

Strong correlations in gauge theories

Jan Martin Pawłowski

Institute for Theoretical Physics
Heidelberg University

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outline

1 Introduction

2 Landau gauge QCD

- Signatures of confinement
- Infrared asymptotics & finite volume effects

3 QCD at finite temperature

- Polyakov loop potential
- confinement-deconfinement phase transition

1 Introduction

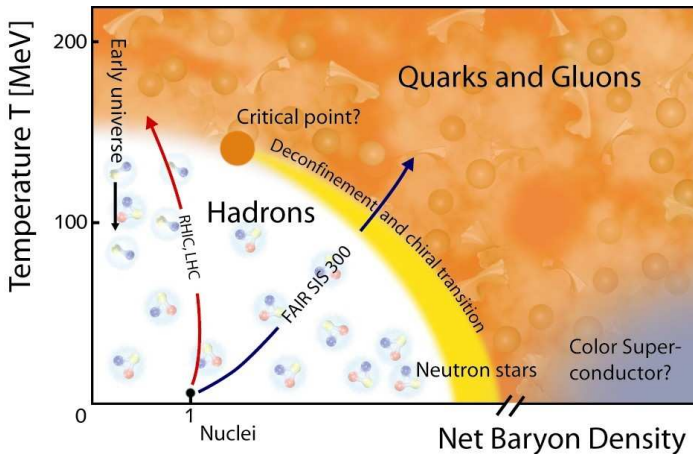
2 Landau gauge QCD

- Signatures of confinement
- Infrared asymptotics & finite volume effects

3 QCD at finite temperature

- Polyakov loop potential
- confinement-deconfinement phase transition

QCD phase diagram



QCD phase diagram

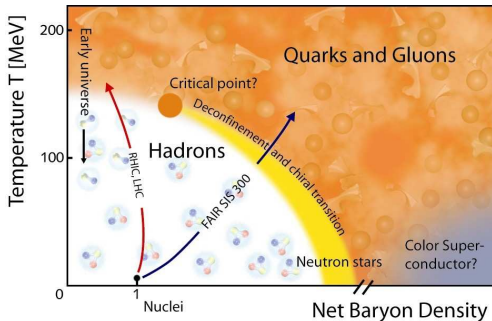
- chiral phase transition

$$SU_L(N_f) \times SU_R(N_f)$$

order parameter:

$$\langle \bar{q} q \rangle = \begin{cases} 0, & T > T_{c,\chi} \\ > 0, & T < T_{c,\chi} \end{cases}$$

talks of J. Braun, B.-J. Schaefer



QCD phase diagram

- chiral phase transition:

$$SU_L(N_f) \times SU_R(N_f)$$

order parameter:

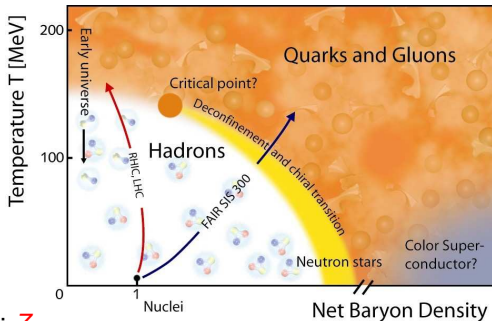
$$\langle \bar{q} q \rangle = \begin{cases} 0, & T > T_{c,\chi} \\ > 0, & T < T_{c,\chi} \end{cases}$$

- confinement-deconfinement: Z_3

order parameter:

$$\Phi = \left\langle \frac{1}{N_c} \text{tr} \mathcal{P} e^{i \int_0^\beta dt A_0} \right\rangle = \begin{cases} > 0, & T > T_{c,\text{conf}} \\ 0, & T < T_{c,\text{conf}} \end{cases}$$

$\Phi = e^{-F_q}$ relates to a static quark state.



outline

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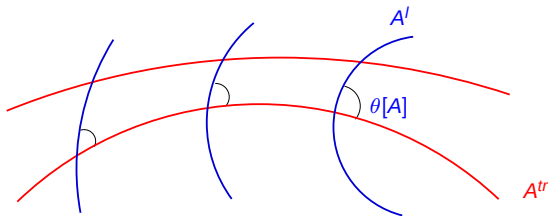
- Signatures of confinement
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3 QCD at finite temperature

