Towards Bridging the Gap Between Quarks and Gluons and Baryonic Degrees of Freedom

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From Microscopic Degrees to Macroscopic DoF

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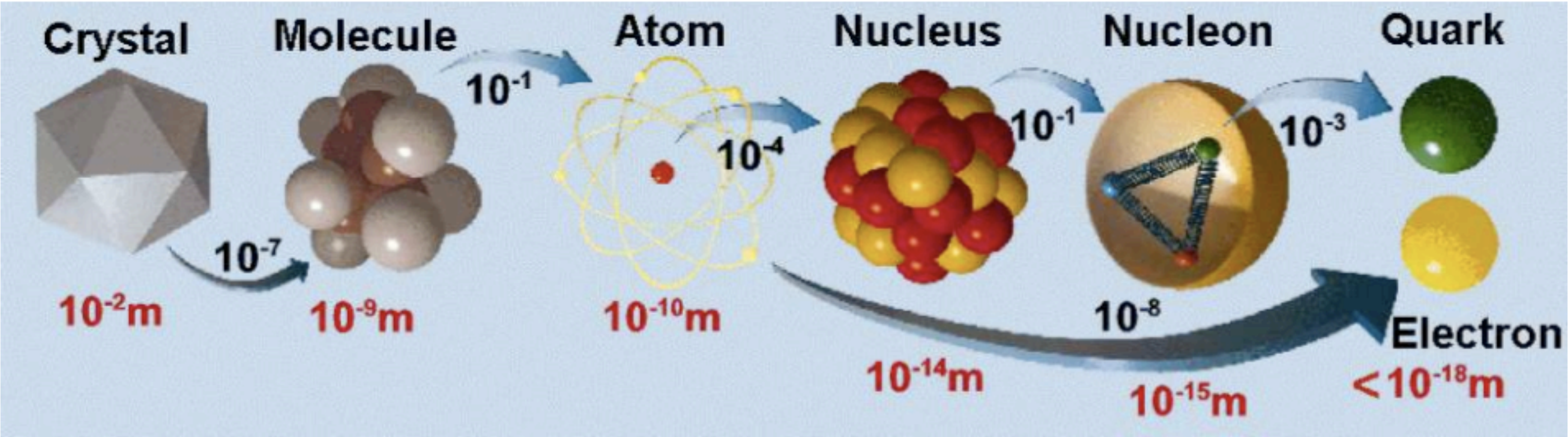

...

large length scales

small length scales

small momentum scales

Renormalization Group

large momentum scales

From Microscopic Degrees to Macroscopic DoF

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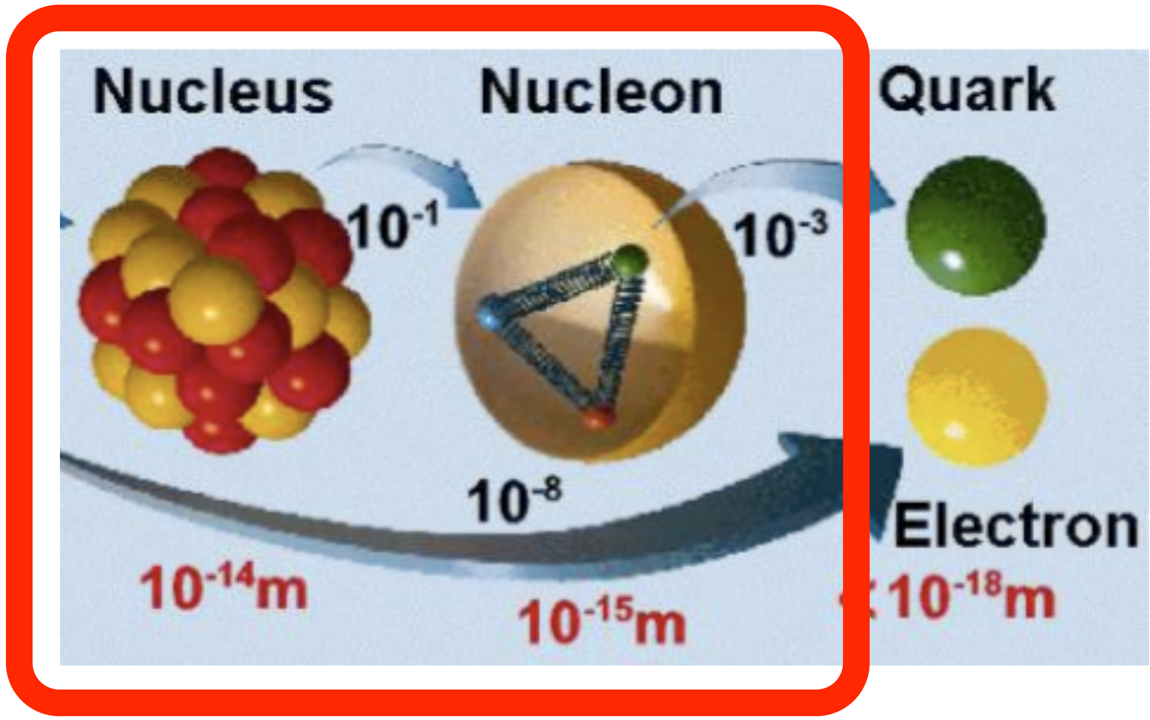

...

large length scales

small length scales

small momentum scales

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Density Functional Renormalization Group flows (JB, J. Polonyi, A. Schwenk)

From Microscopic Degrees to Macroscopic DoF

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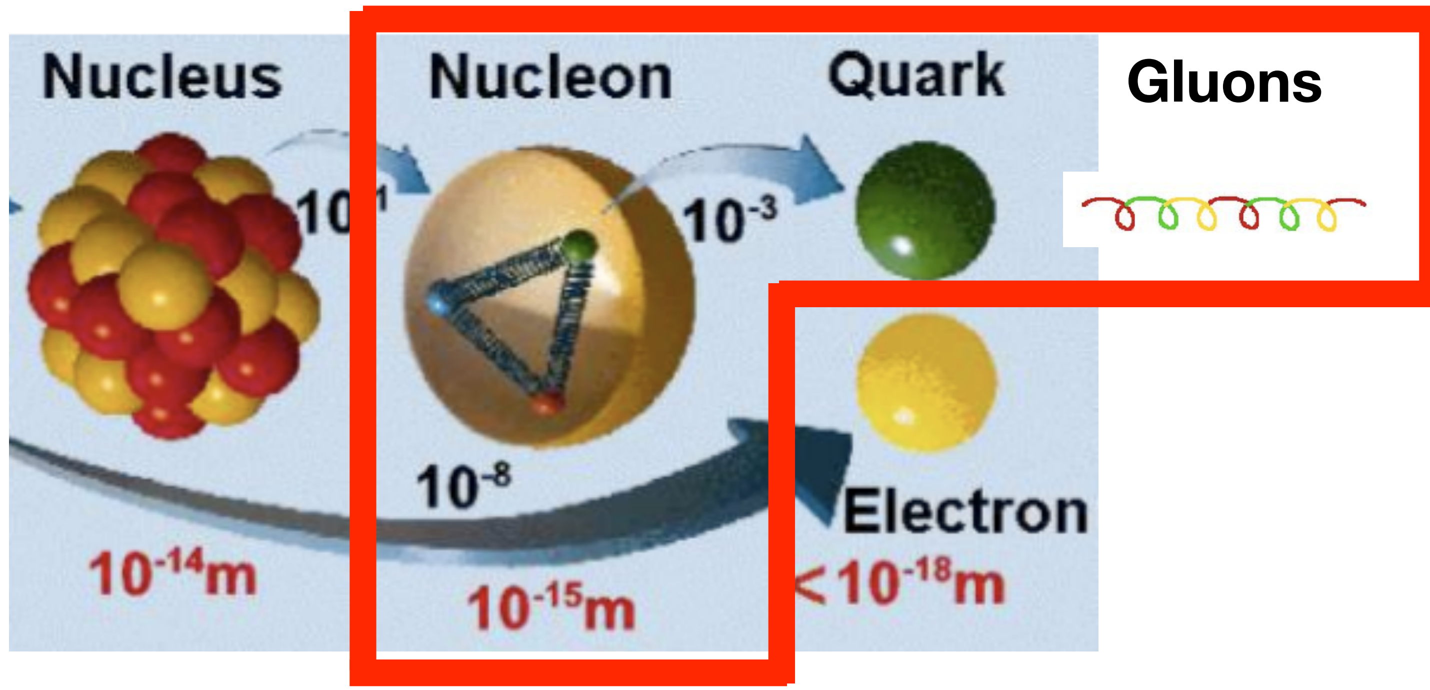

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large length scales

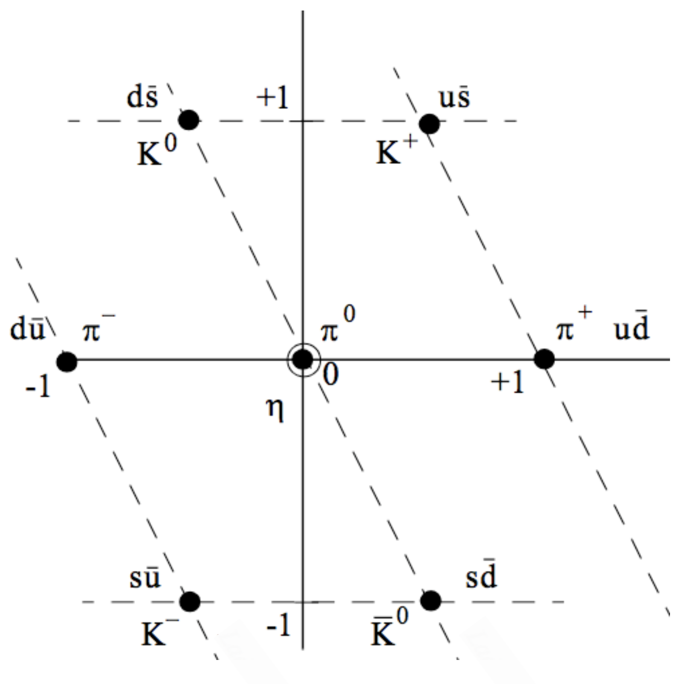
small length scales

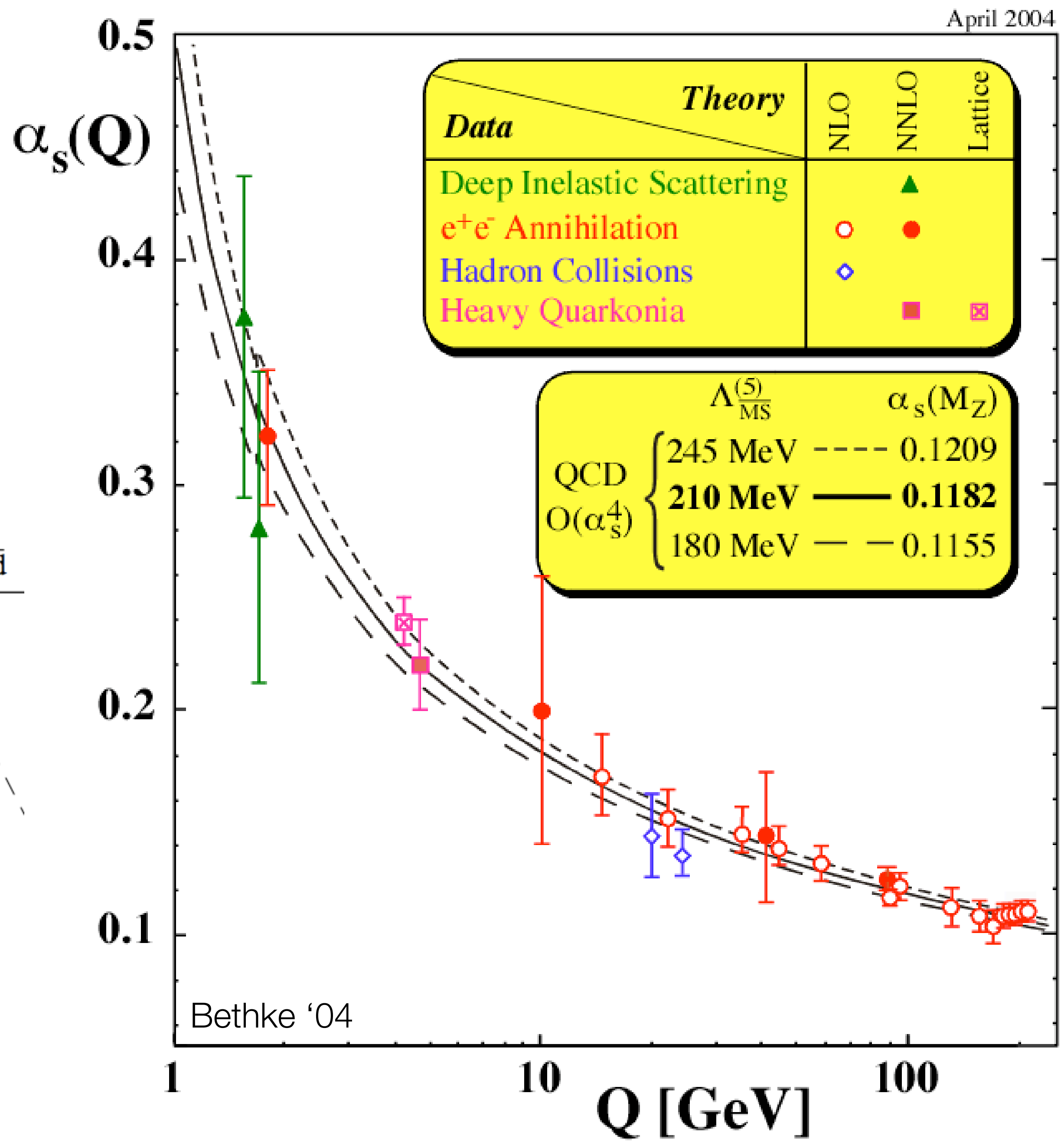
small momentum scales

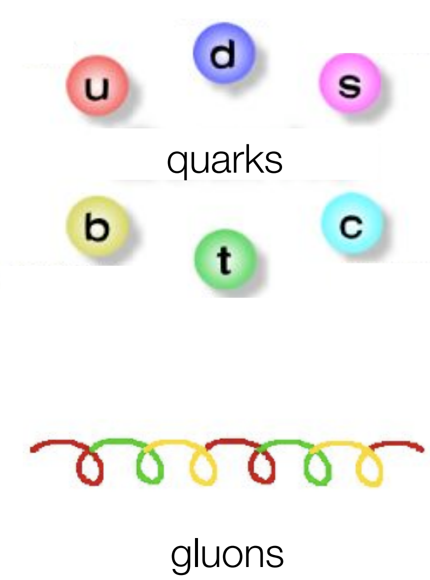
large momentum scales

Functional Renormalization Group flows: Today's talk ...
(JB, H. Gies, J. M. Pawlowski)

Challenges in QCD

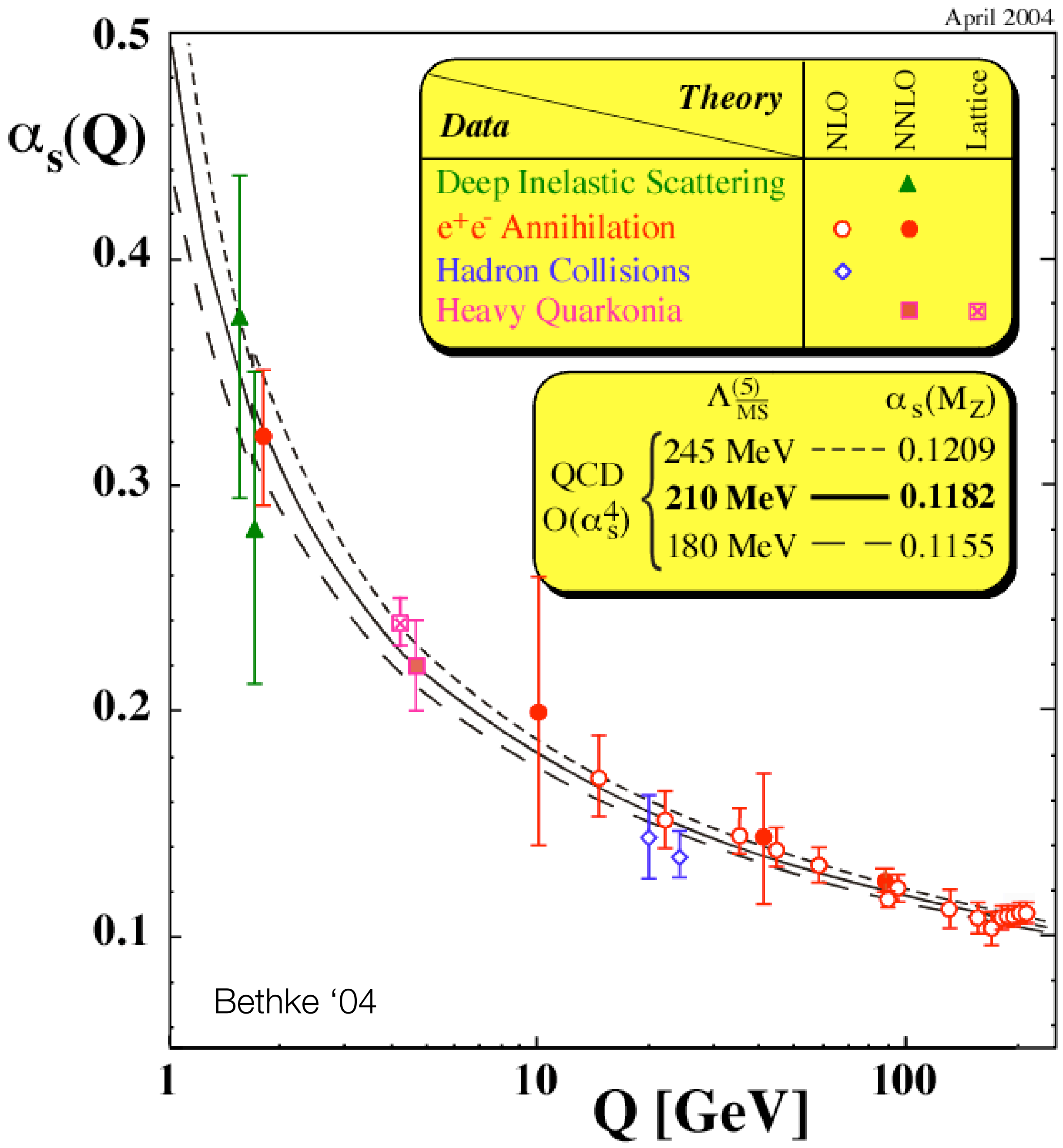






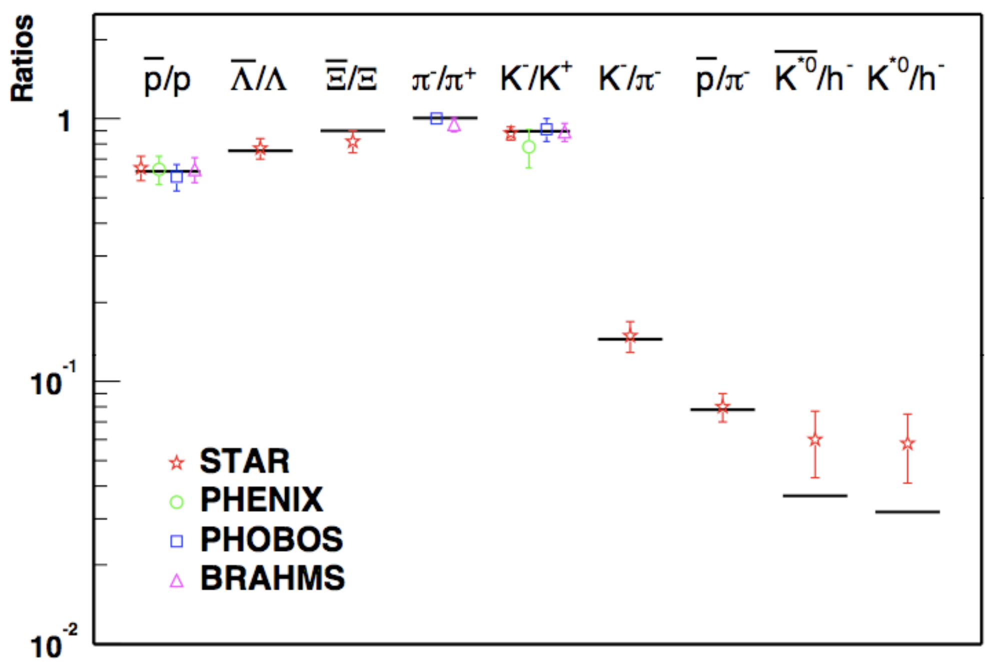
Challenges in QCD

- Asymptotic freedom at high momenta (Gross & Wilczek '73, Politzer '73)
- running coupling exhibits Landau pole at small momenta
  ⟶ pQCD fails
- Understanding of QCD in the mid-momentum regime is needed to study confinement & chiral symmetry breaking



