# Proton scattering on an electron gas

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1. Collision in many-body systems
   with target-beam entanglement and final state interactions
2. Closed Time Path method
   to handle system-envrionment entanglement
3. Reduction formulae
   for transition probabilities
4. Perturbation expansion
5. p+e $\to$ p+e
   - 5.1 entanglement
   - 5.2 decoherence
   - 5.3 real and virtual excitations
6. Summary

# Collision in many-body systems

$e(p_e) + p(p) \to e(q_e) + p(q)$ on electron gas

**Transition amplitude:** $\mathcal{A}_n = \langle \Psi^{gas}_{n,\widehat{\mathbf{q}}_e} | a_p(\mathbf{q}) S a^\dagger_p(\mathbf{p}) | \Psi^{gas}_0 \rangle.$

with or without identifying the state of the participating electron

**Transition probability:** $P = \sum_n |\mathcal{A}_n|^2$

$$
\begin{aligned}
&= \sum_n \langle \Psi^{gas}_0 | a_p(\mathbf{p}) S^\dagger a^\dagger_p(\mathbf{q}) | \Psi^{gas}_{n,\widehat{\mathbf{q}}_e} \rangle \langle \Psi^{gas}_{n,\widehat{\mathbf{q}}_e} | a_p(\mathbf{q}) S a^\dagger_p(\mathbf{p}) | \Psi^{gas}_0 \rangle \\
&= \langle \Psi^{gas}_0 | a_p(\mathbf{p}) S^\dagger a^\dagger_p(\mathbf{q}) a^\dagger_e \widehat{(\mathbf{q}_e) a_e}(\mathbf{q}_e) a_p(\mathbf{q}) S a^\dagger_p(\mathbf{p}) | \Psi^{gas}_0 \rangle \\
&= \mathrm{Tr}\Big[ \underbrace{a^\dagger_p(\mathbf{q}) a^\dagger_e \widehat{(\mathbf{q}_e) a_e}(\mathbf{q}_e) a_p(\mathbf{q})}_{\mathcal{O}} \, S \, \underbrace{a^\dagger_p(\mathbf{p}) | \Psi^{gas}_0 \rangle \langle \Psi^{gas}_0 | a_p(\mathbf{p}_2)}_{\rho_i} \, S^\dagger \Big] \\
&= \mathrm{Tr}[\mathcal{O} S \rho_i S^\dagger] \quad \leftarrow \quad \mathrm{CTP}
\end{aligned}
$$

1. Collision in many body systems
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# Closed Time Path method

## An incomplete list

- QED (Schwinger 1961)
- relaxation in many-body systems (Kadanoff, Baym 1962)
- density matrix by path integral (Feynman, Vernon 1963, Diosi 1986)
- perturbation expansion for retarded Green-functions (Keldysh, 1964)
- time symmetric quantum mechanics
  (Aharonov, Bergmann, Lebowitz 1964)
- thermal field theory (Umezawa 1975)
- consistent histories (Griffith 1984, Gell-Mann, Hartle 1993)
- equations of motion for the expectation value of local operators
  (Jordan 1986)
- non-equilibrium processes (Calzetta, Hu 1988)
- classical mechanics (Polonyi 2012)

# Closed Time Path

## Open systems

**Expectation value** as opposed to transition amplitude

$$\bar{\mathcal{O}} = \mathrm{Tr}[\mathcal{O}U(t_f,t_i)\rho_i U^\dagger(t_f,t_i)],$$

**Generator functional:**

$$\begin{aligned} e^{iW[\hat{j}]} &= \mathrm{Tr}\, T[e^{-i\int dt'[H(t')-j^+(t')x(t')]}]\rho_i T^* e^{i\int dt'[H(t')+j^-(t')x(t')]}] \\ &= \int D[\hat{x}]e^{iS[x^+]-iS^*[x^-]+i\int dt\hat{j}(t)\hat{x}(t)} \\ &= \int D[\hat{x}]e^{iS_{CTP}[\hat{x}]+i\int dt\hat{j}(t)\hat{x}(t)} \end{aligned}$$

reduplication of the source $\hat{j} = (j^+, j^-)$ and
the degrees of freedom $\hat{x} = (x^+, x^-)$