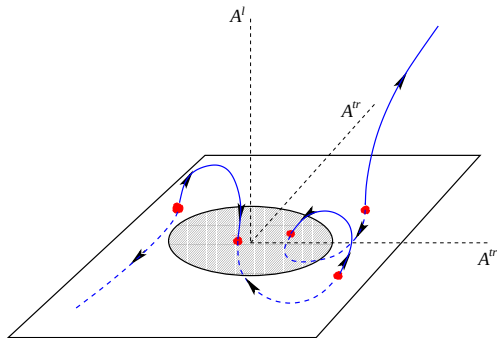
- Polyakov loop potential
- confinement-deconfinement phase transition

$$S_{YM} = \frac{1}{2} \int \text{tr} F^2 = \frac{1}{2} \int A_\mu^a \left(p^2 \delta_{\mu\nu} - p_\mu p_\nu \left(1 + \frac{1}{\xi} \right) \right) A_\nu^a + \dots$$

gauge fixing ensures the existence of the gauge field propagator



Landau gauge $\xi = 0$: $A' = \partial_\mu A_\mu = 0$



Gribov problem

- Observables in Yang-Mills theory: $\mathcal{O}[A^g] = \mathcal{O}[A]$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA \mathcal{O}[A] e^{-S_{YM}[A]}$$

with $S_{YM} = \frac{1}{2} \int \text{tr} F^2$ and $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$

- Observables in Yang-Mills theory: $\mathcal{O}[A^g] = \mathcal{O}[A]$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA \, \delta[\partial_\mu A_\mu] |\det(-\partial_\mu D_\mu)| \mathcal{O}[A] e^{-S_{YM}[A]}$$

Faddeev-Popov

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$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA \delta[\partial_\mu A_\mu] |\det(-\partial_\mu D_\mu)| \mathcal{O}[A] e^{-S_{YM}[A]}$$

Faddeev-Popov

- BRST: $|\det(-\partial_\mu D_\mu)| \rightarrow \det(-\partial_\mu D_\mu)$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA dC d\bar{C} \delta[\partial_\mu A_\mu] \mathcal{O}[A] e^{-S_{YM}[A] - \int \bar{C} \partial_\mu D_\mu C}$$

physical Hilbert space $\leftrightarrow$ nilpotency of BRST transformation s .

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but

$$\int dA \delta[\partial_\mu A_\mu] \frac{\det(-\partial_\mu D_\mu)}{|\det(-\partial_\mu D_\mu)|} = 0$$

- Observables in Yang-Mills theory: $\mathcal{O}[A^g] = \mathcal{O}[A]$

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Faddeev-Popov

- BRST: $|\det(-\partial_\mu D_\mu)| \rightarrow \det(-\partial_\mu D_\mu)$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA dC d\bar{C} \delta[\partial_\mu A_\mu] \mathcal{O}[A] e^{-S_{YM}[A] - \int \bar{C} \partial_\mu D_\mu C}$$

Remedies:

- weighting of copies: loss of BRST-invariance
- topological gauge fixing

loss of nilpotency

Witten index

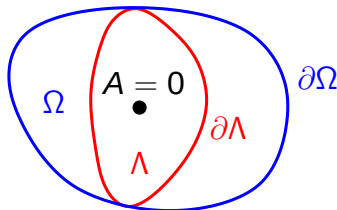
confinement scenario

$$\Omega = \{A \mid \partial_\mu A_\mu = 0, -\partial_\mu D_\mu \geq 0\}$$

- entropy

$$\int dA \det(-\partial D) e^{-S}$$

- entropy ($\int dA$)
 - $\partial\Omega(\cap\partial\Lambda)$ dominates IR
 - ghost IR-enhanced
 - gluonic mass-gap: confined gluons
 - violation of reflection positivity



non-renormalisation of ghost-gluon vertex

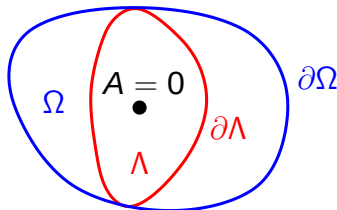
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- entropy ($\int dA$)
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non-renormalisation of ghost-gluon vertex

-
- Kugo-Ojima (in BRST-extended configuration space)
 - gluonic mass-gap + no Higgs mechanism

$$k \partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k \partial_k R_k(p^2)$$

- in Yang-Mills theory: $\phi = (A, C, \bar{C})$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right)$$

Diagram 1: A solid circle with 12 small ovals along its circumference. A cross (X) is at the top, and a solid black dot is at the bottom.

Diagram 2: A dashed circle with 12 small ovals along its circumference. A cross (X) is at the top, and a solid black dot is at the bottom.

$$k \partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k \partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward though 'physically' complicated
 - no sign problem numerics as in scalar theories!
 - chiral fermions reminder: Ginsparg-Wilson fermions from RG argument!
 - bound states via (re-)bosonisation effective field theory techniques applicable!

$$k \partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k \partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward though 'physically' complicated
- gauge invariance Ellwanger '94, Bonini et al '94,
 - loss of BRST-nilpotency
 - flow of modified Slavnov-Taylor identity $\mathcal{W}_k$

$$\partial_t \mathcal{W}_k = -\frac{1}{2} \text{Tr} \left(\frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R \frac{1}{\Gamma_k^{(2)} + R_k} \frac{\delta^2}{\delta \phi^2} \right) \mathcal{W}_k$$

$$k \partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k \partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward though 'physically' complicated
- gauge invariance

talks of Y. Igarashi, E. Itou, H. Sonoda

$$k \partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k \partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward
- gauge invariance
- flows in Landau gauge QCD