Heavy-Ion Collision Experiments

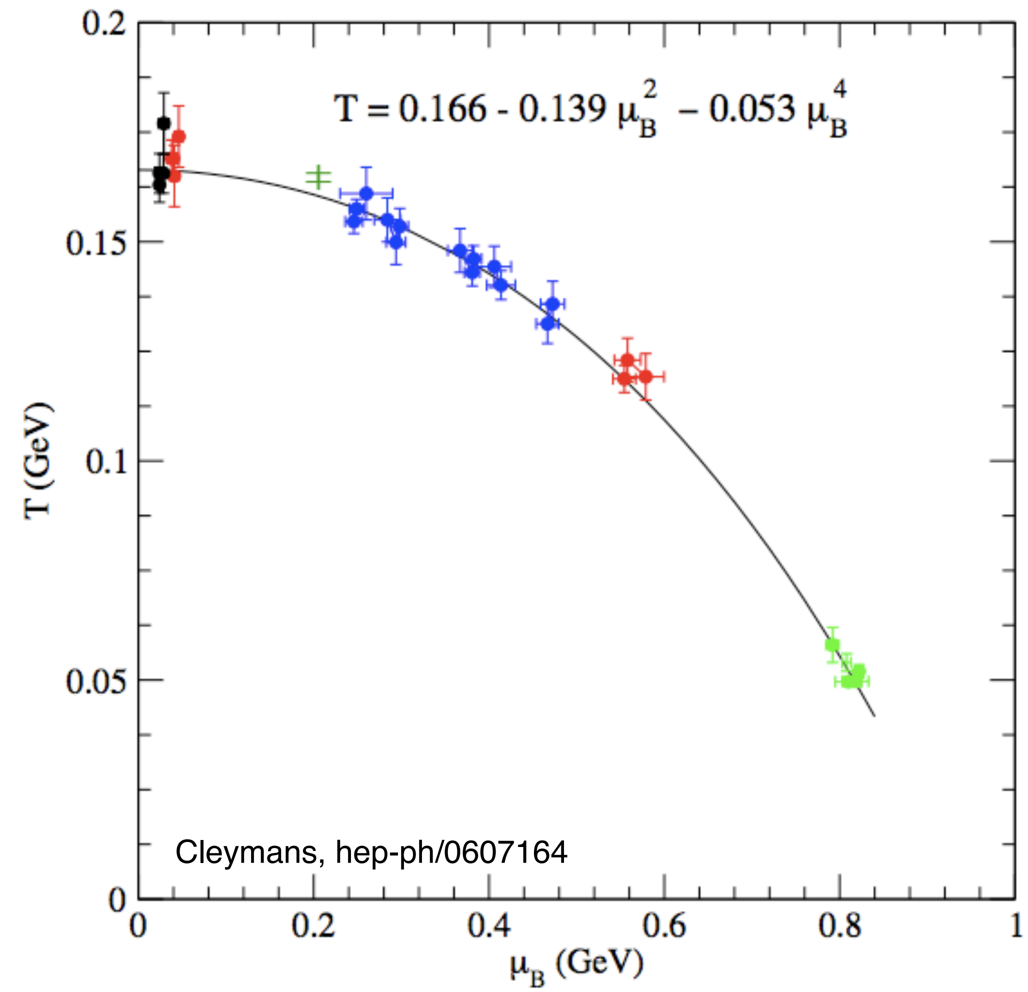
measured relative abundances



fitted very well with Boltzmann-distribution

$\Rightarrow$ (T, μ_B) for given $\sqrt{s}$

chemical freeze-out

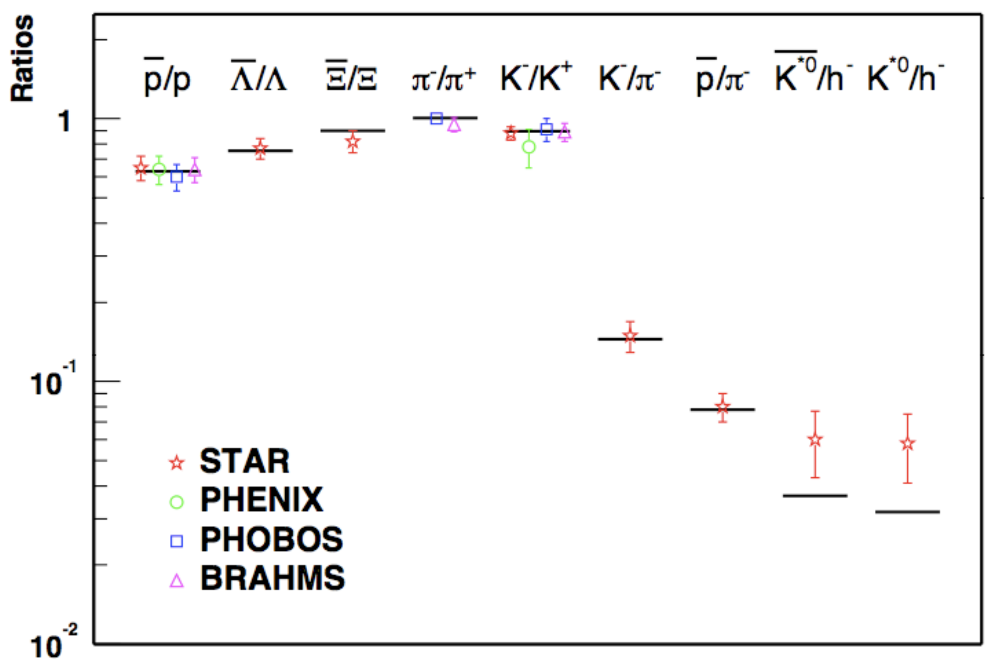


$$T_{\rm exp.} \leq T_\chi$$

P. Braun-Munzinger, J. Stachel, C. Wetterich ’03

Heavy-Ion Collision Experiments

measured relative abundances



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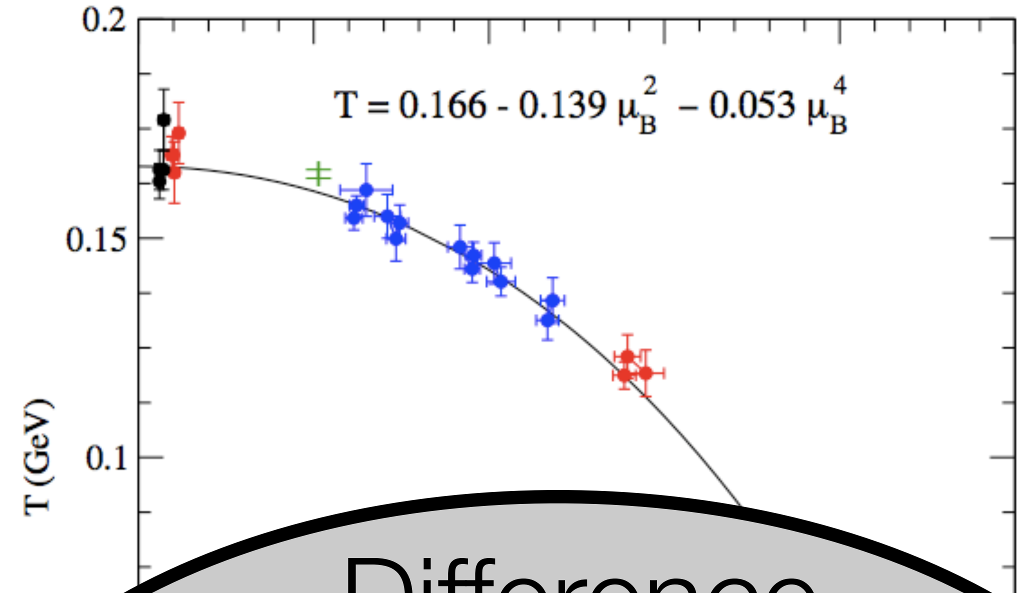


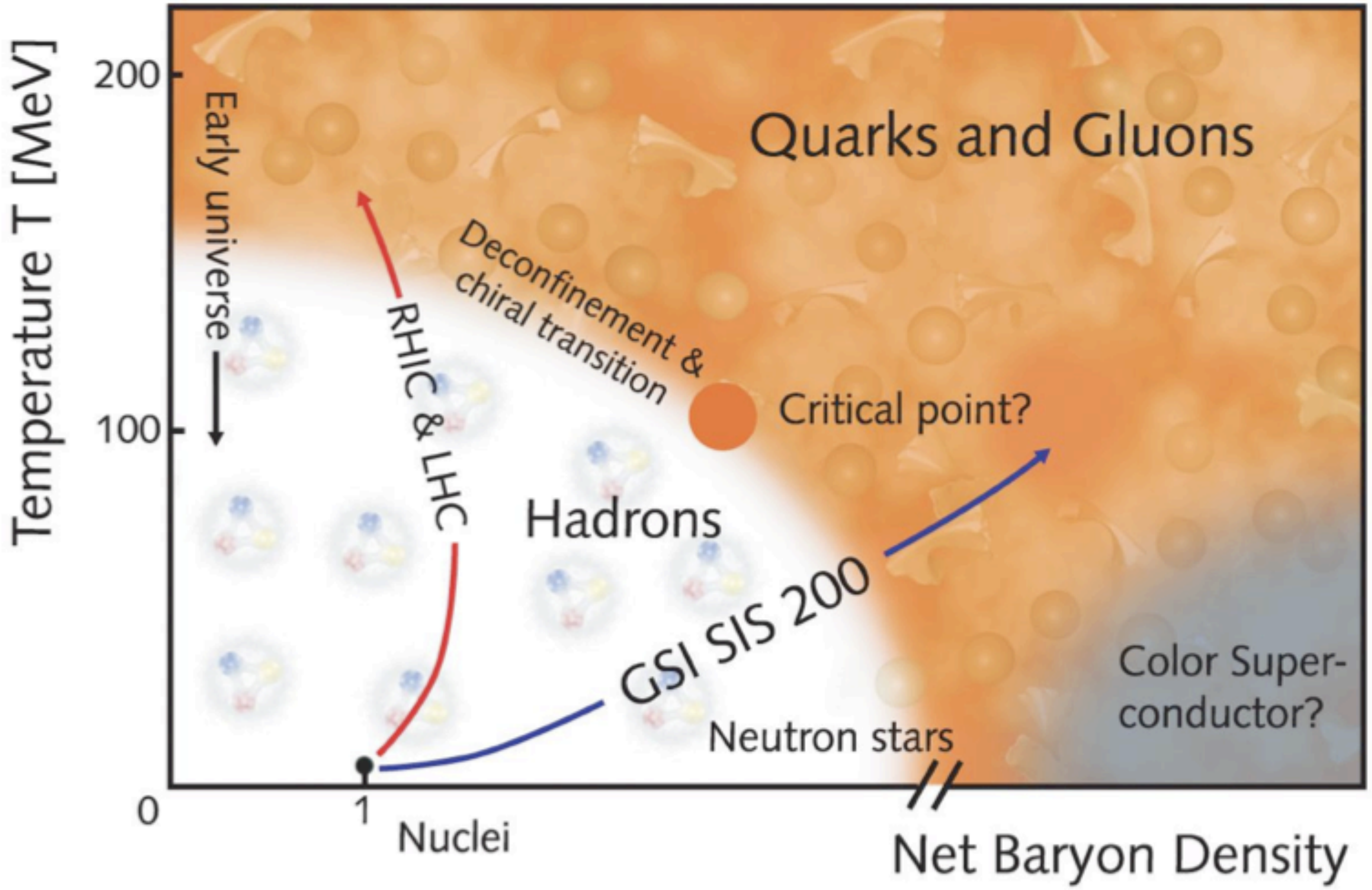
Difference

$$\Delta = T_\chi - T_{\mathrm{exp.}}$$

depends on the order of the phase transition

QCD phase diagram?

FAIR, www.gsi.de

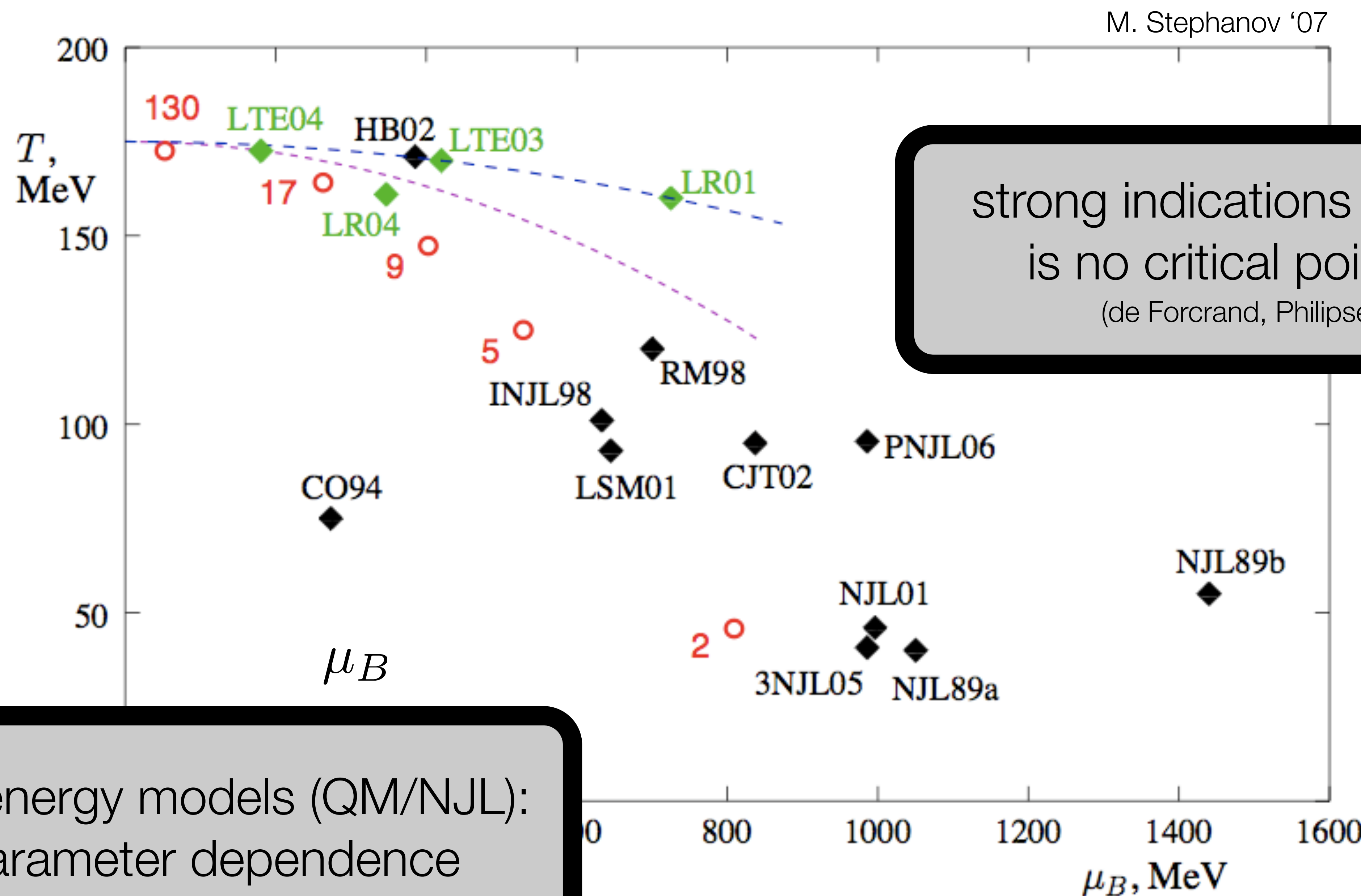


perturbation theory fails:

- not convergent even for very high temperatures: strongly interacting theory even at high T
- phase transitions: long-range fluctuations are important

QCD phase diagram? Many open questions!

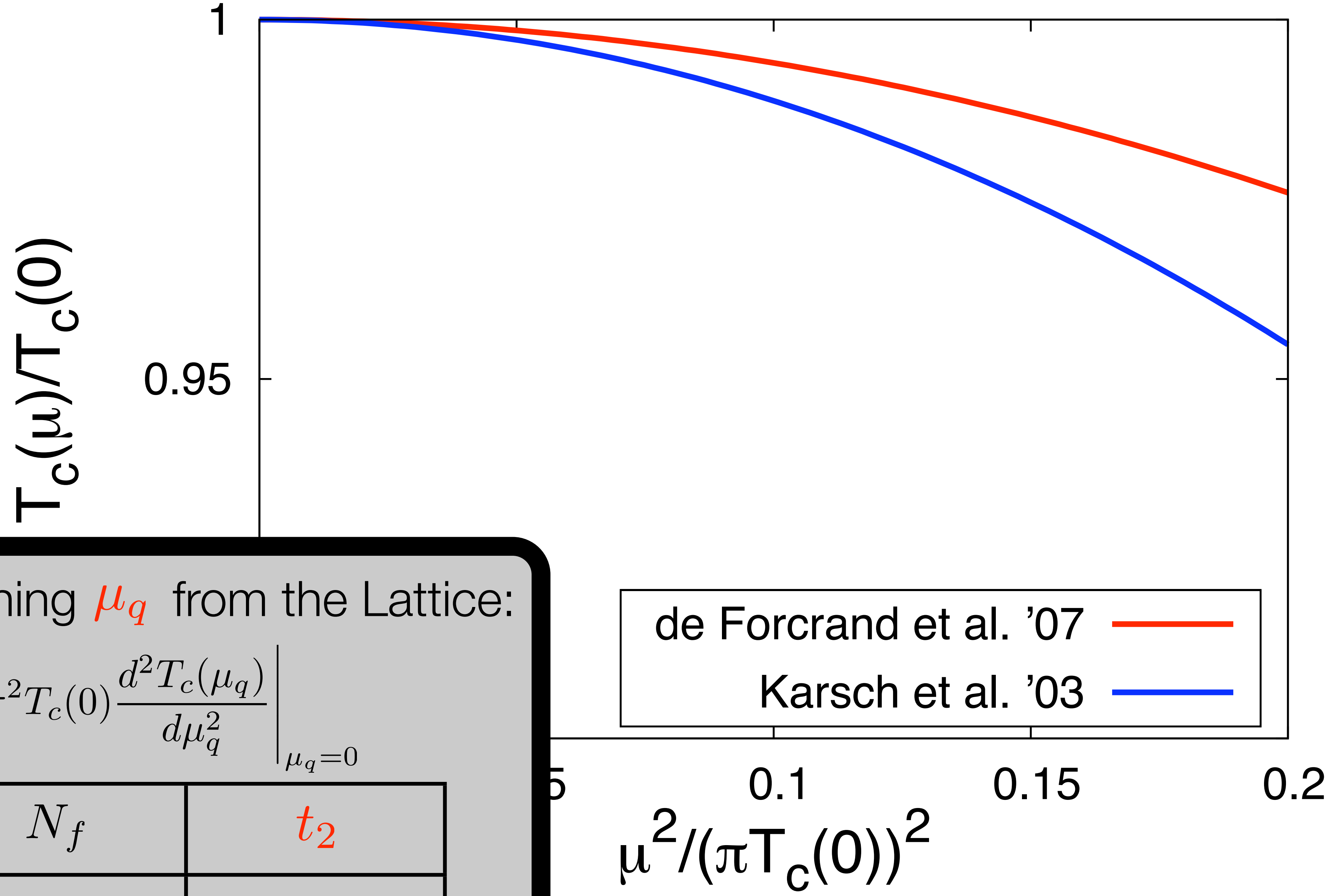
M. Stephanov '07



strong indications that there is no critical point at all
(de Forcrand, Philipsen '07)

low-energy models (QM/NJL): parameter dependence

QCD phase diagram? Many open questions!

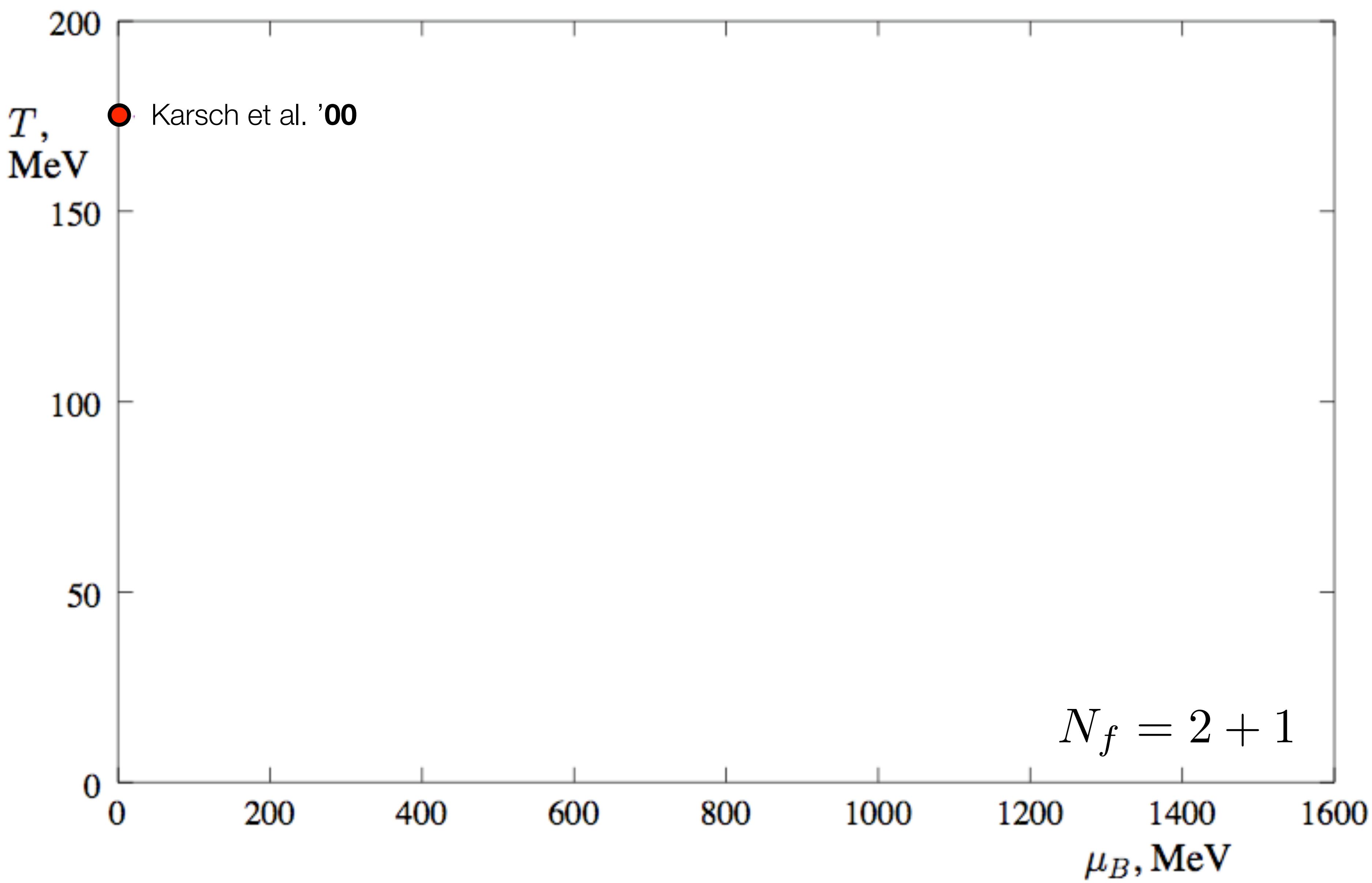


curvature for vanishing μ_q from the Lattice:

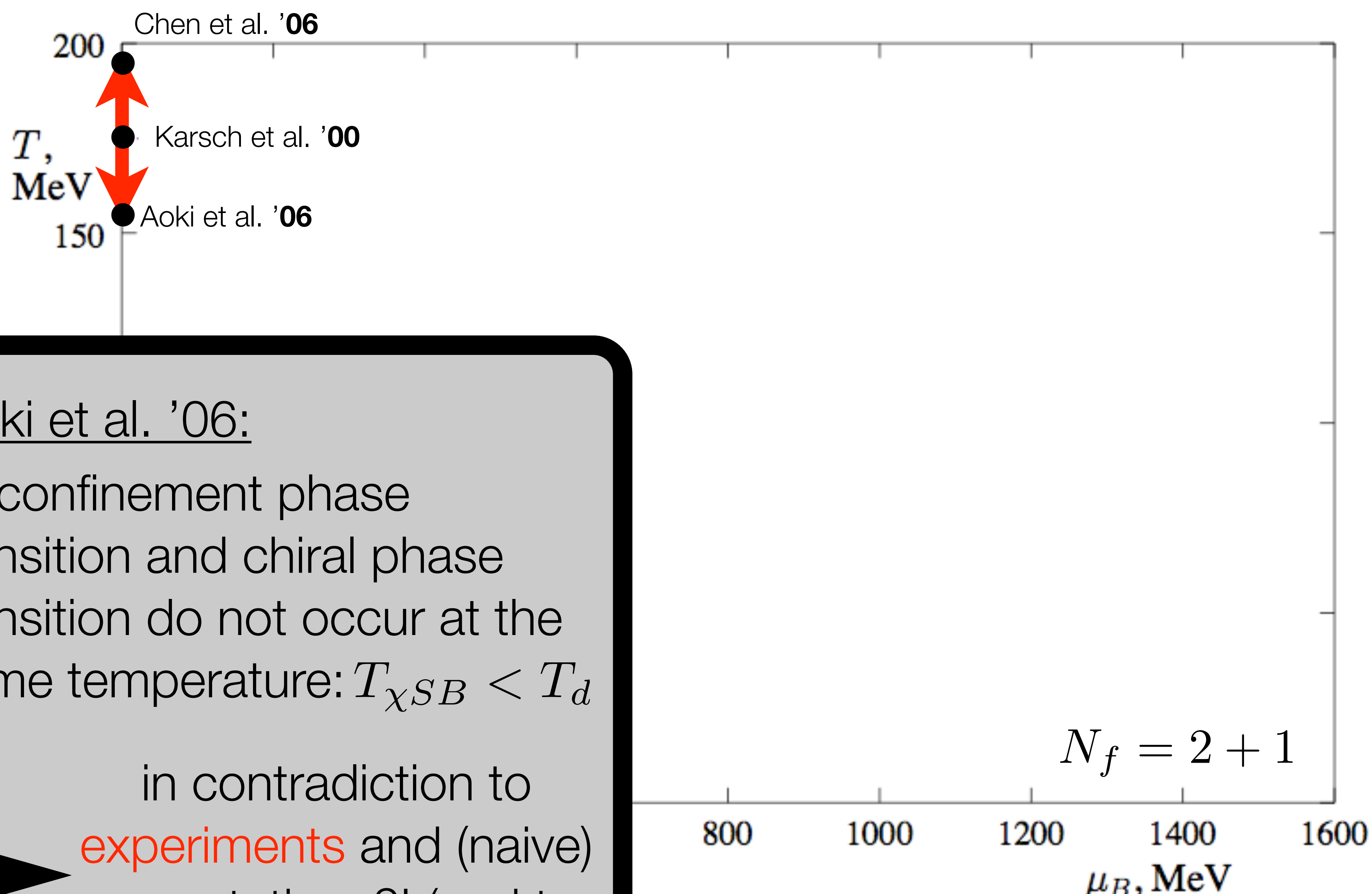
$$t_2 = -\pi^2 T_c(0) \frac{d^2 T_c(\mu_q)}{d\mu_q^2}\bigg|_{\mu_q=0}$$

	N_f	t_2
de Forcrand et al. ‘ 07	3	0.602(9)
Karsch et al. ‘03	3	1.13(45)

QCD phase diagram? Many open questions!



QCD phase diagram? Many open questions!

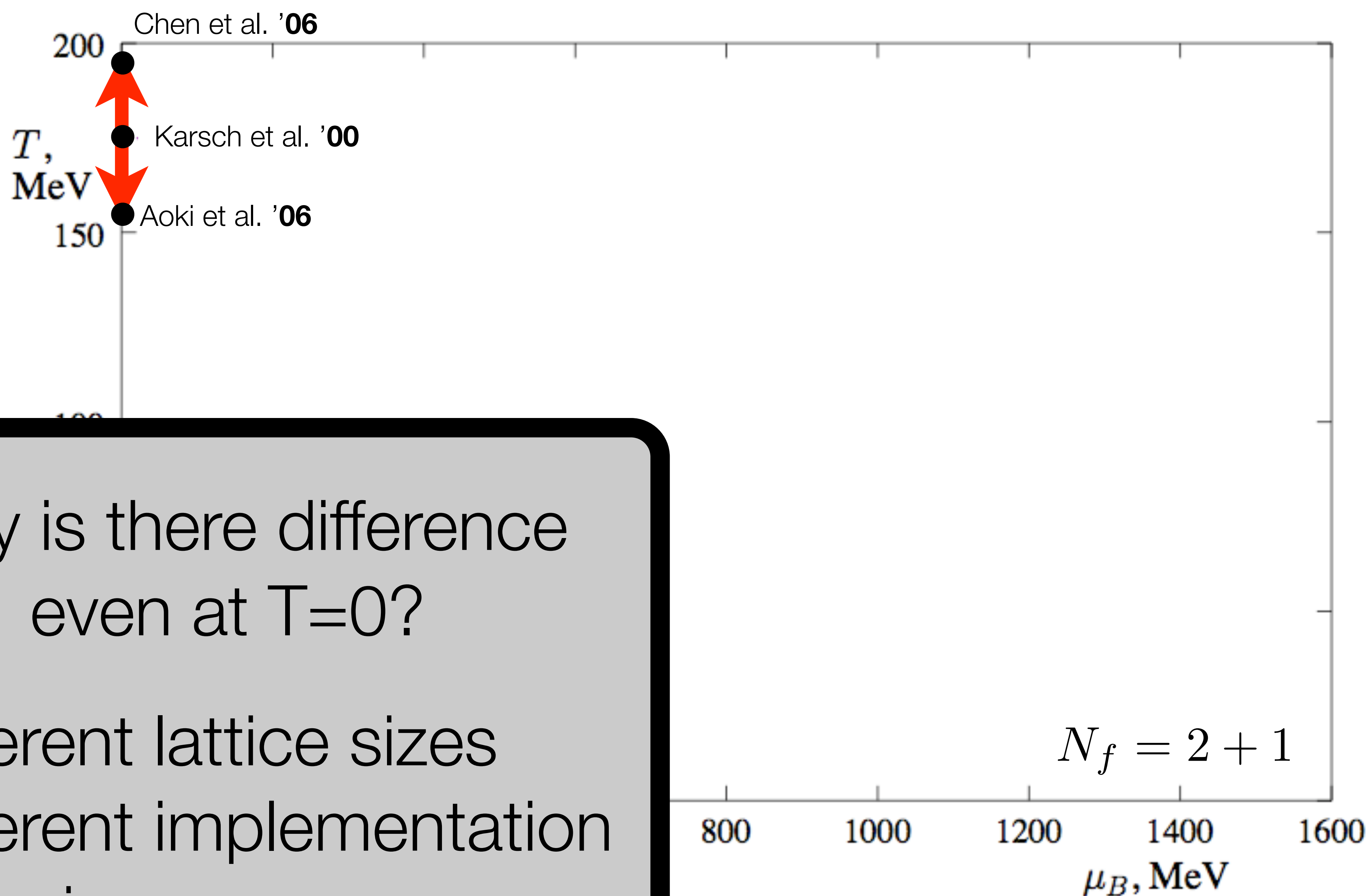


Aoki et al. '06:

Deconfinement phase transition and chiral phase transition do not occur at the same temperature: $T_{\chi SB} < T_d$

→ in contradiction to experiments and (naive) expectations?! (and to Chen et al.)

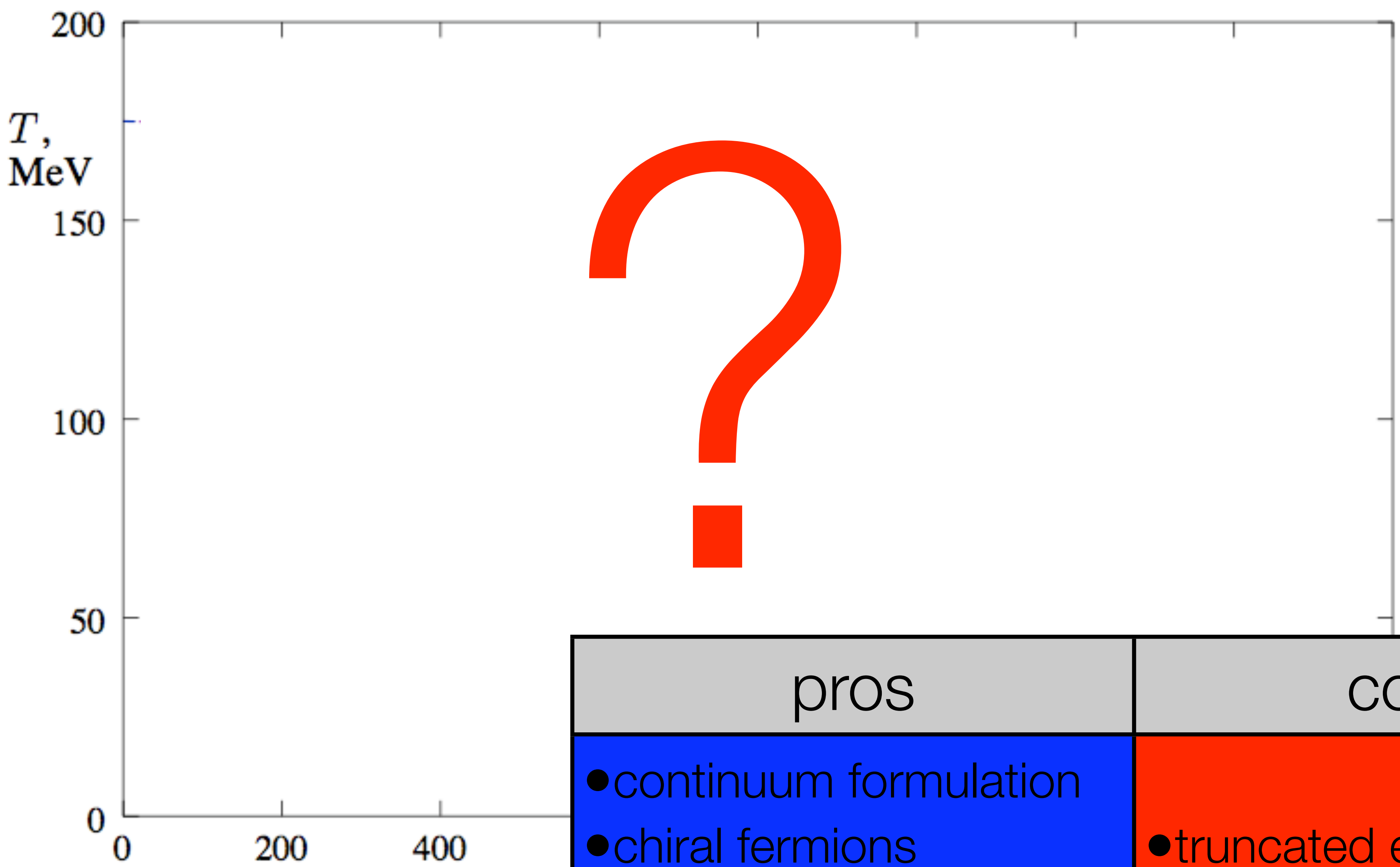
QCD phase diagram? Many open questions!



Why is there difference even at T=0?

- different lattice sizes
- different implementation of fermions

QCD phase diagram from QCD RG flows?



pros	cons
• continuum formulation • chiral fermions • no sign-problem	• truncated eff. action

complementary to Lattice QCD

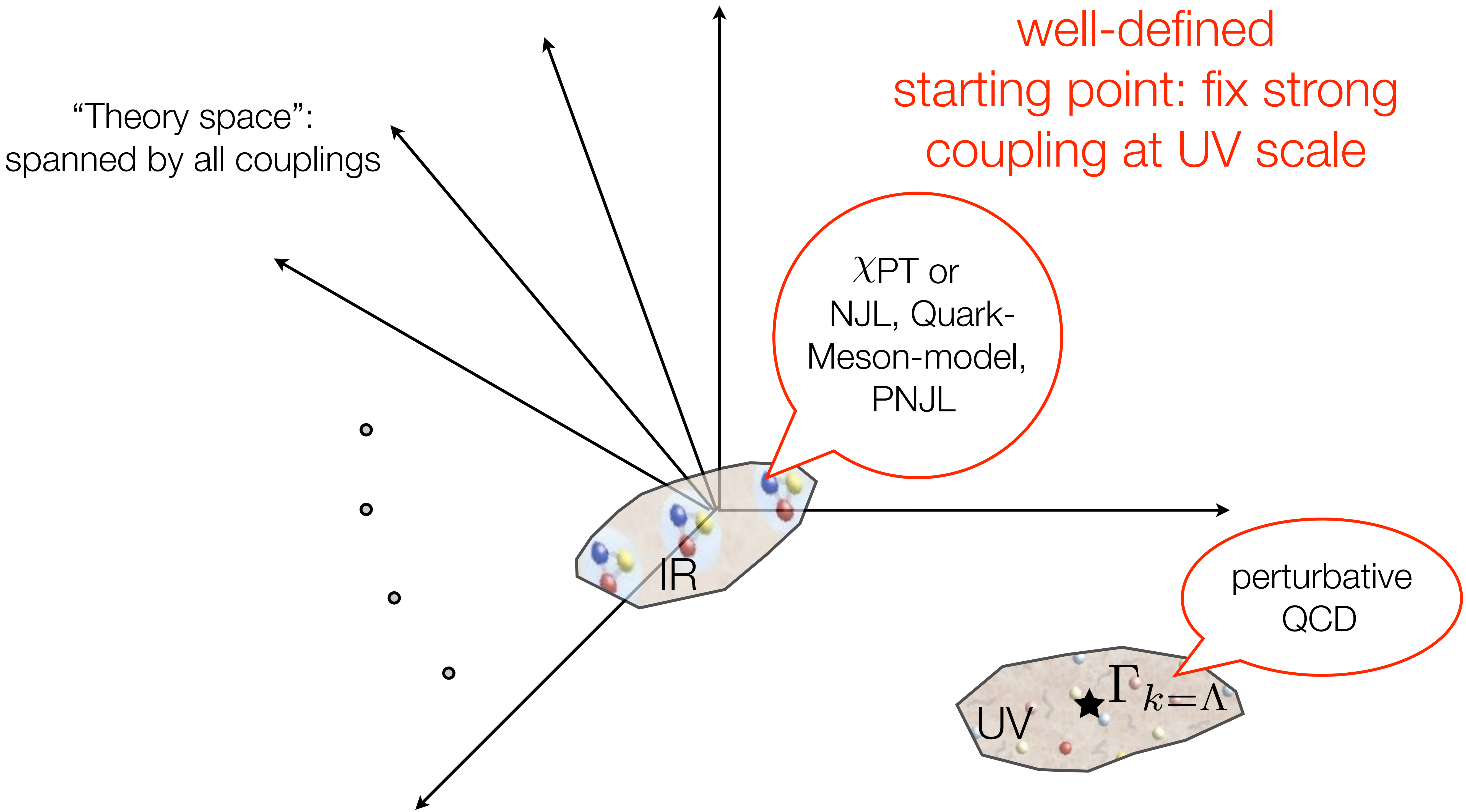
Outline

✓ Motivation

- Functional Renormalization Group
- Chiral Phase Boundary of QCD
- Polyakov-Loop and (De-)Confinement Phase Transition
- Conclusions and Outlook

Functional Renormalization Group

well-defined
starting point: fix strong
coupling at UV scale

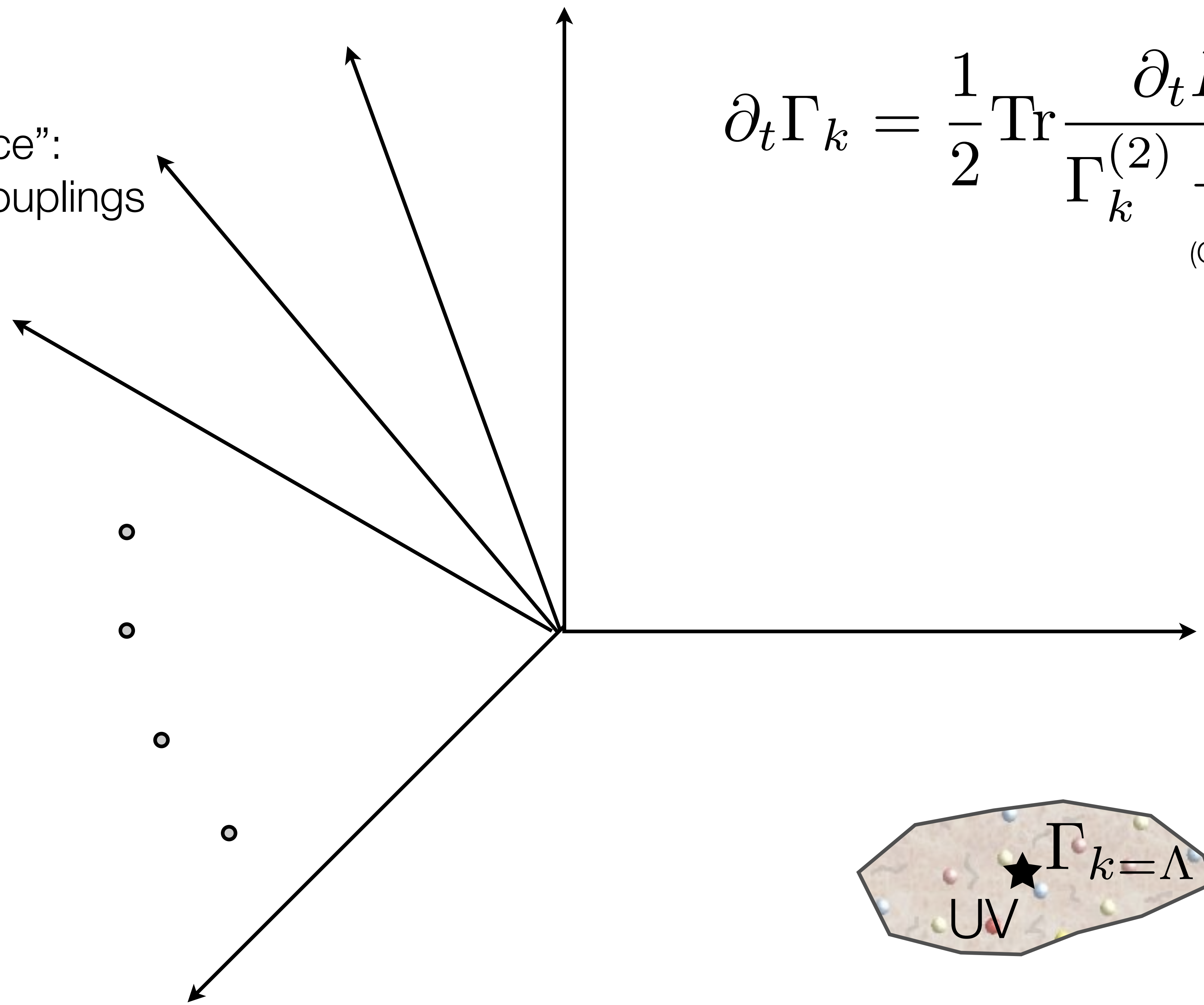


Functional Renormalization Group

"Theory space":
spanned by all couplings

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

(C. Wetterich '93)



