't Hooft Determinant at Finite Temperature with Fluctuations

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Doktoratskolleg Graz "Hadrons in Vacuum, Nuclei and Stars", funded by the Austrian Science Fund: FWF DK W1203-N16.

Heidelberg, January 2013

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Chiral Symmetry and Axial Anomaly

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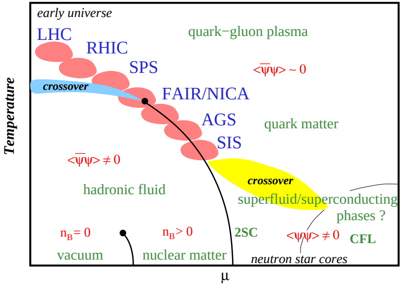
η'-meson is no pseudo-Goldstone boson

- $N_f^2 - 1 + 1$ broken generators
- experiment $N_f = 2$ ($N_f = 3$): 3 pions (+4 kaons and 1 η-meson)
- $U(1)_A$ anomalously broken [Adler, Bell, Jackiw, 1969], [Fujikawa, 1979]

$U(1)_A$ at Chiral Transition

Recent experimental results (PHENIX, STAR): [Csorgo et al., 2010]

- drop $\delta m_{\eta'} \approx 200$ MeV at chiral transition temperature
- partial $U(1)_A$-symmetry restoration at chiral crossover?



[CBM Physics Book, 2011]

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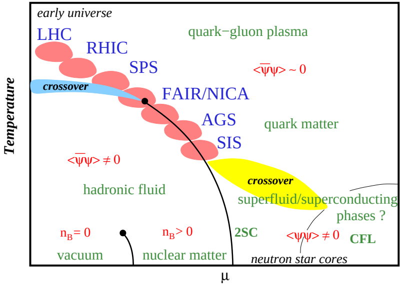

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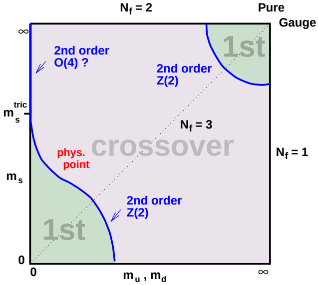

[Laermann, Philipsen, 2003]

Effects of anomaly in chiral limit [Pisarski, Wilczek, 1984]

- $N_f > 2$: 1^{st} order
- $N_f = 2$: $1^{st} \to 2^{nd}$ if insensitive to temperature for $T < T_c$

Effective Description

The Quark-Meson Model (general N_f)

[Jungnickel, Wetterich, 1996]

$$\mathcal{L}_{QM} = \bar{q}\left(\partial^\mu\gamma^\mu - iht^a\left(\sigma^a + i\gamma_5\pi^a\right)\right)q + \mathcal{L}_M$$

$$\mathcal{L}_M = \mathrm{tr}\left[\partial^\mu\Sigma^\dagger\partial^\mu\Sigma\right] + U\left(\{\rho_i\},\xi\right) - \mathrm{tr}\left(C\left(\Sigma+\Sigma^\dagger\right)\right)$$

$$\Sigma = t^a\left(\sigma^a + i\pi^a\right), \quad t^a: \text{ group generators of } U(N_f)$$

$$\rho_i = \mathrm{tr}\left[\left(\Sigma^\dagger\Sigma\right)^i\right], \quad C = \mathrm{diag}\left(c_1,\dots,c_{N_f}\right)$$

$U(1)_A$ anomaly in QM-Model

- "integrate instantons" in QCD Lagrangian ['t Hooft, 1976]
  $\Rightarrow$ contribution to Lagrangian proportional (for QCD with $\theta = 0$)

$$\det\left(q_L\bar{q}_R\right) + \det\left(q_R\bar{q}_L\right)$$

- axial anomaly in QM-model: $\xi = \det(\Sigma) + \det(\Sigma^\dagger)$

Wetterich Equation for Effective Potential

Flow Equation in Leading Order Derivative Expansion

$$\partial_k U_k = \frac{k^4}{12\pi^2} \left[\sum_{i=1}^{2N_f^2} \frac{1}{E_{M,i}} \coth\left(\frac{E_{M,i}}{2T}\right) - 2N_c \sum_{i=1}^{N_f} \frac{1}{E_{Q,i}} \left\{ \tanh\left(\frac{E_{Q,i}+\mu_i}{2T}\right) + \tanh\left(\frac{E_{Q,i}-\mu_i}{2T}\right) \right\} \right]$$

$$E_{M/Q,i} = \sqrt{k^2 + m^2_{M/Q,i}}$$

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Two Approximation Methods

- Taylor expansion of U_k in $(\{\rho_i\}, \xi)$
- discretize U_k:
  - grid $\mapsto (\{\rho_i\}, \xi)$
  - splines,... for derivatives

