboration.**

4. Simulation setup for the crossover temperature(s)

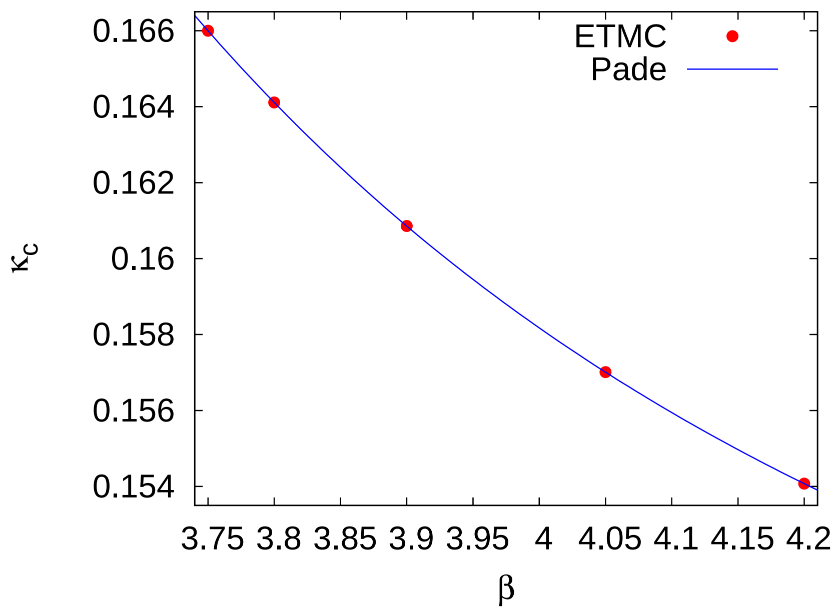
- **Evaluated: three families of ensembles : A12, B12, C12 (one more still in progress : Z12)**
- **populate the three-dimensional phase diagram** β**,** κ**,** μ_0
- **a** β **scan fixes the position of the crossover line**
- **maximal twist: requires choice** $\kappa = \kappa_c(T = 0, \beta)$
- **fixed pion mass: requires choice** $a\mu_0 = a\mu_0(\beta) = C \exp\left(-\beta/(12\beta_0)\right)$**, fitted with** $\beta_0 = \frac{11-2N_f/3}{(4\pi)^2}$**, or by a two-loop formula**
- **these fits for various families of** $T = 0$ **simulations based on data from the ETM-Collaboration [JHEP 08 097 (2010)]**

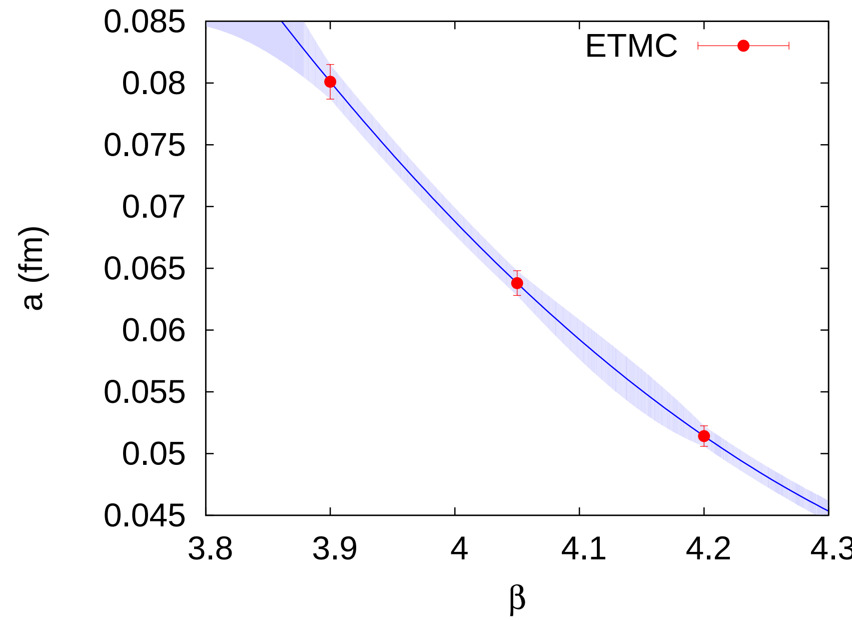

Z
A
B
C
D
$N_\tau = 6$
8
10
12
280
320
400
480
700
m_π[MeV]

List of β-scans

- **A12:** $32^3 \times 12$**,** $3.84 \leq \beta \leq 3.99$**,**
  $m_\pi = 316(16)$ **MeV,** $r_0 m_\pi = 0.673(42)$
  $\beta_\chi = 3.89(3)$**,** $T_\chi = 202(7)$ **MeV**
- **B12:** $32^3 \times 12$**,** $3.86 \leq \beta \leq 4.35$**,**
  $m_\pi = 398(20)$ **MeV,** $r_0 m_\pi = 0.847(53)$
  $\beta_\chi = 3.93(2)$**,** $T_\chi = 217(5)$ **MeV**
  $\beta_{\rm deconf} = 4.027(14)$**,** $T_{\rm deconf} = 249(5)$ **MeV**
- **C12:** $32^3 \times 12$**,** $3.90 \leq \beta \leq 4.07$**,**
  $m_\pi = 469(24)$ **MeV,** $r_0 m_\pi = 0.998(62)$
  $\beta_\chi = 3.97(3)$**,** $T_\chi = 229(5)$ **MeV**
  $\beta_{\rm deconf} = 4.050(15)$**,** $T_{\rm deconf} = 258(5)$ **MeV**

$\kappa_c(T=0,\beta)$ and lattice spacing $a(\beta)$





One T_c or two T_c's (chiral and deconfining ?) localized by considering:

- **chiral susceptibility**

$$\chi_\sigma = \frac{\partial \langle \bar{\psi}\psi \rangle}{\partial m_q}$$

- **disconnected part of it (looking for a Gaussian peak)**

$$\sigma^2_{\bar{\psi}\psi} = \frac{V}{T} \left(\langle (\bar{\psi}\psi)^2 \rangle - \langle \bar{\psi}\psi \rangle^2 \right)$$

with $\langle \bar{\psi}\psi \rangle$ evaluated as stochastic estimator

- **renormalized (subtracted) chiral condensate (defined as the ratio)**

$$R_{\langle \bar{\psi}\psi \rangle} = \frac{\langle \bar{\psi}\psi \rangle (T, \mu_0) - \langle \bar{\psi}\psi \rangle (0, \mu_0) + \langle \bar{\psi}\psi \rangle (0,0)}{\langle \bar{\psi}\psi \rangle (0,0)}$$

- **renormalized Polyakov loop (looking for an inflection point)**

$$\langle \mathrm{Re}(L) \rangle_R = \langle \mathrm{Re}(L) \rangle \exp\left(V(r_0)/2T\right)$$

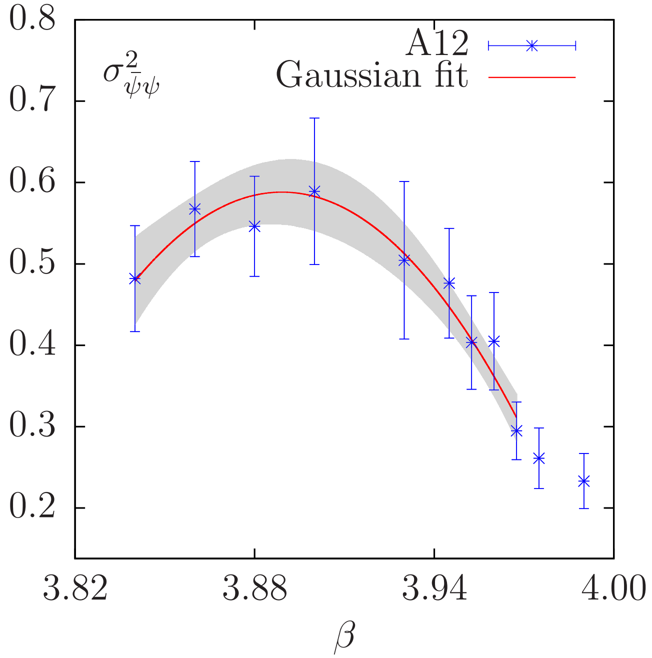
- **and its susceptibility (hardly showing a Gaussian peak)**