Ellwanger, Hirsch, Weber '96

Bergerhoff, Wetterich '97

Pawlowski, Litim, Nedelko, von Smekal '03

Kato '04

Gies, Fischer '04

Pawlowski '05

$$k \partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k \partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward
- functional methods in Landau gauge QCD (**IR**)
 - Dyson-Schwinger equations
 - stochastic quantisation
 - flows in Landau gauge QCD
 - quark confinement from Landau gauge propagators
 - analytic perturbation theory (fixed point for coupling)

von Smekal, Hauck, Alkofer '97

Zwanziger '02

Pawlowski, Litim, Nedelko, von Smekal '03

Braun, Gies, Pawłowski '07

Shirkov, Solovtsov '96

functional RG

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{solid loop with } \otimes \text{ and } \bullet \right) - \left(\text{dashed loop with } \otimes \text{ and } \bullet \right)$$

functional DSE

$$\frac{\delta \Gamma_k[\phi]}{\delta A} = \frac{\delta S[\phi]}{\delta A} + \text{dashed loop with } \bullet \text{ and wavy line} + \text{solid loop with } \bullet \text{ and wavy line} + \text{solid loop with } \bullet \text{ and wavy line and } \bullet \text{ and } \otimes, \quad \frac{\delta \Gamma_k[\phi]}{\delta C} = \frac{\delta S[\phi]}{\delta C} + \text{dashed loop with } \bullet \text{ and wavy line and dashed line}$$

$$\begin{aligned}
 k \partial_k \text{ (gluon line) }^{-1} &= - \text{ (triangle) } - \text{ (triangle) } \\
 &+ \frac{1}{2} \text{ (box) } + \frac{1}{2} \text{ (box) } \\
 &- \frac{1}{2} \text{ (triangle) } + \text{ (triangle) } \\
 k \partial_k \text{ (ghost line) }^{-1} &= \text{ (triangle) } + \text{ (triangle) } \\
 &- \frac{1}{2} \text{ (triangle) } + \text{ (triangle) }
 \end{aligned}$$

$$\begin{aligned}
 \text{ (gluon line) }^{-1} &= \text{ (gluon line) }^{-1} - \frac{1}{2} \text{ (triangle) } \\
 - \frac{1}{2} \text{ (triangle) } &- \frac{1}{6} \text{ (triangle) } \\
 - \frac{1}{2} \text{ (triangle) } &+ \text{ (triangle) } \\
 \text{ (ghost line) }^{-1} &= \text{ (ghost line) }^{-1} + \text{ (triangle) }
 \end{aligned}$$

Unique infrared asymptotics in Landau gauge QCD

- conformal scaling

$$\Gamma^{(2n,m)}(\lambda p_1, \dots, \lambda p_{2n+m}) = \lambda^{\kappa_{2n,m}} \Gamma^{(2n,m)}(p_1, \dots, p_{2n+m})$$

$\Gamma^{(2n,m)}$: vertex with n ghost and anti-ghost lines, m gluons

$$\begin{aligned}
 k \partial_k \text{ (gluon) }^{-1} &= - \text{ (triangle) } - \text{ (triangle) } \\
 &+ \frac{1}{2} \text{ (box) } + \frac{1}{2} \text{ (box) } \\
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 k \partial_k \text{ (ghost) }^{-1} &= \text{ (triangle) } + \text{ (triangle) } \\
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 - \frac{1}{2} \text{ (triangle) } &+ \text{ (triangle) } \\
 \text{ (ghost) }^{-1} &= \text{ (ghost) }^{-1} + \text{ (triangle) }
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$$\Gamma^{(2n,m)}(\lambda p_1, \dots, \lambda p_{2n+m}) = \lambda^{\kappa_{2n,m}} \Gamma^{(2n,m)}(p_1, \dots, p_{2n+m})$$

- decoupling: $\kappa_{2n,m} = 0$ & massive gluon

no confinement!?

$$\begin{aligned}
 k \partial_k \text{ (gluon line) }^{-1} &= - \text{ (triangle) } - \text{ (triangle) } \\
 &+ \frac{1}{2} \text{ (box) } + \frac{1}{2} \text{ (box) } \\
 &- \frac{1}{2} \text{ (pentagon) } + \text{ (pentagon) } \\
 k \partial_k \text{ (ghost line) }^{-1} &= \text{ (triangle) } + \text{ (triangle) } \\
 &- \frac{1}{2} \text{ (box) } + \text{ (box) }
 \end{aligned}$$

$$\begin{aligned}
 \text{ (gluon line) }^{-1} &= \text{ (gluon line) }^{-1} - \frac{1}{2} \text{ (loop) } \\
 &- \frac{1}{2} \text{ (loop) } - \frac{1}{6} \text{ (loop) } \\
 &- \frac{1}{2} \text{ (loop) } + \text{ (loop) } \\
 \text{ (ghost line) }^{-1} &= \text{ (ghost line) }^{-1} + \text{ (loop) }
 \end{aligned}$$

