# Renormalization group flow of $f(R)$-gravity

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M. Reuter, F. S., arXiv:0708.1317 [hep-th]

P. Machado, F. S., Phys. Rev. D77 (2008) 124045, arXiv:0712.0445 [hep-th]

Heidelberg, July 5, 2008

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Challenge: reconcile General Relativity with quantum mechanics

- classical General Relativity (Einstein-Hilbert action)
  - phenomenologically very successful:
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New physics:

- UV: completion of theory at high energies
- IR: possibly: strong RG effects related to cosmological constant problem
  phenomenology: modified GR at long distances

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Gravity: Primary tool for non-perturbative investigation

- flow equation for effective average action $\Gamma_k$
  (C. Wetterich, Phys. Lett. $\mathbf{B301}$ (1993) 90)
- adapted to gravity
  (M. Reuter, Phys. Rev. D $\mathbf{57}$ (1998) 971, hep-th/9605030)

# Exact evolution equation for $\Gamma_k$

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$$\exp\{-S_{\rm grav}[\bar{g}+h] - S_{\rm gf}[h;\bar{g}] - S_{\rm gh}[h,C,\bar{C};\bar{g}] - S_{\rm source} - \Delta_k S[h,C,\bar{C};\bar{g}]\}$$

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3. Effective average action $\Gamma_k$ = (modified) Legendre transform of $W_k$:
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$$\Gamma_k = \int \sqrt{\bar{g}}\left(t^{\mu\nu}\bar{h}_{\mu\nu} + \bar{\sigma}_\mu\xi^\mu + \sigma^\mu\bar{\xi}_\mu\right) - W_k - \Delta S_k \, .$$

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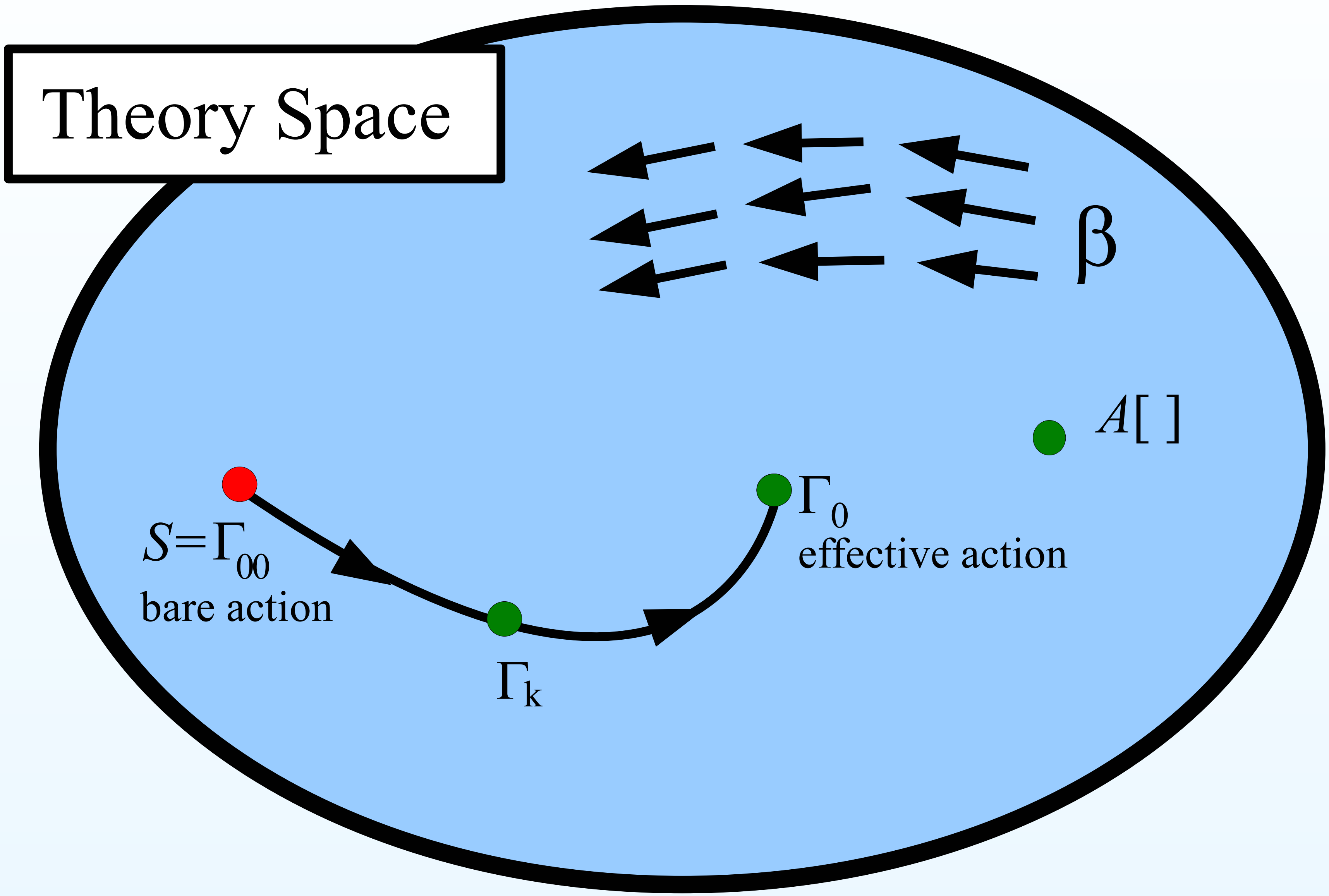
4. functional RG equation for $\Gamma_k$:

$$k\partial_k\Gamma_k = \frac{1}{2}\text{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k(-\bar{D}^2)\right)^{-1} k\partial_k\mathcal{R}_k(-\bar{D}^2)\right] + \text{ghost contribution}$$

## Theory space underlying the Functional Renormalization Group



# Truncating the theory space

- Caveat: FRGE cannot be solved exactly
- non-perturbative approximation scheme

  truncate theory space to "physically most relevant" interactions
  - ansatz for $\Gamma_k$
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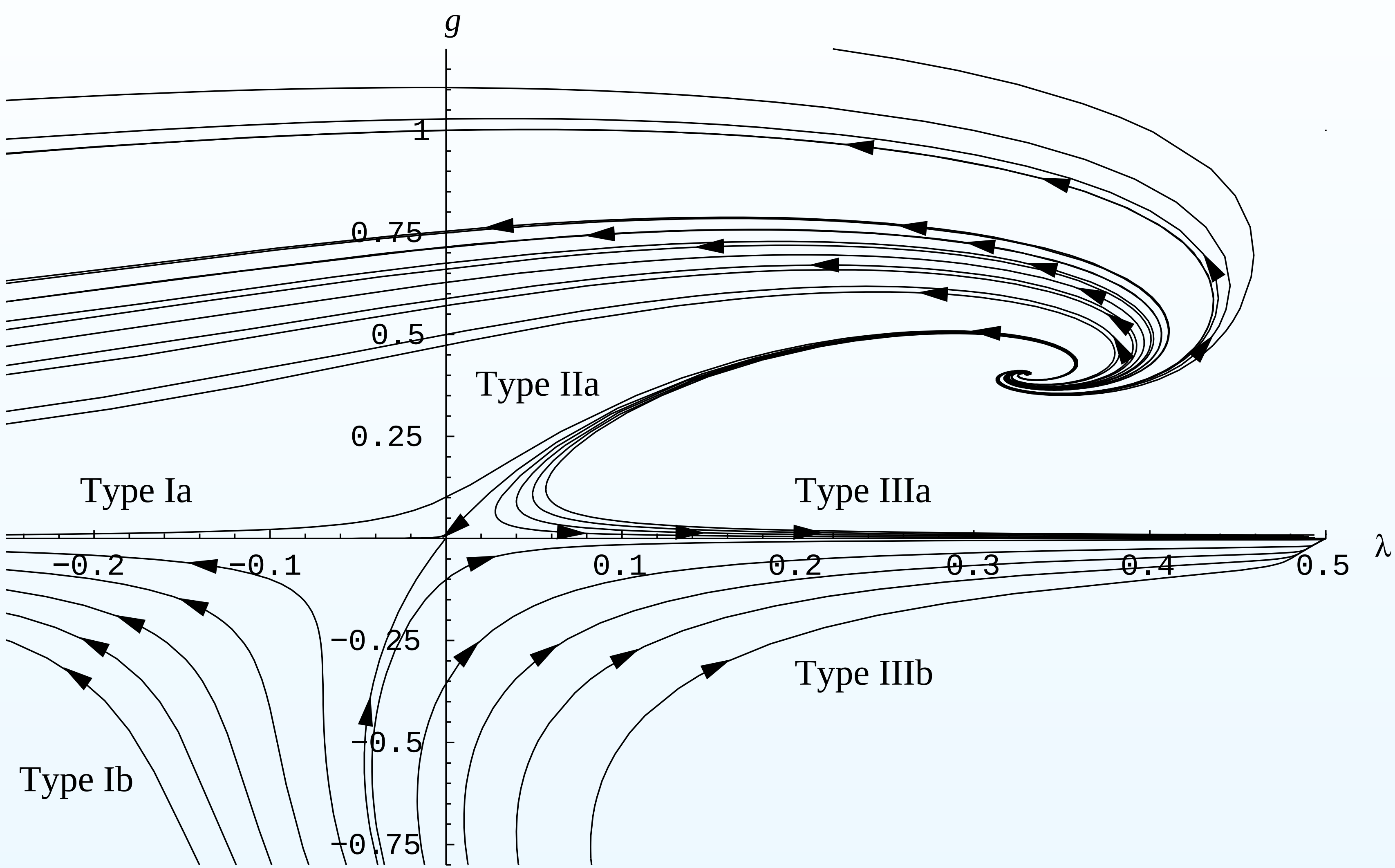
$$k\partial_k g_k = \beta_g(g, \lambda) \, , \quad k\partial_k \lambda_k = \beta_\lambda(g, \lambda)$$

