Critical scaling behavior in the O(N) model in infinite and finite volume

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J. Braun and B. Klein, Phys. Rev. D **77** (2008) 096008.

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Phase transitions in QCD

QCD phase transitions

- de-confinement phase transition
- chiral phase transition

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QCD phase transitions

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Lattice Gauge Theory

- fully non-perturbative method
- finite simulation volume
- explicit symmetry breaking through quark masses
- phase transition order? $\longrightarrow$ (finite-size) scaling analysis

- $N_f = 2$: second order for $m_q = 0$, crossover for $m_q \neq 0$, O(4)

R. D. Pisarski and F. Wilczek, Phys. Rev. D **29** (1984) 338.

- first order phase transition? (confinement dominates?) (staggered fermions)

M. D'Elia, A. Di Giacomo and C. Pica, Phys. Rev. D **72** (2005) 114510 [arXiv:hep-lat/0503030];

G. Cossu, M. D'Elia, A. Di Giacomo, and C. Pica (2007), arXiv:0706.4470 [hep-lat].

- decide by analyzing scaling behavior

A scaling analysis requires *critical exponents* and *scaling functions*

- Scaling functions obtained mostly from O(N) lattice simulations and perturbative RG

  D. Toussaint, Phys. Rev. D55 (1997) 362.

  J. Engels, S. Holtmann, T. Mendes, and T. Schulze, Phys. Lett. **B514** (2001) 299.

  E. Brézin, D. J. Wallace, and K. Wilson, Phys. Rev. B7, 232 (1973).

  F. Parisen Toldin, A. Pelissetto and E. Vicari, JHEP **0307** (2003) 029.

- O(N) critical exponents from FRG calculations

  N. Tetradis and C. Wetterich, Nucl. Phys. **B422** (1994) 541.

  O. Bohr, B.J. Schaefer, and J. Wambach, Int. J. Mod. Phys. **A16** (2001) 3823.

  D. F. Litim and J. M. Pawlowski, Phys. Lett. B **516** (2001) 197.

- few results with explicit symmetry breaking
- no results on finite-size scaling

Functional RG for the O(N)-model

Effective action at scale k ($\Lambda \geq k \geq 0$) in LPA ($\eta = 0$)

$$\Gamma_k[\phi] = \int d^d x \, \frac{1}{2}(\partial_\mu \phi)^2 +$$

$$+ a_1(k)(\phi^2 - \sigma_0^2(k)) + a_2(k)(\phi^2 - \sigma_0^2(k))^2 + \ldots - H(\sigma - \sigma_0(k))$$

$$\phi = (\sigma, \vec{\pi}) \qquad \phi^2 = \sigma^2 + \vec{\pi}^2 \quad O(N)\text{-symmetric}$$

scale-dependent couplings

$\sigma_0(k), a_n(k), \quad n = 1, \ldots, n_{max} \qquad 2a_1(k)\sigma_0(k) = H \equiv const.$

- local expansion around the minimum
- lowest order in local potential approximation
- potential with explicit symmetry breaking
- ERG with optimized cutoff D. F. Litim, Phys. Lett. B486 (2000) 92.
  $\Leftrightarrow$ proper-time RG (infinite volume) S. B. Liao, Phys. Rev. D53 (1996) 2020.
  D. F. Litim and J. M. Pawlowski, Phys. Lett. B 516 (2001) 197.

application to finite volume:

$$\int \frac{d^d p}{(2\pi)^d} \; f(p^2) \longrightarrow \frac{1}{L^d} \sum_{n_1,\ldots,n_d} f\left(\left(\frac{2\pi}{L}\right)^2 (n_1^2 + \ldots + n_d^2)\right)$$

• $d = 3$: $\sigma_0(\Lambda) - \sigma_0^{\mathrm{crit}}(\Lambda) \sim T - T_c$

Scaling for the singular free energy

there is no length scale at a critical point: $\xi \to \infty$
▷ scale invariance of free energy density at critical point
▷ use behavior close to critical point

$$f_s(t, h) = \ell^{-d} f_s(\ell^{y_t} t, \ell^{y_h} h)$$

$$t = (T - T_c)/T_0, \quad h = H/H_0, \quad y_t = \frac{1}{\nu}, \quad y_h = \frac{\nu}{\beta\delta}$$

idea: keep one of the arguments fixed
▷ becomes function of a single scaling variable

