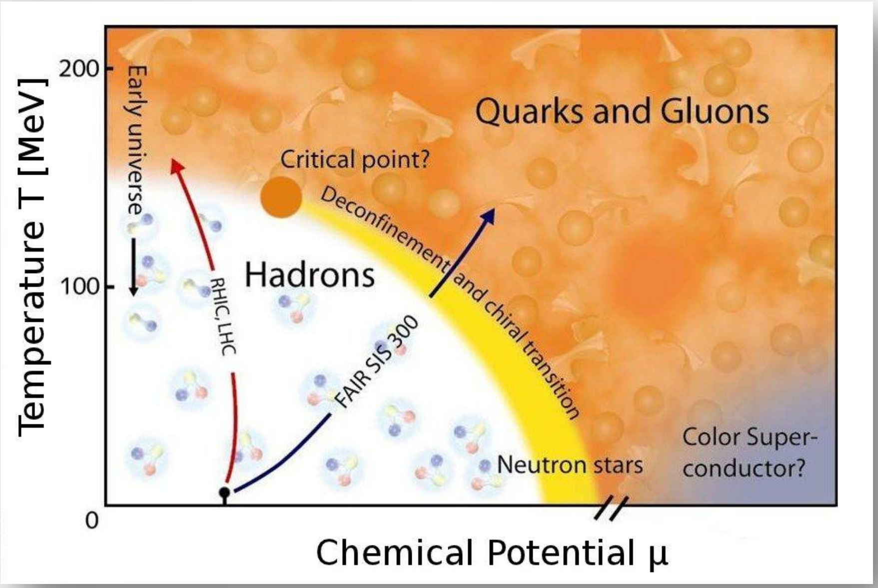
Aspects of QCD-like theories at finite density

Delta Meeting 2013

Nils Strodthoff, U Heidelberg

partially based on Phys. Lett. **B718** (2013) 1044
with K. Kamikado, L. von Smekal and J. Wambach

Motivation



QCD phase diagram

- Sign problem in LQCD
- QCD-like theories at finite μ:
  Lattice simulations available

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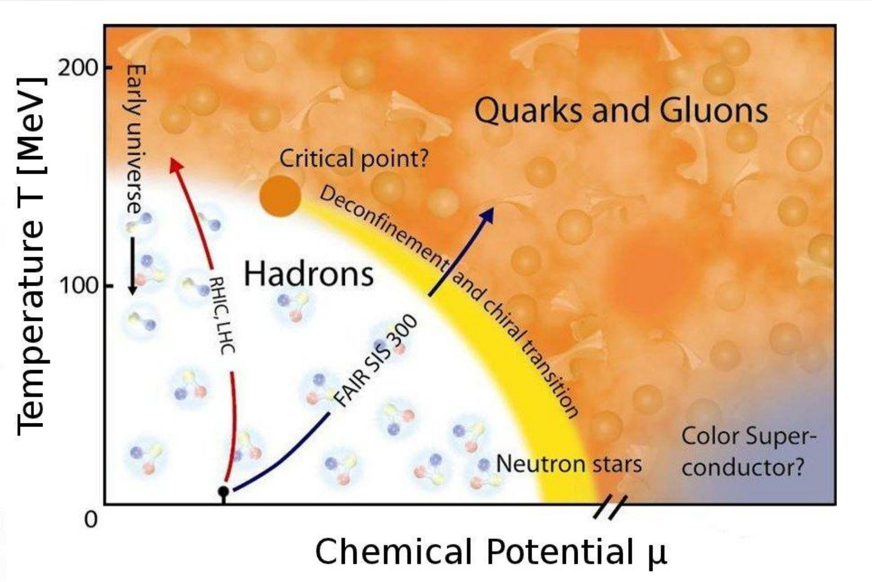

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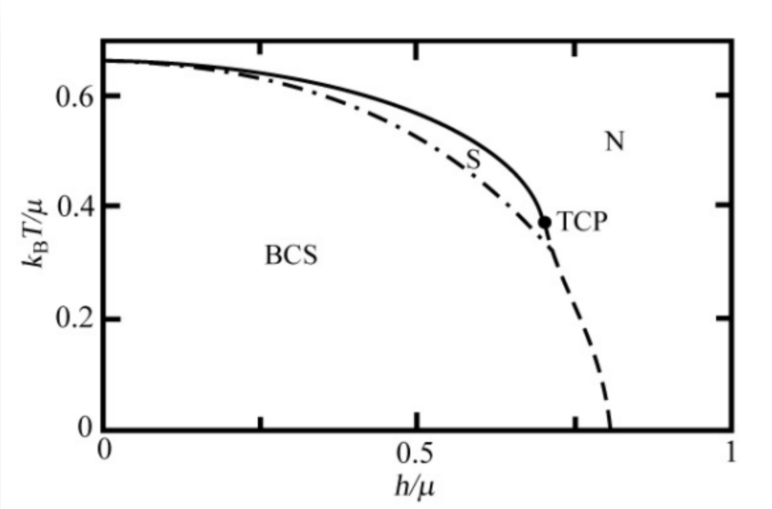
Universal aspects of strongly interacting systems

- QCD
- Ultracold atoms
- Condensed matter

Relativistic vs. non-relativistic



K.B. Gubbels and H.T.C. Stoof arXiv:1205.0568

QCD-like Theories

$$D(\mu) = \not{D} + m - \gamma^0 \mu$$

$$\gamma^5 D(\mu)^\dagger \gamma^5 = D(-\mu^*) \qquad \det D(\mu) = (\det D(-\mu^*))^*$$

Fermion sign problem: $\det D(\mu)$ is complex

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Consider QCD-like theories without a sign problem

1) Dirac Operator has an **antiunitary symmetry** — Dyson index

$$[T, D(\mu)] = 0 \;\Rightarrow\; U^{-1} D(\mu) U = D(\mu)^\dagger \qquad T^2 = (UK)^2 = \pm 1$$

a) Pseudo-real $T^2 = 1$ — $\beta = 1$

2-color QCD $U = \sigma_2 C \gamma^5$

b) Real $T^2 = -1$ — $\beta = 4$

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2) No antiunitary symmetry

QCD with isospin chemical potential — $\beta = 2$

$$\det D(\mu_I) \cdot \det D(-\mu_I) = |\det D(\mu_I)|^2$$

Isospin chemical potential

Imbalance between up and down quarks: $\mu_u = \mu + \mu_I \quad \mu_d = \mu - \mu_I$

