Nuclear Forces and the Renormalization Group

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TRIUMF

CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS

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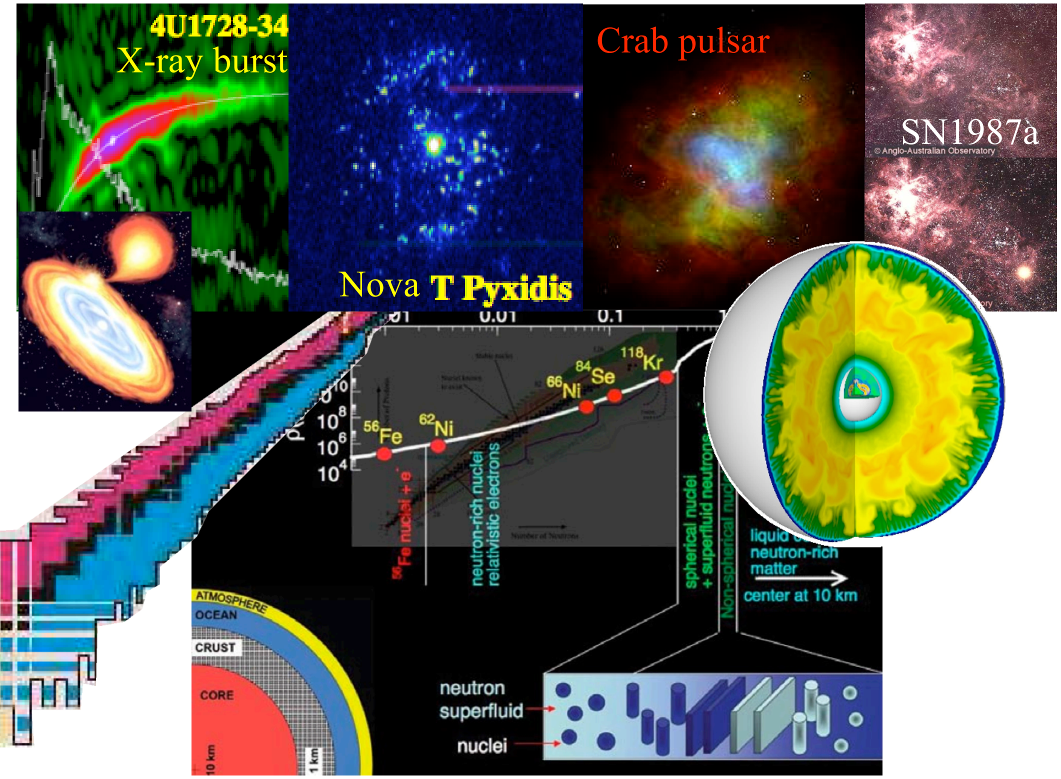
Strong interaction physics in the lab and cosmos

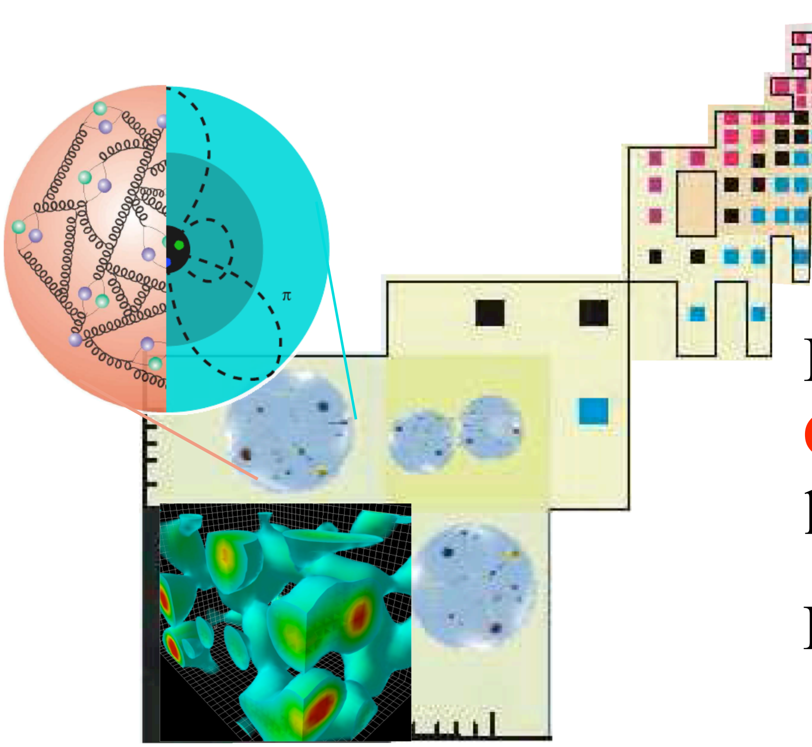
Matter at the extremes:

density $\rho \sim 10^{11} \ldots 10^{15}$ g/cm^3

neutron-rich to proton-rich
Z/N ~0.05…0.6

temperatures T ~…30 MeV





Interaction challenges:
QCD ⇒ chiral EFT ⇒ RG evolved
low-momentum interactions for all nuclei

Many-body challenges

Astrophysics challenges

Outline

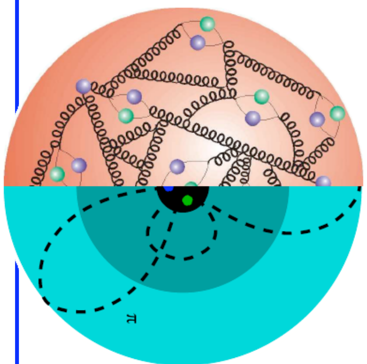
Resolution dependence of nuclear forces

Effective field theory and the renormalization group

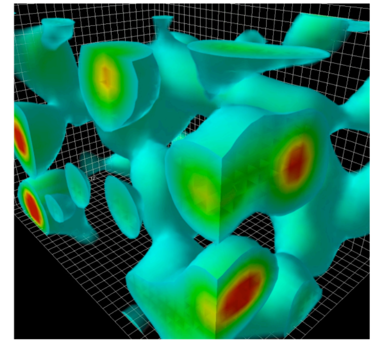
Many-body developments

Λ / Resolution dependence of nuclear interactions

with high-energy probes:
quarks+gluons



at low energies:
complex QCD vacuum



lowest energy excitations:
pions, nearly massless, m_π=140 MeV

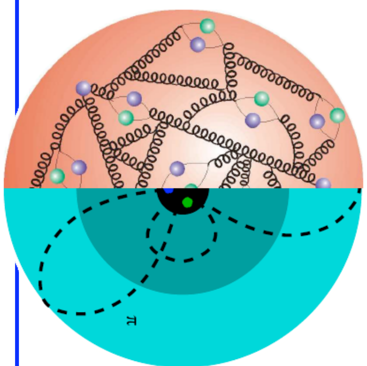
Λ_{chiral}
momenta $Q \sim \lambda^{-1} \sim m_\pi$

$\Lambda_{pionless}$
$Q << m_\pi$=140 MeV

Λ / Resolution dependence of nuclear interactions

with high-energy probes:
quarks+gluons

cf. scale/scheme dependence
of parton distribution functions



Lattice QCD

Effective theory for NN, many-N interactions,
operators depend on resolution scale Λ

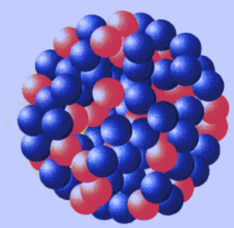
$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

$\Lambda_{\rm chiral}$

momenta $Q \sim \lambda^{-1} \sim m_\pi$: chiral effective field theory

nucleons interacting via pion exchanges + contact interactions

typical Fermi momenta in nuclei $\sim m_\pi$

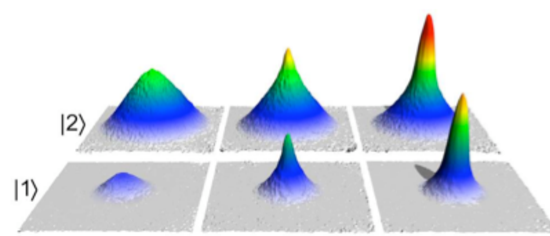


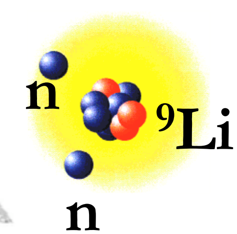
$\Lambda_{\rm pionless}$

$Q << m_\pi$=140 MeV: pion not resolved

pionless effective field theory

large scattering lengths + corrections





applicable to loosely-bound, dilute systems, reactions at astro energies

Lattice QCD and nuclear forces

pion-NN coupling g_a from full QCD
Edwards et al. (2006)

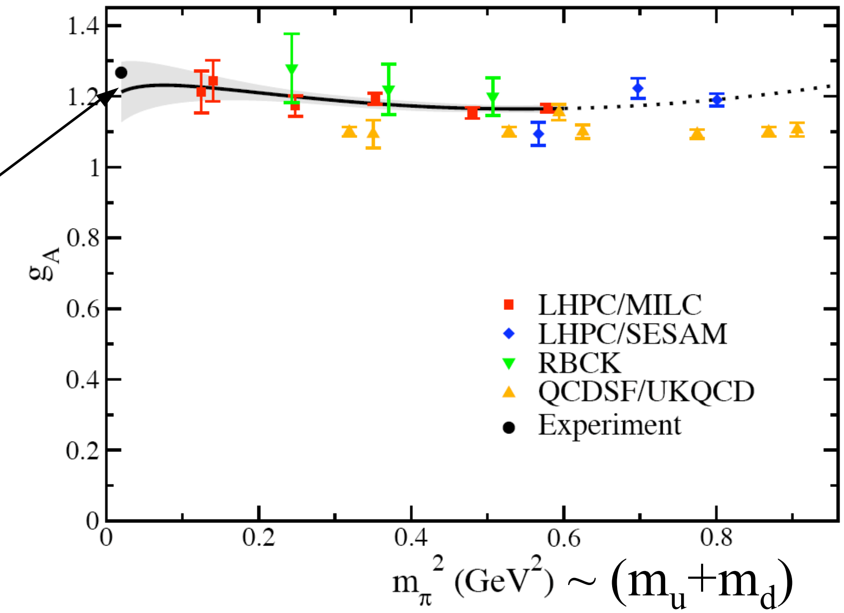
chiral EFT extrapolation to physical pion mass agrees with experiment

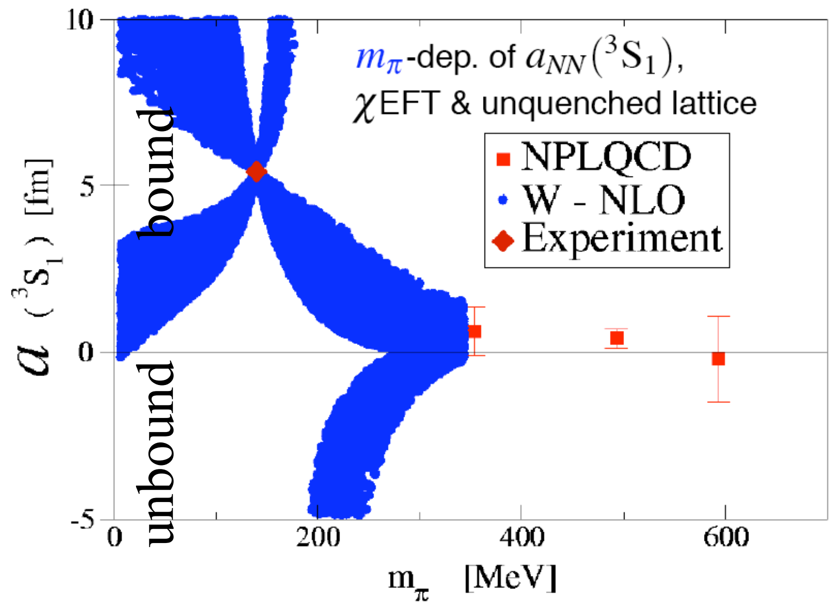
NN scattering lengths from full QCD, dependence on quark masses Beane et al. (2006)