- contain contributions from arbitrary powers $g$:

$$\beta_g(g, \lambda) = (d - 2 + \eta_N) g \, , \quad \eta_N = \frac{g B_1(\lambda)}{1 - g B_2(\lambda)}$$

# RG flow of QEG in the Einstein-Hilbert-truncation

(M. Reuter, F. Saueressig, Phys. Rev. D **65** (2002) 065016 [hep-th/0110054])



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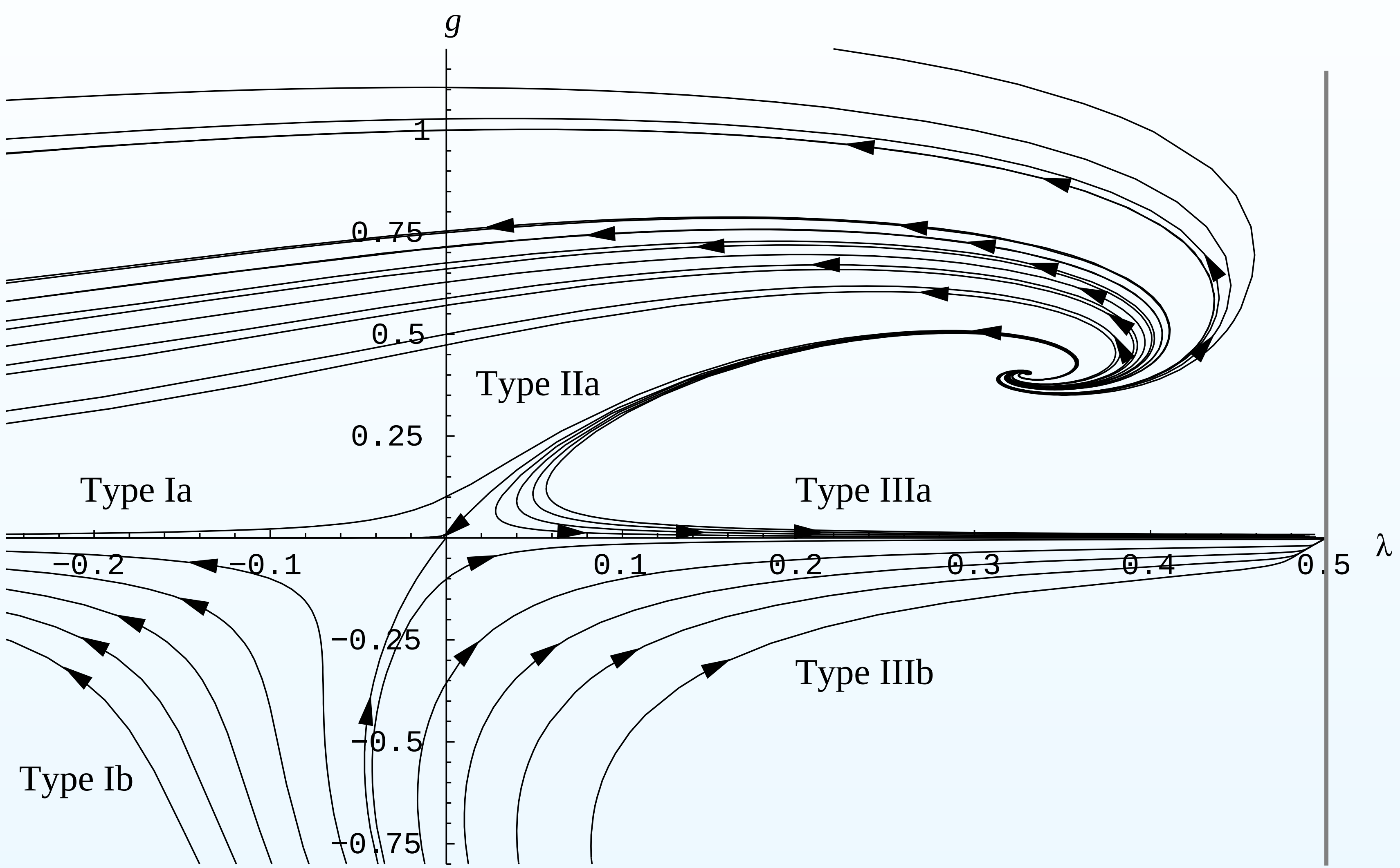

## Questions raised by Einstein-Hilbert truncation

gravitational RG flow in the UV:

- Einstein-Hilbert: controlled by NGFP
  - Robustness of NGFP under extension of truncation space
  - dimension of UV critical surface $\Longleftrightarrow$ relevant parameters of FP action

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Extension of theory space:

- Most sophisticated: flow equation for $f(R)$-gravity

$$\Gamma_k^{\text{grav}} = \frac{1}{16\pi G_k} \int d^d x \sqrt{g} f_k(R)$$

## Flow equation for $f(R)$-gravity

(O. Lauscher, M. Reuter, hep-th/0108040;

