Exploring the QCD Phase Diagram with Functional RG

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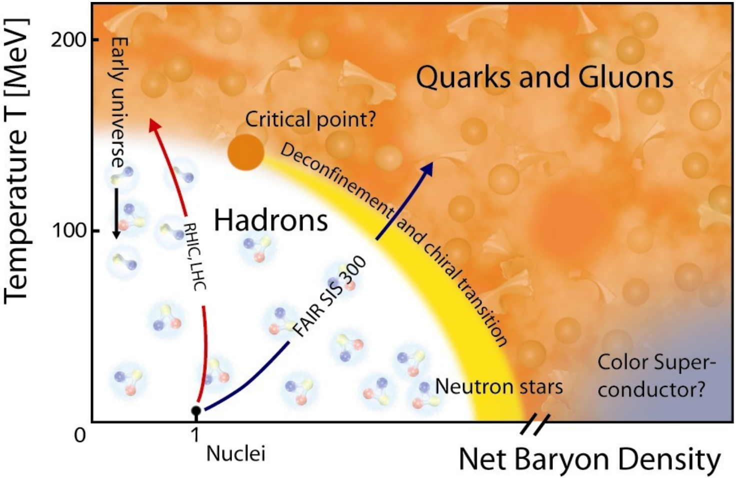
The conjectured QCD Phase Diagram



FAIR, Darmstadt

QCD: two phase transitions:

1. restoration of chiral symmetry

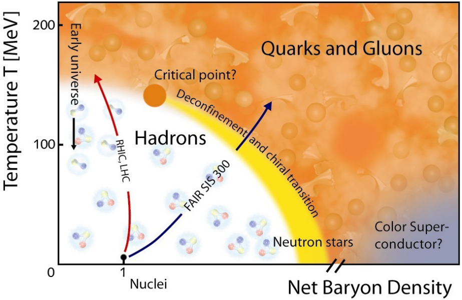
   $SU_{L+R}(N_f) \to SU_L(N_f) \times SU_R(N_f)$

   order parameter:

   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

   associate limit: $m_q \to 0$

FAIR, Darmstadt



chiral transition: spontaneous restoration of global $SU_L(N_f) \times SU_R(N_f)$ at high T

QCD: two phase transitions:

1. restoration of chiral symmetry
2. de/confinement (center symmetry)

order parameter:

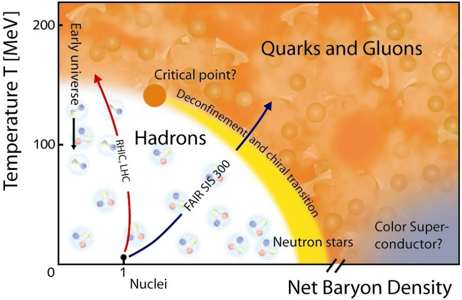
$$\frac{\langle \mathrm{tr}_c \mathcal{P}(\vec{x}) \rangle}{N_c} \begin{cases} = 0 \Leftrightarrow \text{confined phase}, & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase}, & T > T_c \end{cases}$$

$$\mathcal{P}(\vec{x}) = \mathcal{P} e^{\,i \int_0^\beta d\tau A_0(\tau, \vec{x})}$$

associate limit: $m_q \to \infty$

➜ related to free energy of static quark antiquark pair

FAIR, Darmstadt

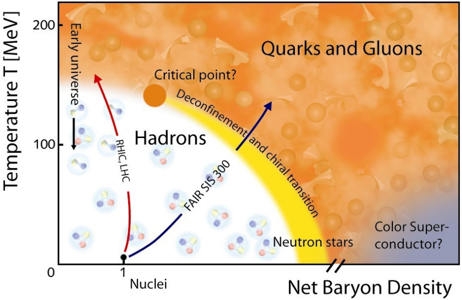


cf. talk by J. Pawlowski and poster by F. Marhauser

QCD: two phase transitions:

1. restoration of chiral symmetry
2. de/confinement

FAIR, Darmstadt



effective models:

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

Outline

◦ Two-Flavor Quark-Meson Model
  ▷ Mean field approximation
  ▷ Renormalization Group study

◦ Polyakov–Quark-Meson Model

◦ Three-Flavor Quark-Meson Model

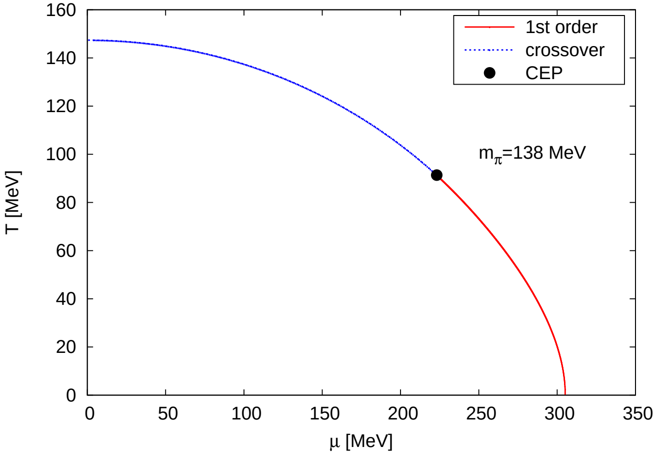
Outline

- Two-Flavor Quark-Meson Model
  - ▷ Mean field approximation
  - ▷ Renormalization Group study
- Polyakov–Quark-Meson Model
- Three-Flavor Quark-Meson Model