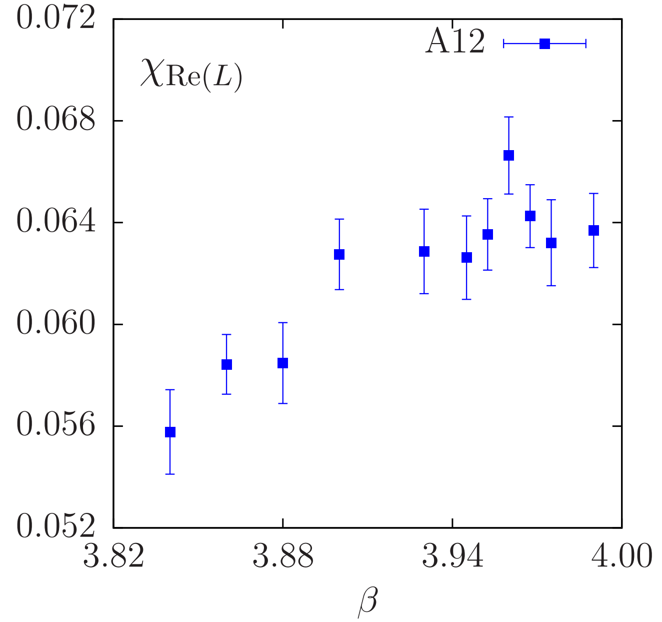
$$\chi_{\mathrm{Re}(L)}$$

m_π**-dependence and chiral extrapolations are discussed in papers :**

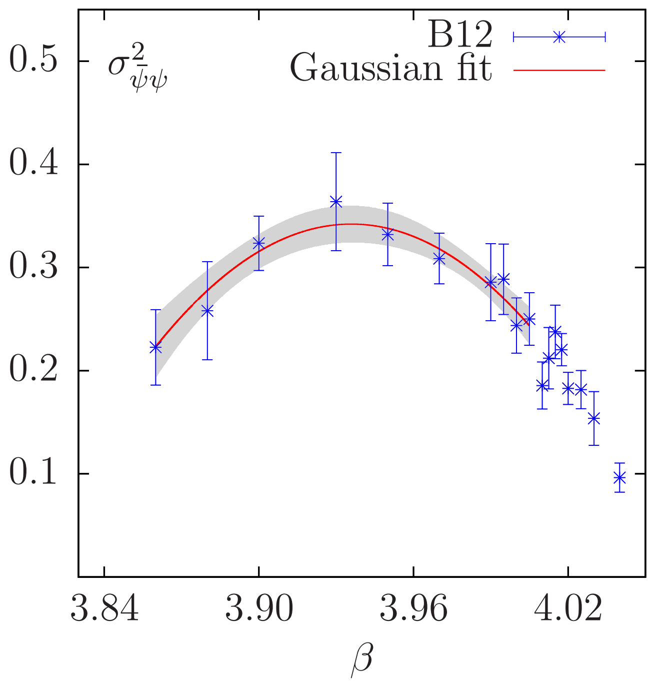
- **arXiv:1102.4530v2 (revised paper, December 2012)**
- **arXiv:1212.0982 (F. Burger at Lattice 2012)**

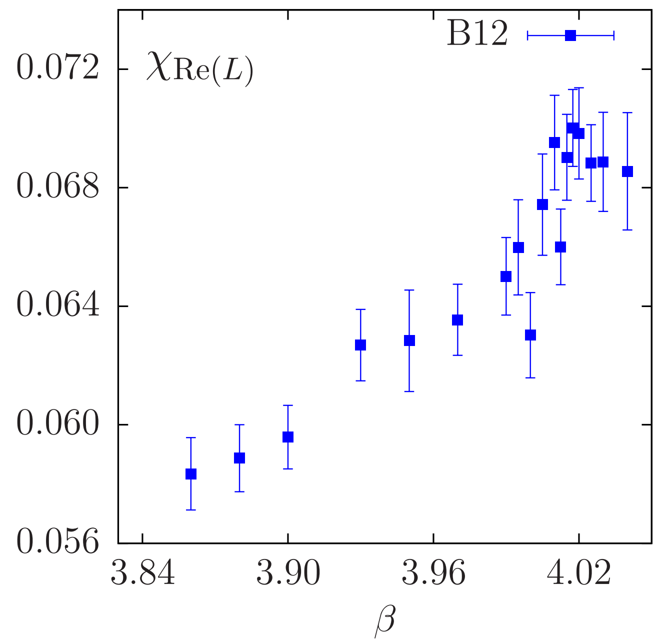
Chiral susceptibility and Polyakov loop susceptibility for A12



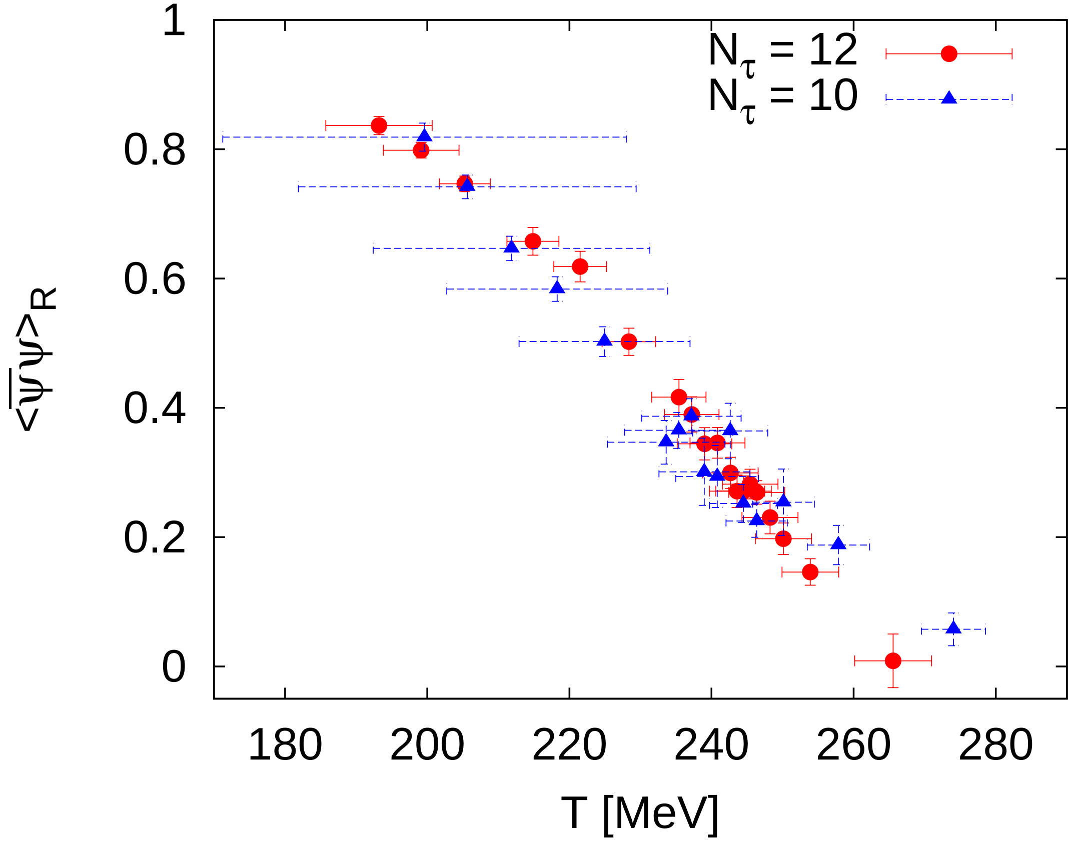


Chiral susceptibility and Polyakov loop susceptibility for B12

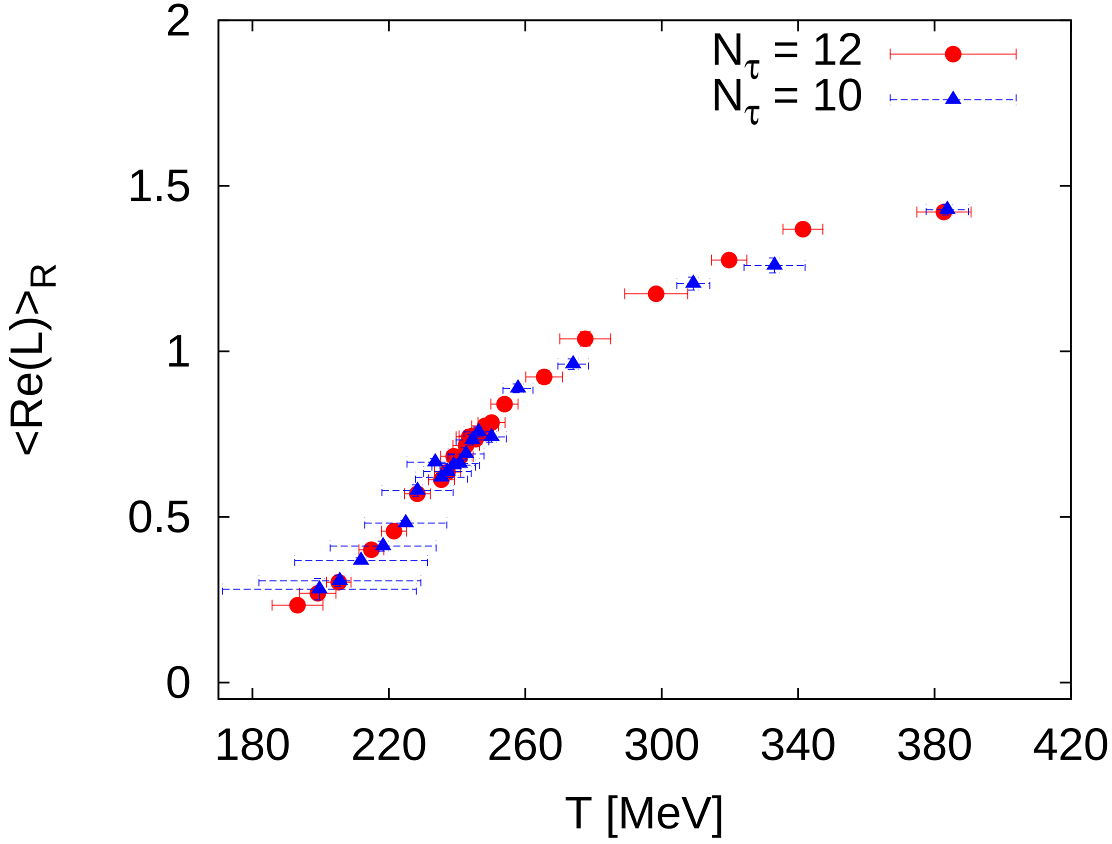




Renormalized $\langle\bar{\psi}\psi\rangle$ for B12 and B10



Renormalized Polyakov loop $\langle \mathrm{Re}(L) \rangle_R$ for B12 and B10



5. Towards the chiral limit

Chiral extrapolations for $T_\chi(m_\pi)$ for various scenarios (χPT)

$$T_\chi(m_\pi) = T_\chi(m_\pi = 0) + A\ m_\pi^{2/(\tilde{\beta}\delta)}$$

with critical indices $\tilde{\beta}$, δ corresponding to the respective equivalence classes of $3d$ spin models.