First coherent effort to connect nuclear forces to underlying QCD

Constrain some low-energy couplings

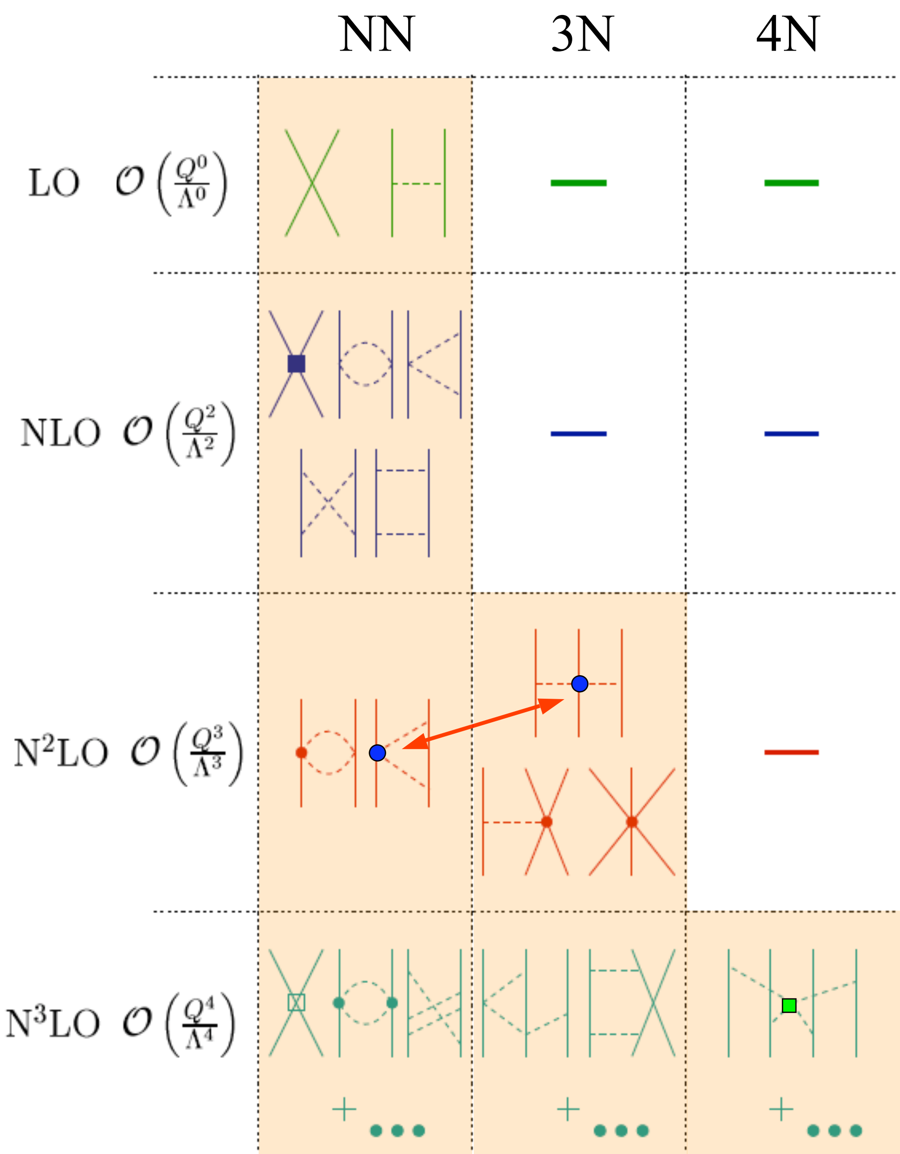
Constrain experimentally difficult observables: 3-neutron properties





Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale Λ_b



limited resolution at low energies, can expand in powers Q/Λ_b

details of short-distance physics not resolved

capture in few short-range couplings, fit to experiment once

include long-range physics explicitly, pions for chiral EFT

systematic: can work to desired accuracy and obtain error estimates

can connect to lattice QCD

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Chiral Effective Field Theory for nuclear forces

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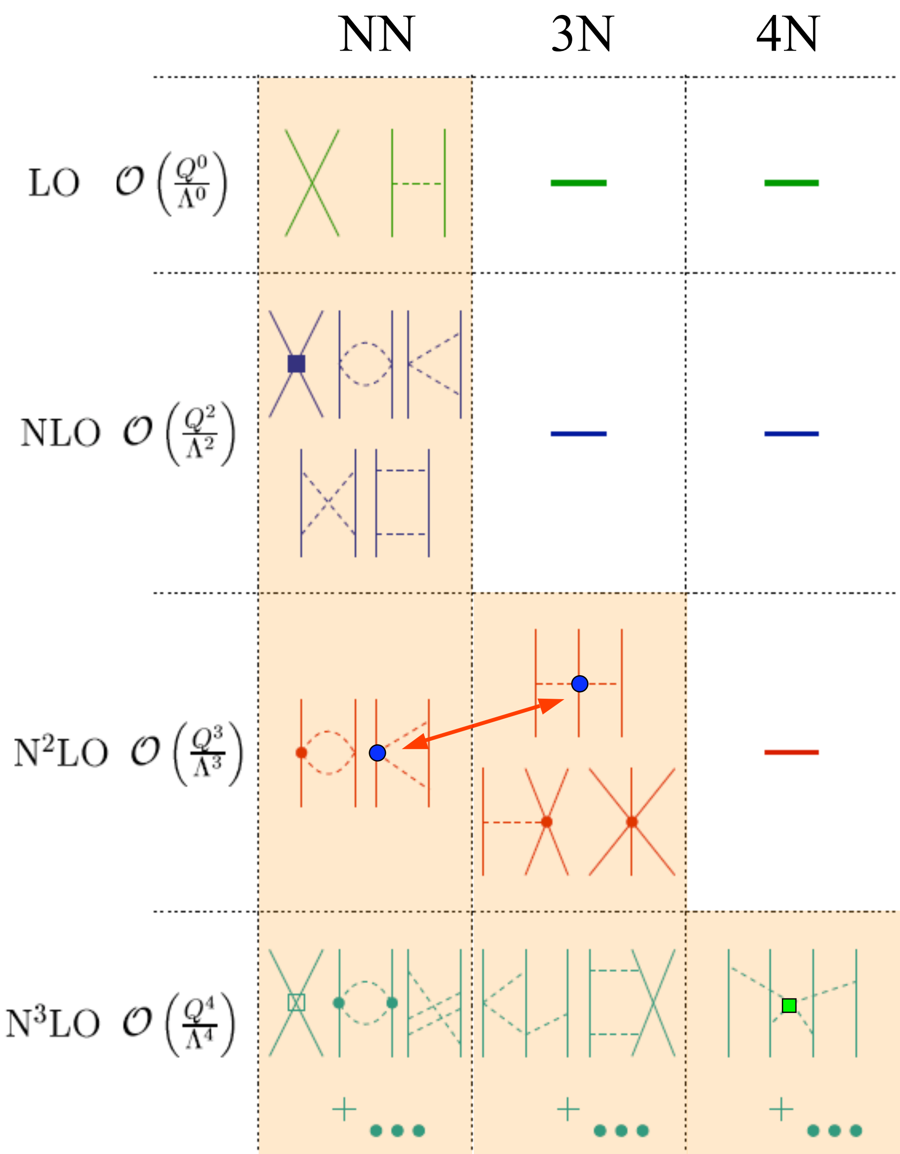


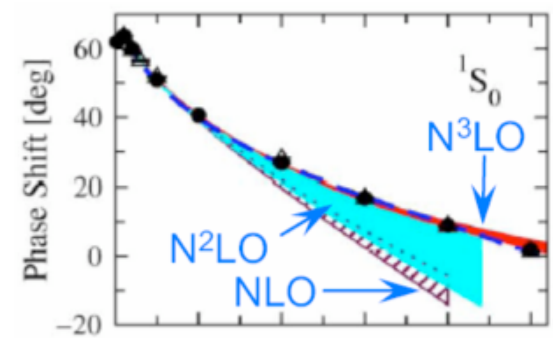
explains pheno hierarchy:
NN > 3N > 4N > …

