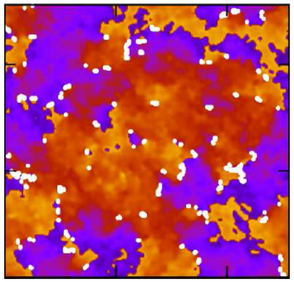
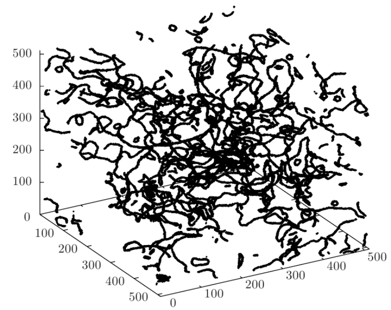


Non-thermal fixed points, vortex dynamics, and superfluid turbulence



Markus Karl

S. Erne, B. Nowak, M. Schmidt, J. Schole, D. Sexty, T. Gasenzer

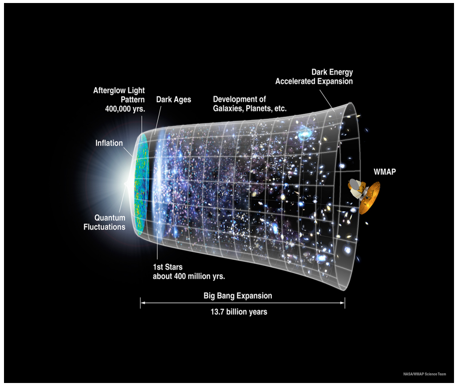
Institut für Theoretische Physik

Ruprecht-Karls Universität Heidelberg

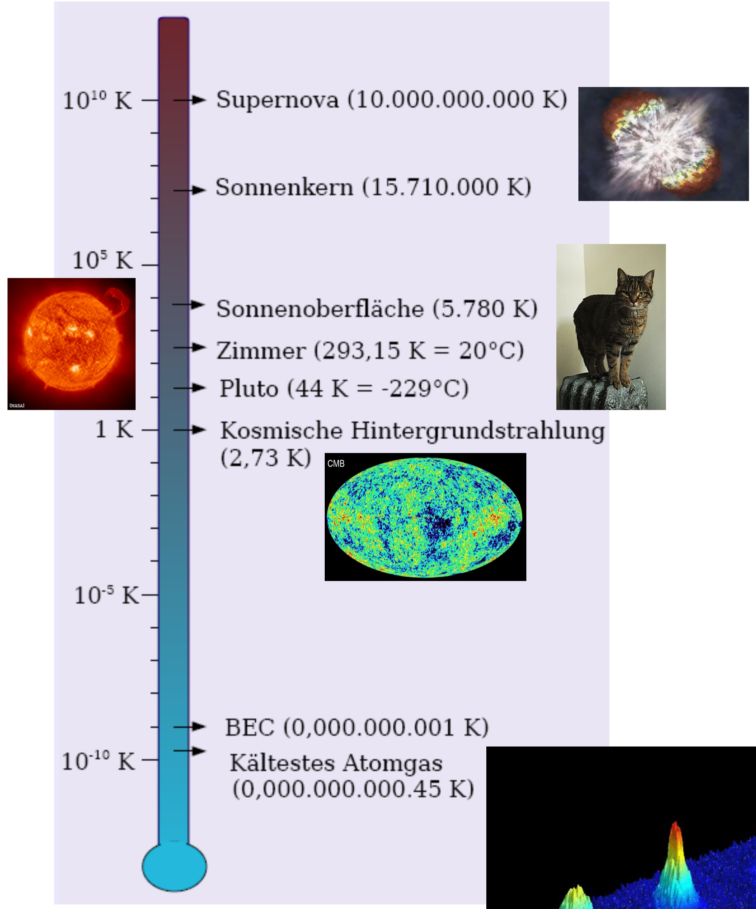
email: m.karl@thphys.uni-heidelberg.de
www: www.thphys.uni-heidelberg.de/~gasenzer

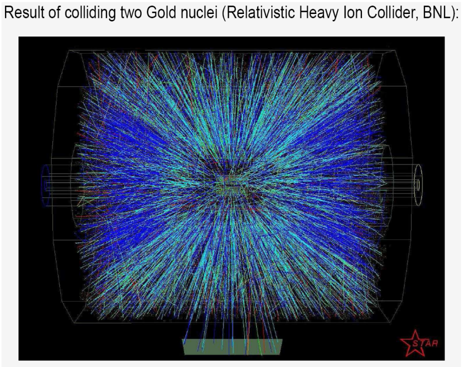
Center for Quantum Dynamics

Non-equilibrium quantum gases

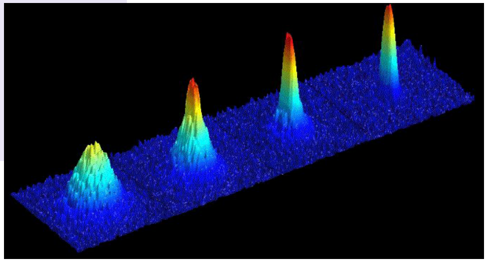


Early universe





Heavy-ion collisions



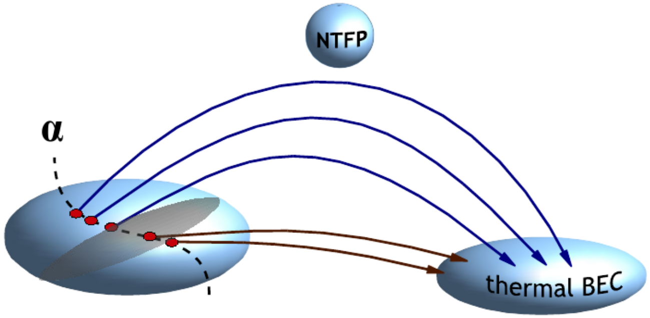
Ultracold gases

Non-equilibrium dynamics

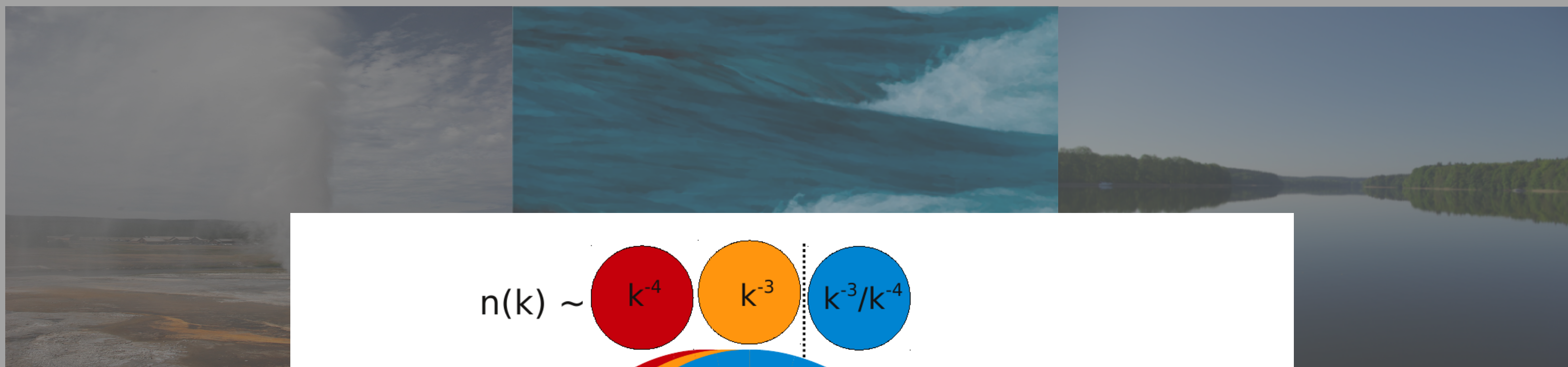
Initial state:
Far from equilibrium

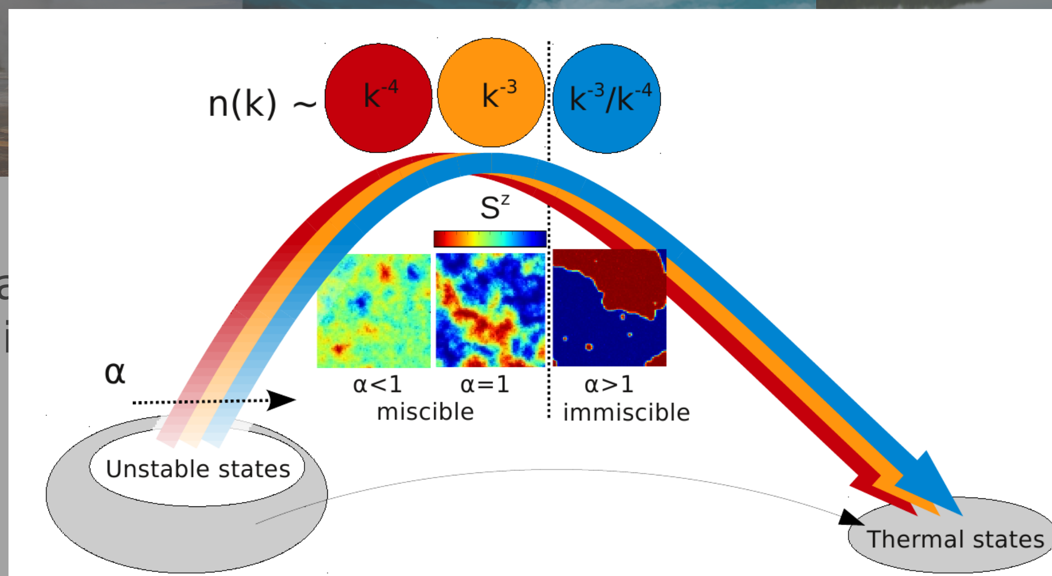
Transient state:
e. g. Turbulence
(Nonthermal fixed point)

Final state:
Thermal equilibrium



Non-equilibrium dynamics

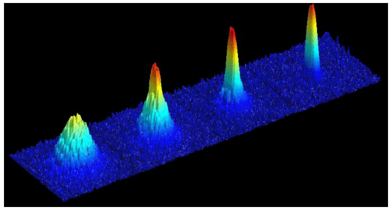




Initial sta
Far from equi

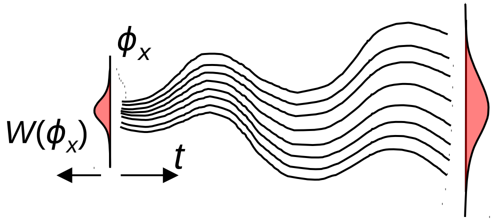
al state:
equilibrium

Semi-classical simulations



Classical field equation for ϕ(x,t):

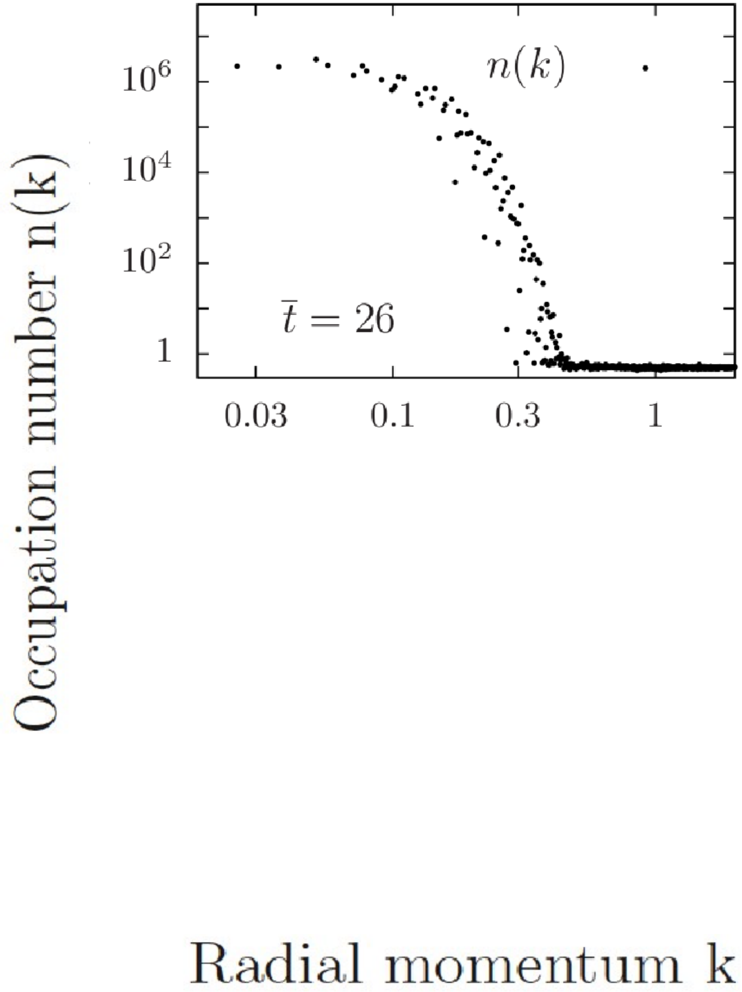
$$\mathrm{i}\partial_t \phi(\mathbf{x},t) = \left[-\frac{\nabla^2}{2m} + g|\phi(\mathbf{x},t)|^2\right]\phi(\mathbf{x},t)$$



Observables: e. g. Momentum distribution

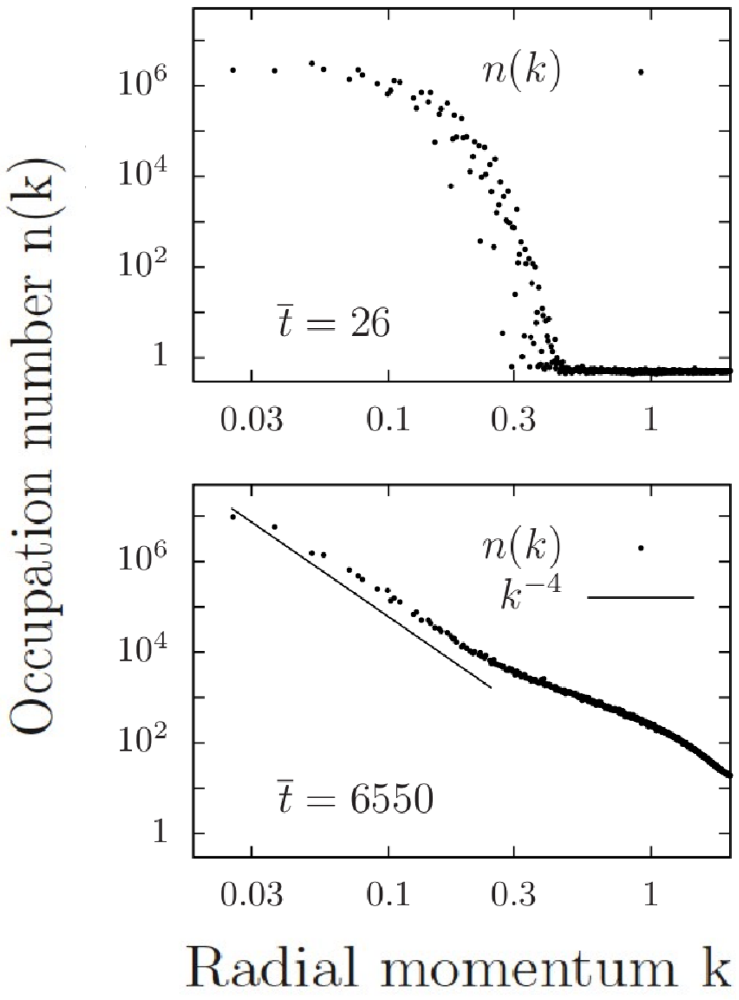
$$n(k) = \int \mathrm{d}^{d-1}\Omega_k \, \langle \phi^*(\mathbf{k})\phi(\mathbf{k})\rangle_{\text{ensemble}}$$

2D: Quench dynamics



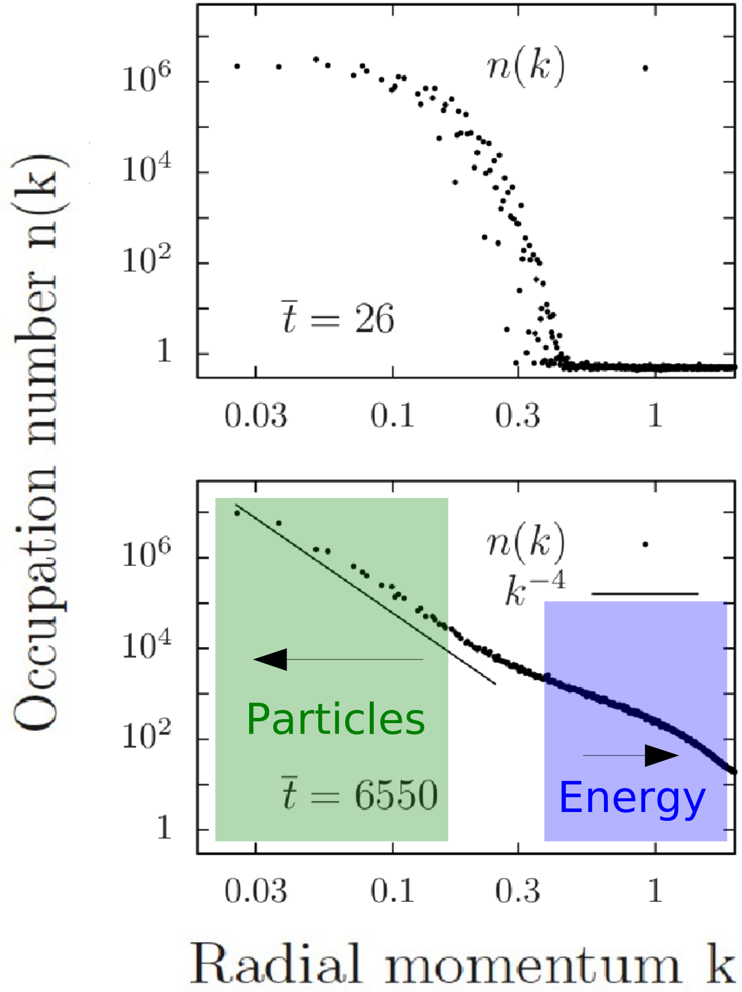
B. Nowak, D. Sexty, T. Gasenzer PRB 84(R) (2011), B. Nowak, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

