Background Independence

and

Asymptotic Safety

in

Quantum Einstein Gravity

(M. Reuter)

(I) The Effective Average Action approach to quantum gravity and Asymptotic Safety

(II) The importance of "Background Independence" for Asymptotic Safety

(or: What is the physical meaning of a coarse graining scale when the metric is quantized?)

Standard quantization of gravity $\widehat{=}$

degrees of freedom carried by:	$g_{\mu\nu}(x)$
bare action:	$\int d^4x \sqrt{-g}\, R$
calculational method:	$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G}\, h_{\mu\nu}$, perturbative quantization, renormalization

What should be given up in order to arrive at a "fundamental" or "microscopic" quantum theory of gravity?

String Theory: d.o.f., action, calc. meth.

Loop Quantum Gravity: d.o.f., calc. meth.

Asymptotic Safety: calc. meth., action

Asymptotic Safety Approach:

- degrees of freedom carried by $g_{\mu\nu}$

- quantization/renormalization is non-perturbative in an essential way

- bare action Γ_* is not an ad hoc assumption, but a prediction:

  $\Gamma_* \sim \int d^4x \sqrt{-g}\, R$ + "more" is a non-Gaussian fixed point of the (∞-dimensional, non-pert.) Wilsonian renormalization group flow

- fixed point "controls" UV divergences

The Effective Average Action $\Gamma_k[g_{\mu\nu}, \cdots]$

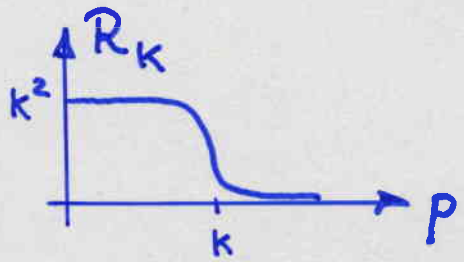
- Scale-dependent (coarse grained) effective action functional for the metric

- Defines family of effective field theories:
  $\{\Gamma_k \mid 0 \leq k < \infty\}$

- Built-in IR cutoff: Only metric fluctuations with cov. momentum $p > k$ are integrated out fully.

  Modes with $p < k$ are suppressed by "mass" term added to the bare action:

  $$(\text{mass})^2 = \mathcal{R}_k(p^2)$$



- $\Gamma_{k\to\infty} = S$ = bare action

  $\Gamma_{k\to 0} = \Gamma$ = standard eff. action

- Γ_k satisfies a FRGE; symbolically:

  $$k\partial_k \Gamma_k = \frac{1}{2}\, \mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\partial_k R_k\right]$$

- Natural (nonperturbative) approximation scheme: project RG flow onto truncated theory space

Construction of Γ_k for Gravity

M.R. 1996

- starting point: $\int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]}$
- decompose $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

  arbitrary backgrd. metric

- add background gauge fixing $S_{gf}[h;\bar{g}]$ + ghost terms
- expand $h_{\mu\nu}$ in $\bar{D}^2$-eigenmodes, and introduce IR cutoff k^2: only modes with generalized momenta ($\bar{D}^2$-eigenvalues) $> k$ are integrated out.
- add sources: generating fctl. $W_k[\text{sources}; \bar{g}]$

$$\xrightarrow{\text{Legendre transf.}}$$

$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle$ $\qquad \Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \text{ghosts}]$

- derive exact RG equation from path integral:

$$k \frac{\partial}{\partial k} \Gamma_k[g, \bar{g}, \cdots] = \mathrm{Tr}(\cdots)$$

- "Ordinary" diffeomorphism invariant action:

$$\Gamma_k[g] = \Gamma_k[g, \bar{g}=g, \text{ghosts}=0]$$

Taking the UV-limit in QEG

If there exists a non-Gaussian Fixed Point Γ_*, $\beta_i(\Gamma_*) = 0$, Quantum Einstein Gravity is nonperturbatively renormalizable ("asymptotically safe").

