Conformal Extension of the Higgs Sector and the Little Hierarchy Problem

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Outline

- Little hierarchy problem
- Higgs with a large anomalous dimension
- A phenomenological model
- EW precision test
- Conclusions

Little hierarchy problem

1. Non-renormalizable operators in the EW theory

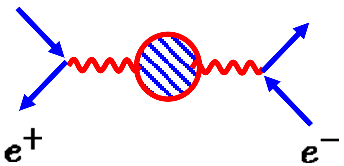
- EW ($SU(2)_W \times U(1)_Y$) gauge invariant dimension 6 operators :

B.Grinstein, M.B.Wise, P.L.B265 (1991)

$$\delta\mathcal{L}_{\rm SM} = \frac{c_{WB}}{M^2} H^\dagger \tau^a H W^a_{\mu\nu} B_{\mu\nu} + \frac{c_H}{M^2} \left| H^\dagger D_\mu H \right|^2$$

Oblique corrections after EWSB by the Higgs VEV ($\langle H \rangle = (v/\sqrt{2}, 0)$)

$$\delta\mathcal{L}_{\rm SM} \sim c_{WB} \frac{v^2}{4M^2} W^3_{\mu\nu} B_{\mu\nu} - c_H \frac{v^4}{16M^2} \left(g W^3_\mu - g' B_\mu \right)^2$$



- S,T parameters and the precision mesurement:

$$\alpha S = 4 \sin 2\theta_W c_{WB} (v^2/M^2) \qquad S = 0.04 \pm 0.10$$

$$\alpha T = -c_H (v^2/M^2) \qquad T = 0.12 \pm 0.10 \text{ (for } m_H = 115 \text{ GeV)}$$

$\Rightarrow$ Deviations from the renormalized trajectory are very small.

Generic oblique corrections ($\Delta T, \Delta S$) $\Rightarrow$ $M > 5 - 10$TeV

Little hierarchy problem

2. Fine-tuning in the Higgs scalar mass

The Higgs scalar mass is relevant and leads to fine-tuning.

$$\mathcal{L} \sim m_H^2 |H|^2 \quad : \text{relevant}$$

- Quadratic divergence in the Higgs mass parameter m_H^2

$$\delta m_H^2 \sim \frac{1}{(4\pi)^2}(-3y_t^2 + 3\lambda_4 + \cdots)\Lambda^2 \qquad (\Lambda : \text{cutoff scale})$$

Note: Top quark contribution is dominant.

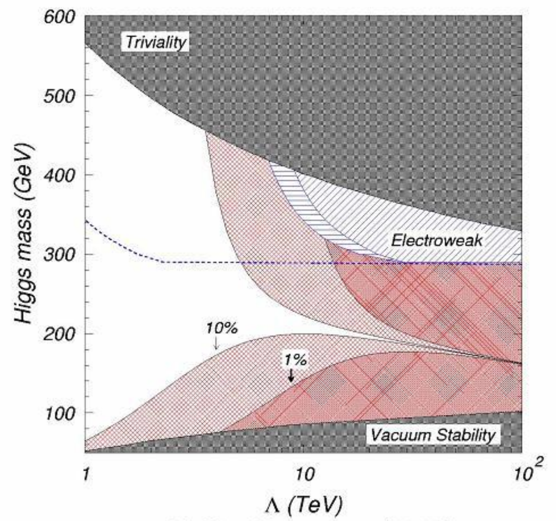
- EW precision test at LEP $\Rightarrow$ light Higgs

$$115\text{GeV} < m_{h^0} < 184\text{GeV}$$

If there is no reason of fine-tuning, δm_H^2 should be on the same order of m_h^2 at most.

$$\Lambda \leq 1\text{TeV} \quad \Rightarrow \quad \text{LHC}$$

Tension between M and Λ : "Little hierarchy problem"



Kolda, Murayama (2000)

Little hierarchy problem

3. Theoretical possibilities above TeV scale

- Cancellation by global symmetries?

  e.g. Supersymmetry $\cdots$ most posturated direction

  – "Supersymmetric little hierarchy problem"

  $$m_{h^0} \leq M_Z + \text{(SUSY breaking loop effects)}$$

  $\Rightarrow$ Fine-tuning of O(1)% is still required.