- Lagrangian:

$$\mathcal{L}_{\text{qm}} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

- Mean field analysis



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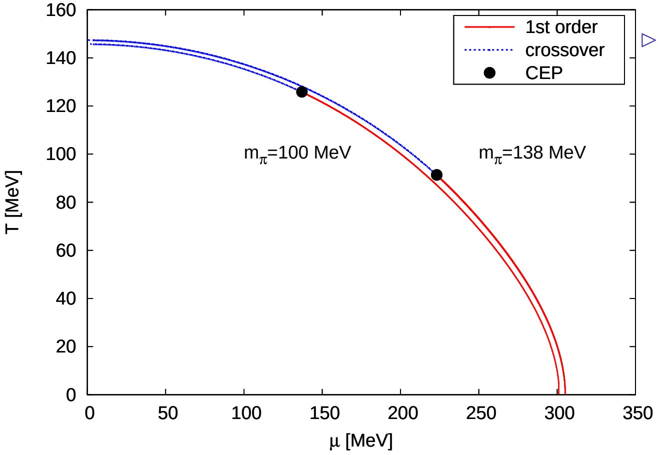

▷ as m_q decreased CEP $\rightarrow$ T-axis

- Lagrangian:

$$\mathcal{L}_{\text{qm}} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

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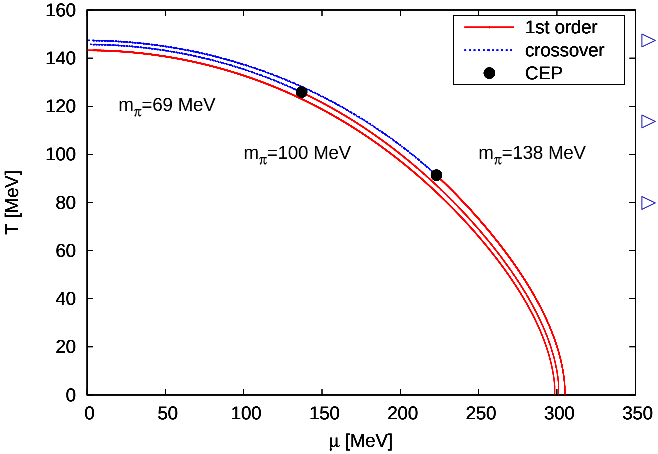


▷ as m_q decreased
CEP $\rightarrow$ T-axis

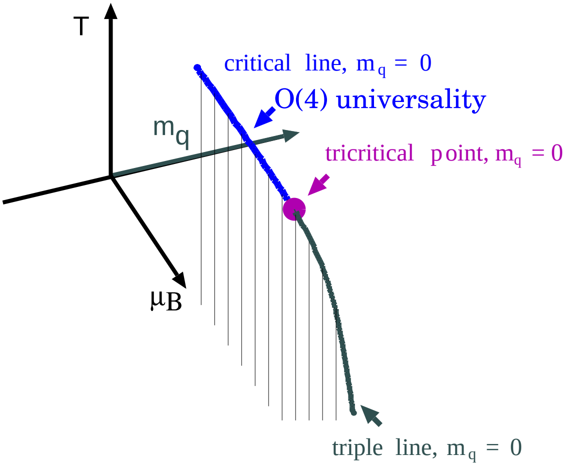
▷ chiral limit
no CEP

▷ at $\mu = 0$
$O(4)$ scaling expected
i.e. 2nd-order PT

$\rightarrow$ truncation effect

Phase diagram in (T, μ_B, m_q)-space

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$-symmetry $\longrightarrow$ 4 modes critical $\sigma, \vec{\pi}$



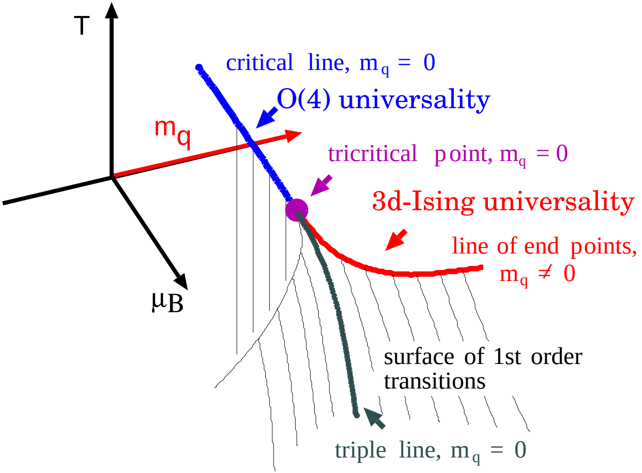
General properties

- chiral limit
  tricritical point (Gaussian fixed point)