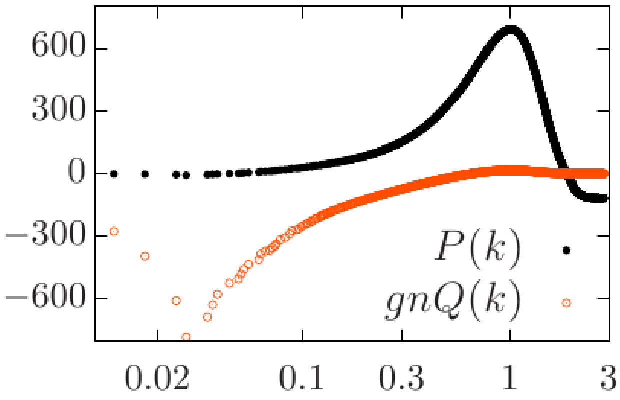
2D: Quench dynamics



B. Nowak, D. Sexty, T. Gasenzer PRB 84(R) (2011), B. Nowak, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

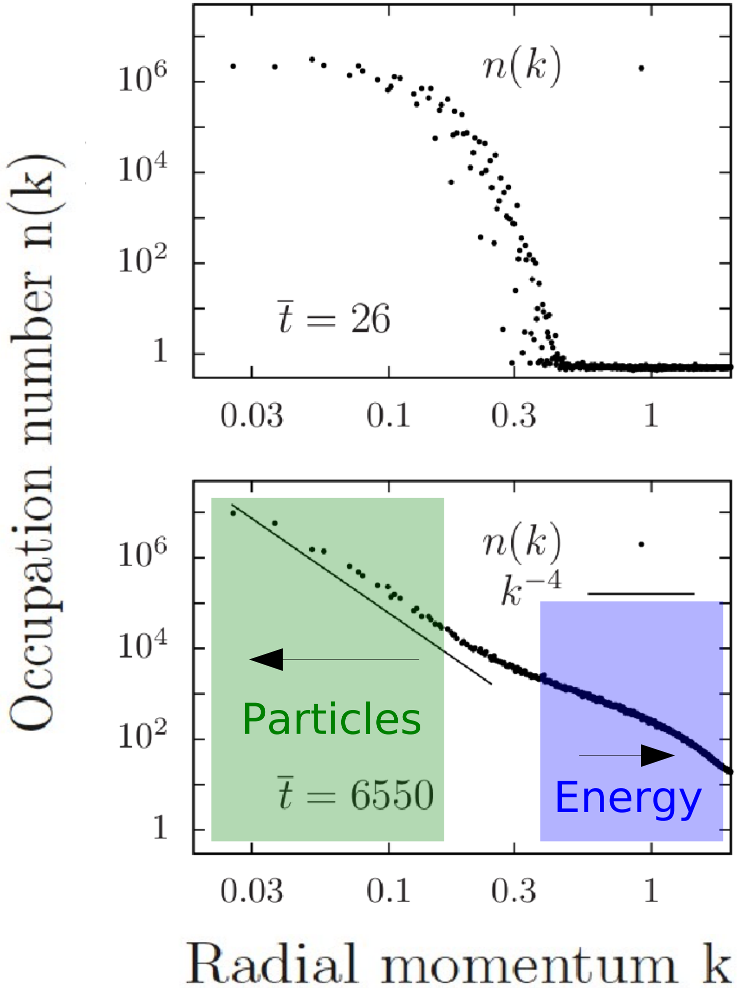
2D: Quench dynamics

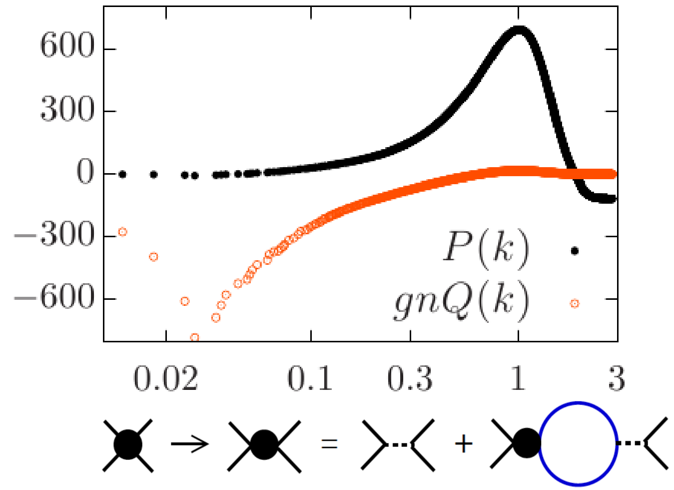




B. Nowak, D. Sexty, T. Gasenzer PRB 84(R) (2011), B. Nowak, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

2D: Quench dynamics



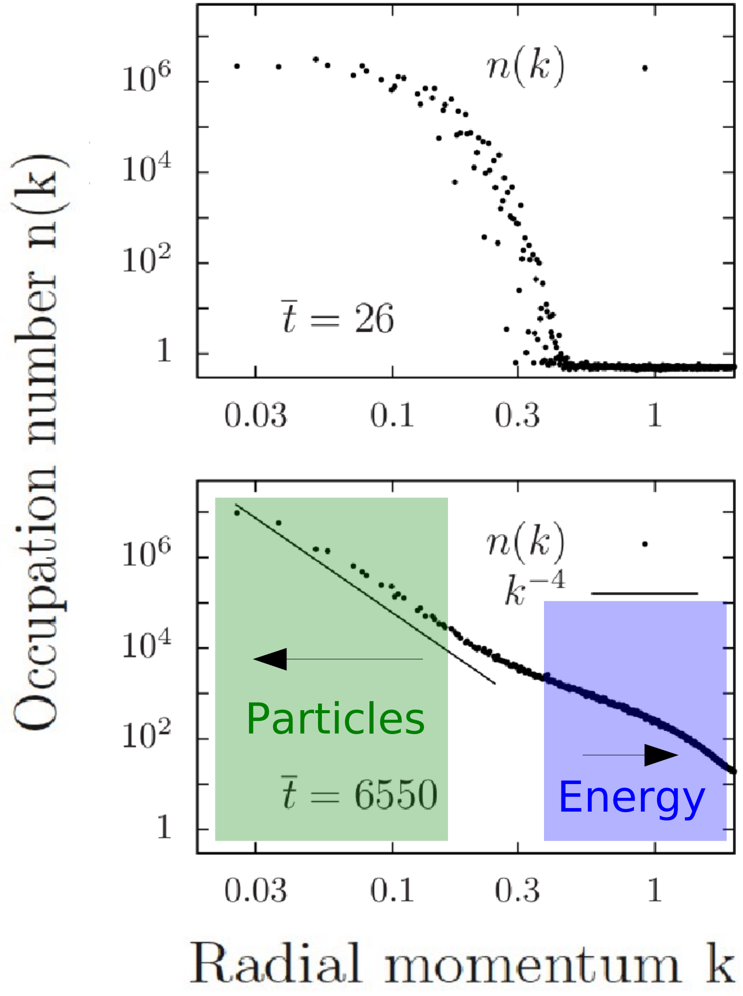


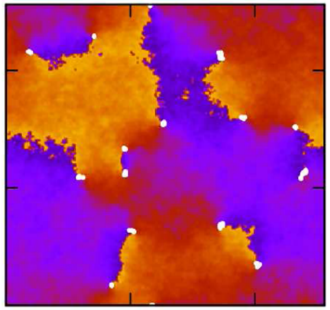
$$n^{\mathrm{IR}} \sim k^{-d-2}$$

J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603,
J. Berges, G. Hoffmeister, NPB **813** (09) 383,

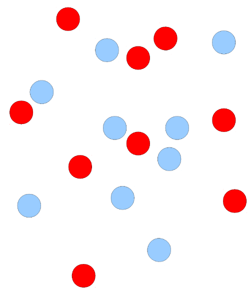
nonrelativ.:
C. Scheppach, J. Berges, T. Gasenzer PRA **81** (10) 033611

2D: Quench dynamics





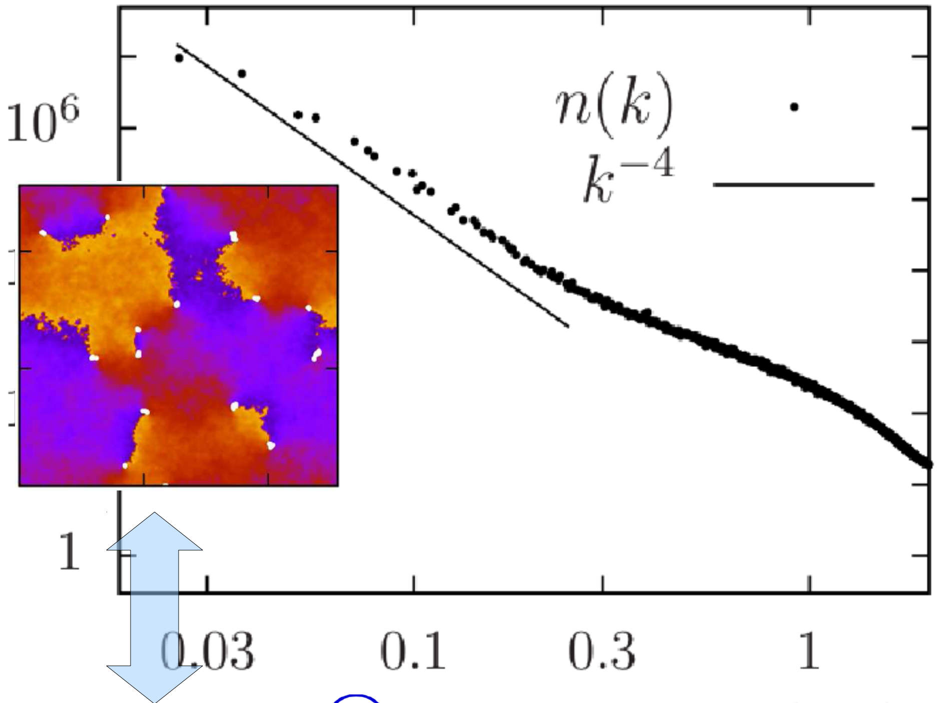
Phase φ

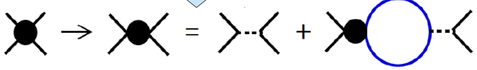


$n_k \sim k^{-4}$

B. Nowak, D. Sexty, T. Gasenzer PRB 84(R) (2011), B. Nowak, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

2D: Wave turbulence / vortex dynamics

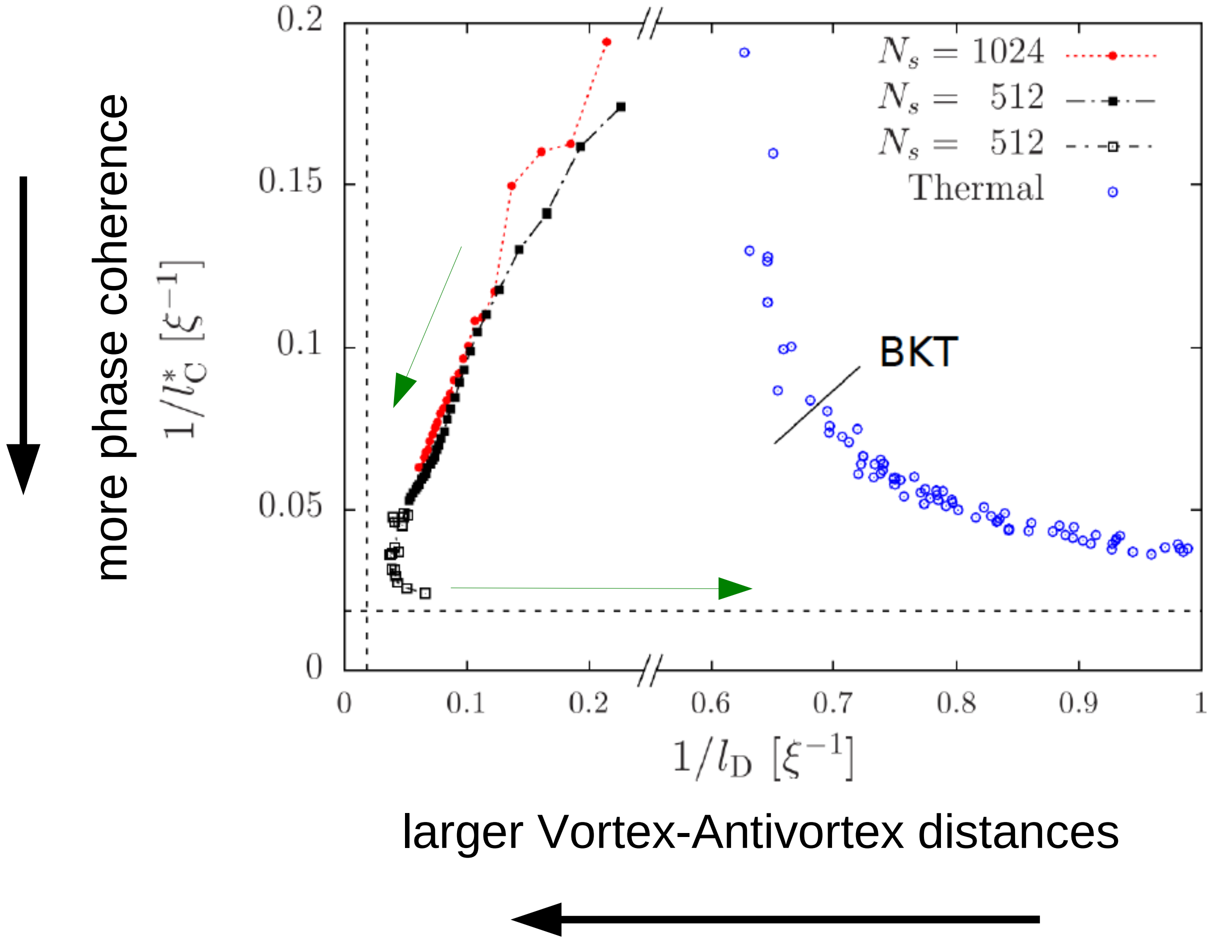


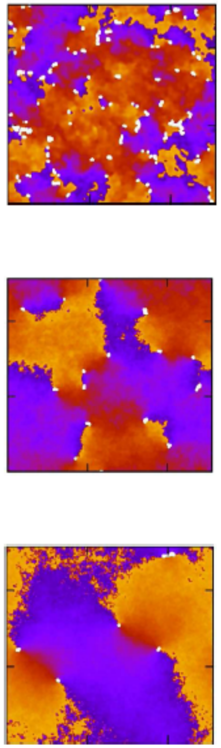


B. Nowak, D. Sexty, T. Gasenzer PRB 84(R) (2011), B. Nowak, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)
J. Schole, B. Nowak, T. Gasenzer, PRA (2012), BN., T. Gasenzer arxiv: 1206.3181

Approach of the NTFP

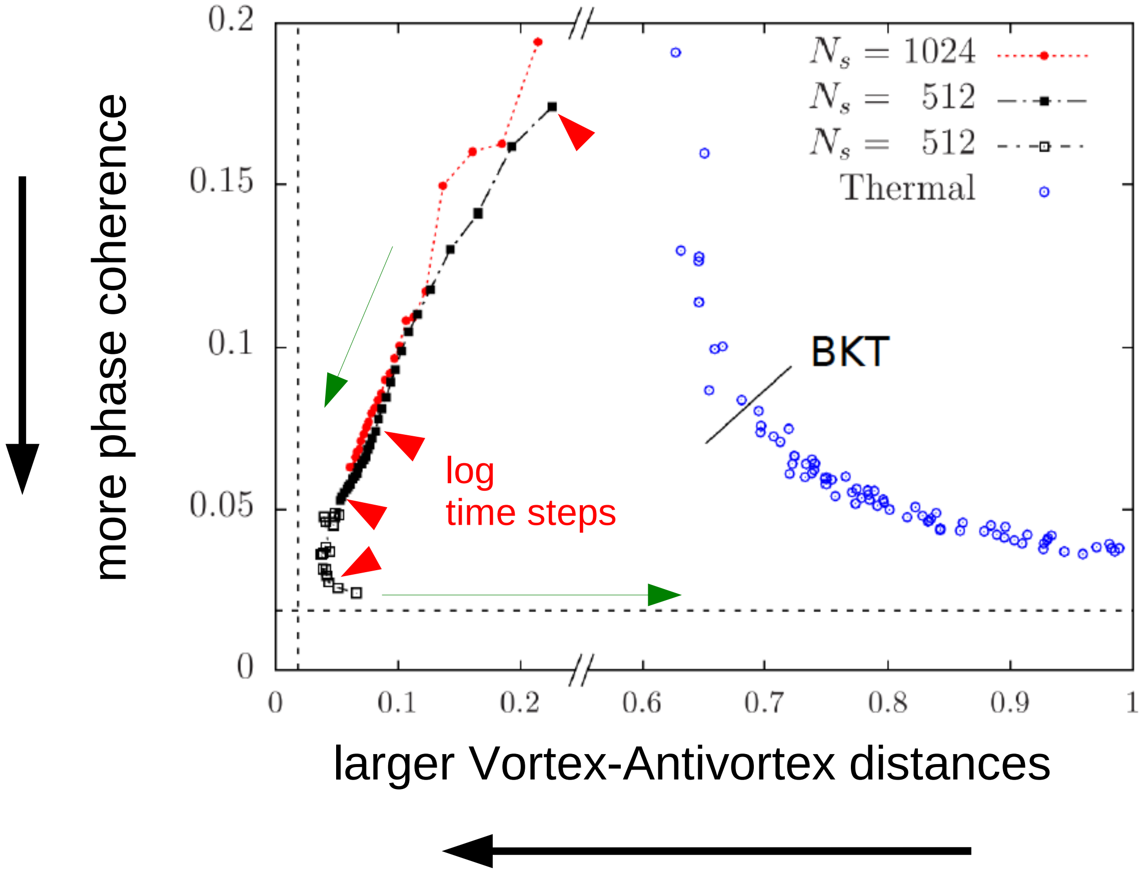
$l_C^* =$ Phase coherence length
$l_D =$ Vortex-Antivortex pair distance