- Higgs as a fermion composite like superconductivity?

  e.g. (Walking) Technicolor

  Condensation of the Techni-fermions : $\langle \bar{\psi}\psi \rangle \sim \Lambda_{\text{EW}}^3 \Rightarrow$ EWSB

  – S-parameter :

  $\Delta S \sim N_f/\pi \cdots$ too large!

  – Top quark mass :

  $$\mathcal{L}_{\text{eff}} \sim \frac{1}{\Lambda_{\text{TC}}^2}\bar{\psi}\psi\bar{t}t \quad \Rightarrow \quad m_t \sim \left(\frac{\Lambda_{\text{EW}}}{\Lambda_{\text{TC}}}\right)^2 \Lambda_{\text{EW}} \text{ or } \left(\frac{\Lambda_{\text{EW}}}{\Lambda_{\text{TC}}}\right) \Lambda_{\text{EW}}$$

  $\Rightarrow \Lambda_{\text{TC}} \leq 10\text{TeV} \cdots$ too low for FCNC!

Higgs with a large anomalous dimension

1. Suppression of the quadratic divergence

Basic idea

- Take the Higgs scalar to be point-like at least near the TeV scale.
- Suppose that the Higgs aquires a large positive anomalous dimension above some TeV scale M.

$$\delta m_H^2 \sim *\Lambda^2 \quad \rightarrow \quad *\left(\frac{\Lambda}{M}\right)^{-2\gamma_H}\Lambda^2 = *\left(\frac{\Lambda}{M}\right)^{\epsilon} M^2 \qquad \epsilon = 2(1-\gamma_H) \ll 1$$

$$\sim \quad *\{1 + \epsilon \ln(\Lambda/M)\} M^2$$

Note :

- The power of divergence is reduced to ϵ for any corrections.
- This is not by cancellation.

How can we realize such a large anomalous dimension?

$\Rightarrow$ Conformal Field Theories (CFTs) $\quad M$: Mass scale of the "CFT" $\sim$ TeV

Higgs with a large anomalous dimension

2. Banks-Zaks fixed point

Banks, Zaks (1982)

$SU(N_c)$ gauge theory with N_f massless flavors

- 2 loop beta function for $\alpha_g = g^2/(4\pi)^2$:

$$\mu\frac{d\alpha_g}{d\mu} \simeq -2\alpha_g^2\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right) - 2\alpha_g^3\left[\frac{34}{3}N_c^2 - \left(2C_2(R) + \frac{10}{3}N_c\right)N_f\right] + \cdots$$

$\Rightarrow$ IR attractive fixed point

exists for $(34/13)N_c < N_f < (11/2)N_c$ at 2 loop.

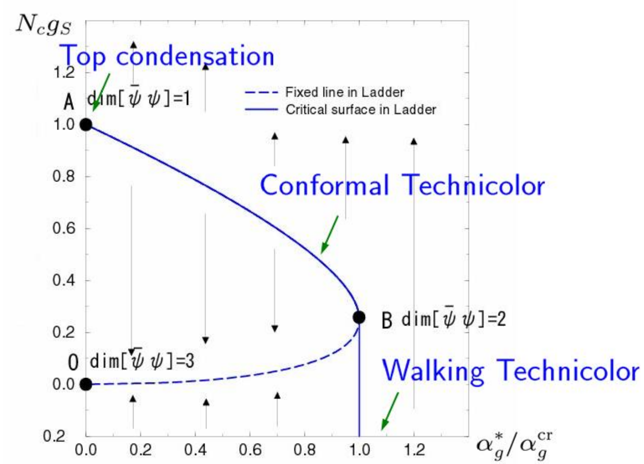
Note : the smaller edge is Not reliable, since the fixed point coupling is non-perturbative.

- The ladder Dyson-Schwinger eqn. :

The chiral symmetry is broken for

$N_f \geq 4N_c$ (for a large N_c, N_f),

$\Rightarrow$ No IR fixed point



Yamawaki,et.al. Appelquist et.al.

