Correlation functions at finite momentum within the eRG

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Outline

- Motivation
- A specific example T_{BEC}
- Summary of the method
- Some results

Work done in collaboration with

- R. Mendez-Galain, N. Wschebor
- F. Benitez, H. Chate, B. Delamotte

The 'magic' of the eRG

Exact flow equation (Wetterich's equation)

$$\partial_\kappa \Gamma_\kappa[\phi] = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} \, \partial_\kappa R_\kappa(q) \, G_\kappa(q)$$

$$G_\kappa(q;\phi)^{-1} = \Gamma_\kappa^{(2)}[\phi] + R_\kappa(q)$$

Local potential approximation

$$\Gamma_\kappa[\phi] = \int d^d x \, \left(\frac{1}{2} (\partial \phi)^2 + V(\phi) \right)$$

Can treat all correlations functions at once (but only at zero external momenta)

We shall look for a generalization that allows calculation of Correlation functions at finite exernal momenta

Motivations : many !

Here, focus on a specific example

$$\frac{\Delta T_c}{T_c^0} = \frac{T_c - T_c^0}{T_c^0} = c\, a\, n^{1/3}$$

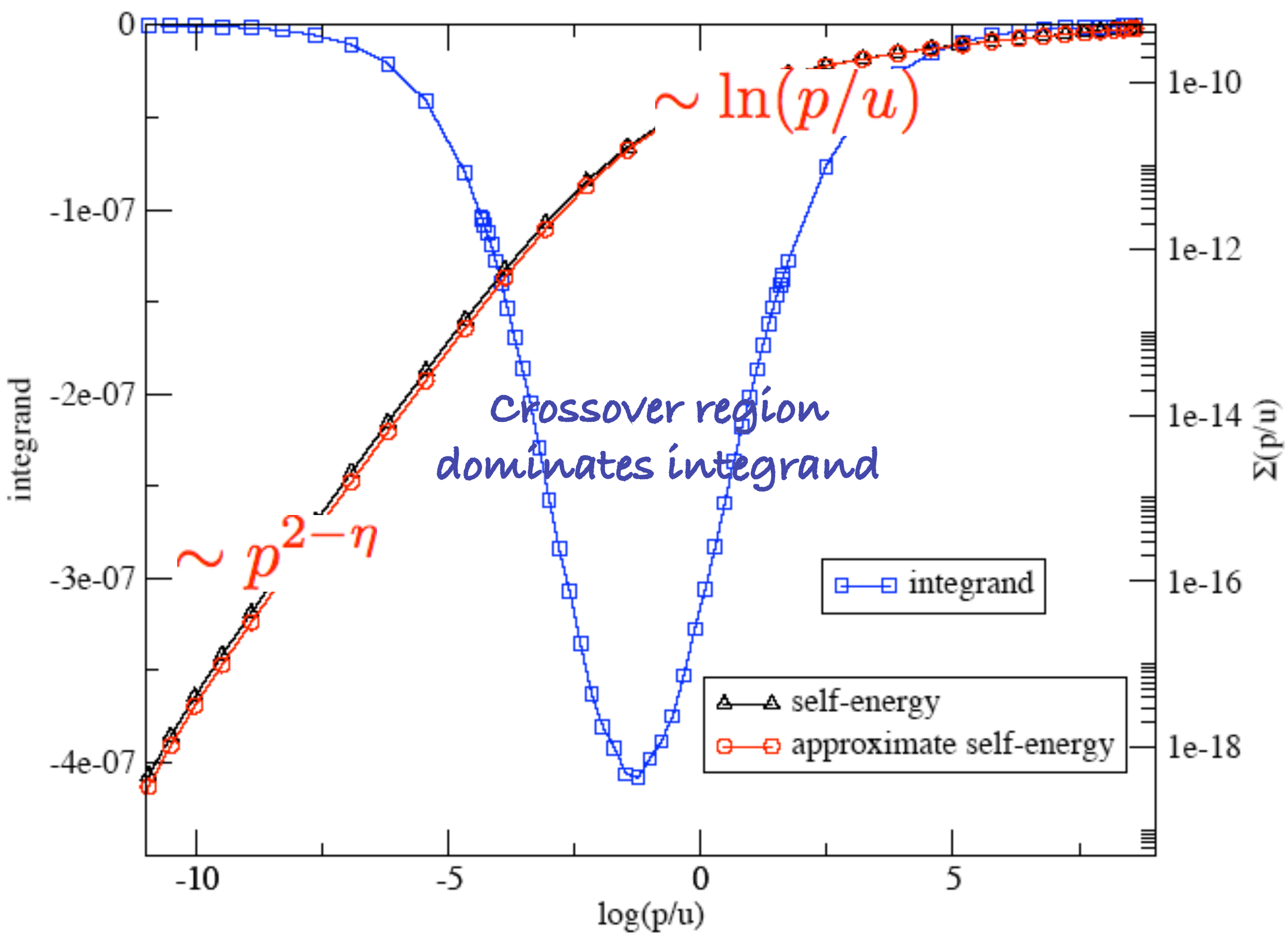
The coefficient c is obtained from the change in the fluctuations in a classical field theory at criticality.