Phase diagram in (T, μ_B, m_q)-space

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$-symmetry $\longrightarrow$ 4 modes critical $\sigma, \vec{\pi}$

$m_q \neq 0$: no symmetry remains $\longrightarrow$ only one critical mode σ (Ising) ($\vec{\pi}$ massive)



General properties

- chiral limit
  tricritical point
  (Gaussian fixed point)
- finite m_q
  critical endpoints
  (3D-Ising class)

Functional RG Approach

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

FRG (average effective action) [Wetterich]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

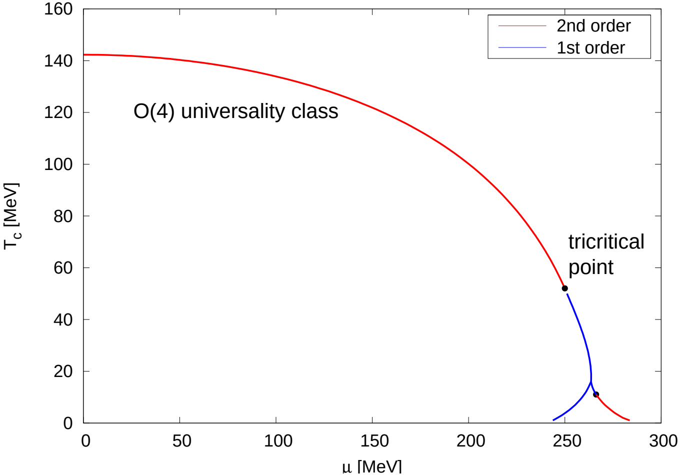
- Ansatz for Γ_k: (LO derivative expansion $\rightarrow$ arbitrary potential V_k)

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + V_k(\phi^2)$$

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

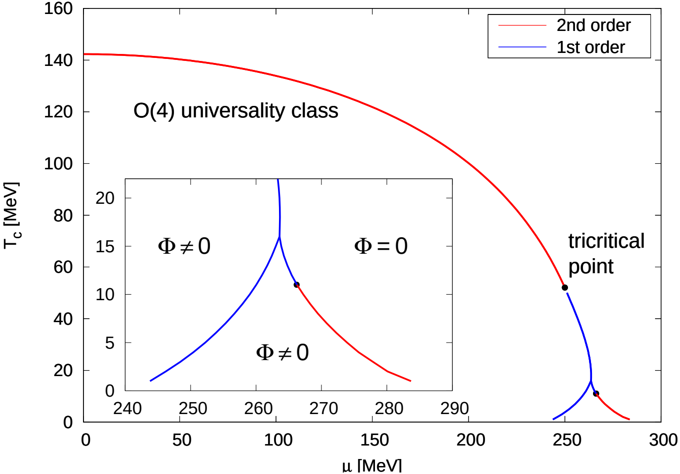
RG analysis:

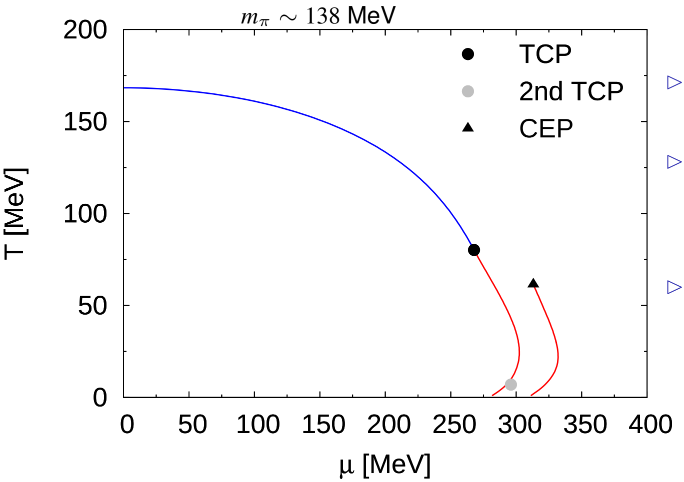
$O(4) \sim SU(2) \times SU(2)$ chiral limit



RG analysis:

$O(4) \sim SU(2) \times SU(2)$ chiral limit no spinodal lines!