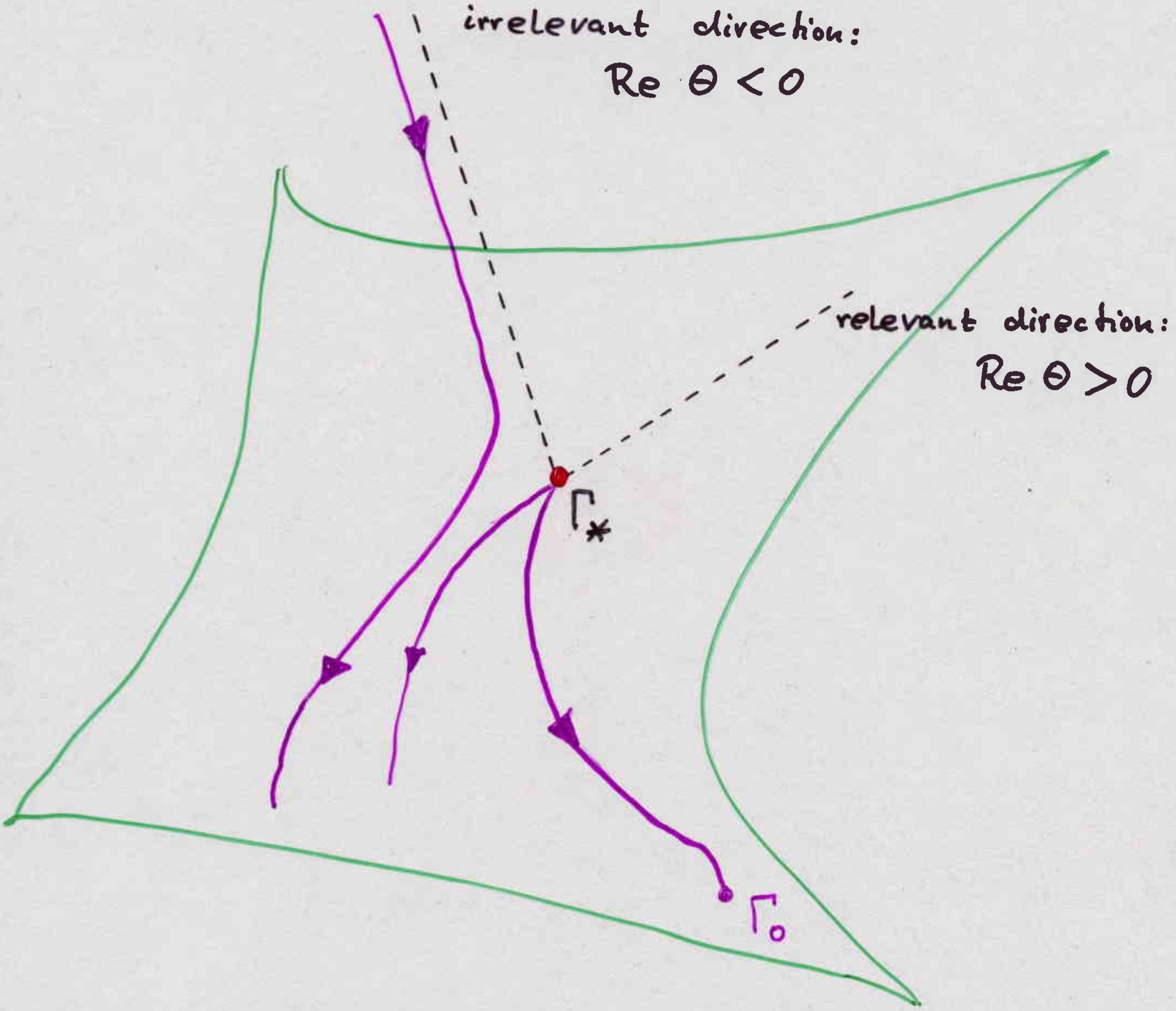
Weinberg 1979

Quantum theory is defined by a RG trajectory running inside the UV-critical hypersurface of the FP, with

initial point = $\Gamma_{k\to\infty} \equiv S$ = action infinitesimally close to Γ_*

end point = $\Gamma_0 \equiv \Gamma$

The UV-critical hypersurface $\mathscr{S}_{UV}$:



$\Delta_{UV} \equiv \dim \mathscr{S}_{UV}$ = # relevant directions

= # free parameters in the a.s. quantum field theory

UV $\longrightarrow$ IR

θ : critical exponent (neg. eigenvalue of lin. flow)

Properties of QEG

- Background-independent quantization scheme:

  No special <u>metric</u> plays any distinguished role!

  <u>The background field method</u>:

  a) Fix arbitrary $\bar{g}_{\mu\nu}$

  b) Quantize (nonlinear) fluctuations $h_{\mu\nu} \equiv \gamma_{\mu\nu} - \bar{g}_{\mu\nu}$ in the backgrd. of $\bar{g}_{\mu\nu}$

  c) Adjust $\bar{g}_{\mu\nu}$ such that $\langle h_{\mu\nu} \rangle = 0$

  $$\rightsquigarrow \quad g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \bar{g}_{\mu\nu}$$

- Fundamental action $S \approx \Gamma_*$ is a prediction:

  No special <u>action</u> plays any distinguished role!

  <u>The only input</u>: field contents + symmetries $\hat{=}$ theory space

  <u>The output</u>: $\Gamma_* = S_{\text{Einstein-Hilbert}}$ + "more"

Einstein-Hilbert action is often a reliable approximation, but not distinguished conceptually.

The Einstein-Hilbert Truncation

(M.R., 1996)

ansatz:

$$\Gamma_k = -\frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left\{ R - 2\Lambda_k \right\}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant G_k, dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k, dimensionless: $\lambda(k) = \Lambda_k / k^2$

insert ansatz into flow equation, expand

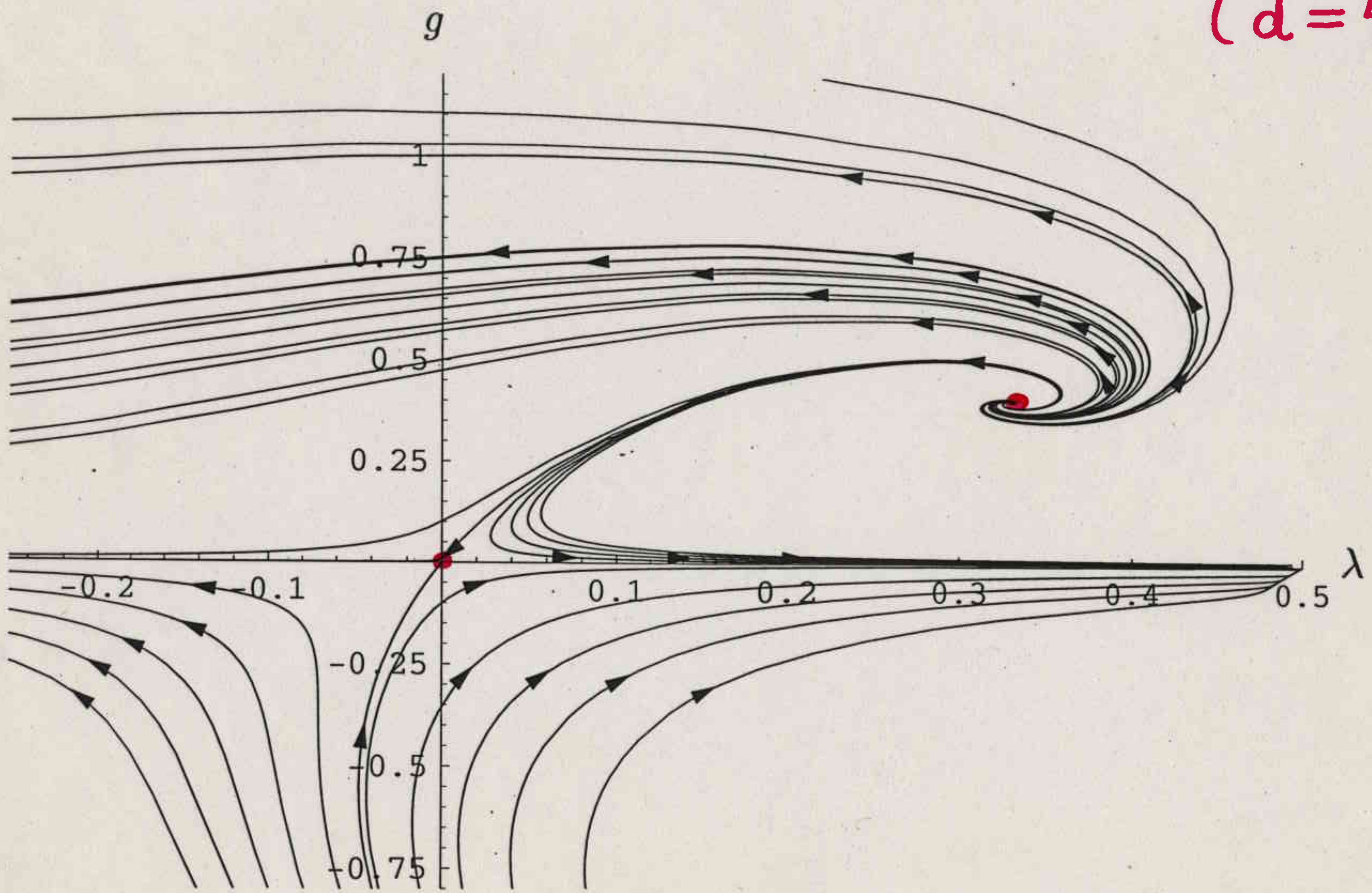
$$\mathrm{Tr}[\cdots] = (\cdots)\int\sqrt{g} + (\cdots)\int\sqrt{g}\,R + \cdots$$