NN-3N, πN, ππ, electro-weak,…
consistency

3N,4N: 2 new couplings to N^{3}LO

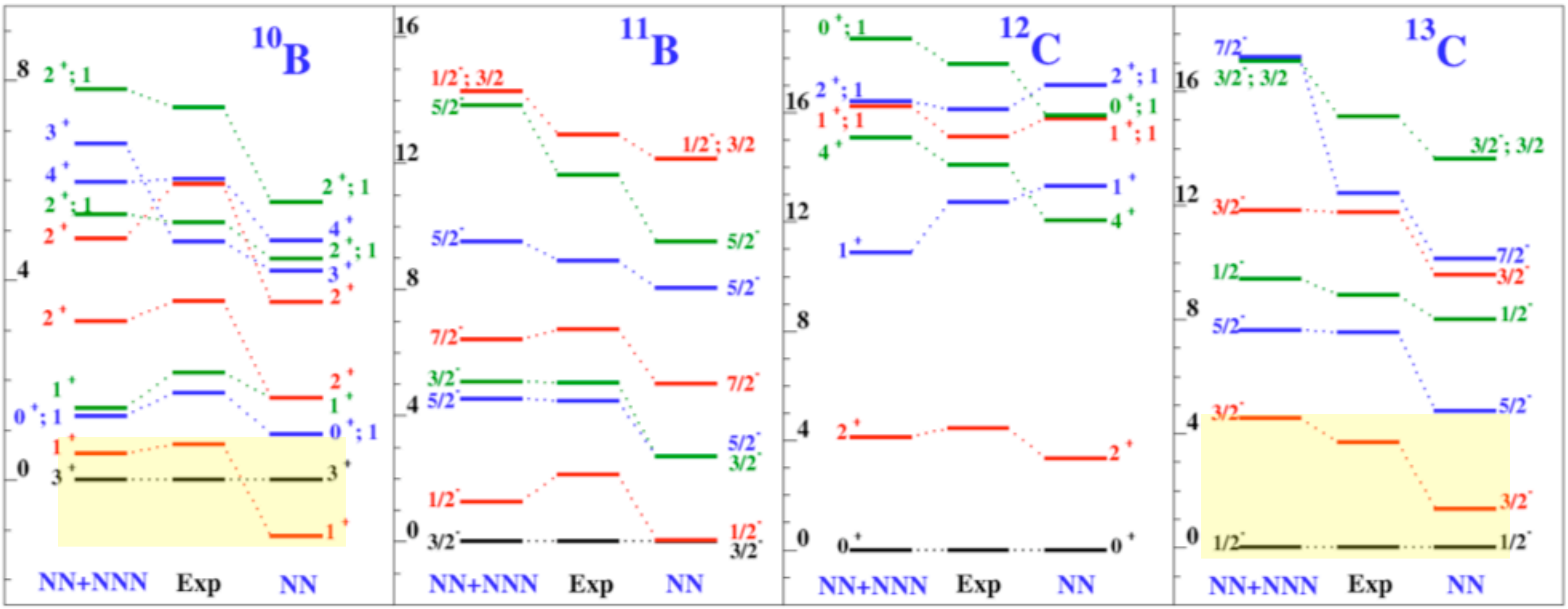
resolution/Λ-dependent couplings

error estimates from truncation order,
lower bound from Λ variation



Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Chiral EFT highlights

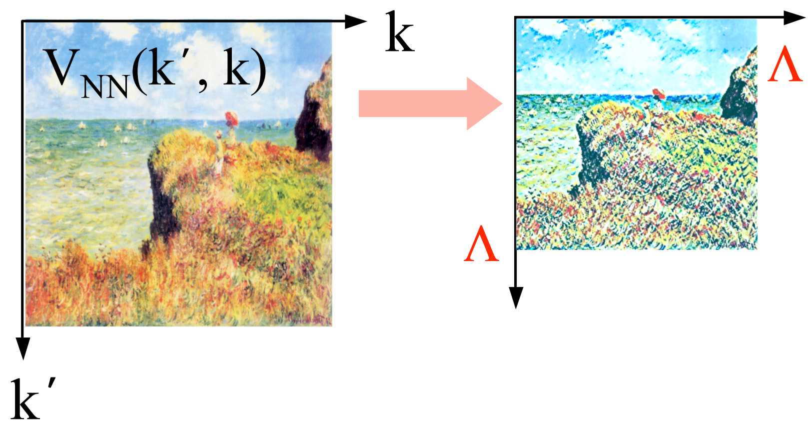


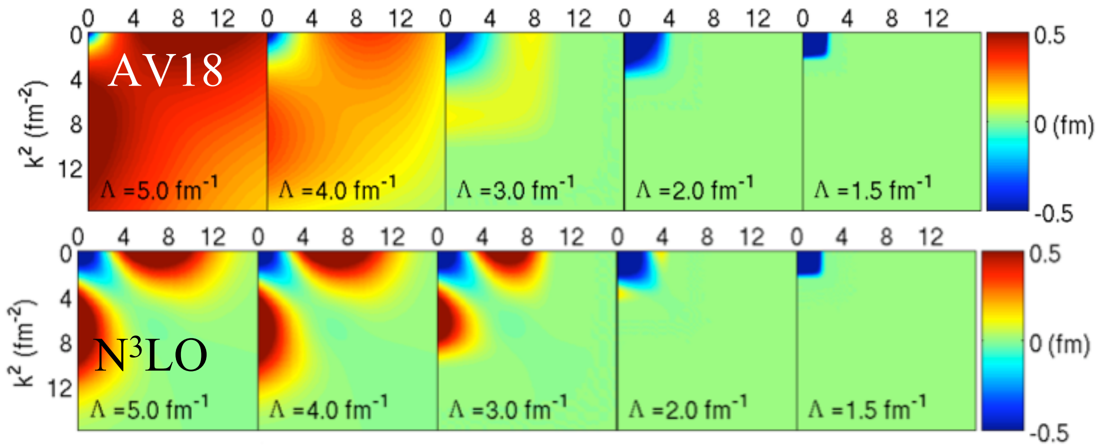
Navratil et al. (2007)

impressive agreement, highlights the importance of 3N interactions

Low-momentum interactions from the Renormalization Group

evolve to lower resolution/cutoffs by integrating out high-momenta,
can be carried out exactly for NN interactions Bogner, Kuo, AS (2003)





implemented by RG equations
or unitary transformation

$$\frac{\mathrm{d}}{\mathrm{d}\Lambda} V^{\Lambda}_{\mathrm{low}\,k}(k',k) = \frac{2}{\pi} \frac{V^{\Lambda}_{\mathrm{low}\,k}(k',\Lambda)\, T^{\Lambda}(\Lambda,k;\Lambda^2)}{1-(k/\Lambda)^2}$$

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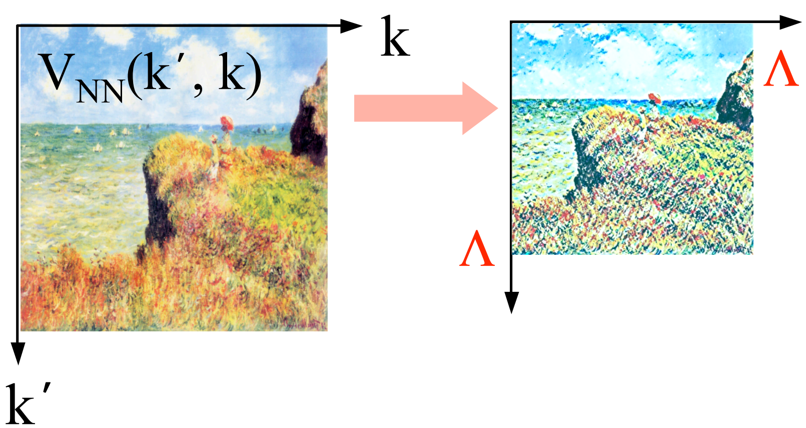
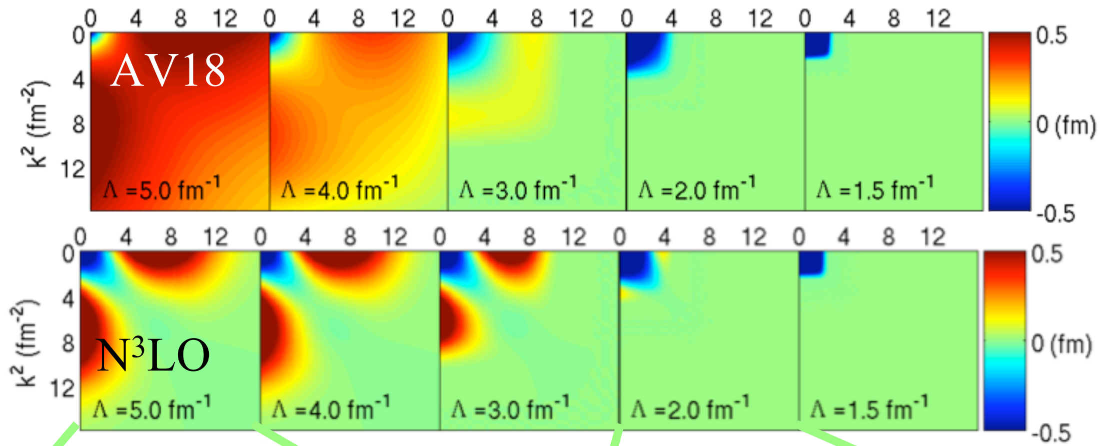
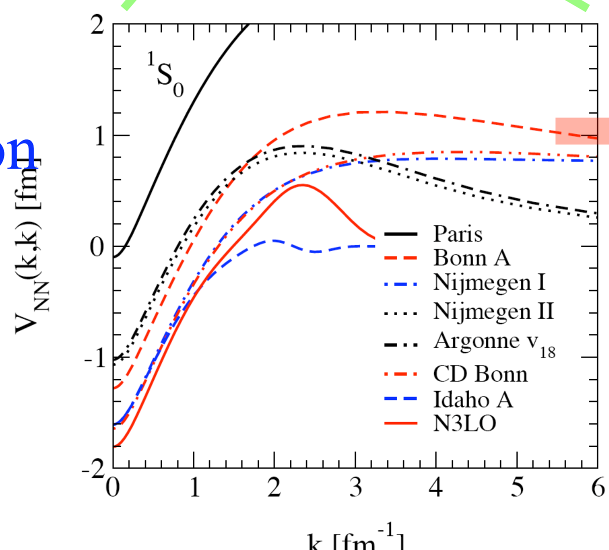