- $O(4)$ **:** $2/(\tilde{\beta}\delta) = 1.08$ **leads to** $T_\chi(m_\pi = 0) = 152(26)$ **MeV**
- $Z(2)$ **: two cases** $m_{\pi,c} = 0$ **or** $m_{\pi,c} \neq 0$**;**
  lead to $T_\chi(m_\pi \to 0)$ **between** $O(4)$ **and 1-st order scenario**
- **first order : in literature formally** $2/(\tilde{\beta}\delta) = 2$ **is taken;**
  leads to $T_\chi(m_\pi = 0) = 182(14)$ **MeV**
  (applicability of these "critical indices" unclear !)

It is also possible to obtain for fixed N_τ a critical $\beta_c(h)$ from the scaling relation

$$\beta_c(h) = \beta_{\rm chiral} + B\ h^{1/(\tilde{\beta}\delta)} \qquad h = 2a\mu_0$$

(gives consistent results for T_χ)

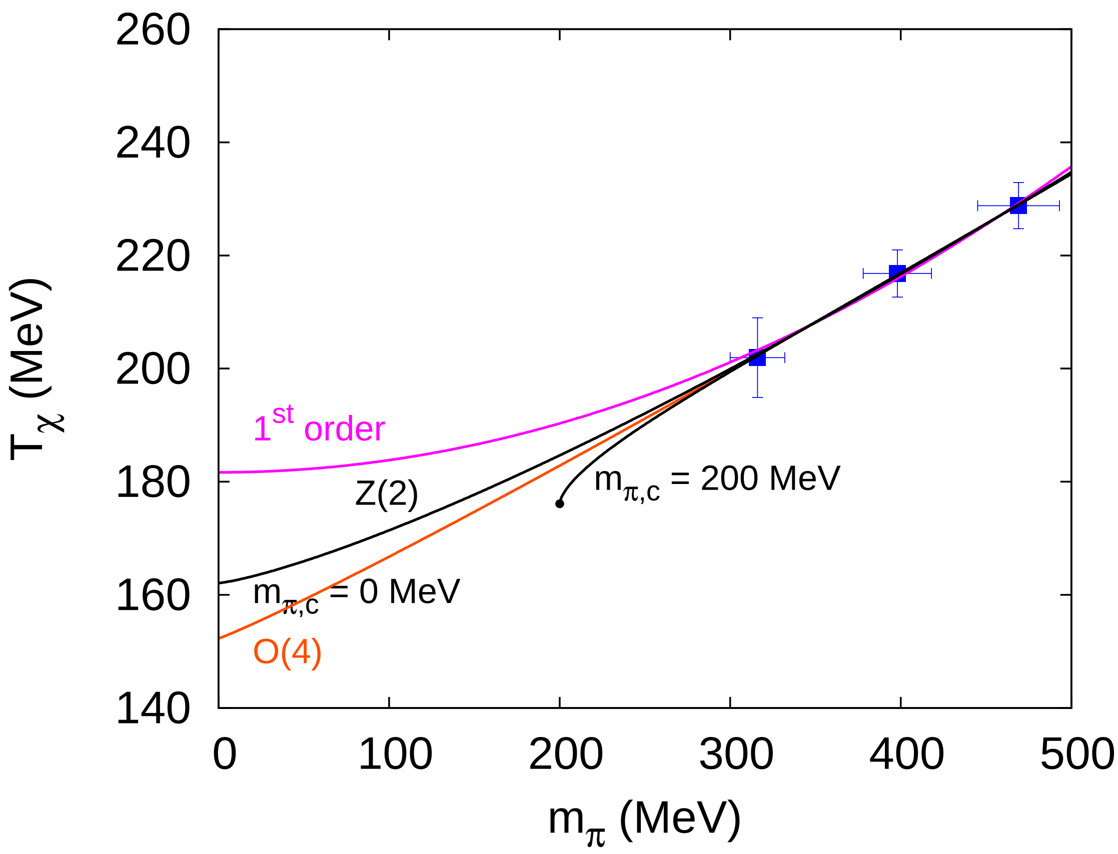
We have also discussed the scaling of the "magnetic EoS"

$$\langle\bar{\psi}\psi\rangle = h^{1/\delta} c f(d\tau/h^{1/(\tilde{\beta}\delta)})$$

$$\tau = \beta - \beta_{\rm chiral}$$

fitting scaling violations due to quark mass.

Chiral extrapolations for $T_\chi(m_\pi)$ for various scenarios



Summary: possible scenarios for the $N_f = 2$ chiral limit

1. **first order transition:**

   fit gives $T_\chi(m_\pi = 0) = 182(14)$ MeV

2. $O(4)$ **second order transition down to** $m_\pi = 0$:

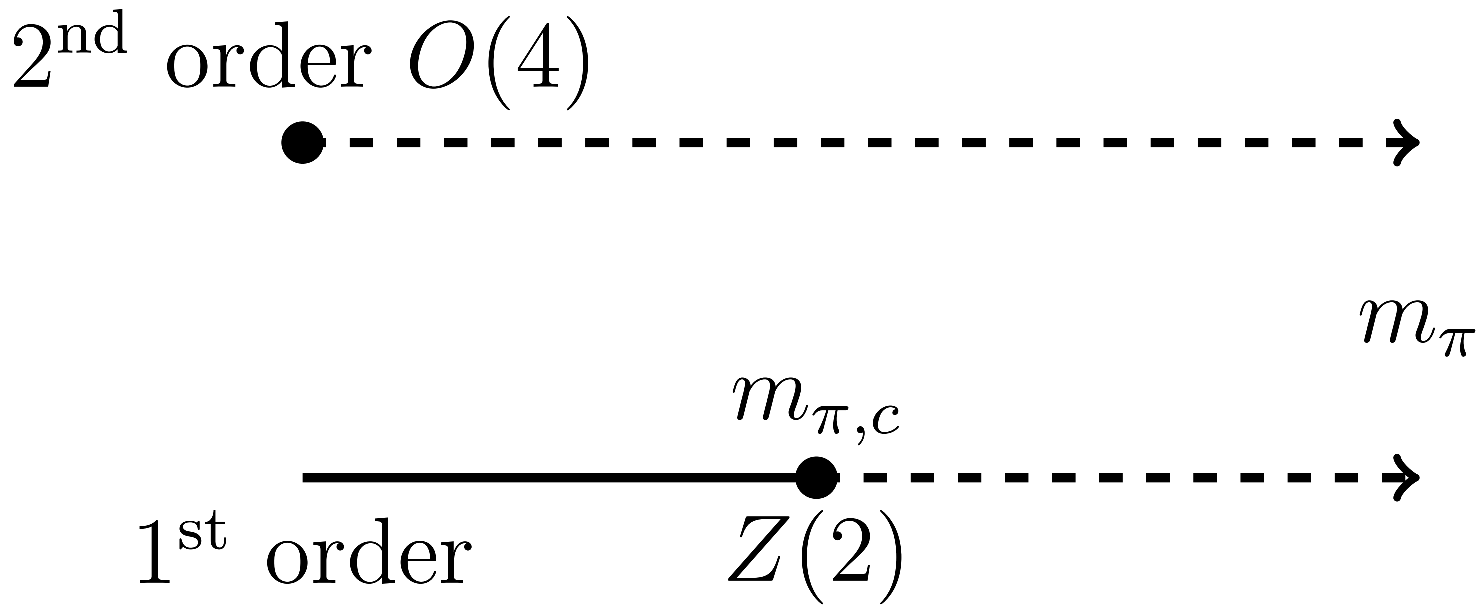
   fit gives $T_\chi(m_\pi = 0) = 152(26)$ MeV

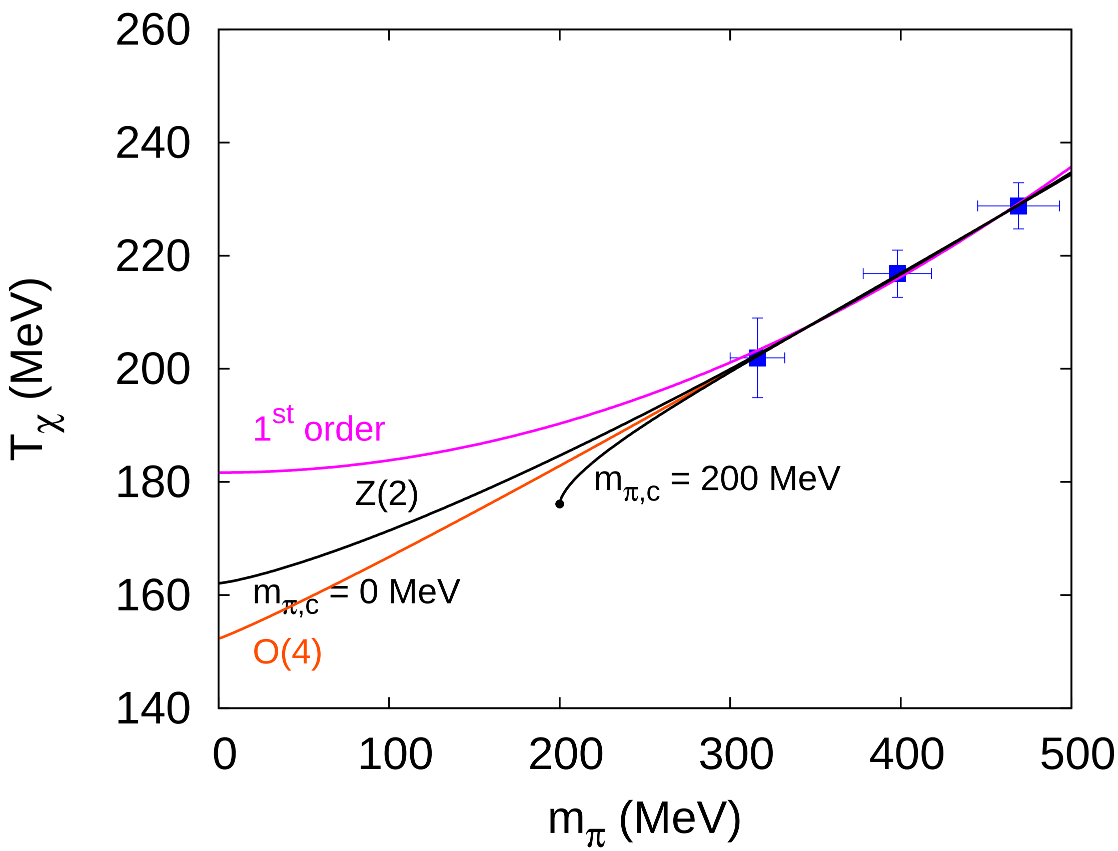
3. **second order** $Z(2)$ **transition, fits give** $T_\chi(m_\pi \to 0)$ **in between.**

   - either with a critical point separating second order ($m_\pi > m_{\pi,c}$) from first order transition ($m_\pi < m_{\pi,c}$) with $m_{\pi,c} \neq 0$

   - or as second order transition down to $m_{\pi,c} = 0$.

