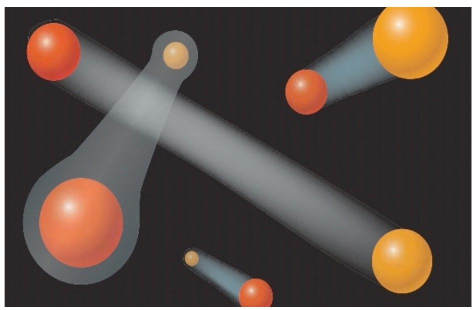
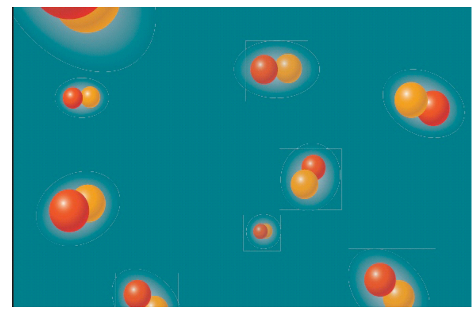
ERG 08, July 04 2008, Heidelberg

Towards precision in the BCS-BEC crossover in ultracold fermion gases



BCS Cooper pairs



BEC of molecules

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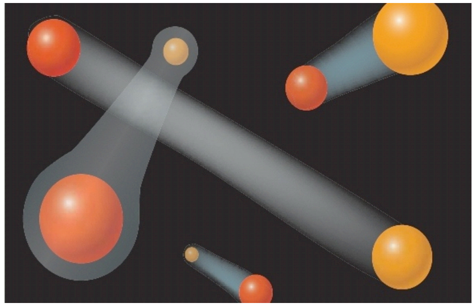
Introduction: BCS-BEC Crossover (Eagles '69; Leggett '80)

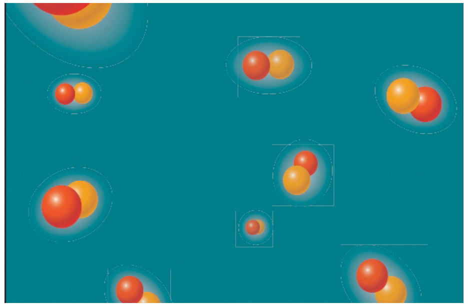
- fermions with attractive interactions

  ➡ BCS superfluidity at low T

- tightly bound microscopic molecules

  ➡ Bose-Einstein Condensate (BEC) of molecules at low T





0 $\qquad (ak_F)^{-1}$

- Localization in position space
- Delocalization in momentum space

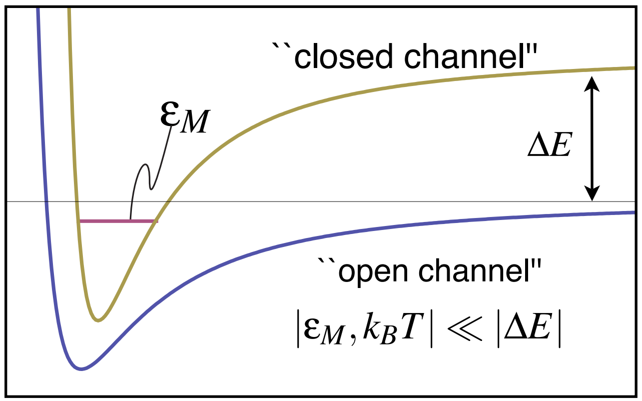
- Crossover: Symmetry properties unchanged
- Experimentally implemented via Feshbach resonances

(Regal& '04; Zwierlein& '04; Kinast& '04; Bourdel& '04)

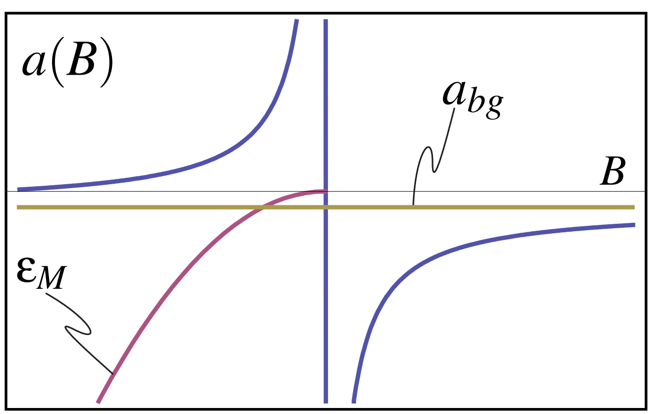
fermions with attractive interactions

Tuneable Interactions: Feshbach resonance

- Physical origin: resonant hyperfine interaction between two electron spins

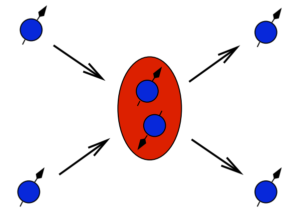


interaction potentials



scattering length a and binding energy ε_M

$$a(B) = a_{bg} + \frac{W}{B - B_0}$$



fermion field

$$S[\psi,\phi] = \int d\tau \int d^3x \Big\{ \psi^\dagger \big(\partial_\tau - \frac{\triangle}{2M}\big)\psi + \frac{\lambda_\psi}{2}(\psi^\dagger\psi)^2$$

$$+\phi^* \big(\partial_\tau - \frac{\triangle}{4M} + \nu\big)\phi \text{- } h_\phi \Big(\phi^*\psi_1\psi_2 - \phi\psi_1^*\psi_2^*\Big)\Big\}$$

molecule field

interconversion term (Feshbach, Yukawa)

parameters:

- (background scattering in open channel) λ_ψ
- Feshbach coupling: width of resonance h_ϕ $\quad W \sim \frac{h_\phi^2}{\mu_B}$
- detuning: distance from resonance $\nu = \mu_B(B - B_0)$

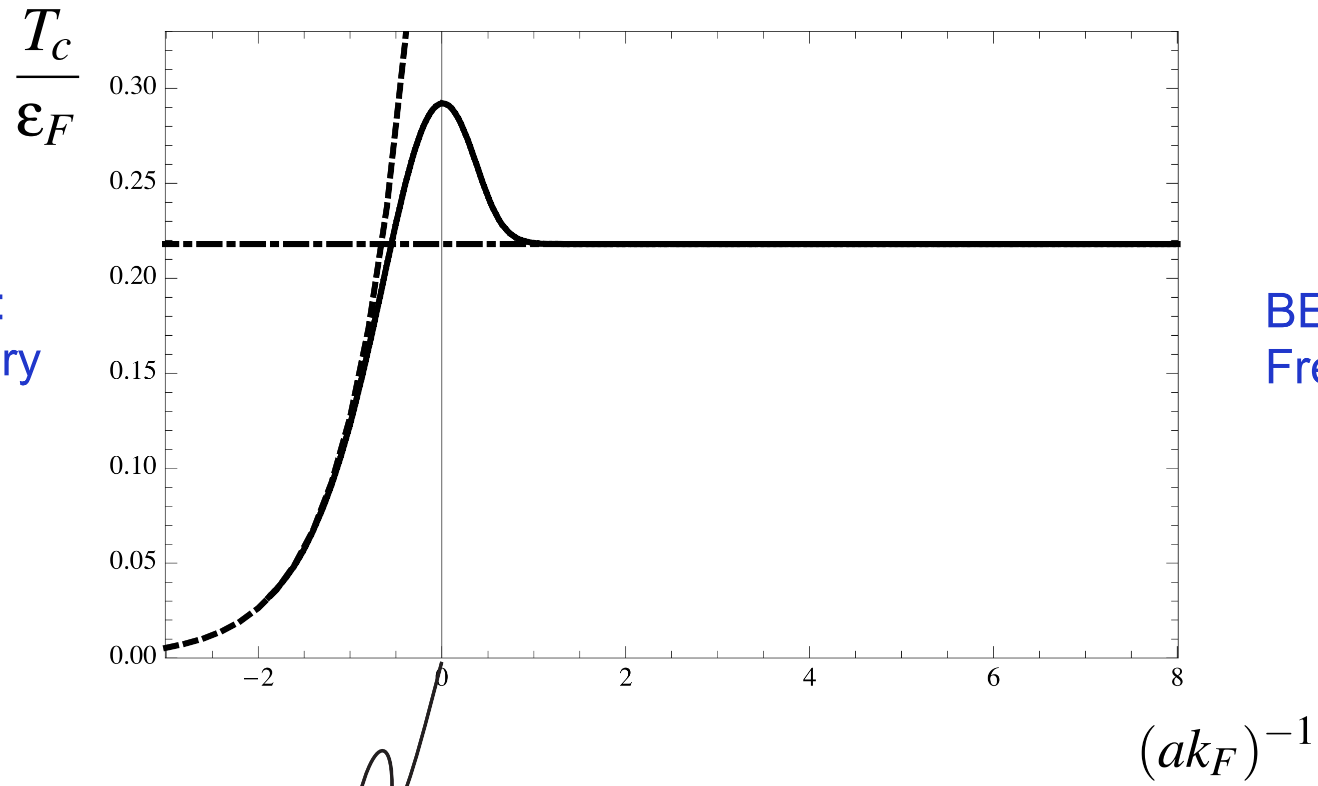
➡ Crossover Parameter: inverse scattering length