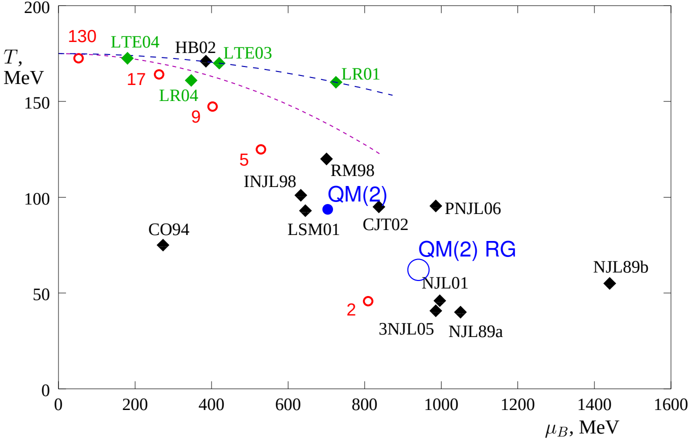
- ▷ bending usual for RG
- ▷ 2nd tricritical point in chiral limit
- ▷ features parameter independent but locations TCP/CEP parameter dependent

[BJS,Wambach '05 & '06]

[see also Tetradis et al.]

model studies vs. lattice simulations

Black points: models | Lines & green points: lattice | Red points: Freezeout points for HIC



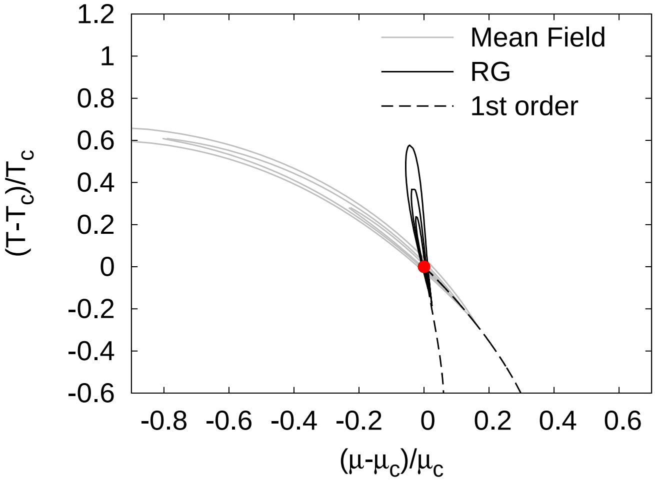
Stephanov '05 & '07

lattice methods:

- reweighting
- imaginary μ_B
- Taylor expansion around $\mu_B = 0$

Comparison with scalar χ_σ : MF↔RG

[BJS, J. Wambach '06]
cf. Tetradis et al.



- ◦ Two-Flavor Quark-Meson Model
  - ▷ Mean field approximation
  - ▷ Renormalization Group study
- ◦ Polyakov–Quark-Meson Model
- ◦ Three-Flavor Quark-Meson Model

- Lagrangian $\mathcal{L}_{\mathrm{PQM}} = \mathcal{L}_{\mathrm{qm}} + \mathcal{L}_{\mathrm{pol}}$
- Polyakov loop potential:

Polyakov 1978

Pisarski 2000

$$\mathcal{L}_{\mathrm{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$$

$$\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

Ratti, Weise et al. 2004

Dumitru, Pisarski 2004

Friman, Redlich, Sasaki 2006

$\Rightarrow$ first-order transition at $T_0 = 270$ MeV

in presence of dynamical quarks: $T_0 = T_0(N_f)$

BJS, Pawlowski, Wambach, 2007

N_f	0	1	2	$2+1$	3
T_0 [MeV]	270	240	208	187	178

cf next talk by J. Braun

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$
- Polyakov loop potential:

Polyakov 1978

Pisarski 2000

$$\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$$

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Ratti, Weise et al. 2004

Dumitru, Pisarski 2004

Friman, Redlich, Sasaki 2006

$\Rightarrow$ first-order transition at $T_0 = 270$ MeV

$\mu \neq 0: \quad T_0 = T_0(N_f, \mu)$

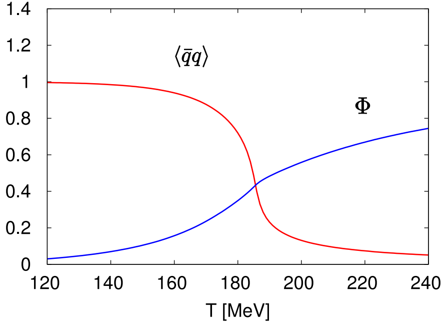
BJS, Pawlowski, Wambach, 2007

$\bar{\phi} \neq \phi^*$

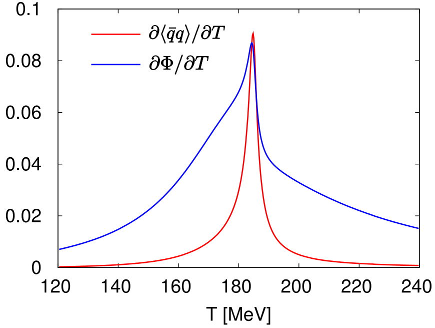
[BJS, Pawlowski, Wambach '07]