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$$SU(2)_L \times SU(2)_R \overset{m>0}{\rightarrow} SU(2)_V \overset{\mu_I>0}{\rightarrow} U(1) \overset{\text{SSB}}{\rightarrow} Z_2$$

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Lagrangian: chiral effective model

$$\begin{aligned}\mathcal{L} =& \bar{\psi}(\partial\!\!\!/ + g(\sigma + i\gamma^5 \vec{\pi}\vec{\tau}) - \mu\gamma^0 - \mu_I \tau_3 \gamma^0)\psi \\ &+ \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi_0)^2 + U(\rho^2, d^2) - c\sigma \\ &+ \frac{1}{2}\left((\partial_\mu + 2\mu_I \delta_\mu^0)\pi_+ (\partial_\mu - 2\mu_I \delta_\mu^0)\pi_-\right)\end{aligned}$$

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➞ $\mu = 0$ map to QMD model for QC_2D:

$N_c : 3 \rightarrow 2 \quad \mu_I \rightarrow \mu \quad \pi_0 \rightarrow \vec{\pi} \quad \pi^-(\pi^+) \rightarrow \Delta(\Delta^*)$

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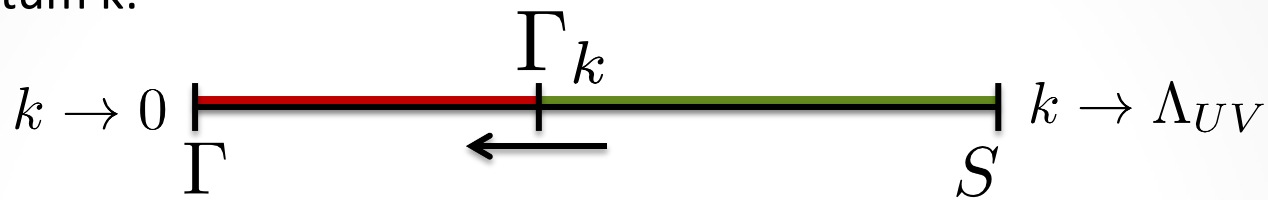
- symmetry breaking pattern requires

  $U \;= U\;(\rho^2, d^2) \quad \rho^2 = \sigma^2 + \pi_0^2, \quad d^2 = \pi_1^2 + \pi_2^2 = \pi^+\pi^-$

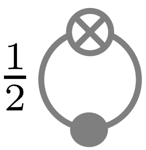
Functional Renormalization Group

Calculate full quantum effective action Γ by integrating fluctuations with momentum k:

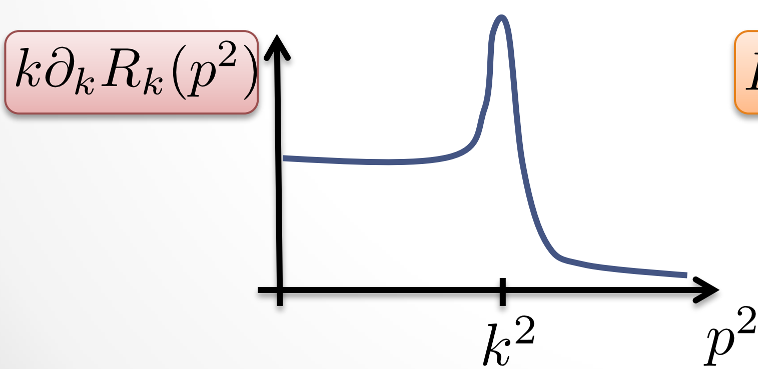


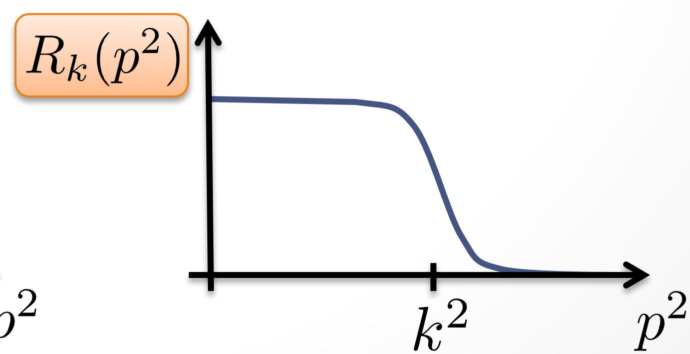
FUNCTIONAL RENORMALIZATION GROUP

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\, k\partial_k R_k \left[\Gamma_k^{(2)} + R_k\right]^{-1} = \frac{1}{2}$$



Wetterich Phys.Lett. **B301** (1993) 90





Isospin chemical potential

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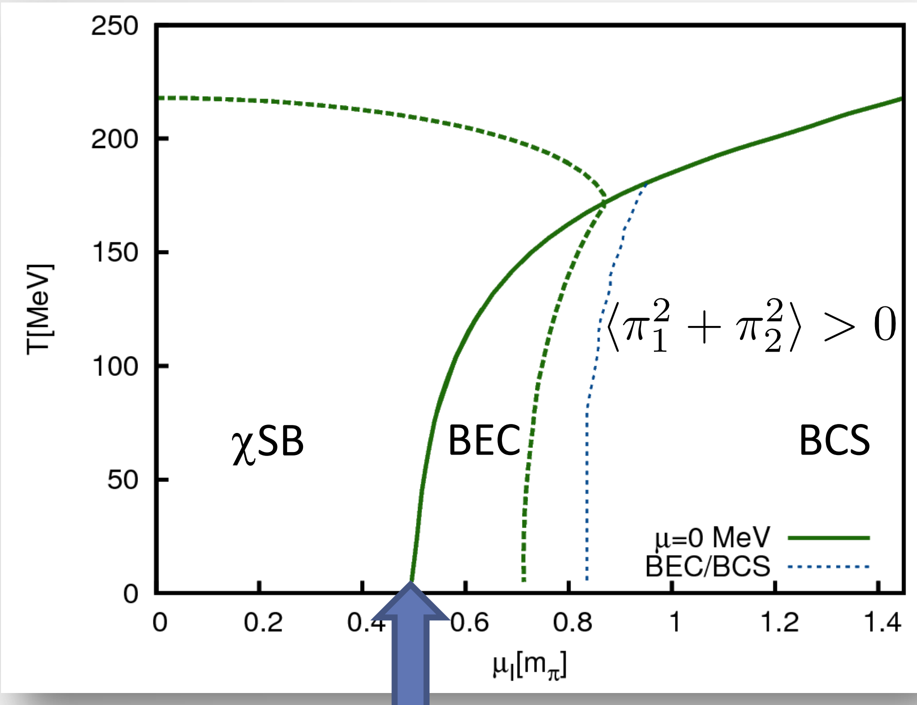
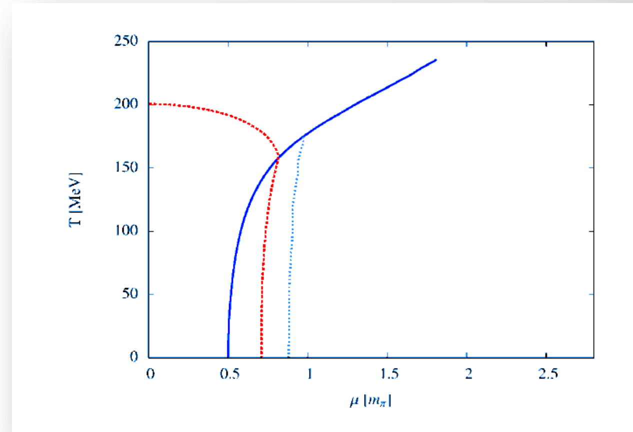


