pulling oneself over the fence... a bootstrap for quantum gravity

Daniel F Litim

University of Sussex

with Kevin Falls, Kostas Nikolakopoulos, and Christoph Rahmede

1301.4191.pdf

DELTA ’13, U Heidelberg
10 Jan 2013

question

is metric quantum gravity fundamental ?

gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

classical action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int \sqrt{\det g}\left(-R(g_{\mu\nu}) + 2\Lambda\right)$$

long distances

gravity not tested beyond $10^{28}\mathrm{cm}$

short distances

gravity not tested below $10^{-2}\mathrm{cm}$

gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

physics of quantum gravity

Planck length $\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\mathrm{cm}$

Planck mass $M_{\mathrm{Pl}} \approx 10^{19}\mathrm{GeV}$

Planck time $t_{\mathrm{Pl}} \approx 10^{-44}\,\mathrm{s}$

Planck temperature $T_{\mathrm{Pl}} \approx 10^{32}\,\mathrm{K}$

expect **quantum modifications** at energy scales M_{Pl}

perturbation theory

- **structure of UV divergences**

  gravity: $[g_{\mu\nu}] = 0\,, \quad [\mathrm{Ricci}] = 2\,, \quad [G_N] = 2 - d$

  effective expansion parameter: $g_{\mathrm{eff}} \equiv G_N\, E^2 \sim \frac{E^2}{M_{\mathrm{Pl}}^2}$

  N-loop Feynman diagram $\sim \int dp\, p^{A\,-[G]N}$

  $[G] > 0:$ superrenormalisable

  $[G] = 0:$ renormalisable

  $[G] < 0:$ dangerous interactions

- **perturbative non-renormalisability**

  gravity with matter interactions

  pure gravity (Goroff-Sagnotti term)

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

- **higher derivative gravity II** (Gomis, Weinberg '96)

  all higher derivative operators

  gravity 'weakly' perturbatively renormalisable

  no unitarity issues at high energies

quantum fields

running couplings

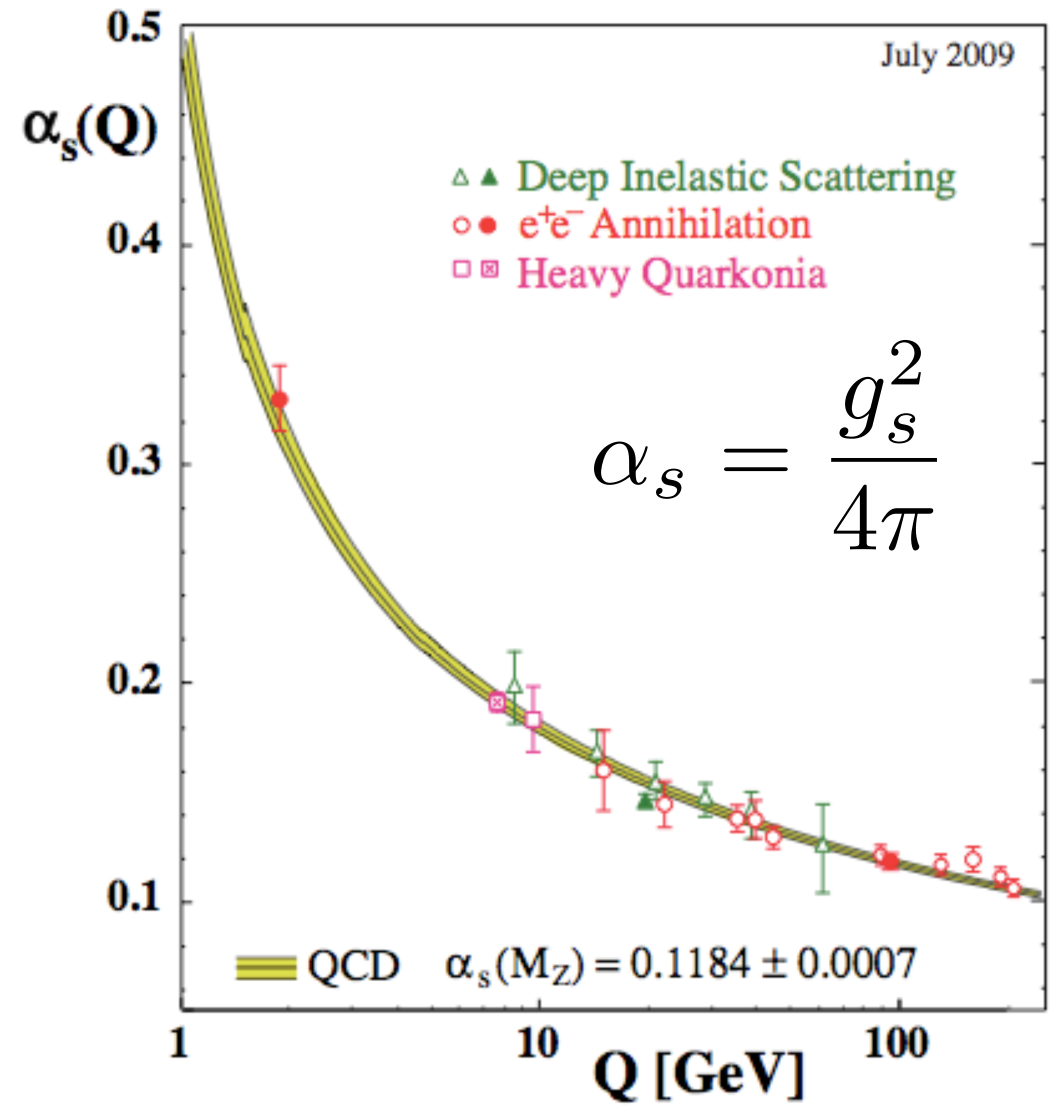
quantum fluctuations modify interactions

couplings depend on eg. energy or distance

asymptotic freedom of the strong force

(taken from PDG)

$$S_{\rm YM} = \frac{1}{4g_s^2}\int F^2$$



quantum fields

running couplings

quantum fluctuations modify interactions

couplings depend on eg. energy or distance

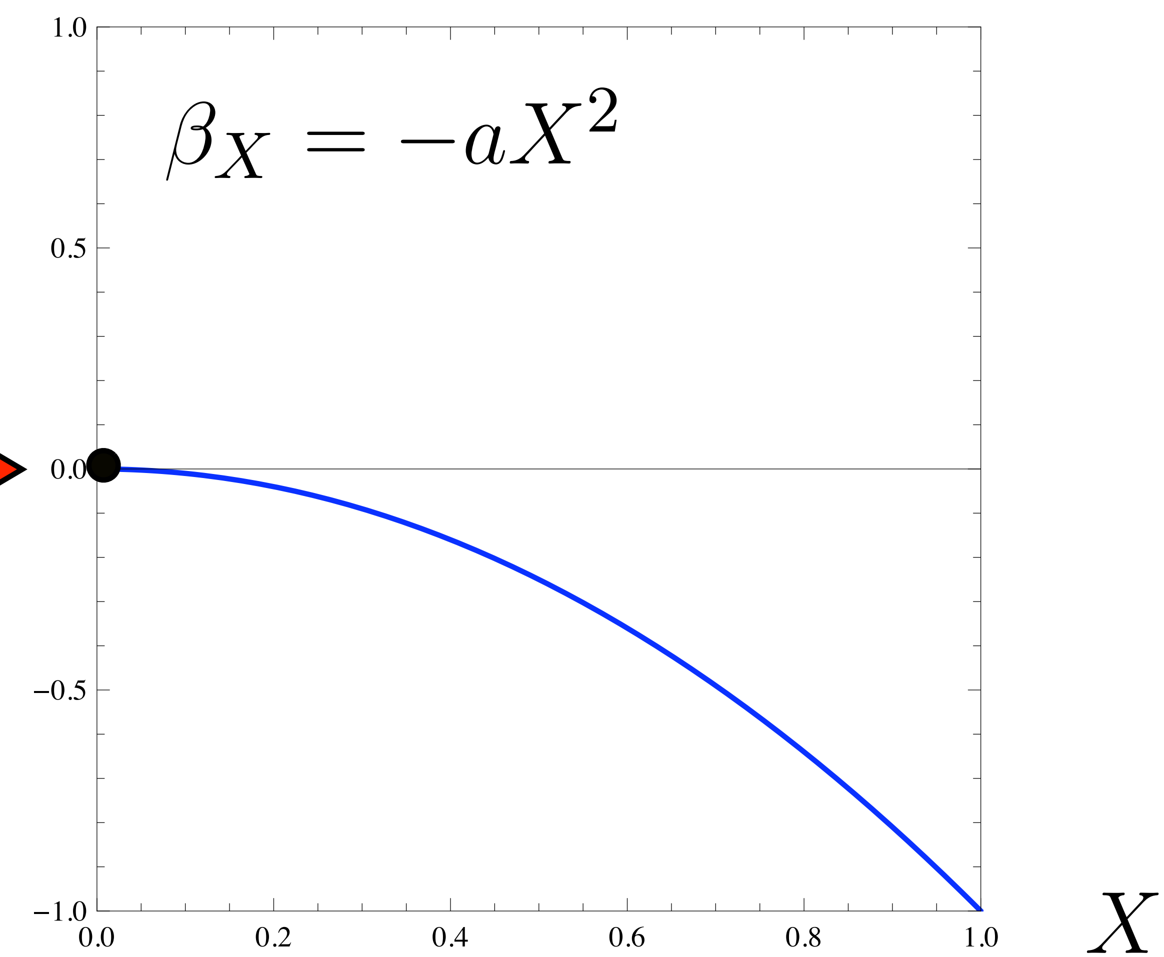
strong nuclear force (QCD)