Unique infrared asymptotics in Landau gauge QCD

$$\Gamma^{(2n,m)} \sim p^{2(n-m)\kappa_C} \quad \text{with} \quad \kappa_C \geq 0$$

$\Gamma^{(2n,m)}$: vertex with n ghost and anti-ghost lines, m gluons

confirms Alkofer, Fischer, Llanes-Estrada, Phys. Lett. **B611** (2005) 279–288
see also Alkofer, Huber, Schwenzer '08

$$\begin{aligned}
 k \partial_k \text{ (gluon line) }^{-1} &= - \text{ (triangle) } - \text{ (triangle) } \\
 &+ \frac{1}{2} \text{ (box) } + \frac{1}{2} \text{ (box) } \\
 &- \frac{1}{2} \text{ (pentagon) } + \text{ (pentagon) } \\
 \\
 k \partial_k \text{ (ghost line) }^{-1} &= \text{ (triangle) } + \text{ (triangle) } \\
 &- \frac{1}{2} \text{ (box) } + \text{ (box) }
 \end{aligned}$$

$$\begin{aligned}
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 &- \frac{1}{2} \text{ (loop) } - \frac{1}{6} \text{ (loop) } \\
 &- \frac{1}{2} \text{ (loop) } + \text{ (loop) } \\
 \\
 \text{ (ghost line) }^{-1} &= \text{ (ghost line) }^{-1} + \text{ (loop) }
 \end{aligned}$$

Unique infrared asymptotics in Landau gauge QCD

$$\Gamma(2n, m, \text{quarks}) \sim p^{2(n-m)\kappa_C + \text{quarks}}$$

QCD: work in progress; QED₃: Nedelko, Pawłowski, in preparation

Truncation

- full momentum dependence of propagators
- vertices momentum-dependent RG-dressing
- **optimisation**

Litim '00, Pawłowski '05

Truncation

- full momentum dependence of propagators
- vertices momentum-dependent RG-dressing
- optimisation

J. M. Pawłowski, Annals Phys. **322** (2007) 2831

- RG-invariance: $D_\mu \Gamma_k = 0$ from $D_\mu \Gamma = 0$ Pawłowski '00,'02

$$(D_\mu + 2\gamma_\phi)R_k \stackrel{!}{=} 0 \quad \rightarrow \quad R_k = \Gamma_k^{(2)} r(x/k^2)$$

with $D_\mu x = 0$, e.g. $x = p^2$, $x = \Gamma_k^{(2)}/Z$.

Truncation

- full momentum dependence of propagators
- vertices momentum-dependent RG-dressing
- optimisation
 - RG-invariance
 - functional optimisation Pawlowski '05

J. M. Pawłowski, *Annals Phys.* **322** (2007) 2831

$$R_{\text{opt}} \simeq (\Gamma_0^{(2)} - \Gamma_k^{(2)})\theta(\Gamma_0^{(2)} - \Gamma_k^{(2)})$$

Truncation

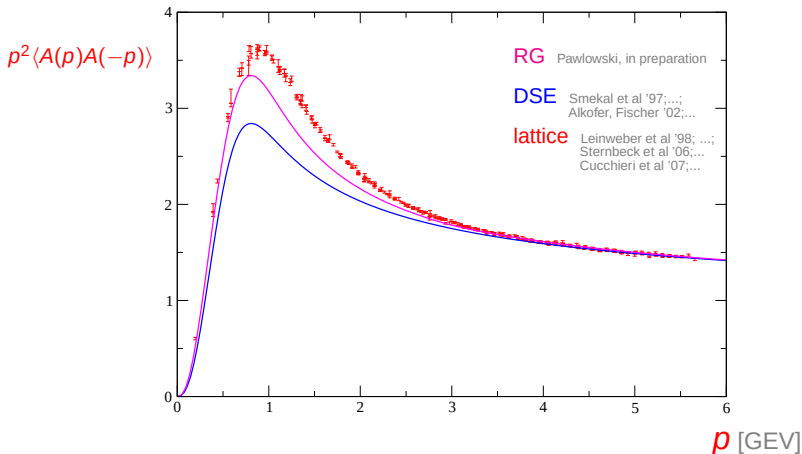
- full momentum dependence of propagators
- vertices momentum-dependent RG-dressing
- optimisation
- functional relations between diagrams: **Flow=Flow(DSE)**

$$\Rightarrow k \partial_k \Gamma_{k,A/C}^{(2)}(p^2) = \text{Flow}_k[\Gamma_{k,A/C}^{(2)}, \Gamma_k^{(3)}, \Gamma_k^{(4)}]$$