# Closed Time Path

## Entanglement

System: $\phi$, environment: $\chi$, $S[\phi,\chi]=S_s[\phi]+S_e[\phi,\chi]$

$$
\begin{aligned}
e^{iW[\hat{j}]} &= \int D[\hat{\phi}]D[\hat{\chi}]e^{iS[\phi^+,\chi^+]-iS[\phi^-,\chi^-]+i\hat{j}\cdot\hat{\phi}} \\
&= \int D[\hat{\phi}]e^{iS_s[\phi^+]-iS_s[\phi^-]-iS_I[\phi^+,\phi^-]+i\hat{j}\cdot\hat{\phi}}
\end{aligned}
$$

Influence functional (Feynman):

$$
e^{iS_I[\phi^+,\phi^-]}=\int D[\hat{\chi}]e^{iS_e[\phi^+,\chi^+]-iS_e[\phi^-,\chi^-]}
$$

$S_I$: system-environment entanglement and mixed system state

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# Reduction formulae

## Generator functional

$$e^{iW[\hat{j},\hat{\bar{\eta}},\hat{\eta}]} = \mathrm{Tr}\left[S(\eta^+,\bar{\eta}^+,j^+)\rho_i S^\dagger(-\eta_-,-\bar{\eta}_-,-j^-)\right]$$
$$= \int D[\hat{\psi}]D[\hat{\bar{\psi}}]D[\hat{A}]e^{iS[\hat{A},\hat{\bar{\psi}},\hat{\bar{\psi}}]+i\int dx[\hat{\bar{\eta}}\hat{\psi}+\hat{\bar{\psi}}\hat{\eta}+\hat{j}\hat{A}]}$$

$$S[\hat{A},\hat{\psi},\hat{\bar{\psi}}] = \int dx\hat{\bar{\psi}}(\hat{G}^{-1}-e\hat{\not{A}})\hat{\psi}+\frac{1}{2}\int dx\hat{A}\hat{D}_0^{-1}\hat{A}$$

$$\hat{G}^{-1} = \begin{pmatrix} i\not{\partial}-m+i\epsilon & 0 \\ 0 & -\gamma^0(i\not{\partial}m+i\epsilon)^\dagger\gamma^0 \end{pmatrix}+\hat{G}_{BC}^{-1}$$

$$\hat{D}_{0\mu\nu}^{-1} = g_{\mu\nu}\begin{pmatrix} \Box+i\epsilon & 0 \\ 0 & -\Box+i\epsilon \end{pmatrix}+\hat{D}_{BC}^{-1},$$

Final conditions on the trajectories:

$$\psi^+(t_f,\mathbf{x}) = \psi^-(t_f,\mathbf{x}),$$
$$\bar{\psi}^+(t_f,\mathbf{x}) = \bar{\psi}^-(t_f,\mathbf{x}),$$
$$A_\mu^+(t_f,\mathbf{x}) = A_\mu^-(t_f,\mathbf{x}),$$

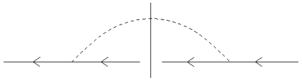
# Reduction formulae

## Transition probability

Electron state not registered: p $\to$ p inelastic collision

$$
\begin{aligned}
P &= Z_p^{-2}\int dx^+ dy^+ dx^- dy^- e^{iq(y^+-y^-)-ip(x^+-x^-)} \\
&\quad (\bar{u}_f K)^+_{y^+}(\bar{u}_i K)^-_{x^-}(K u_i)^+_{x^+}(K u_f)^-_{y^-}\frac{\delta^4 W}{\delta\bar{\eta}_p^-(x^-)\delta\eta_p^-(y^-)\delta\bar{\eta}_p^+(y^+)\delta\eta_p^+(x^+)}
\end{aligned}
$$