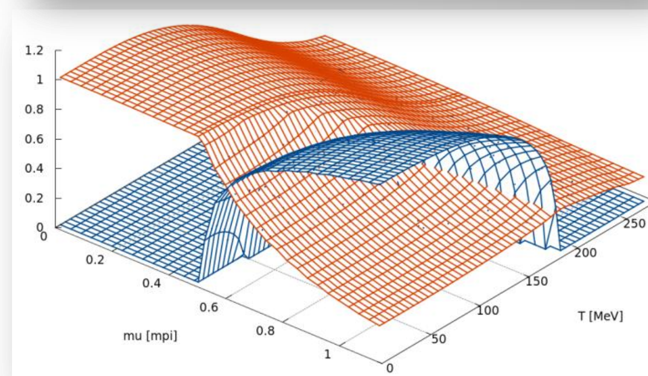
$$\frac{m_\pi}{2}$$

- Pion condensation sets in at at $\mu_I = m_\pi/2$
- Silver Blaze property

  T.D. Cohen Phys.Rev.Lett. **91** (2003)
- BEC/BEC crossover: rule of thumb $\mu_I = g\,\rho$

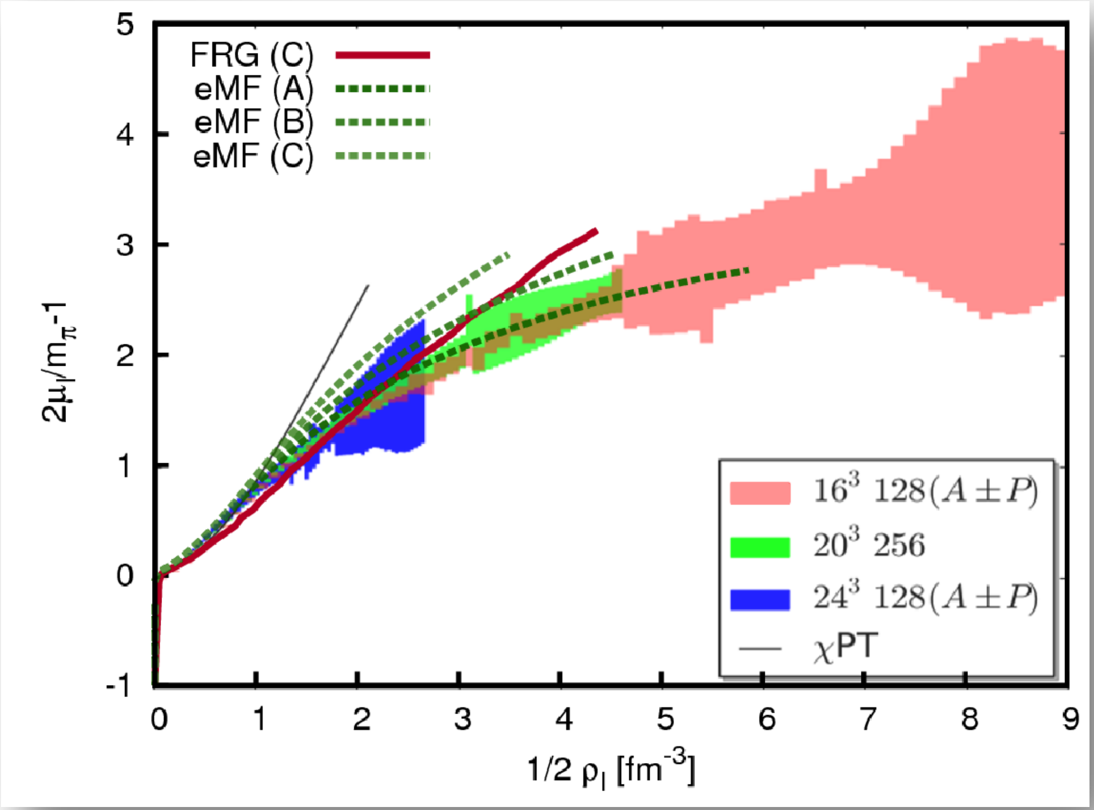
2-color QCD:





NST B.-J. Schaefer and L. von Smekal Phys.Rev. **D85** (2012)