Higgs with a large anomalous dimension

3. ERG with 4-fermi couplings

H.T. Tshuchiya, 2007

$SU(N_f)_L \times SU(N_f)_R$ invariant 4-fermi interactions: $(i, j = 1, \cdots, N_f)$

$$\mathcal{L}_{4\text{fermi}} = -\frac{2G_S}{\mu^2}\bar{\psi}_{Li}\psi^j_R\bar{\psi}_{Rj}\psi^i_L - \frac{G_V}{\mu^2}[\bar{\psi}_{Li}\gamma_\mu\psi^j_L\bar{\psi}_{Lj}\gamma^\mu\psi^i_L + (L \leftrightarrow R)]$$
$$-\frac{2G_{V_1}}{\mu^2}\bar{\psi}_{Li}\gamma_\mu\psi^i_L\bar{\psi}_{Rj}\gamma_\mu\psi^j_R - \frac{G_{V_2}}{\mu^2}[(\bar{\psi}_{Li}\gamma_\mu\psi^i_L)^2 + (L \leftrightarrow R)] + \cdots$$

ERG in the large N_c, N_f leading: $(g_S = G_S/4\pi^2, g_V = G_V/4\pi^2)$

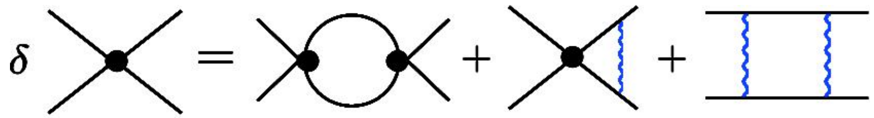
$$\mu\frac{dg_S}{d\mu} = 2g_S - 2N_c g_S^2 + 2N_f g_S g_V - 6N_C\alpha_g g_S - \frac{9}{2}N_c\alpha_g^2$$

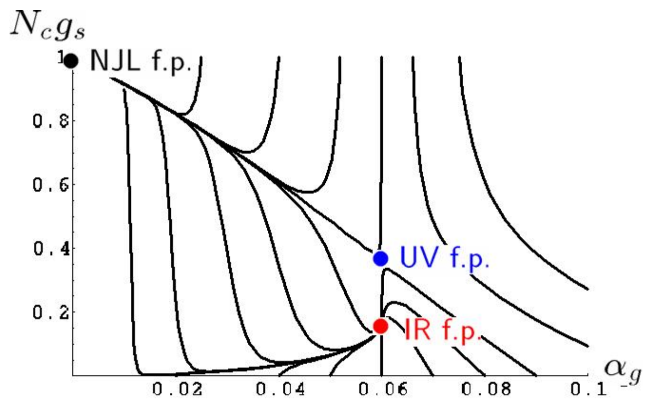
$$\mu\frac{dg_V}{d\mu} = 2g_V + \frac{N_f}{4}g_S^2 + (N_c + N_f)g_V^2 - \frac{3}{4}N_c\alpha_g^2$$ (Landau gauge)

Ladder approximation : Aoki, H.T. et.al. (1997–2001)

$$\mu\frac{dg_S}{d\mu} = 2g_S - 2N_c\left(g_S + \frac{3C_2(N_c)}{N_c}\alpha_g\right)^2$$

$$\mu\frac{d\alpha_g}{d\mu} = -2b_0\alpha_g^3 - 2b_1\alpha_g^3$$





RG flows in the case of $N_c = 3, N_f = 12$.

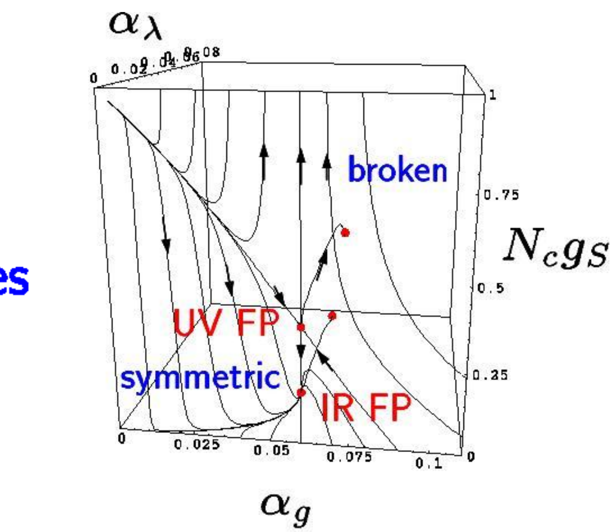
Higgs with a large anomalous dimension

4. IR fixed point with a non-trivial Yukawa coupling

Now let us add Yukawa couplings with a gauge singlet scalar ϕ:

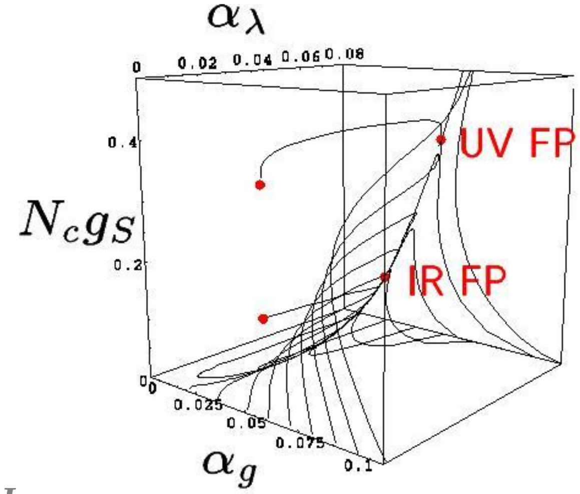
$$\mathcal{L}_{\mathbf{yukawa}} = -\sum_i \lambda_i \phi \bar{\psi}_i \psi^i$$

- The negative anomalous dimension $\gamma_{\bar{\psi}\psi}$ makes the Yukawa coupling relevant.
- A new IR attractive fixed point with non-vanishing Yukawa couplings $\lambda_i = \lambda^*$ appears.



The NPRG eqns. : $(\alpha_{\lambda i} = \lambda_i^2/(4\pi)^2)$

$$\begin{aligned} d\alpha_g/d\ln\mu &= -2b_0\alpha_g^2 - 2b_1\alpha_g^3 + 2N_f\alpha_g^2\alpha_\lambda \\ d\alpha_{\lambda i}/d\ln\mu &= 2\alpha_{\lambda i}(\gamma_\phi + \gamma_{\bar{\psi}\psi}) \\ \gamma_\phi &= 2N_c\sum_j \alpha_{\lambda j} + 3\alpha_{\lambda i} \\ \gamma_{\bar{\psi}\psi} &= -6C_2(R)\alpha_g - 2N_c g_S \end{aligned}$$

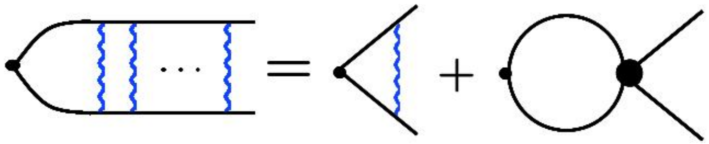


Higgs with a large anomalous dimension

5. Anomalous dimensions at the IR fixed points

Effective 4-fermi couplings are significant for the anomalous dimensions.

$\gamma_{\bar{\psi}\psi} = \Sigma$



- The B-Z fixed point :

$$\gamma^*_{\bar{\psi}\psi} = -6C_2(N_c)\alpha^*_g - 2N_c g^*_S = -1 + \sqrt{\frac{1-\alpha^*_g}{\alpha^{\rm cr}_g}} \geq -1$$

- The IR fixed point with a non-trivial Yukawa coupling :

  At the IR fixed point

$$\gamma^*_\phi \sim 2N_c N_f \alpha^*_\lambda = -\gamma^*_{\bar{\psi}\psi} \quad \Rightarrow \quad 0 < \gamma^*_\phi < 1$$

An explicit case: $N_c = 3, N_f = 12$ seems to be interesting.

$$\alpha_g * = 0.06 \text{ (2 loop)}, \quad \alpha^{\rm cr}_g = 1/16 = 0.0625 \text{ (ladder)}$$

$$\gamma^*_\phi = -\gamma^*_{\bar{\psi}\psi} = 0.8 \;\Rightarrow\; \epsilon = 0.4, \quad \lambda^* = 2\pi/5 \sim 1.3$$

Higgs with a large anomalous dimension

6. Effects of the scalar anomalous dimension

- Scalar mass

ERG eqn for the dimensionless mass parameter $\tilde{m}_\phi^2 = m_\phi^2/\mu^2$:

$$\mu \frac{d\tilde{m}_\phi^2}{d\mu} = -\epsilon(\tilde{m}_\phi^2 - \tilde{m}_\phi^{*2}), \qquad \tilde{m}_\phi^{*2} = 4N_c N_f \alpha_\lambda^*/\epsilon = 4 \quad \text{for } N_c = 3, N_f = 12$$