where $K^{\pm} = \pm i\partial\!\!\!/ - M$



CTP graph structure: $t_f$ $S$ $t_i$ $t_f$ $S^\dagger$ $t_i$

$\uparrow$

particles in the final state

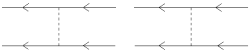
**CTP $\neq$ Thermal Field Theory**

# Reduction formulae

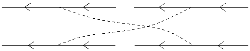
## Transition probability

Electron state registered: e+p $\to$ e + p elastic collision

$$
\begin{aligned}
P \;=\;& Z_e^{-2} Z_p^{-2} \int dx_1^+ dx_2^+ dy_1^+ dy_2^+ dx_1^- dx_2^- dy_1^- dy_2^- \\
& e^{iq_1(y_1^+ - y_1^-) + iq_2(y_2^+ - y_2^-) - ip_1(x_1^+ - x_1^-) - ip_2(x_2^+ - x_2^-)} \\
& (\bar{u}_{ef} K)^+_{y_e^+} (\bar{u}_{pf} K)^+_{y_p^+} (K u_{ef})^-_{y_e^-} (K u_{pf})^-_{y_p^-} \\
& (K u_{ei})^+_{x_e^+} (K u_{pi})^+_{x_p^+} (\bar{u}_{ei} K)^-_{x_e^-} (\bar{u}_{pi} K)^-_{x_p^-} \\
& \frac{\delta^8 W}{\delta\eta_p^-(y_p^-)\delta\eta_e^-(y_e^-)\delta\bar{\eta}_p^-(x_p^-)\delta\bar{\eta}_e^-(x_e^-)\delta\bar{\eta}_e^+(y_e^+)\delta\bar{\eta}_p^+(y_p^+)\delta\eta_e^+(x_e^+)\delta\eta_p^+(x_p^+)}
\end{aligned}
$$



direct



entanglement

# Reduction formulae

## Transition probability

$$
\begin{aligned}
P &= -\frac{ie^2}{2M^2}[p^\mu q^\nu + q^\mu p^\nu + g^{\mu\nu}(M^2 - pq)]D^{+-}_{0\mu\nu}(r) \\
P_d &= \frac{e^4}{4m^2M^2}\{2\Re[(p_1 D^{++} p_2)(q_1 D^{++*} q_2)] + 2\Re[(p_1 D^{++} q_2)(q_1 D^{++*} p_2)] \\
&\quad + \mathrm{tr}[D^{++} D^{++*}](m^2 - p_1 q_1)(M^2 - p_2 q_2) \\
&\quad + 2(M^2 - p_2 q_2)\Re[p_1 D^{++} D^{++*tr} q_1] \\
&\quad + 2(m^2 - p_1 q_1)\Re[p_2 D^{++tr} D^{++*} q_2]\}, \\
P_e &= \frac{e^4}{4m^2M^2}\{2\Re[(p_1 D^{+-} p_2)(q_1 D^{+-*} q_2)] + 2\Re[(p_1 D^{+-} q_2)(q_1 D^{+-*} p_2)] \\
&\quad + \mathrm{tr}[D^{+-} D^{+-\dagger}](m^2 - p_1 q_1)(M^2 - p_2 q_2) \\
&\quad + 2(M^2 - p_2 q_2)\Re[p_1 D^{+-} D^{+-\dagger} q_1] \\
&\quad + 2(m^2 - p_1 q_1)\Re[q_2 D^{-+\dagger}(-r) D^{-+}(-r) p_2]\},
\end{aligned}
$$

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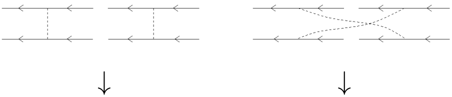
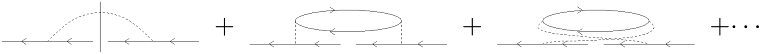
# Perturbation expansion

## *t*-channel resummation

closing electron lines:
$G^{+-}$ on mass shell





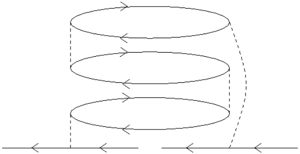
$D^{+-}$ — mass shell→vanishing

$D^{++}\Pi^{+-}D^{--}$

$D^{+-}\Pi^{-+}D^{+-}$ — vanishing

non-vanishing contributions:

$D^{++}\Pi^{+-}D^{--}\Pi^{-+}D^{++}\Pi^{+-}D^{--}$



Schwinger-Dyson resummation: $\hat{D} = \dfrac{1}{\hat{D}_0^{-1} - \hat{\Pi}}$

for homogeneous electron gas, $k_F \neq 0$

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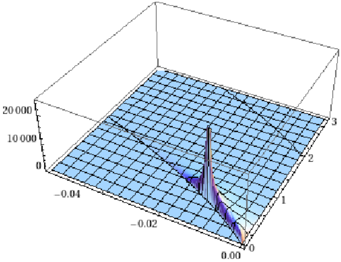
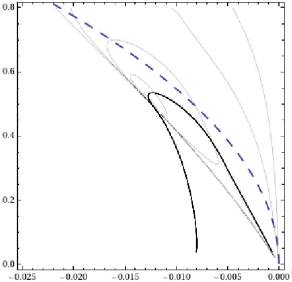
Spectral function, $\Im D_\ell^{+-}(\omega, \mathbf{r})$, of dressed electric photons on the plane $\left(\frac{\omega}{k_F}, \frac{|\mathbf{r}|}{k_F}\right)$