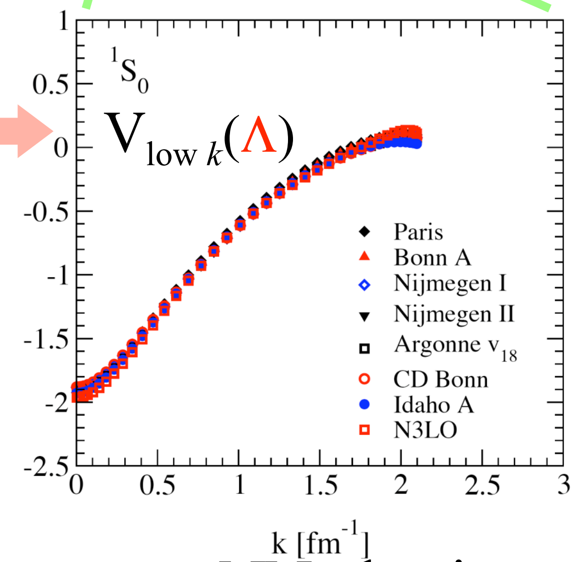


implemented by RG equations
or unitary transformation

find ≈ universal interaction
for low momenta

evolution to $V_{\text{low }k}(\Lambda)$
decouples high momenta



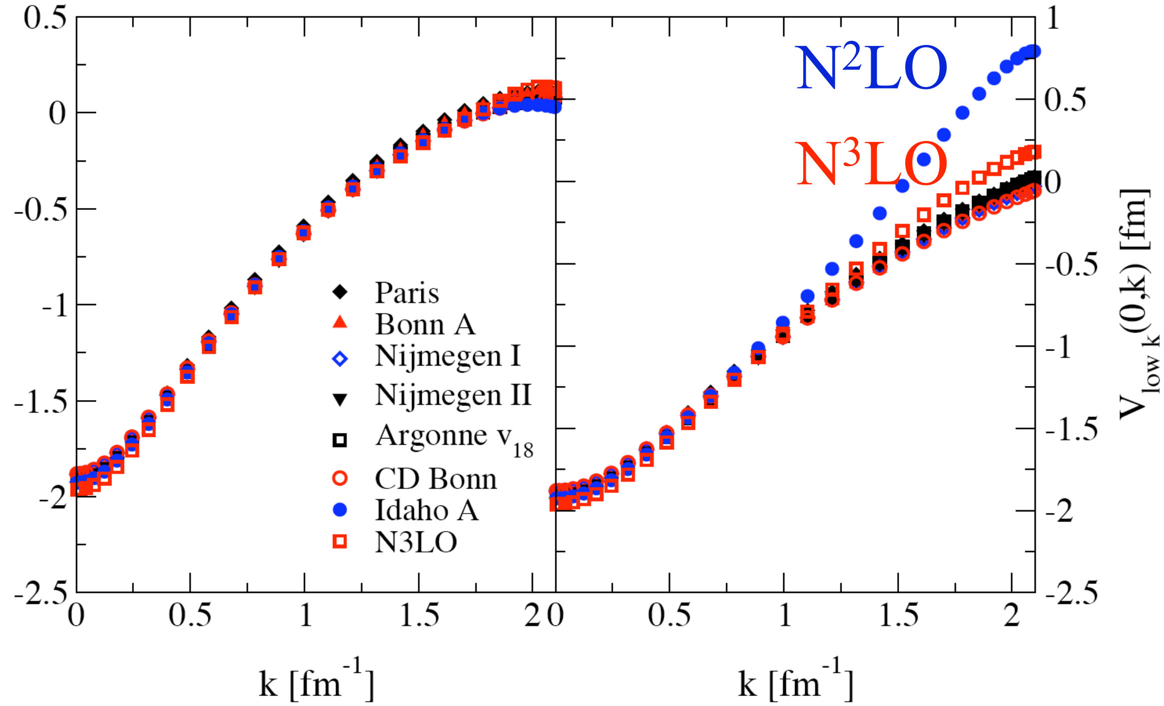


method to vary resolution scale without loss of low-energy NN physics

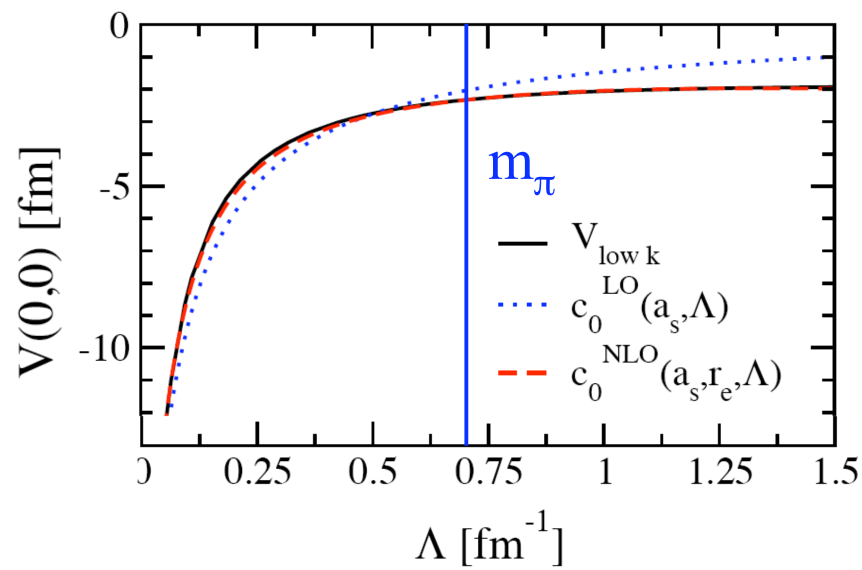
Chiral EFT and RG

Collapse to $V_{\mathrm{low}\,k}$ band for higher orders in chiral EFT

Renormalization generates higher-order contacts



Evolution of $V_{\mathrm{low}\,k}(0,0;\Lambda)$ follows contact interaction $c_0(\Lambda)$ at NLO



Weinberg eigenvalue diagnostic

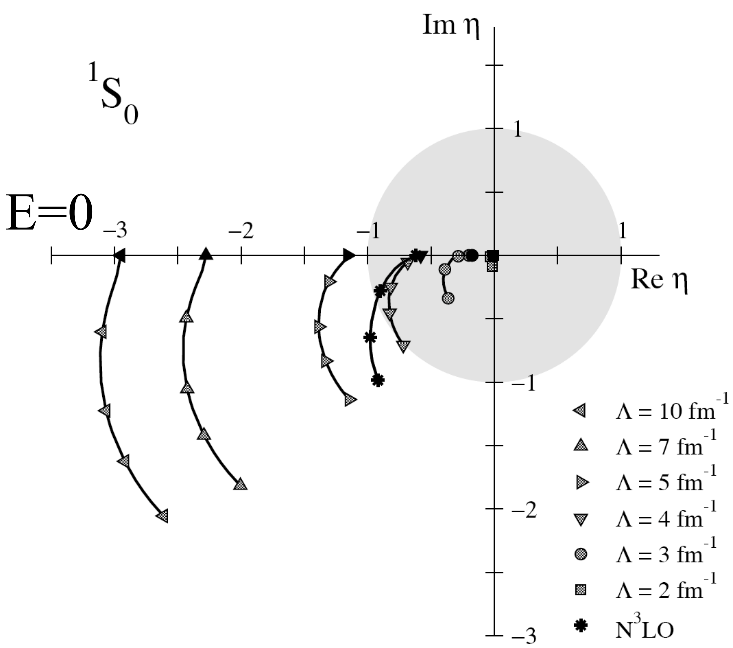
study spectrum of $G_0(z)V\,|\Psi_\nu(z)\rangle = \eta_\nu(z)\,|\Psi_\nu(z)\rangle$ at fixed energy z

governs convergence $T(z)\,|\Psi_\nu(z)\rangle = (1+\eta_\nu(z)+\eta_\nu(z)^2+\ldots)\,V\,|\Psi_\nu(z)\rangle$

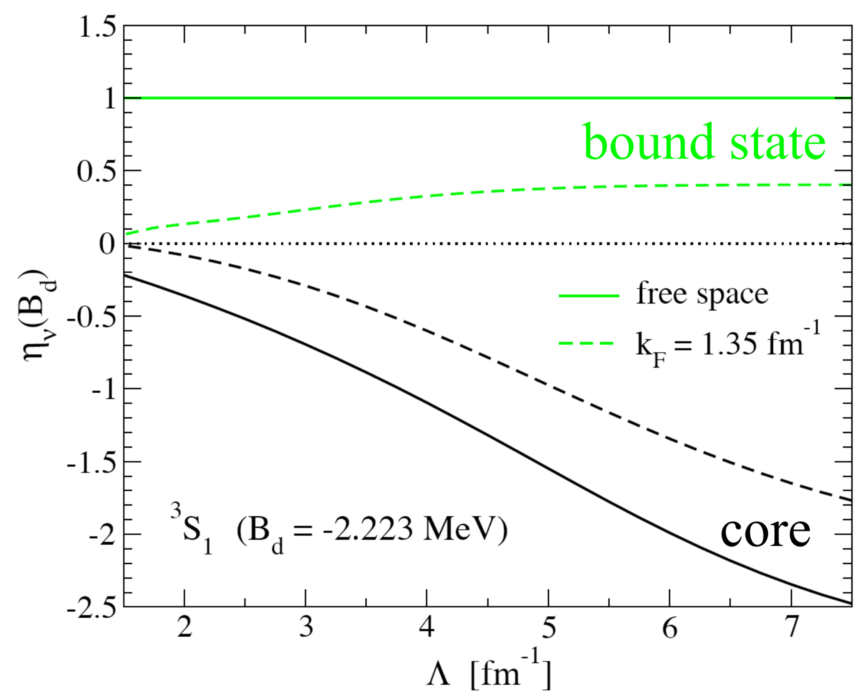
can write as Schrödinger eqn $\left(H_0 + \dfrac{1}{\eta_\nu(z)}V\right)|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$

high momenta/large cutoffs lead to flipped-potential bound states of -λV for small λ/large η ⇒ Born series always nonperturbative with cores

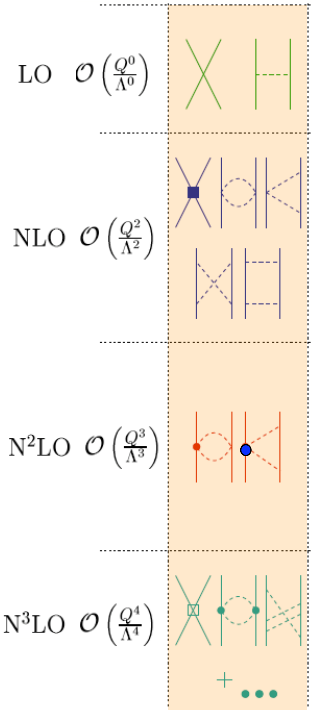
Repulsive core eigenvalues small for lower cutoffs

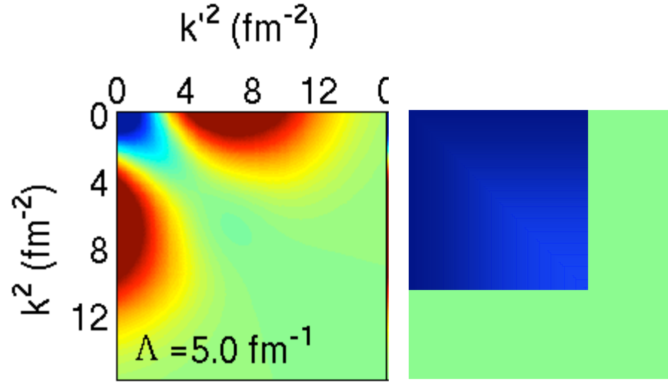


Bogner, AS, Furnstahl, Nogga (2005)

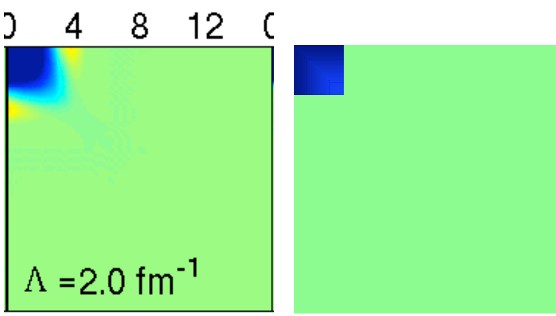


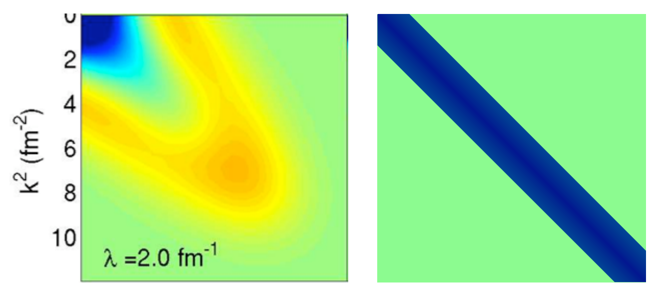
Different schemes: RG and Similarity RG (SRG)





all interactions are low-energy NN equivalent, up to truncation errors





sharp or smooth cutoff $V_{\text{low }k}(\Lambda)$ from RG equations or unitary transformations

smooth cutoff $V_{\text{SRG}}(\lambda)$ evolves towards band diagonal

Bogner et al. (2007)

high momenta decouple

Similarity RG interactions

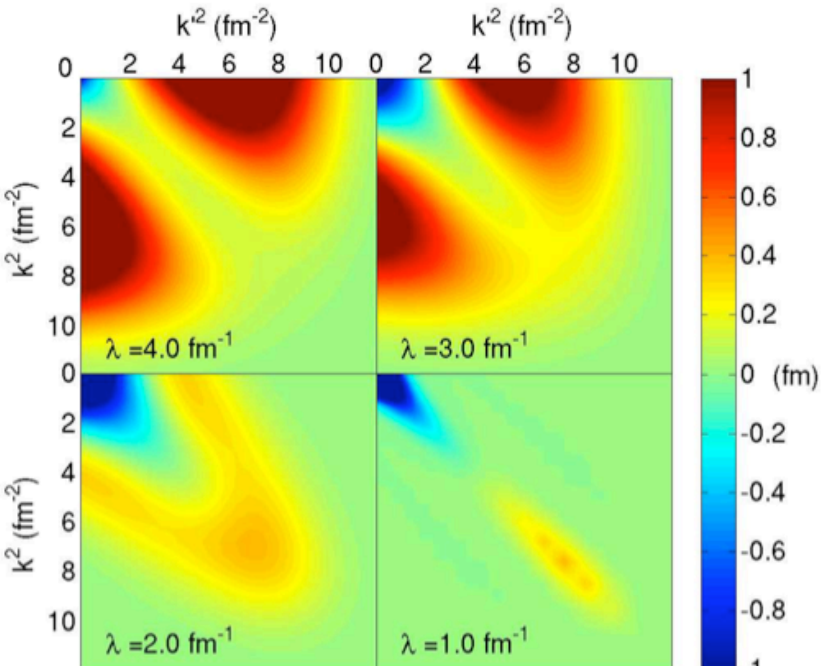
Unitary transformations to band-diagonal $V_{srg}(\lambda)$ from flow equations
Glazek, Wilson (1993), Wegner (1994)