$\rightsquigarrow$

$$\boxed{\begin{aligned} k\,\partial_k\, g(k) &= \beta_g(g,\lambda) \\ k\,\partial_k\, \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

RG - Flow in the Einstein - Hilbert Truncation

(d = 4)



M.R., F. Saueressig, hep-th/0110054

O. Lauscher, M.R.
F. Saueressig, M.R.
R. Percacci, ...
D. Litim, ...
P. Machado

Reliability checks (universality, ...),
more general truncations $\Longrightarrow$

The NGFP seems to exist in
the full un-truncated theory
beyond any reasonable doubt.

Conformally Reduced QEG

(M.R., H. Weyer, 2008)

- Simplified version of full QEG:
  only the conformal factor is quantized

- The same approach as in full QEG is used:
  effective average action,
  background field method

- Disentangles conceptual / technical problems

- Illustrates importance of "background independence" for the RG flow:

  scalar-like theory, but with RG behavior <u>very different</u> from that of a scalar matter field on a rigid spacetime

- Disentangles role of backgrd. field method for gauge invariance / "backgrd. independence"

- Has the same qualitative features as full QEG

  $\rightsquigarrow$ play ground for gaining new conceptual insights

Conformally Reduced QEG

- quantize only the conformal factor:

$$\underbrace{\gamma_{\mu\nu}}_{\text{integration variable}} = \chi^2 \underbrace{\hat{g}_{\mu\nu}}_{\text{class. reference metric,}}$$

$\neq$ backgrd. metric !

- treat scalar-like theory $\int \mathcal{D}\chi \, e^{-S[\chi]}$ in the same way as full QEG:
effective average action $\oplus$ backgrd. field method

- introduce background conf. factor:

$$\bar{g}_{\mu\nu} = \chi_B^2 \, \hat{g}_{\mu\nu}$$

- decompose quantum field:

$$\chi = \chi_B + \underbrace{f}_{\text{"fluctuation"}}$$

- expectation values:

$$\phi \equiv \langle \chi \rangle = \chi_B + \bar{f} \quad , \quad \bar{f} \equiv \langle f \rangle$$

$$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \langle \chi^2 \rangle \, \hat{g}_{\mu\nu} = \langle (\chi_B + f)^2 \rangle \, \hat{g}_{\mu\nu}$$

The innocent first steps:

- define, formally, $e^{W_k[J;\chi_B]} =$

$$= \int \mathcal{D}f \; e^{-S[\chi_B + f] - \Delta_k S[f;\chi_B] + \int d^4_x \sqrt{\hat{g}}\, J\, f}$$

with $\Delta_k S[f;\chi_B] = \frac{1}{2} \int d^4_x \sqrt{\hat{g}}\; f(x)\, \mathcal{R}_k[\chi_B]\, f(x)$

- define $\bar{f} \equiv \langle f \rangle_k = \frac{1}{\sqrt{\hat{g}}} \frac{\delta W_k}{\delta J}$

$$\rightsquigarrow \quad J = \mathcal{J}_k[\bar{f};\chi_B]$$

- define effective average action:

$$\Gamma_k[\bar{f};\chi_B] = \int d^4_x \sqrt{\hat{g}}\; \bar{f}\, \mathcal{J}_k - W_k[\mathcal{J}_k;\chi_B] - \Delta_k S[\bar{f};\chi_B]$$

$$\equiv \Gamma_k[\phi, \chi_B] \qquad \phi \equiv \chi_B + \bar{f}$$

- derive FRGE:

$$\boxed{\begin{aligned} &k\partial_k \Gamma_k[\bar{f};\chi_B] \\ &= \frac{1}{2} \mathrm{Tr}\left[\left(\Gamma_k^{(2)}[\bar{f};\chi_B] + \mathcal{R}_k[\chi_B] \right)^{-1} k\partial_k \mathcal{R}_k[\chi_B] \right] \end{aligned}}$$

Constructing $\mathcal{R}_k$:

"backgrd. independence" vs. rigid backgrd.