coupling $X = g_s^2/(4\pi)$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

trivial UV fixed point

$$X_* = 0$$



quantum fields

running couplings

quantum fluctuations modify interactions

couplings depend on eg. energy or distance

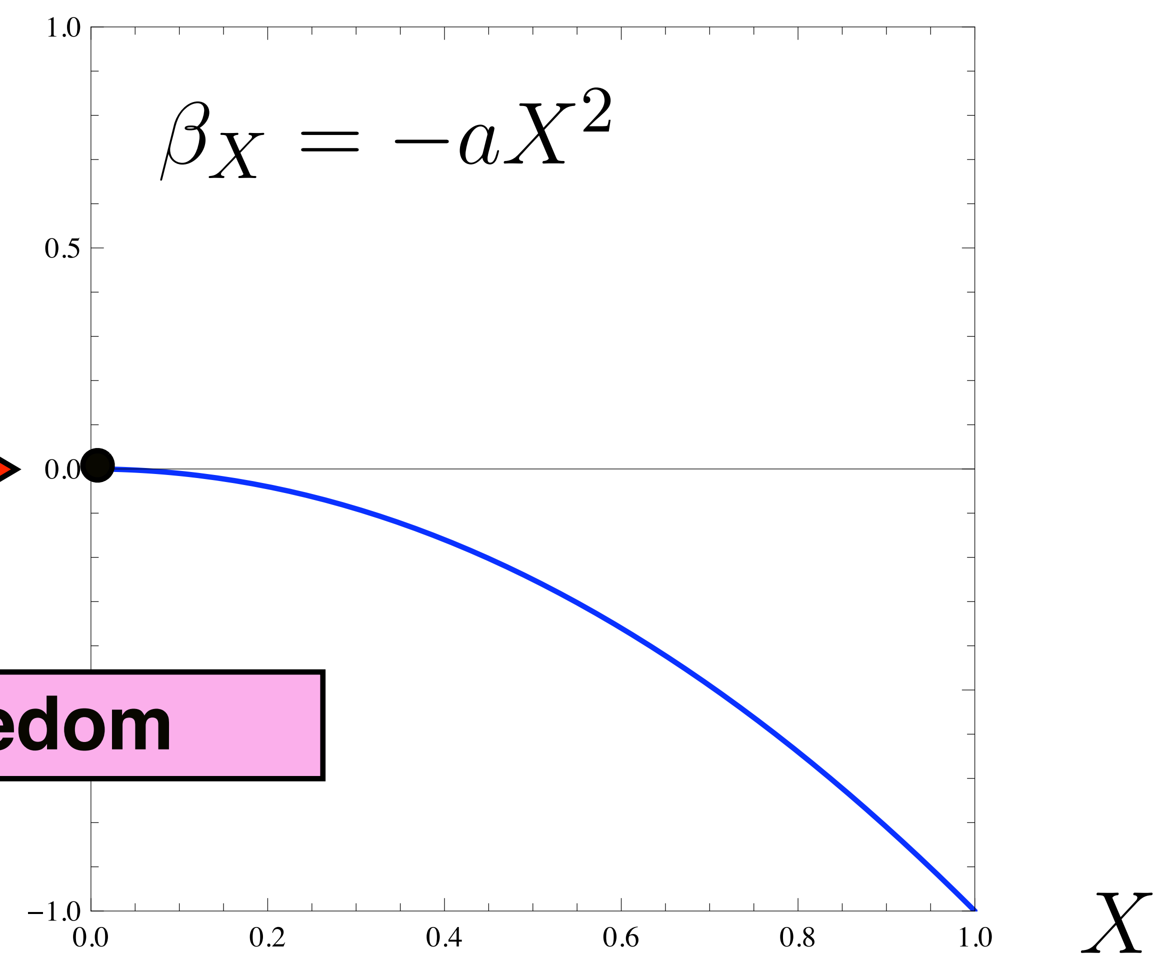
strong nuclear force (QCD)

coupling $X = g_s^2/(4\pi)$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

trivial UV fixed point

$$X_* = 0$$



asymptotic freedom

quantum fields

running couplings

quantum fluctuations modify interactions

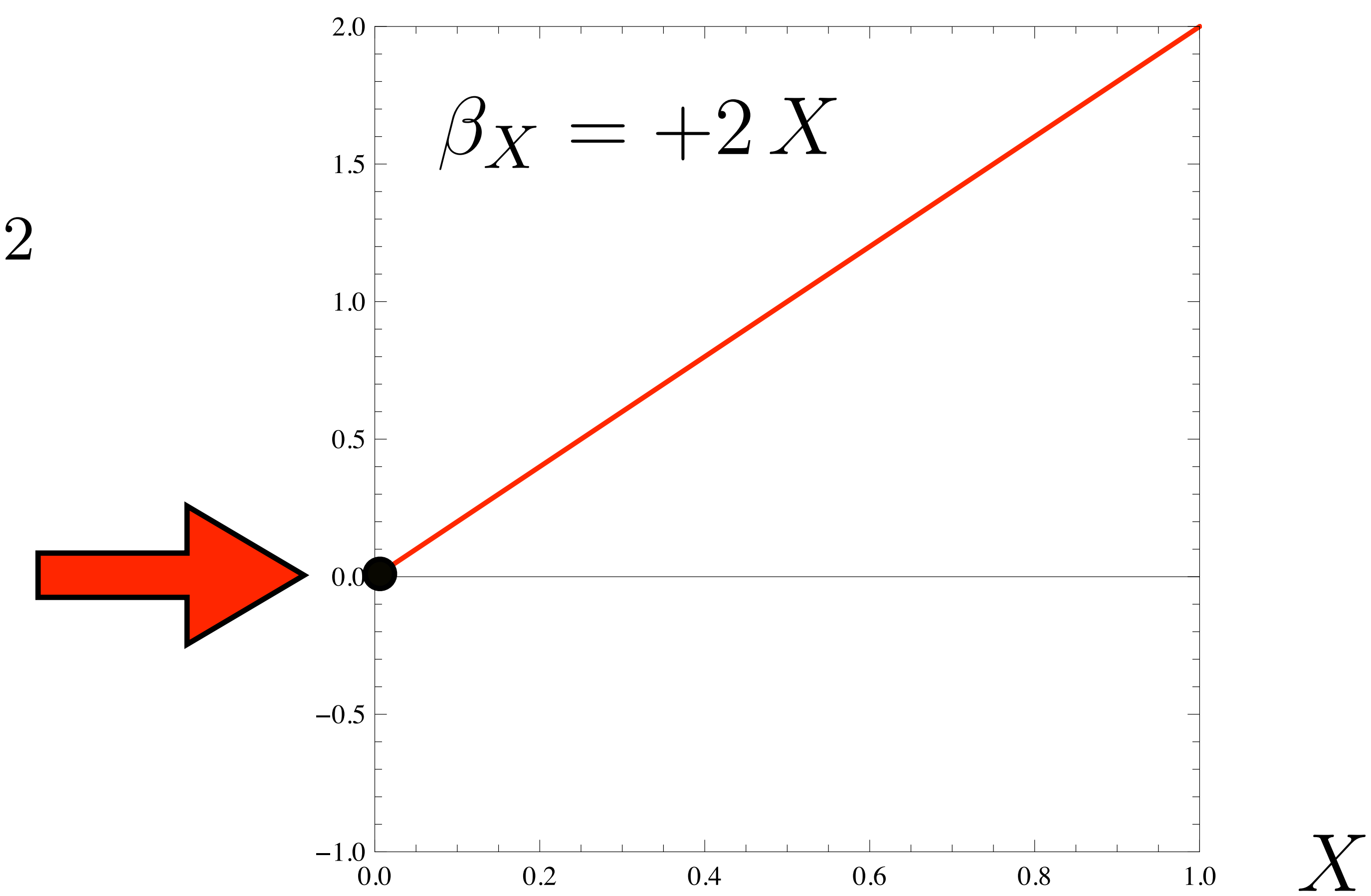
couplings depend on eg. energy or distance