UV-IR flow

$$k \partial_k \text{ (wavy line with a dot) }^{-1} = - \text{ (triangle diagram with 2 wavy, 1 dashed) } - \text{ (triangle diagram with 2 dashed, 1 wavy) } + \frac{1}{2} \text{ (triangle diagram with 3 wavy) } + \frac{1}{2} \text{ (triangle diagram with 3 dashed) } - \frac{1}{2} \text{ (triangle diagram with 3 wavy and a cross) } + \text{ (triangle diagram with 3 dashed and a cross) }$$

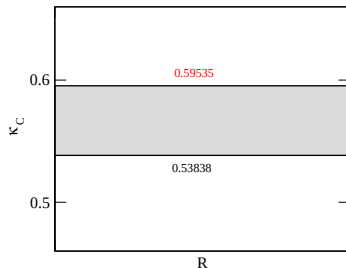
$$k \partial_k \text{ (dashed line with a dot) }^{-1} = \text{ (triangle diagram with 2 dashed, 1 wavy) } + \text{ (triangle diagram with 2 wavy, 1 dashed) } - \frac{1}{2} \text{ (triangle diagram with 3 dashed) } + \text{ (triangle diagram with 3 wavy) }$$



Improvement on FRG & DSE results: J. M. Pawłowski '08, Fischer '08

$$p^2 \langle A(p) A(-p) \rangle = \frac{p^2}{\Gamma_A^{(2)}(p)} \xrightarrow{p \rightarrow 0} (p^2)^{-2\kappa_C}$$

$$\stackrel{\text{DSE}}{=} \frac{D(p^2)}{p^2}$$



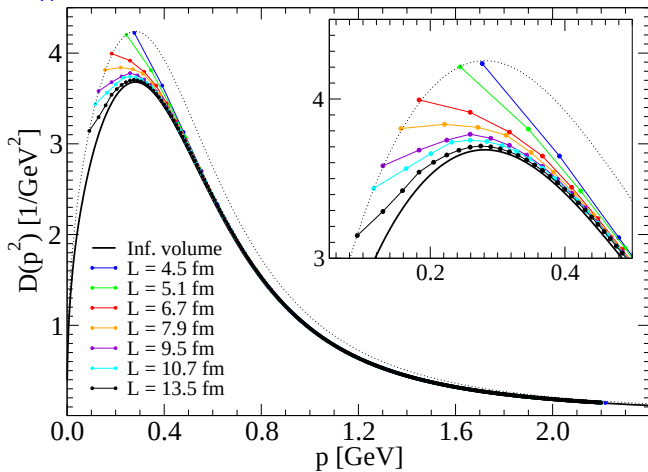
Pawlowski, Litim, Nedelko, von Smekal, Phys. Rev. Lett. **93** (2004) 152002

- functional optimisation: $\kappa_C = 0.59535\dots$, $\alpha_s = 2.9717\dots$

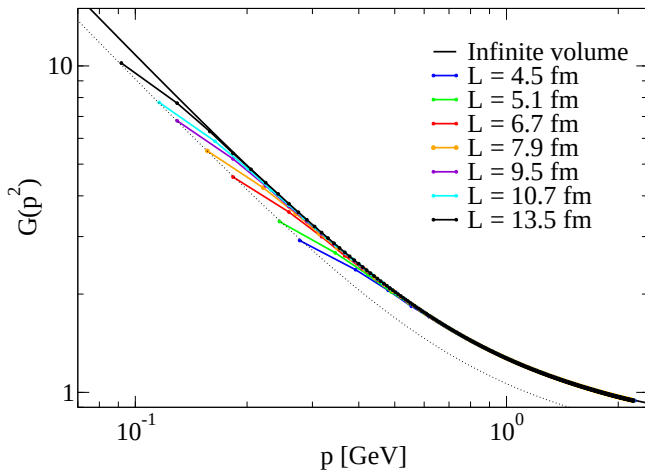
equals DS/StochQuant-result: Lerche, von Smekal, Phys. Rev. D **65** (2002) '02
D. Zwanziger, Phys. Rev. D **65** (2002)

RG-confirmation: C. S. Fischer and H. Gies, JHEP **0410** (2004)

$$D(p^2) = \frac{1}{\Gamma_A^{(2)}(p)}$$



$$G(p^2) = \frac{p^2}{\Gamma_c^{(2)}(p)}$$



Functional methods—lattice puzzle

- lower dimensions
 - quantitative agreement in $d = 2$ Maas '07
 - qualitative agreement in $d = 3$ A. Maas (St Goar '08)
- large volumes on the lattice
 - in $d = 4$ up to 128^4 at $\beta = 2.2$ Cucchieri et al '07
- gauge fixings
 - improved gauge fixings Bogolubsky et al '07, von Smekal et al '07, Maas (St Goar '08)
 - stochastic quantisation with D. Spielmann, I.O. Stamatescu
- $SU(2)$ versus $SU(3)$ Cucchieri et al '07, Sternbeck et al '07
- $\beta = 0$: evidence for gauge fixing/finite size problems von Smekal (St Goar '08)