- require coarse graining scale k^{-1} of $\Gamma_k[g,\bar{g}]$ to be a <u>proper</u> (rather than coordinate) length

- "proper" w.r.t. which metric ?

- "backgrd. independence" $\Longrightarrow$ k^{-1} can be proper only w.r.t. metric given by the arguments $[g,\bar{g}]$, but not w.r.t. any rigid metric (such as $\hat{g}_{\mu\nu}$ in CR-QEG)

- <u>Our choice:</u> k^{-1} is proper w.r.t. $\bar{g}_{\mu\nu}$

  more precisely: $\boxed{-k^2 \text{ is a cutoff in the spectrum of } \bar{\Box} \equiv (D^\mu D_\mu)(\bar{g})}$

  $\Longrightarrow$ Typical structures (periods, $\cdots$) of $\bar{\Box}$-eigenfunction with eigenvalue $-k^2$ have $\bar{g}$-proper size of the order k^{-1}.

  $\Longrightarrow$ $\Gamma_k \mathrel{\hat{=}}$ "effective field theory valid near k"

- <u>Cf. rigid backgrd.</u> : $-k^2$ cutoff in $\hat{\Box}$-spectrum

Implementation (LPA):

- $\mathcal{R}_k$ must be such that $\Gamma_k^{(2)} \longrightarrow \Gamma_k^{(2)} + \mathcal{R}_k$ entails the replacement

$$(-\bar{\Box}) \longrightarrow (-\bar{\Box}) + k^2 \underbrace{R^{(0)}\left(\frac{-\bar{\Box}}{k^2}\right)}_{\stackrel{!}{=} \begin{cases} 0 & \text{if } -\bar{\Box} \gg k^2 \\ 1 & \text{if } -\bar{\Box} \ll k^2 \end{cases}}$$

- Since $\bar{\Box} = \chi_B^{-2}\,\hat{\Box}$ when $\bar{g}_{\mu\nu} = \chi_B^2\,\hat{g}_{\mu\nu}$ with $\chi_B = \text{const}$, this is equivalent to:

$$(-\hat{\Box}) \longrightarrow (-\hat{\Box}) + \chi_B^2\, k^2 R^{(0)}\left(\frac{-\hat{\Box}}{\chi_B^2\, k^2}\right)$$

absent when k is proper w.r.t. $\hat{g}_{\mu\nu}$!

- "Backgrd. independent" choice of $\mathcal{R}_k$ contains additional factors of χ_B compared to standard quantization of scalar matter field on rigid backgrd.:

$$\boxed{\mathcal{R}_k = -\frac{3}{4\pi G_k}\,\chi_B^2\, k^2 R^{(0)}\left(\frac{-\hat{\Box}}{\chi_B^2\, k^2}\right)}$$

Truncations employed :

- Conformally Reduced Einstein-Hilbert ("CREH") truncation :

$$\Gamma_k[\bar{f};\chi_B] \equiv \Gamma_k[\phi,\chi_B]$$

$$= -\frac{1}{16\pi G_k}\int d^4x\sqrt{g}\,\left(R(g)-2\Lambda_k\right)\Big|_{g_{\mu\nu}=\phi^2\hat{g}_{\mu\nu}}$$

$$= \frac{3}{4\pi G_k}\int d^4x\sqrt{\hat{g}}\left\{\frac{1}{2}\phi\hat{\Box}\phi-\frac{1}{12}\hat{R}\phi^2+\frac{1}{6}\Lambda_k\phi^4\right\}$$

depends only on the combination $\phi \equiv \chi_B + \bar{f}$;

2-dimensional theory space : $\{g, \lambda\}$

- Local Potential Approximation (LPA):

$$\Gamma_k[\phi,\chi_B] = \frac{3}{4\pi G_k}\int d^4x\sqrt{\hat{g}}\left\{\frac{1}{2}\phi\hat{\Box}\phi - F_k(\phi)\right\}$$

infinite dimensional theory space :

$$\{G, F(\cdot)\} \sim \{g, Y(\cdot)\}$$

Flow equations and β-functions

$$Y_k(\varphi) \equiv k^2 \, F_k\left(\frac{\varphi}{k}\right), \quad \varphi \equiv k\,\phi \qquad \text{dim. less}$$

$$g_k \equiv k^2 G_k, \quad \lambda_k \equiv \Lambda_k / k^2$$

$$\boxed{\begin{aligned} k\partial_k g_k &= \left[2 + \eta_N\left(g_k, [Y_k]\right)\right] g_k \\ k\partial_k Y_k(\varphi) &= (2+\eta_N)\, Y_k - \varphi\, Y_k' \\ &\quad - \frac{g_k}{24\pi}\left(1 - \frac{1}{6}\eta_N\right) \frac{\varphi^6}{\varphi^2 + Y_k''(\varphi)} \\[1em] &\text{anomalous dimension:} \\ \eta_N(g, [Y]) &= -\frac{g}{24\pi} \frac{\left[\varphi_1^3\, Y'''(\varphi_1)\right]^2}{\left[\varphi_1^2 + Y''(\varphi_1)\right]^4} \end{aligned}}$$