gravitation

coupling $X = G_N \, \mu^2$

$$\beta_X \equiv \frac{dX}{d \ln \mu}$$

trivial IR fixed point



quantum fields

running couplings

quantum fluctuations modify interactions

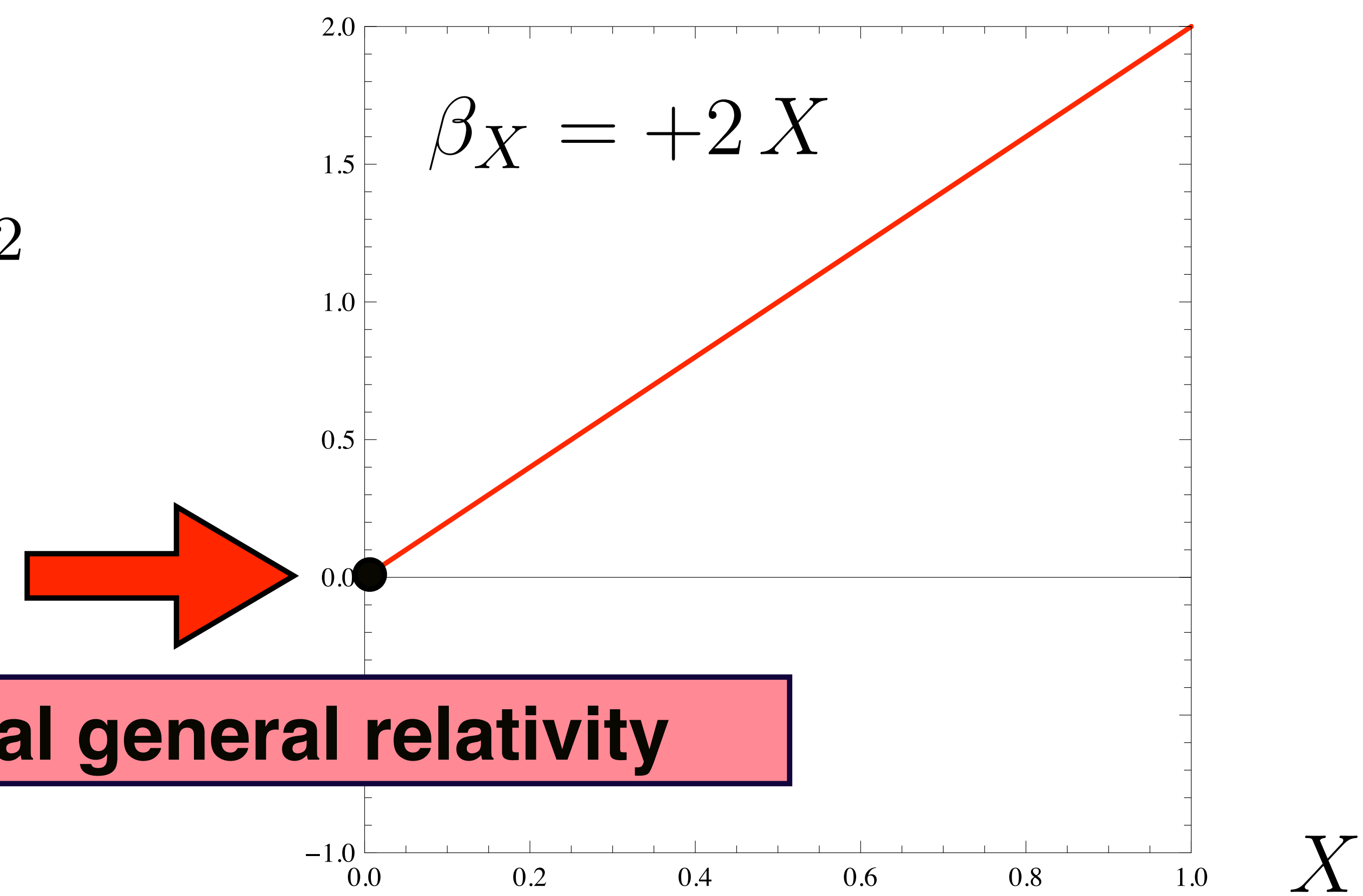
couplings depend on eg. energy or distance

gravitation

coupling $X = G_N \, \mu^2$

$$\beta_X \equiv \frac{dX}{d \ln \mu}$$

trivial IR fixed point



classical general relativity

quantum gravity

running couplings

quantum fluctuations modify interactions

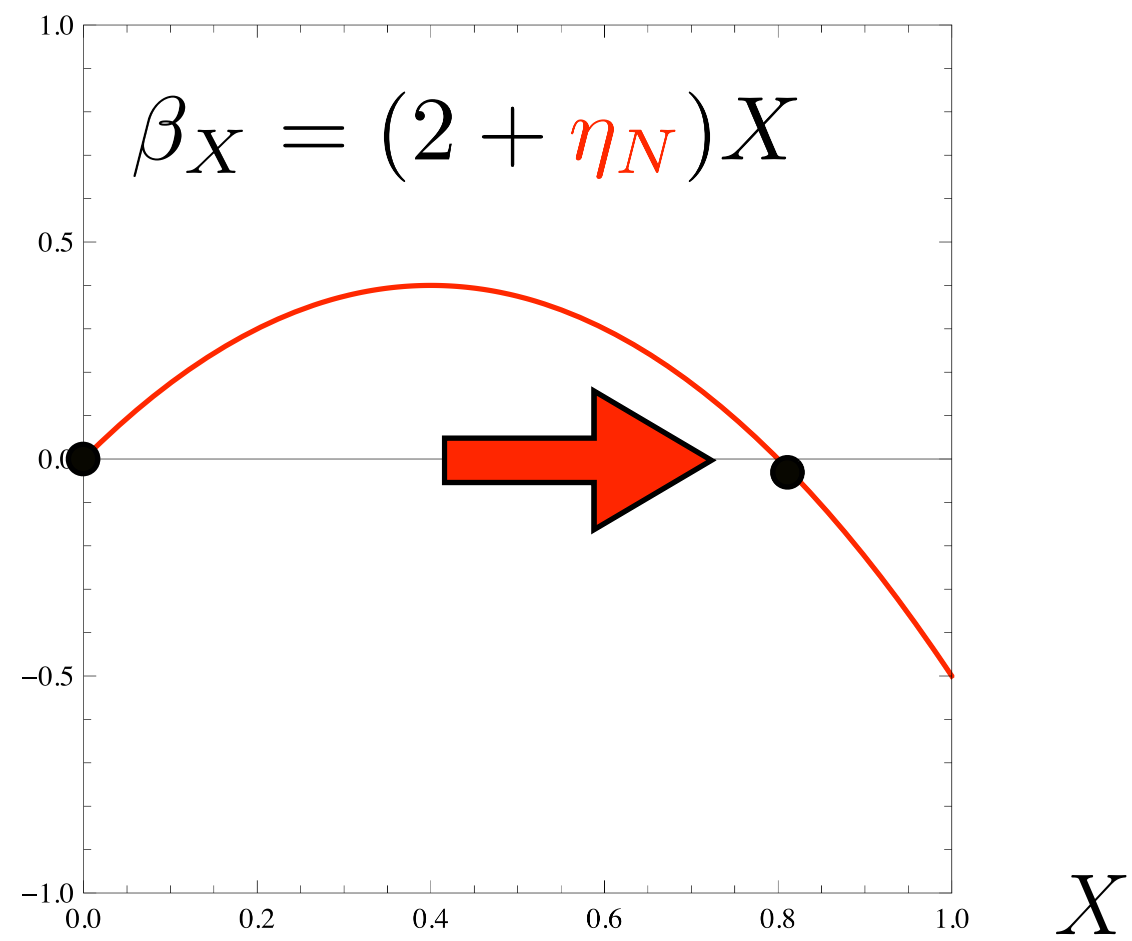
couplings depend on eg. energy or distance

