Quantum Master Equation for Yang-Mills Theory

in ERG

(量子論的マスター方程式)

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Motivation

♢ One of the most important subjects in ERG:

How to realize gauge symmetries, naively not compatible with reg. scheme.

$\Rightarrow$ Symmetries enforced to undergo deformation

such that they become (more or less) compatible with given reg. scheme,

and reduce to the standard realization in cutoff removing limit.

Discuss cutoff-dependent realization of symmetries

- Prototype : chiral symmetry on the lattice

described by Ward-Takahashi (WT) identity $\equiv$ Ginsparg-Wilson relation

Is it possible to formulate cutoff-dependent gauge symmetries in ERG
in parallel with lattice chiral symmetry ?

"yes"

- Need a suitable machinery = Batalin-Vilkovisky (BV) antifield formalism

The presence of local (as well as global) symmetries

$\Leftrightarrow$ Quantum Master Equation (QME) $\Sigma[\phi, \phi^*] = 0$ in BV formalism
(applies not to Legendre average action Γ but to Wilson action S)

- Ginsparg-Wilson (GW) relation for lattice chiral symmetry ('83) (Ukita-So-YI '03)
- Becchi's fine tuning eq. in ERG approach to Yang-Mills (YM) theory ('93)

can be identified with QME.

♢ Summary: Results are complicated but simple

A key concept characterizing cutoff dependent symmetries

≡ composite objects (operators) (Becchi '93, Sonoda '07)

BRS tr. in the standard realization

$$\hat{\delta}\Phi^A = R^{(1)A}{}_B \Phi^B + \frac{1}{2} R^{(2)A}{}_{BC} \Phi^B \Phi^C \, .$$

$\Rightarrow$

$$\delta\Phi^A = R^{(1)A}{}_B [\Phi^B]_{com} + \frac{1}{2} R^{(2)A}{}_{BC} [\Phi^B \Phi^C]_{com} \, .$$

in cutoff-dependent realization.
($\delta^2 \neq 0 \Rightarrow \delta_Q^2 = 0$ if $\Sigma = 0$: only possible in the BV formalism)

What we discuss here:

Express these composite objects in terms of Wilson action,

and see how they emerge in the BV formalism

♣ Other approaches:

1) finding symmetry preserving reg. (Morris *et al.* '00, Rosten '06)

2) using modified Slavnov-Taylor identities = broken Zinn-Justin Eq.

for generator of 1PI cutoff vertex functions Γ : $(\Gamma,\ \Gamma) \propto \left(\Gamma^{(2)}\right)^{-1}$

to control symmetry breaking effects $\Rightarrow$ not algebraic

(Ellwanger, Bonini *et al.*. '94, Litim *et al.*. '99, Morris *et al.* '00, Freire *et al.* '01, Pawlowski '05, Gies '06 $\cdots$)

"modified" or "broken" identities not exclude presence of exact symmetry

♢ Outline

[1] BV formalism and the QME

[2] A toy model for global SU(2)

[3] Structure of Wilson master action for YM theory

([4] Existence of perturbative solutions of QME)

[5] Summary and outlook

Batalin-Vilkovisky (BV) formalism and the QME

◇ For generic gauge fixed action $S_0[\phi]$,

consider "extended" action

$$S[\phi, \phi^*] = S_0[\phi] + \phi^*_A \delta\phi^A \ ,$$

introducing antifields (AF) ϕ^*_A for fields ϕ^A as sources for BRS tr. $\delta\phi^A$:

For QED, $\phi^*_A \delta\phi^A \Rightarrow A^*_\mu \delta A_\mu + \psi^* \delta\psi + \delta\bar{\psi}\bar{\psi}^* = A^*_\mu \partial_\mu C + ie(\psi^* C \psi + \bar{\psi} C \bar{\psi}^*)$

- ϕ^A and ϕ^*_A have opposite Grassmann parity: e.g. $\epsilon(A^*_\mu) = 1$ while $\epsilon(A_\mu) = 0$.