Scalar mass at a low energy scale M:

$$m_\phi^2(M) = \left[\tilde{m}_\phi^{*2} + \left(\frac{\Lambda}{M}\right)^\epsilon (\tilde{m}_\phi^2(\Lambda) - \tilde{m}_\phi^{*2})\right] M^2 \sim O(1-10) \times M^2$$

$$\text{Note}: \ (\Lambda/M)^\epsilon \quad \begin{array}{l} \sim 6.31 \quad \text{for} \quad (\Lambda/M)^2 = 10^4 \\ \sim 2.51 \qquad\qquad\qquad\quad\ = 10^2 \end{array}$$

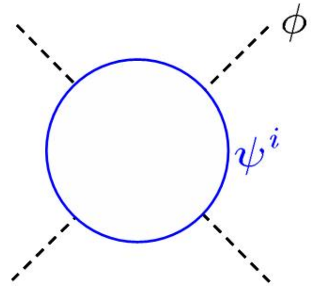
Although the scalar mass term is slightly relevant, the fine-tuning is remarkably ameliorated.

Higgs with a large anomalous dimension

- The quartic coupling $\lambda_{4\phi}$ $\quad (\tilde{\lambda}_{4\phi} = \lambda_{4\phi}/4\pi^2)$

  Note : The anomalous dimension makes scalar couplings highly irrelevant.

$$\mu \frac{d\tilde{\lambda}_{4\phi}}{d\ln\mu} = 4\gamma_\phi \tilde{\lambda}_{4\phi} - 8N_c N_f \alpha_\lambda^{*2}$$



$$\Rightarrow \qquad \lambda_{4\phi} \longrightarrow \lambda^*_{4\phi} = \lambda^{*2} \quad \gg 1 \text{ in general}$$

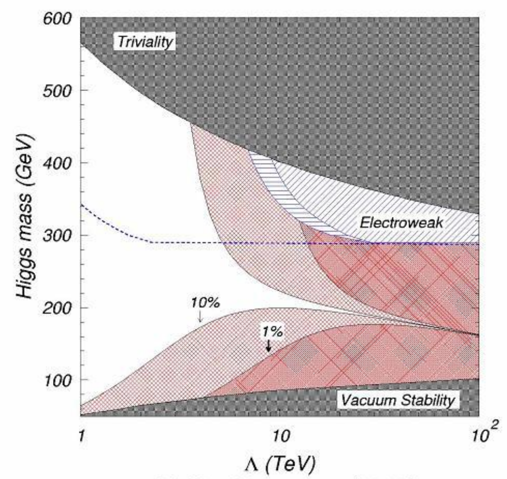
If we regards ϕ as the EW Higgs, then the Higgs mass must be near it's triviality bound. (Ca. 500GeV for $M \sim 2$.)

- Suppression for top Yukawa coupling!

$$\mu \frac{dy_t}{d\mu} \sim \gamma^*_\phi y_t + \cdots, \quad \Rightarrow \quad y_t(M) \sim \left(\frac{M}{\Lambda}\right)^{\gamma^*_\phi} y_t(\Lambda)$$

The origin of top quark mass should be different from other masses.

(Why is top quark as heavy as the EW scale?)



Kolda, Murayama (2000)

A phenomenological model

1. Basic elements of the model

- Dynamical breaking of the conformal symmetry

  Gauge symmetry : $G_{\rm DSB} \times SU(3)_{\rm CFT} \times SU(3)'_C \times SU(2)_W \times U(1)_Y$

  - $G_{\rm DSB}$ interaction becomes strong at scale $M \sim$ a few TeV.
  - The conformal gauge sector decouples through the mass generation.
  - Spontaneous symmetry breaking of $SU(3)_{\rm CFT} \times SU(3)'_C \to SU(3)_C$.
  - Strong dynamics does not break the EW symmetry.

- Generation of the top Yukawa coupling
  - Top Yukawa coupling y_t is suppressed, so we neglect it.
  - A large top Yukawa coupling is induced through mass mixing with the extra fermions of the CFT.