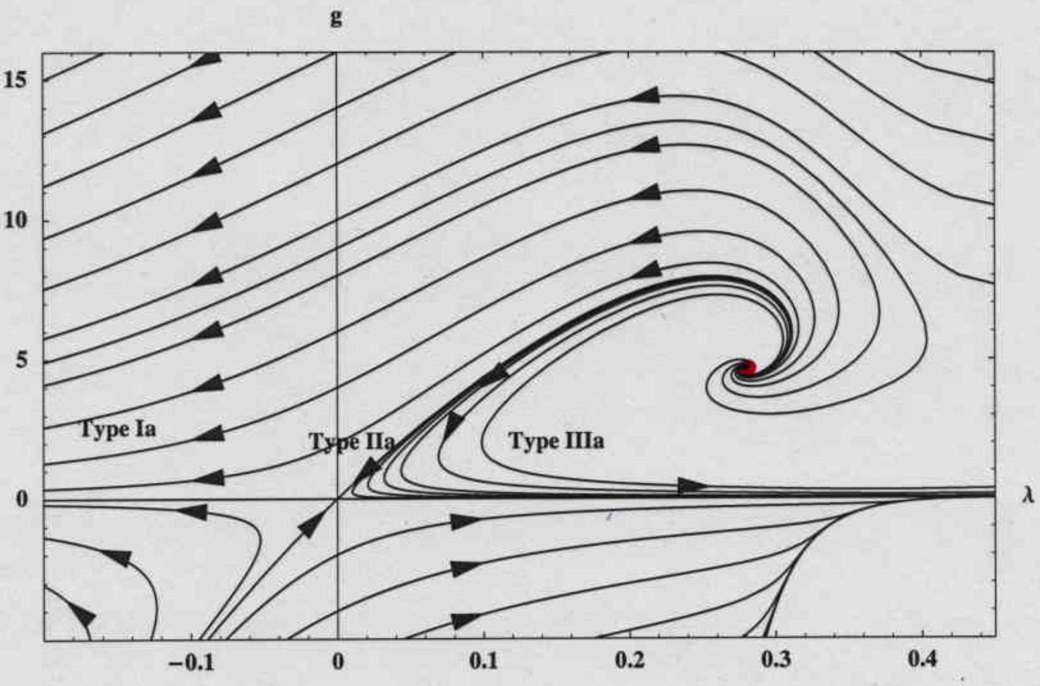
(R^4 topology)

φ_1 : normalization point $(\varphi_1 \to \infty)$.

Compare to standard scalar on rigid backgrd.:

$$\frac{\phi^6\, k^6}{\phi^2 k^2 + F_k''(\phi)} \longrightarrow \frac{k^6}{k^2 + F_k''(\phi)}$$

The CREH flow: $\Upsilon_k(\varphi) = -\frac{1}{6}\lambda_k \varphi^4$



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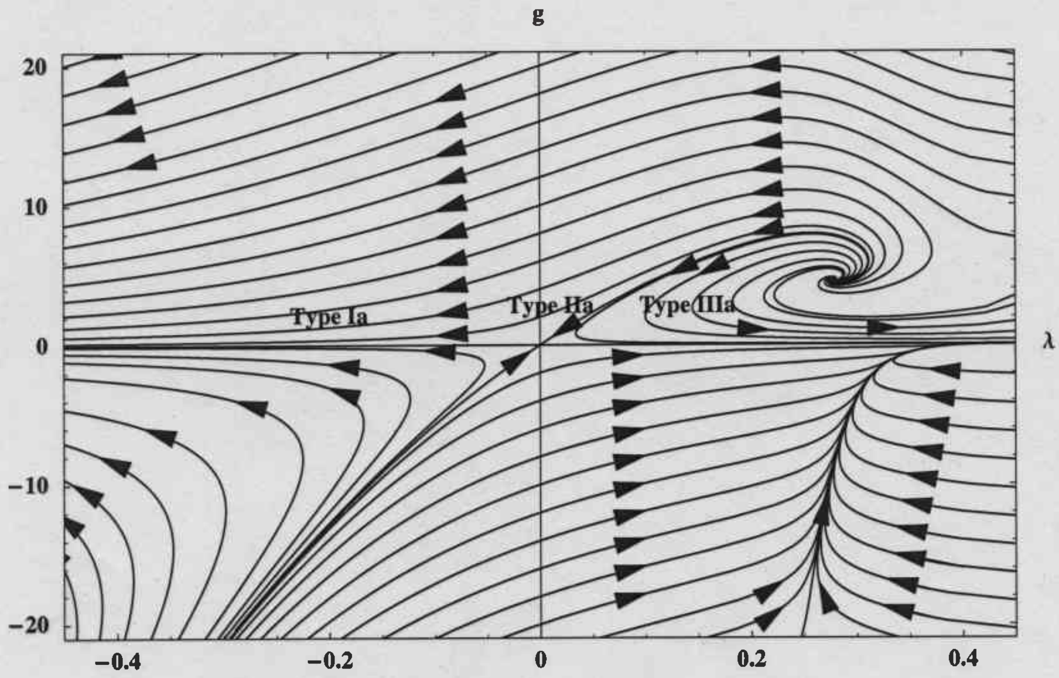


(b)

Figure 1: The figures show the RG flow on the (g, λ)-plane which is obtained from the CREH truncation with $\eta_N^{(\mathrm{kin})}$. The arrows point in the direction of decreasing k.

M. R., H. Weyer, arXiv: 0801.3287

Results:

- Inequivalent quantization schemes:
  - rigid background - quantization
  - "backgrd. independent" quantization

- Resulting RG flows are very different:
  - standard $(-\phi^4)$ - theory : no NGFP (asym. free: Symanzik 1973)
  - NGFP exists : theory is asym. safe!

- Scaling dimensions at the GFP differ by 2 units, e.g.

$$\varphi^n \begin{cases} \Theta = n-4 \\ \Theta = n-2 \end{cases}$$

- Scaling fields / dimensions at the NGFP depend on choice of theory space, e.g.

$$\{\varphi^m\}, \quad m \in \mathbb{N},\ m \in \mathbb{Z},\ m \in \mathbb{R},\ m \in \mathbb{C}, \ldots$$

- β - functions of LPA depend on <u>topology</u>: $R^4, S^4, \ldots$

Example: Running of the cosmological constant near the GFP

obtains from interaction term $\Lambda_k \phi^4$

- rigid background quantization:

standard ϕ^4-theory $\Longrightarrow$ $\boxed{\Lambda_k \sim \log(k)}$

Polyakov (2001)
Jackiw et al. (2005)