Numerical results:

order parameters



T-derivatives of order parameters

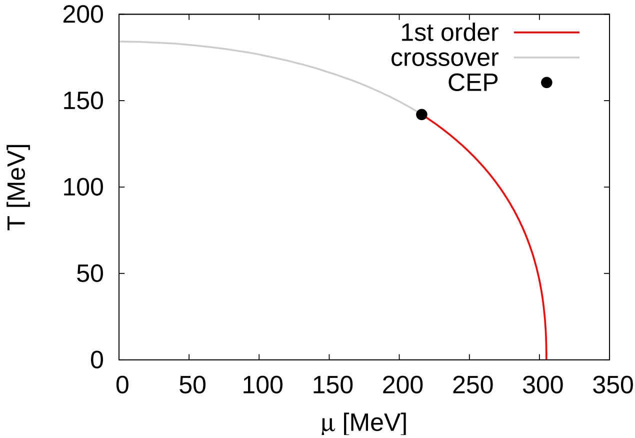


[BJS, Pawlowski, Wambach '07]

- for PQM model

in mean field approximation

chiral transition and 'deconfinement' coincide

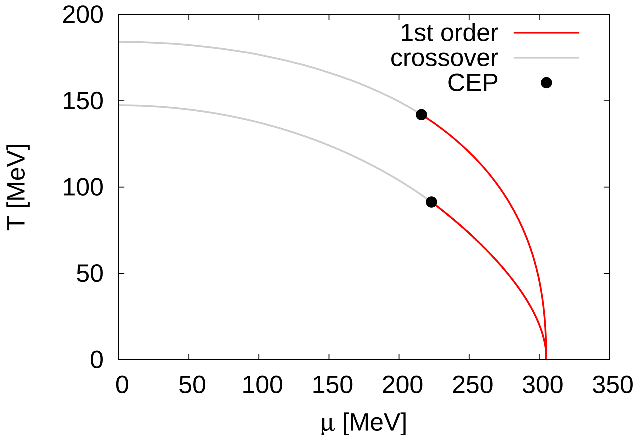


[BJS, Pawlowski, Wambach '07]

in mean field approximation

chiral transition and 'deconfinement' coincide

- for PQM model
- for QM model (lower lines)



[BJS, Pawlowski, Wambach '07]

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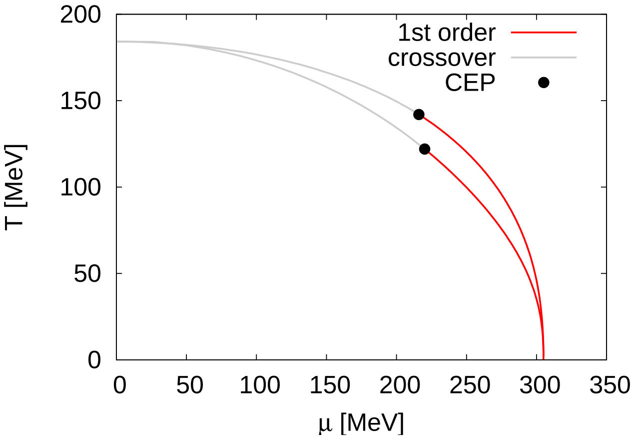


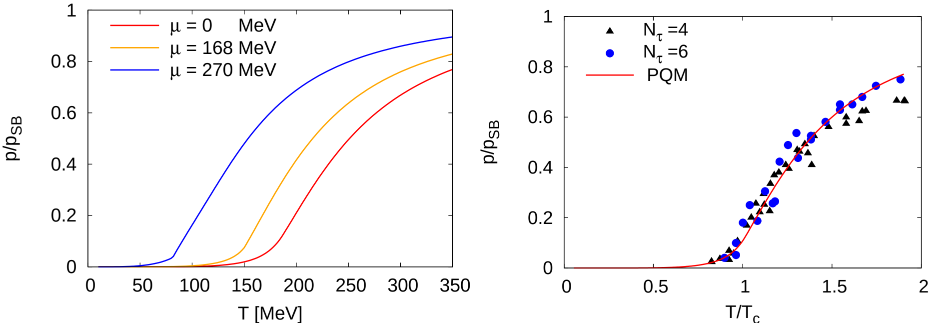
- for PQM model
- for PQM model with μ-modification in Polyakov loop potential

  (lower lines)

- perturbative pressure of QCD with N_f massless quarks

$$\frac{p}{T^4} = (N_c^2 - 1)\frac{\pi^2}{45} + N_f\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu}{T}\right)^4\right] .$$

- $N_f = 2$: lattice: $N_\tau = 4, 6$; $\mu = 0$



[Ali Khan et al. '01]

Outline

◦ Two-Flavor Quark-Meson Model

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◦ Polyakov–Quark-Meson Model

◦ Three-Flavor Quark-Meson Model

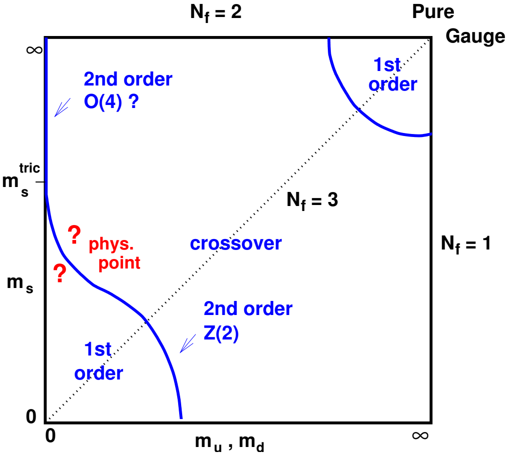
Columbia plot:

[Brown et al. '90]

$T_\chi^{N_f=2} \sim 175$ MeV

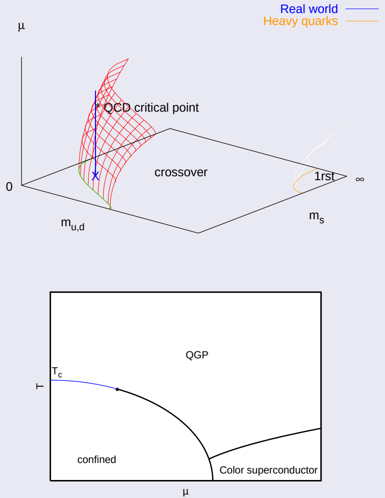
$T_d \sim 270$ MeV

$T_\chi^{N_f=3} \sim 155$ MeV

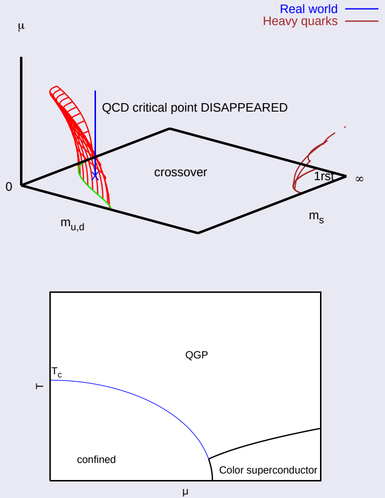


Mass Sensitivity (lattice, $N_f = 3, \mu_B \neq 0$)

Standard scenario: $m_c(\mu)$ increasing



Nonstandard scenario: $m_c(\mu)$ decr.



[de Forcrand, Philipsen: hep-lat/0611027]

- Lagrangian $\mathcal{L} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - g\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

$$\begin{aligned}\mathcal{L}_{\text{meson}} = & \, tr(\partial_\mu\phi^\dagger\partial^\mu\phi) - m^2 tr(\phi^\dagger\phi) - \lambda_1(tr(\phi^\dagger\phi))^2 \\ & -\lambda_2 tr((\phi^\dagger\phi)^2) + c\,(\det(\phi) + \det(\phi^\dagger)) \\ & + trH(\phi + \phi^\dagger)\end{aligned}$$

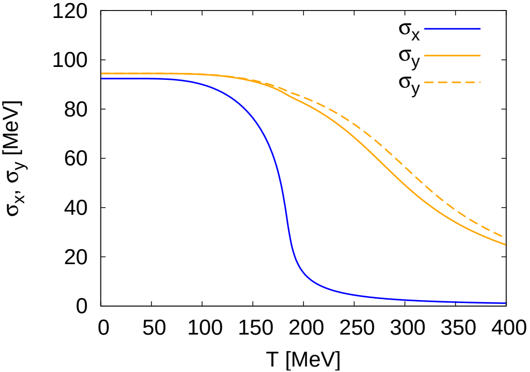
fields: $\phi = \sum_a \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$ and $H = \sum_a \frac{\lambda_a}{2} h_a$

σ_a scalar and π_a pseudoscalar nonet

[BJS, M. Wagner, to be published]

➜ two condesates: nonstrange $\sigma_x(T, \mu_f)$ and strange $\sigma_y(T, \mu_f)$

with (solid) and without (dashed) $U(1)_A$ anomaly



- ▷ almost no effect due $U(1)_A$ anomaly
- ▷ T_c depends on m_σ

[BJS, M. Wagner, to be published]

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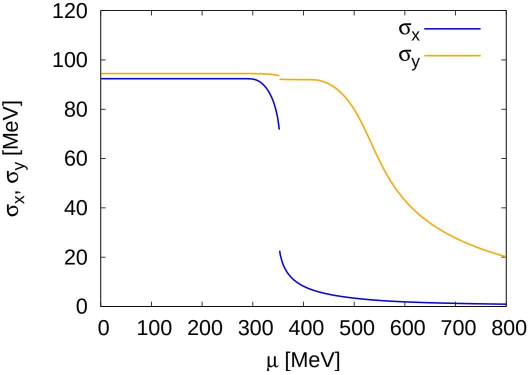