$$(ak_F)^{-1} \sim \frac{\mu_B(B - B_0)}{h_\phi^2}$$

First Look: Crossover Phase Diagram

BCS Mean Field + Gaussian bosonic fluctuations:
(Nozieres, Schmitt-Rink ’81)

BCS limit:
BCS theory



BEC limit:
Free bosons

“Unitarity” Regime
- strong interactions
- no microscopic length scale
- no small parameter

Crossover Phase Diagram

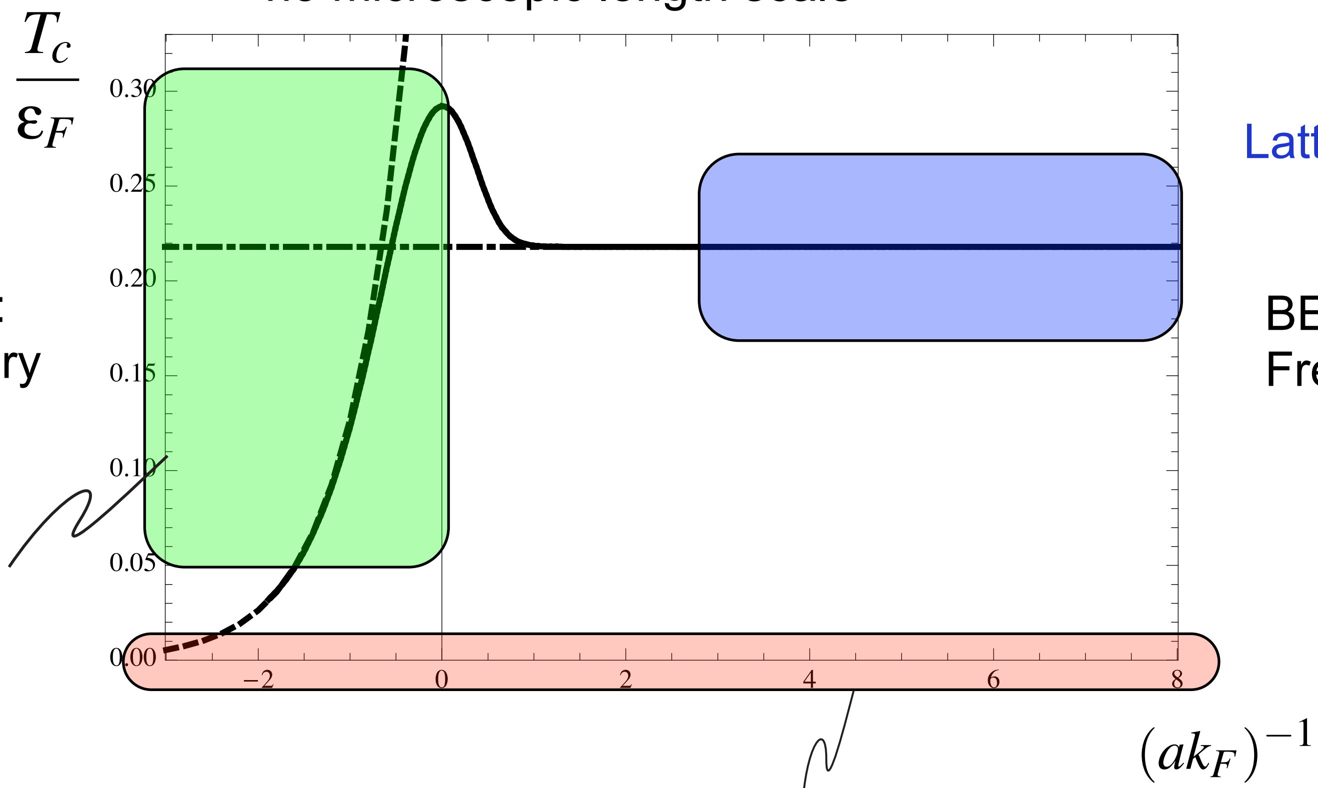
BCS Mean Field + Gaussian bosonic fluctuations:
(Nozieres, Schmitt-Rink ’81)

“Unitarity” Regime
strong interactions,
no microscopic length scale

Lattice O(2) model

BCS limit:
BCS theory

BEC limit:
Free bosons



QMC + DMC

Prokofev&’06
Bulgac& ’06
Trivedi& ‘07

DMC

Giorgini& ’04;
Carlson& ‘04

Semi-analytical Approaches I

Idea from critical phenomena:

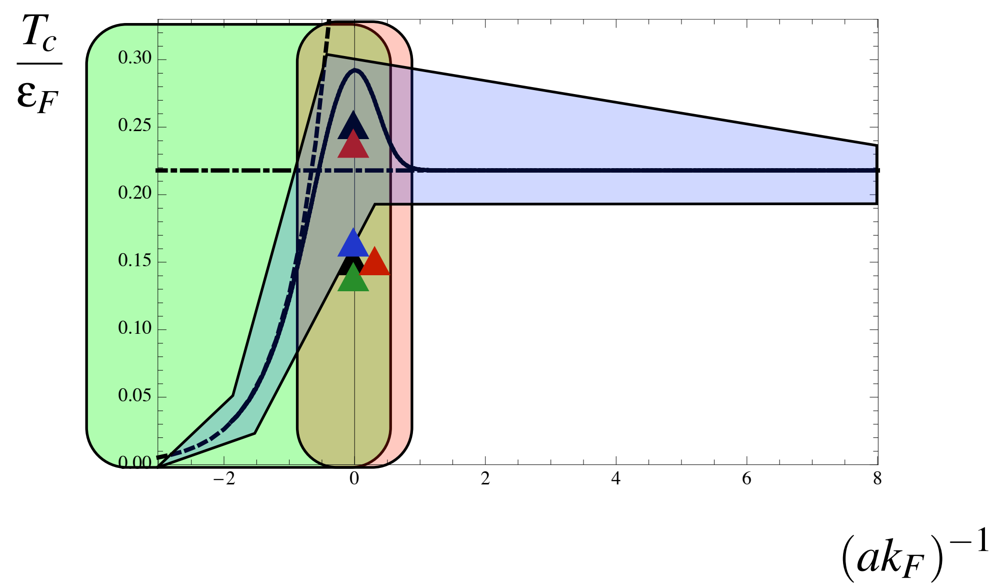
- identify Gaussian fixed point related to the problem
- expand about it
- continue to the interacting fixed point