- "canonical structure" defined by the anti-bracket

$$(X,\ Y) = \frac{\partial^r X}{\partial\phi^A}\frac{\partial^l Y}{\partial\phi^*_A} - \frac{\partial^r X}{\partial\phi^*_A}\frac{\partial^l Y}{\partial\phi^A}, \qquad r(l) : \text{derivatives from right (left)}.$$

Then, $\delta\phi^A = \partial^l S/\partial\phi^*_A = (\phi^A,\ S)$, so that (classical) BRS inv. of the action is expressed by the classical master equation

$$(S,\ S) = 2\,\frac{\partial^r S}{\partial\phi^A}\frac{\partial^l S}{\partial\phi^*_A} = 0$$

♣ More generally, for partition function $\int \mathcal{D}\phi \exp -S$, the total change induced by $\delta S = (\partial^r S/\partial\phi_A)\,\delta\phi^A$ and by $\delta \ln \mathcal{D}\phi = \partial^r \delta\phi^A/\partial\phi^A$ is given by

$$\Sigma[\phi,\phi^*] \equiv \frac{\partial^r S}{\partial\phi^A}\frac{\partial^l S}{\partial\phi^*_A} - \frac{\partial^r}{\partial\phi^A}\delta\phi^A = \frac{1}{2}(S,\ S) - \Delta S \qquad \left(\Delta \equiv (-)^{\epsilon(A)+1}\frac{\partial^r}{\partial\phi^A}\frac{\partial^r}{\partial\phi^*_A}\right)$$

$\Sigma[\phi,\phi^*]$: Quantum Master Operator

$\Sigma[\phi,\phi^*] = 0$ QME : (express quantum BRS inv.)

$S[\phi,\phi^*] = S_M[\phi,\phi^*]$: master action

A toy model for global SU(2) symmetry

♢ Wilson RG approach introduces

UV fields : $\{\phi^A, \ \phi^*_A\}$ and IR fields : $\{\Phi^A, \ \Phi^*_A\}$

- Consider a UV theory described by generating functional

$$\mathcal{Z}_\phi = \int \mathcal{D}\phi^* \prod_A \delta(\phi^*_A) \ \mathcal{D}\phi \exp\left(-\mathcal{S}[\phi, \ \phi^*]\right)$$

For “coarse graining”, $\Phi^A \approx K\phi^A \quad (\Phi^*_A \approx K^{-1}\phi^*_A)$, with momentum cutoff function

$$K(p) = K\left(\frac{p^2}{\Lambda^2}\right) \approx \begin{cases} 1 & \text{for } p^2 < \Lambda^2 \\ 0 & \text{for } p^2 > \Lambda^2 \end{cases},$$

perform blocking (Wilson-Kogut ’74, Ginsparg-Wilson ’82, Wetterich, Bonini *et al.*, Morris, $\cdots$)

$$
\begin{aligned}
\mathcal{Z}_\phi &= \int \mathcal{D}\phi^* \prod_A \delta(\phi^*_A)\ \mathcal{D}\phi \int \mathcal{D}\Phi^* \prod_A \delta(\Phi^*_A - K^{-1}\phi^*_A)\ \mathcal{D}\Phi \\
&\quad \times \exp - \left[\mathcal{S}[\phi,\ \phi^*] + \frac{1}{2}(\Phi - K\phi)^A\, \alpha^\Lambda_{AB}\, (\Phi - K\phi)^B \right] \\
&= \int \mathcal{D}\phi^* \prod_A \delta(\phi^*_A) \mathcal{D}\Phi^* \prod_A \delta(\Phi^*_A - K^{-1}\phi^*_A)\ \mathcal{D}\Phi\ \ \exp - S[\Phi, \Phi^*]
\end{aligned}
$$

where the Wilson action is given by

$$
\exp - S[\Phi, \Phi^*] = \int \mathcal{D}\phi \exp - \left[\mathcal{S}[\phi,\ \phi^*] + \frac{1}{2}(\Phi - K\phi)^A\, \alpha^\Lambda_{AB}\, (\Phi - K\phi)^B \right]
$$

♢ Consider fermionic UV theory described by $\mathcal{S}[\psi, \bar{\psi}]$

which is invariant under global SU(2) tr.

$$\begin{aligned}\delta\psi(p) &= ic^a T^a \psi(p)\\ \delta\bar\psi(p) &= -ic^a\bar\psi(p)T^a\\ \delta c^a &= \frac{1}{2}\varepsilon_{abc}c^b c^c = \frac{1}{2}(c\times c)^a\end{aligned}$$

$T^a = \sigma^a/2$ $\qquad$ c^a : constant "ghosts"