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Order parameter

- Polyakov loop $\Phi(\vec{x}) = \langle L[A_0] \rangle$

$$L[A_0](\vec{x}) = \frac{1}{N_c} \text{tr} \mathcal{P} e^{i \int_0^\beta dt A_0}$$

with $\Phi \simeq e^{-F_q}$

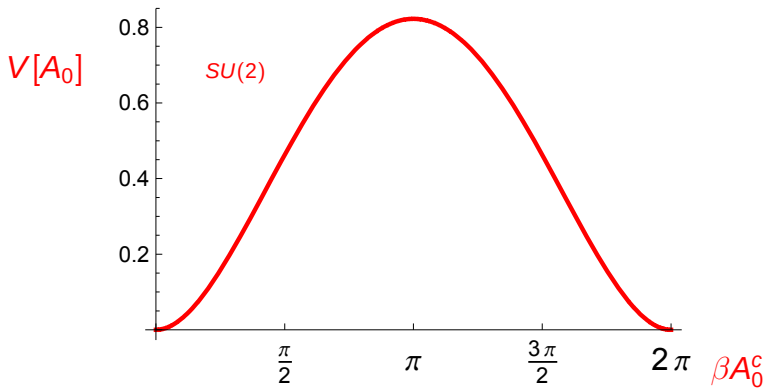
- confinement: $F_q = \infty$
- deconfinement: F_q finite
- string tension

$$\langle L(\vec{x}) L^\dagger(\vec{y}) \rangle \simeq e^{-F_{q\bar{q}}(\vec{x}-\vec{y})}$$

- $\lim_{|\vec{x}-\vec{y}| \rightarrow \infty} F_{q\bar{q}}(\vec{x}-\vec{y}) \simeq \beta\sigma |\vec{x}-\vec{y}|$

Weiss potential

$$V^{\text{UV}}[A_0] = \frac{1}{2\Omega} \text{Tr} \ln S_{AA}^{(2)}[A_0] - \frac{1}{\Omega} \text{Tr} \ln S_{CC}^{(2)}[A_0]$$



$SU(2): L[A_0] = \cos \frac{1}{2} \beta A_0^c$

with

$$A_0 = A_0^c \sigma_3$$

- background field flow for effective potential $V_{\text{eff}}[A_0] = \Gamma_k[A_0]/\Omega$

$$k \partial_k V_{\text{eff}}[A_0] = \frac{1}{2\Omega} \text{Tr} \frac{1}{\Gamma_k^{(2)}[A_0] + R_k(\Gamma_k^{(2)}[A_0])} k \partial_k R_k(\Gamma_k^{(2)}[A_0])$$

- determination of **fluctuation** propagator in Landau-DeWitt gauge

$$\Gamma_k^{(2)}[A] = \Gamma_{k,\text{Landau}}^{(2)}(\mathbf{p}^2 \rightarrow -D^2) + O(F)$$

- background field flow for effective potential $V_{\text{eff}}[A_0] = \Gamma_k[A_0]/\Omega$

$$k \partial_k V_{\text{eff}}[A_0] = \frac{1}{2\Omega} \text{Tr} \frac{1}{\Gamma_k^{(2)}[A_0] + R_k(\Gamma_k^{(2)}[A_0])} k \partial_k R_k(\Gamma_k^{(2)}[A_0])$$

- determination of **fluctuation** propagator in Landau-DeWitt gauge

$$\Gamma_k^{(2)}[A] = \Gamma_{k,\text{Landau}}^{(2)}(\mathbf{p}^2 \rightarrow -D^2) + O(F)$$

- Polyakov loop $L[\langle A_0 \rangle] \geq \langle L[A_0] \rangle$

$$L[\langle A_0 \rangle] \quad \text{from} \quad \left. \frac{\partial V_{\text{eff}}[A_0]}{\partial A_0} \right|_{A_0 = \langle A_0 \rangle} = 0$$

- full effective action

$$\Gamma_0[A] = \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2)}[A] + \int O(\partial_t \Gamma_k^{(2)}) + \text{c.t.}$$

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$$\Gamma_0[A] = \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2)}[A] + \int O(\partial_t \Gamma_k^{(2)}) + \text{c.t.}$$