Isospin density



$$\rho_I = \frac{\partial U}{\partial \mu_I}$$

Lattice: Detmold, Orginos, Shi, Phys. Rev. **D86** (2012) 054507

FRG: Kamikado, NSt, von Smekal, Wambach Phys. Lett. **B718** (2013) 1044

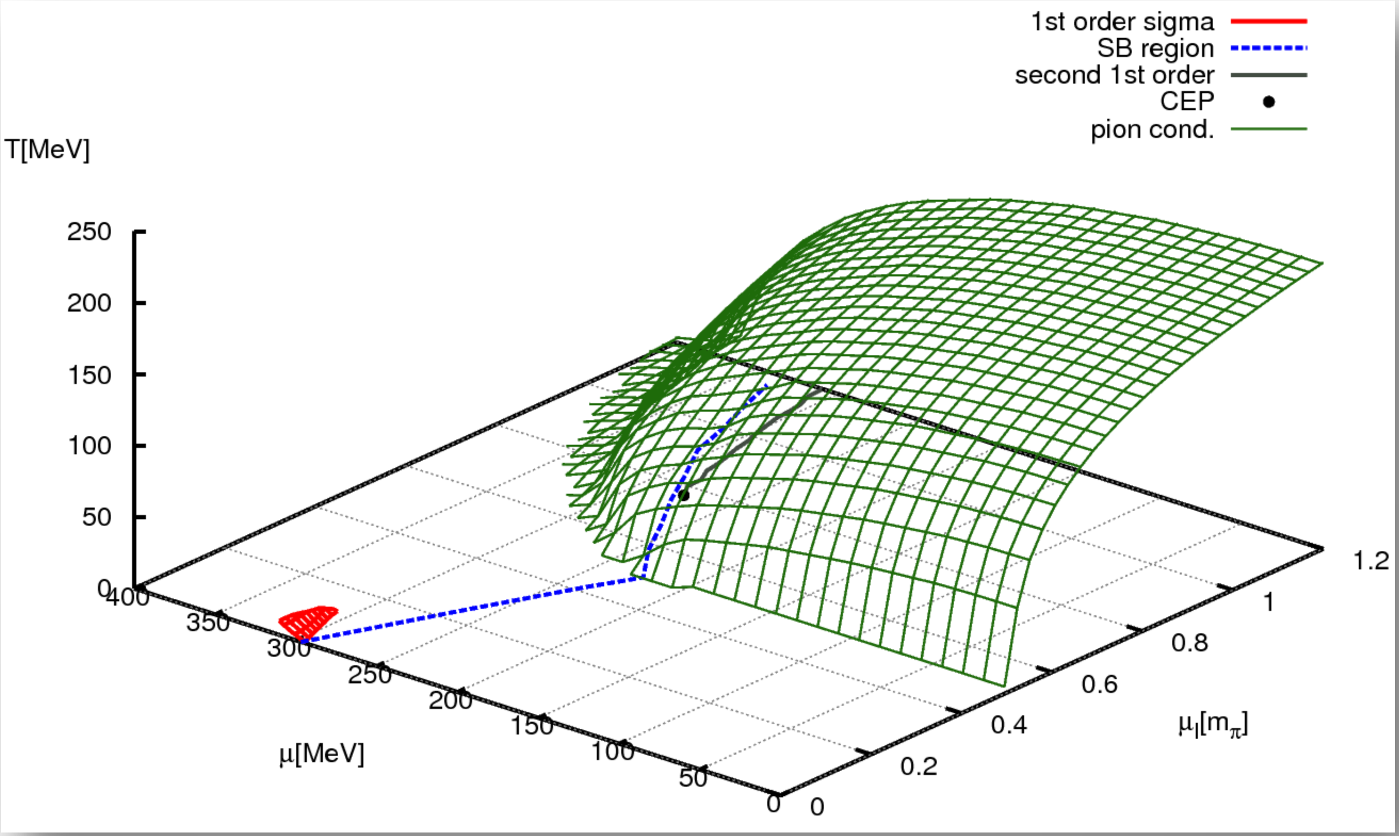
- Shape reproduced by eMF and FRG
- Even simpler: **linear** sigma model $\rho_I = 2f_\pi^2 m_\pi x \left(\frac{y^2-3}{y^2-1} - \frac{1}{x^4} + \frac{2}{y^2-1}x^2\right)$

$$x = \frac{2\mu_I}{m_\pi} \quad y = \frac{m_\sigma}{m_\pi}$$

- χPT fails: importance of σ-π mixing near BEC/BCS crossover

Baryon chemical potential

- Adds another axis to the picture: μ_B induces quark/antiquark imbalance
- Finite μ_I and μ_B : relevant for neutron stars



Kamikado, NSt, von Smekal, Wambach Phys. Lett. **B718** (2013) 1044

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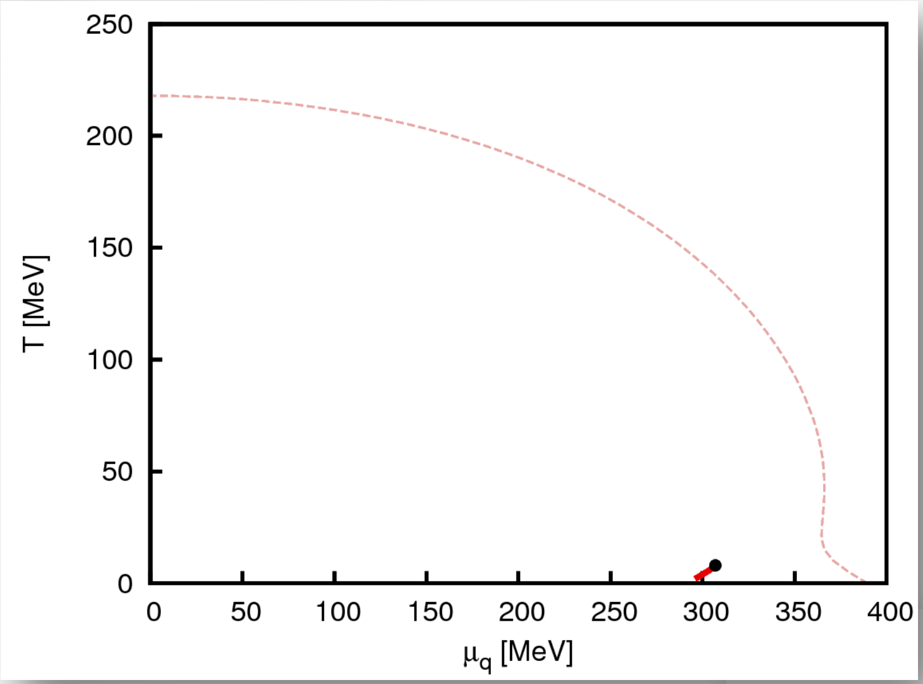