A. Codello, R. Percacci, C. Rahmede, 0705.1769)

transverse-traceless decomposition of metric:

$$h_{\mu\nu} = h^{\mathrm{T}}_{\mu\nu} + \bar{D}_\mu \xi_\nu + \bar{D}_\nu \xi_\mu + \bar{D}_\mu \bar{D}_\nu \sigma - \frac{1}{d} \bar{g}_{\mu\nu} \bar{D}^2 \sigma + \frac{1}{d} \bar{g}_{\mu\nu} \phi$$

- $h^{\mathrm{T}}_{\mu\nu}$ transverse-traceless tensor field
- $\xi_\nu$ transverse vector
- $\sigma$, $\phi$ scalars

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Geometric gauge condition:

$$S_{\mathrm{gf}}[h;\bar{g}] = \frac{1}{2\alpha}\int d^dx\sqrt{\bar{g}}\bar{g}^{\mu\nu}F_\mu F_\nu\,, \quad F_\mu = (16\pi G)^{-1/2}(\bar{D}^\nu h_{\mu\nu} - \frac{1}{d}\bar{D}_\mu h^\nu{}_\nu)$$

- limit $\alpha \to 0$: physical degrees of freedom: $h^{\mathrm{T}}_{\mu\nu}, \phi$

  gauge degrees of freedom $\xi_\mu, \sigma$

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exponentiate resulting Jacobians

employ geometric gauge-condition

Flow equation for $f(R)$-gravity

$$\partial_t \Gamma_k = -\tfrac{1}{2}\mathrm{Tr}_0'' \left[ \frac{\partial_t R_k}{P_k - \frac{1}{d-1} R} \right] - \tfrac{1}{2}\mathrm{Tr}_{1\mathrm{T}}' \left[ \frac{\partial_t R_k}{P_k - \frac{1}{d} R} \right] + D_1(d,0)\, \frac{\partial_t R_k}{P_k} \Big|_{-D^2 = \Lambda_1(d,0)}$$
$$+ \tfrac{1}{2}\mathrm{Tr}_{2\mathrm{T}} \left[ \frac{\partial_t (Z_{Nk} f_k' R_k)}{Z_{Nk} \left( f_k'\, P_k + f_k - \frac{2(d-2)}{d(d-1)}\, R\, f_k' \right)} \right] + \tfrac{1}{2}\mathrm{Tr}_0 \left[ \frac{\partial_t \left( Z_{Nk} \widetilde{\mathcal{R}}_k^{\phi\phi} \right)}{Z_{Nk} \widetilde{\Gamma}_k^{(2)\phi\phi}} \right]$$

- $P_k := -D^2 + R_k$,
- $Z_{Nk} \Leftrightarrow$ running Newton's constant
- $\widetilde{\mathcal{R}}_k^{\phi\phi}, \widetilde{\Gamma}_k^{(2)\phi\phi}$ known functions of $d, R, f_k$

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Important properties:

- only last two terms depend on $f_k(R)$

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Important properties:

- only last two terms depend on $f_k(R)$
- equation is invariant under rescaling $f_k(R) \longrightarrow const \times f_k(R)$

## General properties I: resolving the $\lambda = 1/2$ boundary

$$\partial_t \Gamma_k = -\frac{1}{2}\mathrm{Tr}_0''\left[\frac{\partial_t R_k}{P_k - \frac{1}{d-1}R}\right] - \frac{1}{2}\mathrm{Tr}_{1\mathrm{T}}'\left[\frac{\partial_t R_k}{P_k - \frac{1}{d}R}\right] + D_1(d,0)\,\frac{\partial_t R_k}{P_k}\bigg|_{-D^2=\Lambda_1(d,0)}$$
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3. General resolution: $f_k(R) = -R + 2\Lambda_k + \bar{\mu}_k R^{1-\epsilon}\,, \quad \epsilon > 0$

$$\mathrm{Tr}_{2\mathrm{T}}\left[\frac{Z_{Nk}^{-1}\partial_t\left(Z_{Nk}\,\tilde{f}_k\, R_k\right)}{\tilde{f}_k(P_k - c_d R) - R^{1+\epsilon} + 2\Lambda_k R^\epsilon + \bar{\mu}_k\, R}\right] = \mathrm{Tr}_{2\mathrm{T}}\left[\frac{\partial_t\left(Z_{Nk}\bar{\mu}_k\, R_k\right)}{Z_{Nk}\,\bar{\mu}_k\, P_k} + \mathcal{O}(R^\epsilon, R)\right]$$

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$\lambda = 1/2$-boundary of truncated theory space

Resolved by including non-local operators $\propto R^{1-\epsilon}$, $\epsilon > 0$ in truncation

## Capturing the RG flow in the IR

Toy model truncations:

- including interactions which become important for small curvature $R$:

$$\Gamma_k^{\rm grav} = \frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left\{ -R + 2\Lambda_k + 16\pi G_k \bar{v}_k R^{-n} \right\}$$

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substitute $R^{-n}$ ansatz in flow equation:

- $\beta$-functions for $g_k, \lambda_k, \upsilon_k$
  - removes singularities at $\lambda = 1/2$
  - generically gives rise to NGFP

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- including interactions which become important for small curvature $R$:

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$$\Gamma_k^{\rm grav} = \frac{1}{16\pi G_k} \int d^dx \sqrt{g} \, \{ -R + 2\Lambda_k + 16\pi G_k \bar{v}_k \ln(R/R_0) \}$$

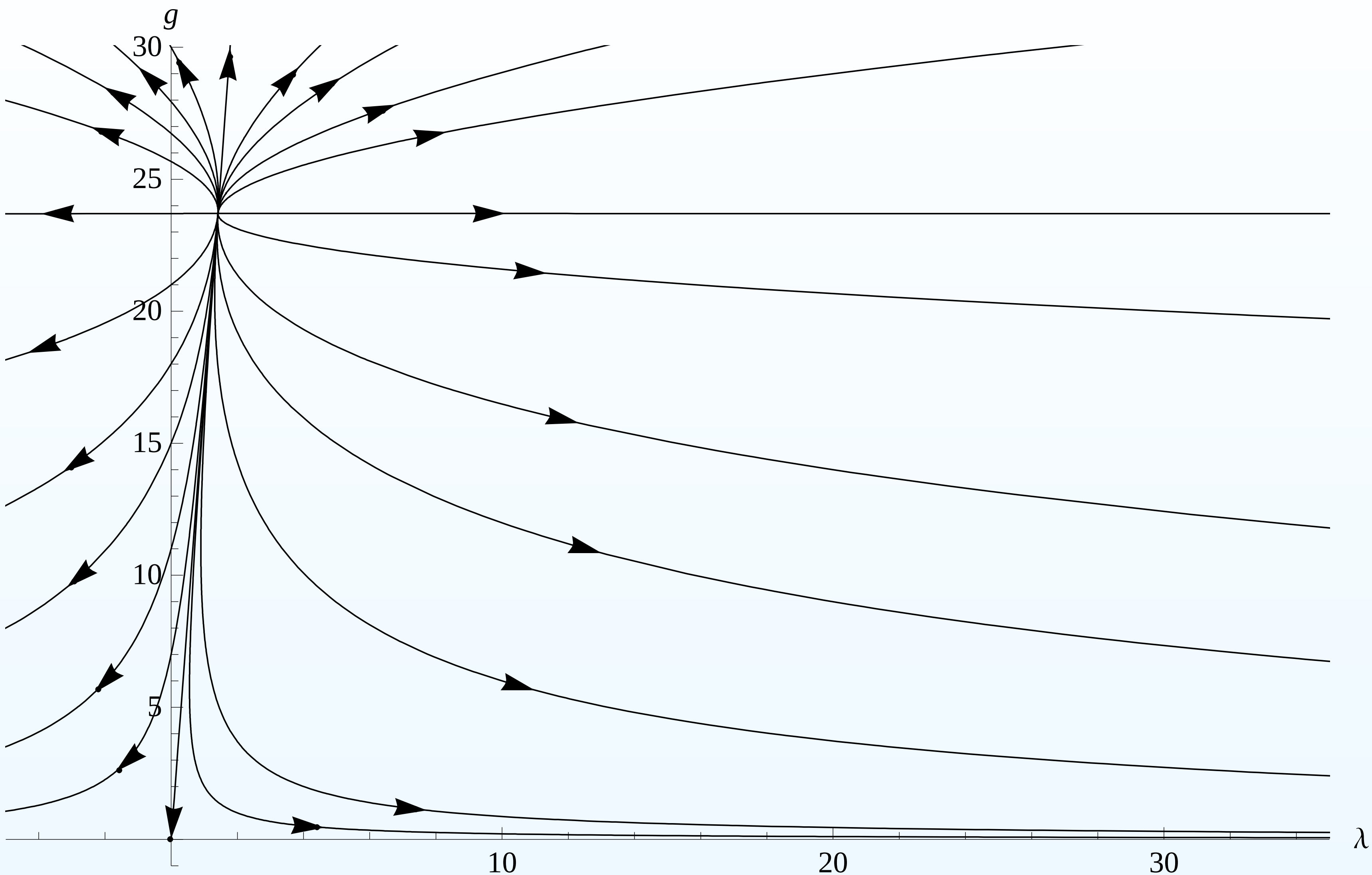
  - phenomenology: new terms drive late-time acceleration in cosmology

substitute $R^{-n}$ ansatz in flow equation:

- $\beta$-functions for $g_k, \lambda_k, \upsilon_k$
  - removes singularities at $\lambda = 1/2$
  - generically gives rise to NGFP

for $n \geq n_{\rm crit}$ “classical” trajectories are well-defined on all RG scales