Examples

- epsilon expansion: noninteracting theory in d=4 or d=2 (Nishida, Son'06)
- 1/N expansion: number of field components (Nicolic, Sachdev '06; Radzihovsky, Sheey '06)
- narrow resonances (SD, Wetterich '05; SD, Gies, Pawlowski, Wetterich '07)

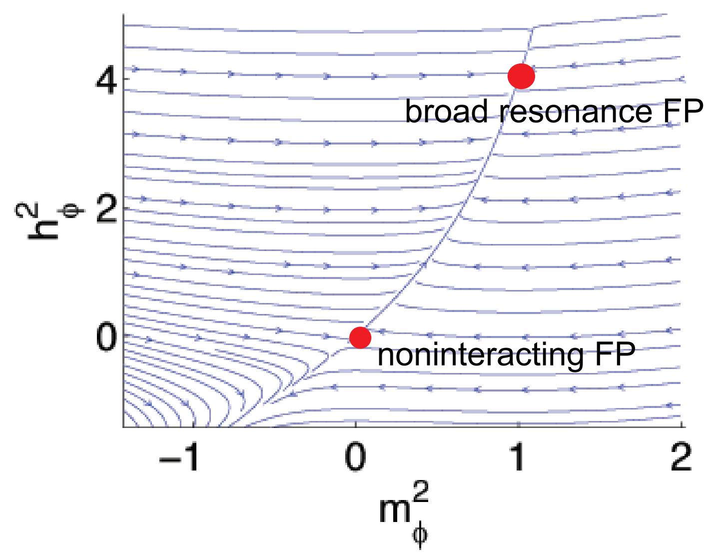
$\frac{T_c}{\varepsilon_F}$ estimate:

▲ epsilon d=4:	0.25	
▲ epsilon d=2:	0.15	
▲ 1/N:	0.14	
▲ Narrow:	0.17	
▲ QMC:	0.152	Prokofev&'06
	0.25	Bulgac& '06
	0.23	Trivedi& '07



Semi-analytical Approaches I: Narrow Resonance Limit

in model with detuning $\nu(B)$ and Feshbach coupling h_ϕ
(or $a^{-1}(B) \sim \nu(B)/h_\phi^2, h_\phi$) and in vacuum:



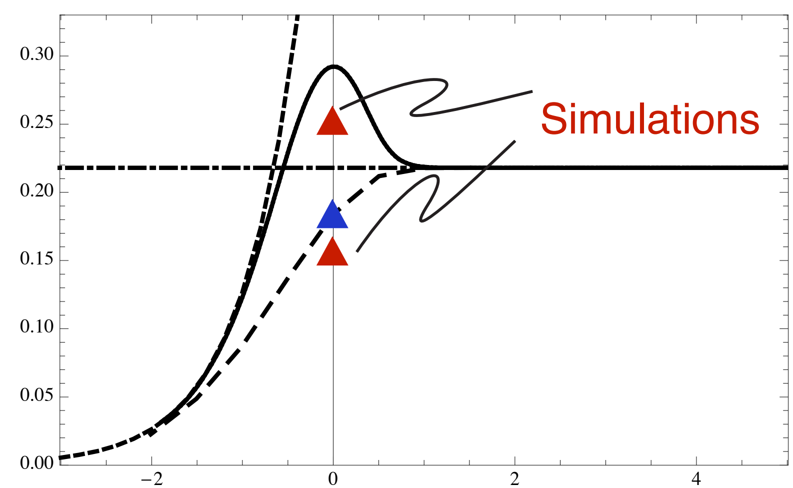
Narrow resonances: Gaussian FP $h_\phi \to 0, a = const.$

- Detuning and Feshbach coupling relevant parameters
- Exact mean field-type solution available (SD, Wetterich '05)

Broad resonances: Interacting FP $h_\phi \to \infty, a = const.$

- Detuning single relevant perturbation: All further microscopic memory lost

Narrow: 0.17



Semi-analytical Approaches II

Address the full many-body problem directly

- Self-consistent Approaches
  - t-matrix (Haussmann ’93; Strinati& ’04)
  - 1PI Effective Action (SD, Wetterich ’05; Randeria& ’07)
  - 2PI Effective Action (Zwerger& ‘06)
- Functional RG (Birse& ’05; SD, Gies, Pawlowski, Wetterich ’07; ongoing with Flörchinger, Krahl, Scherer)

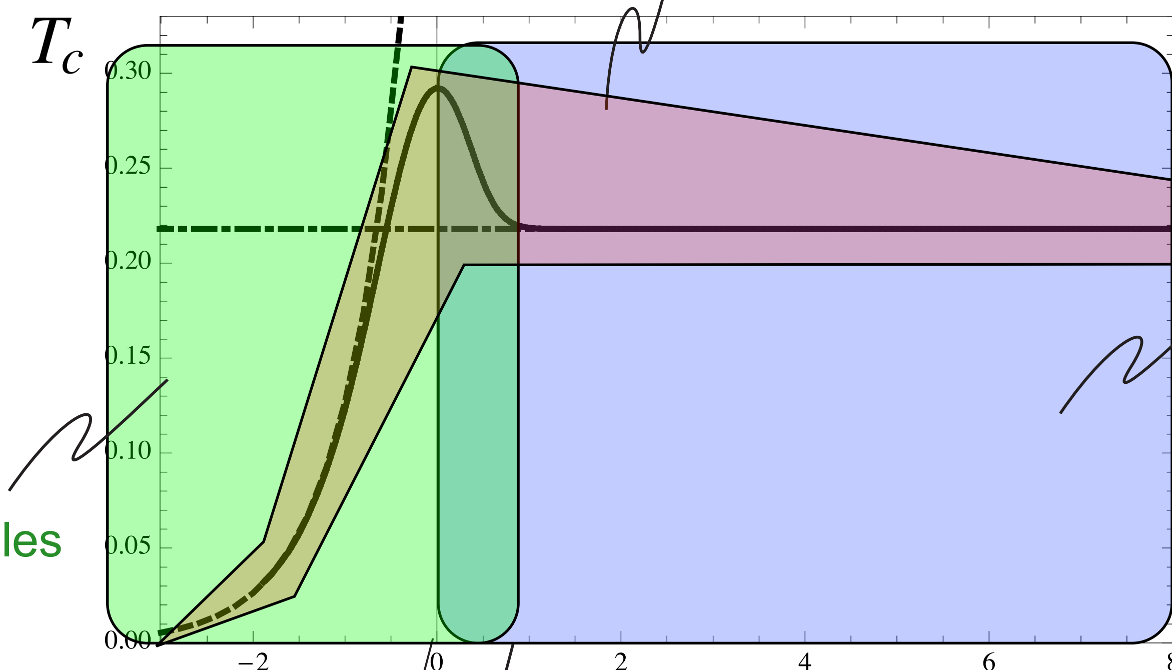
Strategy: Find an interpolation scheme which incorporates known physical effects in the limiting cases

➡ Benchmarking

Challenges

Beyond mean field effects
at very different scales:

critical behavior:
long distance scales $k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$