- "background independent" quantization:

$$\boxed{\Lambda_k \sim G_0 k^4}$$

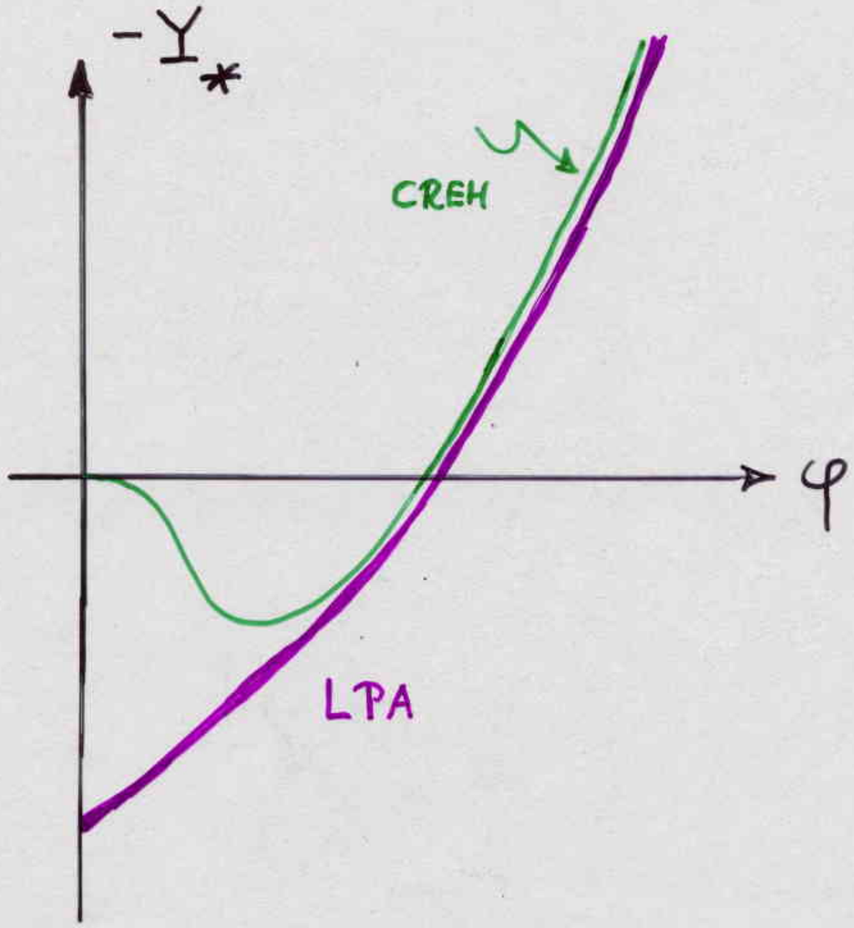
consistent with full QEG

and sum over zero-point energy approach

Non-Gaussian Fixed Point (LPA)

$$\begin{cases} g_*^{NGFP} = g_*^{CREH} \\ Y_*^{NGFP}(\varphi) = y_* - \frac{1}{6}\lambda_*^{CREH}\varphi^4 \end{cases} \qquad (R^4)$$

Numerical solution for S^4 topology :



Corresponds to non-trivial fixed points of infinitely many couplings !

Phase Transitions to a new phase of gravity: Unbroken Diffeomorphism Invariance

- Solve full nonlinear PDE for Υ_k numerically, search for trajectories inside $\mathcal{S}_{UV}$.

- Global minimum $\phi_0(k) \equiv k\,\varphi_0(k)$ of $\mathcal{F}_k(\phi) \sim -\Upsilon_k(\varphi)$ determines expectation value

$$\boxed{\langle \gamma_{\mu\nu} \rangle \equiv \langle g_{\mu\nu} \rangle_k = \phi_0^2(k)\, \hat{g}_{\mu\nu}}$$

- $\phi_0 = 0$: phase with vanishing exp. val. of the metric (vielbein)

  $\phi_0 \neq 0$: exp. val. $\neq 0$, spontaneously breaks group of diffeo.'s to stability group of $\langle g_{\mu\nu} \rangle_k$

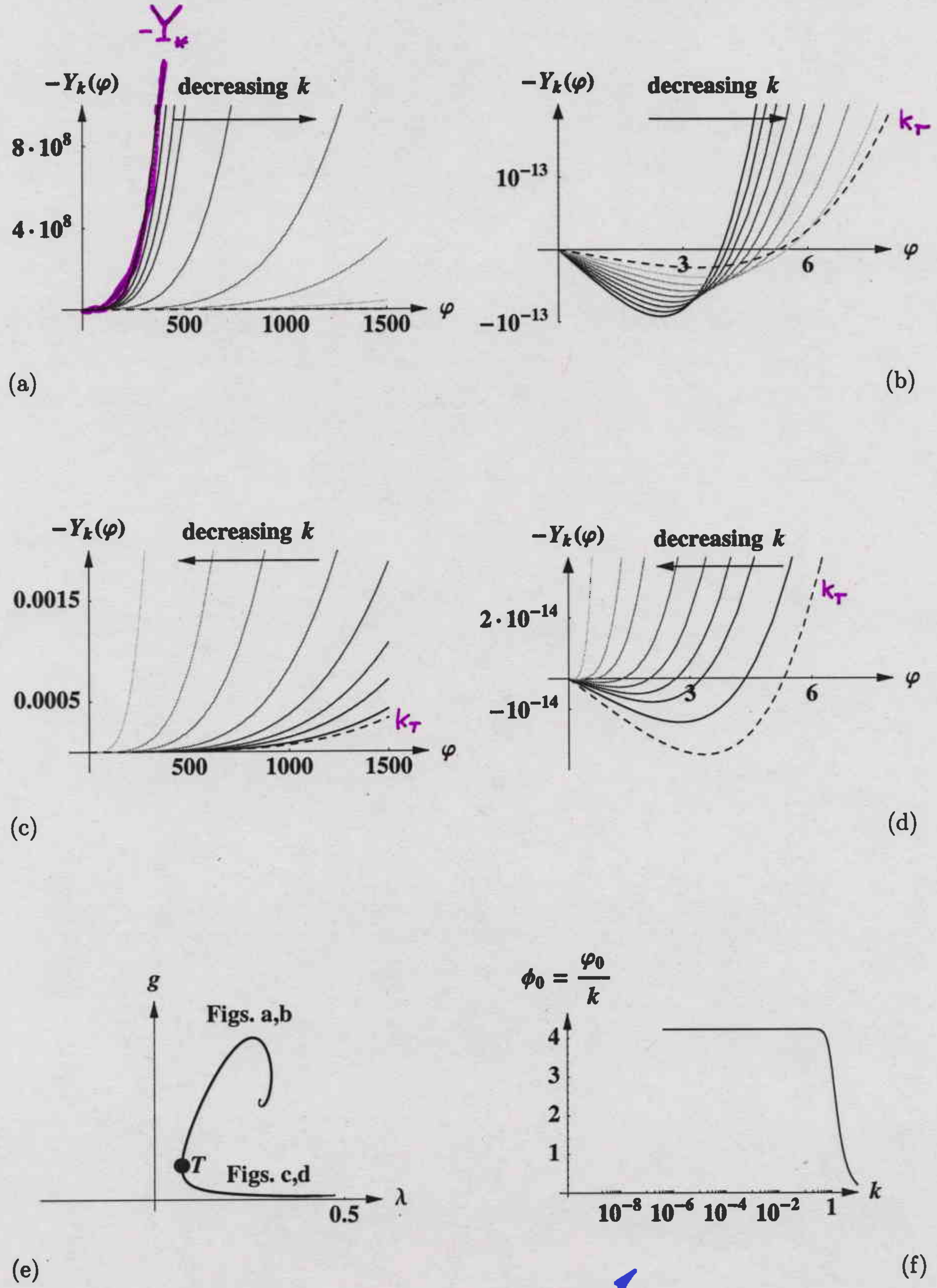
- Forms of phase transitions (w.r.t. scale k):