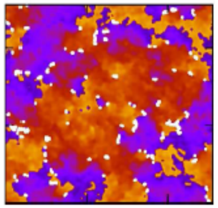


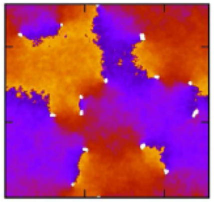


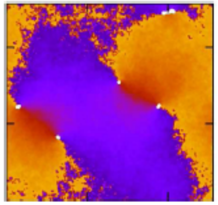
J. Schole, B. Nowak, T. Gasenzer, PRA (2012)

Approach of the NTFP



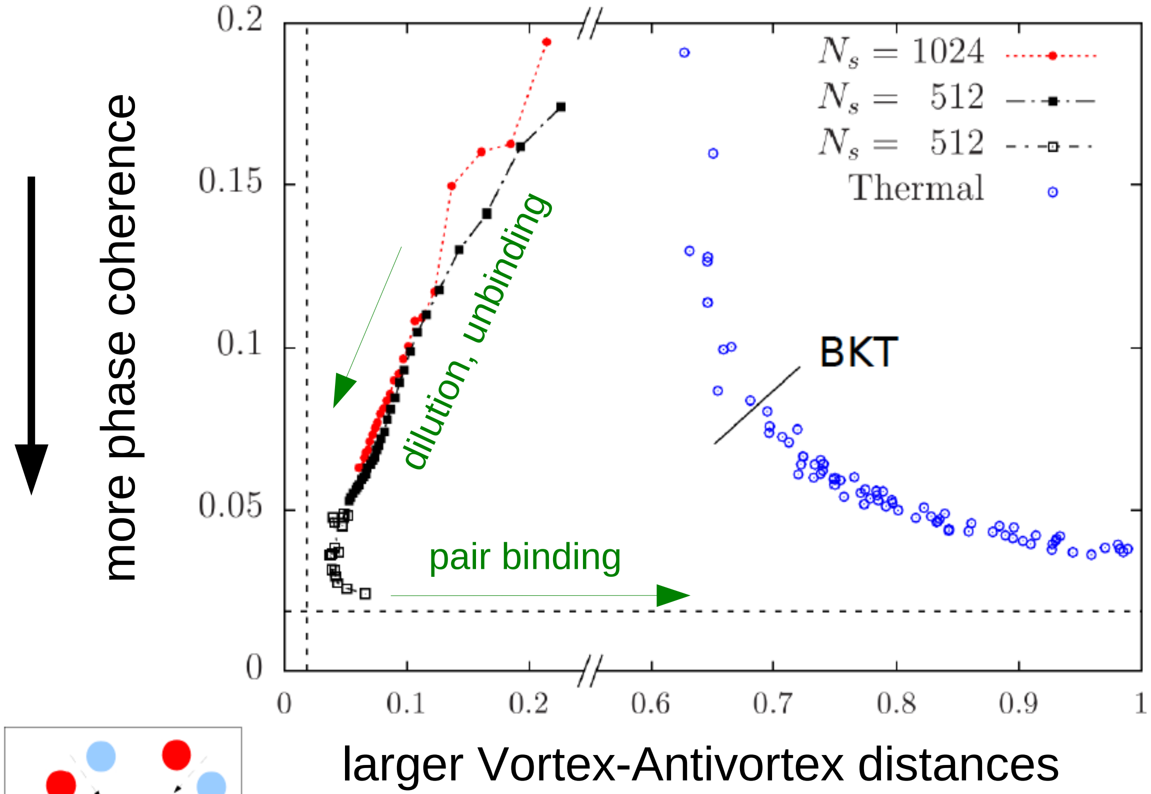


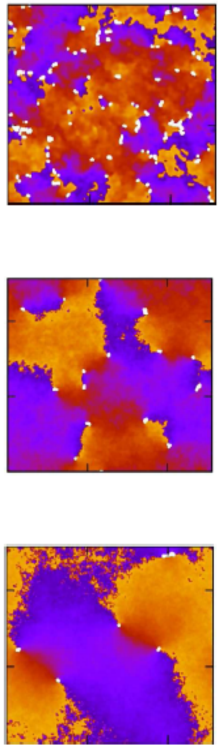


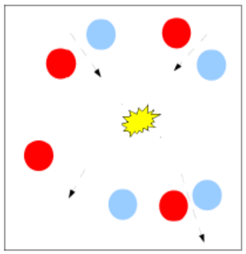


J. Schole, B. Nowak, T. Gasenzer, PRA (2012)

Approach of the NTFP





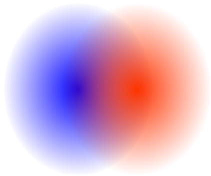


J. Schole, B. Nowak, T. Gasenzer, PRA (2012)

NTFPs in a 2-component Bose gas

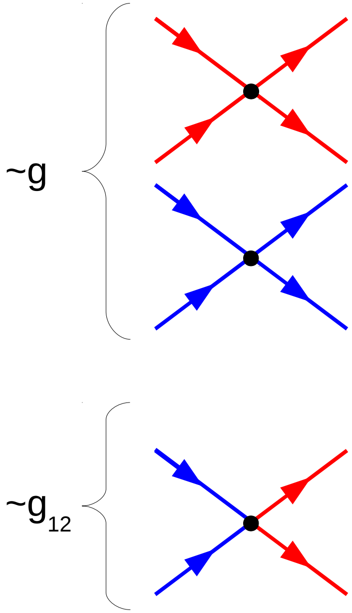
2D: 2-component Bose Gas

immiscible
$g_{12} > g$



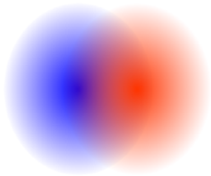
miscible
$g_{12} < g$

$\alpha = g_{12}/g$



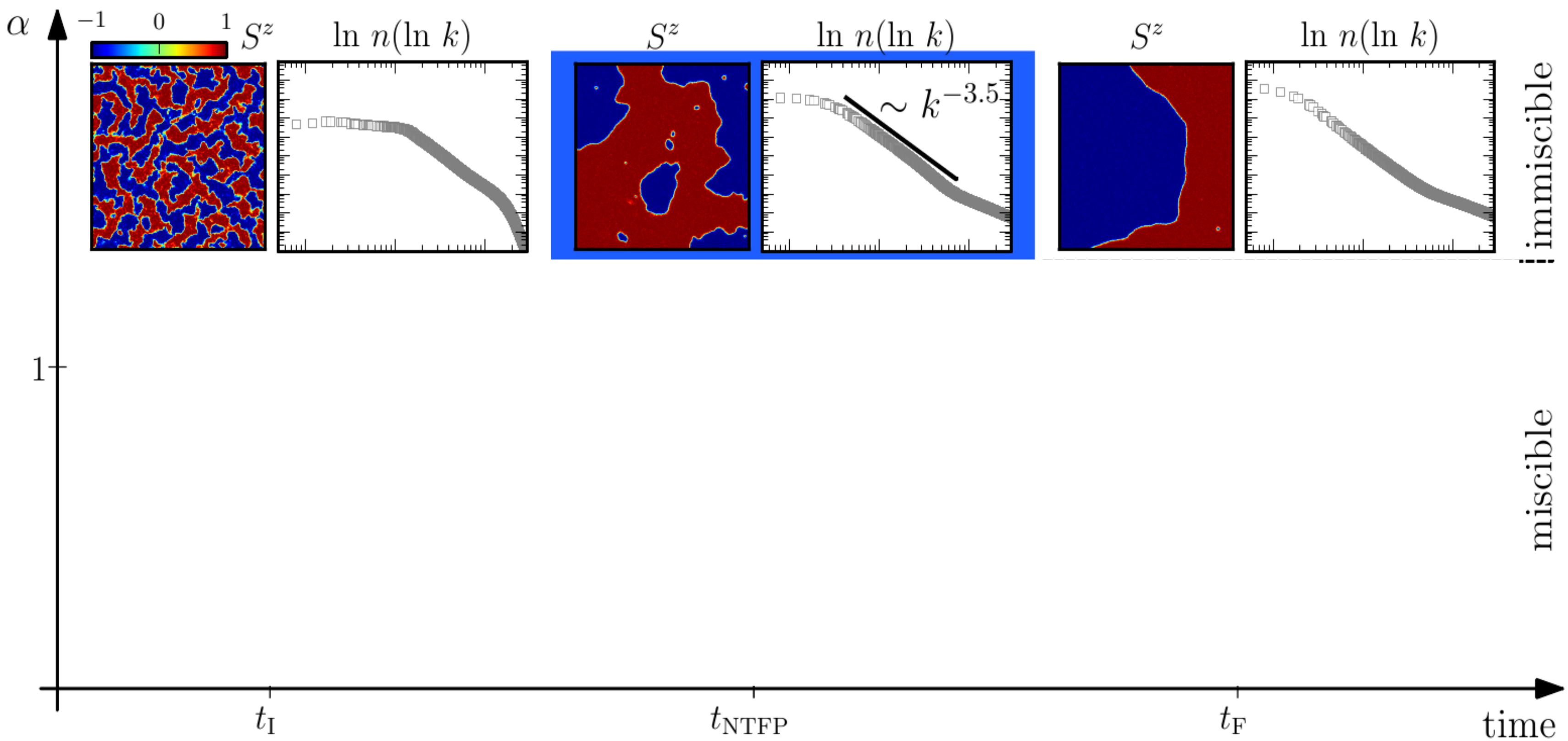
2D: 2-component Bose Gas

immiscible
$g_{12} > g$



miscible
$g_{12} < g$

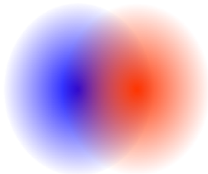
$\alpha = g_{12}/g$



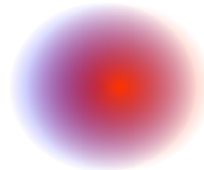
MK, B. Nowak, T. Gasenzer unpublished

2D: 2-component Bose Gas

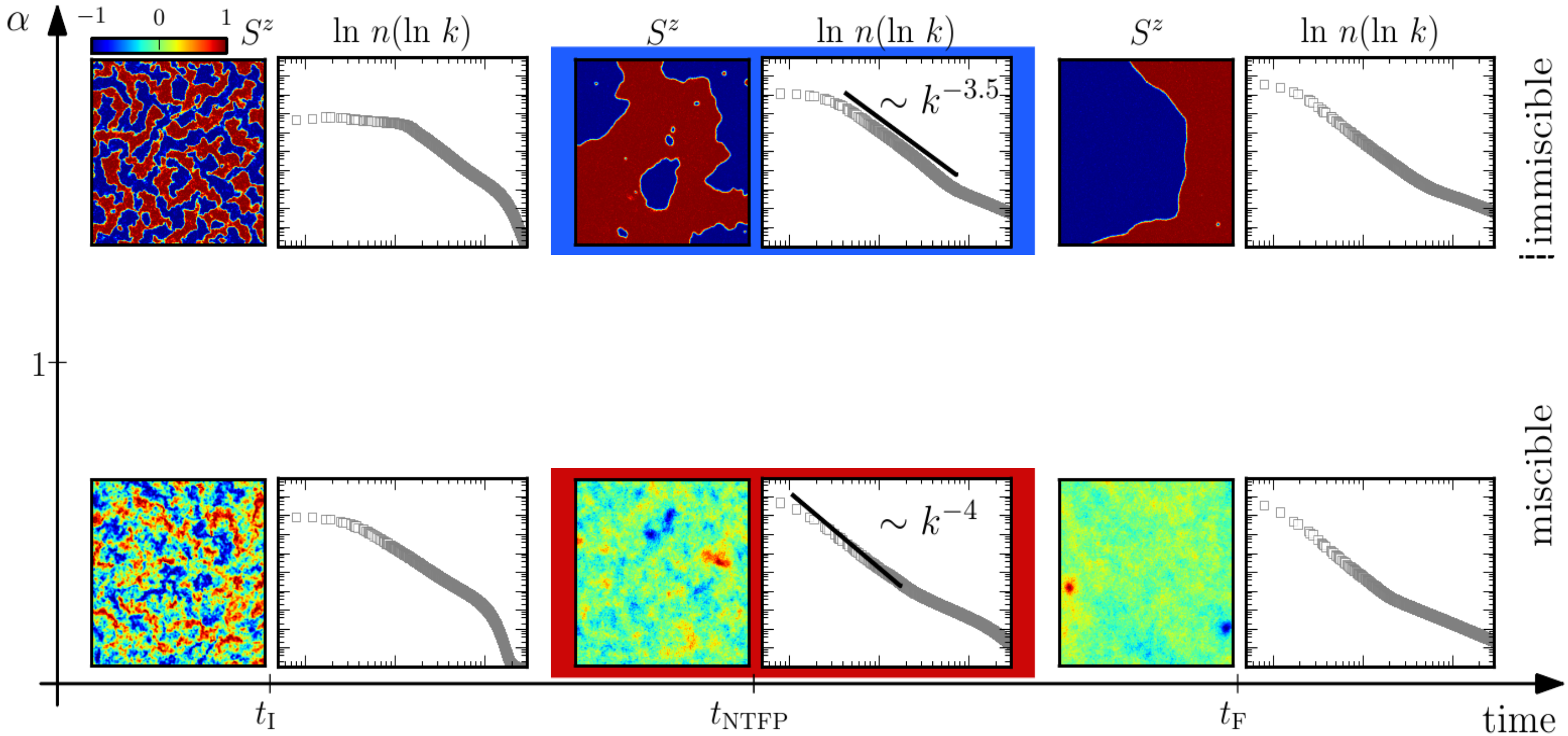
immiscible
$g_{12} > g$



miscible
$g_{12} < g$



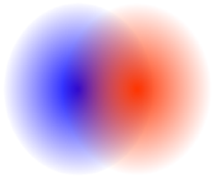
$\alpha = g_{12}/g$



MK, B. Nowak, T. Gasenzer unpublished

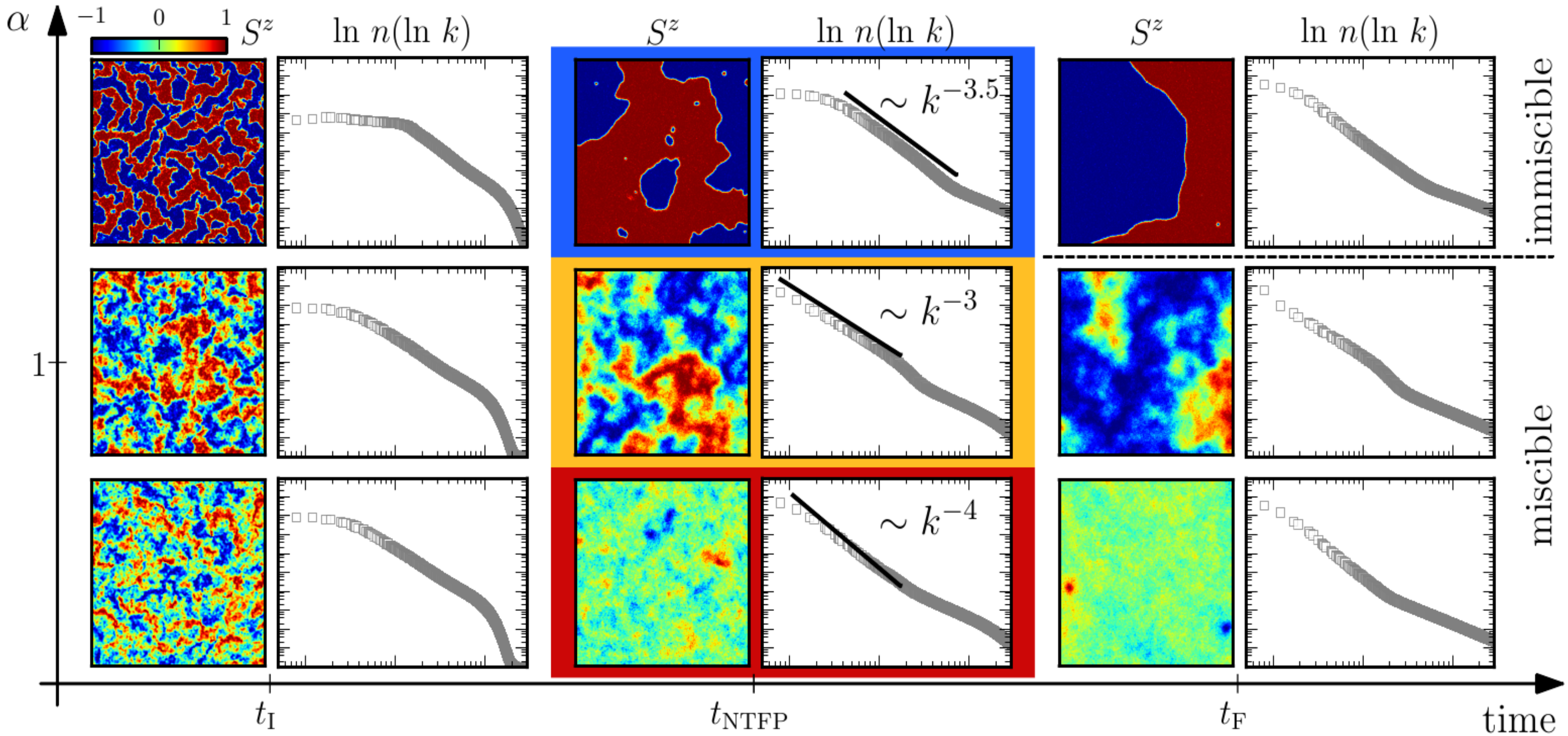
2D: 2-component Bose Gas

immiscible
$g_{12} > g$



miscible
$g_{12} < g$

$\alpha = g_{12}/g$

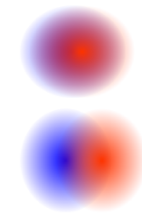


MK, B. Nowak, T. Gasenzer unpublished

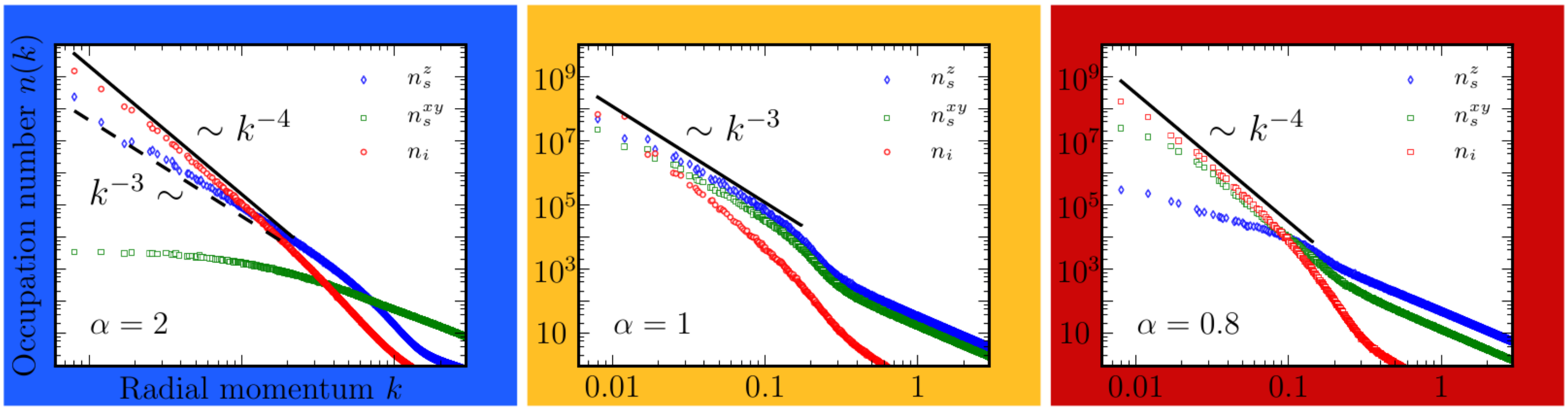
2C-BEC: hydrodynamic aspect

miscible
$g_{12} < g$

immiscible
$g_{12} > g$



immiscible | transition point | miscible



Incompressible energy spectrum

z-Spin excitation spectrum

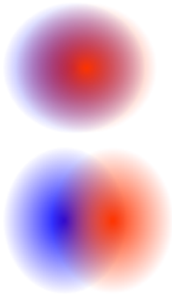
xy-Spin excitation spectrum

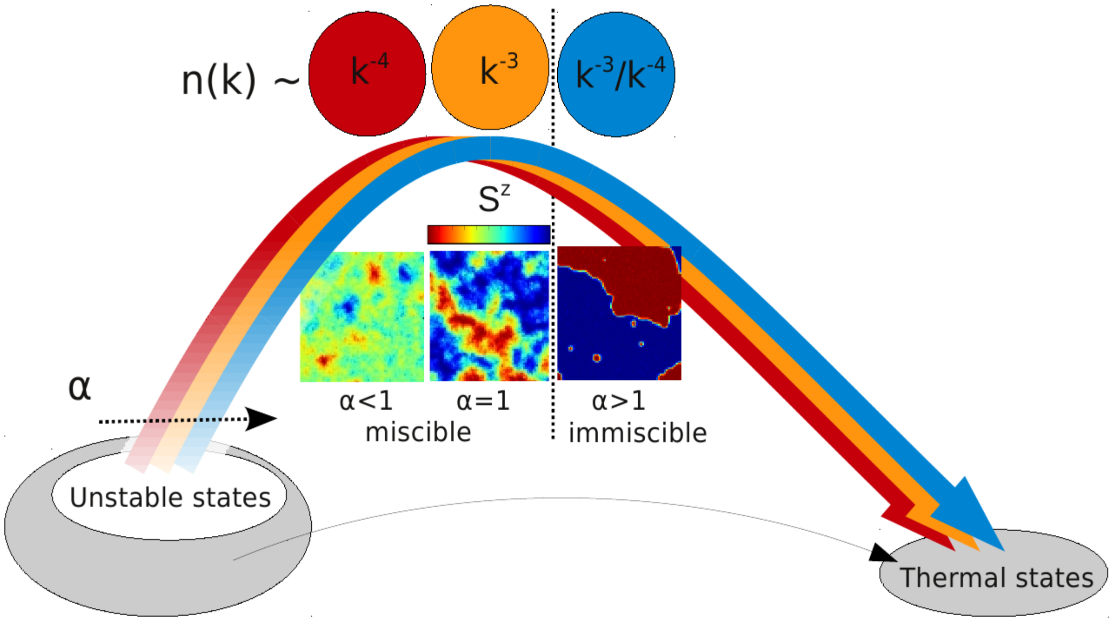