$(ak_F)^{-1}$

few-body physics of
effective dimers:
microscopic scales

$$\varepsilon_M = -\frac{1}{Ma^2} \gg T, \frac{k_F^2}{2M}$$

Many-Body fermion
physics:
Thermodynamic scales

$$n = \frac{k_F^3}{3\pi^2}, T$$

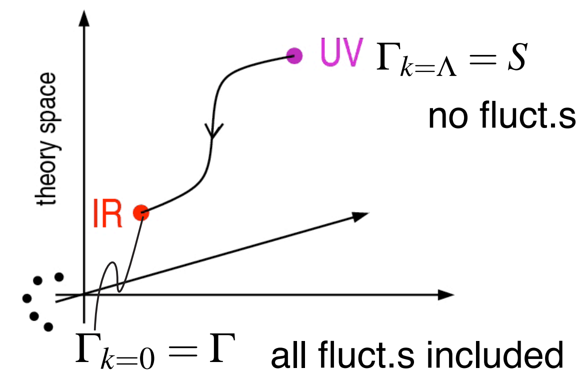
two-body bound state

zero crossing of fermion
chemical potential

Functional RG Approach

Flow of the Effective Action (Wetterich '93):

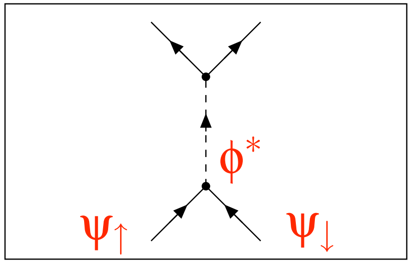
$$k\partial_k\Gamma_k[\phi_0] \equiv \partial_t\Gamma_k[\phi_0] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi_0]+R_k}\partial_t R_k$$



Basic truncation: Systematic and consistent derivative expansion

$$\Gamma[\psi,\phi] = \int_0^{1/T} d\tau \int d^3x \Big\{\psi^\dagger\left(Z_\psi\partial_\tau - A_\psi\triangle - \mu\right)\psi + \phi^*\left(Z_\phi\partial_\tau - A_\phi\triangle\right)\phi + U(\phi^*\phi) - \frac{h_\phi}{2}\left(\phi^*\psi^T\varepsilon\psi - \phi\psi^\dagger\varepsilon\psi^*\right) + \ldots\Big\}$$

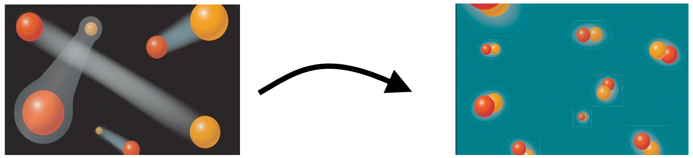
- ψ - stable fermionic atom field
- ϕ - composite bosonic field: Molecules / Cooper pairs
- quartic truncation of the effective potential



$$U(\phi^*\phi) = m_\phi^2\phi^*\phi + \frac{\lambda_\phi}{2}(\phi^*\phi)^2 + \ldots$$

- focus on universal broad resonance limit $h_\phi \to \infty, ak_F$ fixed

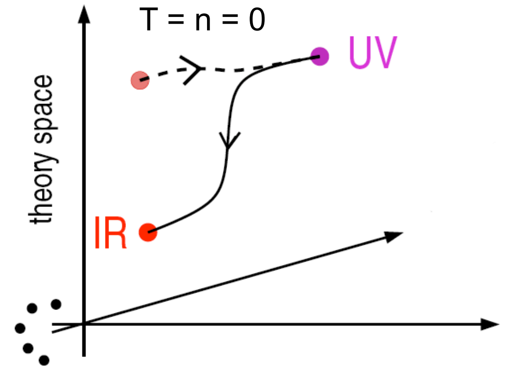
➡ bosons purely auxiliary on initial scale, $P_{\phi,k=\Lambda}(Q) = m^2_{\phi,k=\Lambda}$



Building Blocks for Evaluation

(i) Vacuum Problem:

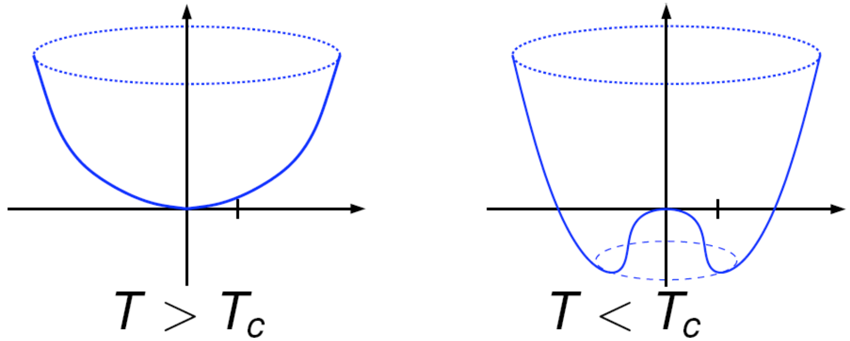
- Fix the observable parameters
- Nontrivial few-body physics



(ii) Many-Body Problem:

New scales: temperature T, density n ($k_F = (3\pi^2 n)^{1/3}$)

- Spontaneous symmetry breaking at the finite temperature phase transition to the superfluid state
- Implement the constraint of a fixed particle number



Spontaneous Symmetry Breaking

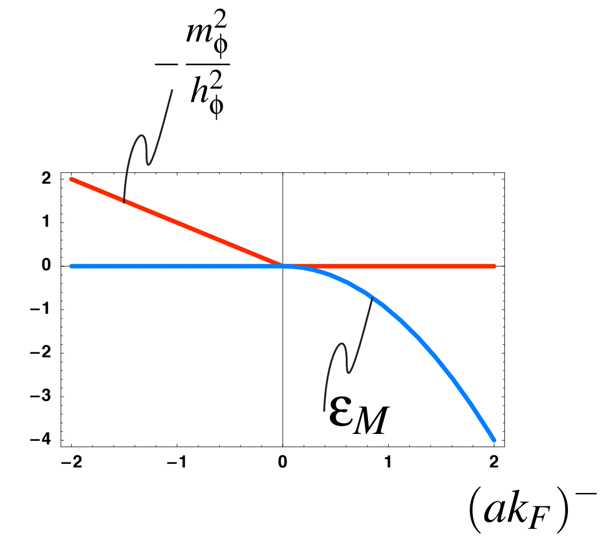
Microscopic Scale: Vacuum Limit

- Project on physical vacuum by

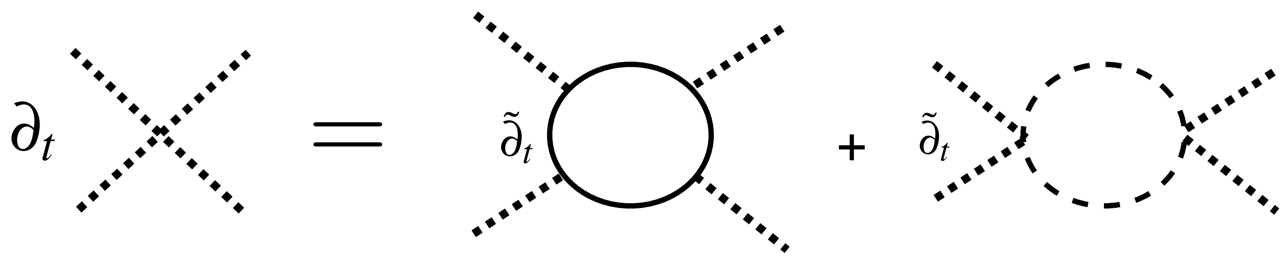
$$n = \frac{k_F^3}{3\pi^2}$$

$$\Gamma_{k\to 0}(vak) = \lim_{k_F \to 0} \Gamma_{k\to 0}\big|_{T/\varepsilon_F > T_c/\varepsilon_F = \mathrm{const.}}$$

  - Diluting procedure: $d \sim k_F^{-1} \to \infty$
  - Getting cold: $T \sim \varepsilon_F$
  - Picture: Smooth crossover terminates in sharp "second order phase transition" in vacuum