The Z12 results ($m_\pi \approx 280$ MeV) will hopefully be able to exclude (at least) the first order scenario !


2$^{\text{nd}}$ order $O(4)$
m_π
$m_{\pi,c}$
1$^{\text{st}}$ order
$Z(2)$



260
240
220
200
180
160
140
0
100
200
300
400
500
T_χ (MeV)
m_π (MeV)
1st order
Z(2)
$m_{\pi,c}$ = 200 MeV
$m_{\pi,c}$ = 0 MeV
O(4)

6. Gluon and ghost propagators: unquenched with twisted mass quarks

Gauge fixing: Landau gauge

$$\nabla_\mu A_\mu(x) = \sum_\mu \left(A_\mu(x+\hat{\mu}/2) - A_\mu(x-\hat{\mu}/2)\right) = 0$$

$$A_\mu(x+\hat{\mu}/2) = \frac{1}{2iag_0}\left(U_{x\mu} - U_{x\mu}^\dagger\right)_{|traceless}$$

established by maximization of

$$F_U[g] = \frac{1}{3}\sum_{x,\mu} \mathrm{Re}\ \mathrm{tr}\left(g_x U_{x\mu} g_{x+\mu}^\dagger\right)$$

with respect to gauge transformations g_x.

Gauge fixing is performed for relevant enembles of MC configurations irrespective of their origin.

Ghosts are not explicit, studied only algebraically, by inversion of the Faddeev-Popov operator.

Problem of Gribov copies serious in IR limit !

Previous work : quenched propagator study at finite T

R. Aouane et al., PRD 85 (2012) 034501 arXiv:1108.1735 [hep-lat]

(with Wilson action, for various lattice sizes)

Gluon propagator in momentum space as ensemble average :

$$D^{ab}_{\mu\nu}(q) = \left\langle \widetilde{A}^a_\mu(k)\widetilde{A}^b_\nu(-k) \right\rangle ,$$

$$q_\mu(k_\mu) = \frac{2}{a}\sin\left(\frac{\pi k_\mu}{N_\mu}\right)$$

For non-zero temperature Euclidean invariance is broken, useful to split $D^{ab}_{\mu\nu}(q)$ into two components, the transversal D_T ("chromomagnetic") and the longitudinal D_L ("chromoelectric") propagator, respectively,

$$D^{ab}_{\mu\nu}(q) = \delta^{ab}\left(P^T_{\mu\nu} D_T(q_4^2, \vec{q}^{\,2}) + P^L_{\mu\nu} D_L(q_4^2, \vec{q}^{\,2})\right) \ .$$

The fourth momentum component q_4 conjugate to the Euclidean time (Matsubara frequency) has been restricted to zero.

For the propagators $D_{T,L}$ (or their respective dimensionless dressing functions $Z_{T,L}(q) = q^2 D_{T,L}(q)$) we find

$$D_T(q) = \frac{1}{2N_g}\left\langle \sum_{i=1}^{3} \widetilde{A}^a_i(k)\widetilde{A}^a_i(-k) - \frac{q_4^2}{\vec{q}^{\,2}}\widetilde{A}^a_4(k)\widetilde{A}^a_4(-k) \right\rangle$$

and

$$D_L(q) = \frac{1}{N_g}\left(1 + \frac{q_4^2}{\vec{q}^{\,2}}\right)\left\langle \widetilde{A}^a_4(k)\widetilde{A}^a_4(-k) \right\rangle \ ,$$

where $N_g = N_c^2 - 1$ **and** $N_c = 3$**.**

The Landau gauge ghost propagator is given by

$$\begin{aligned} G^{ab}(q) &= a^2 \sum_{x,y} \langle e^{-2\pi i (k/N)\cdot(x-y)} [M^{-1}]^{ab}_{xy} \rangle \\ &= \delta^{ab}\ G(q) = \delta^{ab}\ J(q)/q^2 \ , \end{aligned}$$

with the four-vector $(k/N) \equiv (k_\mu/N_\mu)$**.**

$J(q)$ **denotes the ghost dressing function.**

The matrix M **is the lattice Faddeev-Popov operator**

$$M^{ab}_{xy} = \sum_\mu [A^{ab}_{x,y}\delta_{x,y} - B^{ab}_{x,y}\delta_{x+\hat{\mu},y} - C^{ab}_{x,\mu}\delta_{x-\hat{\mu},y}]$$

$$\begin{aligned} A^{ab}_{x,y} &= \quad \mathrm{Re}\ \mathrm{tr}\,[\{T^a, T^b\}(U_{x,\mu} + U_{x-\hat{\mu},\mu})], \\ B^{ab}_{x,y} &= 2 \cdot \mathrm{Re}\ \mathrm{tr}\,[T^b T^a U_{x,\mu}], \\ C^{ab}_{x,y} &= 2 \cdot \mathrm{Re}\ \mathrm{tr}\,[T^a T^b U_{x-\hat{\mu},\mu}]\ , \end{aligned}$$

The corresponding renormalized functions, in momentum subtraction (MOM) schemes, can be obtained from

$$
\begin{aligned}
Z^{ren}_{T,L}(q,\mu) &\equiv \tilde{Z}_{T,L}(\mu) Z_{T,L}(q), \\
J^{ren}(q,\mu) &\equiv \tilde{Z}_J(\mu) J(q)
\end{aligned}
$$

with the $\tilde{Z}$-factors being defined such that $Z^{ren}_{T,L}(\mu,\mu) = J^{ren}(\mu,\mu) = 1$.

For the gluon dressing function we employed the Gribov-Stingl formula

$$
Z_{\text{fit}}(q) = q^2 \frac{c\ (1 + d\, q^{2n})}{(q^2 + r^2)^2 + b^2} \ ,
$$

For the ghost dressing function used a fit formula like

$$
J_{\text{fit}}(q) = \left(\frac{f^2}{q^2}\right)^k + h \qquad (
$$

Our main emphasis: Finite-volume and discretization studies,

providing continuum parametrizations for various temperatures,
as input for finite-T for continuum studies

- **Gribov copy and finite volume effects of minor importance in the momentum range under study**
- **fits of the momentum dependence of the propagators**
  $0.6\ GeV < q < 3.0\ GeV$
- **in the temperature range**
  $0.65 < T/T_{\rm deconf} < 2.97$

Temperature dependence studied at finite scale ($\beta = 6.337$)

T/T_c	N_τ	N_σ	β	$a(\mathrm{GeV}^{-1})$	$a(\mathrm{fm})$	n_{conf}	n_{copy}
0.65	**18**	**48**	**6.337**	**0.28**	**0.055**	**150**	**1**
0.74	**16**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
0.86	**14**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
0.99	**12**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
1.20	**10**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
1.48	**8**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
1.98	**6**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
2.97	**4**	**48**	**6.337**	**0.28**	**0.055**	**210**	**1**

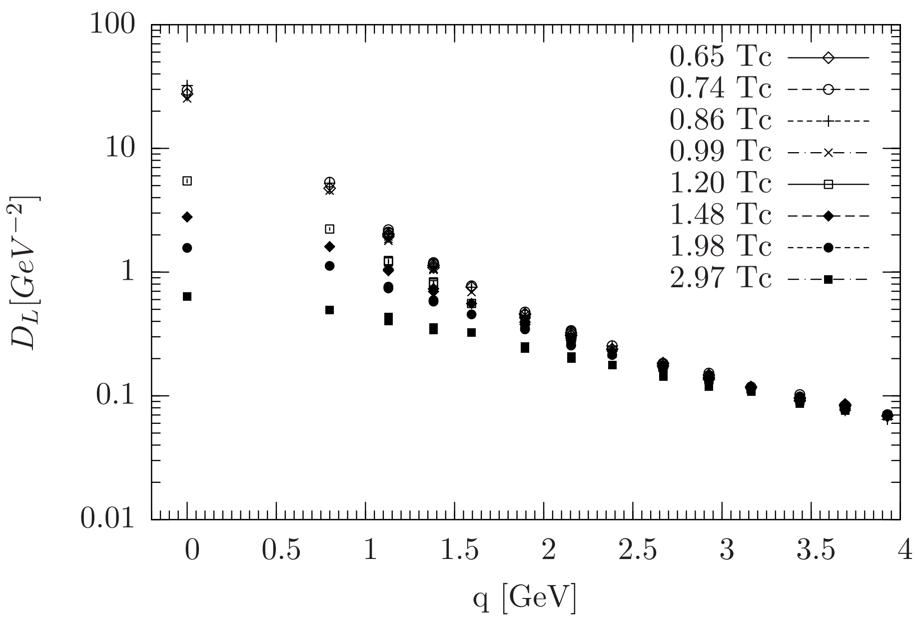
long simulated annealing chains, no copies !