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Scaling function in Fisher parametrization

$$M \sim \frac{\partial}{\partial H} f(t, h)$$

$$t = (T - T_c)/T_0, \qquad h = H/H_0, \qquad z = \frac{t}{h^{1/(\beta\delta)}}, \qquad \xi(t) \sim t^{-\nu}$$

Scaling function for the order parameter M

$$\left.\begin{array}{l} M(t, h = 0) = (-t)^{\beta} \\ M(t = 0, h) = \quad h^{1/\delta} \end{array}\right\} \rightarrow M(t, h) = h^{1/\delta} f(z),\ f(z) \xrightarrow[z \rightarrow -\infty]{} (-z)^{\beta}$$

critical exponent β enters into asymptotic behavior

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Scaling function in Fisher parameterization

Scaling function for the susceptibility χ

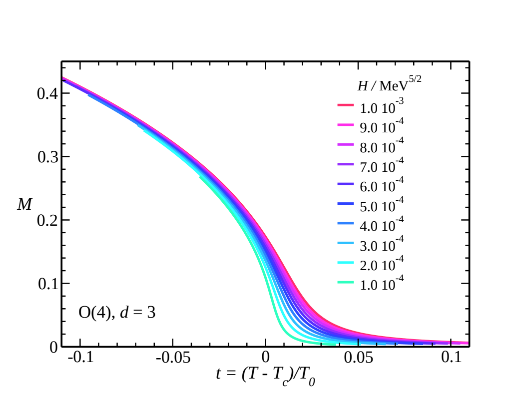
$$\chi = \frac{\partial M}{\partial H} \Rightarrow \chi = \frac{1}{H_0} h^{1/\delta - 1} \frac{1}{\delta} \left[f(z) - \frac{z}{\beta} f'(z) \right] = \frac{1}{H_0} h^{1/\delta - 1} f_\chi(z)$$

▷ consequence of scaling:

$f_\chi(z)$ completely specified by $f(z)$

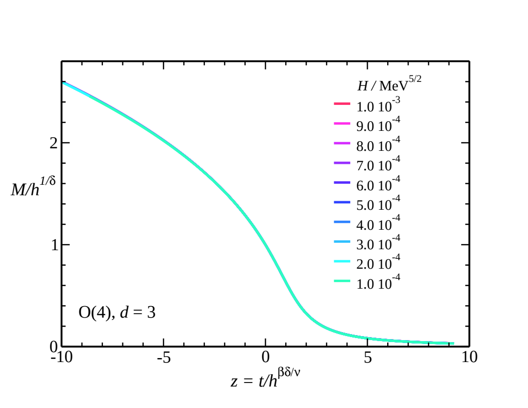
Order parameter *M* as a function of *t*

for small values of *H*



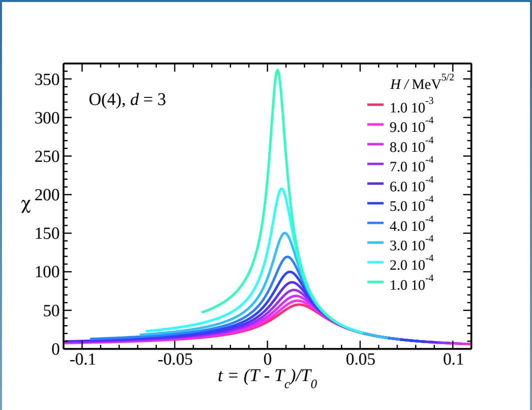
Rescaled order parameter $M/h^{1/\delta}$ as a function of z

for small values of H



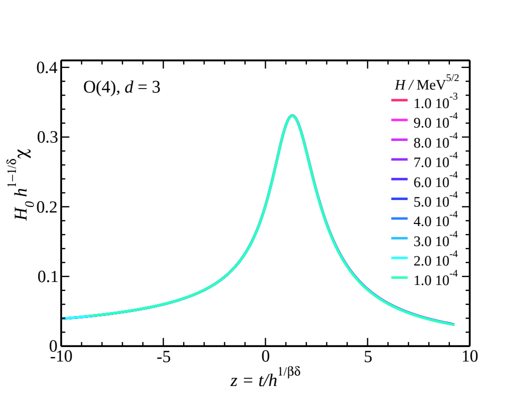
Susceptibility χ as a function of t

for small values of H (note trajectory $t_p/h^{1/(\beta\delta)} = z_p$)