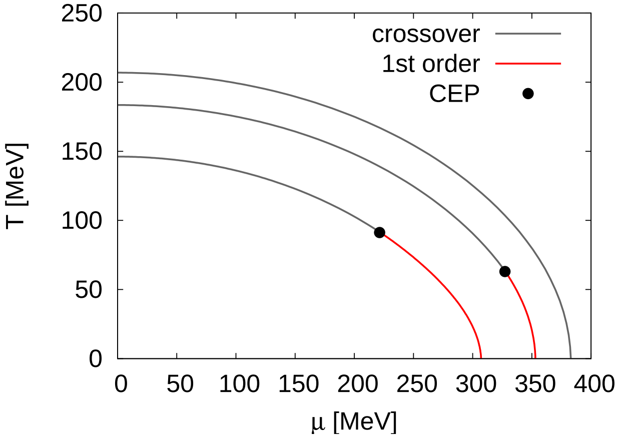
- ▷ almost no effect due $U(1)_A$ anomaly
- ▷ T_c depends on m_σ
- ▷ $\mu \equiv \mu_q = \mu_s$
- ▷ μ_c depends on m_σ

[BJS, M. Wagner, to be published]

$m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV

PDG: $f_0(600)$ mass=$(400 \ldots 1200)$ MeV

➜ influence of existence of CEP!



In-medium meson masses

[BJS, M. Wagner, to be published]

- ▷ genuine problem of linear sigma model w/o quarks at finite T

  $\rightarrow$ negative meson masses

- ▷ but not in this approximation

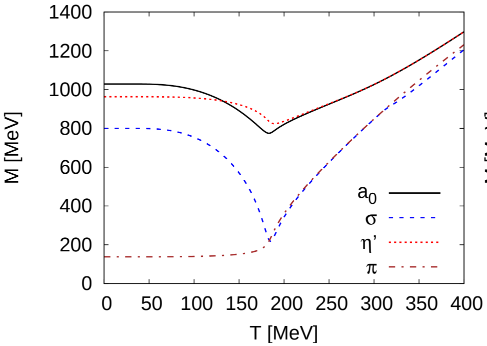
  ➜Ward identities, Goldstone theorem etc. are all valid in-medium e.g.

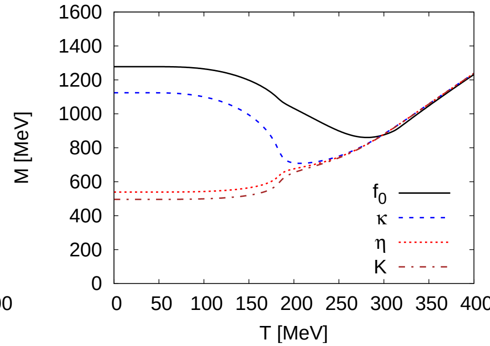
$$h_x = f_\pi m_\pi^2$$

similar in strange sector

[BJS, M. Wagner, to be published]

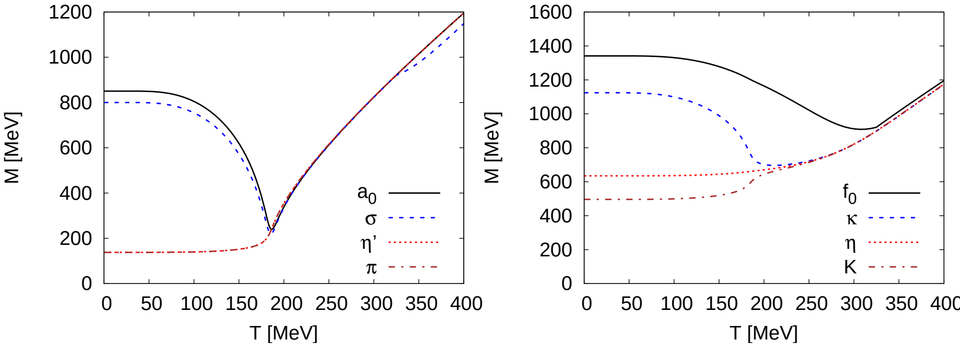
masses with $U(1)_A$ anomaly





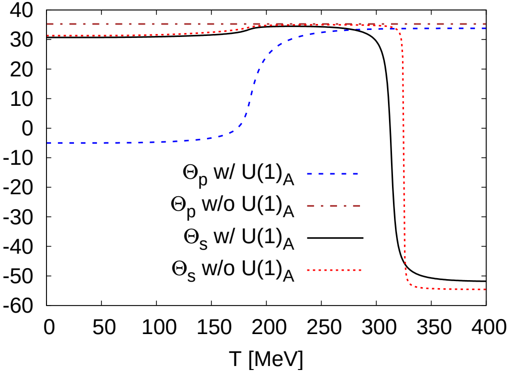
[BJS, M. Wagner, to be published]

masses without $U(1)_A$ anomaly



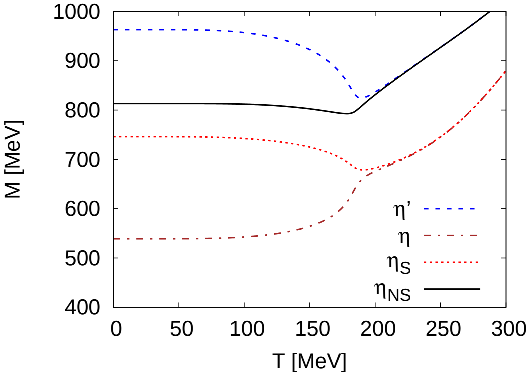
physical $(\eta,\eta') \longleftrightarrow (\eta_8,\eta_0)$ via mixing angle θ_p