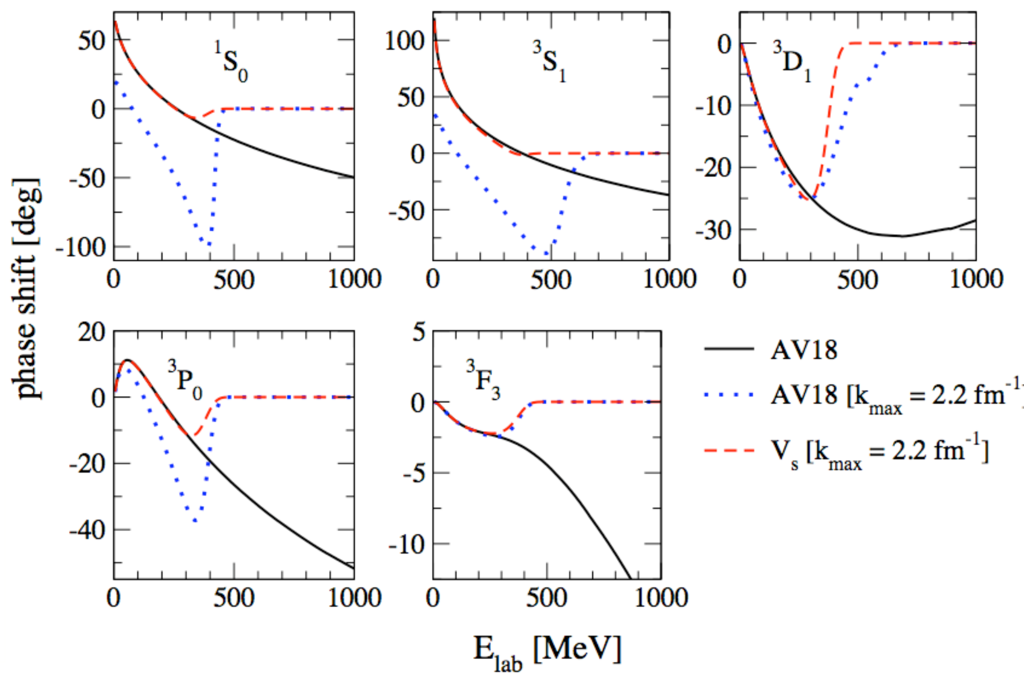
$$\frac{dH_s}{ds} = [\eta_s, H_s] = [[G_s, H_s], H_s]$$

with flow operator $G_s = T_{rel}$ and resolution $\lambda = s^{-1/4}$ Bogner et al. (2007)

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2\, dq\, (k^2 + k'^2 - 2q^2)\, V_s(k,q)\, V_s(q,k')$$

intermediate momenta $k > k_{max} \sim \lambda$ decouple for low energies

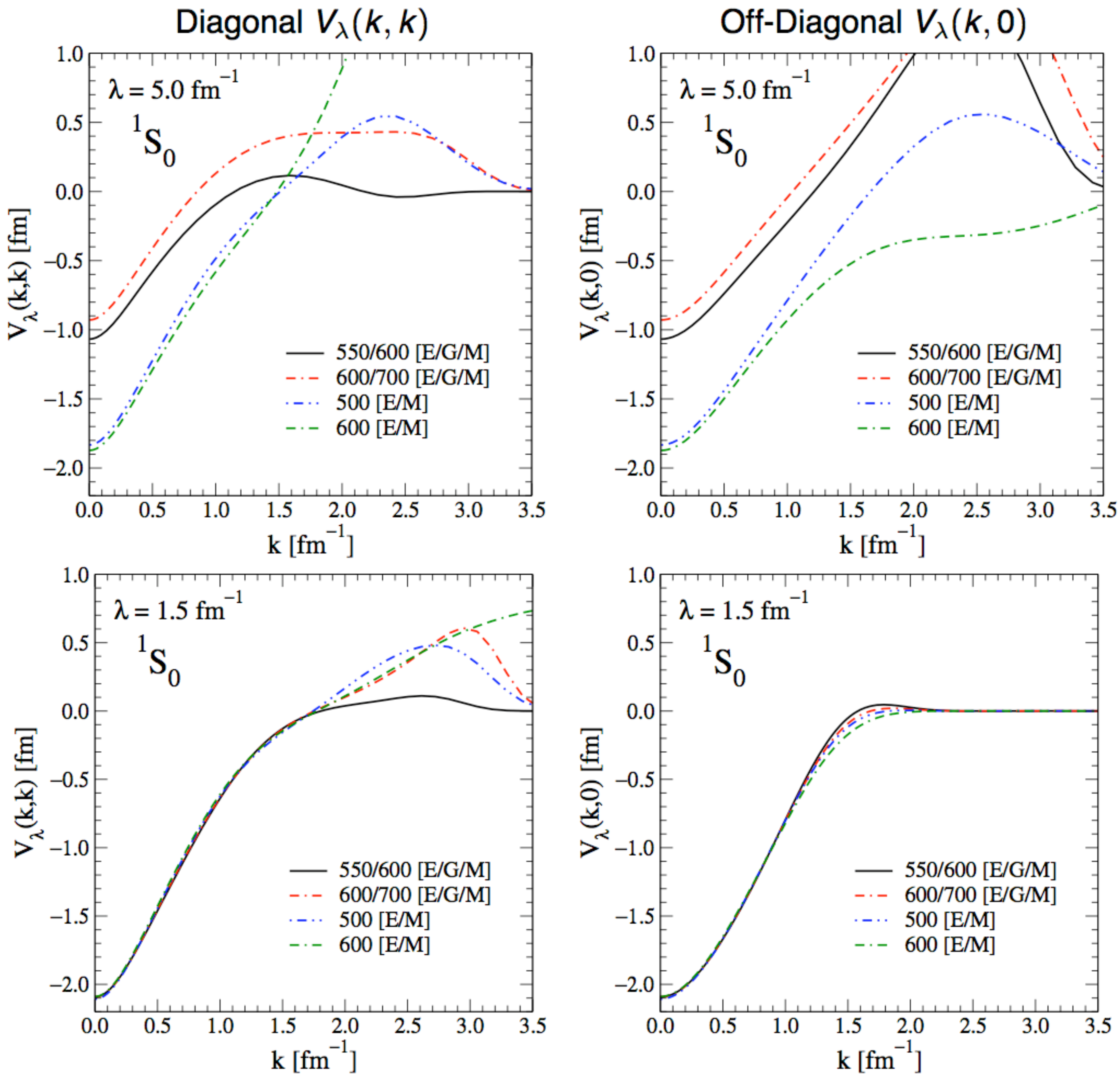




Chiral EFT and RG

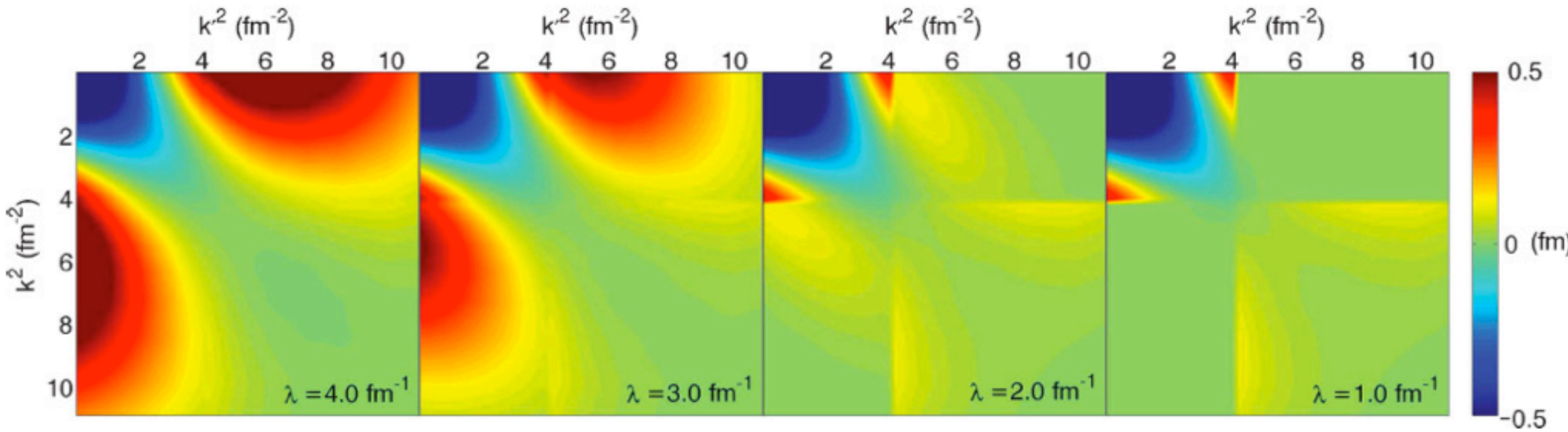
find ≈ universality from different N^3LO potentials

weakens off-diag coupling



Block diagonalization using SRG

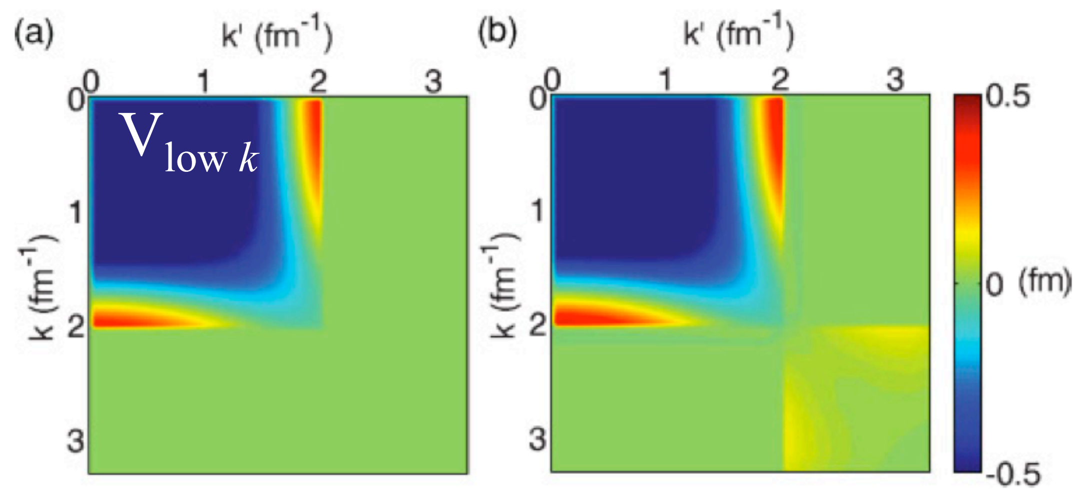
with block-diagonal flow operator Bogner et al. (2008) $G_s = \begin{pmatrix} P H_s P & 0 \\ 0 & Q H_s Q \end{pmatrix}$



low-momentum blocks very similar to $V_{\text{low } k}$

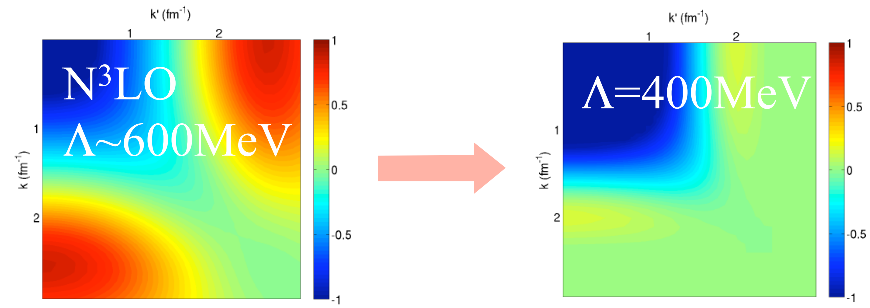
formal equivalence?

connections to EFT?