# RG flow of the $R^{-n}$-truncation



# Capturing the RG flow in the IR: the $\ln(R)$-truncation

Non-generic case "$\epsilon = 1$": the $\ln(R)$-truncation

$$\Gamma_k^{\rm grav} = \frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left\{ -R + 2\Lambda_k + 16\pi G_k \bar{\upsilon}_k \ln(R/R_0) \right\}$$

Projection of RG flow to truncation subspace:

$$k\,\partial_k g_k = \beta_g(g_k, \lambda_k, \upsilon_k)\,, \quad k\,\partial_k \lambda_k = \beta_\lambda(g_k, \lambda_k, \upsilon_k)\,, \quad k\,\partial_k \upsilon_k = \beta_\upsilon(g_k, \lambda_k, \upsilon_k)$$

- Resolve IR singularity of Einstein-Hilbert truncation
- RG flow has IR-fixed point:

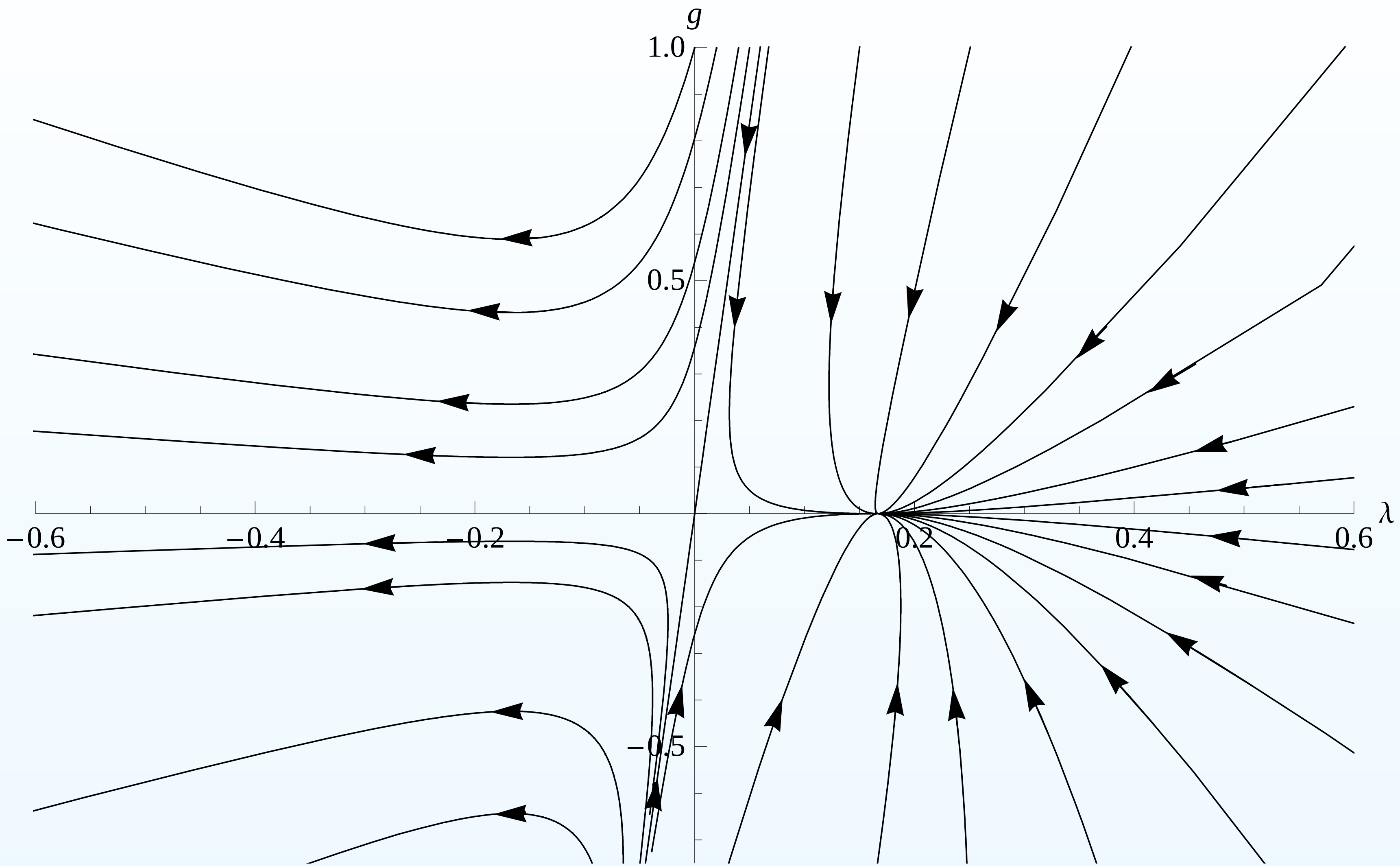
  IR-attractive for Newton's constant and positive cosmological constant

  $$\Lambda_k = c\,k^2\,, \quad c \approx \mathcal{O}(1)$$

  For RG trajectories attracted to IRFP:

  - positive cosmological constant is dynamically driven to zero as $k \to 0$
  - independent of initial value (at, e.g., Planck scale)

## RG flow of the $\ln(R)$-truncation



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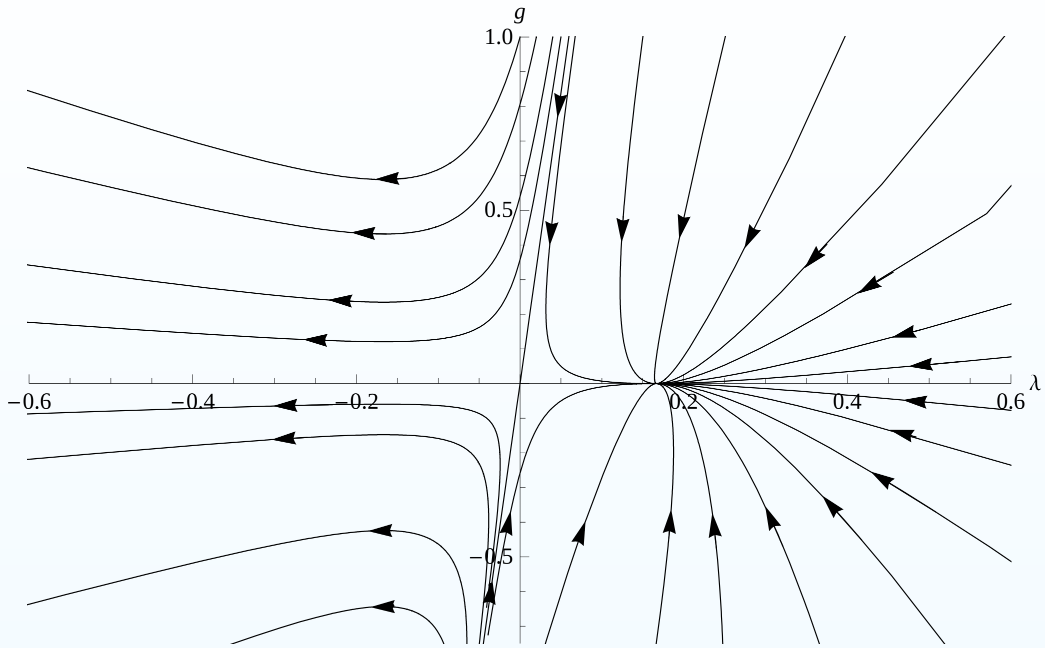


including non-local curvature terms:

- improved description of RG flow in IR

However:

- truncations are inferior to Einstein-Hilbert in UV
- non-local coupling constants $\bar{\upsilon}_k$ are constant along RG trajectory
  non-local interactions are not generated dynamically

## General properties II: decoupling of non-local interactions

$$\partial_t \Gamma_k = -\frac{1}{2}\mathrm{Tr}_0'' \left[\frac{\partial_t R_k}{P_k - \frac{1}{d-1} R}\right] - \frac{1}{2}\mathrm{Tr}_{1\mathrm{T}}' \left[\frac{\partial_t R_k}{P_k - \frac{1}{d} R}\right] + D_1(d,0)\, \frac{\partial_t R_k}{P_k}\Big|_{-D^2 = \Lambda_1(d,0)}$$
$$+ \frac{1}{2}\mathrm{Tr}_{2\mathrm{T}} \left[\frac{\partial_t (Z_{Nk} f_k' R_k)}{Z_{Nk}\left(f_k'\, P_k + f_k - c_d\, R\, f_k'\right)}\right] + \frac{1}{2}\mathrm{Tr}_0 \left[\frac{\partial_t \left(Z_{Nk} \widetilde{\mathcal{R}}_k^{\phi\phi}\right)}{Z_{Nk} \widetilde{\Gamma}_k^{(2)\phi\phi}}\right]$$

1. Homogeneity of trace arguments:
   - Expanding trace-arguments in curvature $\Longrightarrow$ only positive powers!
2. Evaluation of traces $\Longrightarrow$ (early time) heat-kernel expansion
   - gives positive powers of curvature only

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perturbative decoupling:

Interaction monomials not contained in the heat-kernel expansion

can consistently be decoupled from RG flow

## Partial differential equation for $f_k(R)$

Special choice: $d = 4$ and Litim's optimized cutoff $R_k^{\text{opt}}$

- explicit evaluation of traces using finite number of heat-kernel coefficients!