- extended action $\oplus$ blocking term with IR fields $\{\Psi, \bar\Psi\}$ is given by

$$\mathcal{S}[\psi,\bar\psi] + \int_p \left[\psi^*(-p)ic^aT^a\psi(p) - ic^a\bar\psi(-p)T^a\bar\psi^*(p) + (\bar\Psi - K\bar\psi)(-p)\alpha^\Lambda(\Psi - K\psi)(p)\right]$$

$$= \mathcal{S}[\psi,\bar\psi] - \int_p \left[\bar\psi(-p)\alpha^\Lambda K\left(\Psi - i(\alpha^\Lambda)^{-1}c^aT^a\bar\Psi^*\right) + \left(\bar\Psi - i\Psi^* c^aT^a(\alpha^\Lambda)^{-1}\right)\alpha^\Lambda K\psi(p) + \cdots\right]$$

- Wilson action expressed by effective sources for ψ and $\bar\psi$ proportional to :

$\Psi - i(\alpha^\Lambda)^{-1}c^aT^a\bar\Psi^*$, $\qquad$ $\bar\Psi - i\Psi^*c^aT^a(\alpha^\Lambda)^{-1}$ $\qquad$ (shift of variables)

- free-field Wilson action takes the form

$$S[\Phi,\ \Phi^*] \;=\; \int_p \Big[\left(\bar\Psi - i\Psi^* c^a T^a (\alpha^\Lambda)^{-1}\right)(-p)(D-\alpha^\Lambda)(p)\left(\Psi - i(\alpha^\Lambda)^{-1} c^a T^a \bar\Psi^*\right)(p) \\ + \bar\Psi(-p)\alpha^\Lambda \Psi(p)\Big] + c_a^*\,\frac{1}{2}(c\times c)^a$$

- Take SU(2) non-invariant kernel

$$\alpha^\Lambda(p) = \alpha_0^\Lambda(p)\mathbf{1} + \alpha_3^\Lambda(p)\sigma_3$$

- (classical) master equation $\Sigma = (S,\ S)/2 = 0$ gives

$$T^a D(p) - D(p)\hat T^a(p) = 0 \qquad \mathrm{GW-like\ relation\ !}$$

where

$$\hat T^a(p) = T^a + [(\alpha^\Lambda)^{-1}(p),\ T^a]D(p)$$

New generator $\hat{T}^a$ satisfies SU(2) algebra (Itoh-So-YI '01)

$$[\hat{T}^a(p),\ \hat{T}^b(p)] = i\varepsilon_{abc}\hat{T}^c(p)$$

Even if SU(2) symmetry broken explicitly in the standard realization

through blocking, it is realized in a cutoff dependent way !

Structure of Wilson master action for YM theory

♢ For YM theory, we introduce UV cutoff Λ_0 and IR cutoff Λ

via cutoff functions, $K_0(p) = K(p^2/\Lambda_0^2)$ and $K(p) = K(p^2/\Lambda^2)$.

- Consider UV theory and blocking procedure

$$\begin{aligned}
\mathcal{Z}_\phi &= \int \mathcal{D}\phi^* \prod_A \delta(\phi^*_A)\mathcal{D}\phi \exp\left(-\mathcal{S}[\phi,\ \phi^* : \Lambda_0]\right) \\
&= \int \mathcal{D}\phi^* \prod_A \delta(\phi^*_A)\mathcal{D}\phi \int \mathcal{D}\Phi^* \prod_A \delta(\Phi^*_A - K_0 K^{-1}\phi^*_A)\mathcal{D}\Phi \\
&\quad \times \exp -\left[\mathcal{S}[\phi,\ \phi^*] + \frac{1}{2}(K_0\Phi - K\phi)\cdot \frac{D}{K_0 K(K_0 - K)} \cdot (K_0\Phi - K\phi)\right]
\end{aligned}$$

For UV action

$$\mathcal{S}[\phi,\ \phi^* : \Lambda_0] = \frac{1}{2}\phi \cdot K_0^{-1} D \cdot \phi + \mathcal{S}_I[\phi,\ \phi^* : \Lambda_0]\,,$$

$$\mathcal{S}_I[\phi,\ \phi^* : \Lambda_0] = \mathcal{S}_I[\phi : \Lambda_0] + \phi^*_A R^A[\phi]$$
$$R^A[\phi] = \delta\phi^A = R^{(1)A}{}_B \phi^B + \frac{1}{2} R^{(2)A}{}_{BC} \phi^B \phi^C \ .$$