Advantages of low-momentum interactions for nuclei

high momenta/large cutoffs lead to slow convergence for nuclei



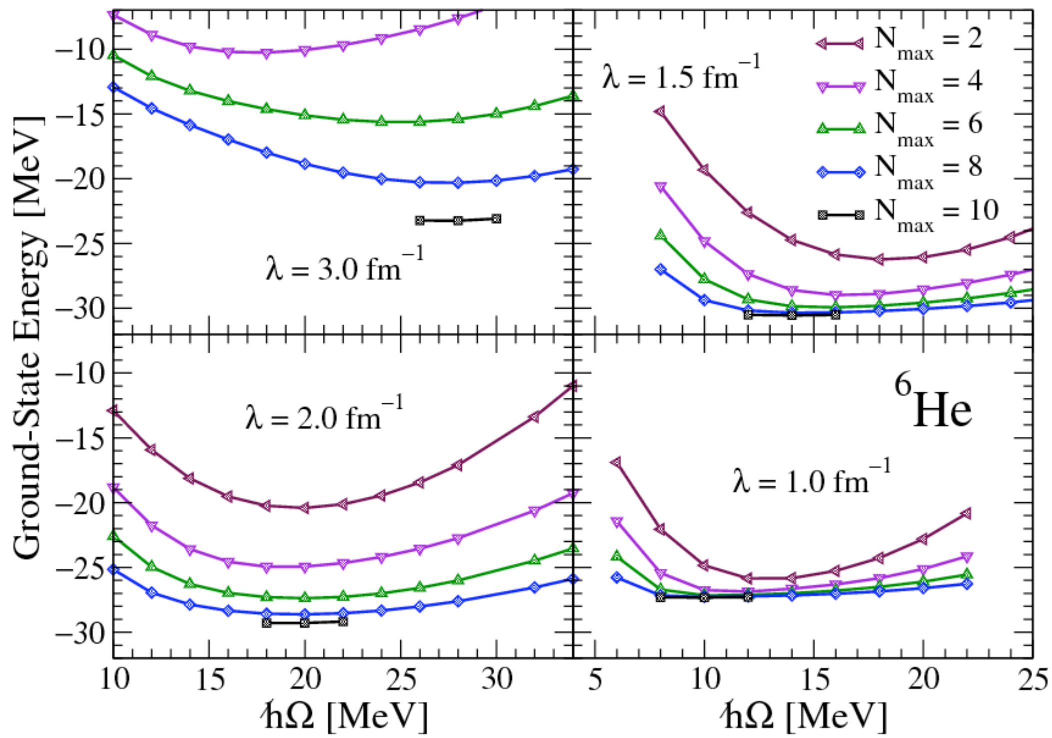
evolution of chiral EFT interactions to low-momentum beneficial

weakens off-diagonal coupling in HO states

lower cutoffs need smaller basis

Bogner et al. (2007)

10^3 states for N_{max}=2 vs.
10^7 states for N_{max}=10



direct convergence in structure calcs

Pushing the limits

First ab-initio calculations for heavier systems:

Coupled-cluster theory based on $V_{\text{low k}}(\Lambda)$

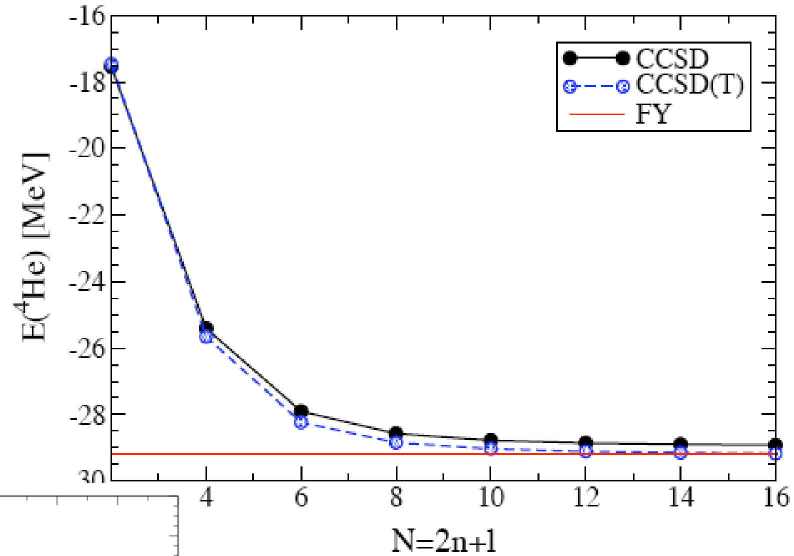
Hagen, Dean, Hjorth-Jensen, Papenbrock, AS (2007)

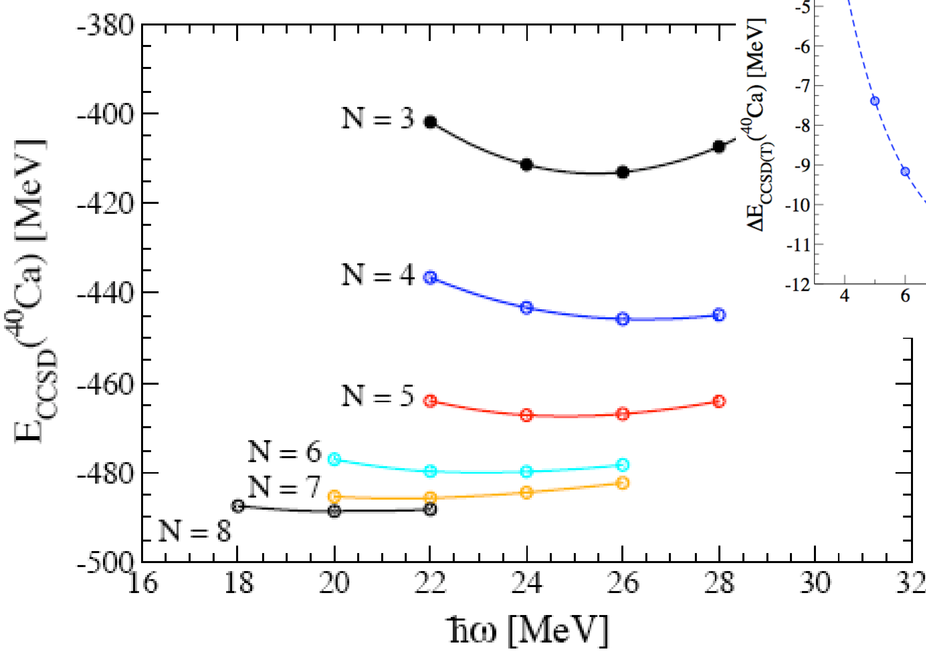
meets and sets benchmarks:

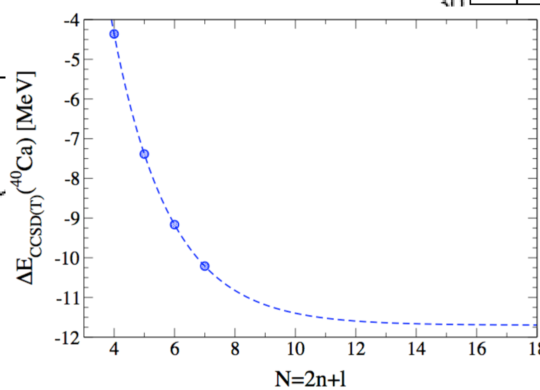
within 10 keV of exact FY for ^{4}He

accurate for ^{16}O and ^{40}Ca

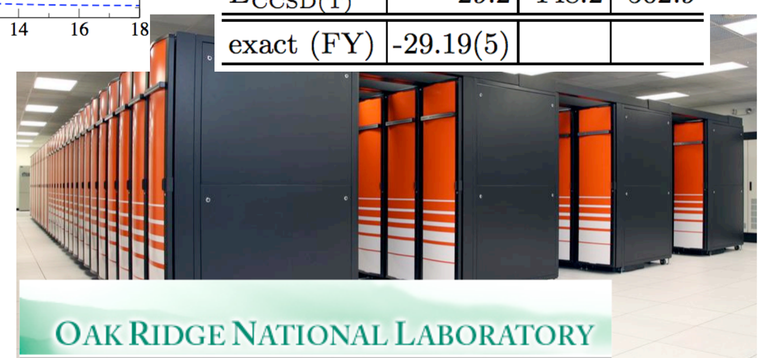
N=8: basis dimension ~10^{63}







	^{4}He	^{16}O	^{40}Ca
E_0	-11.8	-60.2	-347.5
ΔE_{CCSD}	-17.1	-82.6	-143.7
$\Delta E_{\text{CCSD(T)}}$	-0.3	-5.4	-11.7
$E_{\text{CCSD(T)}}$	-29.2	-148.2	-502.9
exact (FY)	-29.19(5)		



Impact on binding energies

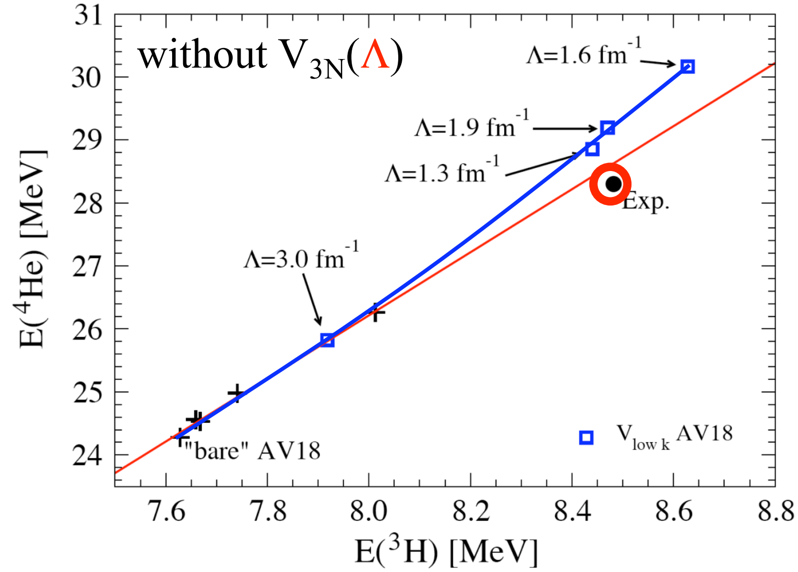
$V_{\text{low }k}(\Lambda)$ defines class of NN interactions with cutoff-independent low-energy NN observables

cutoff variation estimates errors due to neglected parts in $H(\Lambda)$

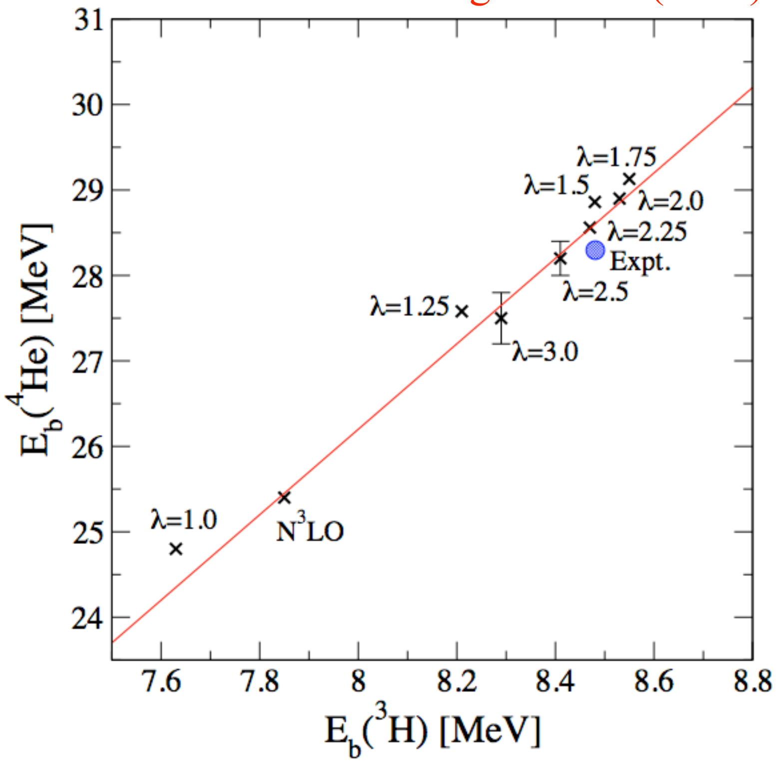
cutoff dependence explains "Tjon line", 3N required by renormalization