- Consistency with EW precision test
  - The oblique corrections ΔT and ΔS by the extra fermions
  - Z-boson decay width

A phenomenological model

2. An explicit model

- Extra vectorlike fermions ($N_f = 12$)

	$SU(3)_{\rm DSB}$	$SU(3)_{\rm CFT}$	$SU(3)'_C$	$SU(2)_W$	$U(1)_Y$
ψ^A	**3**	**3**	**1**	**1**	any
η^a	**3**	**1**	**3**	**1**	any
Φ^{iA}	**1**	**3**	**1**	**2**	1/6
ϕ^{iA}	**1**	**3**	**1**	**1**	2/3

$(i = 1, 2, 3)$

When the DSB interaction is weak, the model shows the CFT with a large anomalous dimension, $\gamma_H^* = 0.8$.

- Dynamical symmetry breaking

Let us assume that the SSB of $SU(3)_{\rm CFT} \times SU(3)'_C \to SU(3)_C$ is induced by condensation of the $G_{\rm DSB}$ charged fermions ψ, η:

$$\left.\begin{array}{lcl} \frac{1}{\Lambda^2}\langle\bar{\psi}_A \cdot \eta^a\rangle & \sim & \omega\delta^a_A \\ \frac{1}{\Lambda^2}\langle\bar{\eta}_a \cdot \psi^A\rangle & \sim & \bar{\omega}\delta^A_a \end{array}\right\} \rightarrow \text{ SSB } \quad \frac{1}{\Lambda^2}\langle\bar{\eta}_a \cdot \eta^a\rangle \sim M$$

A phenomenological model

- The effective Lagrangian $(\lambda^* = 2.64)$

$$
\begin{aligned}
-\mathcal{L}_{\text{int}} &= \lambda^* \bar{\Phi}_{iA}\phi^{iA}\tilde{H} + \frac{c}{\Lambda^2}\bar{Q}_{3La}\Phi_R^{3A}(\bar{\psi}_A \cdot \eta^a) + \frac{\bar{c}}{\Lambda^2}\bar{\phi}_{3A}u_R^{3a}(\bar{\eta}_a \cdot \psi^A) \\
&\quad + \frac{c_\Phi}{\Lambda^2}\bar{\Phi}_{iA}\Phi^{iA}(\bar{\eta}_a \cdot \eta^a) + \frac{c_\phi}{\Lambda^2}\bar{\phi}_{iA}\phi^{iA}(\bar{\eta}_a \cdot \eta^a) + V(H) \\
&= \lambda_*(\bar{\Phi}_{iL}\phi_R^i + \bar{\Phi}_R\phi_L)\tilde{H} + V(H) \\
&\quad + M_\Phi\left(\bar{\Phi}_{3L} + \frac{\omega}{M_\Phi}\bar{Q}_{3L}\right)\Phi_R^3 + M_\phi\bar{\phi}_{3L}\left(\bar{\phi}_R^3 + \frac{\bar{\omega}}{M_\phi}u_R^3\right)
\end{aligned}
$$

- Mass mixing between Top quark and the extra fermions

$$
\begin{pmatrix} Q_3' \\ \Phi' \end{pmatrix}_L = \begin{pmatrix} \cos\theta_L & \sin\theta_L \\ \sin\theta_L & \cos\theta_L \end{pmatrix}\begin{pmatrix} Q_3 \\ \Phi \end{pmatrix}_L, \quad \tan\theta_L = \frac{\omega}{M_\Phi}
$$

$$
\begin{pmatrix} u_3' \\ \phi' \end{pmatrix}_R = \begin{pmatrix} \cos\theta_R & \sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{pmatrix}\begin{pmatrix} u_3 \\ \phi \end{pmatrix}_R, \quad \tan\theta_R = \frac{\bar{\omega}}{M_\phi}
$$

massless modes (Q'_{3L}, u'_{3R}) : top quarks

massive modes M'_Φ for (Φ'_L, Φ_R), M'_ϕ for (ϕ_L, ϕ'_R)

$\Rightarrow$ Effective top Yukawa coupling : $y_t^{\text{eff}} \sim \lambda_* \sin\theta_L \sin\theta_R$