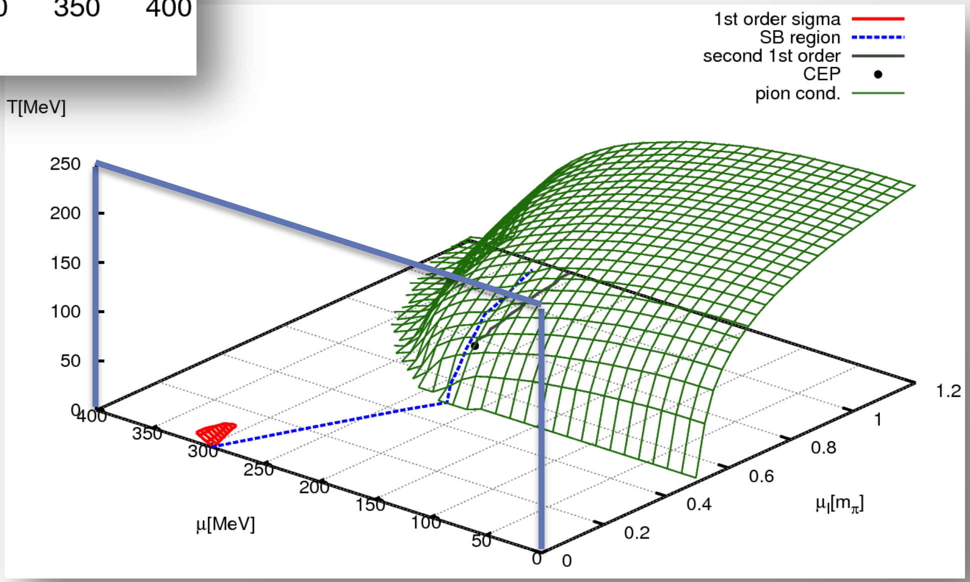

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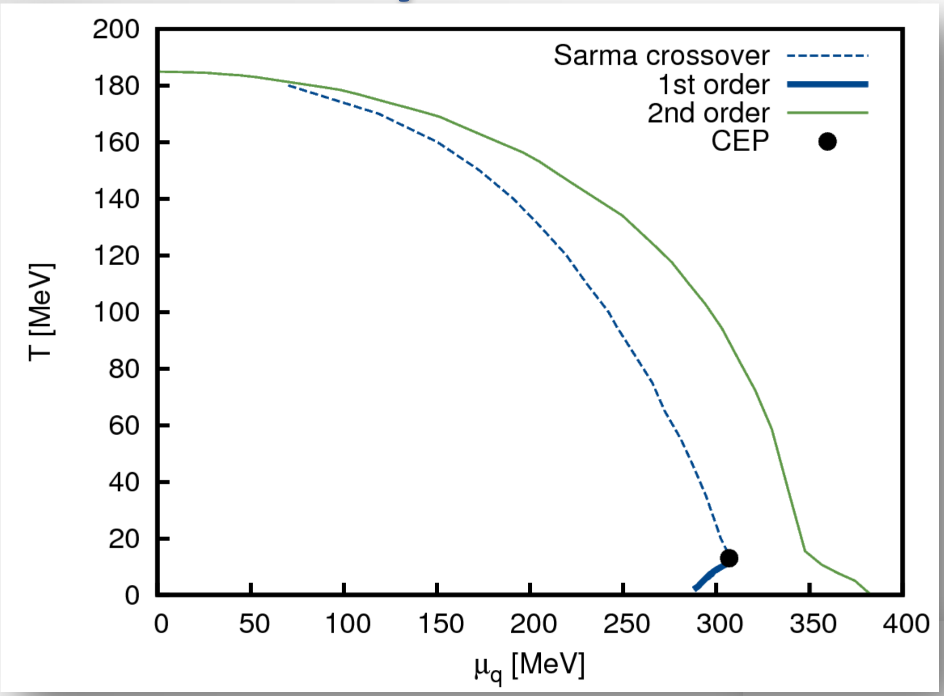
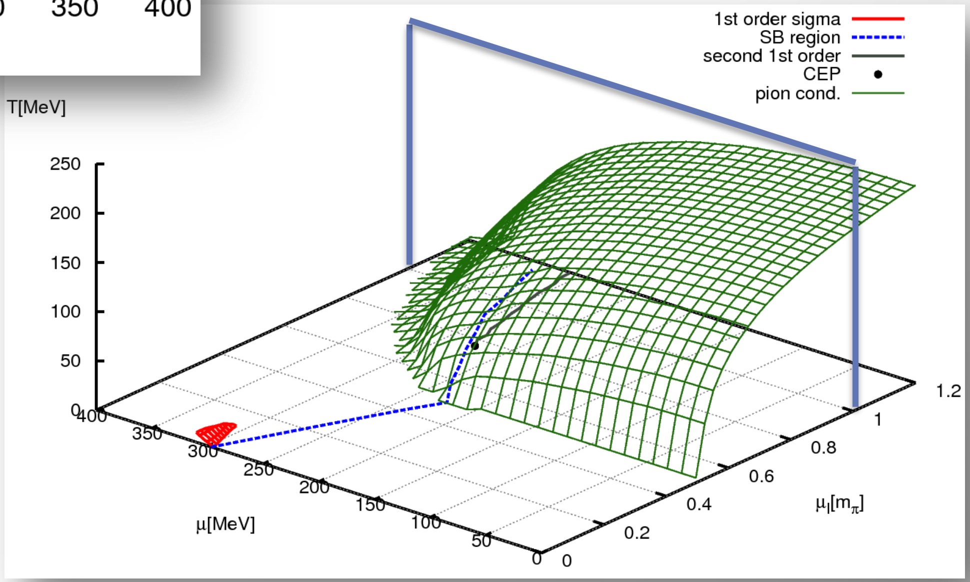
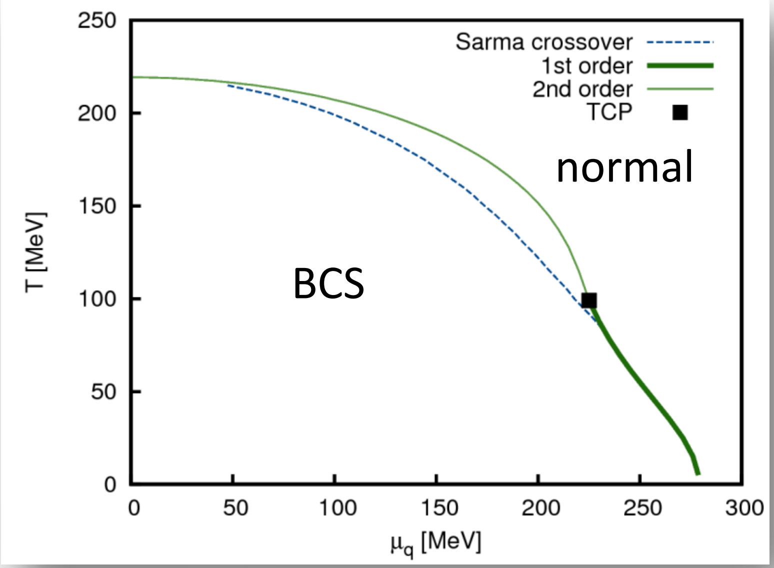