➜ pseudoscalar [and scalar] mixing angles
as a function of T (for $\mu = 0$)
with and without $U(1)_A$ anomaly



physical $(\eta,\eta') \longleftrightarrow (\eta_8,\eta_0)$ via mixing angle θ_p

η_{S}: strange η and η_{NS}: nonstrange η

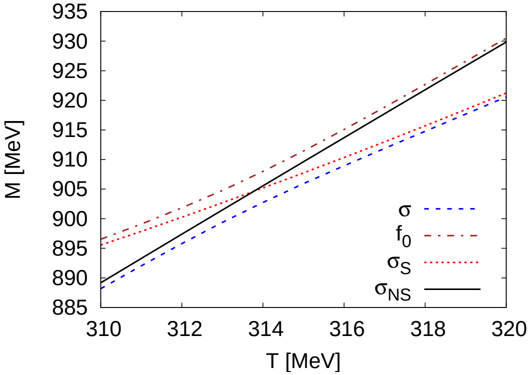


- ▷ $\eta' \to \eta_{\mathrm{NS}}$
- ▷ $\eta \to \eta_{\mathrm{S}}$ for $T > 200$ MeV
- ▷ no Witten-Veneziano relation has been used

physical (σ,f_0) $\longleftrightarrow$ (σ_8,σ_0) via mixing angle θ_s

physical masses anticross

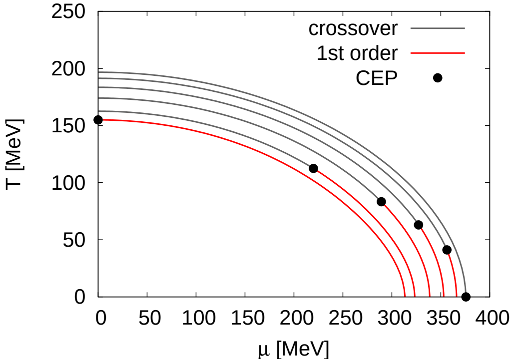
strange-nonstrange state crosses



RG arguments predict for $N_f = 3$ 1st-order in chiral limit

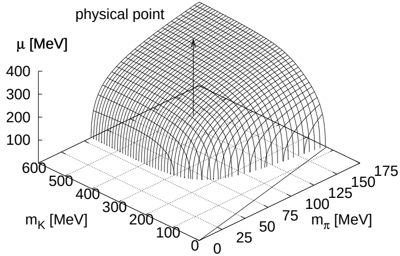
▷ $m_\pi/m_\pi^* = 0.49$ (lower line), $0.6, 0.8\ldots, 1.36$ (upper line) $m_\pi^* = 138$ MeV

with $U(1)_A$, $m_\sigma = 800$ MeV

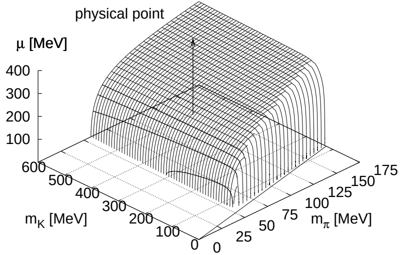


- chiral critical surface in (m_π, m_K) plane

with $U(1)_A$



without $U(1)_A$



Quark-meson model study for $N_F = 2$

➜ Mean field versus RG

Influence of fluctuations on phase diagram

Findings:

- ▷ MF phase diagram: no TCP (in chiral limit) found
- ▷ RG phase diagram: two TCP's (in chiral limit) & CEP found
- ▷ Size of critical region via susceptibilities: "compressed" with fluctuations

Quark-meson model study for $N_F = 3$

➜ Mean field approximation

no need for Optimized Perturbation Theory

with and without axial anomaly

Polyakov–quark-meson model study for $N_F = 2$

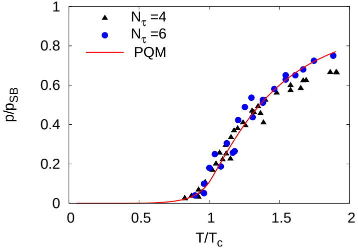
➜ so far mean-field approximation

Findings:

- ▷ Parameter in Polyakov loop potential: $T_0 \Rightarrow T_0(N_f, \mu)$
  - pure gauge: $T_0 \sim 270$ MeV
  - $N_f = 2$: $T_0 \sim 210$ MeV
- ▷ Chiral & deconfinement transition coincide
- ▷ Mean-field approximation encouraging

  Quark-meson model is renormalizable

  $\rightarrow$ no UV cutoff parameter (cf. PNJL model)



Outlook

- ▷ include quark-meson dynamics in PQM model and for $N_f = 3$ with FRG