Solid line: Electric dispersion relation $\Re D^{++-1}(\omega, \mathbf{r}) = 0$

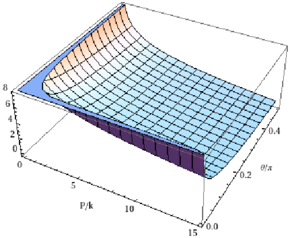
Thin lines: The contour plot of $\Im D_\ell^{+-}$



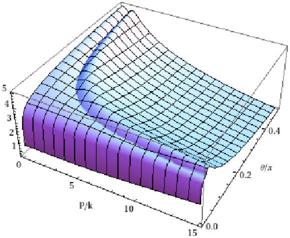
# p+e $\rightarrow$ p+e

## Cross section

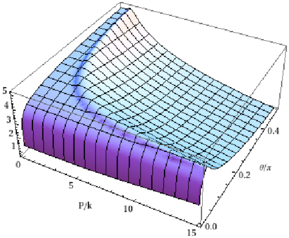
$\frac{d^2\sigma}{dPd\Theta}$ as function of $\frac{P}{k_F}$ and $\frac{\Theta}{\pi}$:




$\log \sigma_{vac}$

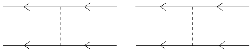



$\log \sigma_{direct}$



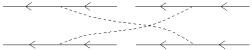

$\log(\sigma_{direct} + \sigma_{entangl})$

$\sigma_{vac}$ or $\sigma_{direct}$



factorizable

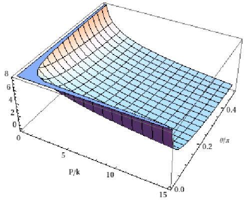
$\sigma_{entangl}$

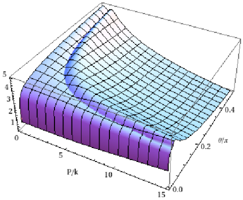


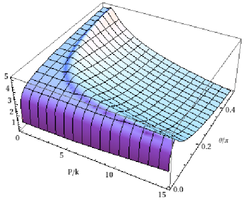
photon-charge entanglement

# p+e $\to$ p+e

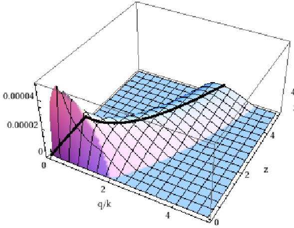
## Spectral strengths at $\Theta = 0.5\pi$

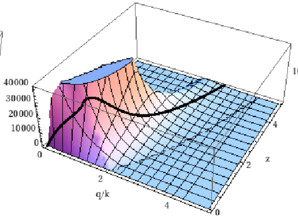


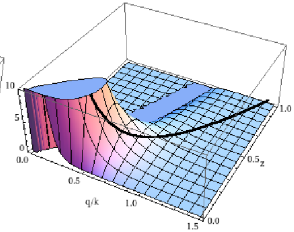




$\log \sigma_{vac}$ $\log \sigma_{direct}$ $\log(\sigma_{direct} + \sigma_{entangl})$




$z = \frac{m\omega}{k_F^2}$ current electric photon magnetic photon

spectral functions

# p+e $\to$ p+e

## Effective theory of non-relativistic charges

$$
\begin{aligned}
Z &= \int D[\hat{A}]D[\hat{\mathbf{x}}]e^{iS[\mathbf{x}^+]-iS[\mathbf{x}^-]+\frac{i}{2}\hat{A}\hat{D}_0^{-1}\hat{A}-ij^+A^++ij^-A^-} = \int D[\hat{\mathbf{x}}]e^{iS_{eff}[\hat{\mathbf{x}}]} \\
e^{iW_0[\hat{j}]} &= \int D[\hat{A}]e^{\frac{i}{2}\hat{A}\hat{D}_0^{-1}\hat{A}+i\hat{j}\hat{A}} = e^{-\frac{i}{2}\hat{j}\hat{D}_0\hat{j}} \\
S_{eff}[\hat{\mathbf{x}}] &= S[\mathbf{x}^+] - S[\mathbf{x}^-] + W_0[-ej[\mathbf{x}^+], ej[\mathbf{x}^-]]
\end{aligned}
$$

Parametrization: $x^\pm = x \pm \frac{x^d}{2}$, $j^\pm = j \pm \frac{j^d}{2}$

Real part: expectation values

$$
\Re S_{eff} = S[\mathbf{x}^+] - S[\mathbf{x}^-] - \frac{e^2}{2}[jD^aj^d - j^dD^rj]
$$