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$$
\begin{aligned}
&384\pi^2\left(\partial_t\mathcal{F}_k + 4\mathcal{F}_k - 2\rho\mathcal{F}_k'\right) = \\
&\quad\Big[5\rho^2\theta\left(1-\tfrac{\rho}{3}\right) - \left(12 + 4\,\rho - \tfrac{61}{90}\,\rho^2\right)\Big]\Big[1-\tfrac{\rho}{3}\Big]^{-1} + 10\,\rho^2\,\theta\left(1-\tfrac{\rho}{3}\right) \\
&+\Big[10\,\rho^2\,\theta(1-\tfrac{\rho}{4}) - \rho^2\,\theta(1+\tfrac{\rho}{4}) - \left(36 + 6\,\rho - \tfrac{67}{60}\,\rho^2\right)\Big]\Big[1-\tfrac{\rho}{4}\Big]^{-1} \\
&+\Big[\eta_f\,\left(10 - 5\rho - \tfrac{271}{36}\rho^2 + \tfrac{241}{168}\rho^3\right) + \left(60 - 20\rho - \tfrac{271}{18}\rho^2\right)\Big]\Big[1 + \tfrac{\mathcal{F}_k}{\mathcal{F}_k'} - \tfrac{\rho}{3}\Big]^{-1} \\
&+\frac{5\rho^2}{2}\Big[\eta_f\,\left((1+\tfrac{\rho}{3})\theta(1+\tfrac{\rho}{3}) + (2+\tfrac{\rho}{3})\theta(1+\tfrac{\rho}{6})\right) + 2\theta(1+\tfrac{\rho}{3}) + 4\theta(1+\tfrac{\rho}{6})\Big]\Big[1 + \tfrac{\mathcal{F}_k}{\mathcal{F}_k'} - \tfrac{\rho}{3}\Big]^{-1} \\
&+\Big[\mathcal{F}_k'\,\eta_f\,\left(6 + 3\rho + \tfrac{29}{60}\rho^2 + \tfrac{37}{1512}\,\rho^3\right) + \left(\partial_t\mathcal{F}_k'' - 2\rho\mathcal{F}_k'''\right)\left(27 - \tfrac{91}{20}\rho^2 - \tfrac{29}{30}\,\rho^3 - \tfrac{181}{3360}\,\rho^4\right) \\
&\quad+\mathcal{F}_k''\left(216 - \tfrac{91}{5}\,\rho^2 - \tfrac{29}{15}\,\rho^3\right) + \mathcal{F}_k'\left(36 + 12\,\rho + \tfrac{29}{30}\,\rho^2\right)\Big]\Big[2\mathcal{F}_k + 3\mathcal{F}_k'(1-\tfrac{2}{3}\rho) + 9\mathcal{F}_k''(1-\tfrac{\rho}{3})^2\Big]
\end{aligned}
$$

- dimensionless quantities

$$
\rho \equiv R/k^2\,, \quad \mathcal{F}_k(\rho) \equiv \frac{1}{16\pi k^4 G_k}\, f_k(R)\,, \quad \eta_f \equiv \frac{1}{\mathcal{F}_k}\left(\partial_t\mathcal{F}_k' + 2\mathcal{F}_k' - 2\rho\mathcal{F}_k''\right)
$$

## UV properties of $f(R)$-gravity: polynomial expansion

(A. Codello, R. Percacci, C. Rahmede, 0705.1769; P. Machado, F.S., 0712.0445)

- Polynomial expansion: $\mathcal{F}_k(\rho) = \sum_{i=0}^{n} u_i \, \rho^i + \ldots$

$$k\partial_k u_i = \beta_{u_i}(u_0, u_1, \ldots)\, , \; i = 0, \ldots, n$$

- reduces search for NGFP to algebraic problem

| $n$ | $u_0^*$ | $u_1^*$ | $u_2^*$ | $u_3^*$ | $u_4^*$ | $u_5^*$ | $u_6^*$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.00523 | −0.0202 | | | | | |
| 2 | 0.00333 | −0.0125 | 0.00149 | | | | |
| 3 | 0.00518 | −0.0196 | 0.00070 | −0.0104 | | | |
| 4 | 0.00505 | −0.0206 | 0.00026 | −0.0120 | −0.0101 | | |
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NGFP is stable under extension of truncation subspace

## UV properties of $f(R)$-gravity: polynomial expansion

- linearize RG flow at NGFP

$$k\partial_k u_i \approx B_{ij}(u_j - u_j^*)\,, \quad B_{ij} = \frac{\partial \beta_{u_i}}{\partial u_j}$$

- eigenvalues $-\theta_i$ of $[B_{ij}] \Longrightarrow$ three UV relevant directions

| $n$ | Re $\theta_{0,1}$ | Im $\theta_{0,1}$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ |
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NGFP is stable under extension of truncation subspace

good evidence: fundamental theory has finite number of relevant parameters

## Summary . . .

Used FRGE to construct a flow equation for $f(R)$-gravity:

- Gravitational RG flow in the IR:
  - non-local curvature terms generically cure IR singularities ($\lambda = 1/2$)
  - . . . but are not generated dynamically
- Gravitational RG flow in the UV:
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## Summary and outlook

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Open questions:

- Gravitational RG flow in the IR:
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  - non-perturbative treatment of 4-derivative terms $\int d^dx \sqrt{g} R_{\mu\nu} R^{\mu\nu}, \ldots$

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Thank you!