gravitation

coupling $X = G(\mu)\,\mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

non-trivial UV fixed point



quantum gravity

running couplings

quantum fluctuations modify interactions

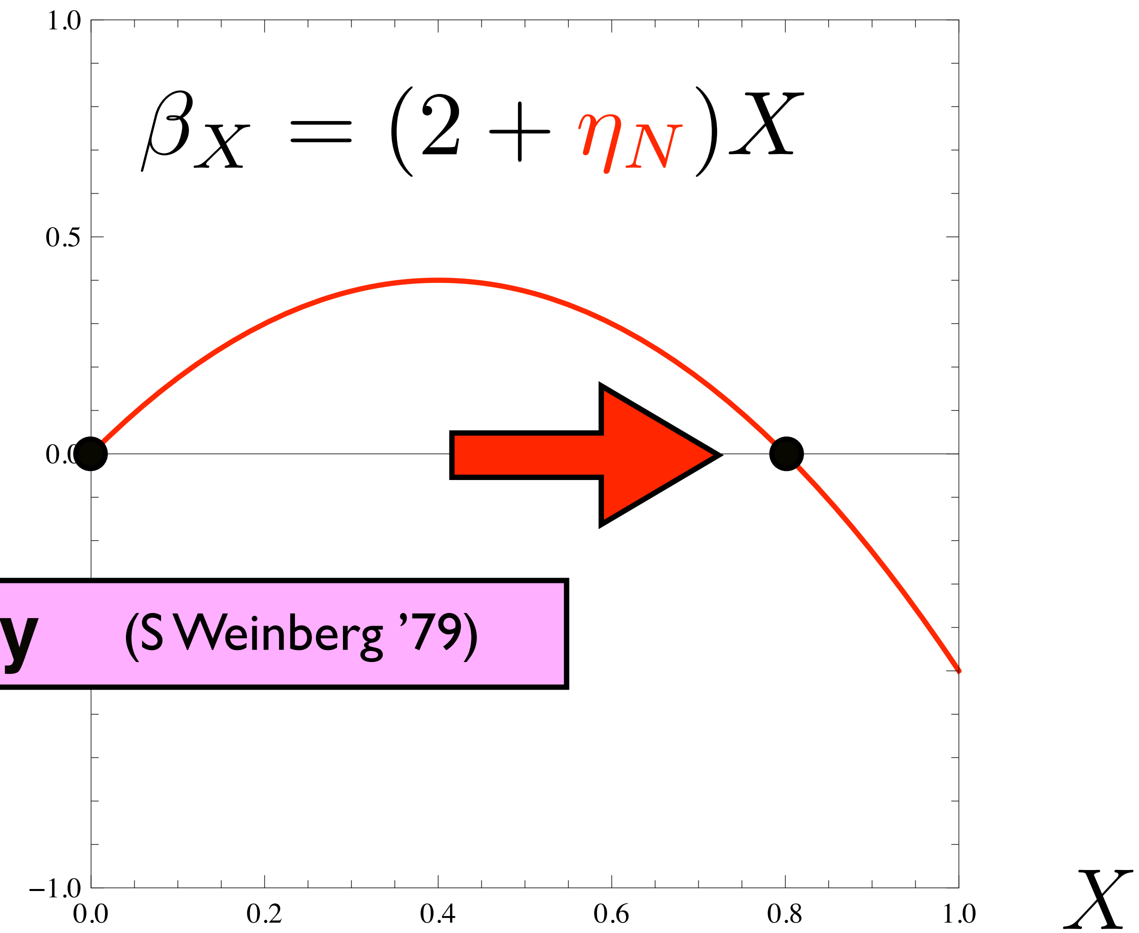
couplings depend on eg. energy or distance

gravitation

coupling $X = G(\mu)\,\mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

non-trivial UV fixed point



asymptotic safety (S Weinberg '79)

quantum gravity

running couplings

quantum fluctuations modify interactions

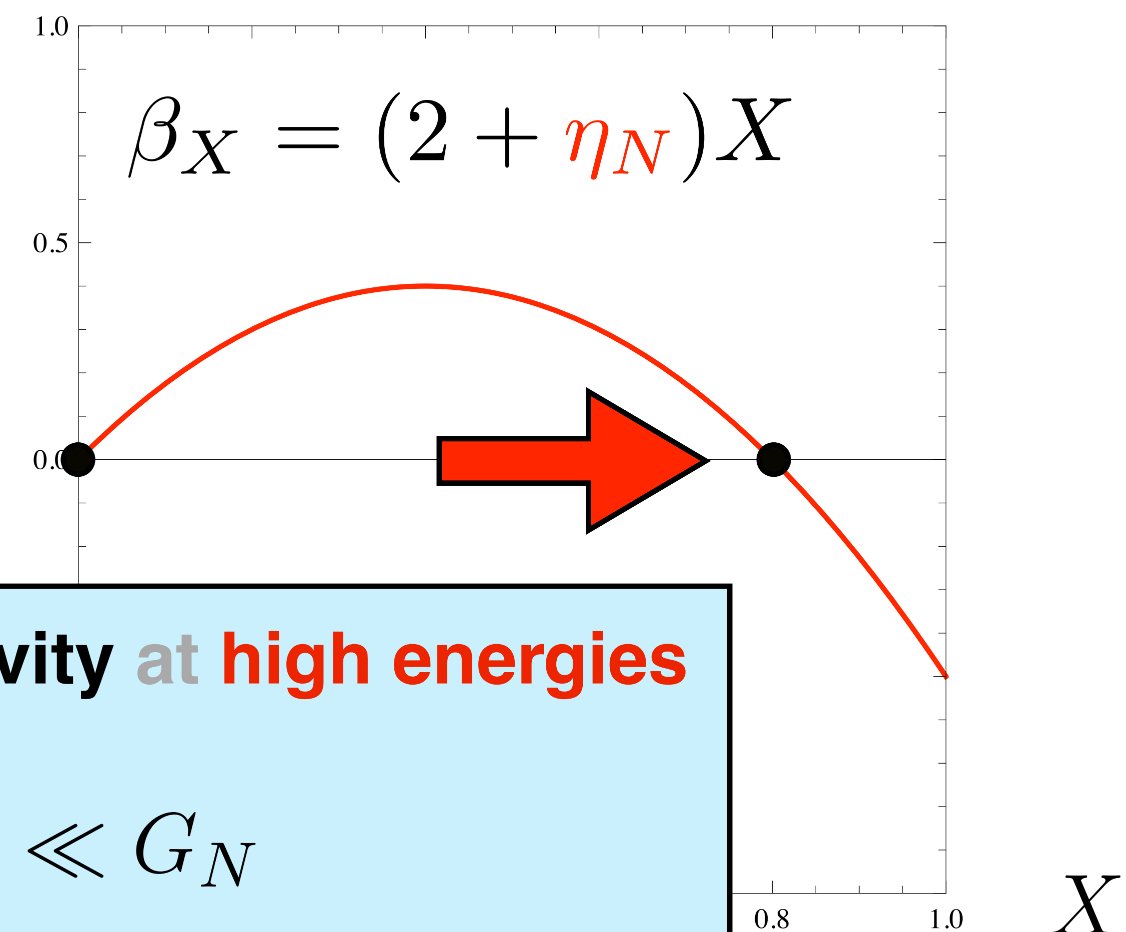
couplings depend on eg. energy or distance