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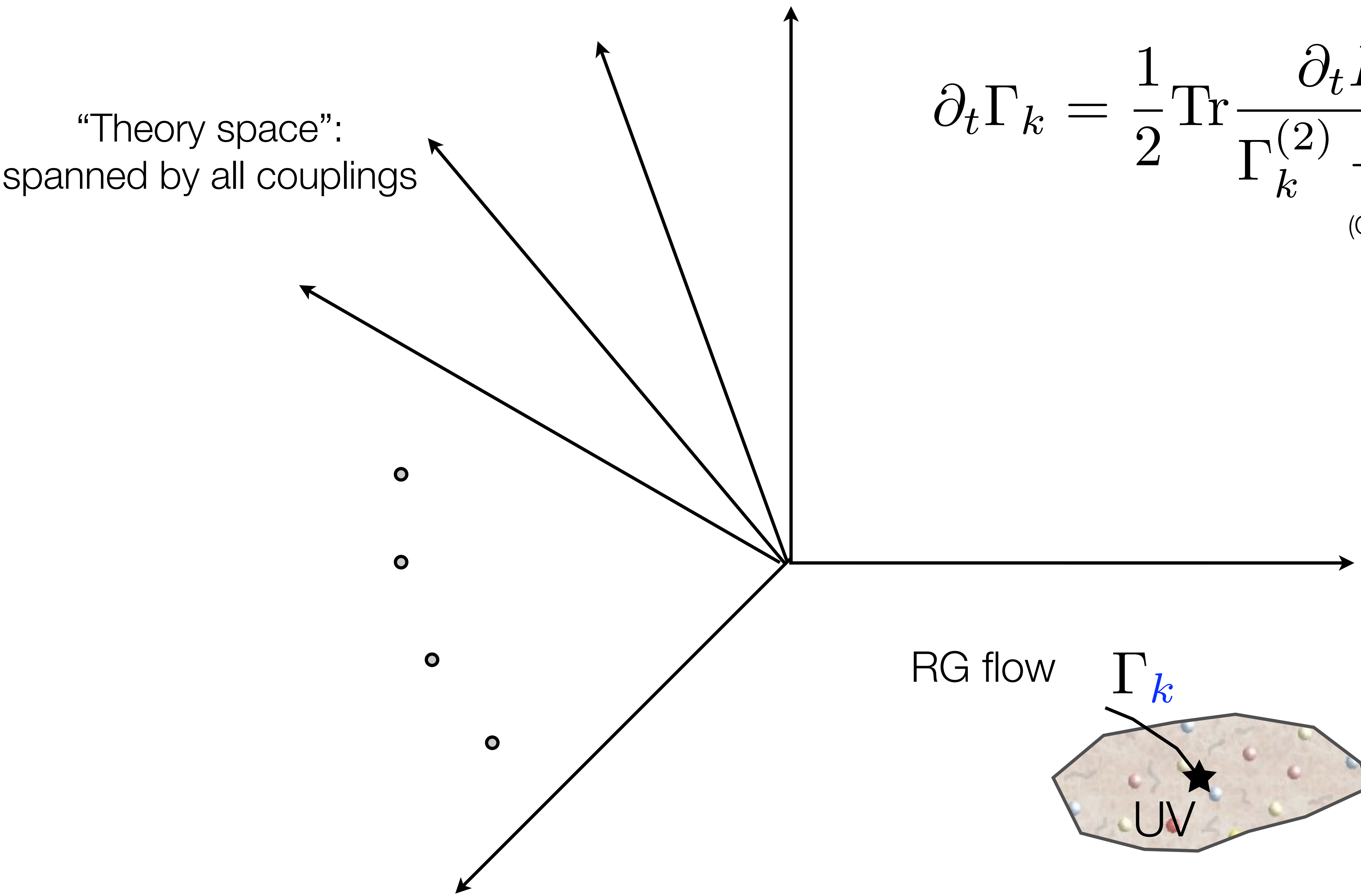

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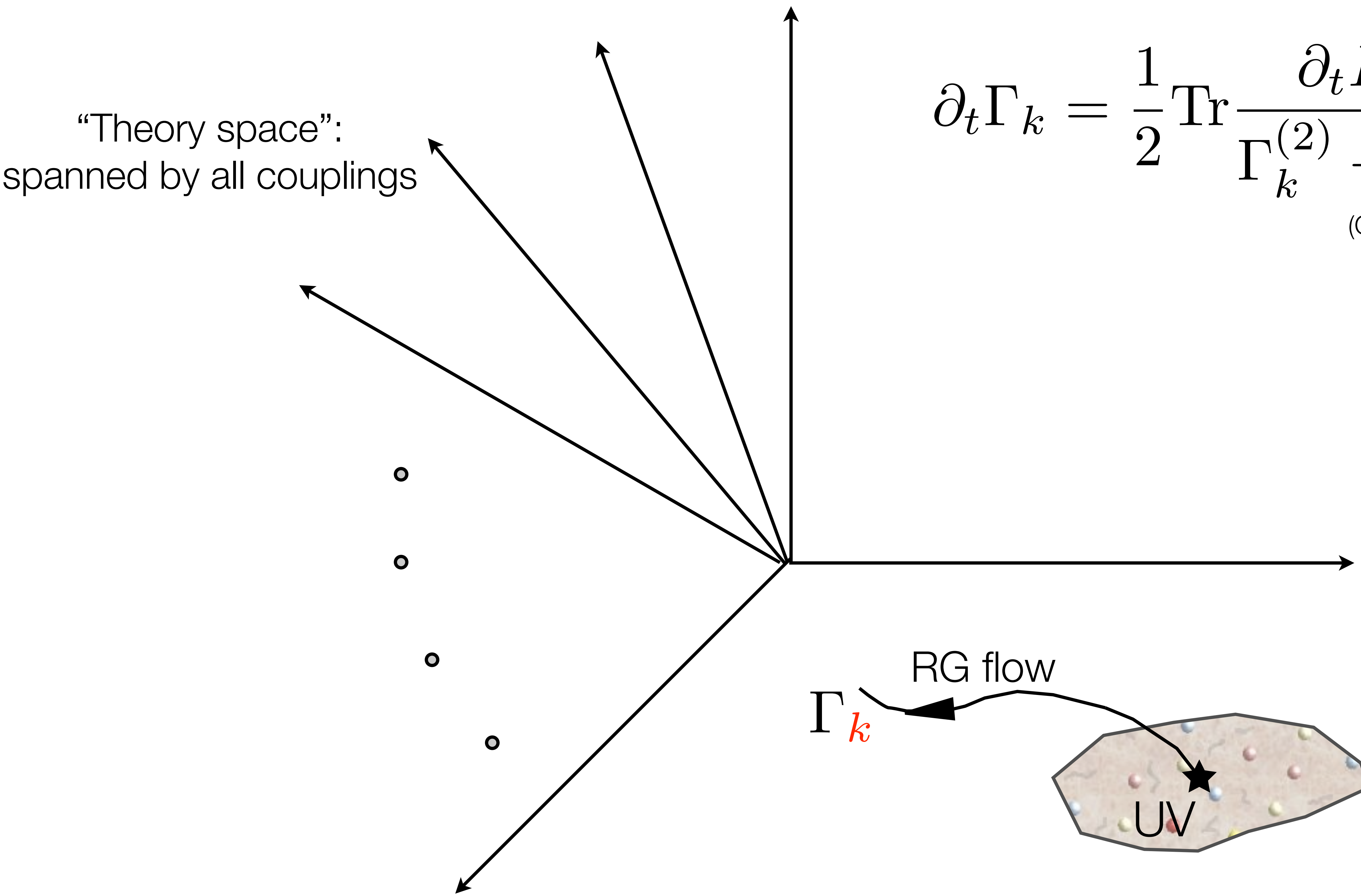

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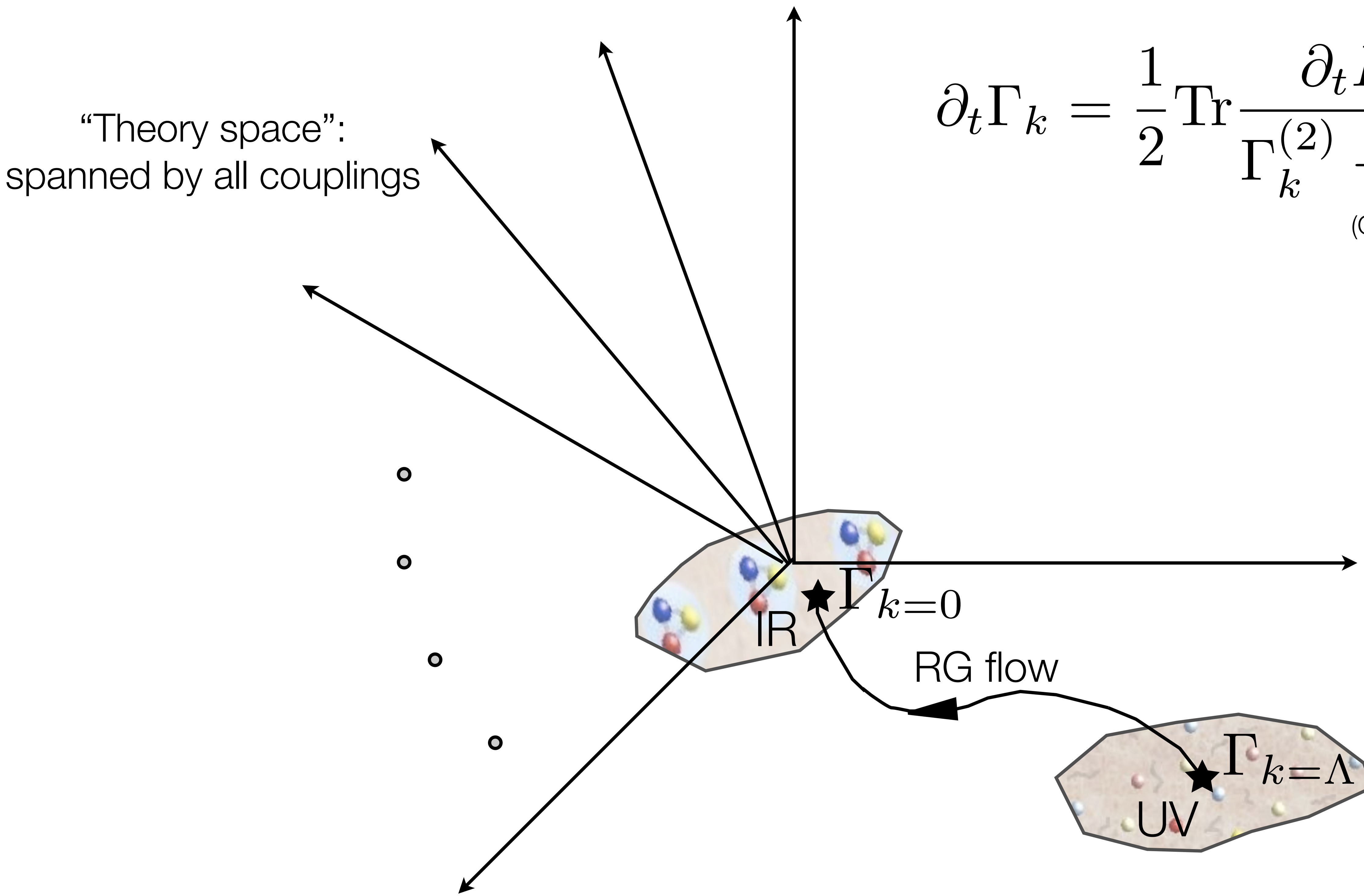


Outline

✓ Motivation

✓ Functional Renormalization Group

- Chiral Phase Boundary of QCD
    - Quark-gluon dynamics and the chiral phase boundary
    - QCD with one quark flavor: from quarks and gluons and mesons
- Polyakov-Loop and (De-)Confinement Phase Transition
- Conclusions and Outlook

Aspects of the NJL model

- classical action of the NJL model:

$$S = \int_x \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\lambda}_\sigma [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] \right\}$$

- spontaneous symmetry breaking if quark condensate is non-vanishing: $\langle \bar{\psi}\psi \rangle \neq 0$

Aspects of the NJL model

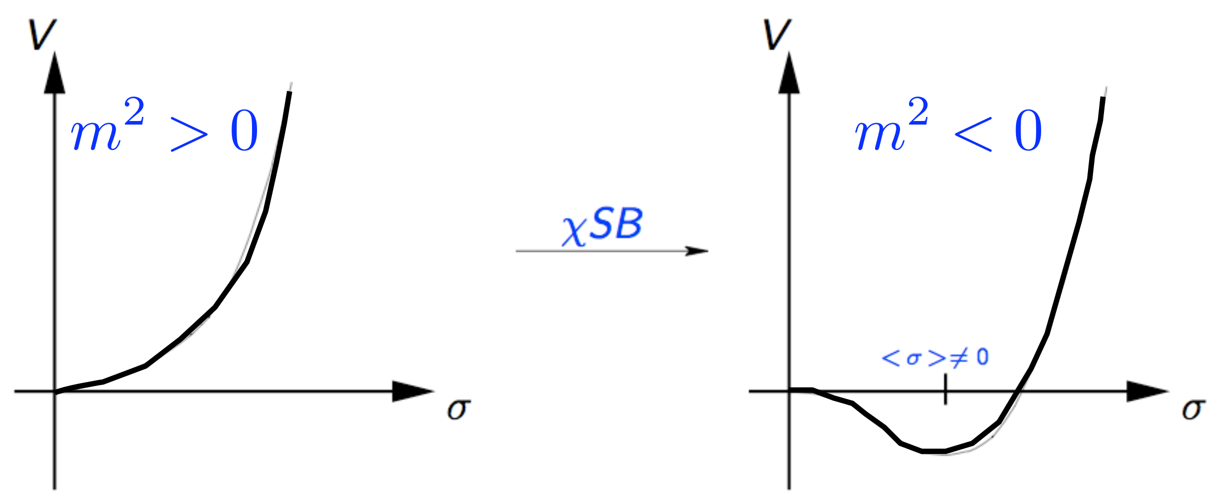
- classical action of the NJL model:

$$S = \int_x \{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\lambda}_\sigma [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] \}$$

- bosonization of the NJL model yields $(\sigma = -2\bar{\lambda}_\sigma \bar{\psi}\psi \, , \pi = -2\bar{\lambda}_\sigma \bar{\psi}\gamma_5\psi)$

$$S = \int_x \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\psi}(\sigma + \mathrm{i}\gamma_5\pi)\psi - \frac{1}{\bar{\lambda}_\sigma}(\sigma^2 + \pi^2) \right\}$$

⇒ $\bar{\lambda}_\sigma$ is inverse proportional to the scalar mass parameter, $m^2 \propto \frac{1}{\bar{\lambda}_\sigma}$



Four-Fermion Interactions in QCD

- at the UV scale $(k = \Lambda \gg \Lambda_{\mathrm{QCD}})$:

$$\Gamma_{\Lambda} = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/)\psi \right\}$$

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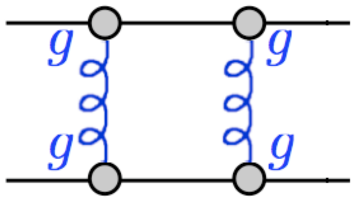
$$k = \Lambda - \delta k$$



$$\Gamma_{\Lambda-\delta k} = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/)\psi + \frac{\lambda_\sigma}{2k^2} [(\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2] + \ldots \right\}$$

- quark-gluon dynamics generate four-fermion interactions

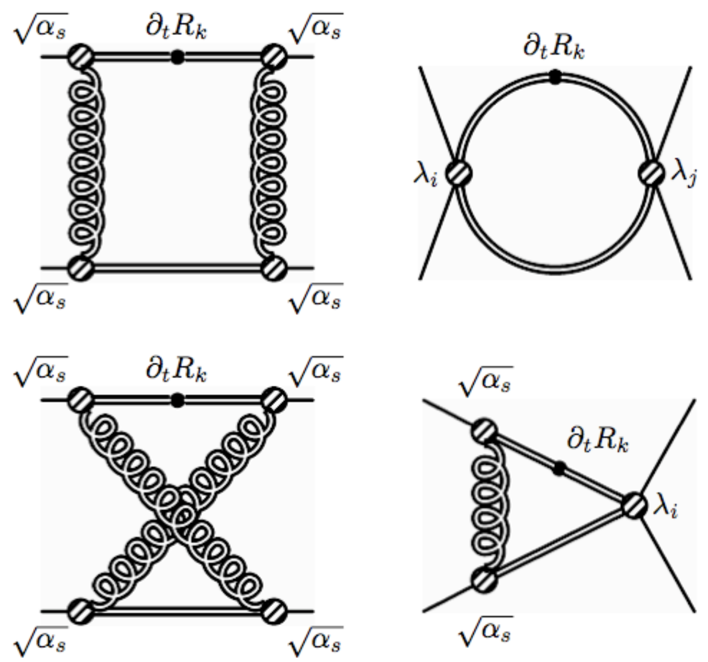
RG flow for the chiral QCD sector

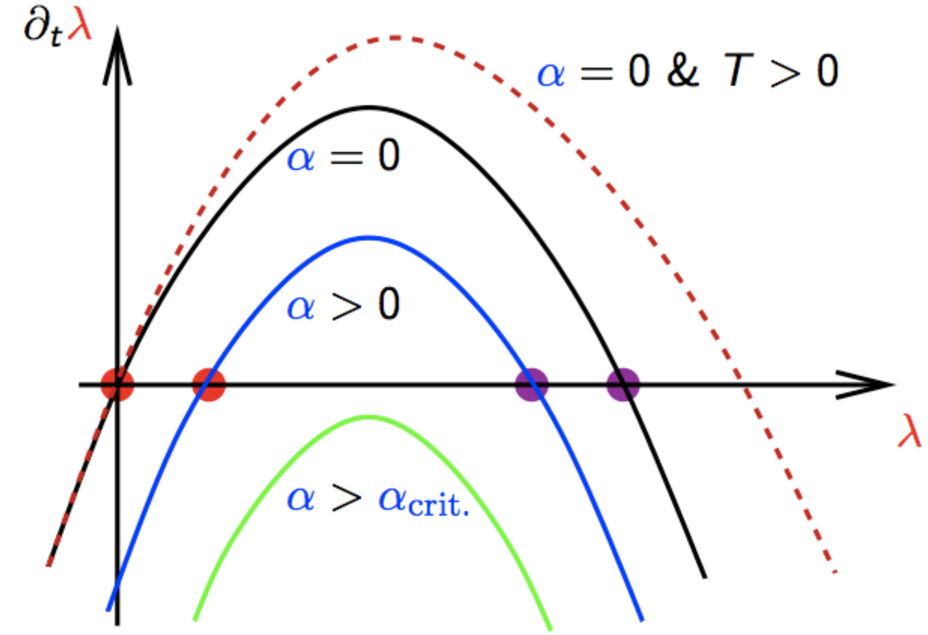
- effective action:

$$\Gamma_k = \int_x \left\{ \frac{\bar{g}^2}{g^2} F^a_{\mu\nu} F^a_{\mu\nu} + w_2 (F^a_{\mu\nu} F^a_{\mu\nu})^2 + w_3 (F^a_{\mu\nu} F^a_{\mu\nu})^3 + \dots \right\}$$
$$+ \int_x \left\{ \bar{\psi}(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar{g} A\!\!\!/)\psi + \frac{1}{2}\left[\frac{\lambda_-}{k^2}(\mathrm{V}-\mathrm{A}) + \frac{\lambda_+}{k^2}(\mathrm{V}+\mathrm{A}) \right.\right.$$
$$\left.\left. + \frac{\lambda_\sigma}{k^2}(\mathrm{S}-\mathrm{P}) + \frac{\lambda_{\mathrm{VA}}}{k^2}[2(\mathrm{V}-\mathrm{A})^{\mathrm{adj}} + (1/N_c)(\mathrm{V}-\mathrm{A})] \right] \right\}$$

- no Fierz-ambiguity
- four-fermion interactions $\left(\lim_{\Lambda\to\infty} \lambda_i = 0 \right)$
- truncation checks: momentum dependencies, regulator dependencies, higher order interactions (H. Gies, J. Jaeckel, C. Wetterich '04)

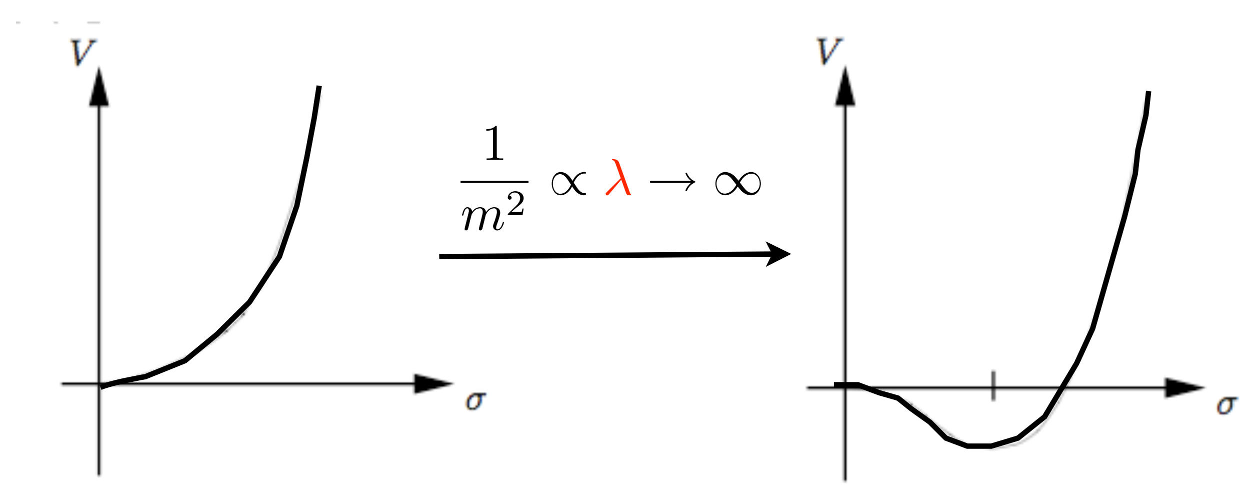
"Criticality" at zero and finite temperature





- flow of four-fermion couplings:

$$\partial_t \lambda = 2\lambda - \lambda A(\tfrac{T}{k})\lambda - b(\tfrac{T}{k})\lambda\alpha_s - c(\tfrac{T}{k}){\alpha_s}^2$$