- full effective potential in the deep infrared, $\Gamma_{0,A/C}^{(2)} \sim (-D^2)^{1+\kappa_{A/C}}$

$$V^{\text{IR}}[A_0] \simeq \left\{ \frac{d-1}{2} (1 + \kappa_A) + \frac{1}{2} - (1 + \kappa_C) \right\} \frac{1}{\Omega} \text{Tr} \ln (-D^2[A_0])$$

- full effective action

$$\Gamma_0[A] = \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2)}[A] + \int O(\partial_t \Gamma_k^{(2)}) + \text{c.t.}$$

- full effective potential in the deep infrared

$$V^{\text{IR}}[A_0] \simeq \left\{ 1 + \frac{(d-1)\kappa_A - 2\kappa_C}{d-2} \right\} V^{\text{UV}}[A_0]$$

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- confinement criterion with sum rule $\kappa_A = -2\kappa_C - \frac{4-d}{2}$

$$\kappa_C > \frac{d-3}{4}$$

no confinement with **background** field propagators

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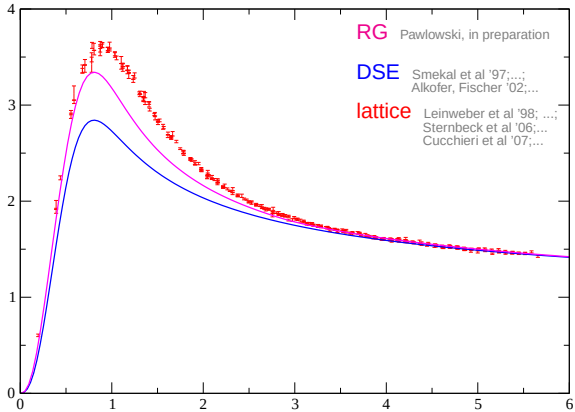
$$V^{\text{IR}}[A_0] \simeq \left\{ 1 + \frac{(d-1)\kappa_A - 2\kappa_C}{d-2} \right\} V^{\text{UV}}[A_0]$$

- bounds on κ_C in $d = 4$ Eichhorn, Gies, Pawłowski, poster

$$\frac{1}{4} < \kappa_C < \frac{41}{42}$$

• determination of $L(\langle A_0 \rangle)$

$$\frac{p^2}{\Gamma_A^{(2)}(p)}$$



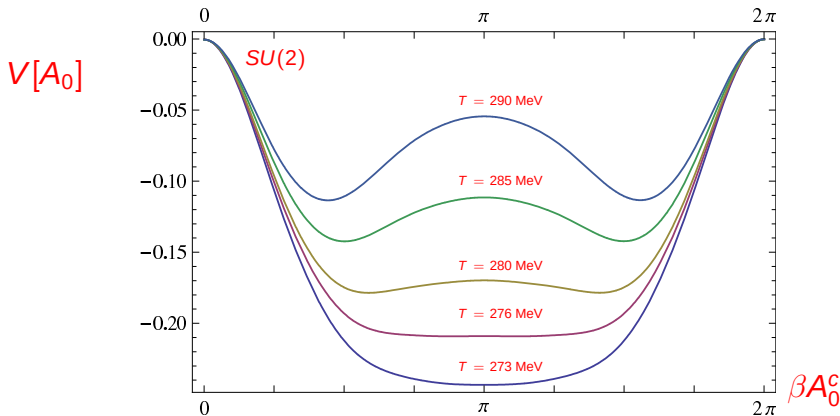
Polyakov loop potential: $SU(2)$

Braun, Gies, Pawłowski, arXiv:0708.2413 [hep-th]

$$T_c \simeq 276 \pm 10 \text{ MeV}$$

$$T_c/\sqrt{\sigma} = 0.627 \pm 0.023$$

$$\text{lattice: } T_c/\sqrt{\sigma} = .709$$

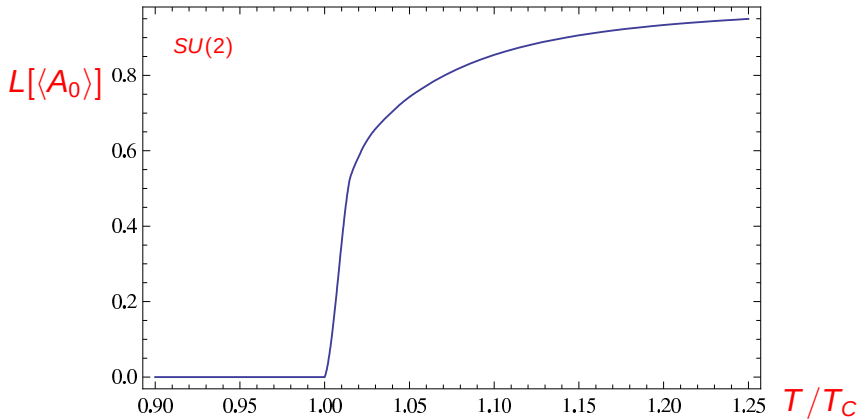


$$L[A_0] = \cos \frac{1}{2} \beta A_0^c$$

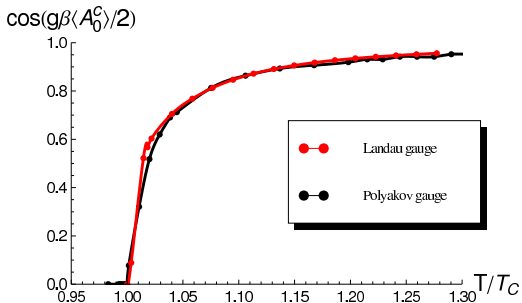
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flow in Polyakov gauge: $A_0 = A_0^c(\vec{x})\sigma_3$