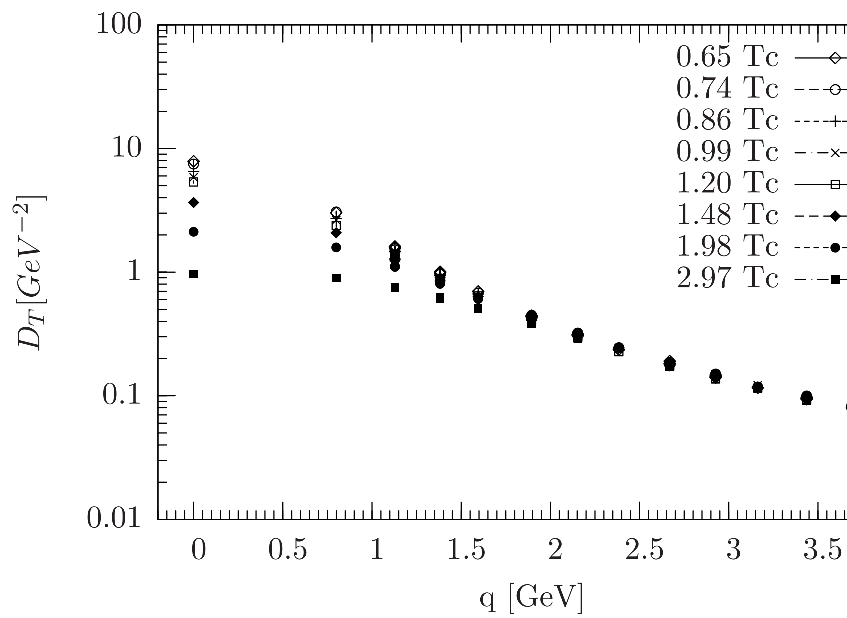
Volume and discretization dependence at two fixed temperatures

T/T_c	N_τ	N_σ	β	$a(\mathrm{GeV}^{-1})$	$a(\mathrm{fm})$	n_{conf}	n_{copy}
0.86	**8**	**28**	**5.972**	**0.49**	**0.097**	**200**	**27**
0.86	**12**	**41**	**6.230**	**0.33**	**0.064**	**200**	**1**
0.86	**16**	**55**	**6.440**	**0.24**	**0.048**	**200**	**1**
1.20	**6**	**28**	**5.994**	**0.47**	**0.094**	**200**	**27**
1.20	**8**	**38**	**6.180**	**0.35**	**0.069**	**200**	**1**
1.20	**12**	**58**	**6.490**	**0.23**	**0.045**	**200**	**1**
0.86	**14**	**56**	**6.337**	**0.28**	**0.055**	**200**	**1**
0.86	**14**	**64**	**6.337**	**0.28**	**0.055**	**200**	**1**
1.20	**10**	**56**	**6.337**	**0.28**	**0.055**	**200**	**1**
1.20	**10**	**64**	**6.337**	**0.28**	**0.055**	**200**	**1**

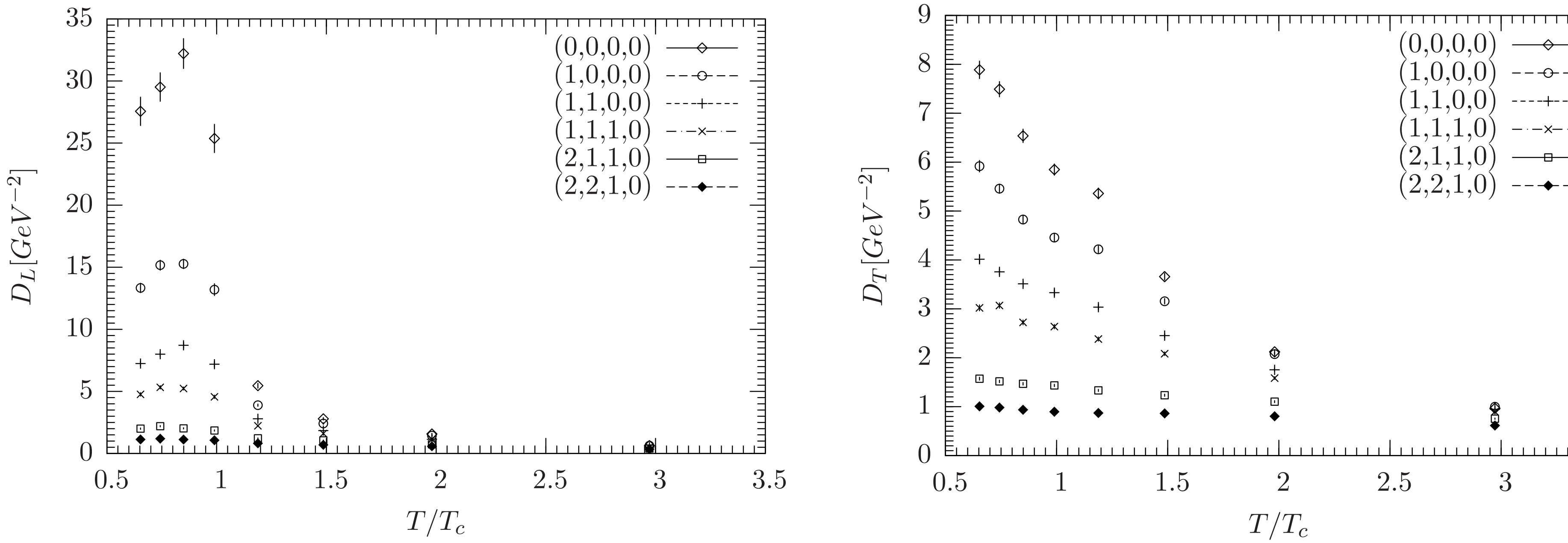
Gribov effect studied partly with 27 **copies !**

q dependence for various temperatures





T dependence for various lattice momenta



"Order parameters" constructed from the longitudinal propagator D_L

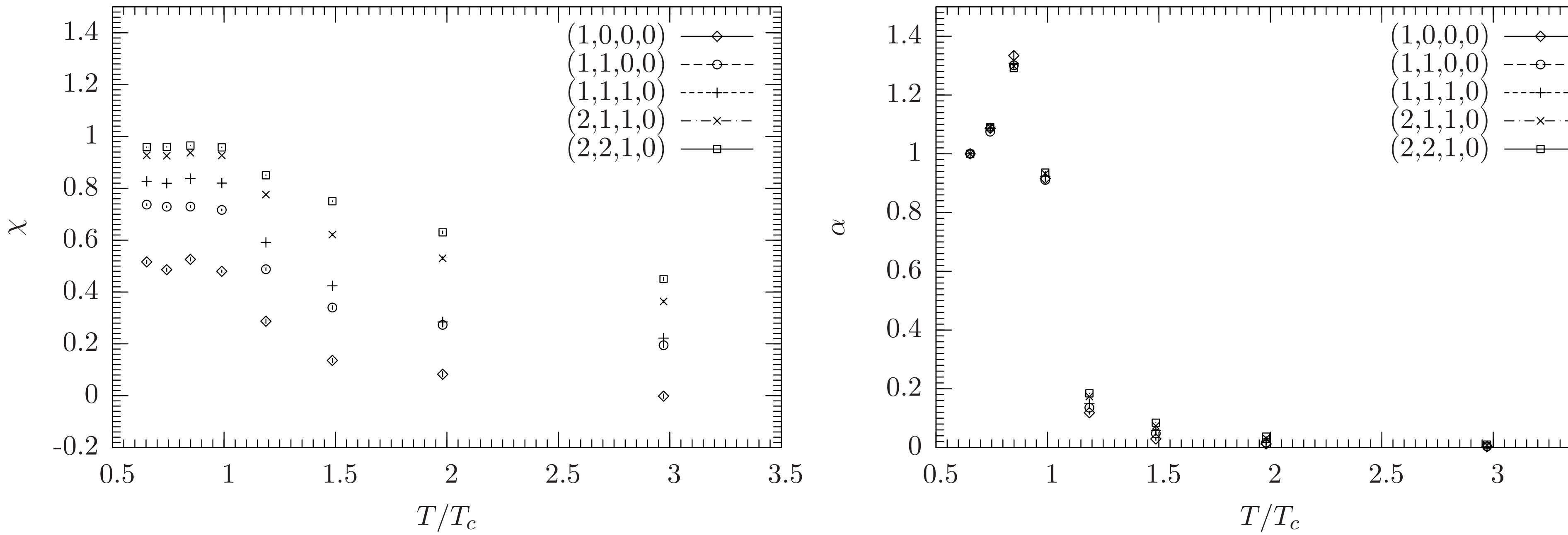
$$\chi = \frac{D_L(0,T) - D_L(q,T)}{D_L(0,T)}$$

$$\alpha = \frac{D_L(0,T) - D_L(q,T)}{D_L(0,T_{\mathrm{min}}) - D_L(q,T_{\mathrm{min}}}$$