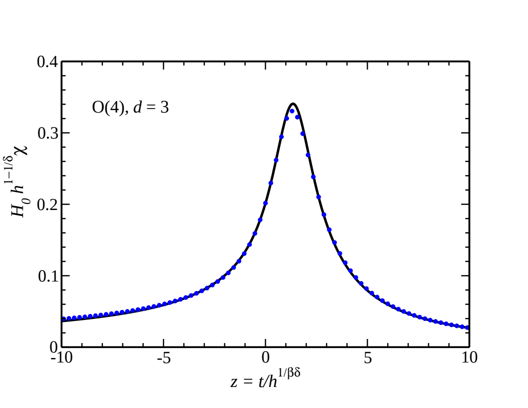
Rescaled Susceptibility $H_0 h^{1-1/\delta}\chi$ as a function of z

for small values of H



Test of scaling function

Is it true that $f_\chi(z) = \frac{1}{\delta}\left[f(z) - \frac{z}{\beta}f'(z)\right]$? Yes!



Finite-Size Scaling

- Universal behavior requires divergence of the correlation length ξ
- Finite volume size L cuts off the critical fluctuations
- Universal scaling behavior is therefore affected if correlation length $\xi \sim L$, depends on ratio ξ/L

Finite-Size Scaling hypothesis (Fisher): The ratio of thermodynamic quantities $(M, \chi, \ldots)$
in the finite-size system and the infinite-size system is a function of *only* the ratio ξ/L:

$$\frac{M_L(t)}{M_\infty(t)} = \mathcal{F}\left(\frac{L}{\xi(t)}\right), \quad \xi(t) \sim t^{-\nu}, \quad M(t,h) = h^{1/\delta} f(z)$$

Finite-Size Scaling Functions

Idea for obtaining the universal Finite-Size Scaling functions:

- keep $L/\xi = \text{const.}$ $\rightarrow$ vary $t \sim L^{1/\nu}$
- keep $z = t/h^{1/(\beta\delta)} = \text{const.}$ $\rightarrow$ vary $h \sim L^{-\beta\delta/\nu}$

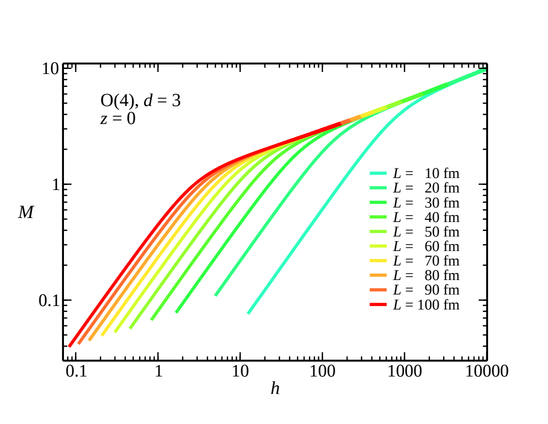
$$M(t,h) = h^{1/\delta} f(z) \quad \rightarrow \quad L^{-\beta/\nu}(hL^{\beta\delta/\nu})^{1/\delta} f(z)$$

Finite-Size Scaling Functions depend only on $hL^{\beta\delta/\nu}$ (for any given value of z):

$$L^{\beta/\nu} M = Q_M(z, hL^{\beta\delta/\nu})$$
$$L^{\gamma/\nu} \chi = Q_\chi(z, hL^{\beta\delta/\nu})$$

Finite-Size Scaling

Order parameter $M(h)$ vs. h for $L = 10 - 100$ fm

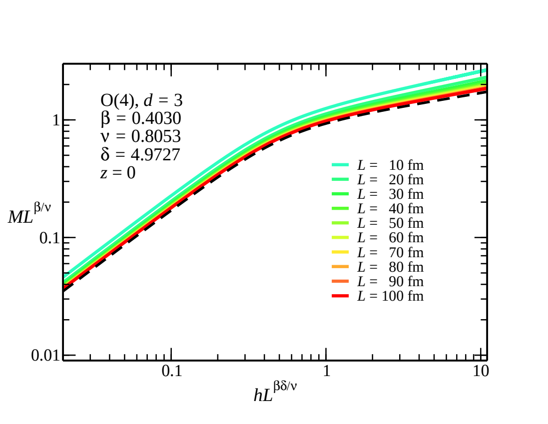


at the critical temperature ($z = 0$).