MK, B. Nowak, T. Gasenzer unpublished

Tunable universality

miscible
$g_{12} < g$

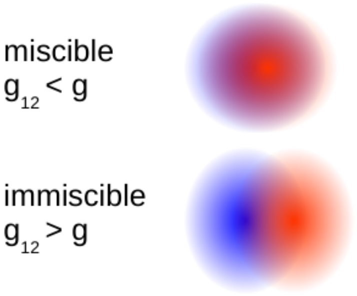
immiscible
$g_{12} > g$

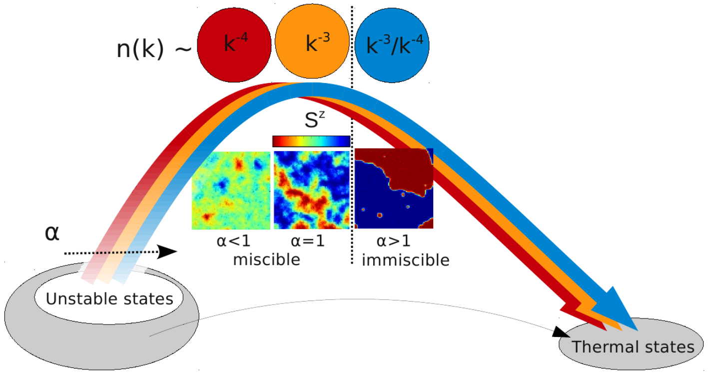




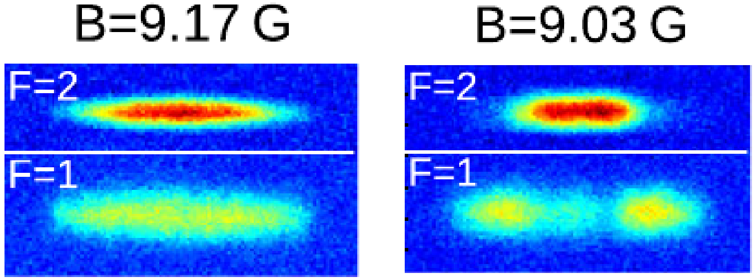
MK, B. Nowak, T. Gasenzer unpublished

Tunable universality





^{87}Rb-Experiment:
Oberthaler group (Heidelberg)

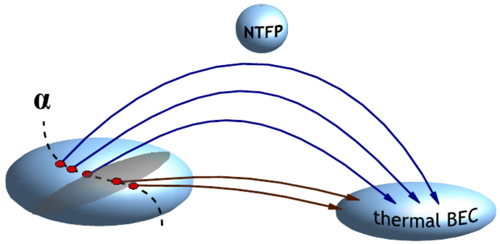


E. Nicklas et al. PRL (2012)

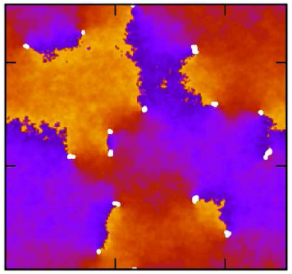
see talk of
Prof Markus Oberthaler

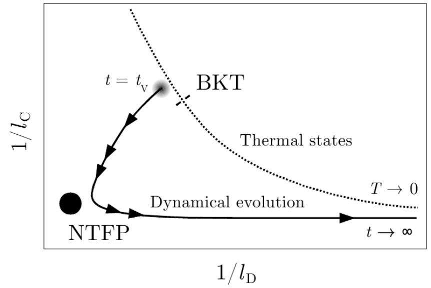
Summary

Non-thermal fixed points (NTFP)

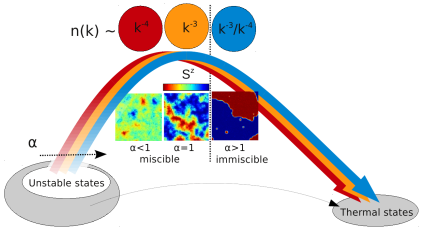


Superfluid turbulence in 2D



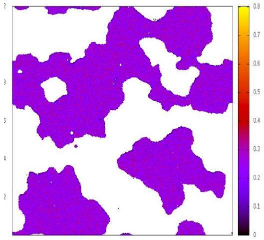


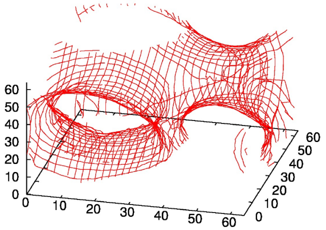
Dynamics near the NTFP in 2D

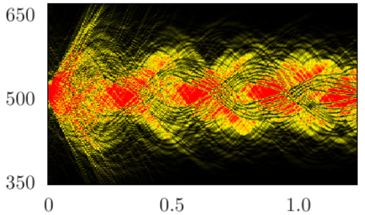


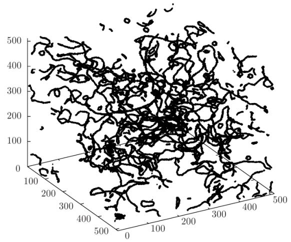
Refined situation in 2-comp BEC

NTFP concept is applicable to various systems



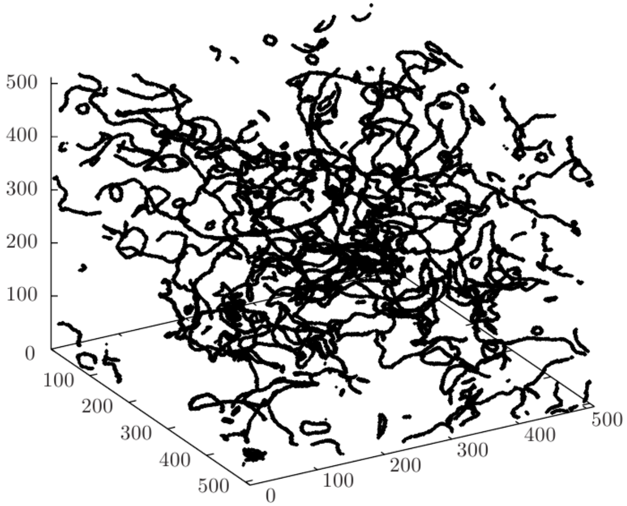


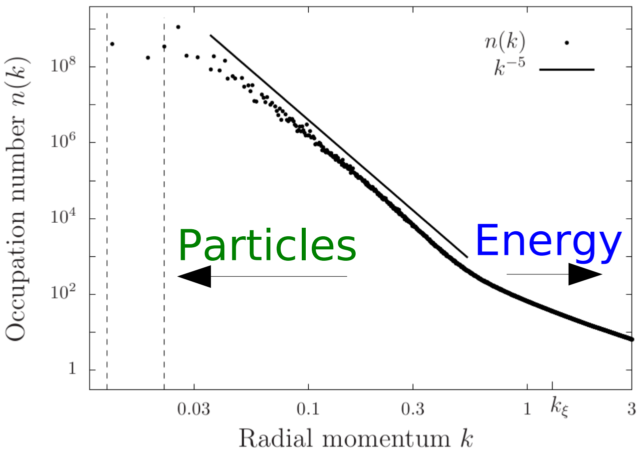




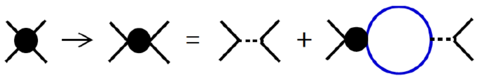
Supplementary Slides I

3D: Non-thermal fixed point





IR: $\varsigma = d+2$
UV: $\varsigma = d$

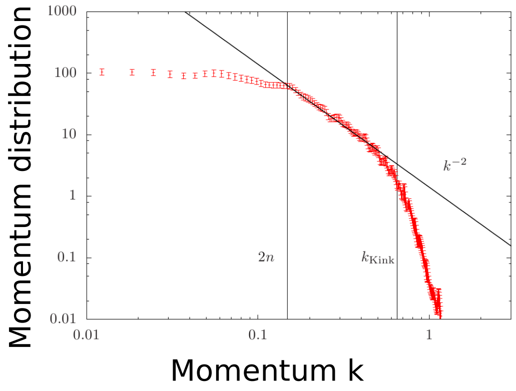


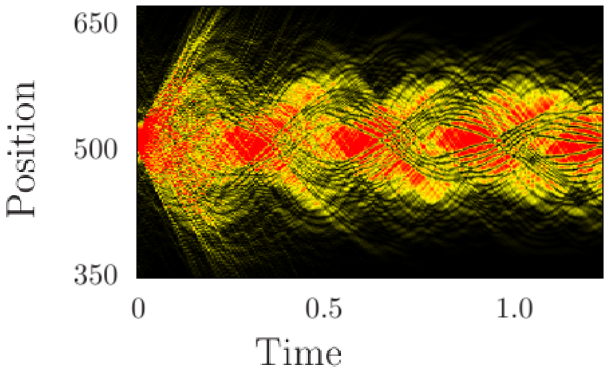
Berges, Rothkopf, Schmidt PRL (2008)
Scheppach, Berges, Gasenzer PRA (2010)

Vortices ← Spectrum n(k) → QFT

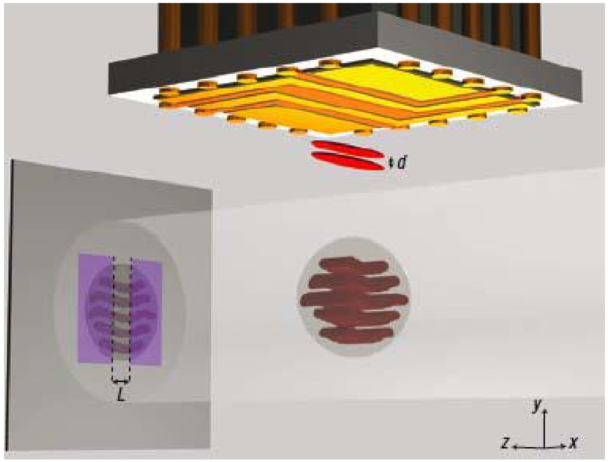
B. Nowak, D. Sexty, T. Gasenzer PRB (2011), B. Nowak, J. Schole, D. Sexty, T. Gasenzer PRA (2012)
B. Nowak, T. Gasenzer arxiv: 1206.3181

1D: Quench dynamics and solitons



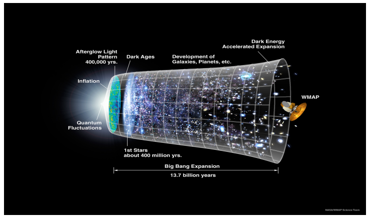


^{87}Rb-Experiment:
Schmiedmeyer group (Vienna)



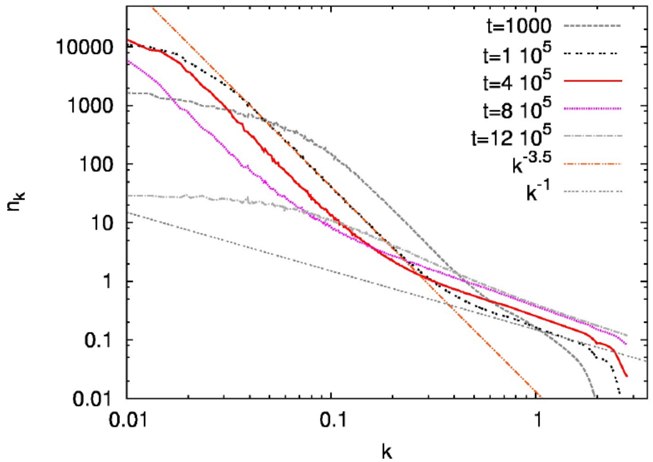
M. Schmidt, S. Erne, B. Nowak, D. Sexty, T. Gasenzer NJP (2012)