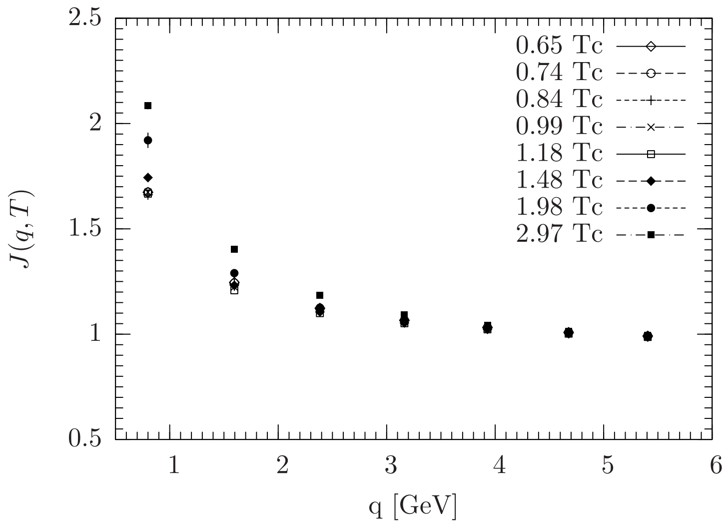
with $T_{\mathrm{min}} = 0.65\ T_c$

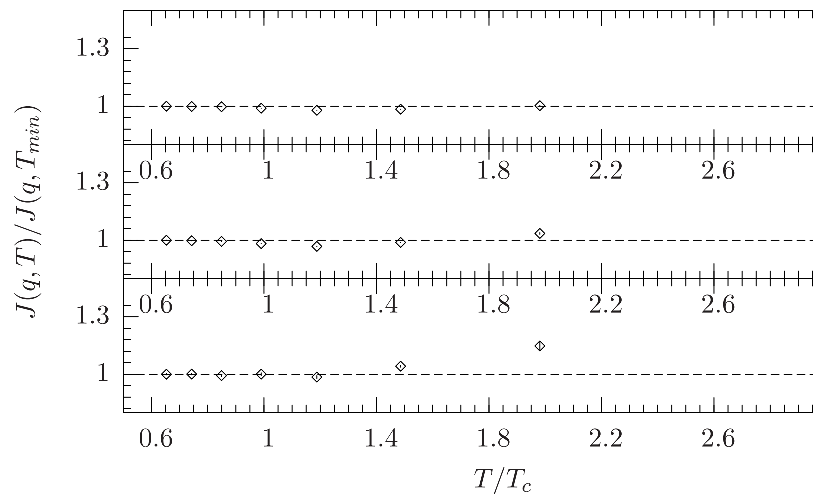
for various fixed lattice momenta as function of T

T dependence of "order parameters" for various lattice momenta



q and T dependence of the ghost dressing function





Our finite-temperature results for pure Yang-Mills theory were used by Kenji Fukushima and Kouji Kashiwa, arXiv:1206.0685 [hep-ph]

- **in the effective potential for the Polyakov loop**
- **for reconstructing the Equation of State (EoS)**

In leading order of the 2PI formalism, the thermodynamical potential can be approximated as follows in terms of the gluon and ghost propagators :

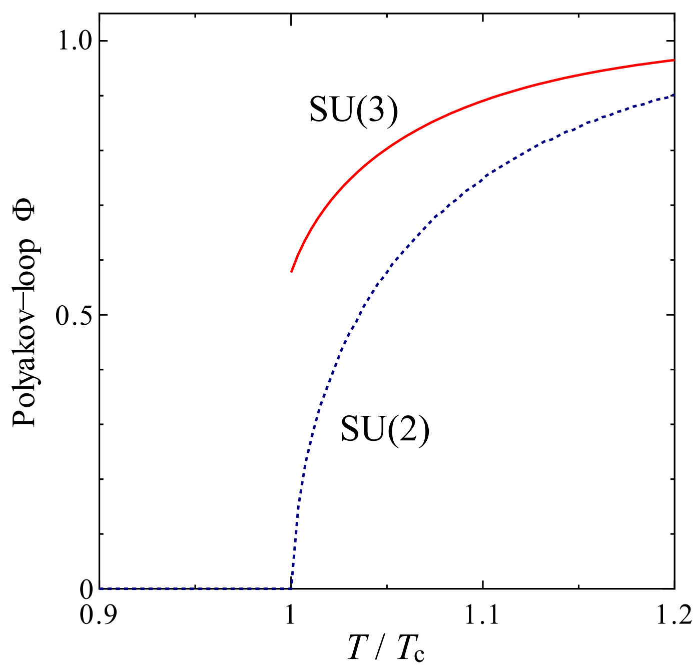
$$\frac{1}{T}\Omega_{\mathbf{glue}} \simeq -\frac{1}{2}\mathrm{tr}\,\ln D_{\mathrm{gl}}^{-1} + \mathrm{tr}\,\ln D_{\mathrm{gh}}^{-1}$$

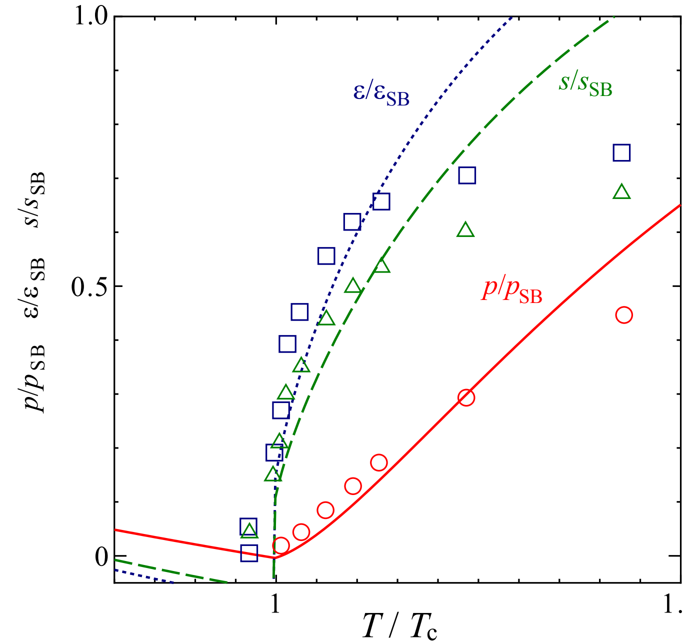
for example, the inverse gluon propagator has been extracted from our data for $T < 1.2\ T_c$

$$D_{\mathrm{gl}}^{-1}(p^2) = \left[p^2 Z_T(p^2) T_{\mu\nu} + \xi^{-1} p^2 Z_L(p^2) L_{\mu\nu}\right]\delta^{ab}$$

Order parameter and EoS of pure Yang-Mills

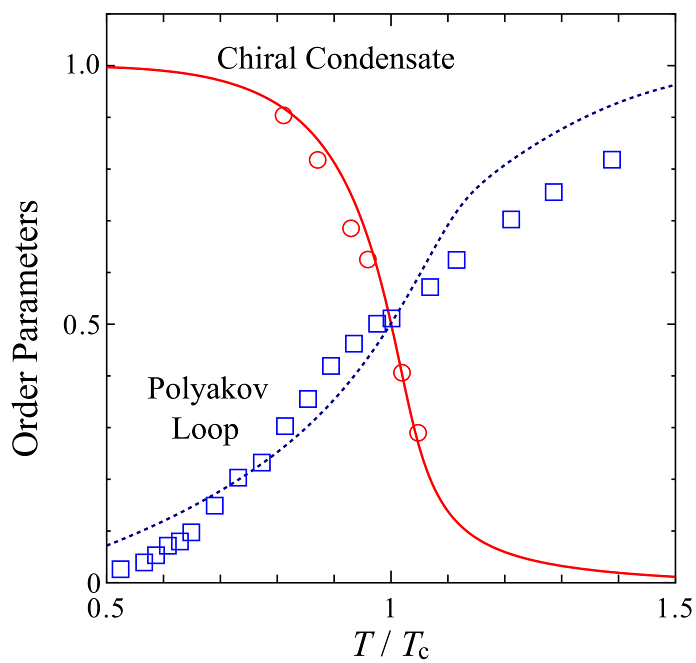
Success mainly in recovering the transition temperature !

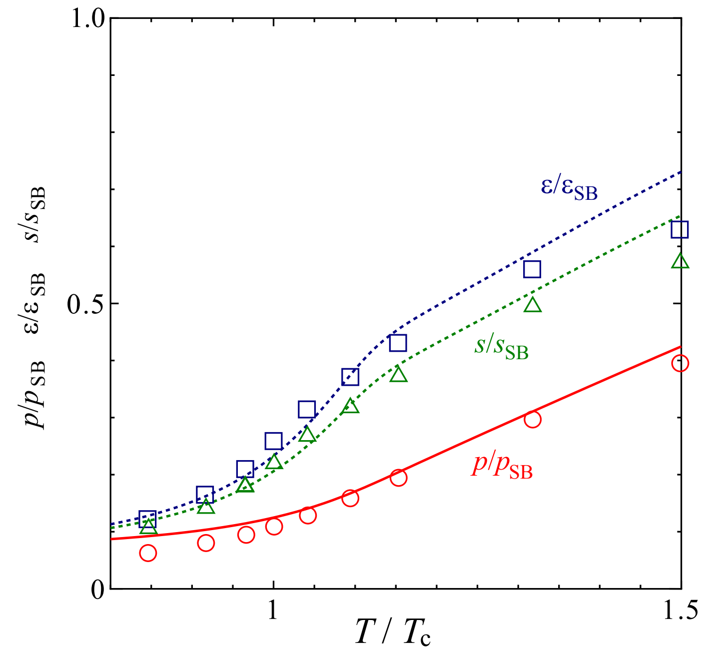




In the same paper, based on schematic lattice propagators, the "order parameters" and EoS of full QCD have been presented :

Order parameters and EoS of full QCD





Our recent paper : arXiv:1212.1102

"Landau gauge gluon and ghost propagators from lattice QCD with $N_f = 2$ twisted mass fermions at finite temperature"