Decompose ϕ^A as $\phi^A = \Phi^A + \chi^A$ to obtain expression for the Wilson action

$$S[\Phi,\ \Phi^*] = \frac{1}{2}\Phi \cdot K^{-1} D \cdot \Phi + S_I[\Phi,\ \Phi^* : \Lambda]$$
$$\exp - S_I[\Phi,\ \Phi^*] \equiv \int \mathcal{D}\chi \exp - \Big(\frac{1}{2}\chi \cdot (K_0 - K)^{-1} D \cdot \chi + \mathcal{S}_I[\Phi + \chi,\ \phi^* : \Lambda]\Big)$$

To exract AF dependence $(\Phi^* = (K_0)^{-1} K \phi^*)$, expand w.r.t χ

$$\phi^*_A R^A[\phi = \Phi + \chi] = \phi^*_A R^A[\Phi] + \mathcal{J}_A \chi^A + \frac{1}{2}\phi^*_A R^{(2)A}{}_{BC} \chi^B \chi^C$$

where effective sources for χ^A given by $\quad \mathcal{J}_A = \phi^*_B \left(R^{(1)B}{}_A + R^{(2)B}{}_{CA} \Phi^C \right) \ .$

$$\phi^*_A R^A[\Phi] \quad \to \quad \phi^*_A R^A[\Phi] \int \mathcal{D}\chi \exp[\ \cdots\ ]$$

$$\phi^*_A R^{(2)A}{}_{BC} \chi^B \chi^C \quad \to \quad \phi^*_C R^{(2)C}{}_{AB} \frac{\partial^l}{\partial \mathcal{J}_A} \frac{\partial^l}{\partial \mathcal{J}_B} \int \mathcal{D}\chi \exp[\ \cdots\ ]$$

- Rewrite the gaussian as complete square form

$$\begin{aligned}&\frac{1}{2}\chi \cdot (K_0 - K)^{-1} D \cdot \chi + \mathcal{J} \cdot \chi \\ &= \frac{1}{2}\chi' \cdot (K_0 - K)^{-1} D \cdot \chi' - \frac{1}{2}(-)^{\epsilon(\mathcal{J})} \mathcal{J} \cdot (K_0 - K) D^{-1} \mathcal{J}\end{aligned}$$

where $\chi' = \chi + \mathcal{J}(K_0 - K)D^{-1}$.

Introduce new variables according to Higashi, Itou and Kugo

$\Phi' = \Phi - \mathcal{J}(K_0 - K)D^{-1}$ for which $\chi' + \Phi' = \chi + \Phi$, We find

$$
\begin{aligned}
&\exp -S_I[\Phi,\ \Phi^* : \Lambda] \\
&= \exp -\Big(\phi^*_A R^A[\Phi]\Big)\ \exp\left(-\frac{1}{2}\phi^*_C R^{(2)C}{}_{AB}\frac{\partial^l}{\partial \mathcal{J}_A}\frac{\partial^l}{\partial \mathcal{J}_B}\right) \\
&\qquad \times \exp\ \left(\frac{1}{2}(-)^{\epsilon(\mathcal{J})}\mathcal{J}\cdot(K_0 - K)D^{-1}\mathcal{J}\right) \\
&\qquad \times \underline{\int \mathcal{D}\chi' \exp -\Big(\frac{1}{2}\chi'\cdot(1-K)^{-1}D\cdot\chi' + \mathcal{S}_I[\Phi' + \chi' : \Lambda_0]\Big)} \\
&= \exp -\Big(\phi^*_A R^A[\Phi]\Big)\ \exp\left(-\frac{1}{2}\phi^*_C R^{(2)C}{}_{AB}\frac{\partial^l}{\partial \mathcal{J}_A}\frac{\partial^l}{\partial \mathcal{J}_B}\right) \\
&\qquad \times \exp\ \left(\frac{1}{2}(-)^{\epsilon(\mathcal{J})}\mathcal{J}\cdot(K_0 - K)D^{-1}\mathcal{J}\right)\underline{\exp -S_I[\Phi' : \Lambda]}
\end{aligned}
$$