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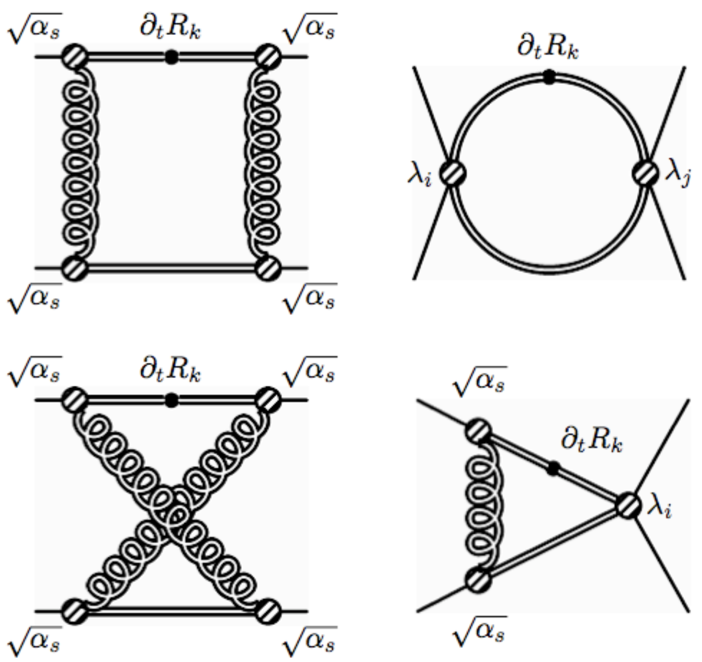
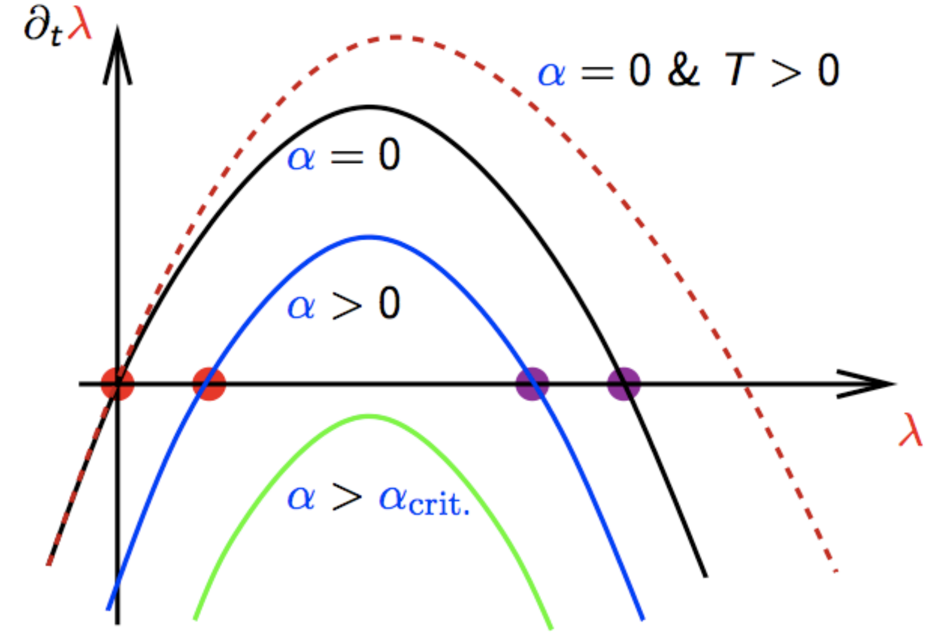


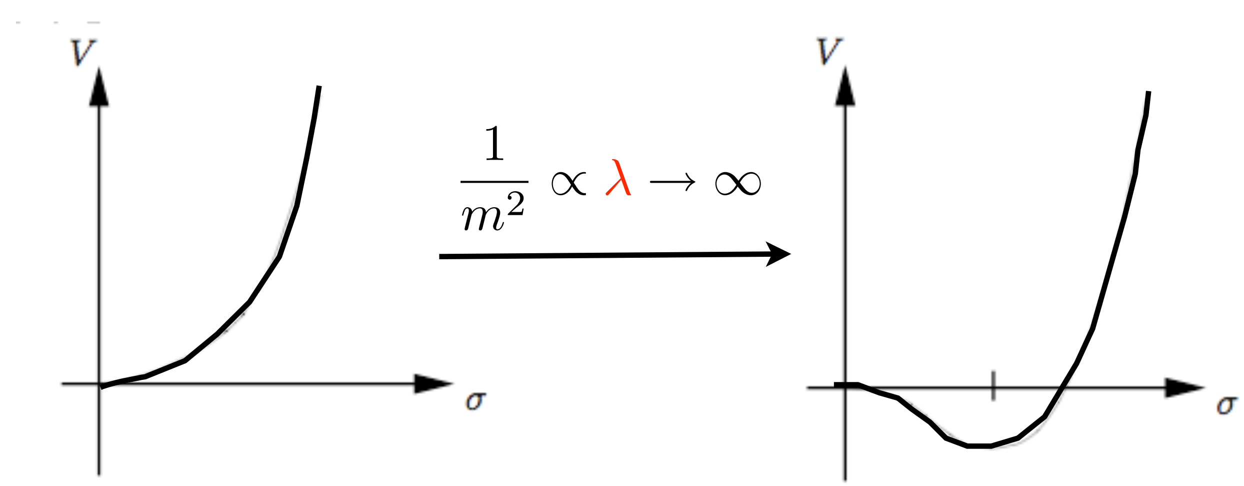


- critical gauge coupling α_{cr} :

  if $\alpha_s > \alpha_{cr}$ ⟹ no fixed points ⟹ χSB

- at zero temperature: (H. Gies, J. Jaeckel '05)

$$\alpha_{cr} \overset{N_c = N_f = 3}{\approx} 0.85$$



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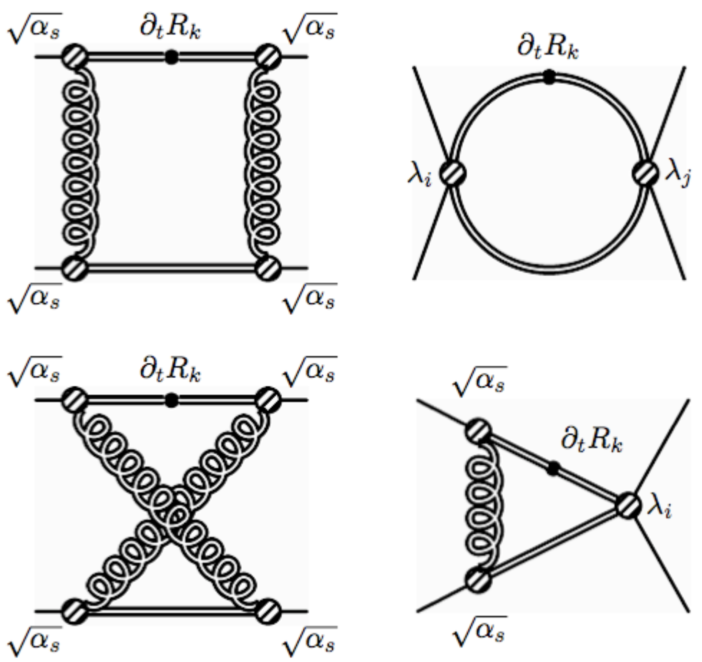
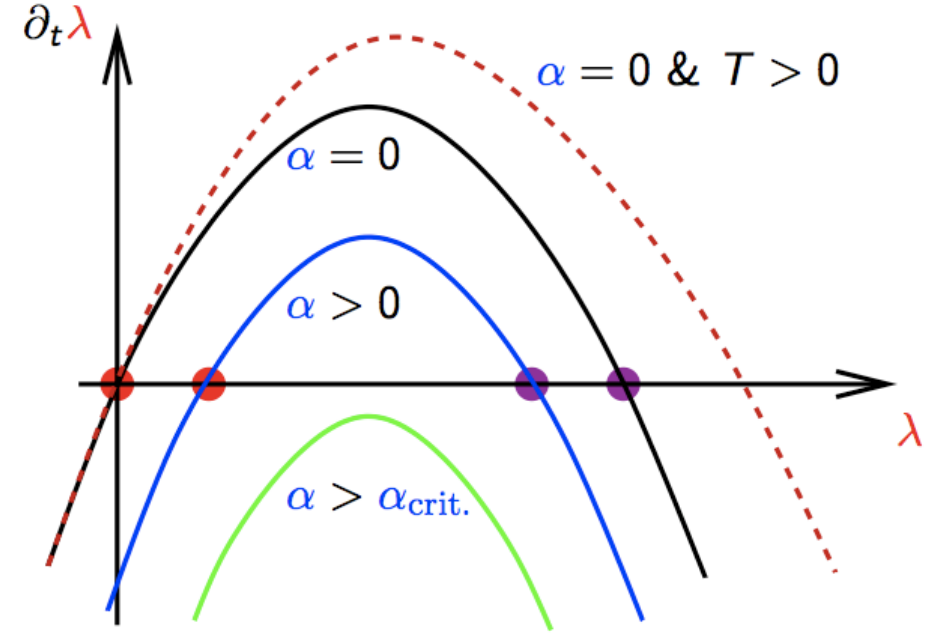




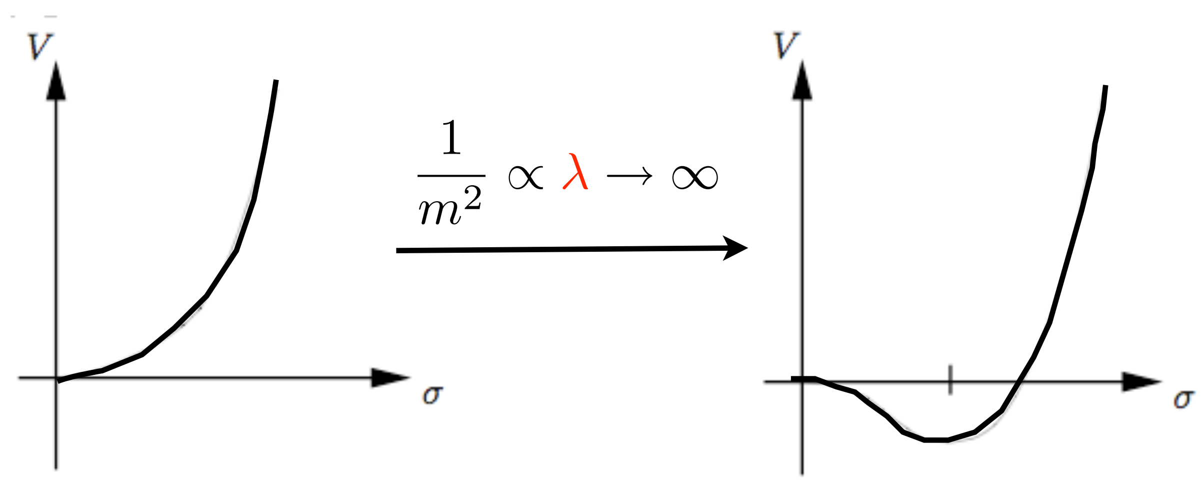
- critical gauge coupling α_{cr} :

  if $\alpha_s > \alpha_{cr}$ ⟹ no fixed points ⟹ χSB

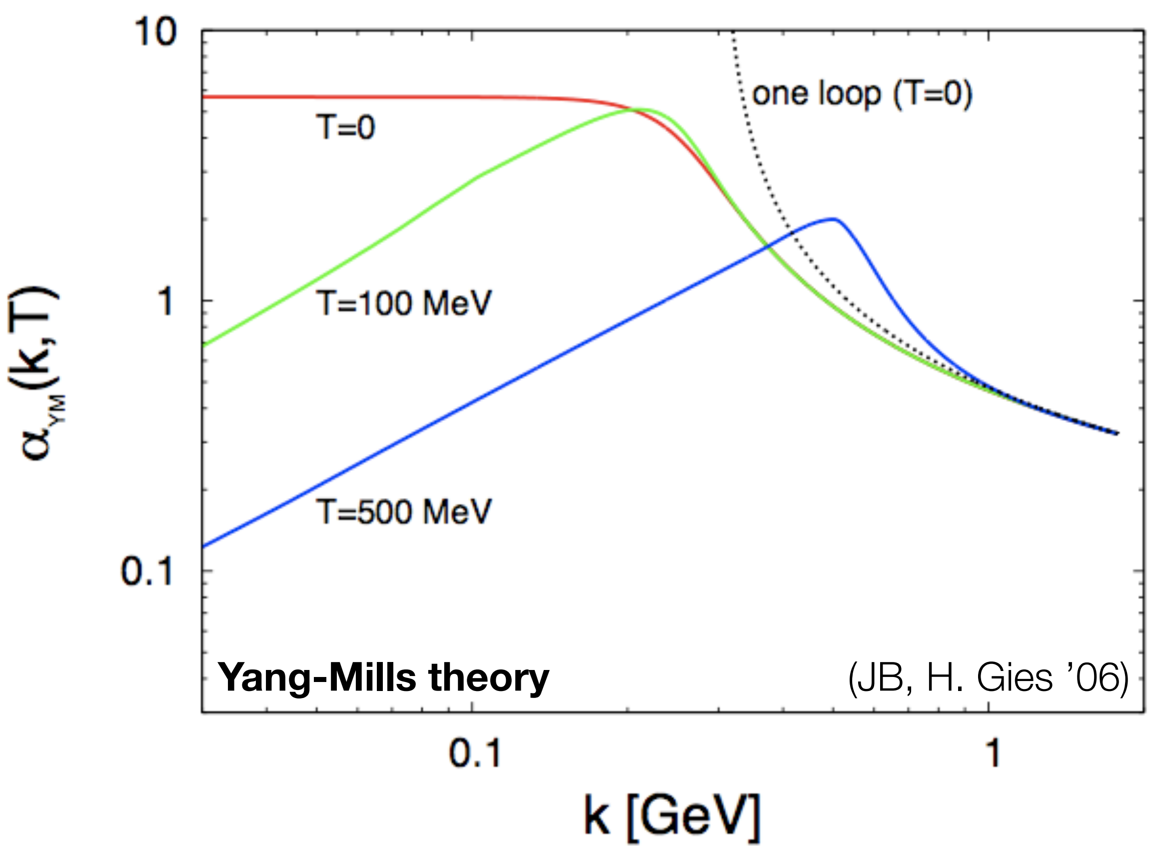
- at finite temperature: (JB, H. Gies ’05)

$$\alpha_{cr}(T/k) > \alpha_{cr}(T=0)$$

quarks acquire a thermal mass



RG flow of gluodynamics



cf. vertex expansion in **Landau-gauge QCD**:

SDE: v. Smekal et al. '97, Fischer et al. '02;
RG: Pawlowski et al. '04, Fischer&Gies '04;
Gies '02; Gies&Braun '05/'
Lattice: e. g. Sternbeck et al. '05; ...

- $k_{max} \propto T$ decoupling of hard gluonic modes ⇨ "finite-size" effect:

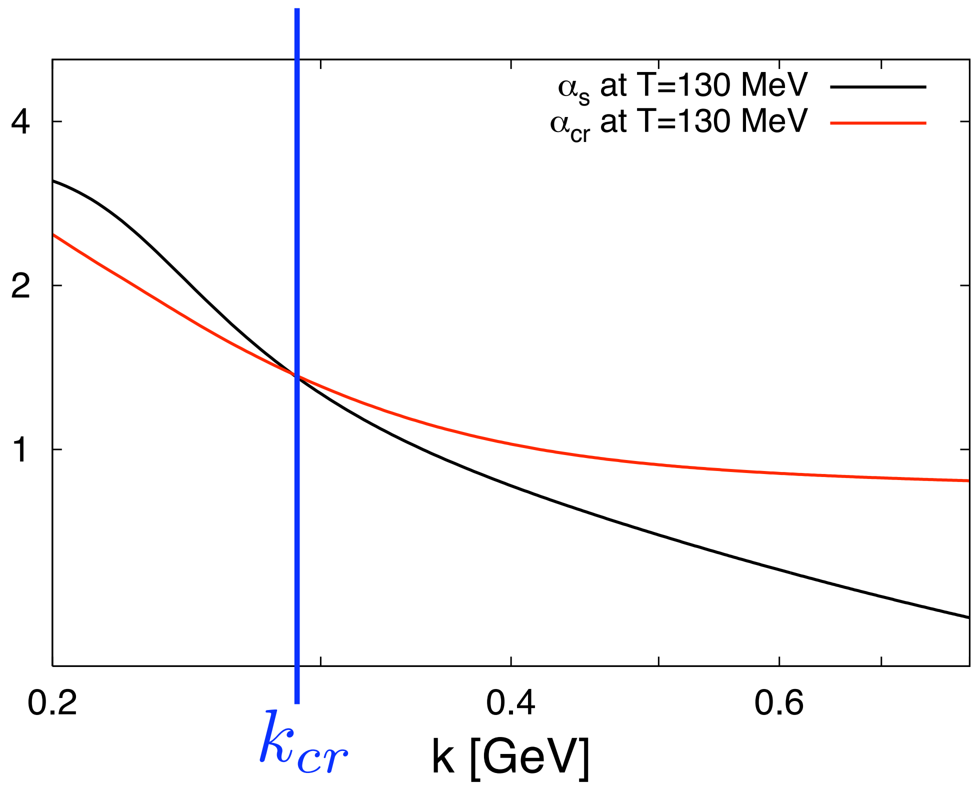
$$p_{g,0}^2 \equiv \omega_n^2 = 4n^2\pi^2T^2 \quad \rightarrow \quad \omega_0^2 = 0$$

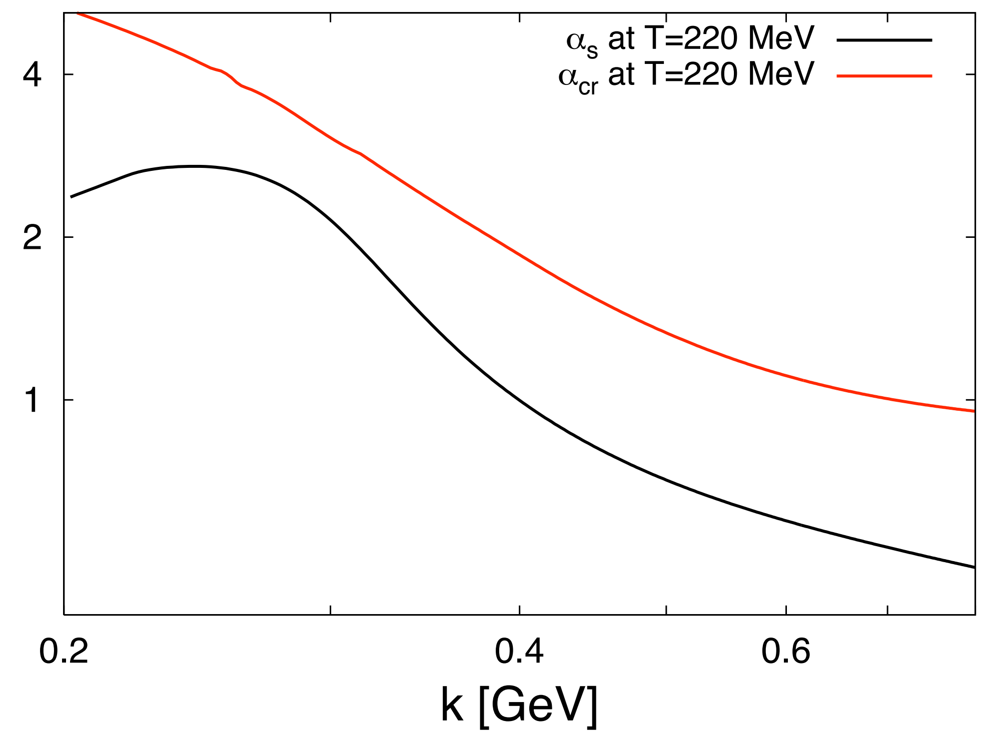
- decrease for $T \gtrsim k$ due to existence of a non-trivial IR fixed point in 3d Yang-Mills theory: strong interactions at high temperatures (JB, H. Gies '06; Lattice: Cucchieri et al. '07)

$$\alpha_{4D} \approx \alpha_{3D}^* \frac{k}{T} + \mathcal{O}\left((k/T)^2\right) \quad \text{with} \quad \alpha_{3D}^* \approx 2.7\,;\ \eta_{3d} \rightarrow 1$$

Chiral Phase Transition in QCD

- study: $\alpha_{cr}(T/k)$ vs. $\alpha_s(T/k)$
- intersection point of α_{cr} and α_s indicates onset of χSB





- single input parameter: $\alpha_s(m_\tau) = 0.322$

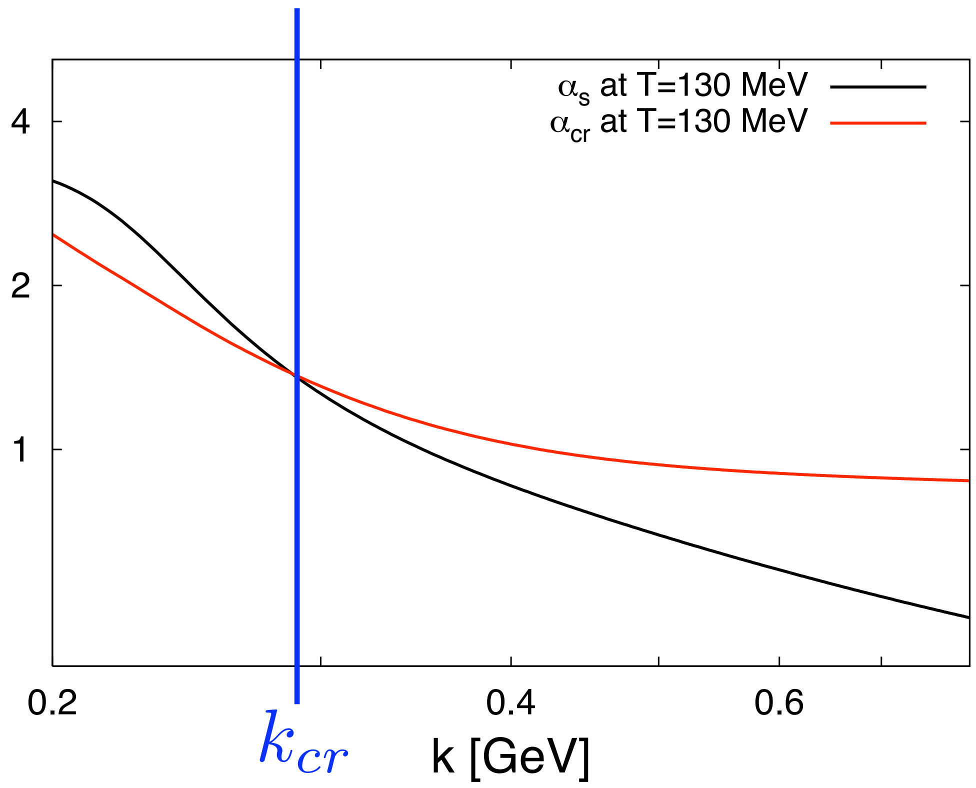
N_f	T_{cr}
2	$172\,\mathrm{MeV}$
3	$148\,\mathrm{MeV}$