- Few-body scattering: dimer-dimer on BEC side a > 0



dimer-dimer scattering length

➡ $\frac{a_M}{a} =$ 2 0.75

microscopic

$\varepsilon_M = -\frac{1}{Ma^2}$

thermodynamic

$n = \frac{k_F^3}{3\pi^2}, T$

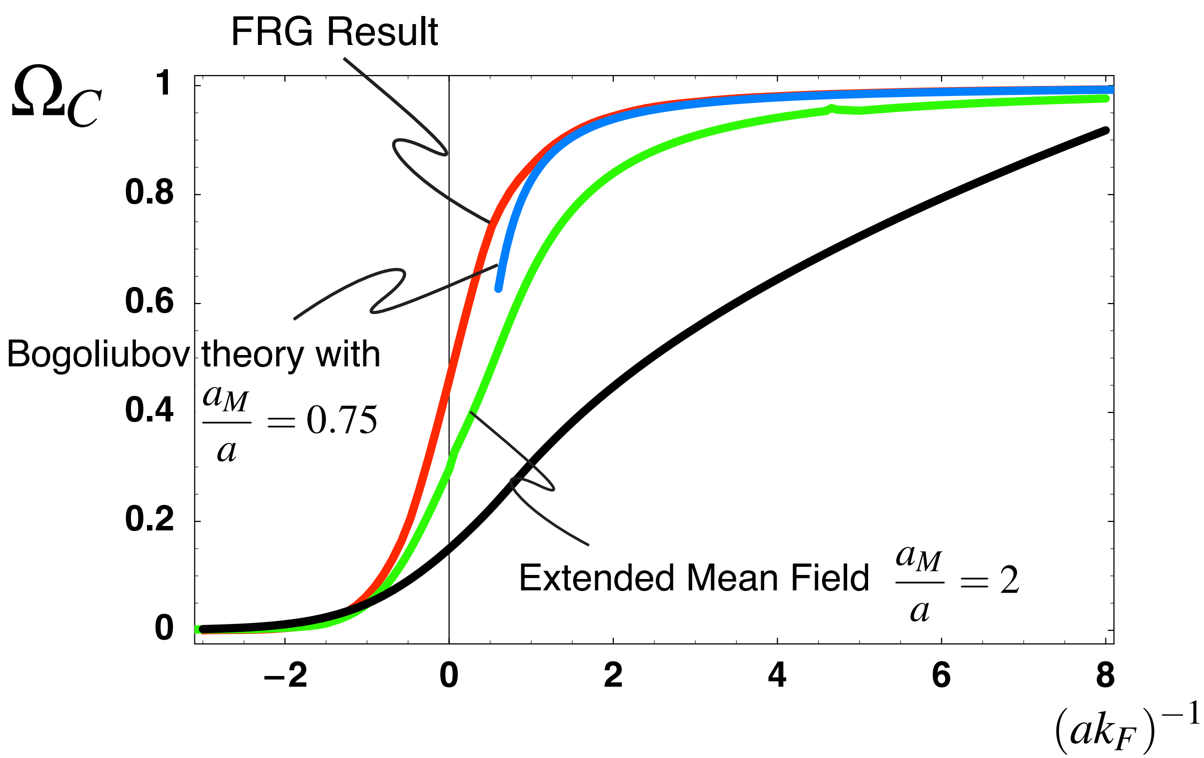
long distance

$k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$

... and impact on thermodynamics

Picture: Tightly bound molecules deep on BEC side:
effective pointlike dof.s interacting via effective scattering length a_M

- Condensate Fraction at T=0:



microscopic

$\varepsilon_M = -\frac{1}{Ma^2}$

thermodynamic

$n = \frac{k_F^3}{3\pi^2}, T$

long distance

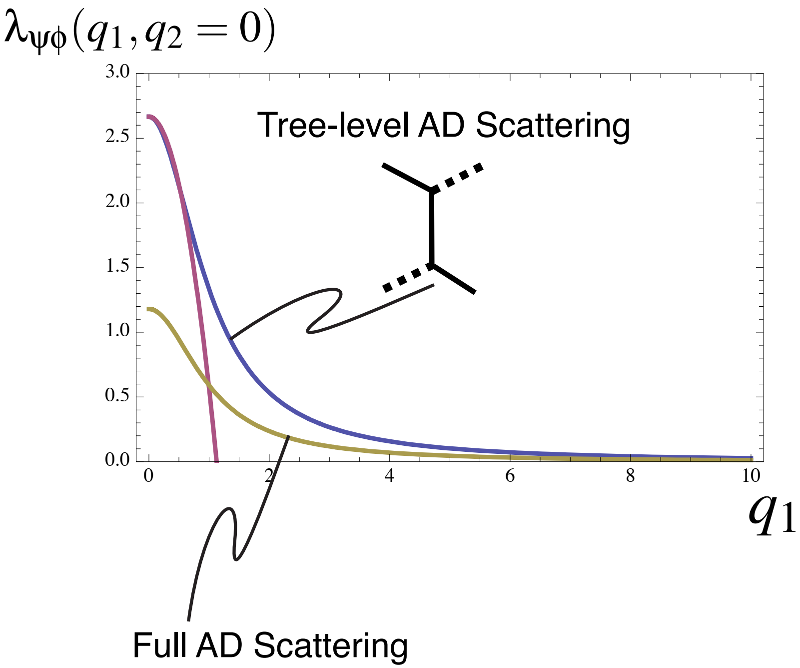
$k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$

Extensions (with H.C. Krahl, M.Scherer)

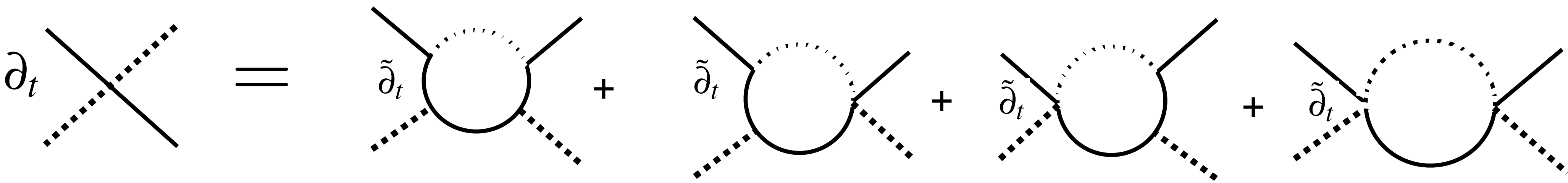
- Few-body scattering impacts on thermodynamics
- Extend the truncation with atom-dimer scattering:

$$\Delta\Gamma_k = \int \lambda_{\psi\phi,k}\phi^*\phi\psi^\dagger\psi$$

- Flow: need (s-wave projected) momentum dependence



$\lambda_{\psi\phi}(q_1, q_2)$ ➡ Solve Matrix Differential Equation



➡ Fermion-boson flow: relative cutoff scale

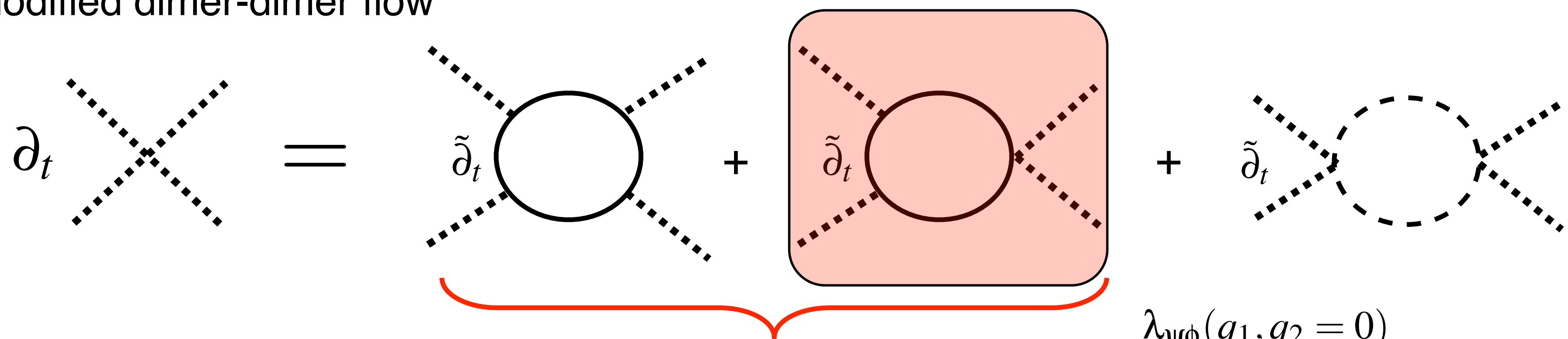
➡ integrate fermions prior to bosons:

- Differential equation can be integrated analytically: $\lambda_{\psi\phi} = (\mathbf{1} + \lambda_{\psi\phi}^{(tree)} \cdot M)^{-1}\lambda_{\psi\phi}^{(tree)}$
- Equivalent to STM integral equation (Nuclear Physics)

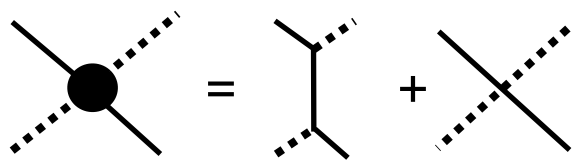
$$\frac{a_{ad}}{a} = 1.12$$

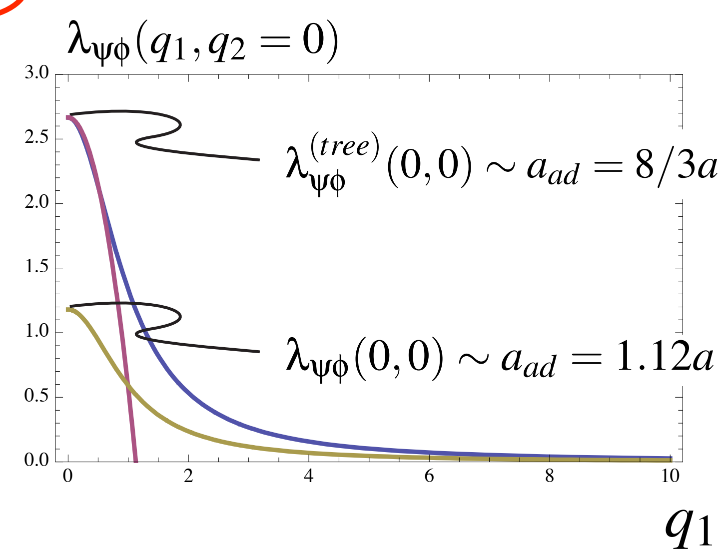
Extensions (with H.C. Krahl, M.Scherer)

- modified dimer-dimer flow



- Full vertex:





- Observation: $$\frac{\lambda_{\psi\phi}(q_1, q_2)}{\lambda^{(tree)}_{\psi\phi}(q_1, q_2)} \approx \frac{1.12}{8/3}$$

➡ estimate for dimer-dimer scattering

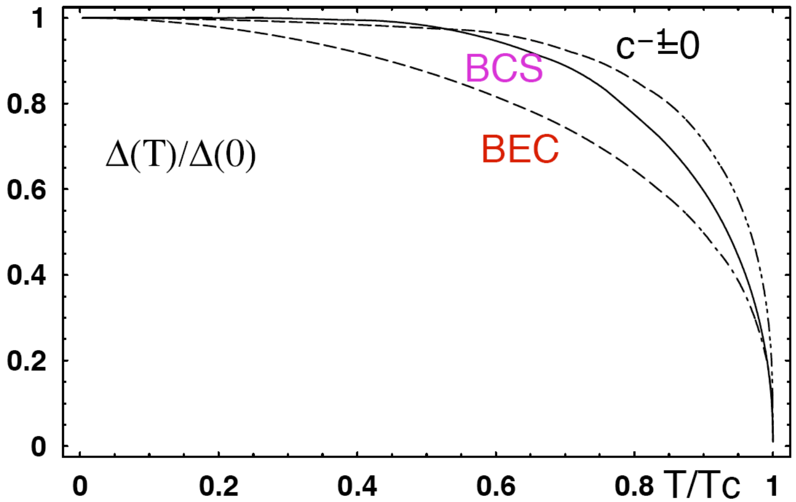
$$\frac{a_M}{a} = 0.65$$

- cf: solution of 4-body Schrödinger Eq. (Shlyapnikov& '04): $\frac{a_M}{a} = 0.6$

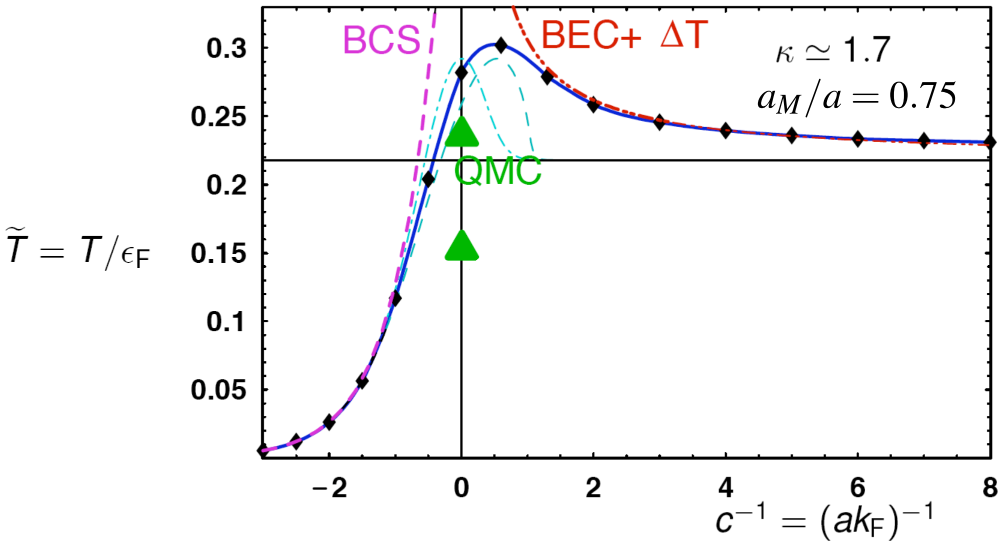
Long Distance Physics

Close to (expected!) second order phase transition: Deep IR physics important

Gap parameter in various regimes



Phase Diagram



- Second order PT throughout crossover
- Universal critical behavior of O(2) universality class from fermionic microscopic model:

$$\eta(1/(ak_F)) = 0.05 \quad \text{for all} \quad ak_F$$

- continuous change of relevant dof.s!