gravitation

coupling $X = G(\mu)\,\mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

non-trivial UV fixed point



UV fixed point implies **weakly coupled gravity** at **high energies**

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

asymptotic safety

effective action for gravity

$$\Gamma_k = \sum_i \bar{\lambda}_i \int d^4x \; \mathcal{O}_i$$

high-energy limit

$$\Gamma_k \to \Gamma_*$$

UV fixed point

low energy limit

$$\Gamma \approx \int d^4x \sqrt{g} \left[\frac{\Lambda}{8\pi\, G} - \frac{R}{16\pi G} \right]$$

classical GR

asymptotic safety

running couplings

$$k\partial_k \, \lambda_i = \sum_j \mathbb{B}_{ij} \, (\lambda_j - \lambda_j^*) + \text{subleading}$$

vicinity of fixed point

$$\lambda_i(k) = \lambda_i^* + \sum_n c_n \, V_i^n \, k^{\vartheta_n} + \text{subleading}$$

scaling exponents
$$\begin{cases} \vartheta_n > 0 & \text{irrelevant} \\ \vartheta_n < 0 & \text{relevant} \end{cases}$$

power counting

$$[g_{\mu\nu}] = 0 \qquad [D_\mu] = 1 \qquad \Box = g^{\mu\nu} D_\mu D_\nu$$

invariants

$$\lambda_i \int d^4x \sqrt{\det g_{\mu\nu}}\, \mathcal{O}_i(D_\rho, g_{\sigma\tau})$$

RG flow

$$\frac{d\lambda_i}{d\ln k} = -d_i\,\lambda_i + \textbf{quantum corrections}$$

canonical dim.

$$d_i = 4 - 2n$$

n=2: $\Box R, \quad R_{\mu\nu}R^{\mu\nu}, \quad R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

n=3: $R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\lambda\tau} R_{\lambda\tau}{}^{\mu\nu}$

power counting

$$[g_{\mu\nu}] = 0 \qquad [D_\mu] = 1 \qquad \Box = g^{\mu\nu} D_\mu D_\nu$$

invariants $\quad \lambda_i \int d^4x \sqrt{\det g_{\mu\nu}}\, \mathcal{O}_i(D_\rho, g_{\sigma\tau})$

RG flow $\quad \dfrac{d\lambda_i}{d\ln k} = -d_i\, \lambda_i +$ **quantum corrections**

canonical dim. $\quad d_i = 4 - 2n$

classical scaling $\quad \vartheta_{\mathrm{G},n} = 2n - 4$

knowns and unknowns

asymptotic freedom

$$g_* = 0$$

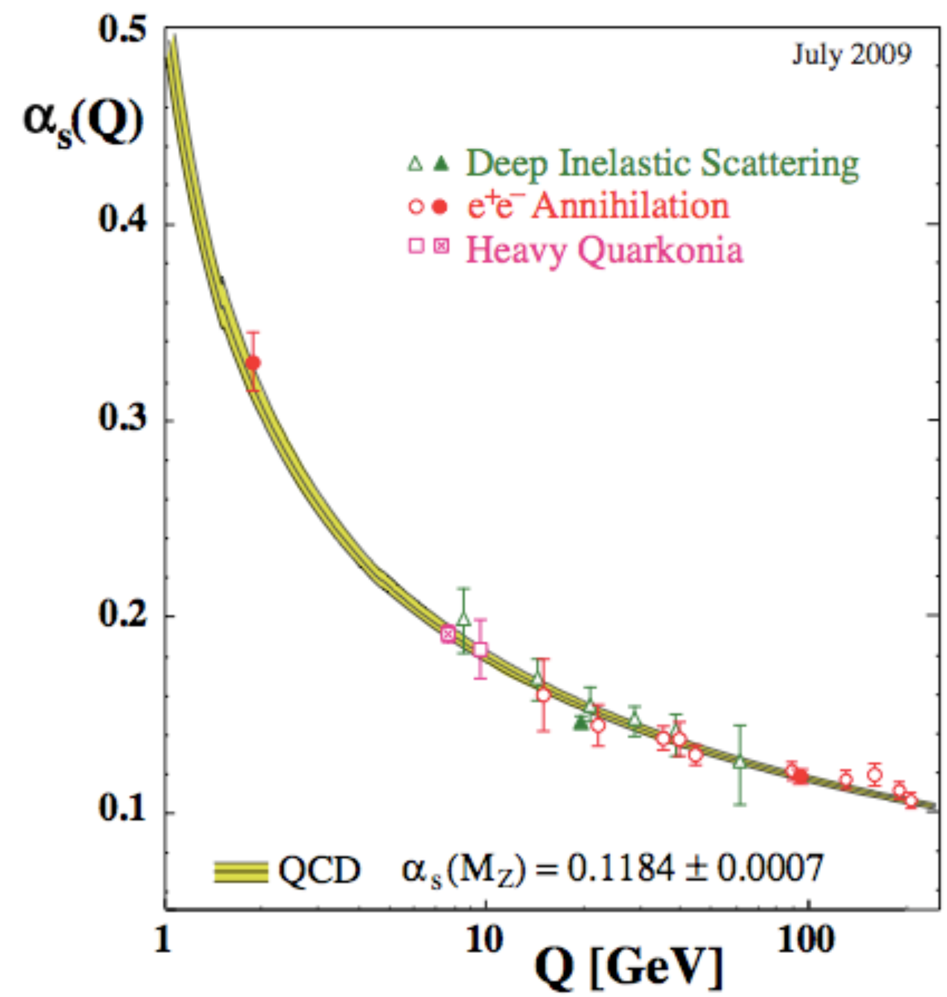
anomalous dimensions

$$\eta_A = 0$$

canonical power counting

$\{\vartheta_{\mathrm{G},n}\}$ are known

F^{256} irrelevant !