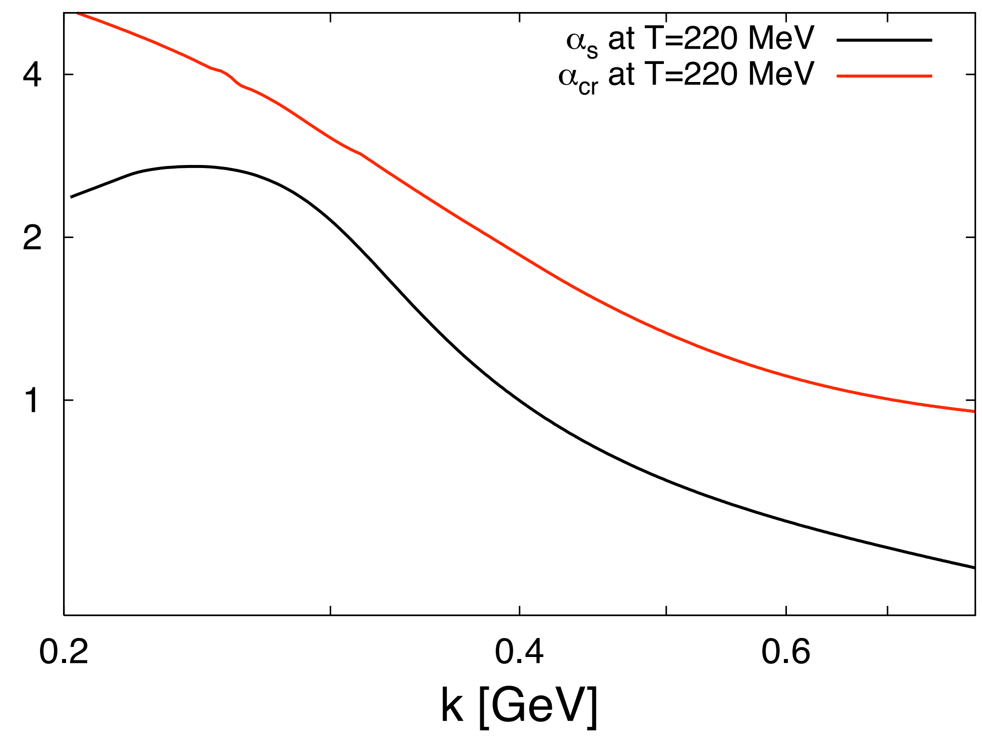
(JB, H. Gies '06)

$N_f = 2+1$	T_{cr}
Lattice (Chen et al. '06)	$192\,\mathrm{MeV}$
Lattice (Aoki et al. '06)	$151\,\mathrm{MeV}$

Chiral Phase Transition in QCD: Error estimate

- study: $\alpha_{cr}(T/k)$ vs. $\alpha_s(T/k)$
- intersection point of α_{cr} and α_s indicates onset of χSB





- single input parameter: $\alpha_s(m_\tau) = 0.322 \pm 0.03$

N_f	T_{cr}
2	172^{+40}_{-34} MeV
3	148^{+32}_{-31} MeV

(JB, H. Gies '06)

$N_f = 2+1$	T_{cr}
Lattice (Chen et al. '06)	192 MeV
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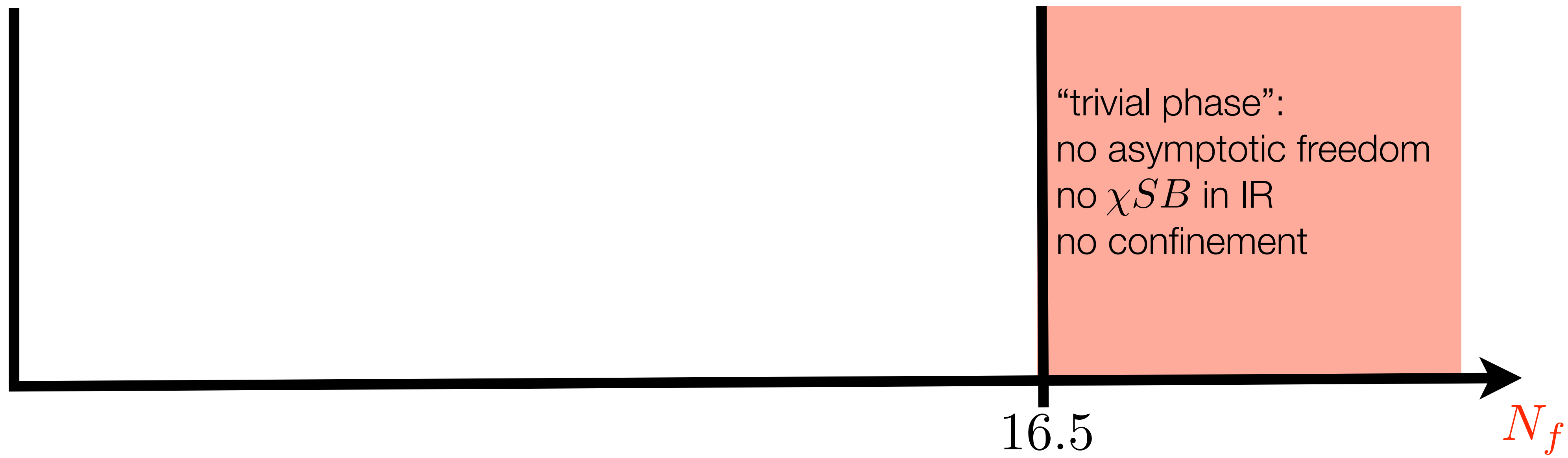
What to expect for QCD with many flavors?



- one-loop β-function

$$\partial_t \alpha \equiv \beta(\alpha) = -\overbrace{\frac{1}{6\pi}(11N_c - 2N_f)}^{b_1}\,\alpha^2$$

- $b_1 < 0 \Longrightarrow N_f > \frac{11}{2}N_c \overset{N_c=3}{=} 16.5$ (QCD is NOT asymptotically free)

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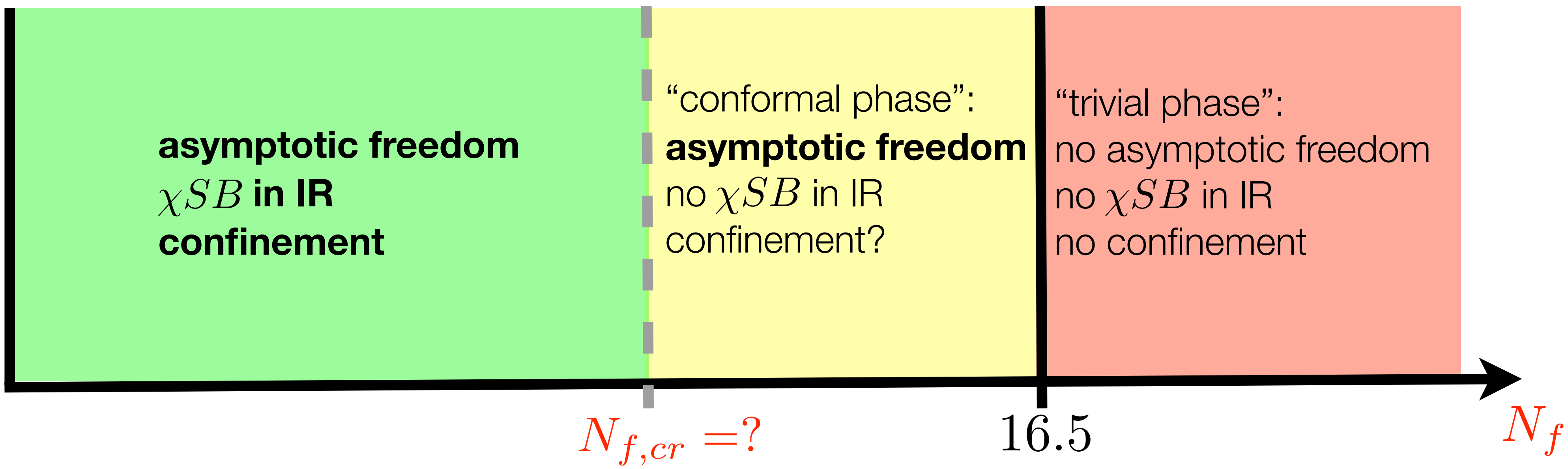

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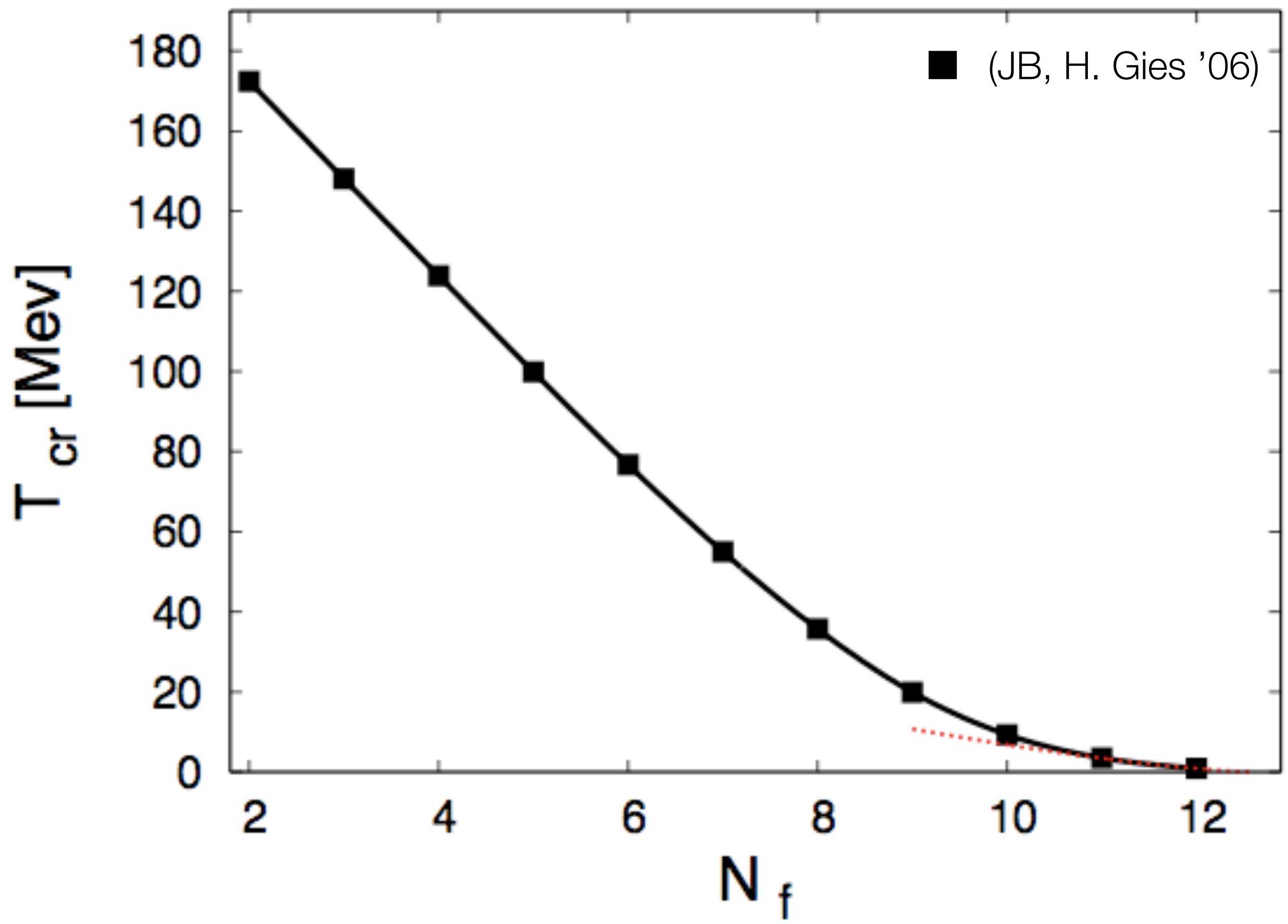
- $b_1 < 0 \Longrightarrow N_f > \frac{11}{2} N_c \stackrel{N_c=3}{=} 16.5$ (QCD is NOT asymptotically free)

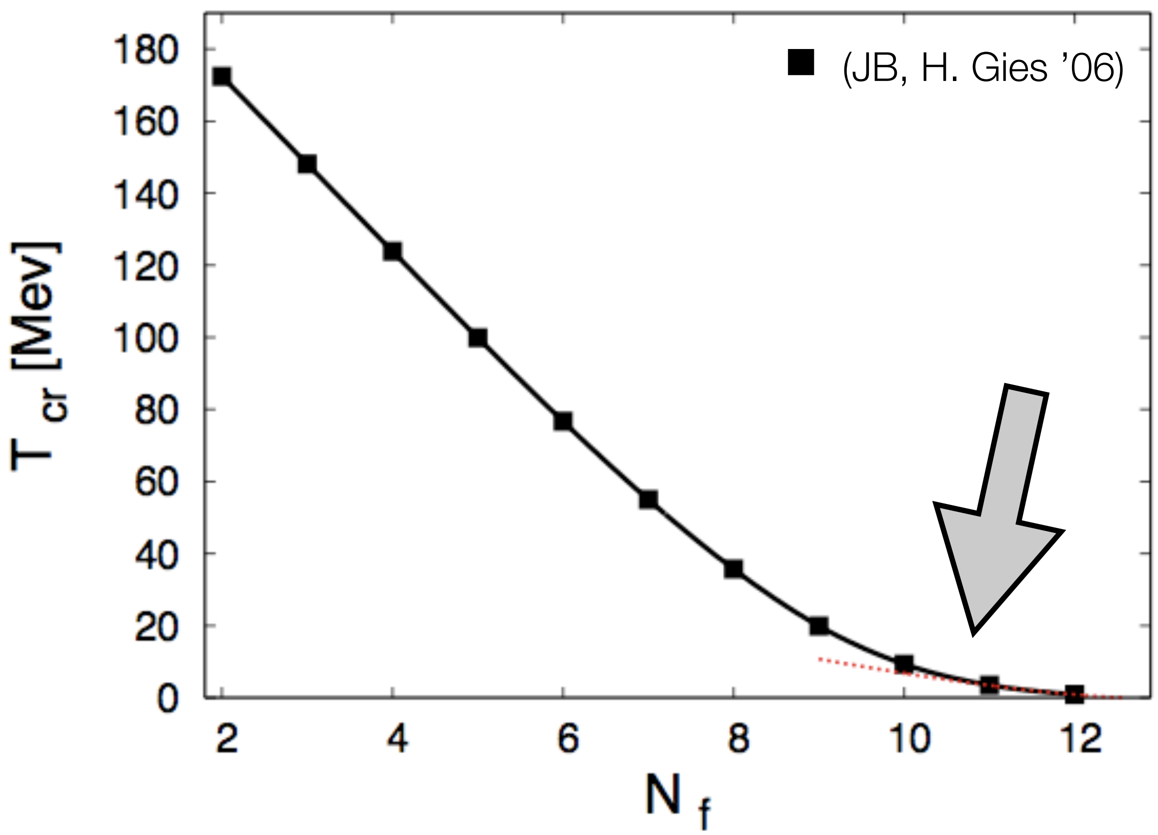
- $b_1 > 0$: QCD is asymptotically free

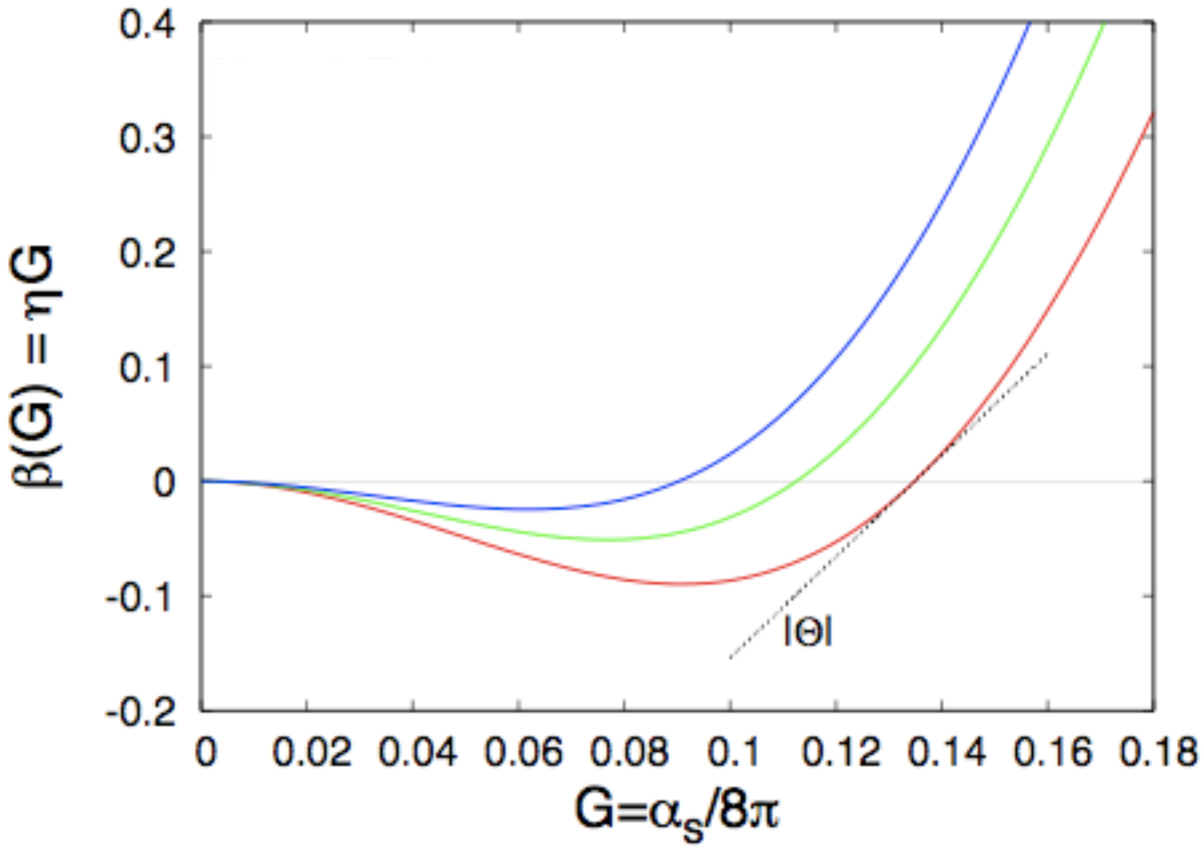
Many-flavor QCD



- small N_f : fermionic screening
- critical number of quark flavors: $N_{f,cr} = 12$ (cf. e. g. Appelquist '07 & '96)
- "conformal phase" for $N_{f,cr} < N_f < 16.5$: asymptotic freedom but no χSB

Many-flavor scaling regime





- fixed-point regime for large N_f: critical exponent $|\Theta|$

$$\partial_t g^2 \approx |\Theta|(g^2 - g_*^2)$$

- shape of the phase boundary for $N_f \approx N_{f,cr}$ (JB, H. Gies '06)

$$T_{cr} \propto |N_f - N_{f,cr}|^{\frac{1}{|\Theta|}} \quad \text{with} \quad |\Theta| \approx 0.71$$

(currently under investigation on the lattice, Deuzeman et al. '08)

Outline

✓ Motivation

✓ Functional Renormalization Group

- Chiral Phase Boundary of QCD
  - ✓ Quark-gluon dynamics and the chiral phase boundary
  - QCD with one quark flavor: from quarks and gluons and mesons
- Polyakov-Loop and (De-)Confinement Phase Transition
- Conclusions and Outlook

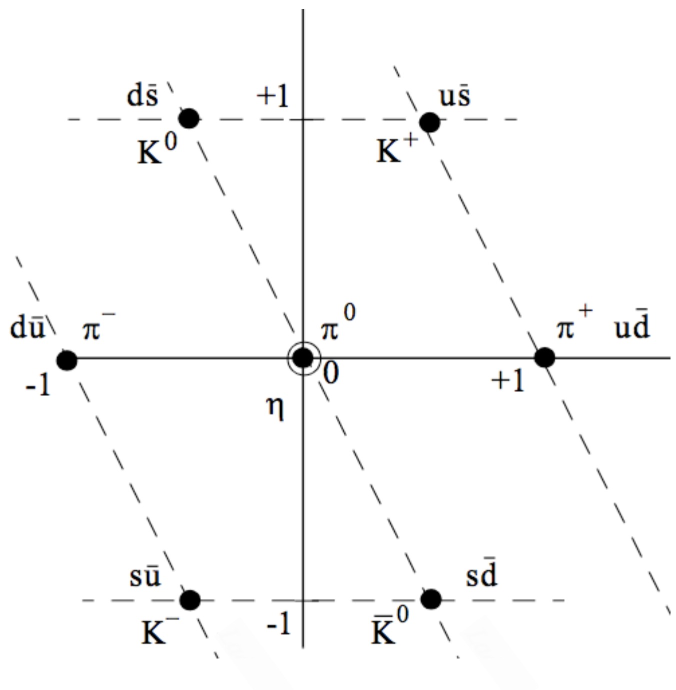
Challenge:

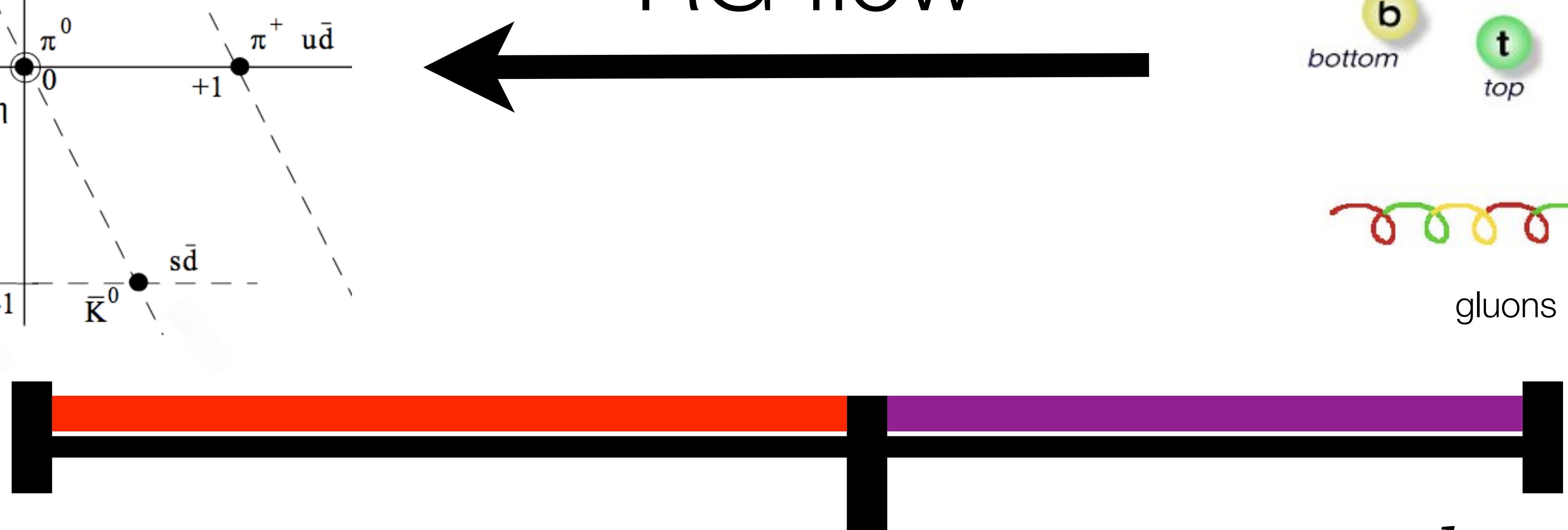
How to penetrate the phase boundary in order to get access to the low-energy observables?