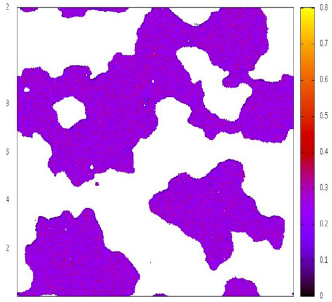
Relativistic simulations O(2)



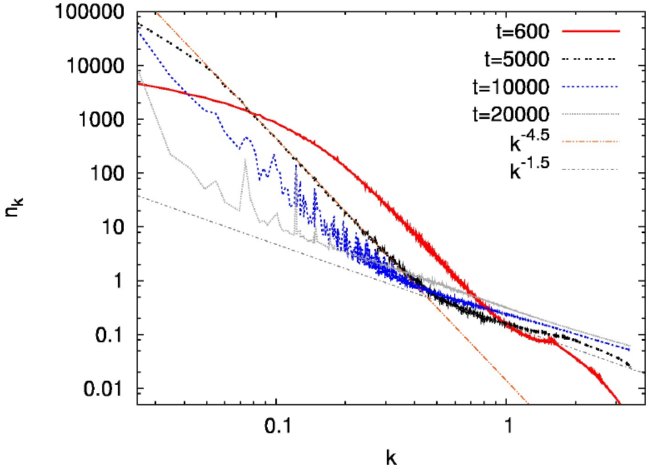
Classical field equation: $$\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$$

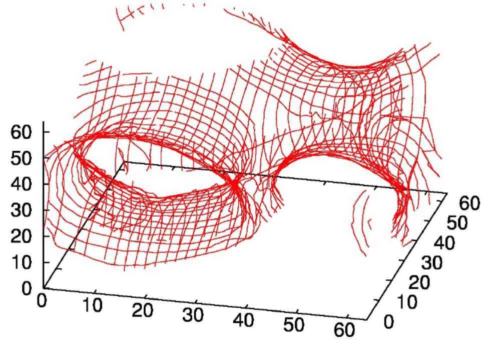
d=2





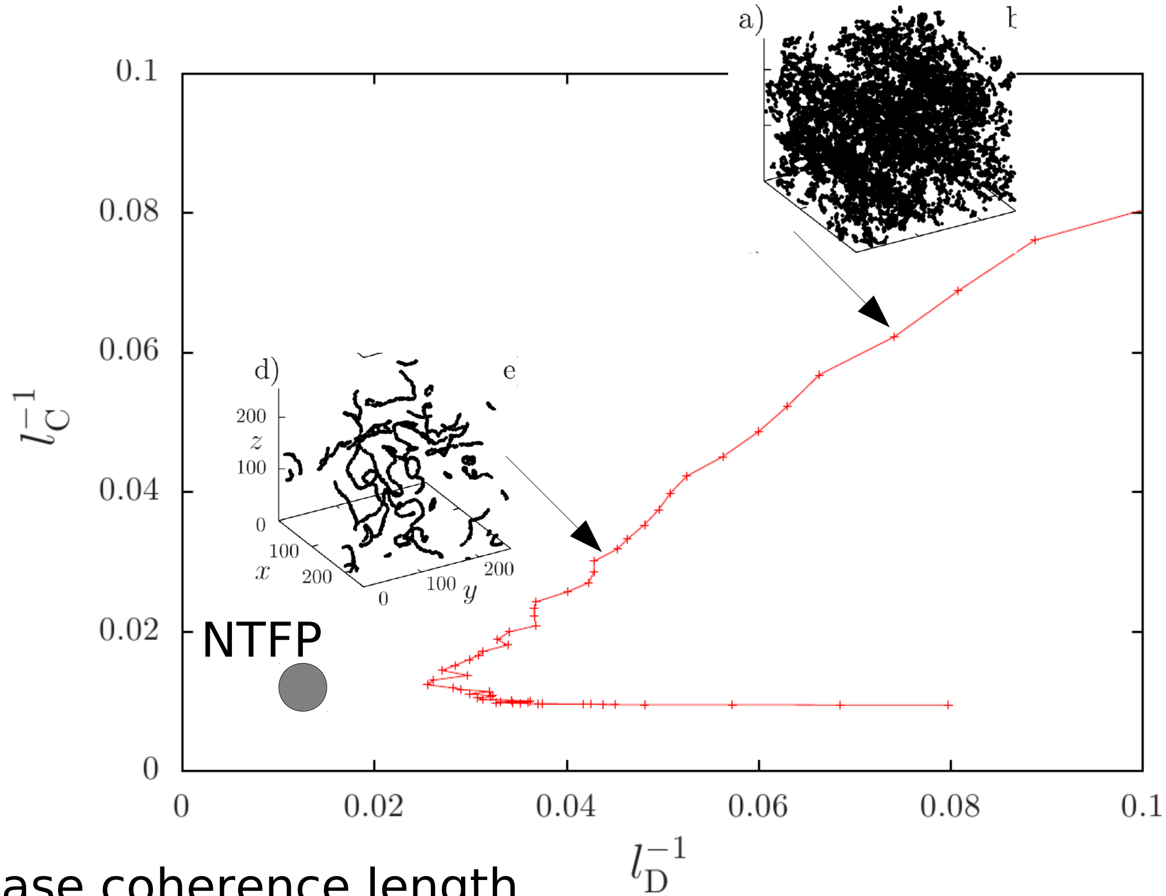
d=3





T. Gasenzer, B. Nowak, D. Sexty PLB (2012)

3D: Non-thermal fixed point

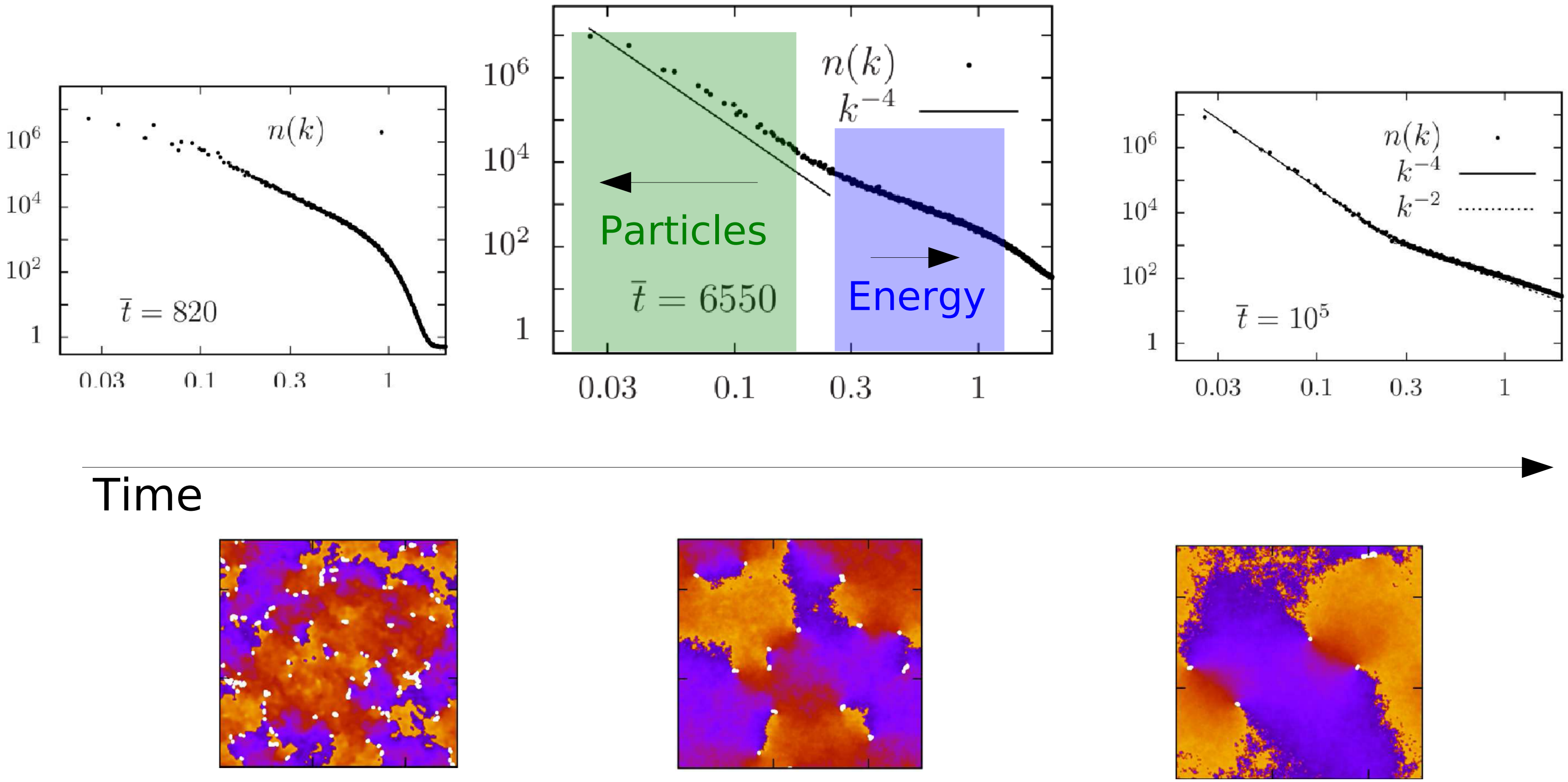


l_c Phase coherence length

l_D Vortex ring radius

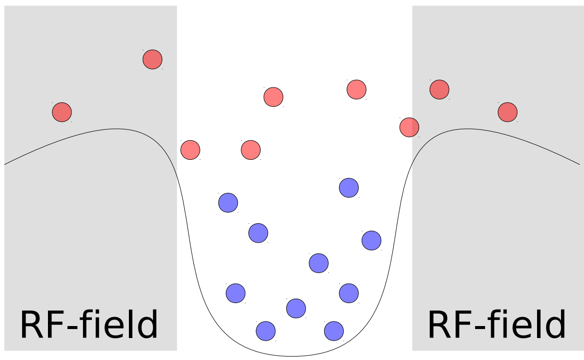
B. Nowak, J. Schole, T. Gasenzer in preparation

2D: Phase ordering dynamics

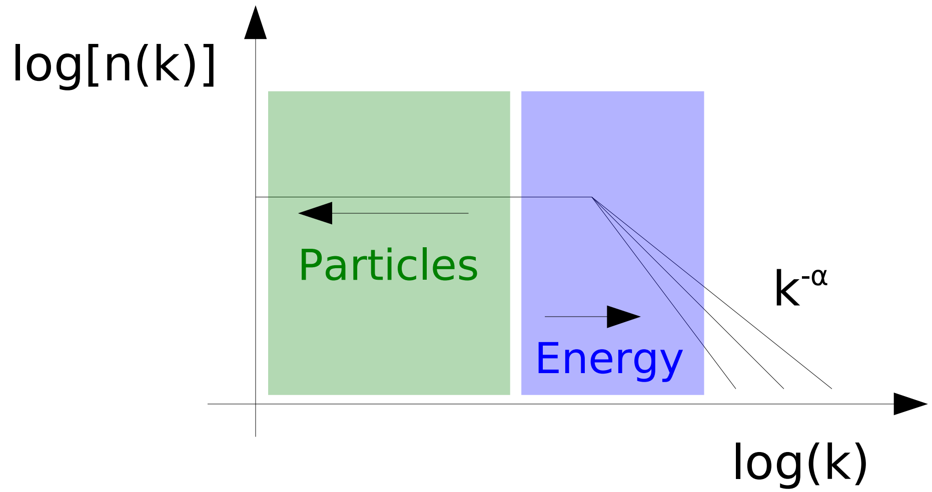


Supplementary Slides II

3D: Bose condensation



Evaporative cooling

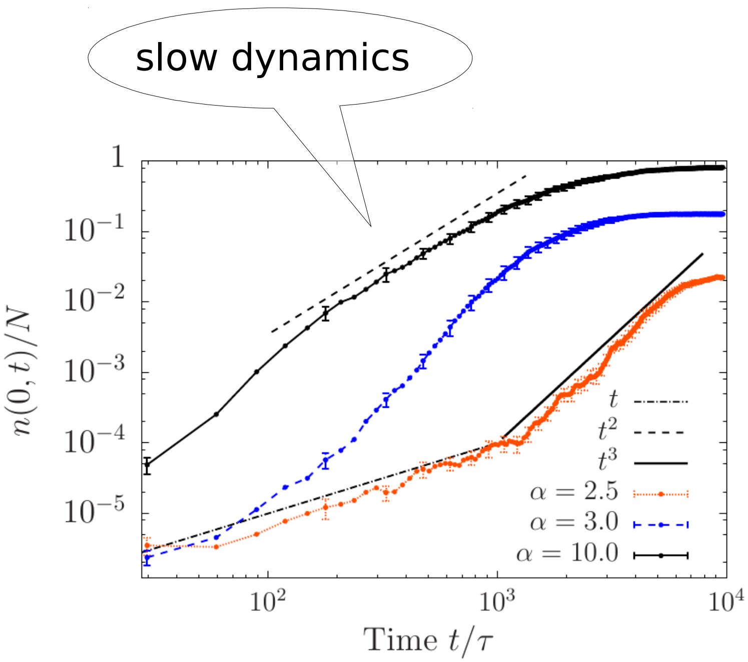


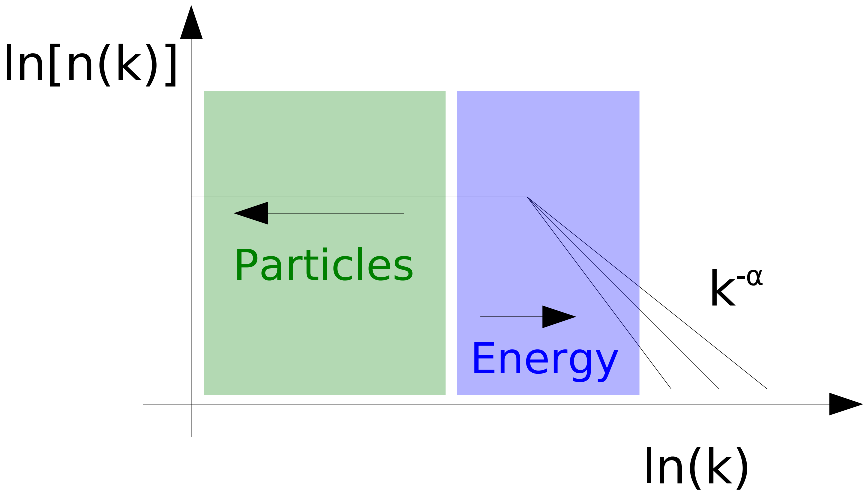
Experiments: Hänsch, Esslinger (2002), Esslinger (2007), Hadzibabic (2012)

Condensation dynamics: Kagan, Svistunov, Shlyapnikov ('90s), Semikoz, Tkachev (1995), Berloff (2002), Anderson, Davis (2008), Blaizot, McLerran(2012), Berges, Sexty (2012)

BN., T. Gasenzer arxiv: 1206.3181

3D: Bose condensation

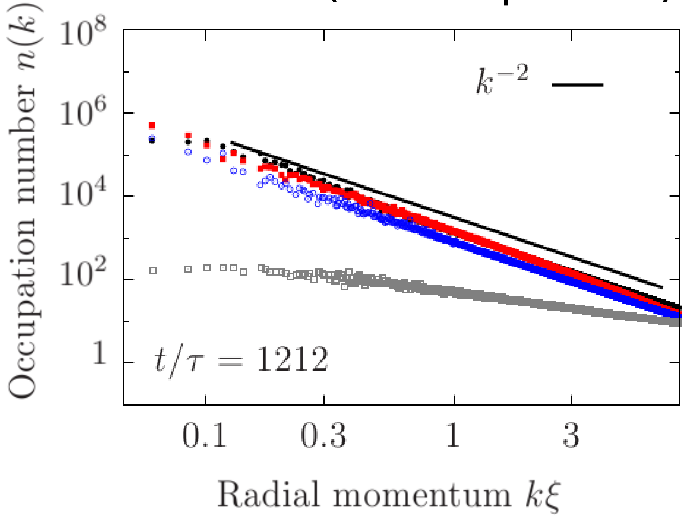




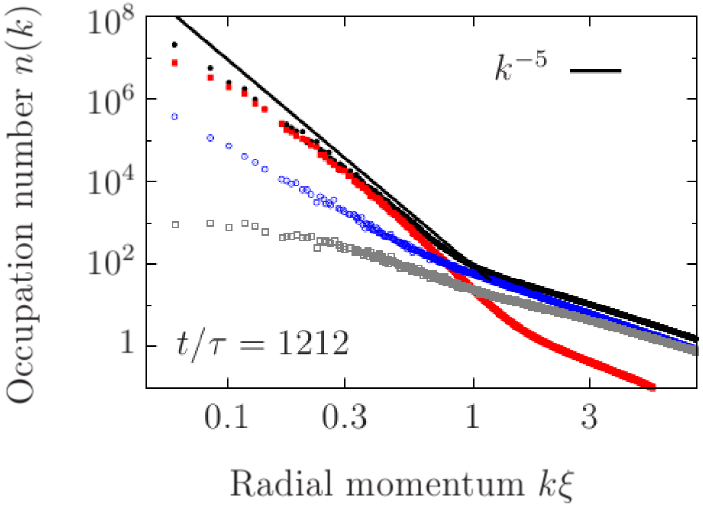
BN., T. Gasenzer arxiv: 1206.3181

3D: Bose condensation

α=2.5 (weak quench)