knowns and unknowns

asymptotic freedom

$$g_* = 0$$

anomalous dimensions

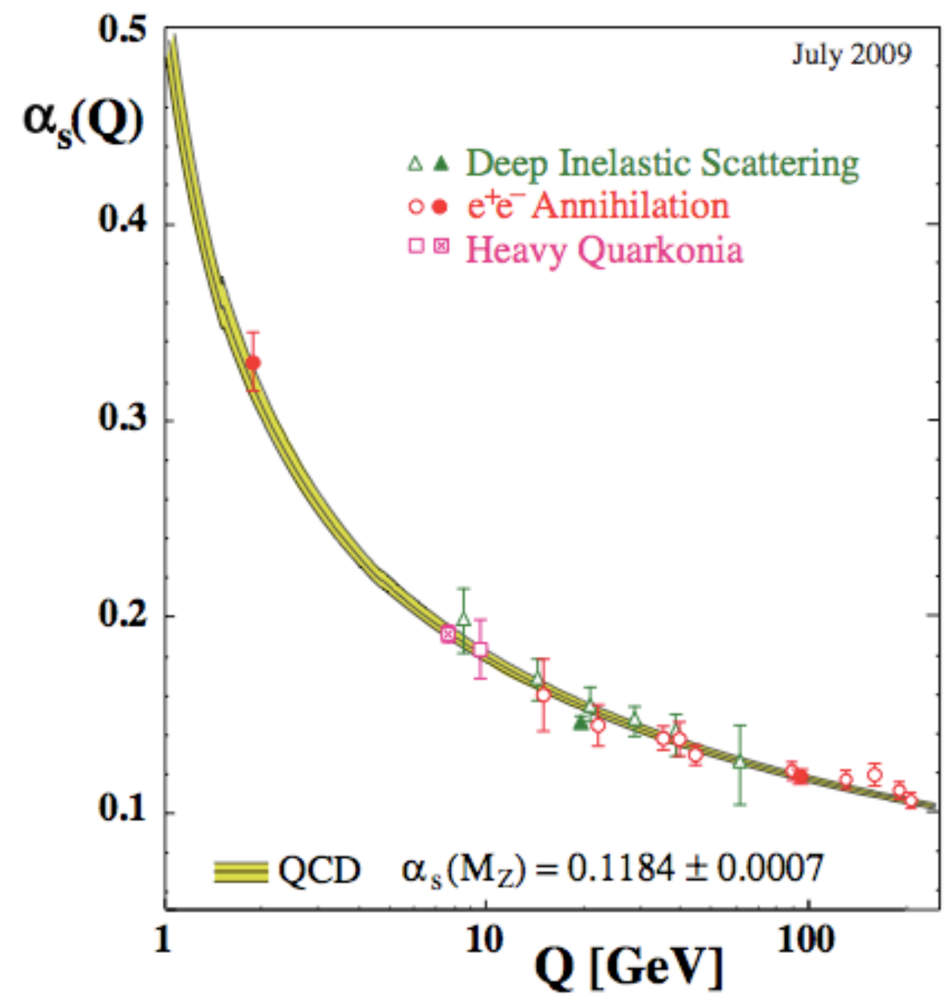
$$\eta_A = 0$$

canonical power counting

$\{\vartheta_{\mathrm{G},n}\}$ are known

F^{256} irrelevant !

go and climb Mount Everest



knowns and unknowns

asymptotic freedom

$$g_* = 0$$

anomalous dimensions

$$\eta_A = 0$$

canonical power counting

$\{\vartheta_{\mathrm{G},n}\}$ are known

F^{256} irrelevant !

asymptotic safety

$$g_* \neq 0$$

anomalous dimensions

$$\eta_N \neq 0$$

non-canonical power counting

$\{\vartheta_n\}$ are not known

R^{256} relevant / marginal / irrelevant ?

bootstrap

hypothesis ordering follows canonical dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

convergence (no convergence) of the iteration:

hypothesis supported (refuted)

bootstrap

hypothesis ordering follows canonical dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

testing ground

f(R) quantum gravity

f(R) quantum gravity

Step 1

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g} \, R^n$$

invariants up to $D = 2(N-1)$

Step 2

RG flow
fixed point
scaling exponents

Step 3

enhance $N \to N+1$
& iterate

f(R) quantum gravity

Step 1

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g} \, R^n$$

invariants up to $D = 2(N-1)$

Step 2

RG flow
fixed point
scaling exponents

Step 3

enhance $N \to N+1$
& iterate

iterate Step 1, 2 and 3

how often is enough?

well, it depends...

here:
34 consecutive orders

f(R) quantum gravity

Step 1

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n\, k^{d_n} \int d^4x \sqrt{g}\, R^n$$

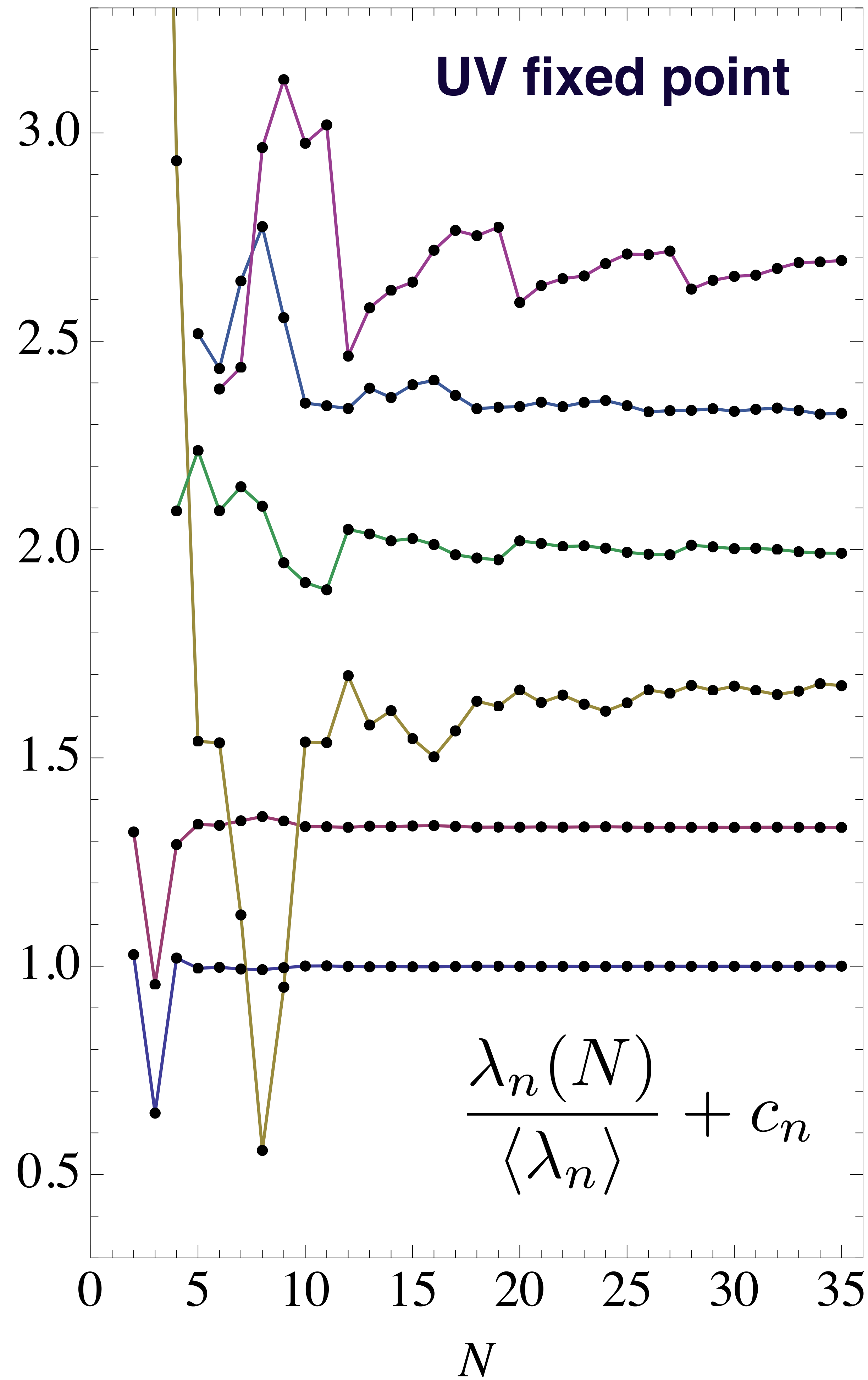
invariants up to $D = 2(N-1)$

Step 2

RG flow
fixed point
scaling exponents

Step 3

enhance $N \to N+1$
& iterate



f(R) quantum gravity

UV fixed point

Step 1

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g}\, R^n$$

invariants up to $D = 2(N-1)$

$$\begin{aligned}
\langle\lambda_0\rangle &= 0.25574 && \pm\, 0.015\% \\
\langle\lambda_1\rangle &= -1.02747 && \pm\, 0.026\% \\
\langle\lambda_2\rangle &= 0.01557 && \pm\, 0.9\% \\
\langle\lambda_3\rangle &= -0.4454 && \pm\, 0.70\% \\
\langle\lambda_4\rangle &= -0.3668 && \pm\, 0.51\% \\
\langle\lambda_5\rangle &= -0.2342 && \pm\, 2.5\%
\end{aligned}$$