- ξ small for large h
- deviations for $\xi \sim L$
- asymptotic behavior given by $1/\delta$

Finite-Size Scaling

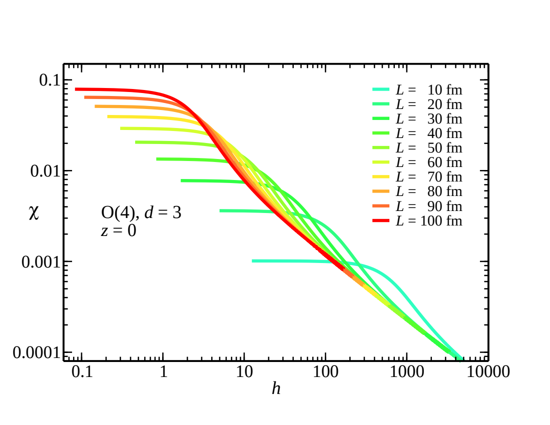
Finite-size scaled order parameter $ML^{\beta/\nu}$ vs. $hL^{\beta\delta/\nu}$



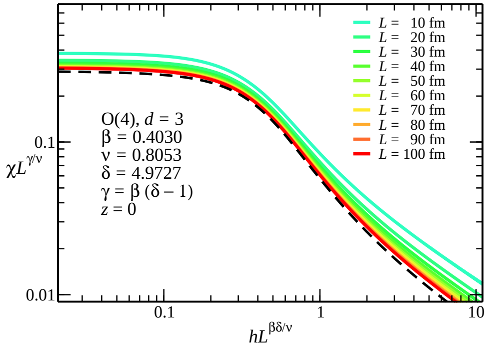
for $L = 10 - 100$ fm.

- scaling deviations for large fields h
- controlled by sub-leading operator
- consistent with RG prediction for ω
- extrapolate to obtain scaling function

Susceptibility $\chi(h)$ vs. h for $L = 10 - 100$ fm

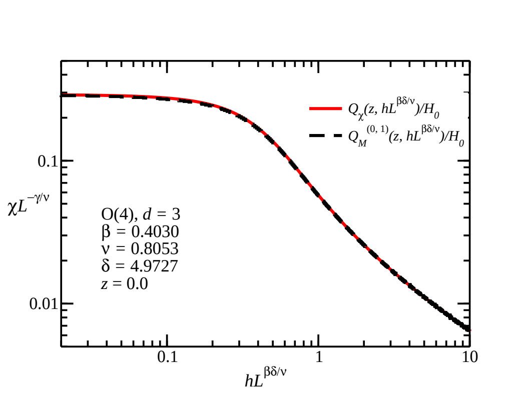


Finite-size scaled susceptibility $\chi L^{\gamma/\nu}$ vs. $hL^{\beta\delta/\nu}$