EW precision test

1. The oblique corrections

- Heavy Higgs and ΔT

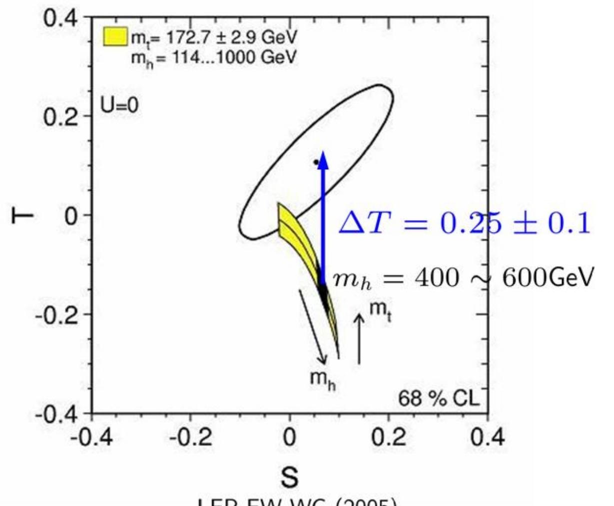
  Higgs mass bound by the EW precision test

  $m_{h^0} < 184\text{GeV}$ excluding a heavy Higgs?

  If a suitable amount of ΔT is present,
  then heavy Higgs is also allowed.

  e.g. for $m_{h^0} = 400 - 600\text{GeV}$

  $$\Delta T = 0.25 \pm 0.1, \quad \Delta S \sim 0$$

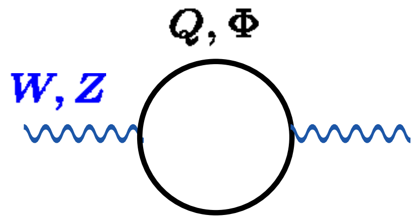


LEP EW WG (2005)

- Explicit evaluation by one-loop

$$\Delta T \sim \frac{3}{16\pi^2\alpha}\left(\ln\frac{M'^2}{m_t^2} + \lambda^*\right)\left(\frac{x}{M'}\right)^2(\lambda^* - 1)$$

$$\Delta S < 0.12\Delta T \quad (x = \lambda_* v \sin\theta\cos\theta, \quad \lambda^* = 2.63)$$



Note: The custodial symmetry is explicitly broken.

The heavy Higgs near the triviality bound is allowed, when

$$1.5\text{TeV} < M' < 2.5\text{TeV}.$$

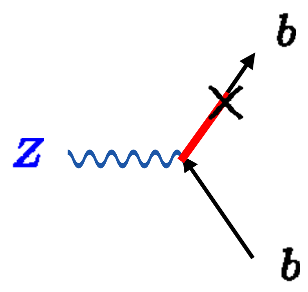
EW precision test

2. Z-boson decay width

- Experimental constraint

$$R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons}) \quad \Rightarrow \quad \delta g_b < 10^{-3}$$

- Mass mixing of bottom quarks through EWSB



$$-\mathcal{L}_{\text{Yukawa}} = y_t(\sin\theta_L \bar{\Phi}'_L + \cos\theta_L \bar{Q}'_{3L})t_R\tilde{H} + y_b(\sin\theta_L \bar{\Phi}'_L + \cos\theta_L \bar{Q}'_{3L})b_R H$$

$$\Rightarrow \quad \delta g_{bL} \simeq \delta g_{bR} \simeq (m_b/M)^2 \leq 10^{-5}$$

Mixing of the bottom quarks (b_L, b_R) with the heavy fermions is very tiny! Contribution for the coupling of $Z \to b\bar{b}$ is negligible.

Summary

- **We considered the scenario in which**
  - The Higgs sector is approximately conformal invariant above TeV (``Conformal Higgs model").
  - Fine-tuning can be ameliorated by a large anomalous dimension.
  - The Higgs mass is predicted to be heavy (about 500GeV).
- **Simple ERG analysis of the gauge-Yukawa theory shows**
  - Existence of an IR fixed point with the Yukawa coupling.
  - Large anomalous dimensions of the scalar fields.
- **We also presented a explicit model in which**
  - Top quark mass can be explained by mixing with the extra quarks in the CFT sector.
  - The model can be consistent with the EW precision test.
  - Extra heavy quarks exist near 2TeV.

Let's wait for what LHC will tell us!

- **Further issues**
  - Explicit dynamics of the DSB of the CFT.
  - Masses and mixings of other quarks/leptons than top.
  - 5 dim picture or the AdS/CFT correspondence.