Clogston-Chandrasekhar (MF)

Fixed isospin chemical potential $\mu_I = m_\pi$:

Quark chemical potential μ_q



Silver Blaze property (T=0):

No dependence on μ_q

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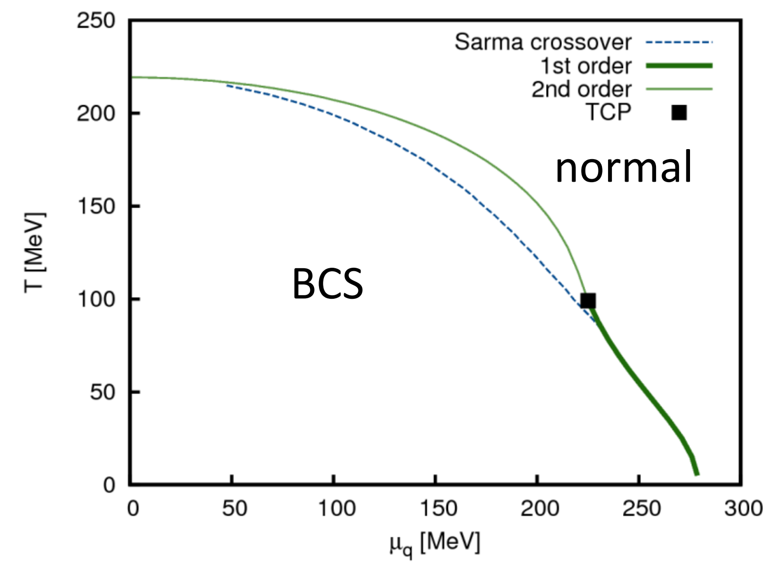

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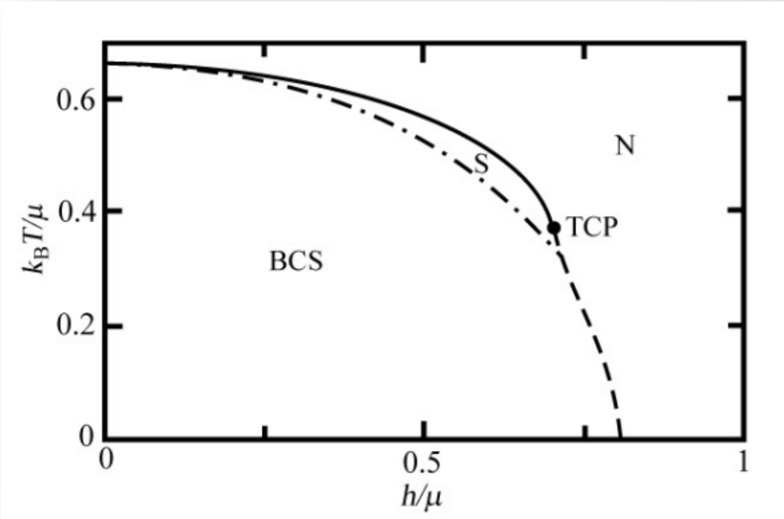
Imbalanced Fermi gas (MF):

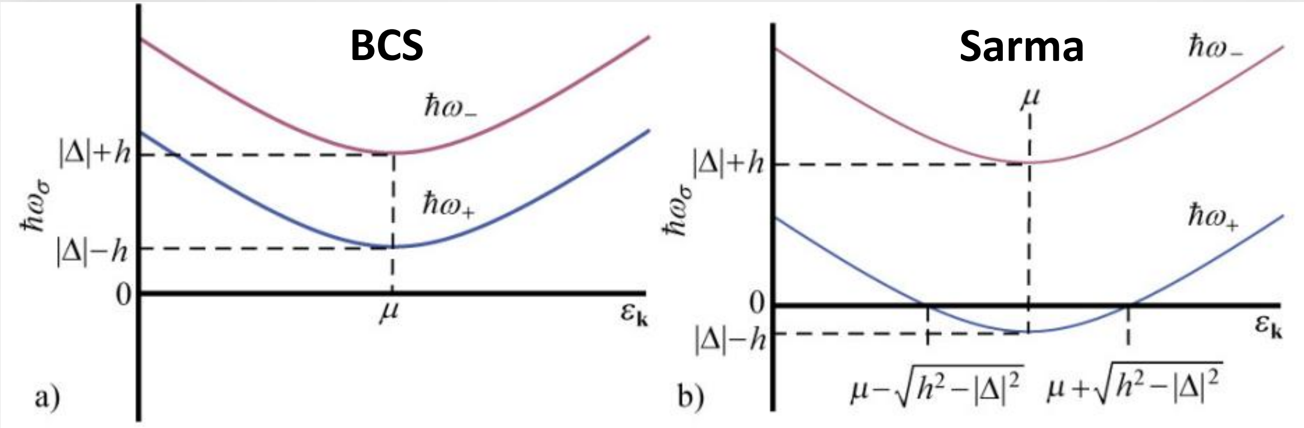
Population Imbalance
Polarization: $h = (\mu_\uparrow - \mu_\downarrow)/2$



K.B. Gubbels & H.T.C. Stoof arXiv:1205.0568

Clogston-Chandrasekhar (1962):
Superfluid phase robust against spin polarization

Sarma Phases



"Phase separation in momentum space"