■ "1^{st} order" $\longleftrightarrow$ "2^{nd} order"

(φ_0 discontinuous) (φ_0 continuous)

■ at k "$=$" ∞ $\longleftrightarrow$ at $k = k_c < \infty$

2^{nd} order transition at k "=" ∞ (R^4)

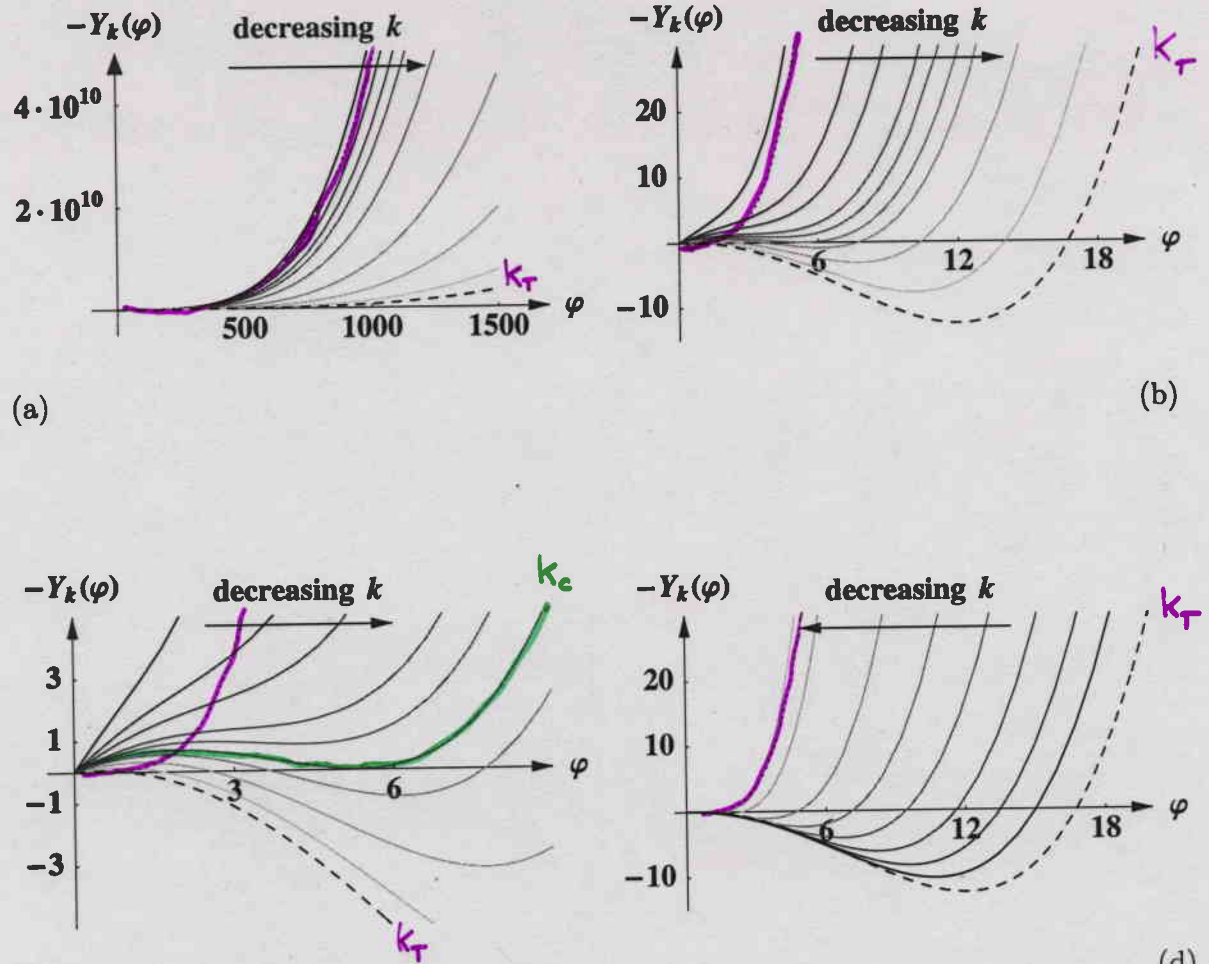


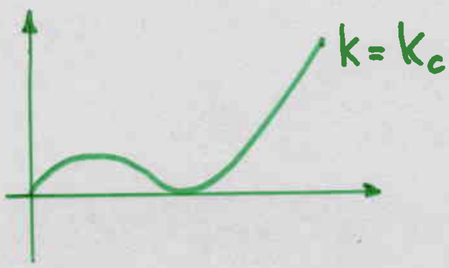
classical spacetime emerges:

$\phi_0 \approx \text{const} \neq 0$

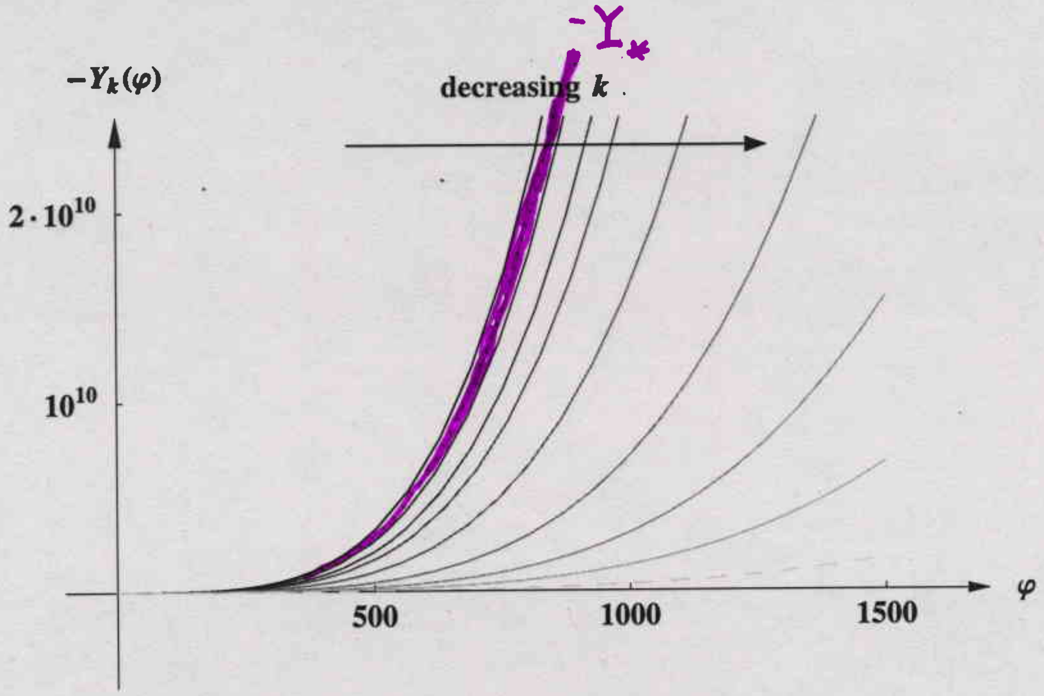
M. R., H. Weyer, arXiv: 0804.1475

1^{st} order transition at $k_c < \infty$ (R^4)





1^{st} order transition at $k_c < \infty$ (S^4)



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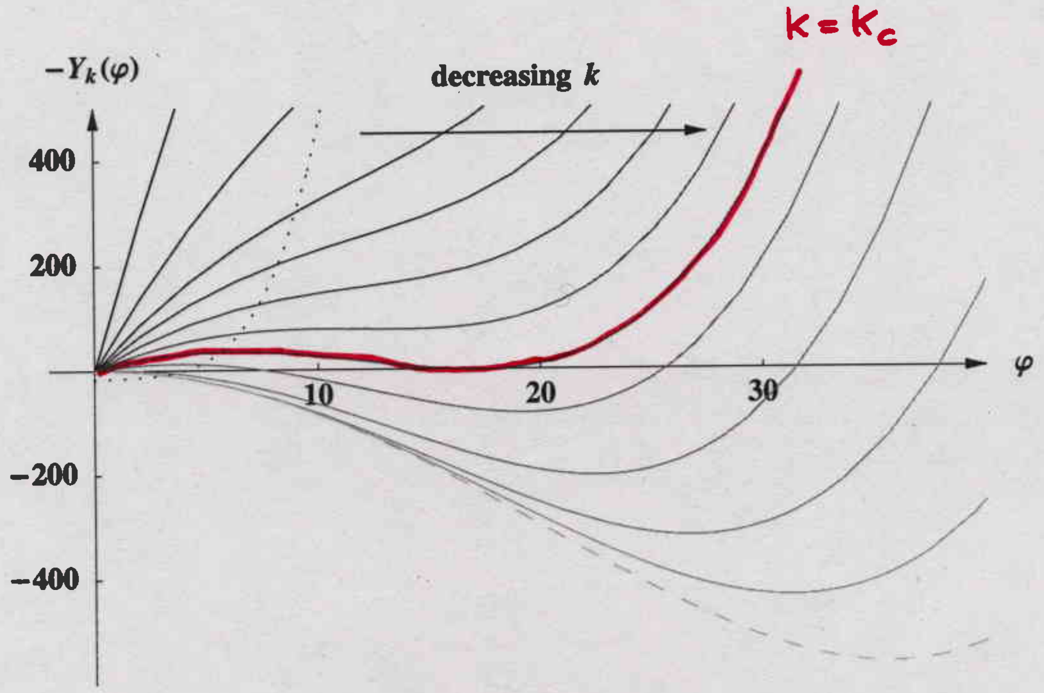

(b)

Summary

- "Backgrd. independence" has a crucial impact on the RG flow of the eff. average action:

  - rigid backgrd: standard $-\phi^4$ theory (asymptotically free: Symanzik '73)

  - "backgrd. indep.": NGFP forms $\Rightarrow$ A.S.

- RG flow due to the conformal factor is <u>typical</u> of the full set of metric degrees of freedom:

  "backgrd. indep." seems to be more important to A.S. than spin-2 excitations, their complicated self-interactions, etc. !