R. Aouane, F. Burger, E.-M. I., M. Müller-Preussker, A. Sternbeck

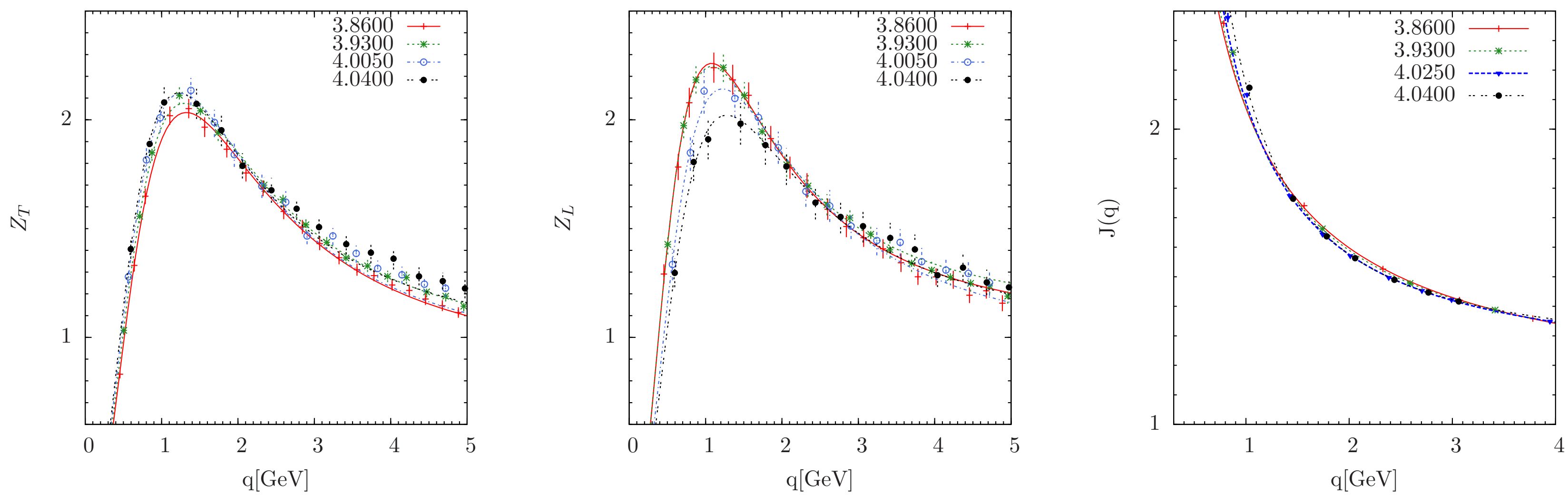
is an unquenched propagator study for twisted mass ensembles of the tmfT collaboration, leading to continuum parametrizations in the momentum ranges :

- $0.4\ GeV < q < 3.0\ GeV$ **for the gluon propagators (perfect fit !) fitting parameter b^2 in the Grivov-Stingl fit compatible with zero (no splitting in complex conjugate poles visible in this momentum range !)**
- $0.4\ GeV < q < 4.0\ GeV$ **for the ghost propagator (less good, fit correct within few percent, a mass term $m_{\rm gh}$ would't help),**

Done for various temperatures in the range $180\ MeV < T < 260\ MeV$**.**

Renormalized propagators for renormalization scale $\mu = 2.5\ GeV$

The unrenormalized dressing functions,
for the transverse gluon Z_T (left panel),
for the longitudinal gluon Z_L (middle panel)
and for the ghost J (right panel) for various temperatures,
B12 for $m_\pi = 398$ MeV

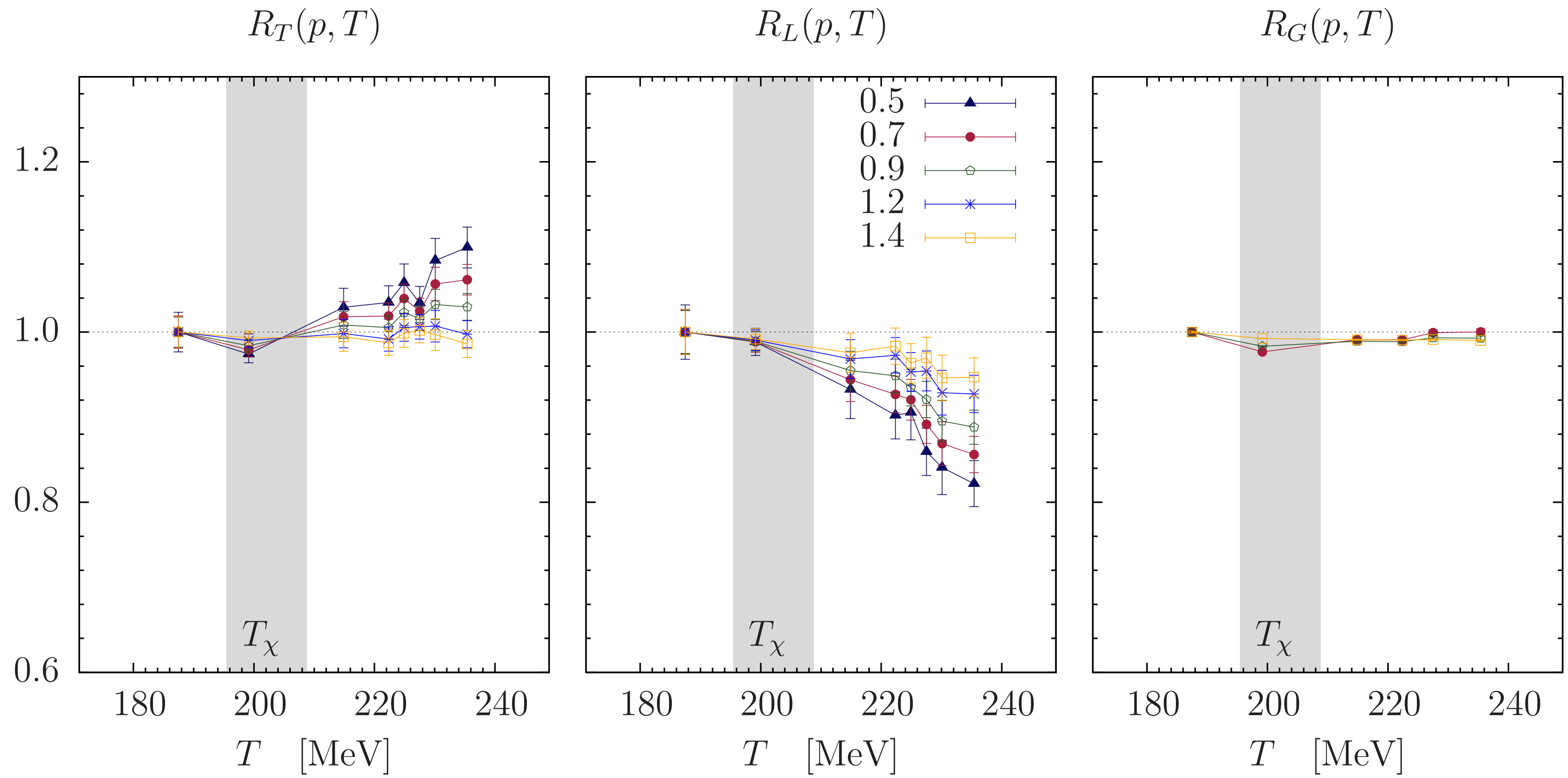


To describe the T-dependence, ratios of the renormalized dressing functions or propagators

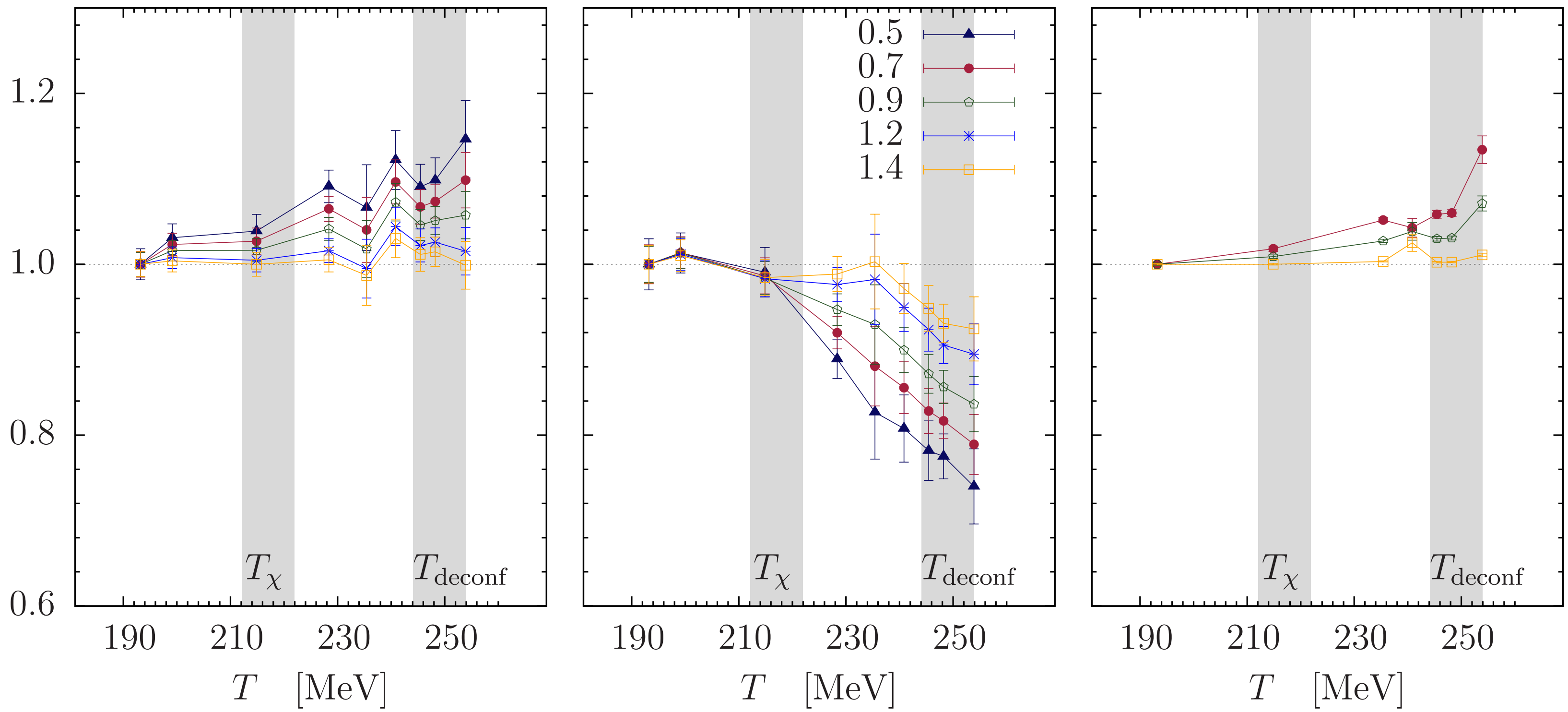
$$\begin{aligned} R_{T,L}(q,T) &= D^{ren}_{T,L}(q,T)/D^{ren}_{T,L}(q,T_{\min}), \\ R_G(q,T) &= G^{ren}(q,T)/G^{ren}(q,T_{\min}) \end{aligned}$$