- Shift in T_c (Baym, Blaizot& '01)

$$(T_c - T_c^{\mathrm{BEC}})/T_c^{\mathrm{BEC}} = \kappa \cdot a_M \cdot n^{1/3}$$

- low momentum dependence of bosonic self energy at
- lattice result (O(2) model, fundamental bosons): (Arnold& '01) $\kappa = 1.3$

microscopic

$$\varepsilon_M = -\frac{1}{Ma^2}$$

thermodynamic

$$n = \frac{k_F^3}{3\pi^2}, T$$

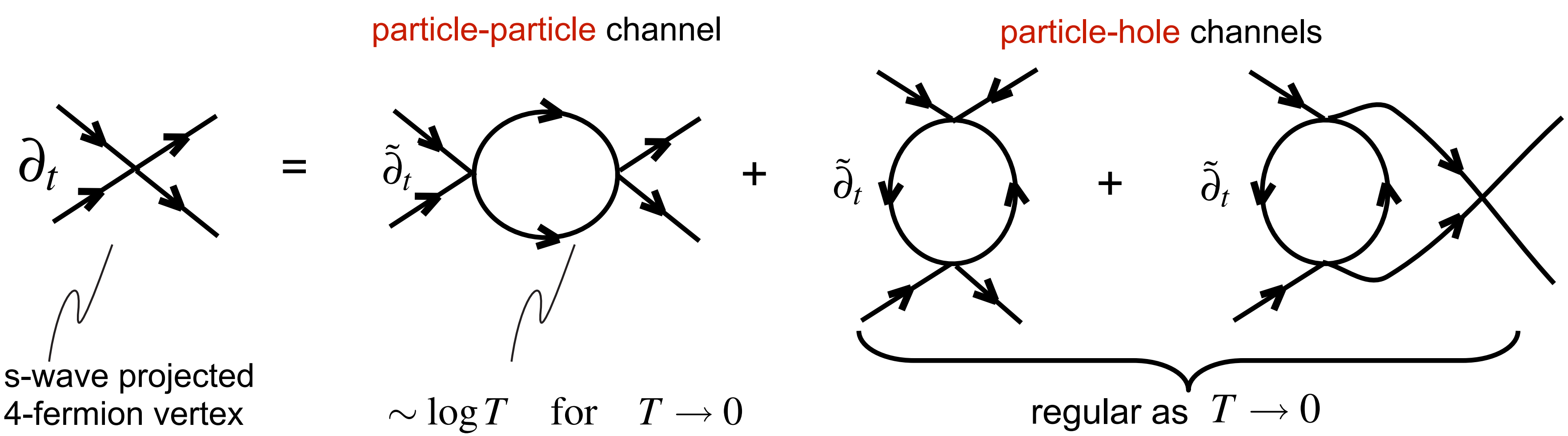
long distance

$$k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$$

Many-Body Fermion Physics (with S. Flörchinger, M. Scherer, C. Wetterich)

Particle-Hole Fluctuations for weakly interacting fermions:

- Purely fermionic description $S[\psi,\phi] = \int d\tau \int d^3x \left\{ \psi^\dagger (\partial_\tau - \frac{\triangle}{2M} - \mu)\psi + \frac{\lambda}{2}(\psi^\dagger\psi)^2 \right\}$
- Simple RG Equation



- Screening effect with impact on critical temperature at weak interaction
  - Thouless criterion $\lambda^{-1}_{k\to 0}(T,n) = 0$
  - result

$$T_c^{(BCS)} = 0.61\varepsilon_F e^{-\frac{\pi}{2ak_F}}, \qquad \frac{T_c^{(BCS)}}{T_c^{(Gorkov)}} = 2.2$$

Gorkov effect

microscopic	thermodynamic	long distance
$\varepsilon_M = -\frac{1}{Ma^2}$	$n = \frac{k_F^3}{3\pi^2}, T$	$k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$

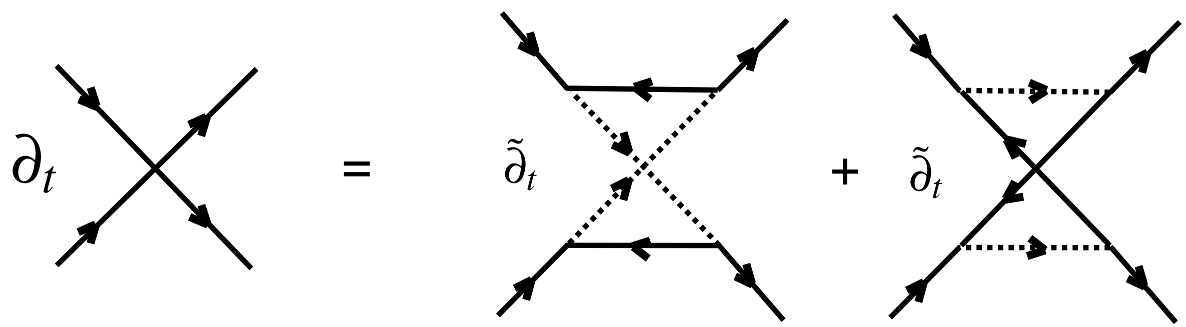
Many-Body Fermion Physics (with S. Flörchinger, M. Scherer, C. Wetterich)

- Hubbard-Stratonovich transformation: Decoupling into particle-particle channel
- essential: describe the bound state generation
- how to reconstruct the lost particle-hole channel?
- Study flow of newly generated 4-fermion vertex

  - extend truncation: $\Delta\Gamma_k = \int \lambda_{\psi_k} (\psi^\dagger \psi)^2$

  - initial condition: $\lambda_{\psi_{k=\Lambda}} = 0$

  - flow:



✓ s-wave projected
✓ included via rebosonization technique
(Gies, Wetterich ’02)