K.B. Gubbels & H.T.C. Stoof arXiv:1205.0568

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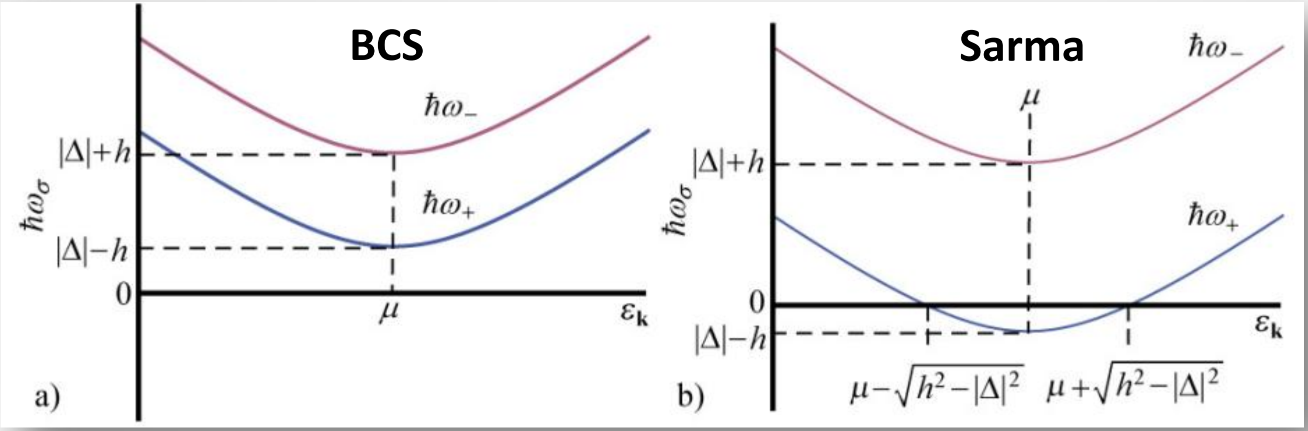

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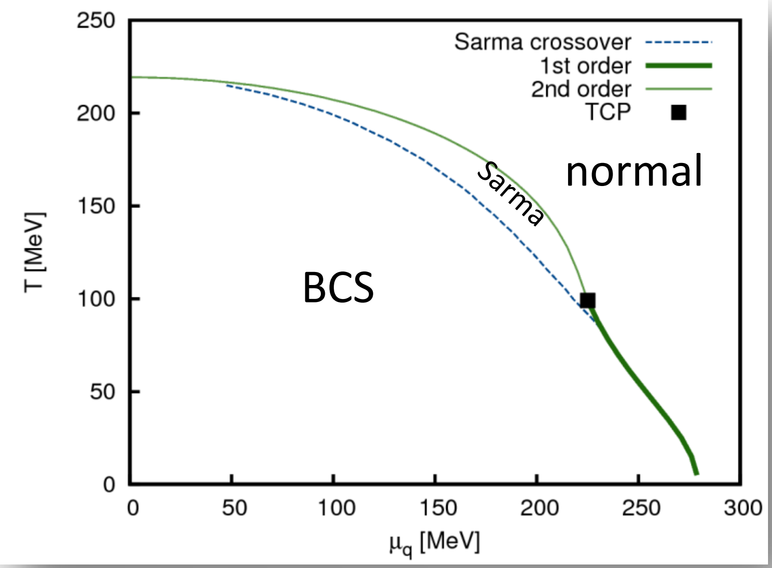
Mean-field:



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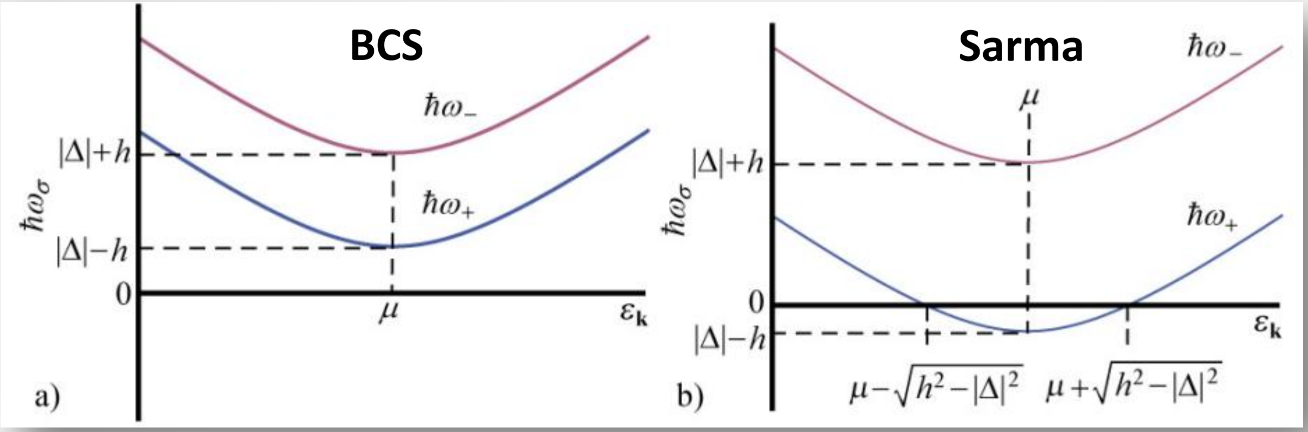

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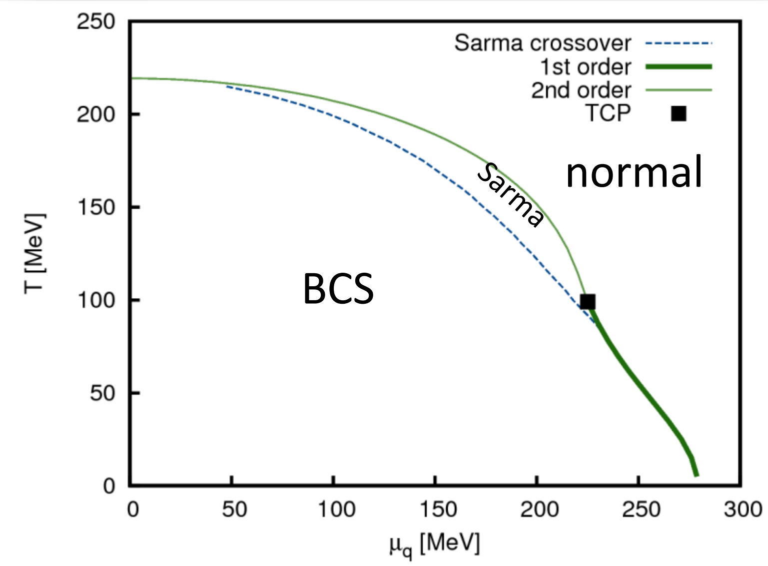
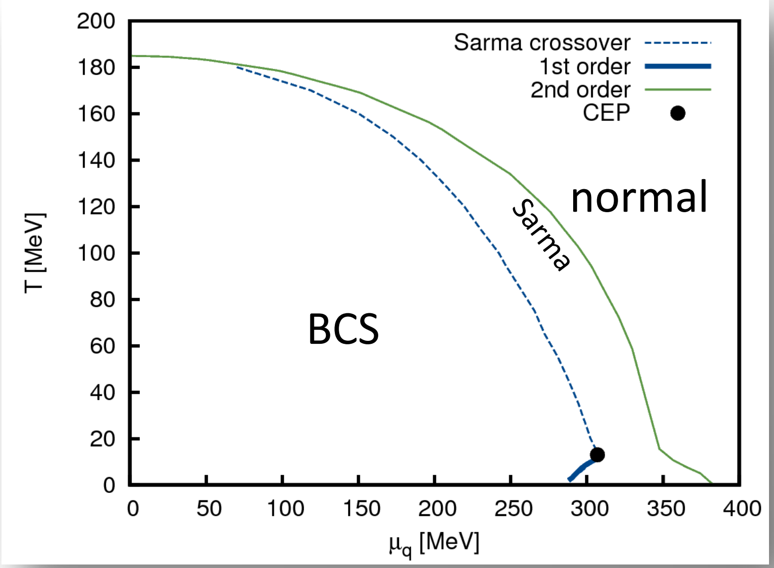