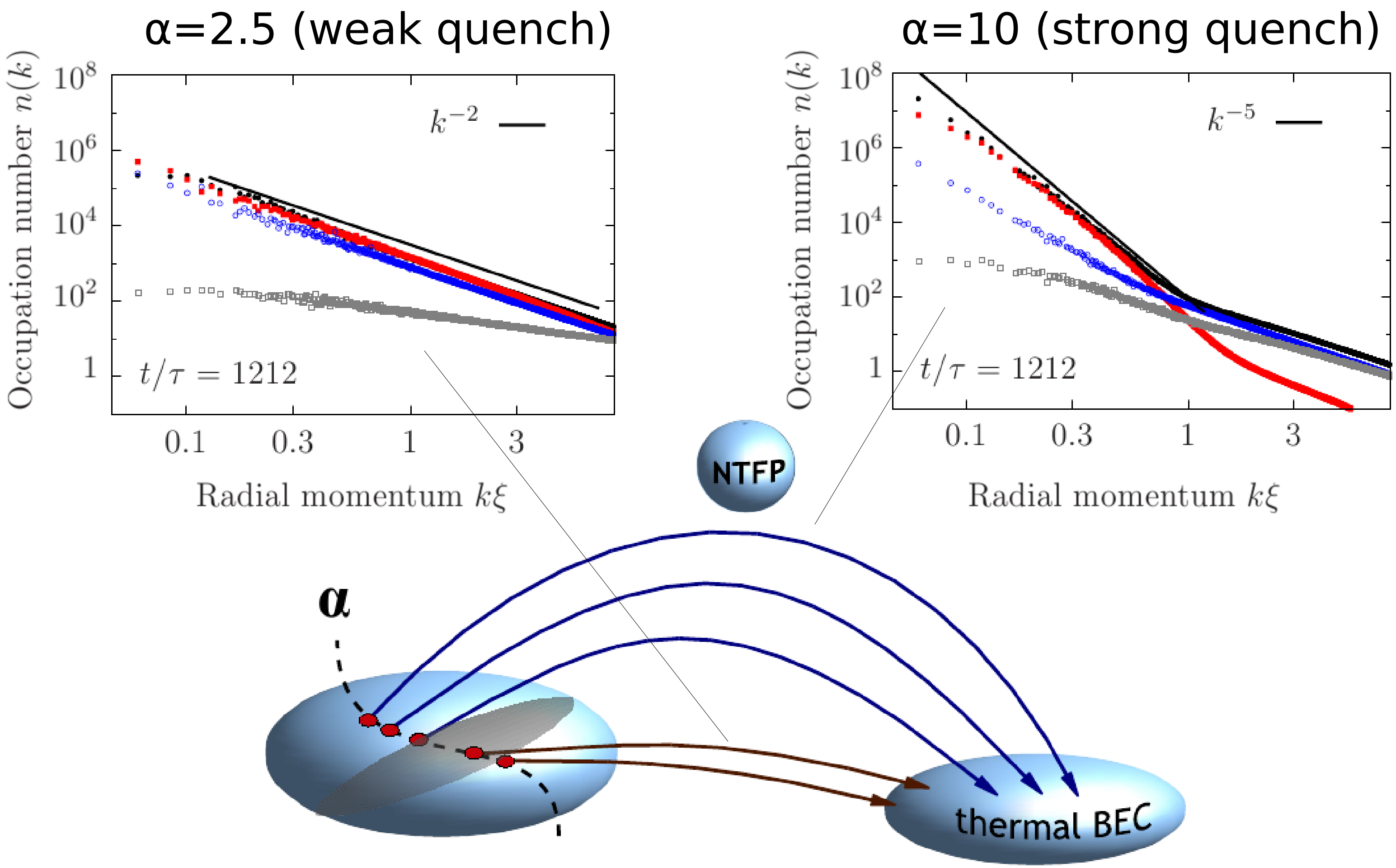


α=10 (strong quench)



BN., T. Gasenzer arxiv: 1206.3181

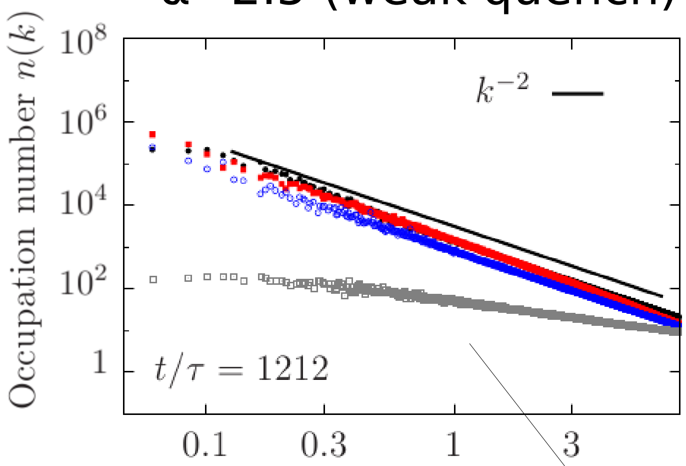
3D: Bose condensation



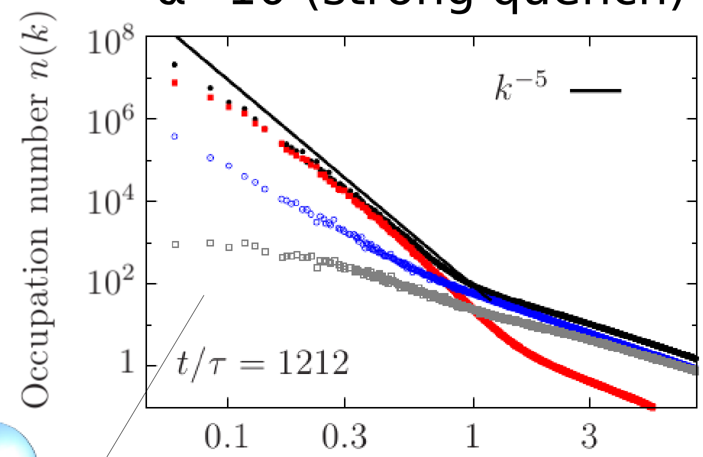
BN., T. Gasenzer arxiv: 1206.3181

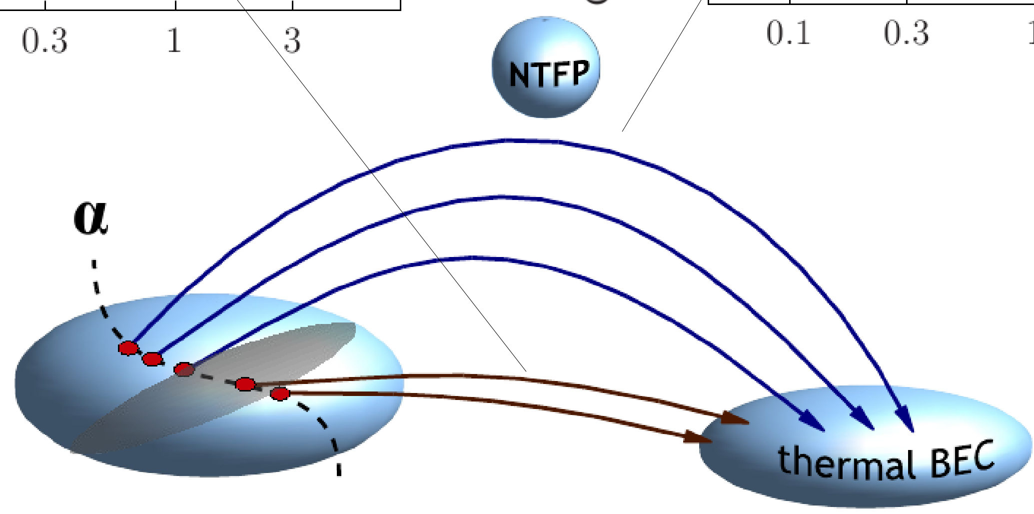
3D: Bose condensation

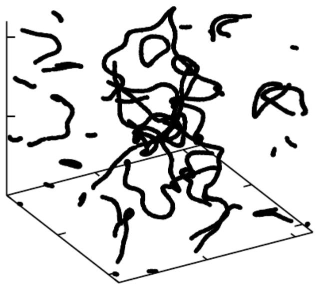
α=2.5 (weak quench)



α=10 (strong quench)

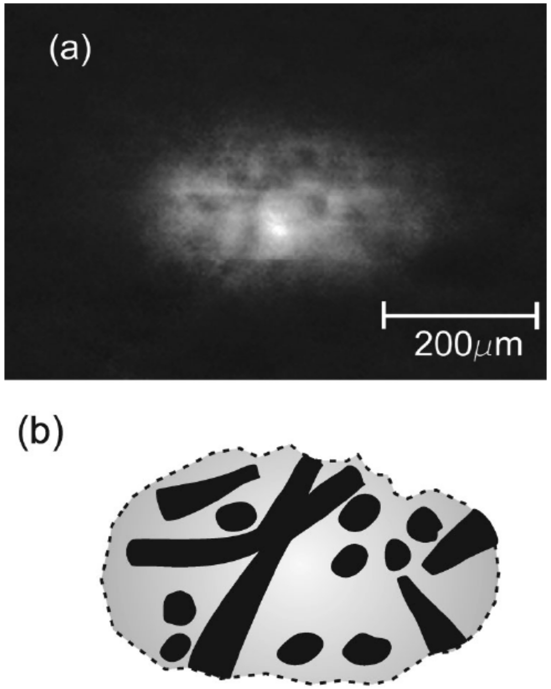




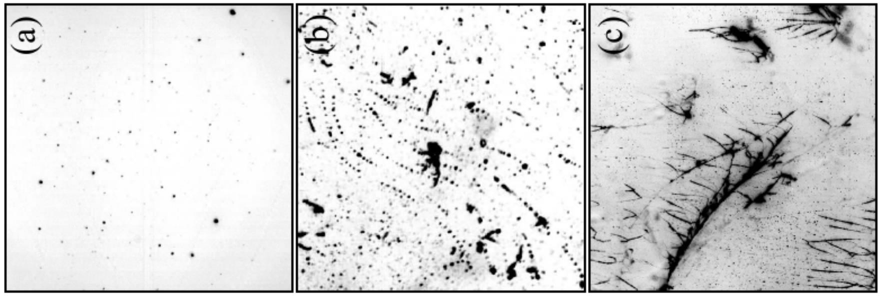


BN., T. Gasenzer arxiv: 1206.3181

Turbulence experiments

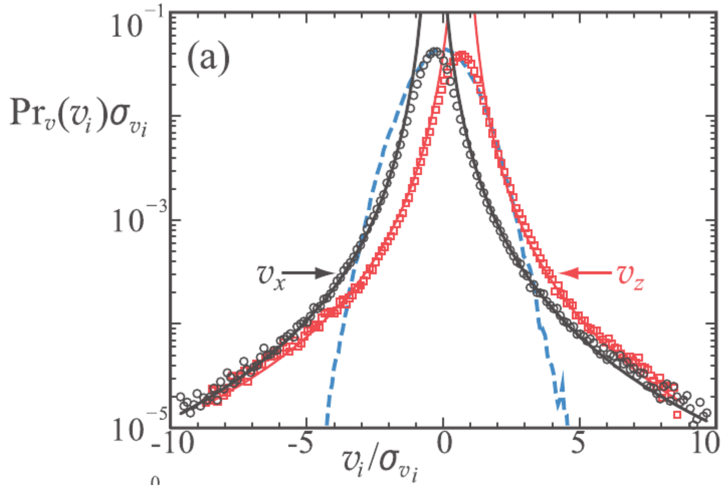


V. Bagnato (Brazil)



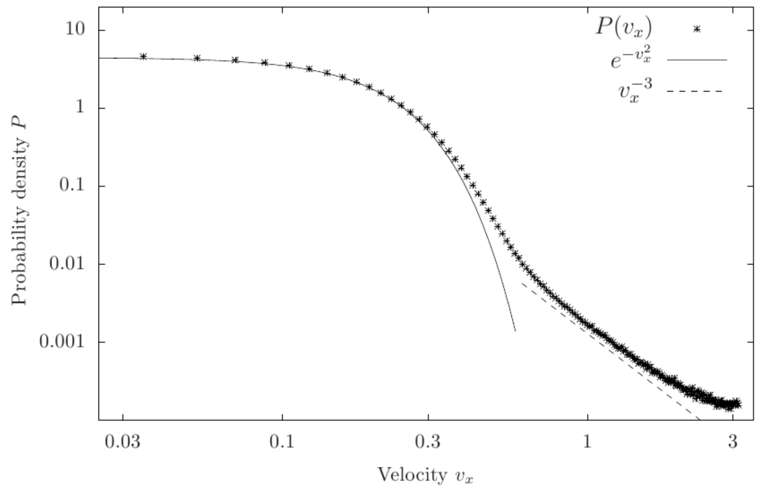
D. Lathrop (USA)

Velocity distribution

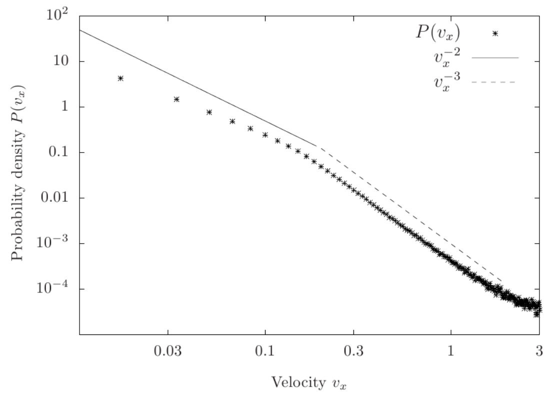


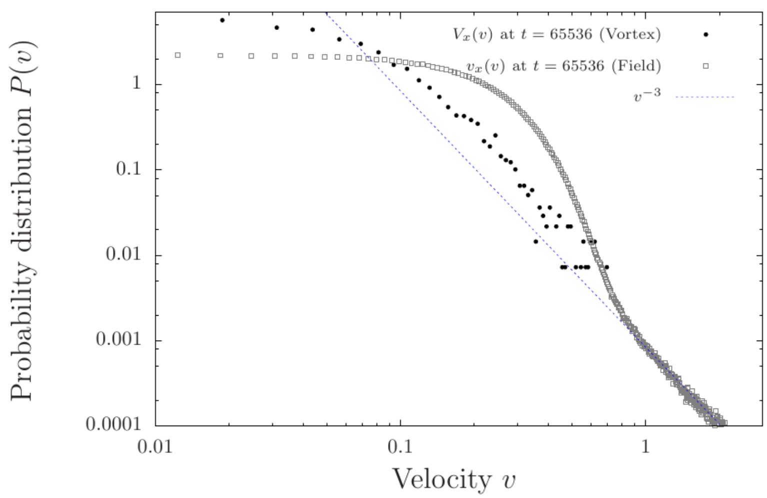
M. S. Paoletti, M. E. Fisher,

K. R. Sreenivasan, D. P. Lathrop: PRL (2008)



Pairing:





Quantum turbulence

$$E_{tot} = \int \left(\frac{1}{2} |\nabla \sqrt{n} e^{-i\varphi}|^2 + \frac{1}{2} g n^2 \right) d\boldsymbol{\rho}$$

$$= E_{kin} + E_q + E_{int}$$

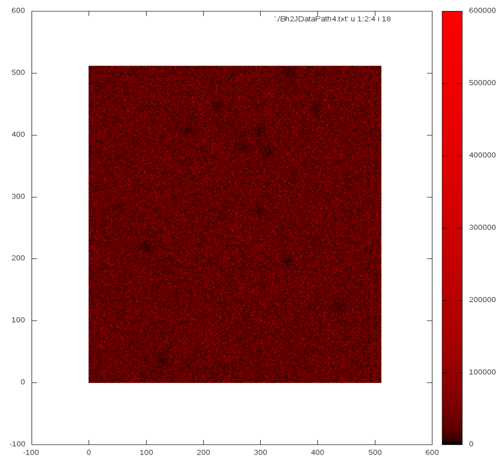
$$E_{kin} = \frac{1}{2} \int |\sqrt{n}\mathbf{u}|^2 d\boldsymbol{\rho} = E_{kin}^i + E_{kin}^c$$

$$\mathbf{u}(\boldsymbol{\rho}, t) = \nabla \varphi(\boldsymbol{\rho}, t)$$

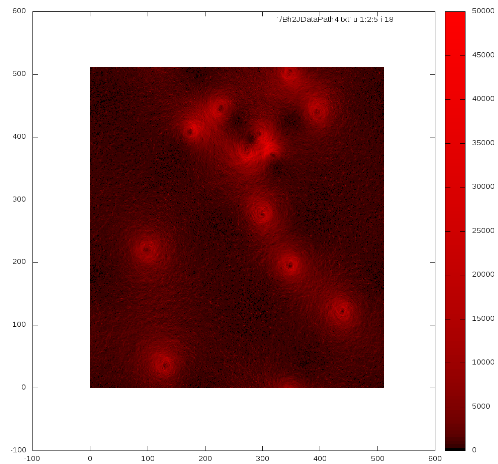
$$\nabla \times (\sqrt{n}\mathbf{u})^c = 0$$