microscopic

$\epsilon_M = -\frac{1}{Ma^2}$

thermodynamic

$n = \frac{k_F^3}{3\pi^2}, T$

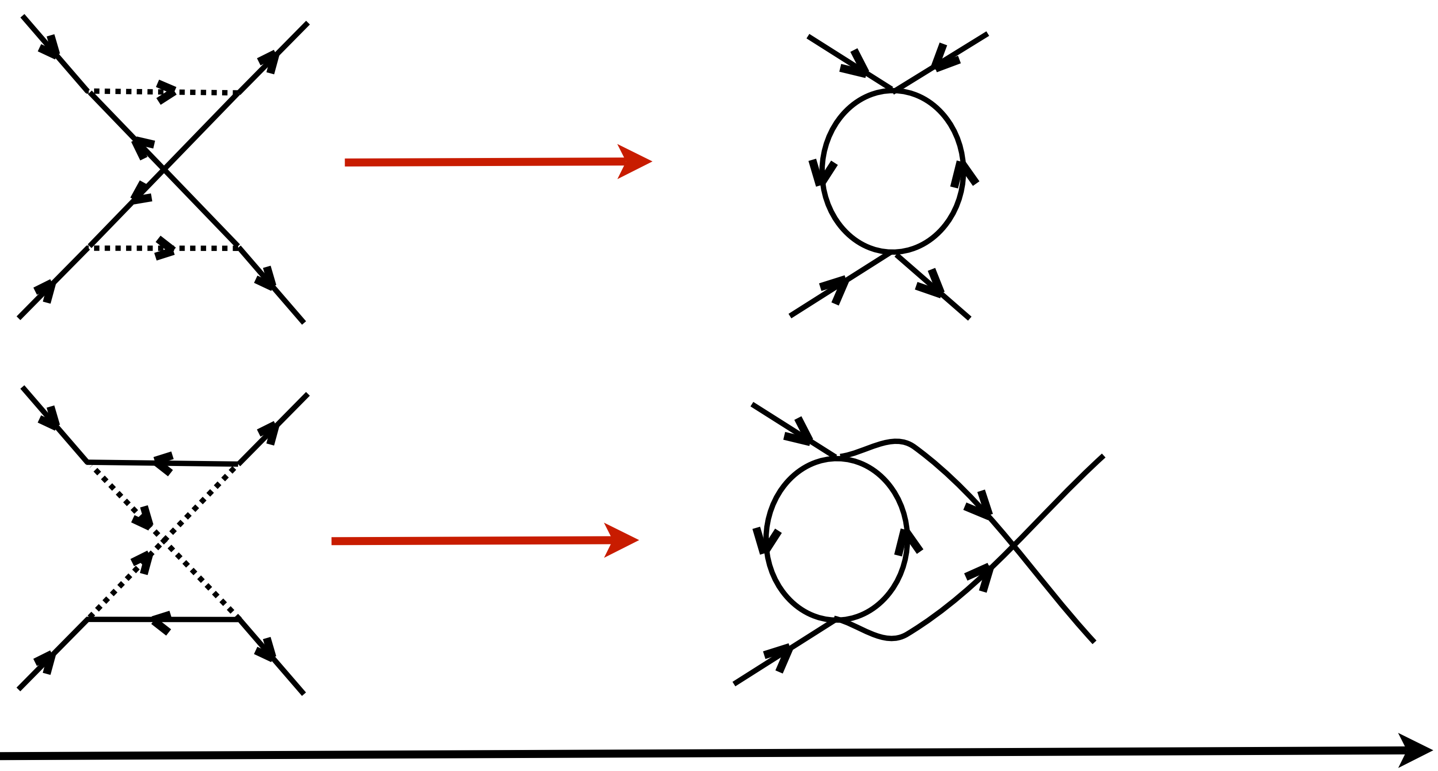
long distance

$k_{ld} \gg n^{1/3}, T^{1/2}, \epsilon_M^{1/2}$

Many-Body Fermion Physics (with S. Flörchinger, M. Scherer, C. Wetterich)

Interpretation

- assume massive bosons $P_{\phi,k}(Q) \approx m^2_{\phi,k}$
- contract boson lines $\lambda_{ph,k} \approx \dfrac{h^2_{\phi,k}}{m^2_{\phi,k}}$



microscopic

$$\varepsilon_M = -\frac{1}{Ma^2}$$

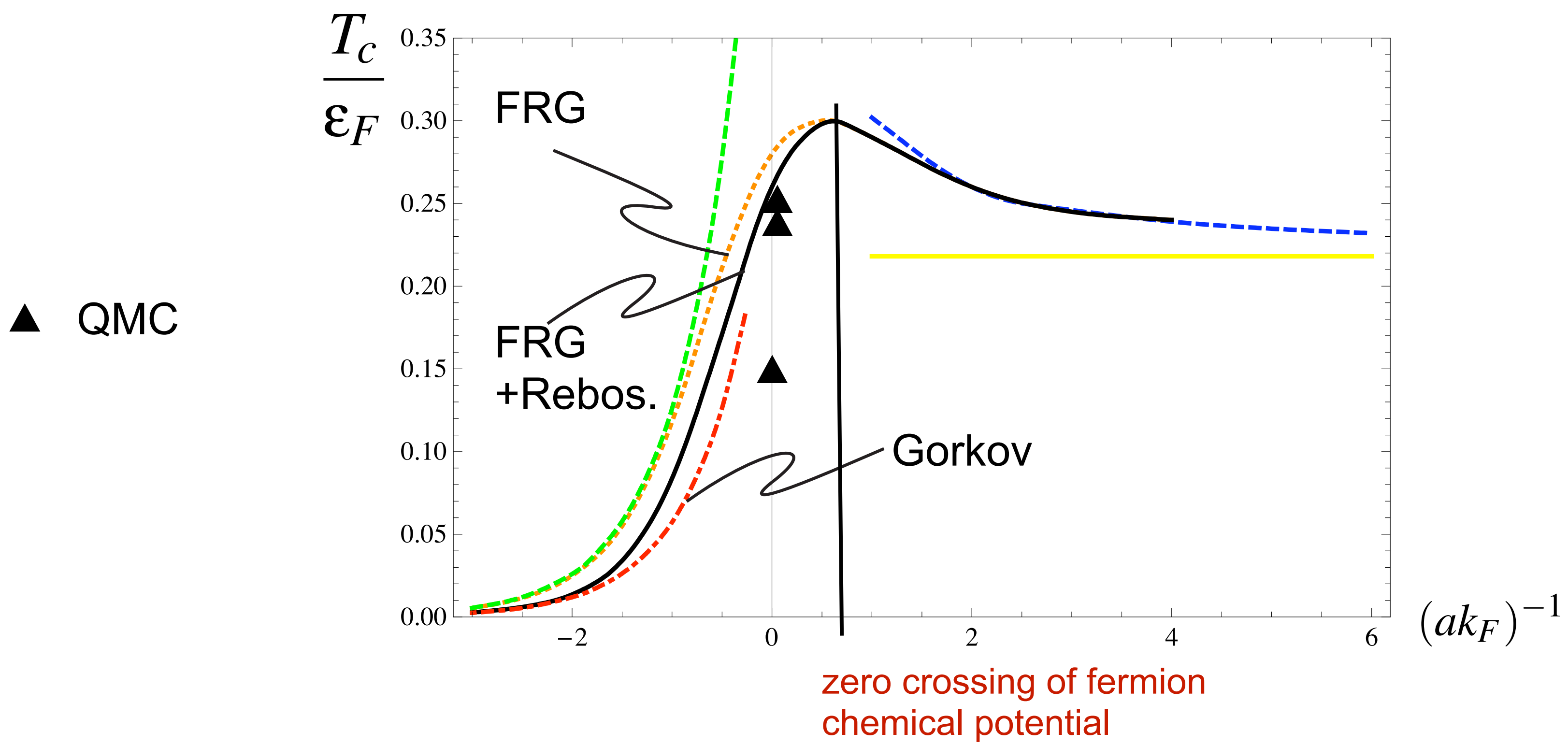
thermodynamic

$$n = \frac{k_F^3}{3\pi^2}, T$$

long distance

$$k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$$

Result (preliminary; with S. Flörchinger, M. Scherer, C. Wetterich)



- Accurately reproduce Gorkov effect in the BCS regime from rebosonization procedure: bosons massive even close to phase transition
- Fermion many-body effect: vanishes at zero crossing of chem. pot.

microscopic

$$\varepsilon_M = -\frac{1}{Ma^2}$$

thermodynamic

$$n = \frac{k_F^3}{3\pi^2}, T$$

long distance

$$k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$$

Conclusions

• RG put to work for universal aspects (BR universality, critical behavior at T_c...) and nonuniversal observables (gap, condensate fraction, critical temperature...)

microscopic

$$\varepsilon_M = -\frac{1}{Ma^2}$$

thermodynamic

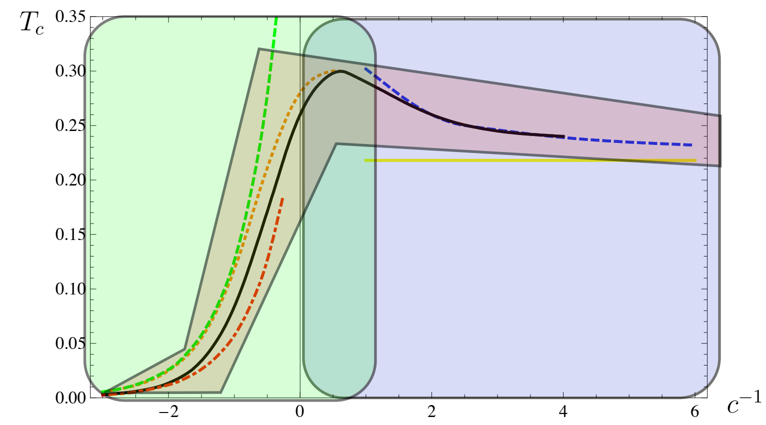
$$n = \frac{k_F^3}{3\pi^2}, T$$

long distance

$$k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$$

• Use FRG to head towards quantitative accuracy combined with analytical insight for the crossover:

- Precision estimate for few body scattering lengths.
- Shift in T_c in BEC regime
- Improved estimate of T_c in strongly interacting and BCS regime (preliminary; see talk by Flörchinger, poster by Scherer)



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