$$\int \mathrm{d}^d x \left\{ \frac{1}{2} \left[\nabla \varphi(x)\right]^2 + \frac{1}{2} r \varphi^2(x) + \frac{u}{4!} \left[\varphi^2(x)\right]^2 \right\}$$

$$c \propto \frac{\Delta \langle \varphi_i^2 \rangle}{N u}$$

Why is c difficult to calculate ?

$$\frac{\Delta\langle\varphi_i^2\rangle}{N} = \int \frac{d^3p}{(2\pi)^3} \left(\frac{1}{p^2+\Sigma(p)} - \frac{1}{p^2}\right)$$



Quest for precision

Look for an approximation to the eRG

- that yields accurate determination of $\Gamma_\kappa^{(2)}(p)$ for all p

- that is simple conceptually, and in its numerical implementation

- that can be systematically improved

Summary of the method

Based on two observations

The vertex functions depend weakly on the loop momentum

$$\Gamma_\kappa^{(n)}(p_1, p_2, ..., p_{n-1} + q, p_n - q; \phi) \sim \Gamma_\kappa^{(n)}(p_1, p_2, ..., p_{n-1}, p_n; \phi)$$

The hierarchy can be closed by exploiting the dependence on the field

$$\Gamma_\kappa^{(n+1)}(p_1, p_2, ..., p_n, 0; \phi) = \frac{\partial \Gamma_\kappa^{(n)}(p_1, p_2, ...p_n; \phi)}{\partial \phi}$$

J.-P. B, R. Mendez-Galain, N. Wschebor (PLB, 2006)

Start at bottom of hierarchy

$$\partial_\kappa V_\kappa(\phi) = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\,\partial_\kappa R_\kappa(q)\,G_\kappa(q)$$

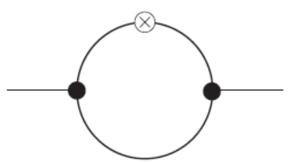
$$G_\kappa(q;\phi)^{-1} = q^2 + \frac{\partial^2 V_\kappa}{\partial\phi^2} + R_\kappa(q)$$

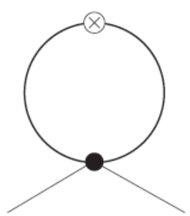
(Local potential approximation)

Two-point function

$$\partial_\kappa \Gamma^{(2)}_\kappa(p,-p;\phi) = \int \frac{d^d q}{(2\pi)^d}\,\partial_\kappa R_\kappa(q^2)\;G^2_\kappa(q^2;\phi)$$

$$\times\left\{\left(\frac{\partial\Gamma^{(2)}_\kappa(p,-p;\phi)}{\partial\phi}\right)^2 G_\kappa((p+q)^2;\phi)\;-\;\frac{1}{2}\frac{\partial^2\Gamma^{(2)}_\kappa(p,-p;\phi)}{\partial\phi^2}\right\}$$