where

$$
S_I[\Phi' : \Lambda] \equiv S_I[\Phi',\ \Phi^* = 0 : \Lambda].
$$

- The Wilson master action :

$$
\begin{aligned}
S_M[\Phi,\ \Phi^* : \Lambda] = &\frac{1}{2}\Phi\cdot K^{-1}D\cdot\Phi + \phi^*_A R^A[\Phi] - \frac{1}{2}(-)^{\epsilon(\mathcal{J})}\mathcal{J}\cdot(K_0-K)D^{-1}\mathcal{J} + S_I[\Phi' : \Lambda] \\
&-\log\left(\exp\left(S_I[\Phi' : \Lambda] - (-)^{\epsilon(\mathcal{J})}\mathcal{J}\cdot(K_0-K)D^{-1}\mathcal{J}/2\right)\exp\left(-\frac{1}{2}\phi^*_C {R^{(2)C}}_{AB}\frac{\partial^l}{\partial\mathcal{J}_A}\frac{\partial^l}{\partial\mathcal{J}_B}\right)\right. \\
&\left.\times\exp-\left(S_I[\Phi' : \Lambda] - (-)^{\epsilon(\mathcal{J})}\mathcal{J}\cdot(K_0-K)D^{-1}\mathcal{J}/2\right)\right) .
\end{aligned}
$$

- Define BRS tr. by

$$
\delta\Phi^A = \left[\frac{\partial^l S_I}{\partial\Phi^*_A}\right]_{\Phi^*=0} = K_0^{-1}K\left({R^{(1)A}}_B\left[\Phi^A\right]_{\rm com} + \frac{1}{2}{R^{(2)A}}_{BC}\left[\Phi^A\ \Phi^B\right]_{\rm com}\right)
$$

$$
\begin{aligned}
\left[\Phi^A\right]_{\rm com} &\equiv \Phi^A - (K_0-K)\left(D^{-1}\right)^{AB}\frac{\partial^l S_I}{\partial\Phi^B} \\
\left[\Phi^A\ \Phi^B\right]_{\rm com} &\equiv \left[\Phi^A\right]_{\rm com}\ \left[\Phi^B\right]_{\rm com} - (K_0-K)\left(D^{-1}\right)^{AC}(K_0-K)\left(D^{-1}\right)^{BD}\frac{\partial^l\partial^l S_I}{\partial\Phi^C\partial\Phi^D}
\end{aligned}
$$

• $\mathcal{O}$: $\left[\Phi^A\right]_{\rm com}$ and $\left[\Phi^A\ \Phi^B\right]_{\rm com}$ are composite op. satisfying RG flow eq.

$$\dot{\mathcal{O}} = \frac{\partial^r \mathcal{O}}{\partial \Phi^A} \left(\dot{K} D^{-1}\right)^{AB} \frac{\partial^l S_I}{\partial \Phi^B} - (-)^{\epsilon_A(\epsilon_{\mathcal{O}}+1)} \left(\dot{K} D^{-1}\right)^{AB} \frac{\partial^l \partial^r \mathcal{O}}{\partial \Phi^B \partial \Phi^A}$$

♢ BRS tr. for YM theory obtained our general formula

$$\begin{aligned}
\delta A_\mu(p) &= \frac{K(p)}{K_0(p)}\left(-ip_\mu K(p)[C(p)]_{\rm com} + \int_q \left[A_\mu(q) \times C(p-q)\right]_{\rm com}\right) \\
\delta \bar{C}(p) &= i\frac{K(p)}{K_0(p)}[B(p)]_{\rm com} \\
\delta C(p) &= \frac{K(p)}{2K_0(p)} \int_q \left[C(q) \times C(p-q)\right]_{\rm com}
\end{aligned}$$

where

$$[A_\mu(p)]_{\rm com} = A_\mu(p) - \frac{K_0(p) - K(p)}{p^2} \left(\delta_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right) \frac{\partial S_I}{\partial A_\mu(-p)}$$

$$[B(p)]_{\rm com} = B(p) + i\ p_\mu \frac{K_0(p) - K(p)}{p^2} \frac{\partial S_I}{\partial A_\mu(-p)}$$

$$[C(p)]_{\rm com} = C(p) + i \frac{K_0(p) - K(p)}{p^2} \frac{\partial S_I}{\partial \bar{C}(-p)}$$