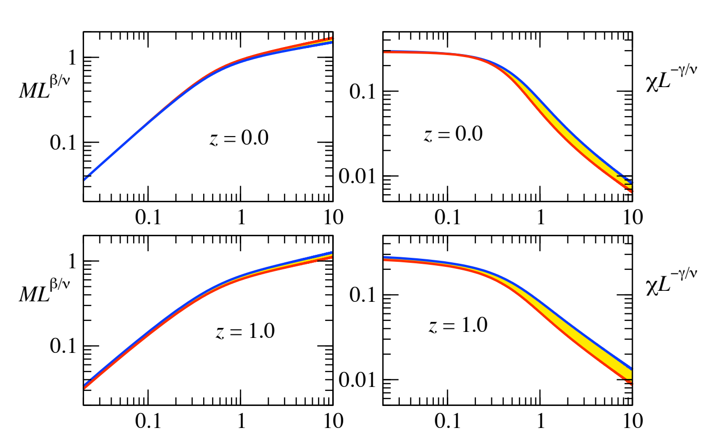


Test of the scaling function

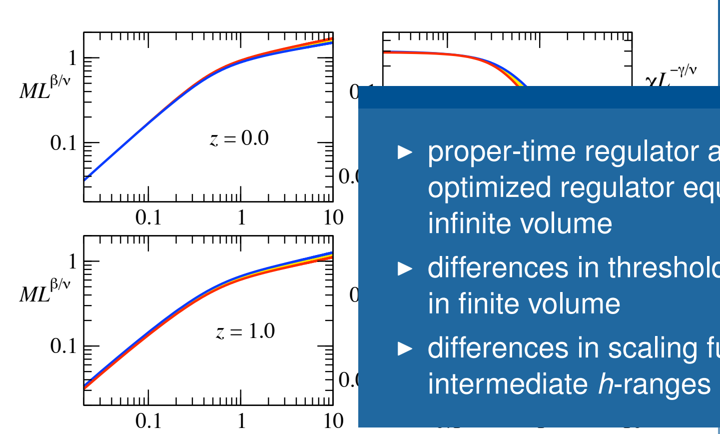
Is it true that $Q_\chi(z, hL^{\beta\delta/\nu}) = \frac{\partial}{\partial(hL^{\beta\delta/\nu})} Q_M(z, hL^{\beta\delta/\nu})$? Yes!



Comparison of cutoff schemes



Comparison of cutoff schemes



- proper-time regulator and optimized regulator equivalent in infinite volume
- differences in threshold functions in finite volume
- differences in scaling functions in intermediate *h*-ranges

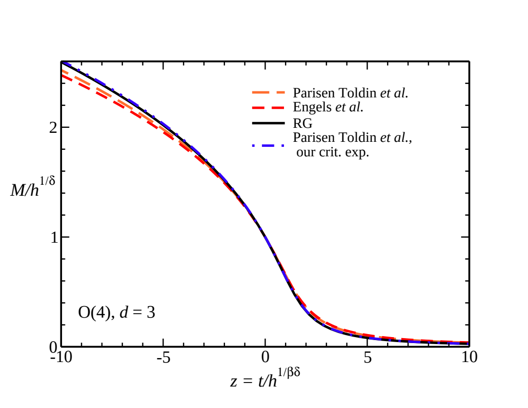
Conclusions

- Scaling with explicit symmetry breaking works in the FRG as expected
- Finite-Size Scaling works
- Critical exponents consistent with finite-size scaling
- Scaling functions have been obtained
- Finite-Size scaling functions have been obtained
- results are consistent with expectations and spin model lattice results

Outlook

- comparison to lattice QCD results:
  are calculations in the "interesting" region?
- obtain O(2) scaling functions
- go beyond local potential approximation

Comparison to lattice spin-model O(4) result

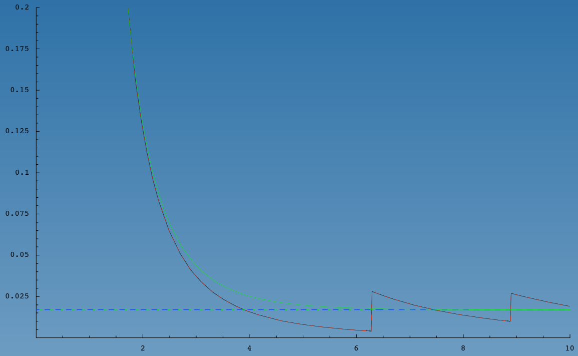


J. Engels, S. Holtmann, T. Mendes, and T. Schulze, Phys. Lett. **B514** (2001) 299

F. Parisen Toldin, A. Pelissetto and E. Vicari, JHEP **0307** (2003) 029.

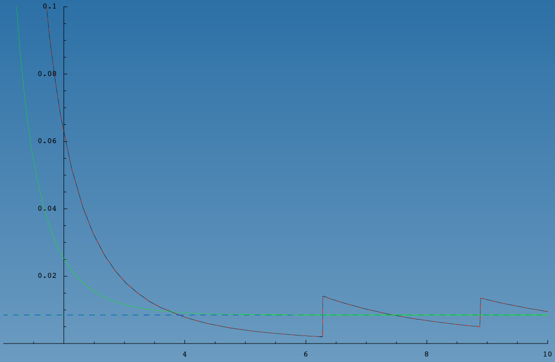
Comparison of Threshold functions

$k = 1.0, m = 0.0$ as a function of L



Comparison of Threshold functions

$k = 1.0, m = 1.0$ as a function of L



Comparison of Threshold functions

$k = 1.0, m = 10.0$ as a function of L