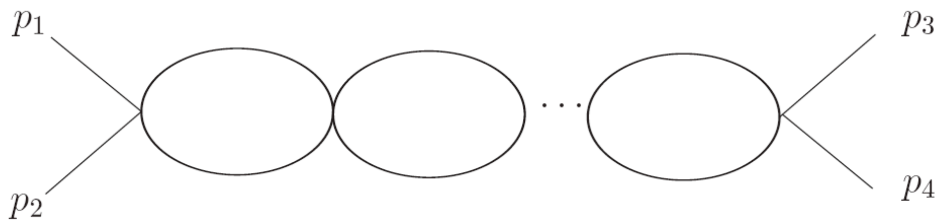


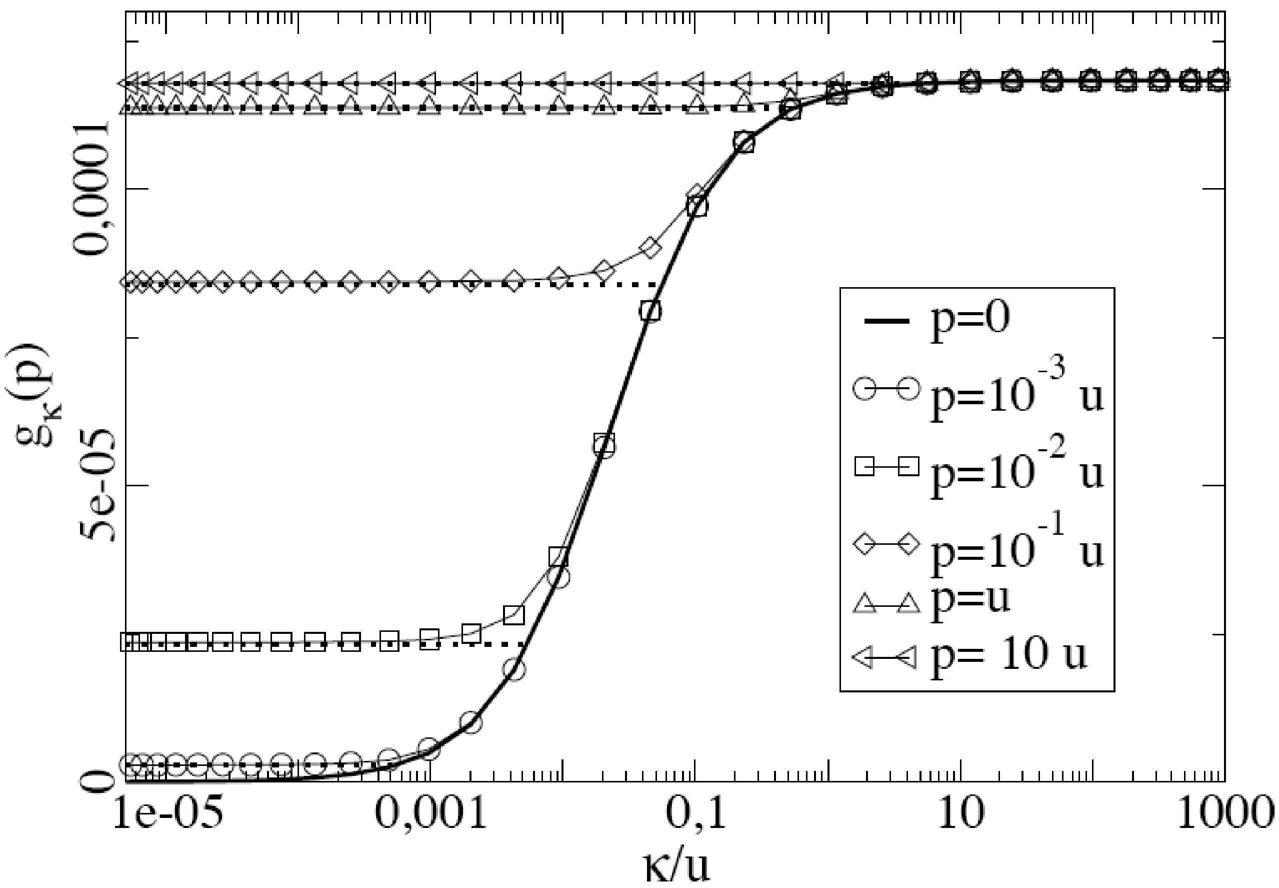


Etc.

Role of the external momentum

$g_\kappa(p_1 + p_2)$





Some improvements

$$\Gamma^{(3)}(p,-p,0;\phi) = \frac{\partial \Gamma^{(2)}(p,-p;\phi)}{\partial \phi}$$