$N_f = 2$

effective potential ($\sigma_0 \leftrightarrow \bar{u}u + \bar{d}d$, $\sigma_3 \leftrightarrow \bar{d}d - \bar{u}u$):

$$U_k(\Sigma) = \tilde{U}_k(\rho_1, \xi) - c_0\sigma_0 - c_3\sigma_3 \ ,$$

first chiral invariant and 't Hooft determinant:

$$\rho_1 = \frac{\sigma_0^2 + \sigma_3^2}{2} \ , \qquad \xi = \frac{\sigma_0^2 - \sigma_3^2}{2} \ ,$$

Fix Parameters in Vacuum $T = 0$

- initial potential:
  $U_\Lambda = a_{\Lambda,10}(\rho_1 - \rho_{1,0}) + a_{\Lambda,01}(\xi - \xi_0) + \frac{a_{\Lambda,20}}{2}(\rho_1 - \rho_{1,0})^2 - c_0\sigma_0 - c_3\sigma_3$
- Yukawa coupling h

- m_π, f_π, $m_d + m_u$, $m_{\eta'}$ and m_σ at $k \to 0$

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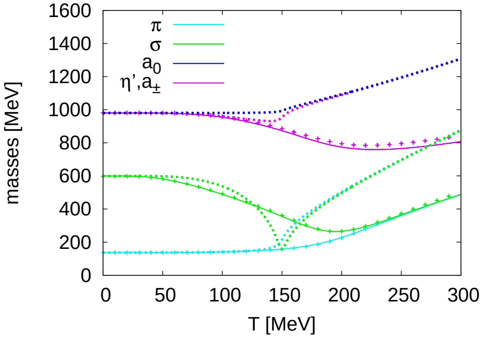
- Yukawa coupling h
- m_π, f_π, $m_d + m_u$, $m_{\eta'}$ and m_σ at $k \to 0$
- different isospin breakings c_3

Mesonic Masses

- realistic ($N_f = 3$) $m_{\eta'}$: $m_{\eta'}(k \to 0) = 980$ MeV
- explicit breaking: $c_3 = c_0$



solid: Taylor RG, crosses: grid RG, short dashed: MF

[MM, Schaefer, Strodthoff, von Smekal, in preparation 2013]

Condensates

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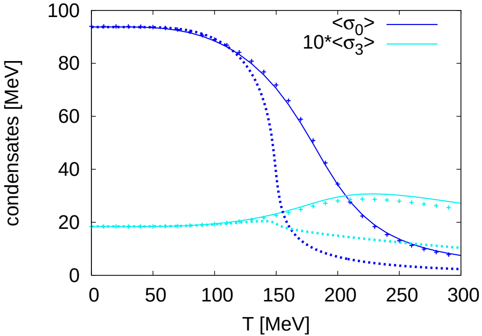

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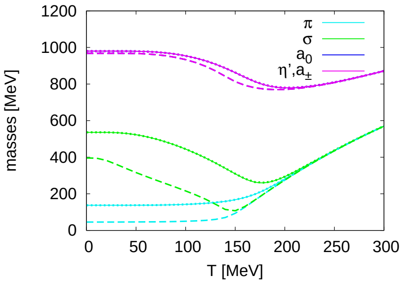
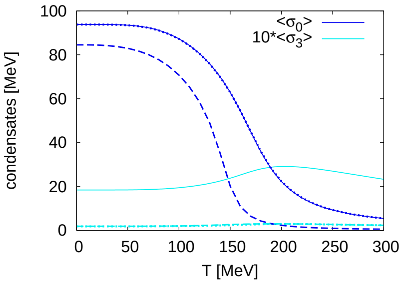
Influence of Explicit Symmetry Breaking

- realistic ($N_f = 3$) $m_{\eta'}$: $m_{\eta'}(k \to 0) = 980$ MeV
- solid: $c_3 = c_0$, dotted: $c_3 = 0.1 \cdot c_0$
  (dashed: $10 \cdot c_3 = 10 \cdot c_0 = c_{phys}$)





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Explicit Symmetry Breaking and Vacuum Alignment

- $\rho_1 \propto \sigma_0^2 + \sigma_3^2$ respects rotations in σ_0, σ_3 plane
- absence of determinant terms ξ: $(\langle\sigma_0\rangle, \langle\sigma_3\rangle) \propto (c_0, c_3)$

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- physical: $c_0 \approx 2 \cdot c_3$ (or $m_d \approx 2 \cdot m_u$)
- but: $\langle\sigma_0\rangle \gg \langle\sigma_3\rangle$ (or $\langle\bar{d}d + \bar{u}u\rangle \gg \langle\bar{d}d - \bar{u}u\rangle$)

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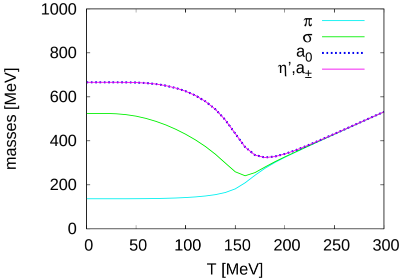
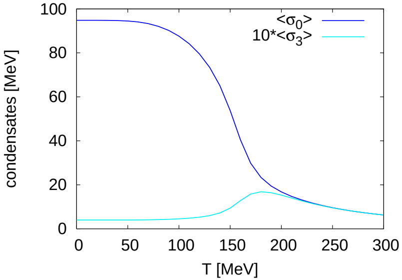
- suppression of $\langle\sigma_3\rangle$ requires other operator than ρ_1