Step 2

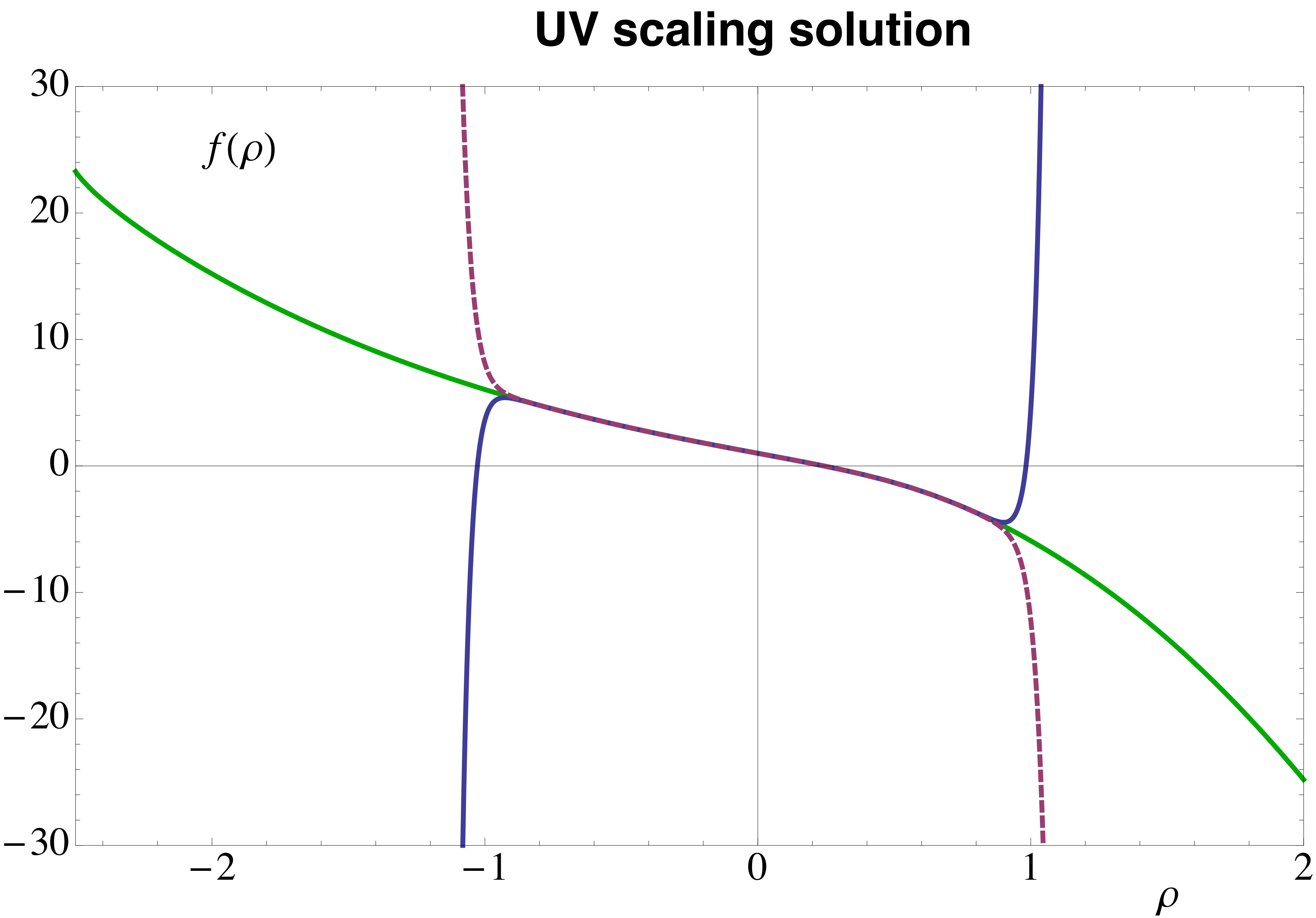
RG flow
fixed point
scaling exponents

fixed point coordinates become **independent** of the approximation order

Step 3

enhance $N \to N+1$
& iterate

f(R) quantum gravity

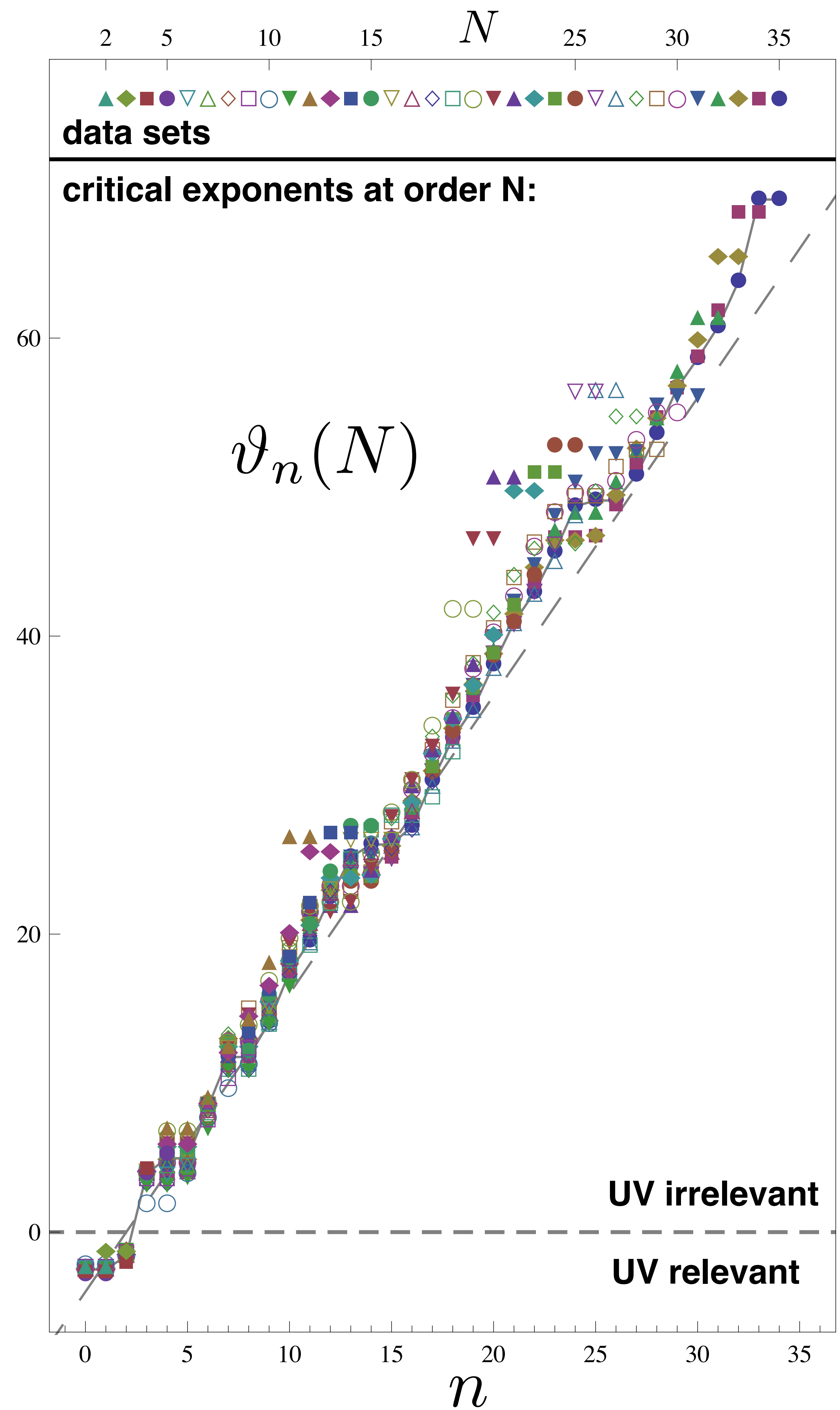


radius of convergence $\rho_c \approx 0.82 \pm 5\%$

f(R) quantum gravity

evaluate sets of eigenvalues

$$\{\vartheta_n(N), 0 \leq n \leq N-1\}$$



f(R) quantum gravity

evaluate sets of eigenvalues

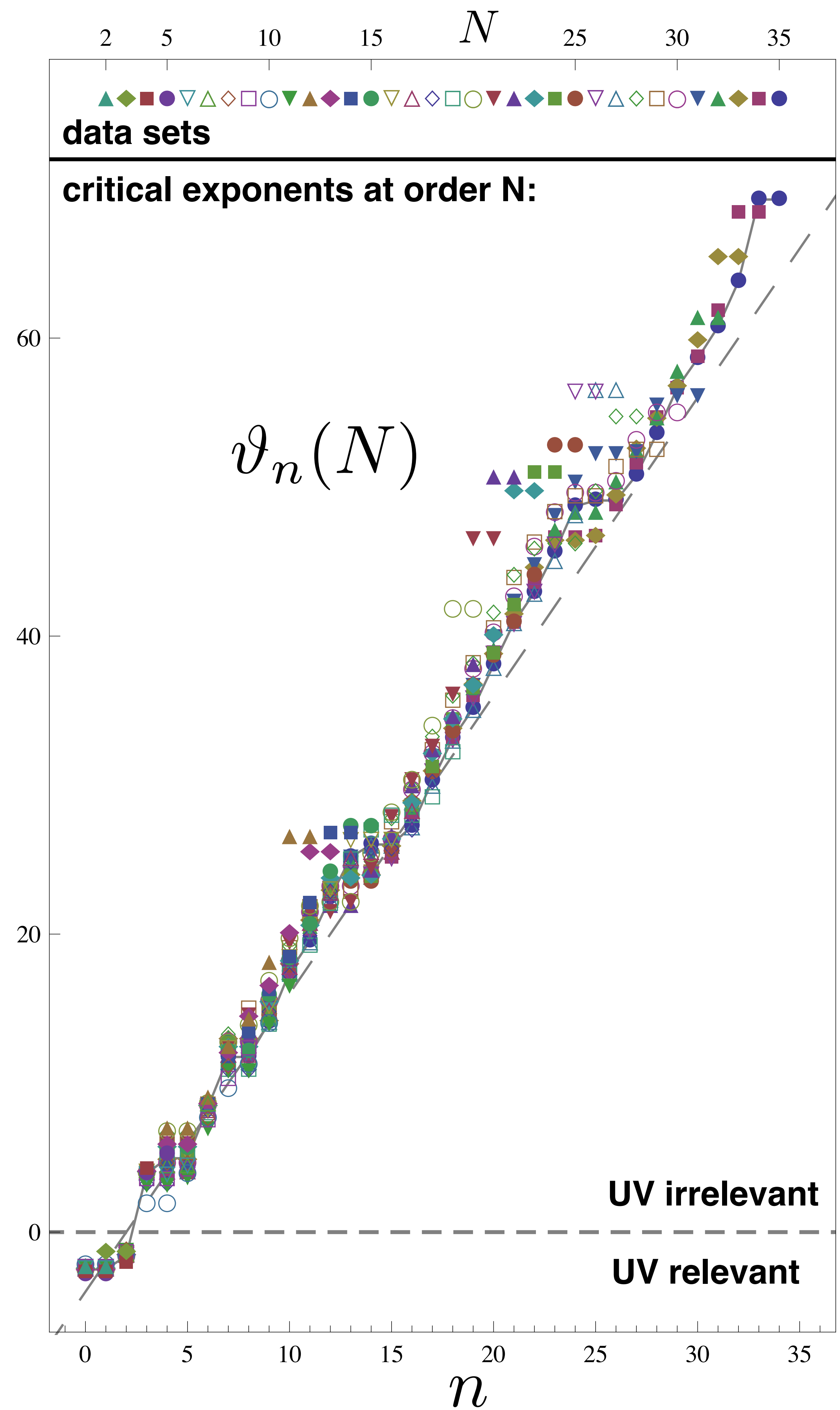
$$\{\vartheta_n(N), 0 \le n \le N-1\}$$

linear least-square fit

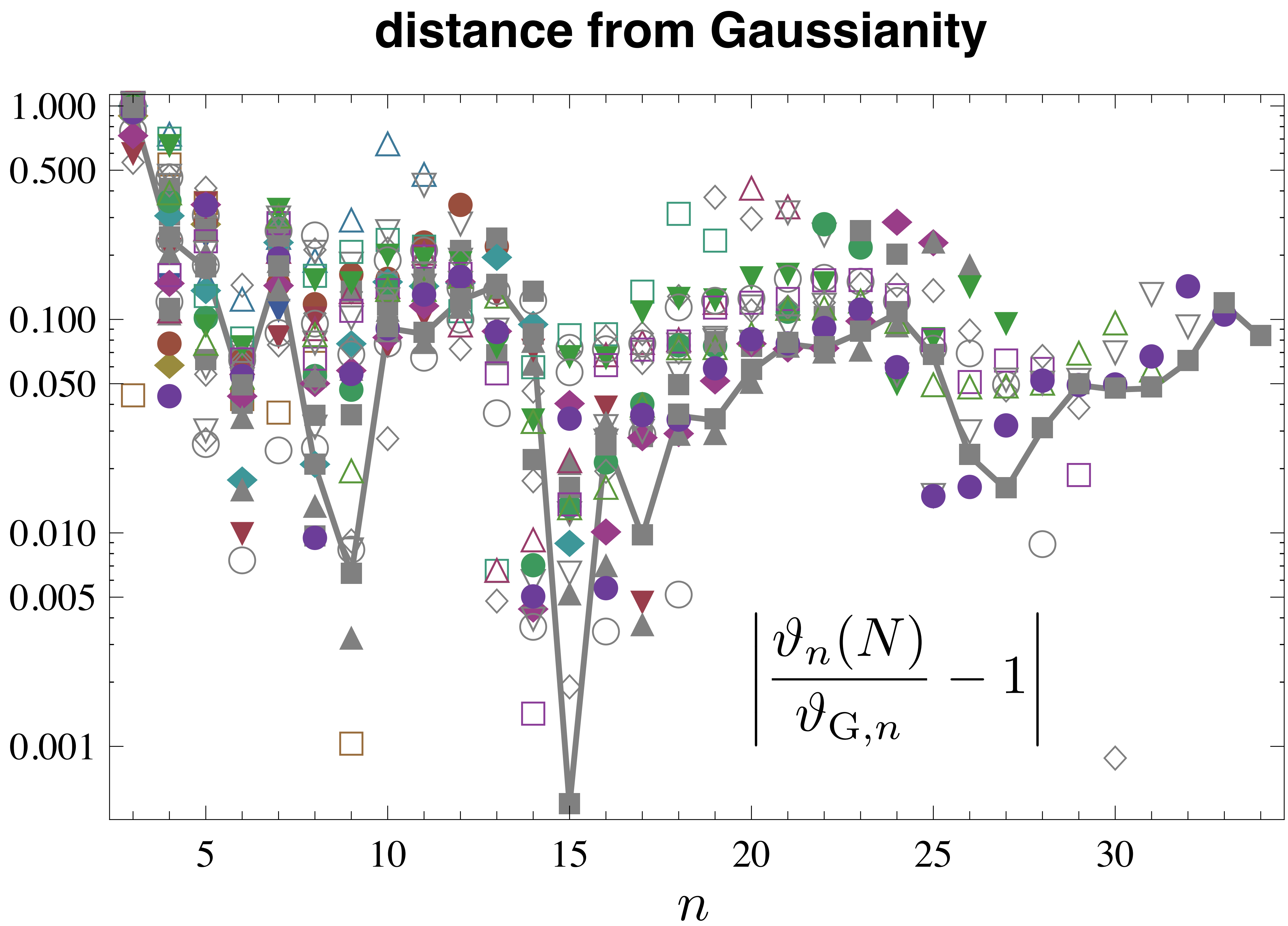
$$\vartheta_n \approx a \cdot n - b$$

with

$$\begin{aligned} a_{\rm UV} &= 2.17 \pm 5\% \\ b_{\rm UV} &= 4.06 \pm 10\% \end{aligned}$$



f(R) quantum gravity



question

is metric quantum gravity fundamental ?

answer

perhaps.

systematic search strategy available

very encouraging results from model studies

conclusions

systematic bootstrap strategy

justifies **canonical dimension** as guiding principle

f(R) quantum gravity

stability of asymptotic safety

near-Gaussian scaling dimensions

R^{256} relevant / marginal / irrelevant **?**

irrelevant at self-consistent fixed point **!**

extendable to other asymptotically safe theories