$$\nabla \cdot (\sqrt{n}\mathbf{u})^i = 0$$

C. Nore, M. Abid, M. Brachet: Phys. Fluids (1997)



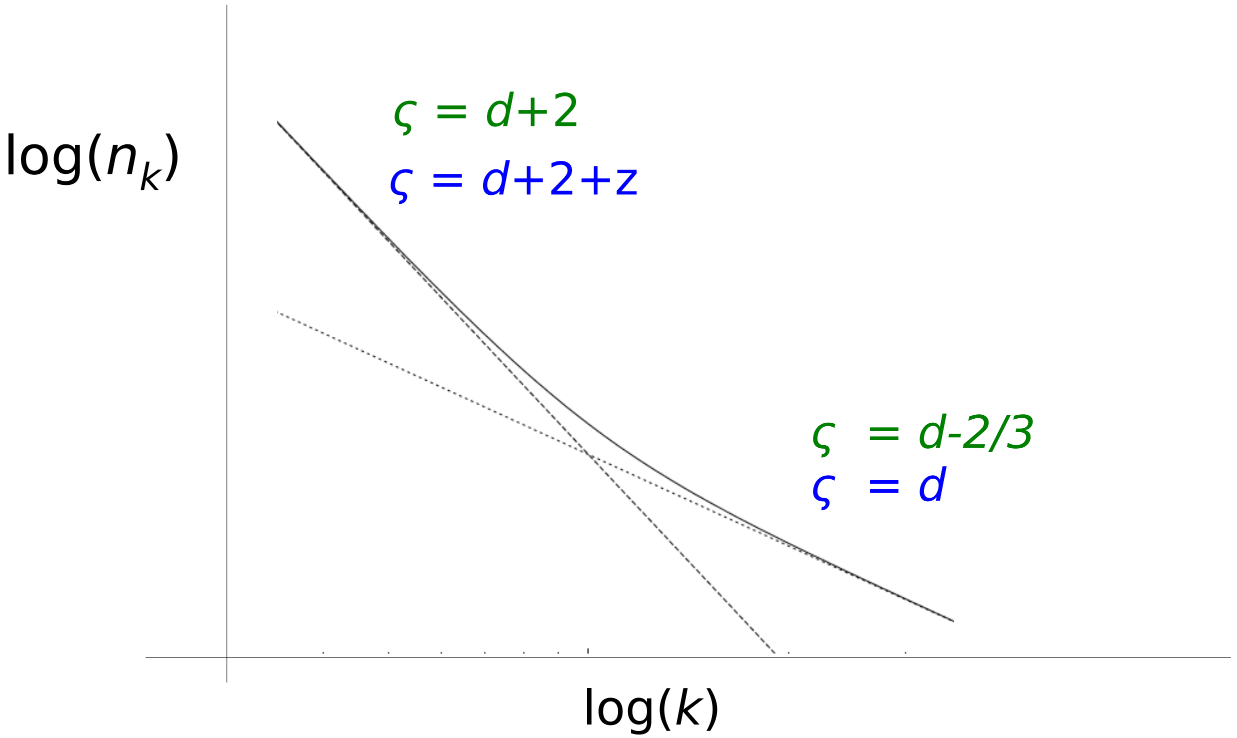
E_{kin}^c

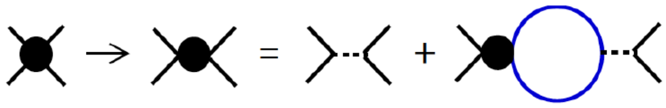


E_{kin}^i

Nonthermal fixed points $n_k \sim k^{-\varsigma}$

from dynamical quantum field theory

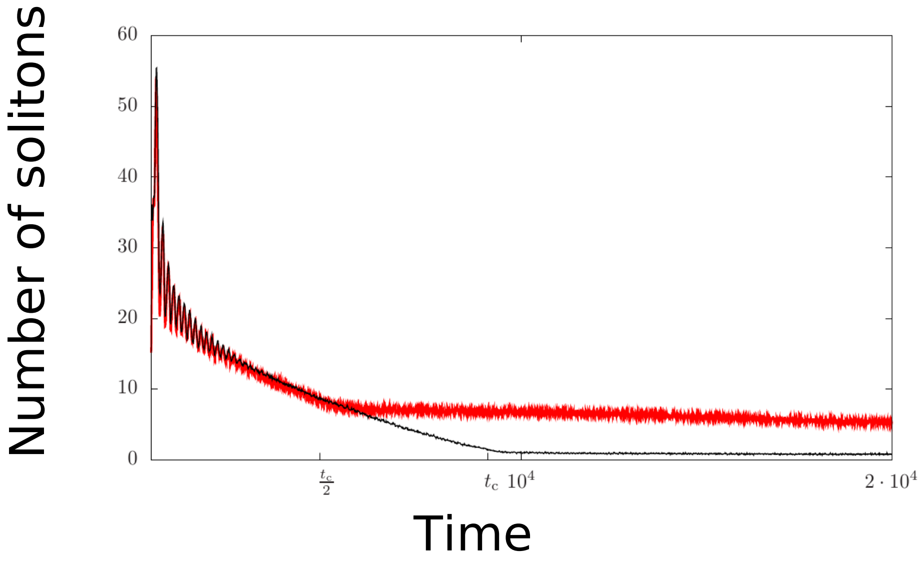




J. Berges, A. Rothkopf, J. Schmidt, PRL (2008)

C. Scheppach, J. Berges, T. Gasenzer, PRA (2010)

Soliton density decay

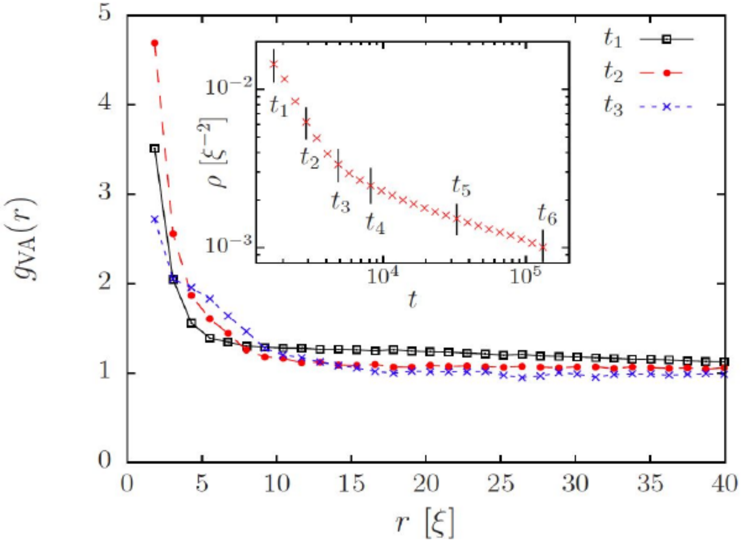


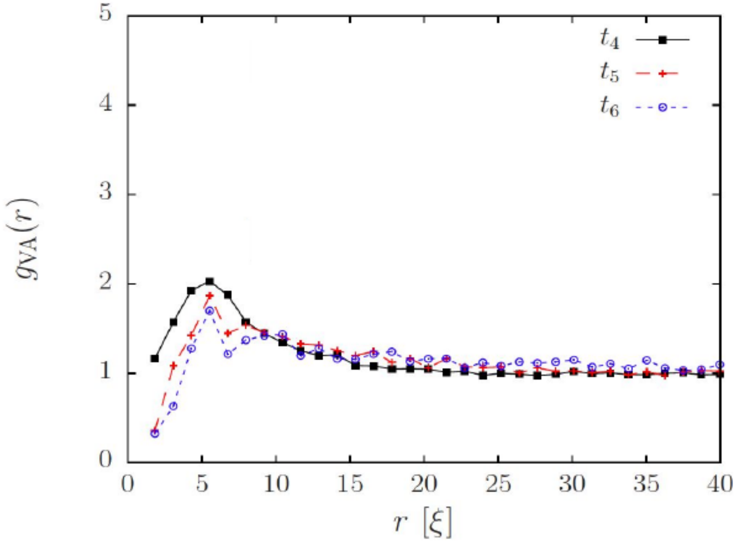
Time Scales:

- Sound propagation a_{ho}/c_s ~ 10
- Inverse trap frequency 1/ω~ 10^2
- Damping of breathing mode ~ 10^3
- Cooling time ~ 10^4
- Soliton lifetime > 10^6

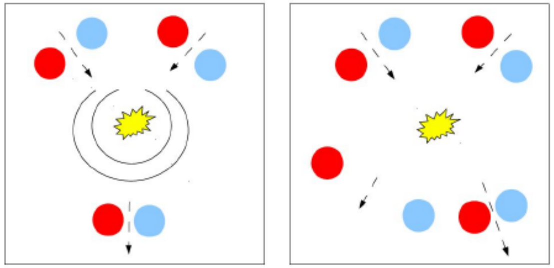
2D: Vortex correlations

B. N. et al. PRA 85, 043627 (2012)
B. N. et al. PRB 84, 020506(R) (2011)





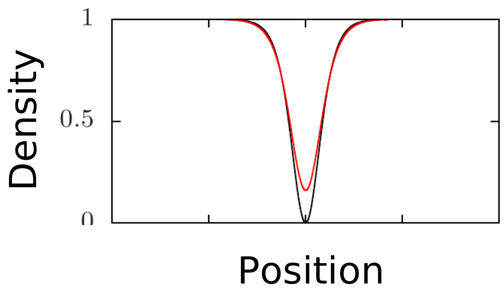
$$g_{\mathrm{VA}}(\mathbf{x},\mathbf{x}',t) = \frac{\langle \rho^{\mathrm{V}}(\mathbf{x},t)\rho^{\mathrm{A}}(\mathbf{x}',t)\rangle}{\langle \rho^{\mathrm{V}}(\mathbf{x},t)\rangle\langle \rho^{\mathrm{A}}(\mathbf{x}',t)\rangle}$$

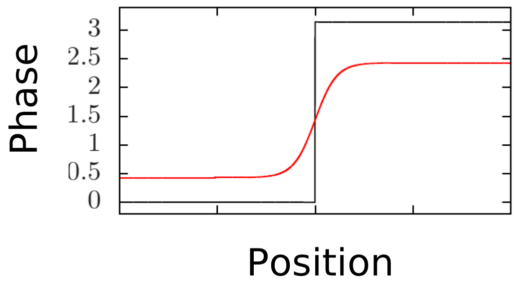


1D: Solitons replace vortices

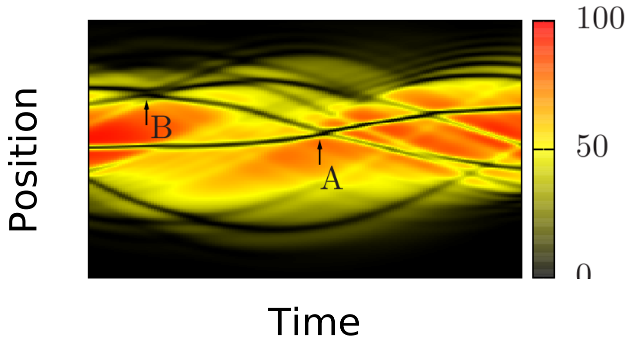
M. Schmidt, S. Erne, B. N., D. Sexty, T. Gasenzer, arXiv:1203.3651 (2012)

- Stationary solutions of non-linear wave equation:





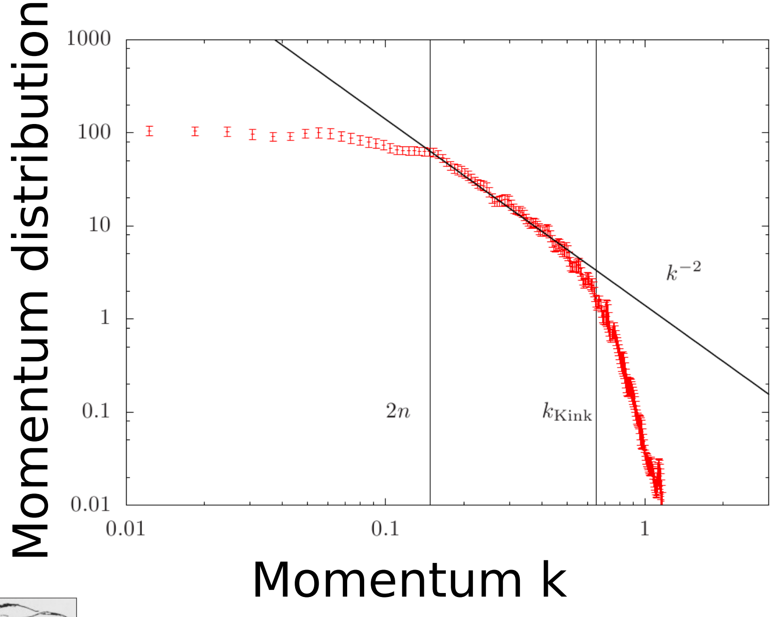
- Soliton-Soliton interactions:

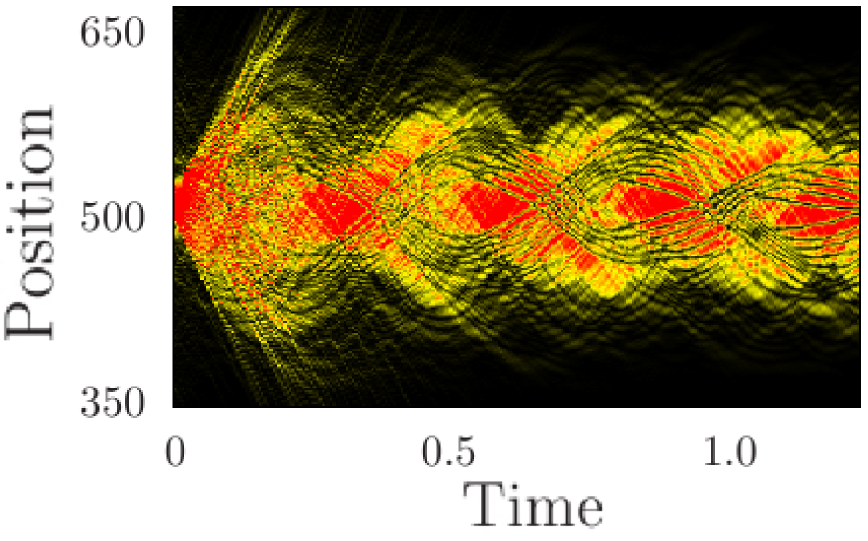


→ Turbulence (NTFP) as a random state of solitons

1D: Quench dynamics

Features:
- Quasi-stationary profile
- Scaling



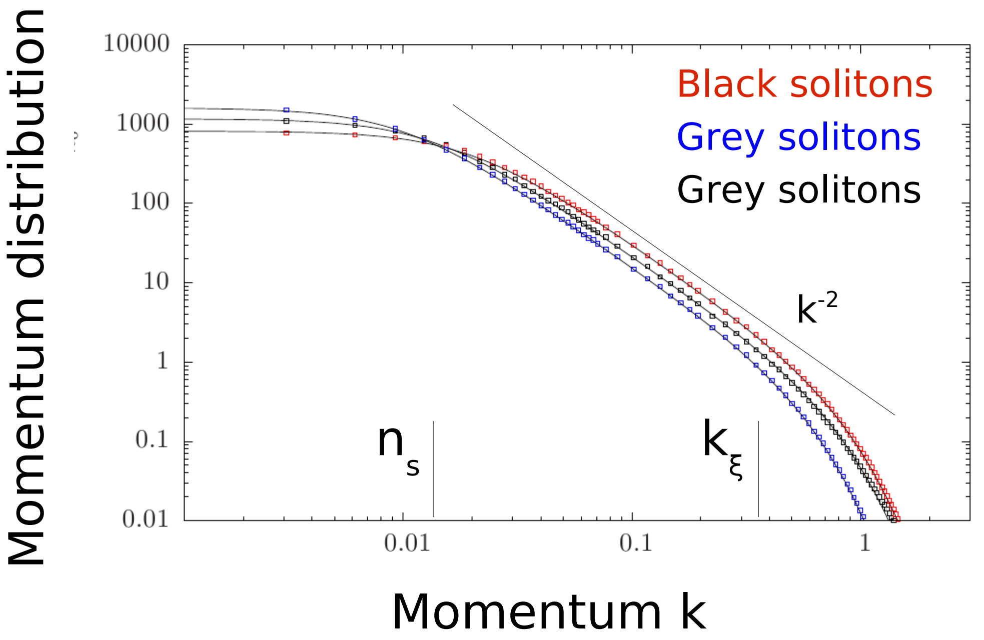


M. Schmidt, S. Erne, BN, D. Sexty, T. Gasenzer NJP (2012)

1D: Momentum distribution

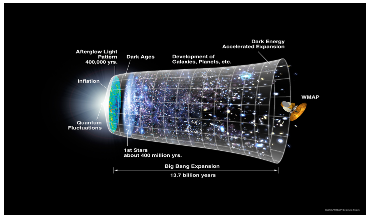
- Random soliton model for black/grey solitons (in a trap):

e. g. $$n(k)|_{\nu=0} = \frac{4n_s n}{4n_s^2 + k^2} \frac{(\pi k \xi)^2/2}{\sinh^2\left(\pi k \xi/\sqrt{2}\right)}$$ (Black Solitons)



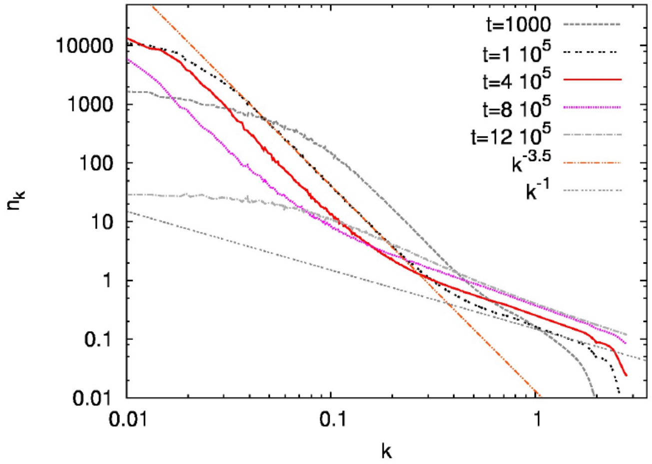
M. Schmidt, S. Erne, BN, D. Sexty, T. Gasenzer NJP (2012)

Relativistic simulations O(2)