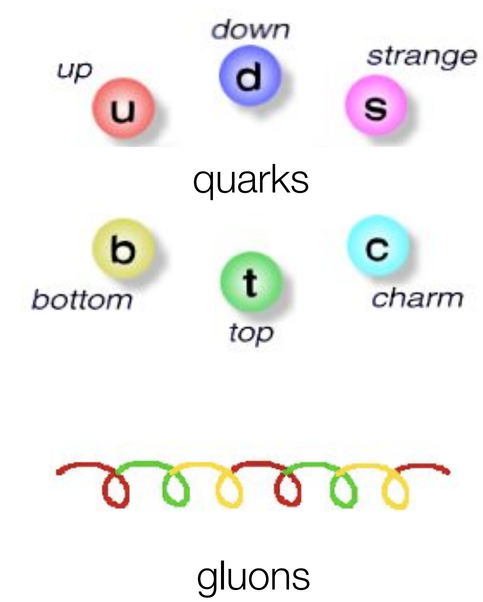
From microscopic to macroscopic DoFs

macroscopic DoF

microscopic DoF







$k \to 0$ | k | $k \to \Lambda$

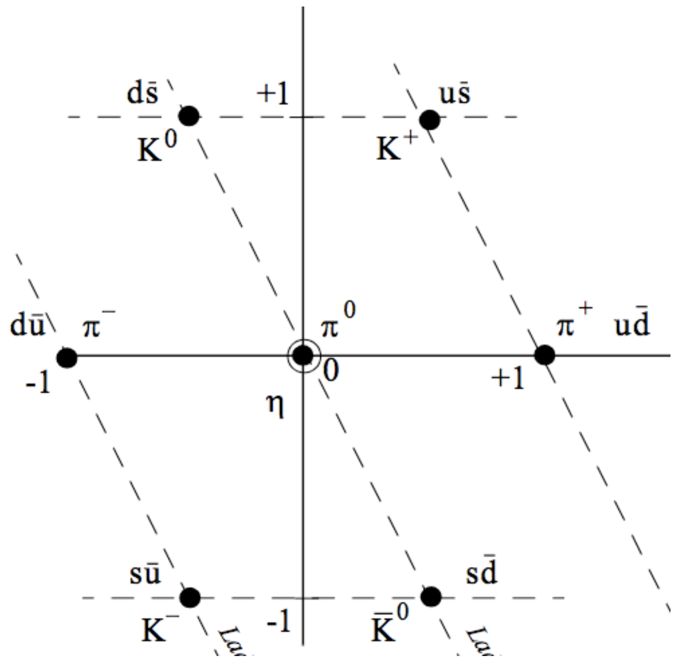
large length scales | small length scales

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k}{\Gamma_k^{(2)}[\phi] + R_k}$$

From microscopic to macroscopic DoFs: Do it by hand

macroscopic DoF

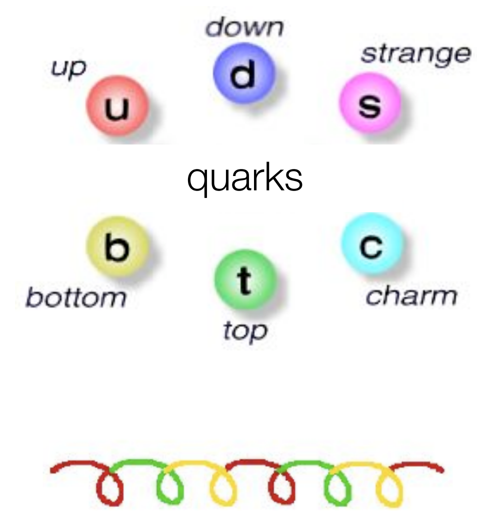
For example:
(constituent-quark-) meson-model



RG flow | RG flow

microscopic DoF

quark-gluon dynamics

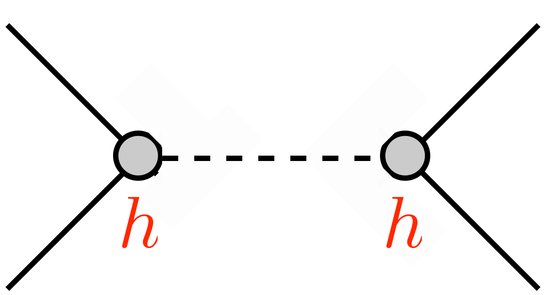


gluons

e. g.: $k \approx k_{cr}$
"set" by diverging $\bar{\lambda}_\sigma$

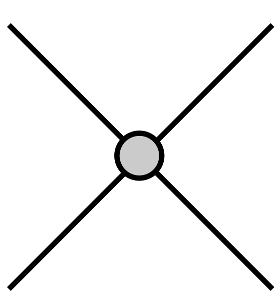
$$h\bar{\psi}(\sigma + \mathrm{i}\gamma_5\pi)\psi + m^2(\sigma^2 + \pi^2)$$

$$\bar{\lambda}_\sigma[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$$



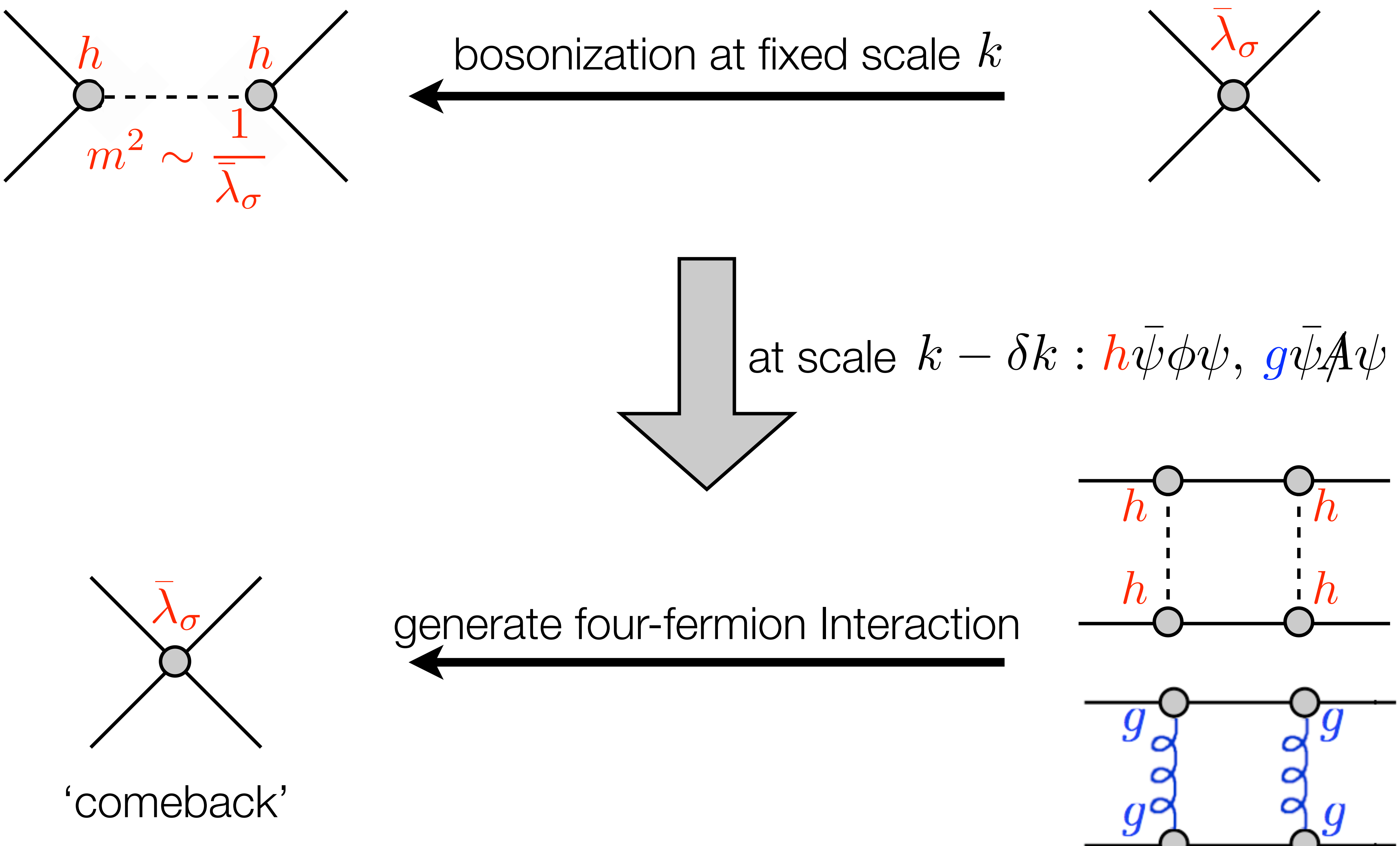
bosonization at fixed scale

Hubbard-Stratonovich transformation



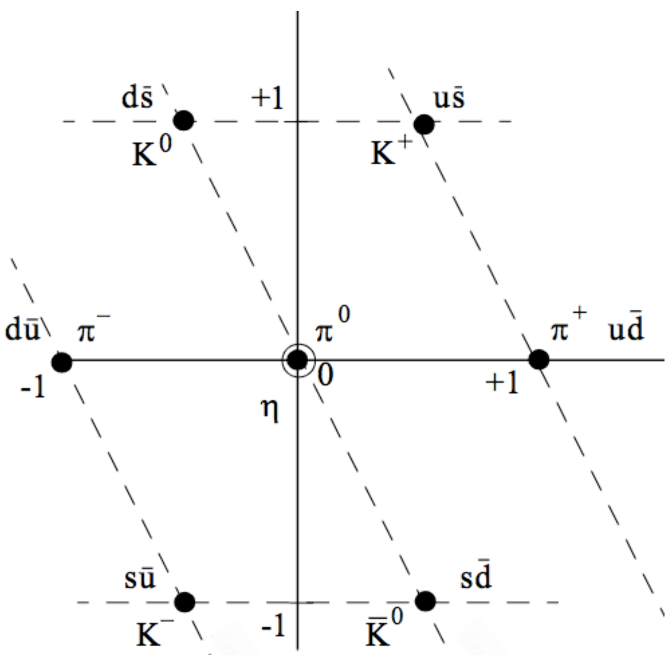
From microscopic to macroscopic DoFs

- problem:



From microscopic to macroscopic DoFs

macroscopic DoF

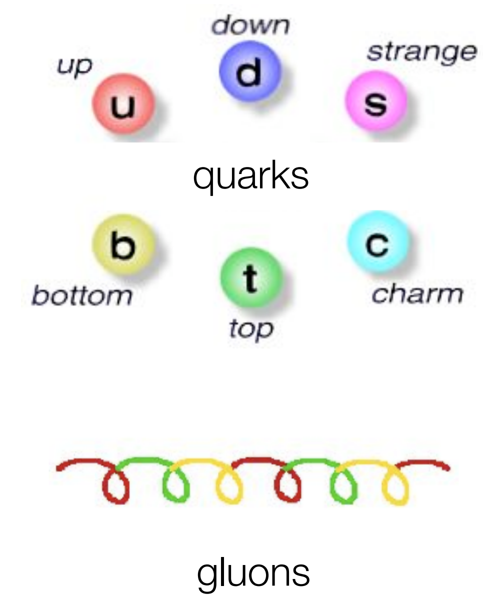


solution:

scale-dependent degrees of freedom

$$\partial_t \phi_k \sim \bar{\psi}_L \psi_R$$

microscopic DoF



$k \to 0$ $\qquad k \qquad$ $k \to \Lambda$

$$\partial_t \Gamma_k[\phi_k] = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)}[\phi_k] + R_k} - \int_x \frac{\delta \Gamma_k[\phi_k]}{\delta \phi_k} \partial_t \phi_k$$

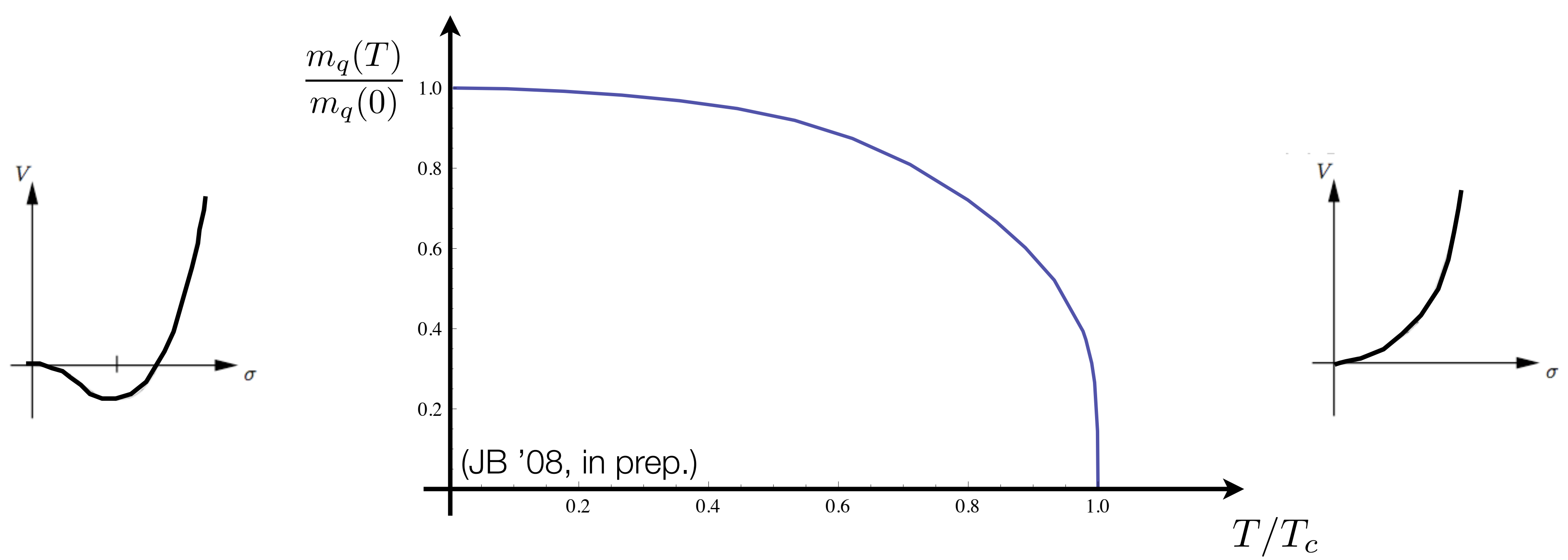
(H. Gies, C. Wetterich '01, '02; J. M. Pawlowski '05)

QCD with one quark flavor

- ansatz:

$$\Gamma_k = \int_x \Big\{ \bar{\psi}(\mathrm{i}\not{D} + \mathrm{i}\gamma_0\mu_q)\psi + \frac{\bar{\lambda}_\sigma}{2}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right] + Z_\phi \partial_\mu \phi^* \partial_\mu \phi + U(\phi^2)$$
$$+ \bar{h}\left[(\bar{\psi}_R\psi_L)\phi - (\bar{\psi}_L\psi_R)\phi^*\right] \Big\} + \Gamma_{gauge}$$

- initial conditions: $\bar{\lambda}_\sigma|_\Lambda = 0,\ \bar{\lambda}_\phi|_\Lambda = 0,\ \bar{h}|_\Lambda = 0,\ Z_\phi|_\Lambda = 0,\ \alpha_s(M_Z) = 0.117$
- allows to include (momentum-dependent) four-fermion interactions to arbitrary order can be easily included
- serves as a check for the approach incorporating “only” quark-gluon dynamics



QCD with one quark flavor: phase boundary

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - t_2 \left(\frac{\mu_q}{\pi T_c(0)} \right)^2 + \dots$$

- large N_c expansion: $t_2 \sim \frac{N_f}{N_c}$
- results from different approaches:

Method	$N_f = 1$	$N_f = 2$	$N_f = 3$	
FRG: QCD flow	0.97	---	---	(JB '08, in prep.)
Lattice: imag. μ	0.398(75)	0.50	0.602(9)	(de Forcrand et al. '03, '07)
Lattice: Taylor+Rew.	---	---	1.13(45)	(Karsch et al. '03)

red: obtained from extrapolation

QCD with one quark flavor: phase boundary

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red: obtained from extrapolation

- only one single input parameter: $\alpha_s(M_Z)$

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- ✓ Motivation
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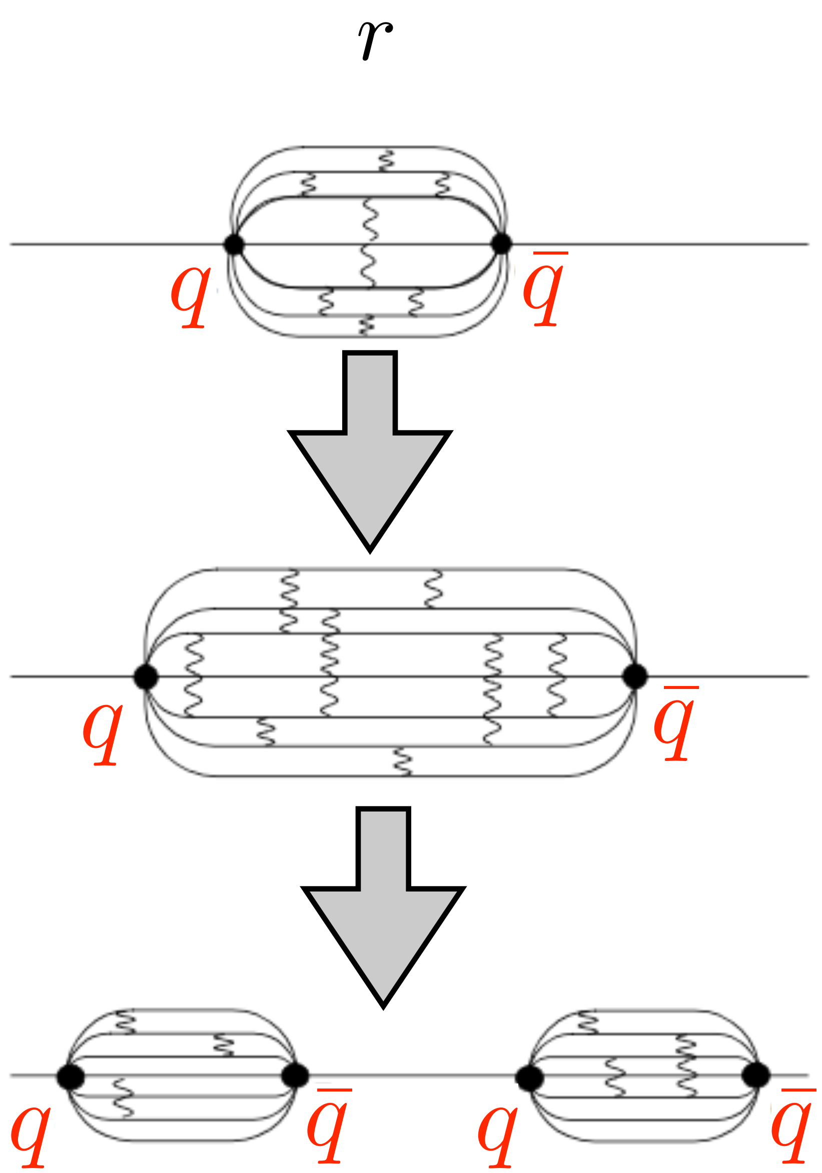
Confinement at zero temperature

potential of a quark-antiquark pair: $\mathcal{F}_{q\bar{q}}(r) \propto \sigma r$



confinement at finite temperature

- Hamiltonoperator for an electron in an EM-field:

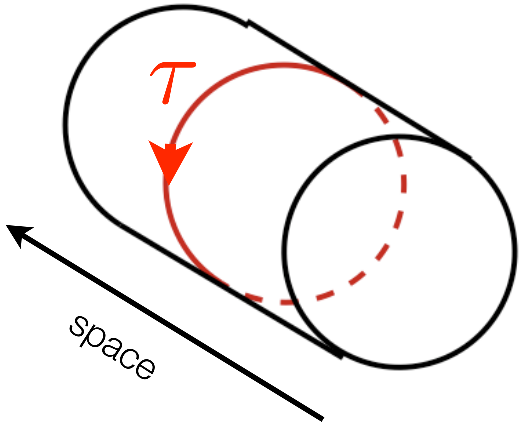
$$H_{ED} = \frac{1}{2m}\left(-\mathrm{i}\vec{\nabla} - e\vec{A}\right)^2 + e\Phi$$

confinement at finite temperature

- infinitely heavy quark moving in Euclidean time direction:

$$\frac{\partial \Psi_q}{\partial \tau} = \mathrm{i}\bar{g} A_0 \Psi_q \quad \Longrightarrow \quad \Psi_q(\vec{x},\tau) = \left[\mathrm{P}\exp\left(i\bar{g}\int_0^\tau dt A_0\right)\right]\Psi_q(\vec{x},0)$$

infinitely heavy quark propagating in (Euclidean) time direction



- Polyakov-Loop: $\tau = \beta = 1/T$ (Polyakov '78, Susskind '79)

$$\mathcal{P}(\vec{x}) = \frac{1}{N_c}\mathrm{P}\exp\left(i\bar{g}\int_0^\beta dt A_0(t,\vec{x})\right)$$

confinement at finite temperature

- expectation value of Polyakov-loop is related to the quark free energy $\mathcal{F}_q$:

$$\langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle \sim \int \mathcal{D}A \, \mathrm{Tr}_F \mathcal{P}(\vec{x}) \, e^{-S} \sim \mathrm{e}^{-\beta \mathcal{F}_q}$$