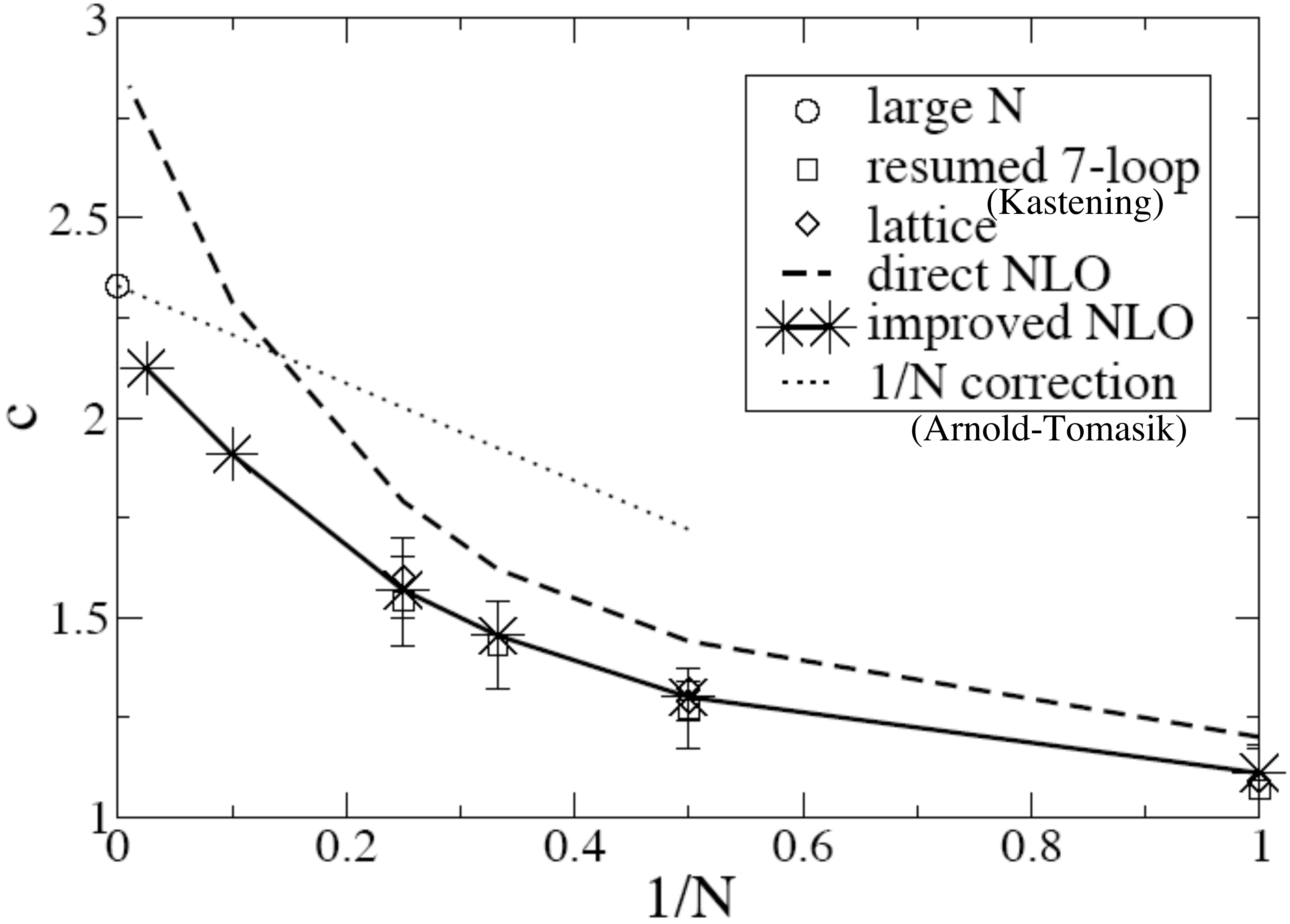
$$\Gamma^{(3)}(p,q,k) = \frac{1}{2}\frac{\partial \Gamma^{(2)}(p,-p)}{\partial \phi} + \frac{1}{2}\frac{\partial \Gamma^{(2)}(q,-q)}{\partial \phi} + \frac{1}{2}\frac{\partial \Gamma^{(2)}(k,-k)}{\partial \phi} - \frac{1}{2}\frac{\partial \Gamma^{(2)}(0,0)}{\partial \phi}$$

$$\Gamma^{(4)}(p,-p,0,0;\phi) = \frac{\partial^2 \Gamma^{(2)}(p,-p;\phi)}{\partial \phi^2}$$

$$\Gamma^{(4)}(p,-p,q,-q) = \frac{1}{2}\frac{\partial^2 \Gamma^{(2)}(p+q,-p-q)}{\partial \phi^2} + \frac{1}{2}\frac{\partial^2 \Gamma^{(2)}(p-q,-p+q)}{\partial \phi^2}$$

Allows one to recover 2-loop accuracy in UV

c as a function of N (2006)



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N	lattice	7-loops [14]	this work*	(2006)
1	1.09 ± 0.09 [13]	1.07 ± 0.10	1.15	(1.11)
2	1.32 ± 0.02 [12]	1.27 ± 0.10	1.37	(1.30)
	1.29 ± 0.05 [11]			
3		1.43 ± 0.11	1.50	(1.45)
4	1.60 ± 0.10 [13]	1.54 ± 0.11	1.63	(1.57)
5			1.75	
10			2.02	(1.91)
50			2.31	
100			2.36	

$N \to \infty, \qquad c \to 2.33$ (J.-P. B., G. Baym and J. Zinn-Justin (2000))

[13] X. Sun (2000)
[12] P. Arnold and G. Moore (2001)
[11] V.A. Kashurnikov, N.V. Prokof'ev, B.V. Svistunov (2001)
[14] B. Kastening (2004)