• —: Polyakov gauge: crit. exp. $\nu = 0.65$

$\nu_{\text{Ising}} = 0.63$

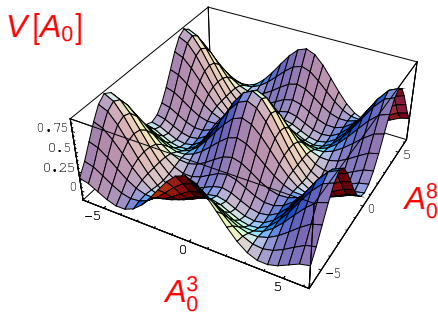
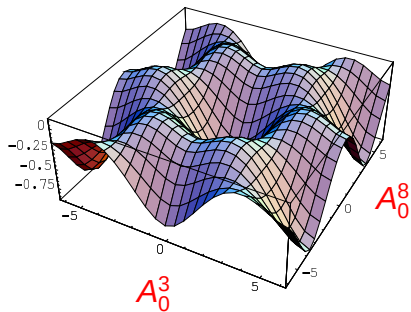
• —: Landau gauge propagators

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$$T_c \simeq 284 \pm 10 \text{ MeV}$$

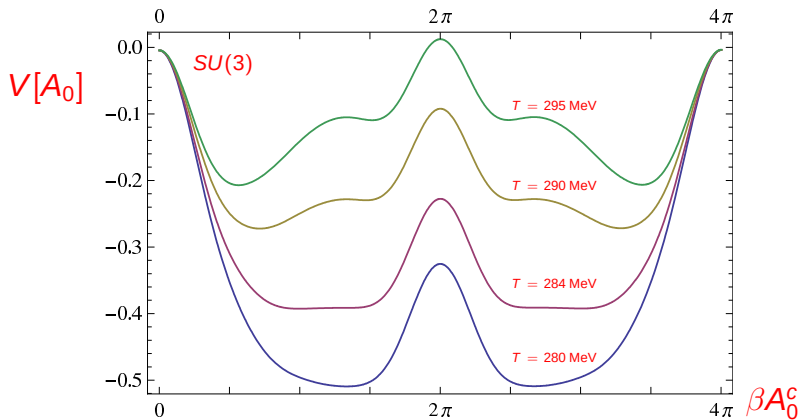
$$T_c / \sqrt{\sigma} = 0.646 \pm 0.023 \quad \text{lattice: } T_c / \sqrt{\sigma} = .646$$



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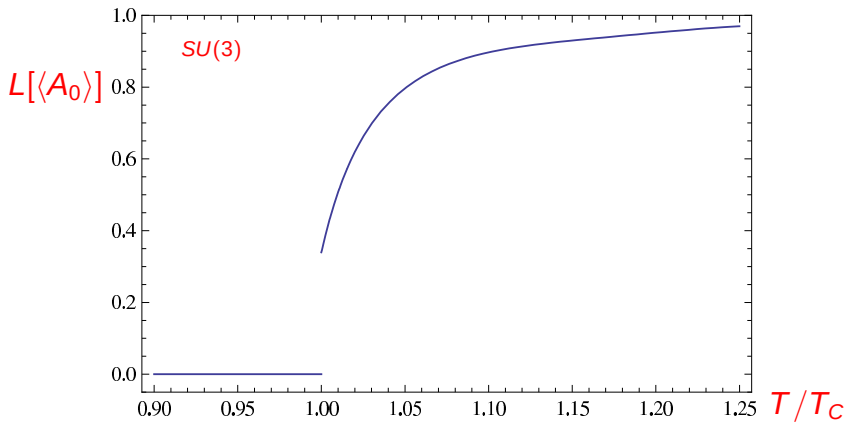


$$L[\beta A_0^c = \frac{4}{3}\pi] = 0$$

$$T_c \simeq 284 \pm 10 \text{ MeV}$$

$$T_c/\sqrt{\sigma} = 0.646 \pm 0.023$$

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- confinement-deconfinement phase transition from KO/GZ
- dynamical chiral symmetry breaking
- 'QCD phase diagram' from models

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- flow of Wilson loops & Polyakov loops: area law
- QCD at finite temperature & density

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