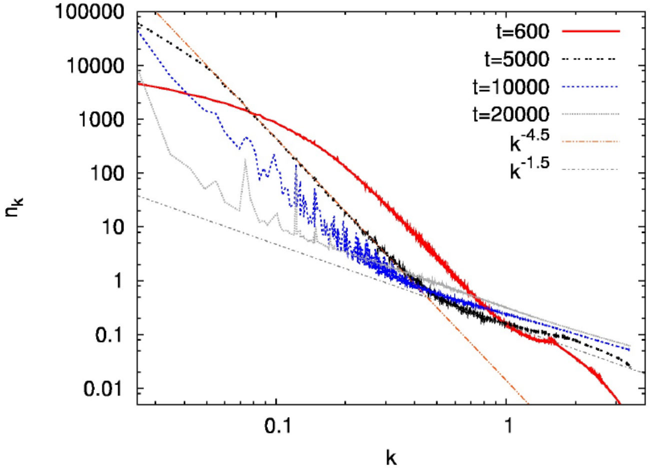


Classical field equation: $\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$

d=2



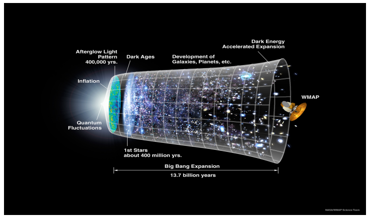
d=3



T. Gasenzer, B. Nowak, D. Sexty PLB (2012)

S. Khlebnikov, I. Tkachev PRL (1996)
J. Berges, A. Rothkopf, J. Schmidt PRL (2008)
J. Berges, D. Sexty PRD (2011)

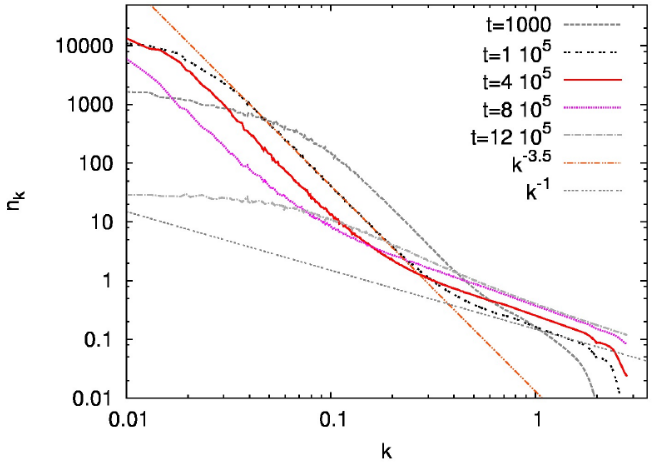
Relativistic simulations O(2)



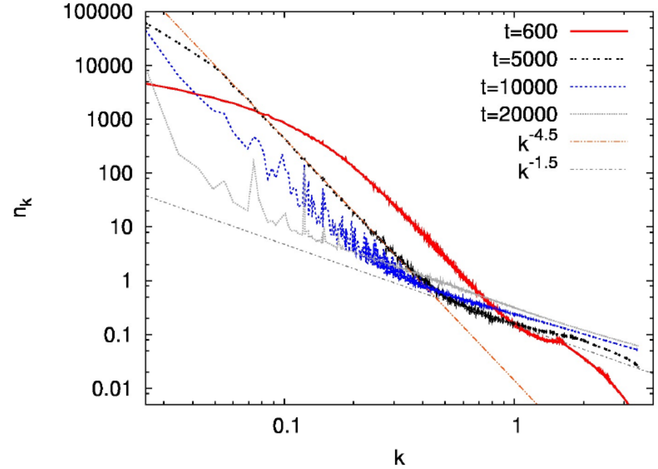
Classical field equation: $\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$

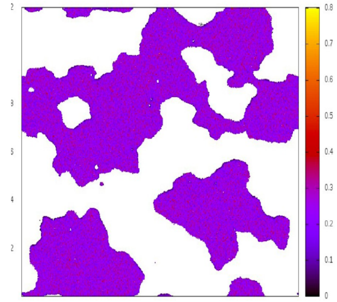
$-m^2$

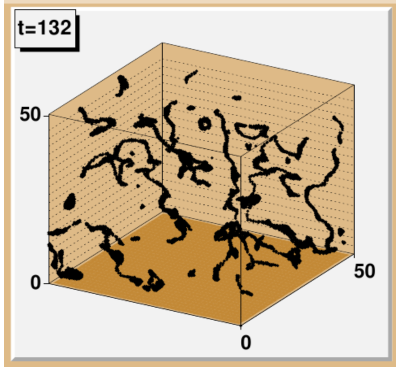
d=2

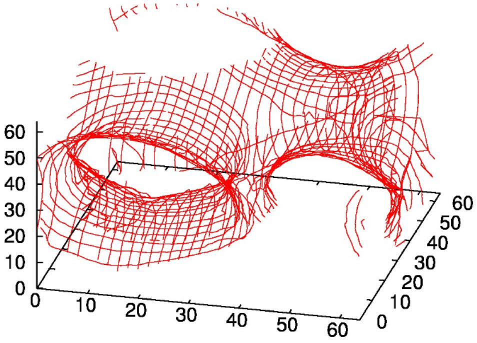


d=3





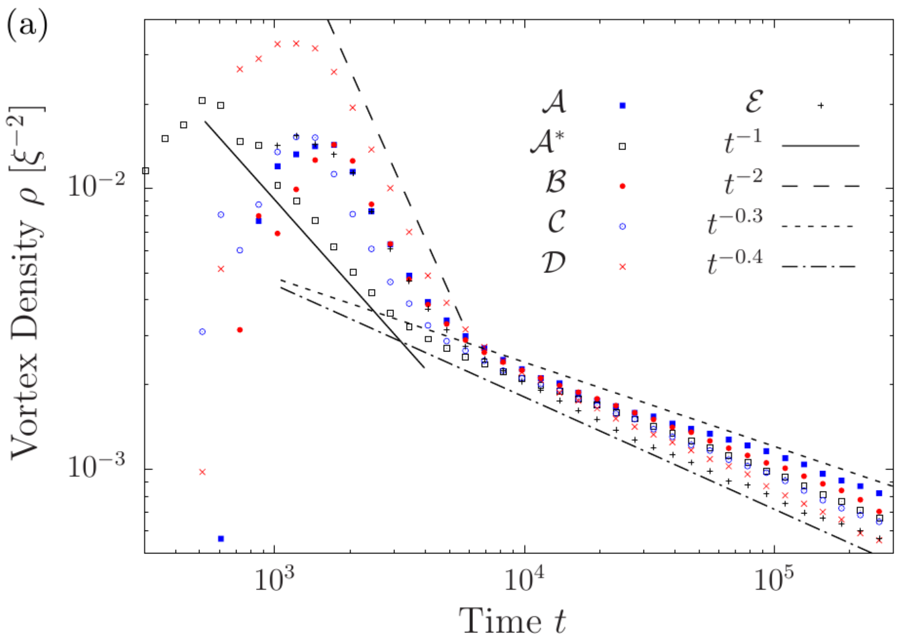




T. Gasenzer, B. Nowak, D. Sexty PLB (2012)

I. Tkachev, S. Khlebnikov, L. Kofman, A. Linde PL (1998)

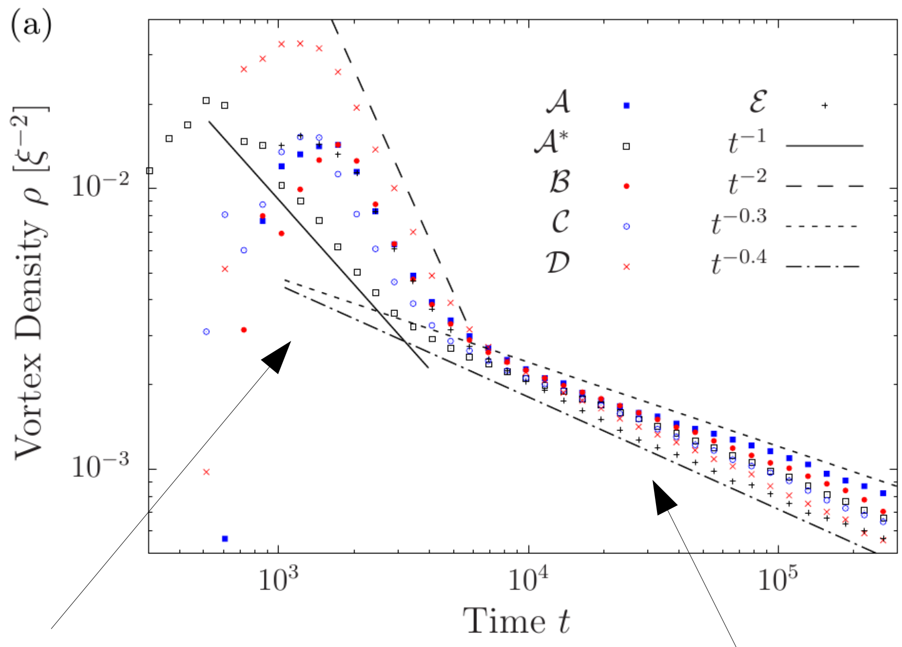
2D: Phase ordering dynamics



J. Schole, B. Nowak, T. Gasenzer, arXiv:1204.2487

2D: Phase ordering dynamics

Scaling needs
vortex unbinding

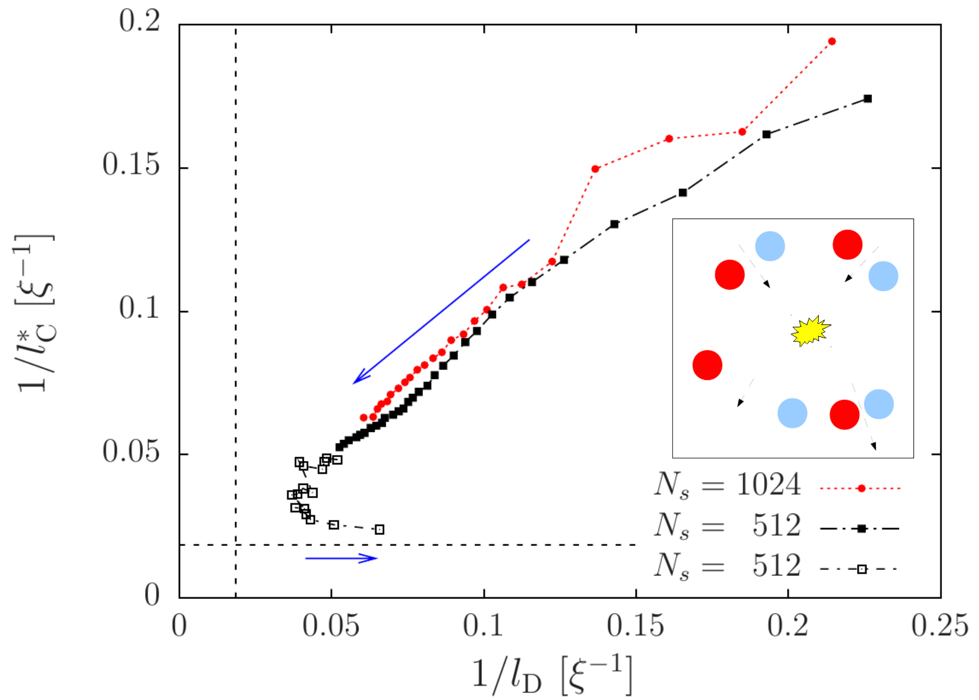


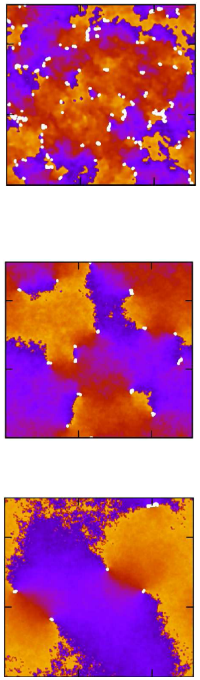
Non-universal decay law
(Initial vortex distribution dependent)
Kinetic gas theory for dipoles

Universal decay regime
Strongly correlated, dilute vortex gas
Scaling $n(k) \sim k^{-4}$

J. Schole, B. Nowak, T. Gasenzer, arXiv:1204.2487

Correlations near the NTFP



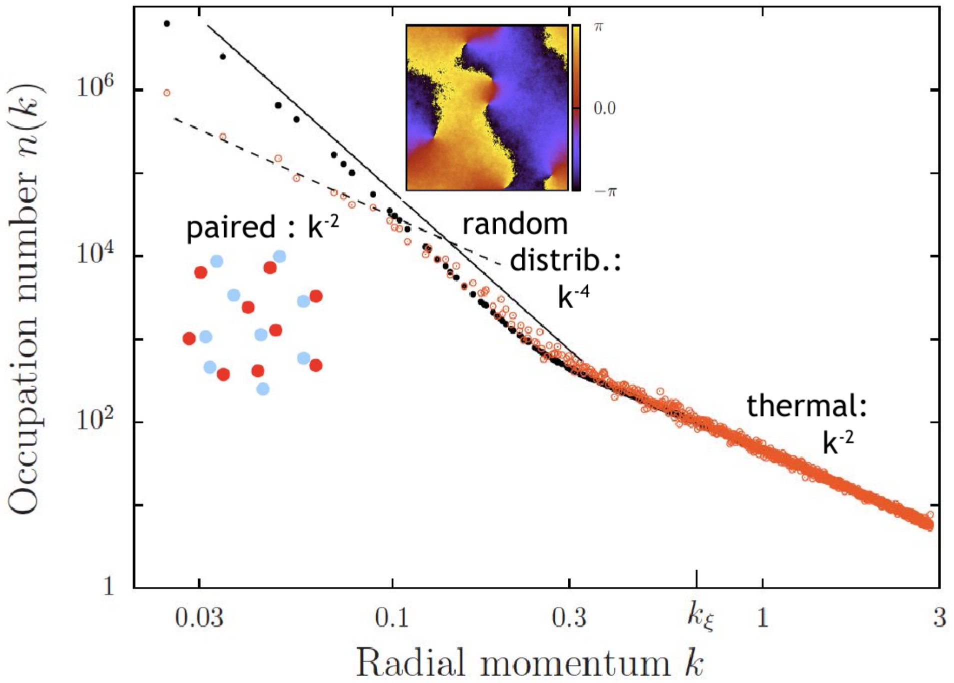


$l_C^\star$ Phase coherence length

l_D Vortex-antivortex pair distance

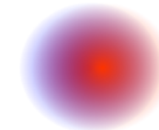
J. Schole, B. Nowak, T. Gasenzer, PRA (2012)

Vortex Pairing Spectrum in2D

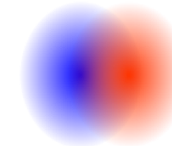


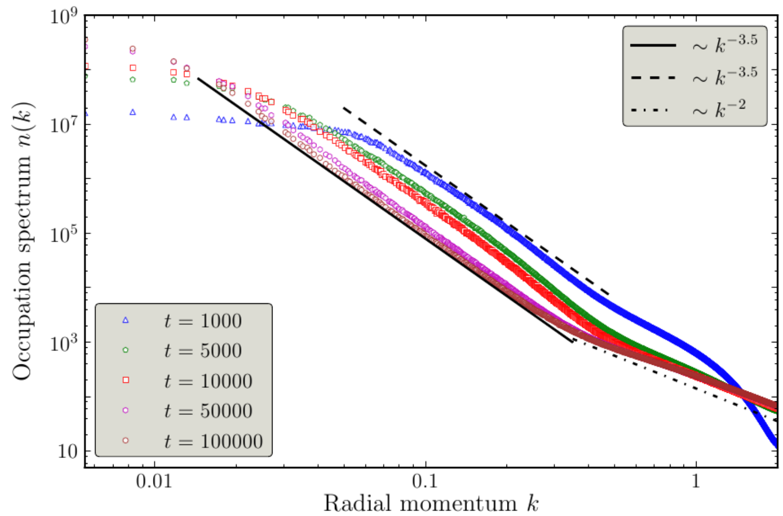
2D: 2-component BEC

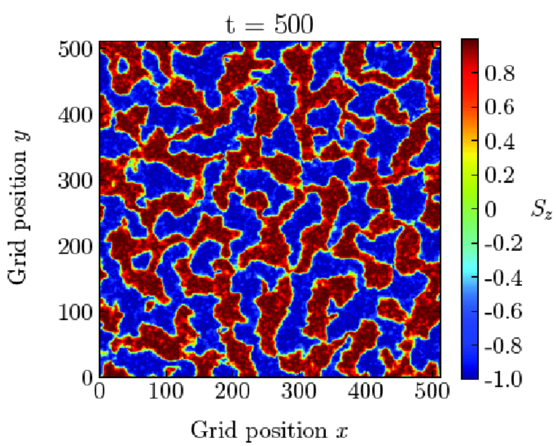
miscible
$g_{12} < g$

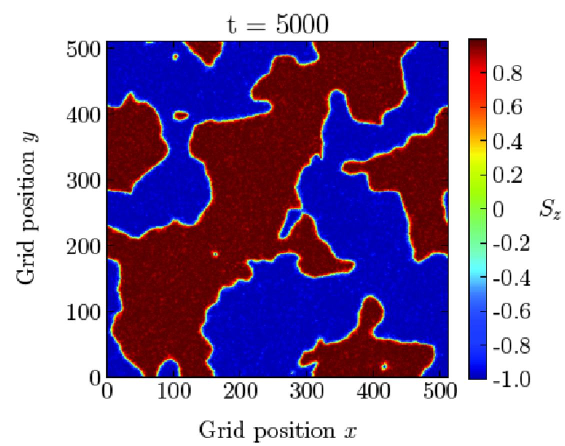


immiscible
$g_{12} > g$









MK, B. Nowak, T. Gasenzer in preparation