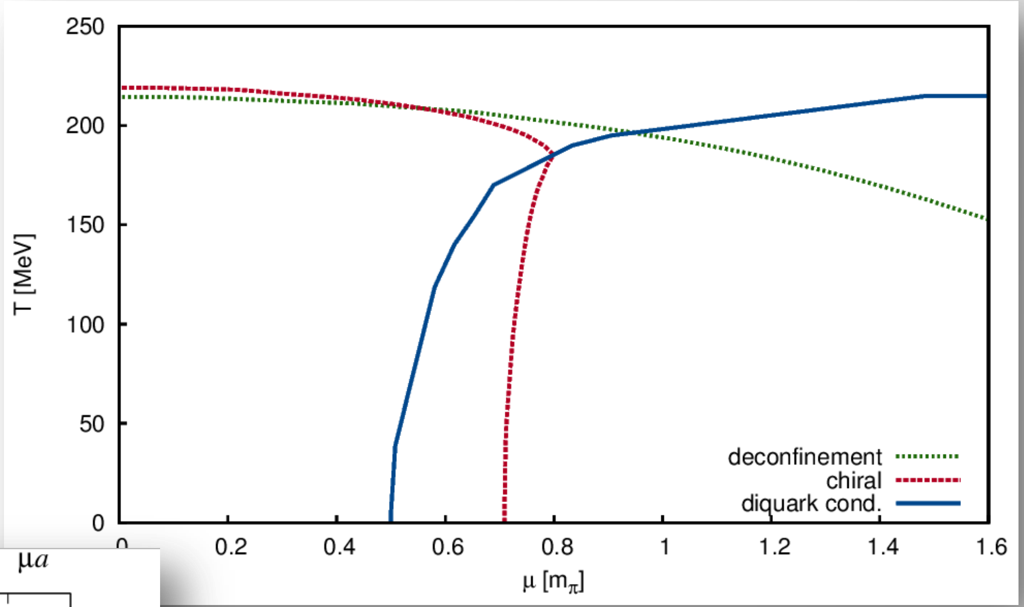
RG:



Sarma phase stabilized by fluctuations

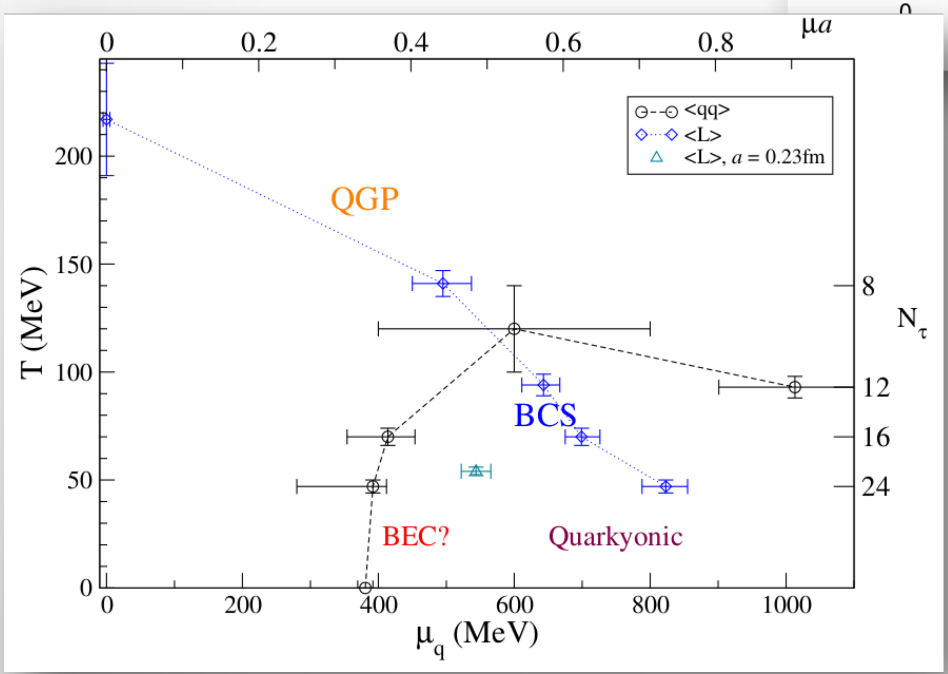
Outlook: 2-color QCD

QMD model + phenomenological Polyakov loop potential



NST L. von Smekal in prep.

NST J.M. Pawlowski, L. von Smekal work in progress

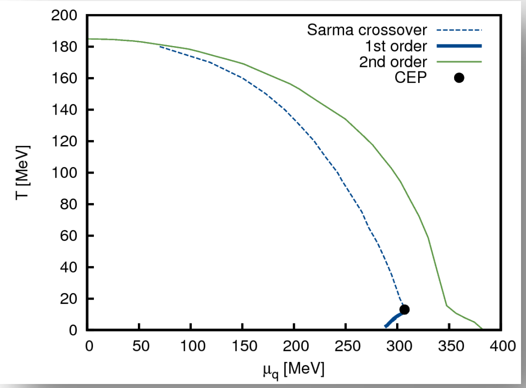


Qualitative agreement with recent lattice results

Cotter, Giudice, Hands & Skullerud, arXiv:1210.4496

Summary

- **QCD-like theories without a fermion sign problem**
  QC_2D, G_2-QCD, QCD with isospin chemical potential…



- **QCD with Isospin chemical potential**
  - for μ_B=0 equivalent to 2-color QCD
  - Close analogy to Imbalanced Fermi gases: Chandrasekhar Clogston & Sarma phases

Thank you for your attention!