⇨ deconfinement:

$$\mathcal{F}_q \ \text{finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0$$

⇨ confinement:

$$\mathcal{F}_q \to \infty \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0$$

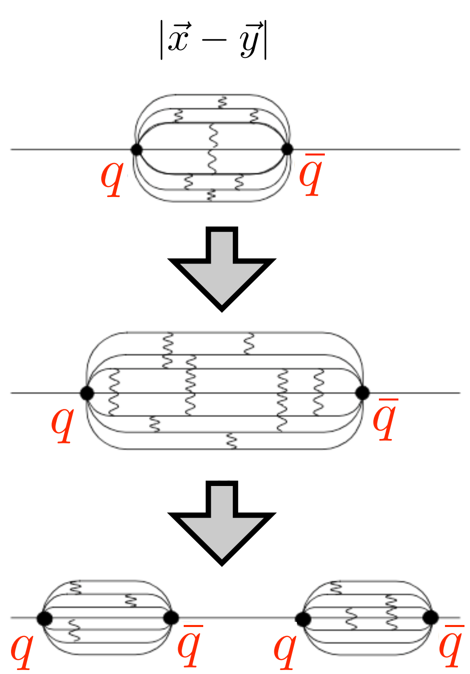
confinement at finite temperature

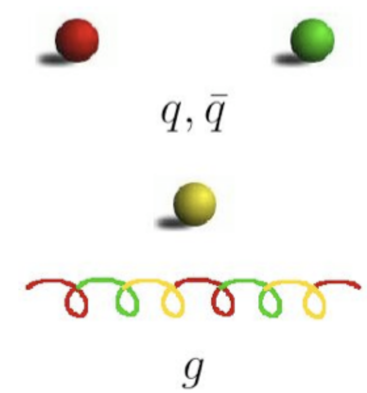
- quark-antiquark correlator:

$$\lim_{|\vec{x}-\vec{y}|\to\infty} \mathrm{e}^{-\beta\mathcal{F}_{q\bar{q}}} \sim \lim_{|\vec{x}-\vec{y}|\to\infty} \langle \mathrm{Tr}\,\mathcal{P}(\vec{x}) \cdot \mathrm{Tr}\,\mathcal{P}^{\dagger}(\vec{y})\rangle \leq |\mathrm{e}^{-\beta\mathcal{F}_q}|^2$$

$T < T_c$ | $T \geq T_c$

$$\mathcal{F}_{q\bar{q}} \sim \sigma|\vec{x}-\vec{y}|$$





deconfinement

$$\mathcal{F}_q \to \infty \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0$$

$$\mathcal{F}_q \text{ finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

confinement criterion at vanishing temperature

(JB, H. Gies, J. M. Pawlowski '07)

- (RG) Polyakov-loop potential in Landau-background-field-gauge

$$V(\beta\langle A_0\rangle) = \frac{1}{\Omega T^4}\left(\frac{1}{2}\mathrm{Tr}\ln\Gamma^{(2)}_{\mathrm{A}}[\beta\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma^{(2)}_{\mathrm{gh}}[\beta\langle A_0\rangle]\right) + \mathcal{O}(\partial_t\Gamma^{(2)})$$

- (very) high-temperature: potential is dominated by modes $k \sim p \sim T$

$$(\Gamma^{(2)}_{A}) \sim D^2[\langle A_0\rangle]\,, \qquad (\Gamma^{(2)}_{\mathrm{gh}}) \sim D^2[\langle A_0\rangle]$$

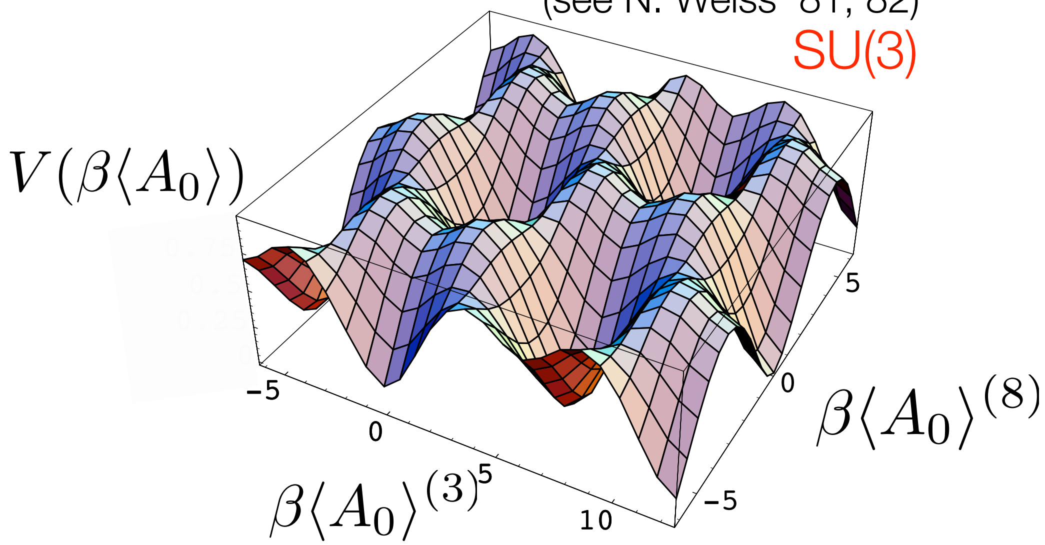
perturbative Polyakov-Loop potential

(JB, H. Gies, J. M. Pawlowski '07)

- perturbative Polyakov-loop potential in background-field gauge, $A_\mu = \delta_{\mu 0}\langle A_0 \rangle$

(see N. Weiss '81,'82)

SU(3)



minimum at $\beta\langle A_0\rangle = 0$:
deconfinement (broken Z_3-symmetry)

$$\mathcal{F}_q \text{ finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

confinement criterion at vanishing temperature

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- low-temperature: $k \sim p \sim T \lesssim \Lambda_{\mathrm{QCD}}$

$$(\Gamma^{(2)}_{A}) \sim (D^2[\langle A_0\rangle])^{1+\kappa_A}\,, \qquad (\Gamma^{(2)}_{\mathrm{gh}}) \sim (D^2[\langle A_0\rangle])^{1+\kappa_{gh}}$$

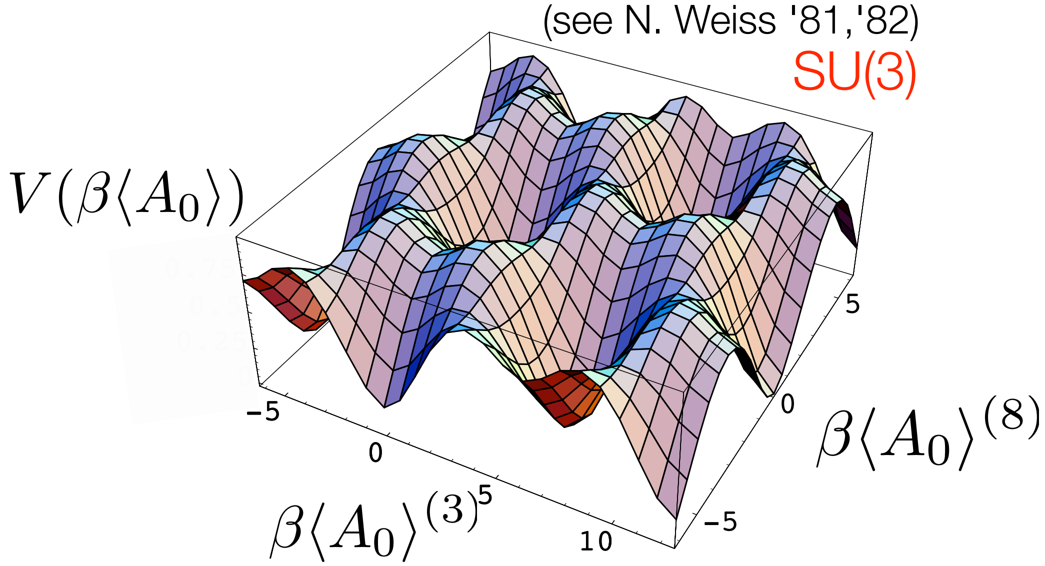
- **what if ...**

$$3\kappa_A - 2\kappa_{gh} < -2 \quad \Rightarrow \quad \kappa_{gh} > \frac{d-3}{4}$$

perturbative Polyakov-Loop potential

(JB, H. Gies, J. M. Pawlowski '07)

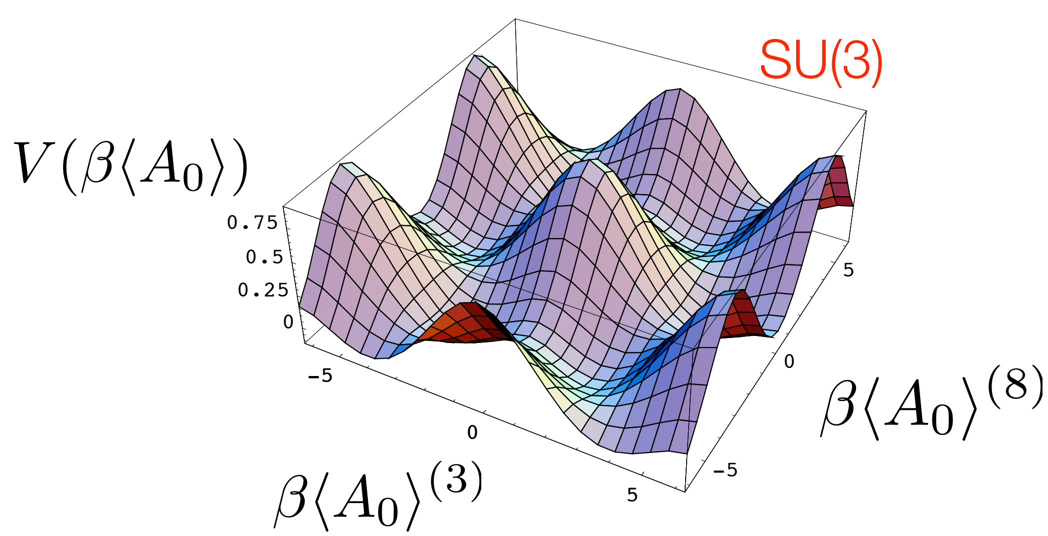
- perturbative Polyakov-loop potential in background-field gauge, $A_\mu = \delta_{\mu 0}\langle A_0\rangle$



minimum at $\beta\langle A_0\rangle = 0$:
deconfinement (broken Z_3 -symmetry)

$$\mathcal{F}_q \text{ finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

- for $T < T_c$



minimum at $\beta\langle A_0\rangle = (2/3)2\pi$:
deconfinement (broken Z_3 -symmetry)

$$\mathcal{F}_q \to \infty \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0$$

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- low-temperature: $k \sim p \sim T \lesssim \Lambda_{\mathrm{QCD}}$

$$(\Gamma^{(2)}_A) \sim (D^2[\langle A_0\rangle])^{1+\kappa_A}\,, \qquad (\Gamma^{(2)}_{\mathrm{gh}}) \sim (D^2[\langle A_0\rangle])^{1+\kappa_{gh}}$$

- **quark confinement criterion (Landau gauge)**:

$$\langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0\ :\ 3\kappa_A - 2\kappa_{gh} < -2$$

⇒

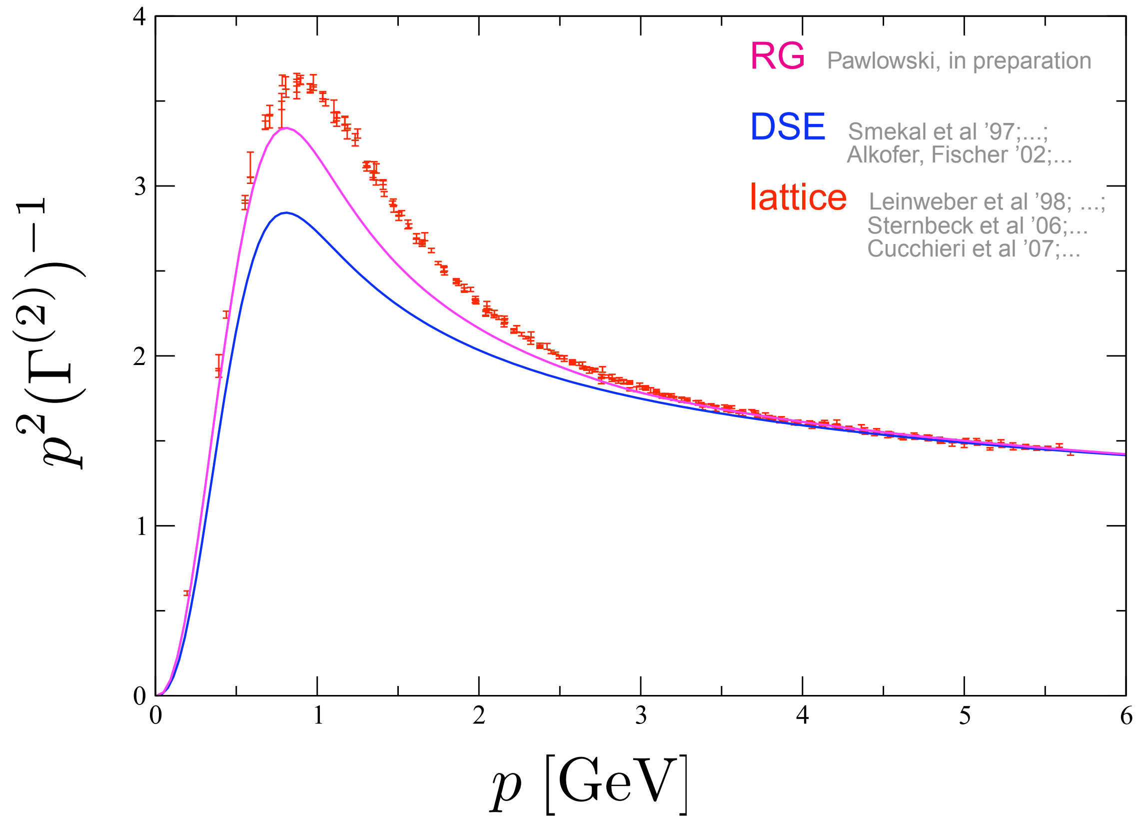
$$\langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0\ :\ \kappa_{gh} > \frac{d-3}{4}$$

- quark confinement induced by IR gluon suppression

- confer: Kugo-Ojima criterion: $\kappa_{gh} > 0$ (Kugo, Ojima '79) Gribov-Zwanziger condition: $\kappa_{gh} > \frac{1}{2}$ (Gribov '78; Zwanziger '94,'03)

Landau-gauge propagators & color confinement

$$(\Gamma_A^{(2)})^{-1} \xrightarrow{\text{IR}} \frac{1}{(p^2)^{1-2\kappa}}$$



- results for κ_{gh}

Method	κ_{gh}
DSE/SQ	0.595
FRG	$0.539 \leq \kappa \leq 0.595$
Lattice	...

(Lerche, v. Smekal '02; Zwanziger '02)

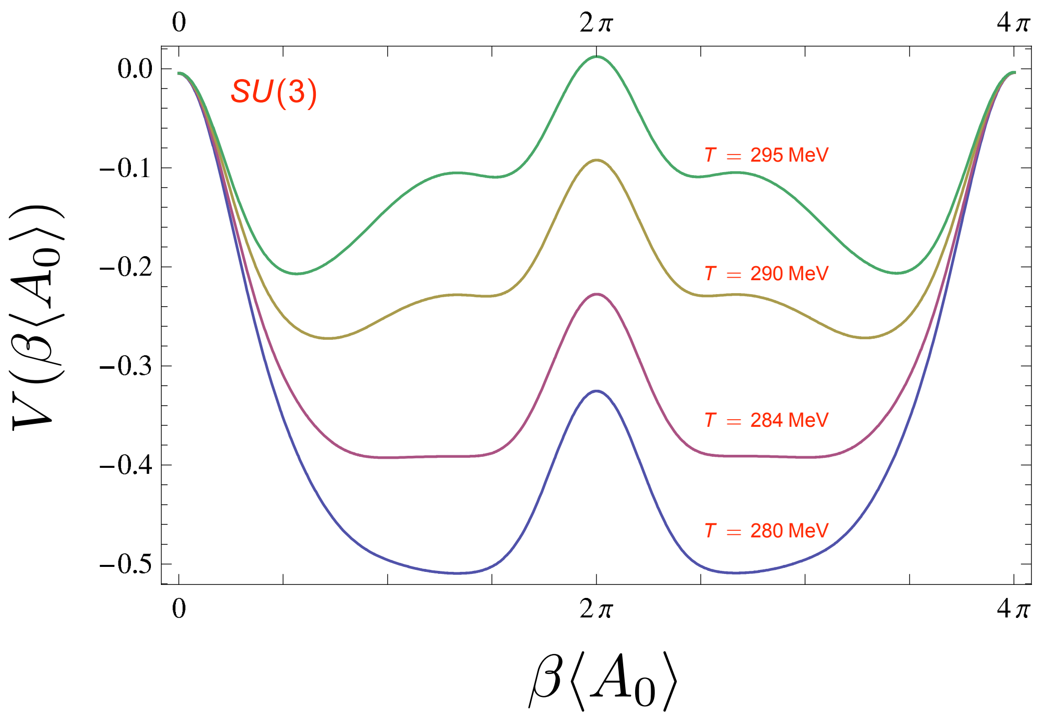
(Pawlowski, Litim, Nedelko, v. Smekal '03; Fischer, Gies '04)

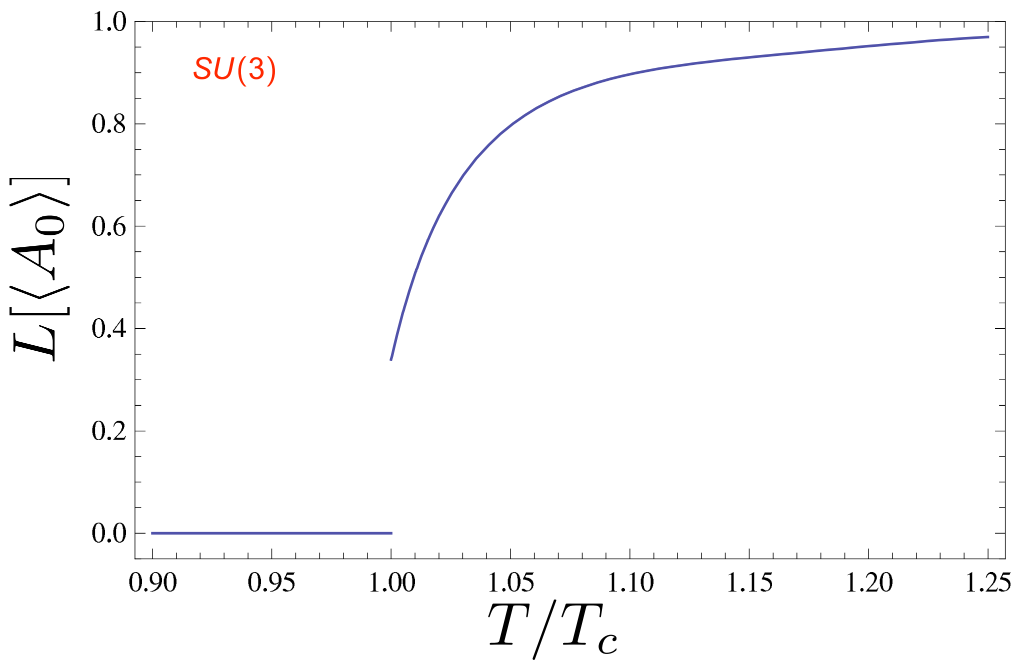
(Sternbeck et al.'05; Olivera, Silva '06; Cucchieri, Mendes '06;Cucchieri, Mendes '07; Sternbeck et al.'07)

Polyakov-Loop Potential in Landau-gauge

(JB, H. Gies, J. M. Pawlowski '07)

- order parameter $L[\langle A_0 \rangle] = \frac{1}{N_c} \mathrm{Tr}_F \exp\left(i \int_0^\beta dt \langle A_0 \rangle \right)$





- first order phase transition for SU(3) (and second order for SU(2))

- SU(3): $T_c = 284\,\mathrm{MeV} (= 0.646\sqrt{\sigma})$ Lattice QCD: $T_c = 0.646\sqrt{\sigma}$ (Kaczmarek et al.)

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Conclusions

- FRG allows to bridge the gap between regimes with different DoF
- good agreement with Lattice QCD studies for chiral as well as deconfinement phase transition
- critical number of quark flavors for SU(3): $N_{f,cr} = 12$
- shape of the phase boundary near $N_{f,cr}$ is determined by the underlying IR fixed point scenario (testable prediction!)
- promising results for finite chemical potential
- criterion for quark confinement

Outlook

- study finite chemical potential for $N_f = 2$ and $N_f = 3$

    ➡ first ‘shot’: phase diagram from bosonization at a fixed scale (with H. Gies, J. M. Pawlowski, B. J. Schaefer)

- order of the phase transition? (from finite-volume scaling, with B. Klein)

- deconfinement and chiral phase transition at the same temperature?

- ...