Vanishing Initial Anomaly

- $m_\pi - m_{\eta'} = -2a_{01} \to 0$ at initial scale Λ
- solid: $10 \cdot c_3 = c_0$





[MM, Schaefer, Strodthoff, von Smekal, in preparation 2013]

Z_2 Symmetric Limit

- initial potential:
  $U_\Lambda = a_{\Lambda,10}(\rho_1 - \rho_{1,0}) + a_{\Lambda,01}(\xi - \xi_0) + \frac{a_{\Lambda,20}}{2}(\rho_1 - \rho_{1,0})^2 - c_0\sigma_0 - c_3\sigma_3$
- $\rho_1 \propto \sigma_0^2 + \sigma_3^2$ and $\xi^{2n} \propto (\sigma_0^2 - \sigma_3^2)^{2n}$ respect $Z_2:\ \sigma_0 \leftrightarrow \sigma_3$

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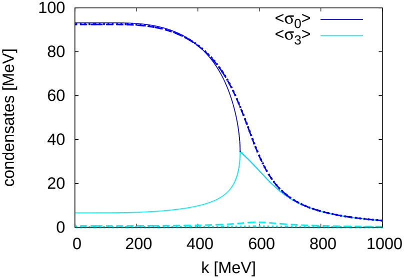
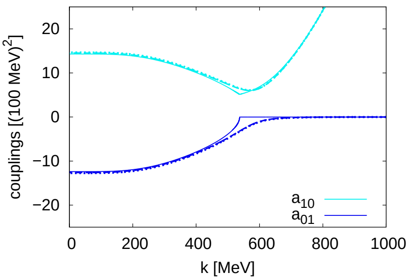
- physical infrared observables require Z_2 breaking via $\langle\sigma_0\rangle \gg \langle\sigma_3\rangle$

Spontaneous Breaking of Z_2

- ultraviolet: $\det(k=\Lambda)=0$, $m_{\eta'}(k=\Lambda)=m_\pi(k=\Lambda)$
- $U_k(\rho_1,\xi) = \sum_{i,j} \frac{a_{ij}}{i!j!}(\rho_1-\rho_{0,1})^i(\xi-\xi_0)^j$





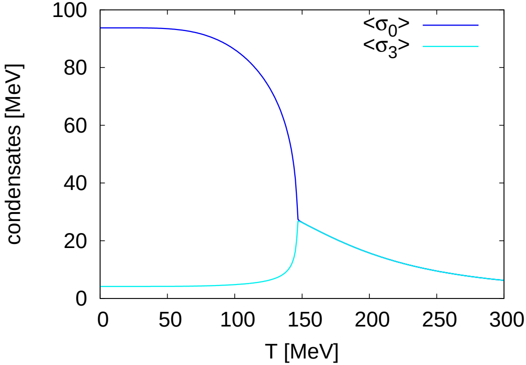
solid: $c_3 = c_0$, dashed: $c_3 = 0.1 \cdot c_0$, dotted: $c_3 = 0.02 \cdot c_0$, [MM, Schaefer, Strodthoff, von Smekal, in preparation 2013]

New Phase transition: Z_2 Universality Class

- ultraviolet: $\det(k = \Lambda) \approx 0$,

  $m_{\eta'}(k = \Lambda) \approx m_\pi(k = \Lambda)$

- $m_\pi^2 \propto a_{10} + a_{01}$
- $m_{\eta'}^2 \propto a_{10} - a_{01}$



[MM, Schaefer, Strodthoff, von Smekal, in preparation 2013]

- $\beta = 0.37$ (Z_2 universality class: 0.326)

Z_2 Transition: Couplings

- ultraviolet: $\det(k = \Lambda) \approx 0$,

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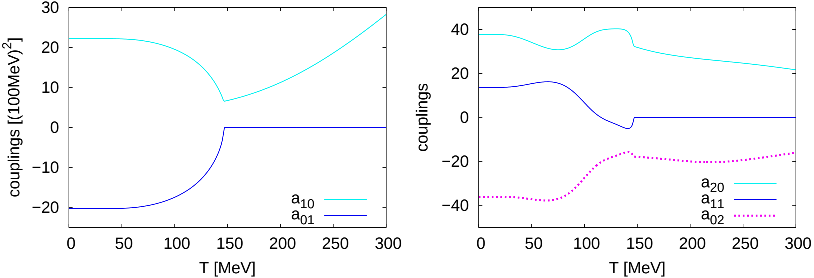

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Summary and Outlook

- temperature dependence: $m_{\eta'}$
- 't Hooft term $\leftrightarrow$ isospin breaking
- new transition: Z_2 universality class

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- influence of chiral invariant ρ_2
- rebosonization, determinant from QCD