$$[A_\mu(q) \times C(p-q)]_{\rm com} = [A_\mu(q)]_{\rm com} \times [C(p-q)]_{\rm com}$$

$$+ i \frac{K_0(q) - K(q)}{q^2} \left(\delta_{\mu\nu} - (1-\xi) \frac{q_\mu q_\nu}{q^2} \right) \frac{K_0(p-q) - K(p-q)}{(p-q)^2} \frac{\partial^l}{\partial A_\nu(-q)} \times \frac{\partial^l S_I}{\partial \bar{C}(-p+q)}$$

$$[C(q) \times C(p-q)]_{\rm com} = (C(q))_{\rm com} \times (C(p-q))_{\rm com}$$

$$+ \frac{K_0(q) - K(q)}{q^2} \frac{K_0(p-q) - K(p-q)}{(p-q)^2} \frac{\partial^l}{\partial \bar{C}(-q)} \times \frac{\partial^l S_I}{\partial \bar{C}(-p+q)}$$

- The WT identity for YM theory

$$\Sigma[\Phi,\ \Lambda] \equiv \int_p \Bigg(\frac{\partial S}{\partial A_\mu(p)} \delta A_\mu(p) + \frac{\partial^r S}{\partial \bar{C}(p)} \delta \bar{C}(p) + \frac{\partial^r S}{\partial C(p)} \delta C(p)$$

$$- \frac{\partial}{\partial A_\mu(p)} \delta A_\mu(p) - \frac{\partial^r}{\partial C(p)} \delta C(p) \Bigg) = 0$$

Existence of perturbative solutions of QME (Becchi '93, Sonoda 07)

◇ Introduce equality $X \approx 0 \quad \Leftrightarrow \quad X = \mathcal{O}(1/\Lambda_{UV}^2)$.

In perturbative computation, we can take $\Lambda_0 \to \infty$ and $\Lambda = \Lambda_{UV}$.

Make loop expansion

$$S = \sum_{l=0}^{\infty} S_l \ , \qquad \Sigma = \sum_{l=0}^{\infty} \Sigma_l \, .$$

$S_0 \approx$ the standard YM classical action: $(S_0,\ S_0) \approx 0$

Define the classical BRS tr. by $\delta_c Y \approx (Y, S_0) \qquad \delta_c^2 \approx 0$

Assume the QME satisfied up to $(l-1)$-th loop

$$\Sigma_{l-1}[\Phi, \Phi^*] = \frac{1}{2} \sum_{i=0}^{l-1} (S_i,\ S_{l-1-i}) - \Delta S_{l-2} \approx 0$$

Decompose the Quantum Master Operator at l-th loop Σ_l as

$$\Sigma_l[\Phi, \Phi^*] \quad = \quad (S_l,\ S_0) + \frac{1}{2}\sum_{i\neq 0}(S_i,\ S_{l-i}) - \Delta S_{l-1} = \delta_c S_l + \Sigma'_l\,.$$

and fix unknown S_l as follows.

The identity $\delta_Q \Sigma = 0$ gives at l-loop level

$$\delta_c \Sigma_l = \delta_c^2 S_l + \delta_c \Sigma'_l = \delta_c \Sigma'_l \approx 0\,.$$

Σ'_l : dimension 5, ghost number 1 and closed under δ_c

- General cohomological argument on pure YM theory (Barnich, Brandt and Henneaux '93)

$\Leftrightarrow \Sigma'_l \approx \delta_c \exists(-S_l) \quad \Leftrightarrow \quad \Sigma_l \approx 0.$

$\Rightarrow$ Existence of perturbative solutions to QME.

Summary and outlook

◇ General method to construct QME for gauge theories.

◇ BRS tr. characterized by $[\Phi]^A_{com}$ and $[\Phi^A\Phi^B]_{com}$ expressed in terms of Wilson action.

$\Phi^A \longrightarrow [\Phi]^A_{com}$: shift of variables $\quad \Phi^A \rightarrow \Phi'^A$

$\Phi^A\Phi^B \longrightarrow [\Phi^A\Phi^B]_{com}$: exponentiate $\quad \Phi^*_C L^C{}_{AB}\Phi^A\Phi^B \rightarrow \exp(\Phi^*_C L^C{}_{AB}\cdots)$

◇ Application to QCD and Super-YM

◇ How to combine QME (WT) analysis with RG flow ?