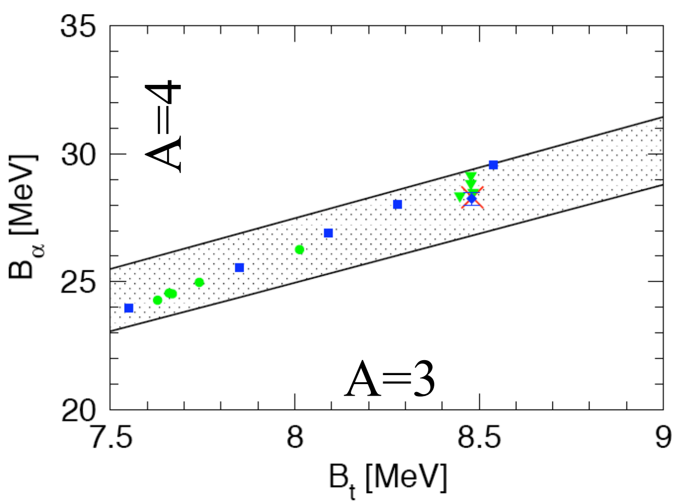
large scattering lengths drive correlation
Platter et al. (2005)

Nogga, Bogner, AS (2004)



SRG interactions: Bogner et al. (2007)

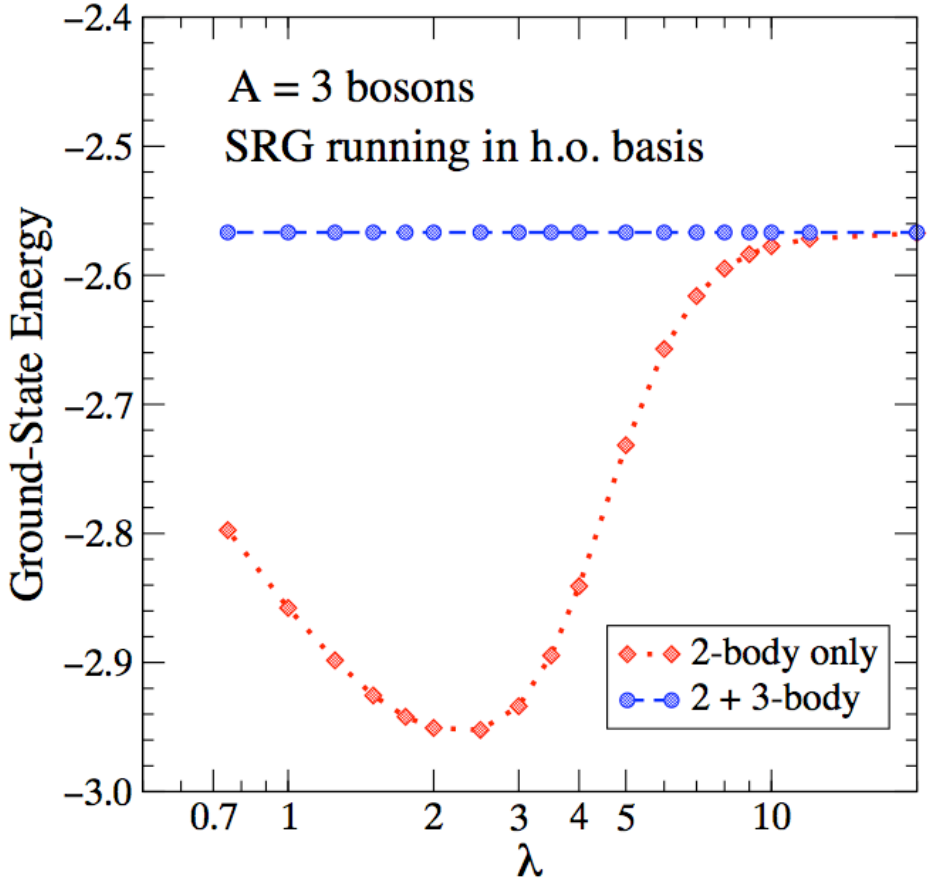


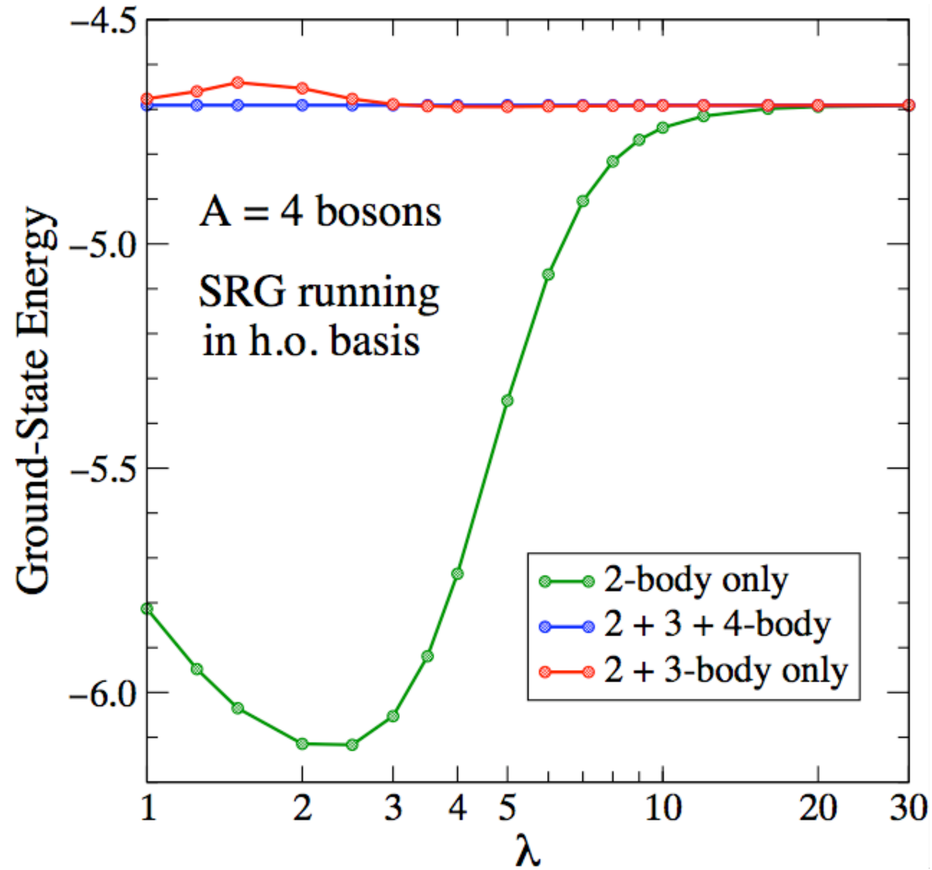


Towards evolving 3N interactions

SRG evolution for 1d systems with T_{rel} in Jacobi HO basis

Jurgenson et al., in prep.





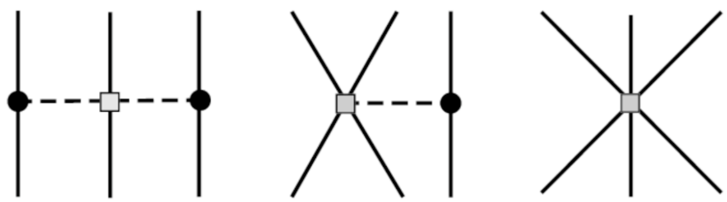
gearing up to evolve chiral 3N interactions

for now: use chiral EFT is complete basis

Low-momentum 3N interactions

from leading N^2LO chiral EFT $\sim (Q/\Lambda)^3$ van Kolck (1994), Epelbaum et al. (2002)

long (2π) intermed. (π) short-range



c_1,c_3,c_4 terms $D(\Lambda)$ term $E(\Lambda)$ term

$$= \frac{g_A^2}{8F_\pi^4}\frac{(\vec\sigma_1\cdot\vec q_1)(\vec\sigma_3\cdot\vec q_3)}{[\vec q_1^{\,2}+M_\pi^2][\vec q_3^{\,2}+M_\pi^2]}\Big\{\boldsymbol\tau_1\cdot\boldsymbol\tau_3\,(-4c_1M_\pi^2+2c_3\,\vec q_1\cdot\vec q_3)+c_4\,[\boldsymbol\tau_1\times\boldsymbol\tau_3]\cdot\boldsymbol\tau_2\,[\vec q_1\times\vec q_3]\cdot\vec\sigma_2\Big\} + \text{perm.}$$

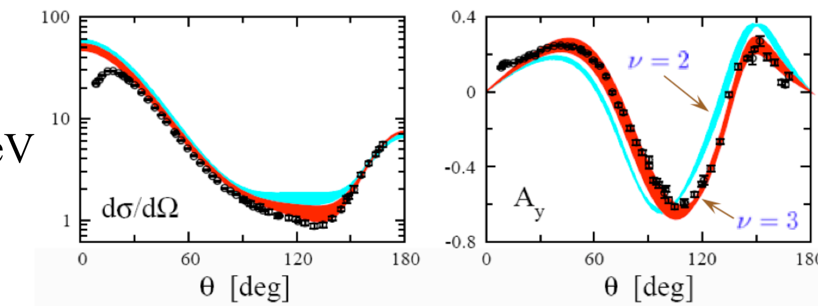
$$= -\frac{g_A D}{8F_\pi}\frac{(\vec\sigma_3\cdot\vec q_3)}{\vec q_3^{\,2}+M_\pi^2}(\boldsymbol\tau_2\cdot\boldsymbol\tau_3)\,(\vec\sigma_2\cdot\vec q_3) + \text{perm.}$$

$$= \frac{1}{2}E\,(\boldsymbol\tau_1\cdot\boldsymbol\tau_3) + \text{perm.}$$

c_i from πN, consistent with NN $\quad c_1 = -0.9^{+0.2}_{-0.5}\,,\ c_3 = -4.7^{+1.2}_{-1.0}\,,\ c_4 = 3.5^{+0.5}_{-0.2}$