are shown as functions of the temperature T for 6 fixed momentum values $q \neq 0$, and for different pion masses. For better visibility, ratios are taken with respect to the lowest temperature $T_{\min}$.

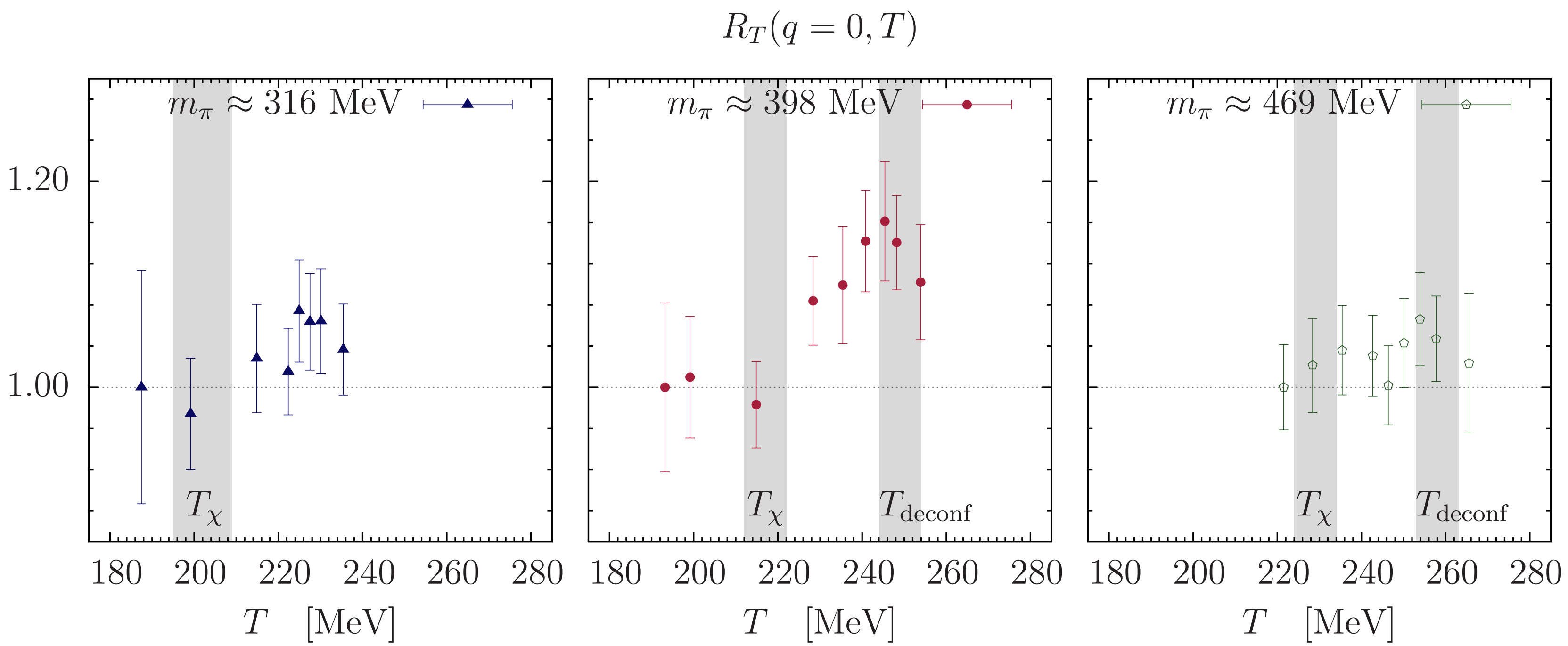
A12 with $m_\pi = 316$ MeV



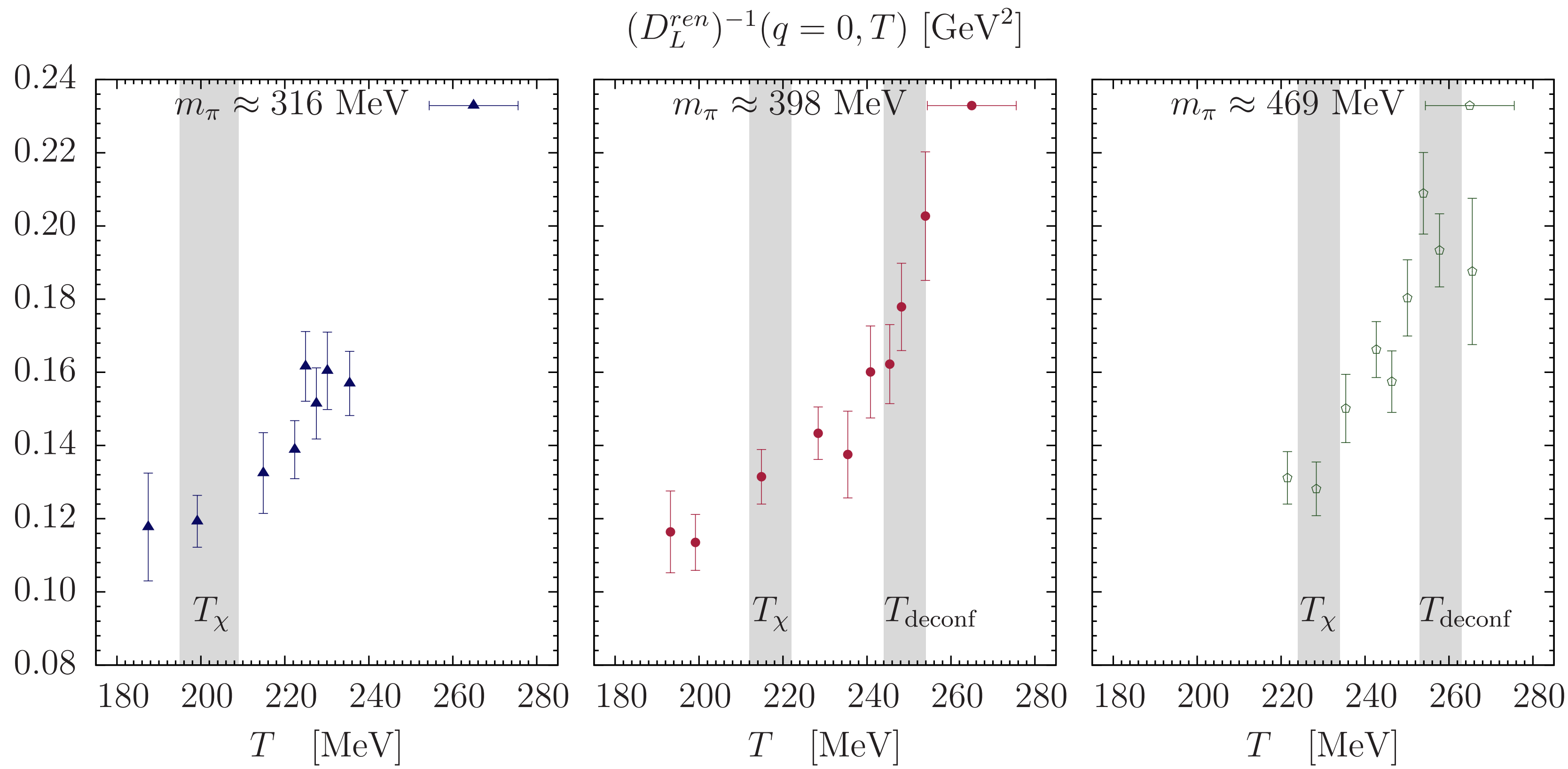
B12 with $m_\pi = 398$ MeV



The ratio R_T at zero momentum for the three pion mass values



The inverse renormalized longitudinal gluon propagator $(D_L^{ren})^{-1}$ at zero momentum



7. Conclusions and Outlook

- **The $N_f = 2$ crossover structure and the investigation of its chiral limit are close to completion.**
- **The results are in fair agreement with other results with Wilson fermions (DIK collaboration) and other lattice fermions.**
- **The EoS is will be presented soon in full detail (F. Burger). First reliable results have been presented at Lattice 2012.**
- **Chiral and deconfinement crossover are not strictly coincident.**
- **The effect of the crossover on the gluon propagator is weak. Masses too large ! Longitudinal gluon propagator most sensitive.**
- **Future orientation of the tmfT collaboration ? Probably, we will turn to $N_f = 2 + 1 + 1$ simulations.**