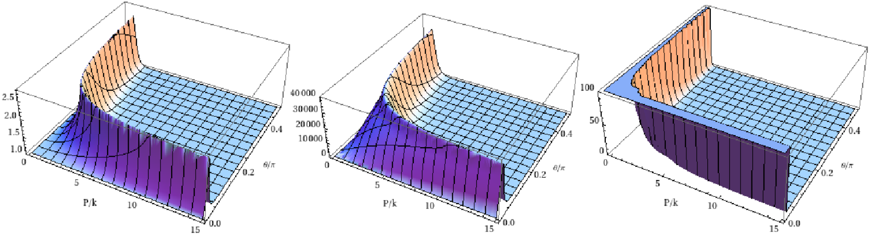
Imaginary part: decoherence, consistence

$$
\Im S_{eff} = -\frac{e^2}{2}j^dD^ij^d
$$

# p+e $\to$ p+e

## Entanglement and decoherence

$\left(\frac{P}{K_F}, \frac{\Theta}{\pi}\right)$ plane:




$\frac{\sigma_{direct}+\sigma_{entangl}}{\sigma_{direct}}$ $D^i_\ell$ $D^i_t$

weight of entanglement

inverse width of $\rho(x+\frac{x^d}{2}, x-\frac{x^d}{2})$ in $x^d$

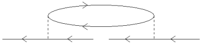
$$\left|e^{iS_{eff}}\right| = e^{-\frac{e^2}{2}j^d D^i j^d}$$

# p+e → p+e

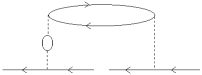
## Real vs. virtual excitations

Bloch, Nordsieck, Kinoshita, Lee, Nauenberg:
Cancellation of collinear divergences between real and virtual soft photons
(need of degenerate perturbation expansion)

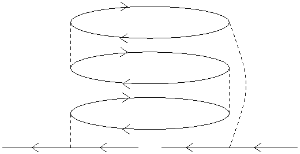
Leading order:



Virtual excitations:
within $S$ or $S^\dagger$



Real excitations:
at $t_f$



Is there a partial cancellation between real and virtual degenerate excitations?

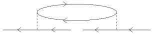
# p+e $\to$ p+e

## Real vs. virtual excitations

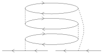
$$\hat{\Pi} = \hat{\Pi}_d + \hat{\Pi}_{nd}, \quad \hat{D}_d = \frac{1}{\hat{D}_0^{-1} - \hat{\Pi}_d} = \begin{pmatrix} D_F & 0 \\ 0 & -D_F^\dagger \end{pmatrix}$$

$$\hat{D} = \frac{1}{\hat{D}_d^{-1} - \hat{\Pi}_{nd}}$$

$$\hat{D}^{+-} = \sum_{n=0}^{\infty} [\hat{D}_d(\hat{\Pi}_{nd}\hat{D}_d)^{2n+1}]^{+-} = \hat{D}_d\hat{\Pi}_{nd}\hat{D}_d + \sum_{n=1}^{\infty} [\hat{D}_d(\hat{\Pi}_{nd}\hat{D}_d)^{2n+1}]^{+-}$$



virtual

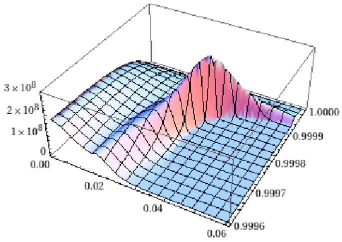


real

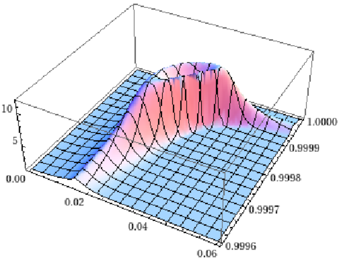
# p+e → p+e

## Real vs. virtual excitations

p → p inelastic collision: in the plane $(\frac{p}{M}, \frac{q}{p})$ for $\theta = 0.4 \cdot 10^{-4}\pi$




$\sigma$




$\frac{\sigma_{virt}}{\sigma}$

strong cancellation between
soft real and virtual excitations

# Summary

1. Open systems $\leftrightarrow$ QFT remade in CTP
2. e + p $\rightarrow$ e + p
   2.1 Solution of the CTP Schwinger-Dyson equation
   2.2 Relation between quasi-particles, entanglement and decoherence
   2.3 Reduction formulae for transition probability is unitarity protected
3. (Partial) Cancellations among degenerate excitations
   An extension of the Kinoshita-Lee-Nauenberg theorem?