Meissner (2007)

generally improves 3N scattering pd @ 65MeV



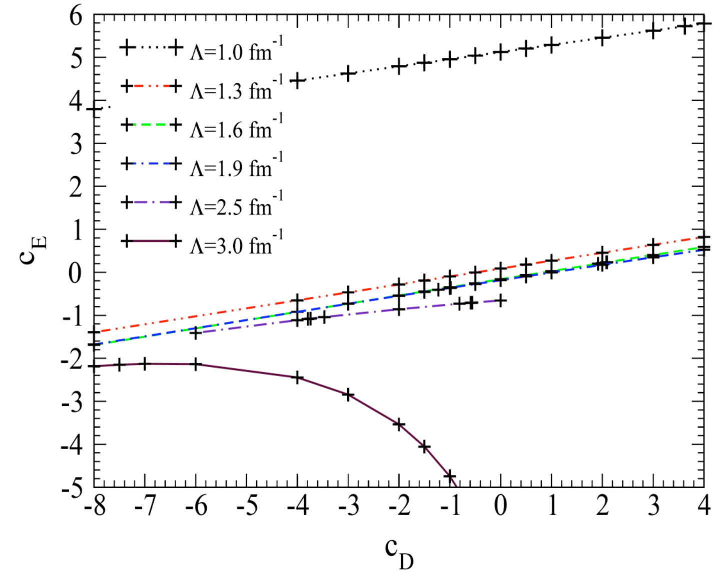
c_3,c_4 important for structure, large uncertainties at present

chiral EFT is complete basis → 3N up to truncation errors

D term could be fixed by tritium beta decay

Low-momentum 3N fits

fit D,E couplings to A=3,4 binding energies for range of cutoffs



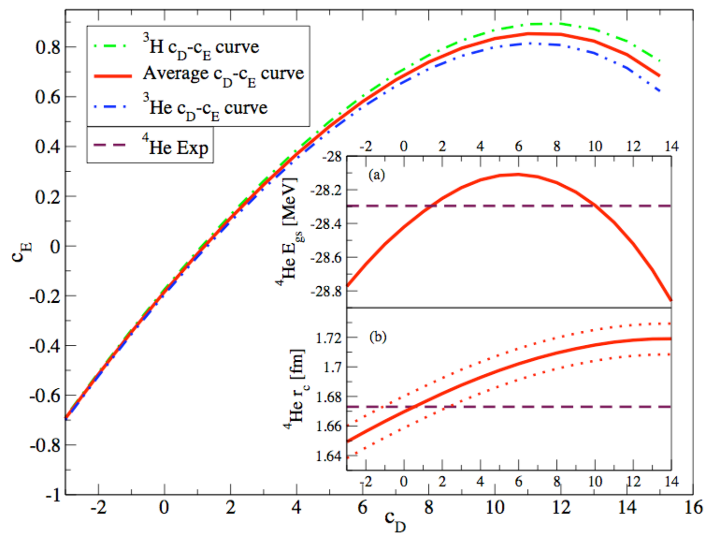
linear dependences in fits to triton binding

3N interactions perturbative for $\Lambda \lesssim 2\,\mathrm{fm}^{-1}$

Nogga, Bogner, AS (2004)

nonperturbative at larger cutoffs

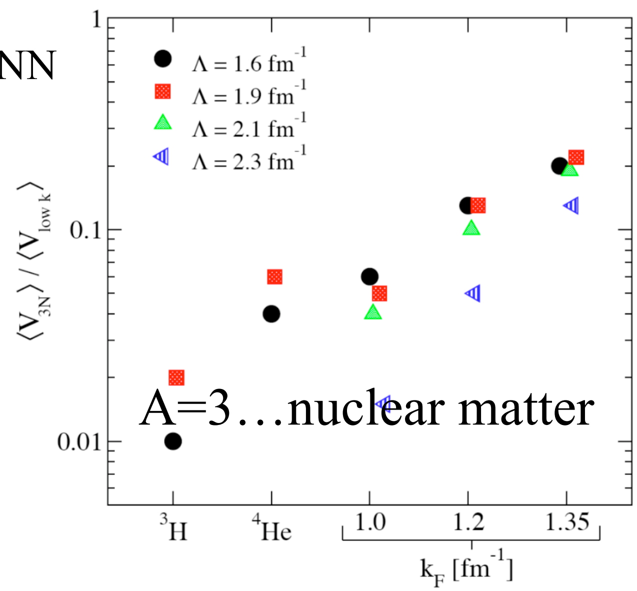
cf. chiral EFT $\Lambda \approx 3\ \mathrm{fm}^{-1}$



Navratil et al. (2007)

3N exp. values natural

$\sim(Q/\Lambda)^3\ V_{NN} \sim 0.1\ V_{NN}$



Subleading chiral EFT 3N interactions

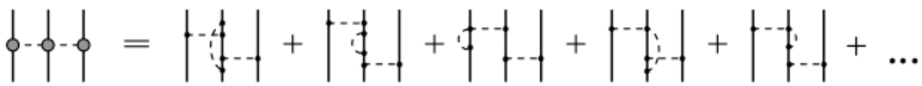
parameter-free $N^3LO \sim (Q/\Lambda)^4$ Status from Epelbaum @ TRIUMF 3N workshop (2007)

- $1/m$-corrections to 1 insertion from $\mathcal{L}^{(2)}_{1/m}$ =  $+ \mathcal{O}(\pi^3)$

  — rich operator structure (includes spin-orbit interactions)

- 1-loop diagrams with all vertices from $\mathcal{L}^{(0)}_{\text{eff}}$

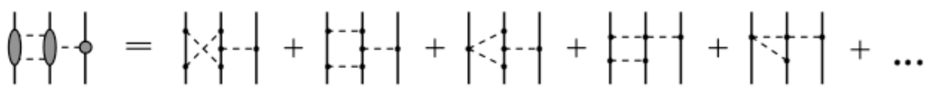
2π – exchange



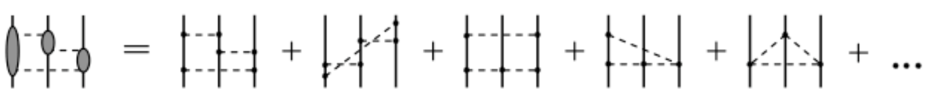
The calculated corrections simply shift the LECs c_i as follows:

$$\delta c_1 = \frac{g_A^2 M_\pi}{64\pi F_\pi^2} \sim 0.13\ \text{GeV}^{-1} \qquad \delta c_3 = \frac{3 g_A^4 M_\pi}{16\pi F_\pi^2} \sim 2.5\ \text{GeV}^{-1} \qquad \delta c_4 = -\frac{g_A^4 M_\pi}{16\pi F_\pi^2} \sim -0.85\ \text{GeV}^{-1}$$

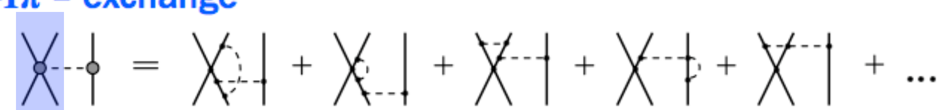
2π-1π – exchange



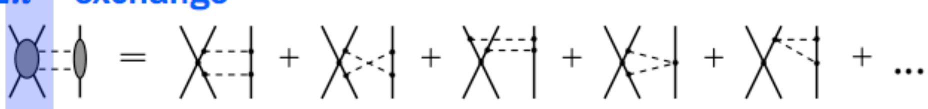
ring diagrams



contact-1π – exchange



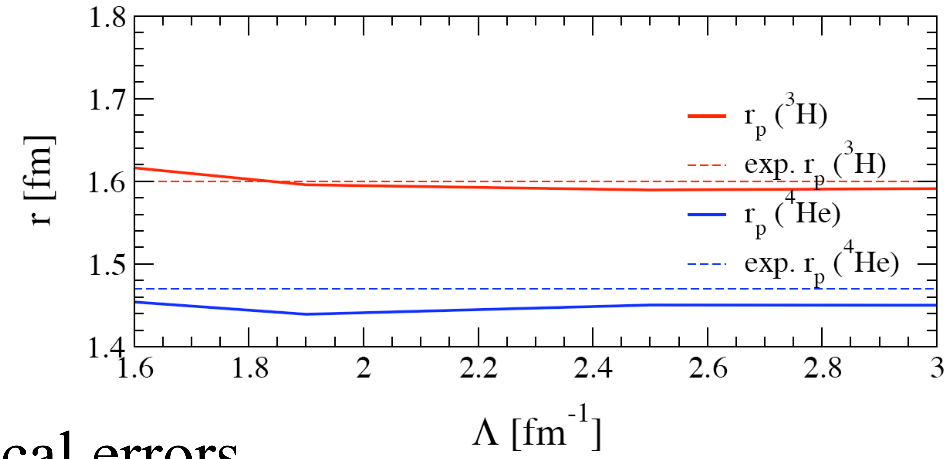
contact-2π – exchange



Theoretical uncertainties

Cutoff variation estimates errors due to neglected parts in H(Λ)

Radii of light nuclei approximately cutoff-independent, agree with exp.



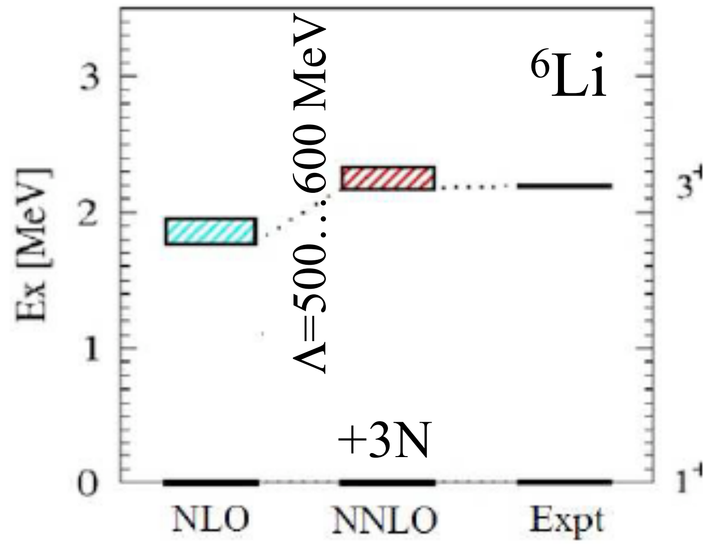
Can provide lower limits on theoretical errors

goal: uncertainties of matrix elements needed in fundamental symmetry tests

isospin-symmetry breaking corrections
V_{ud}=0.97416(13) (14/18)theo.

neutrinoless double-beta decay

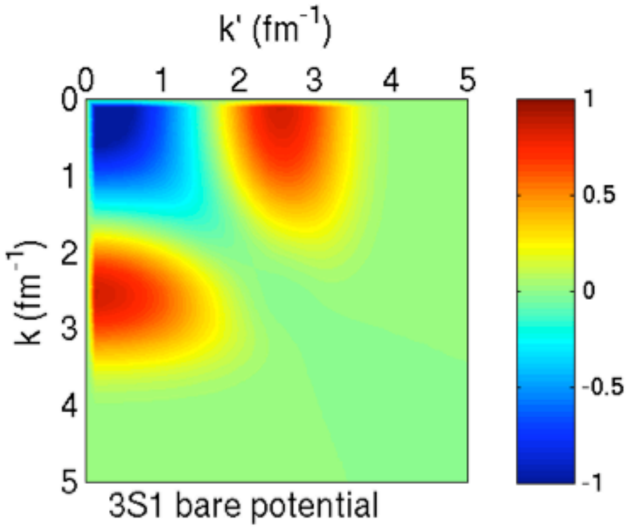
atomic EDMs ……

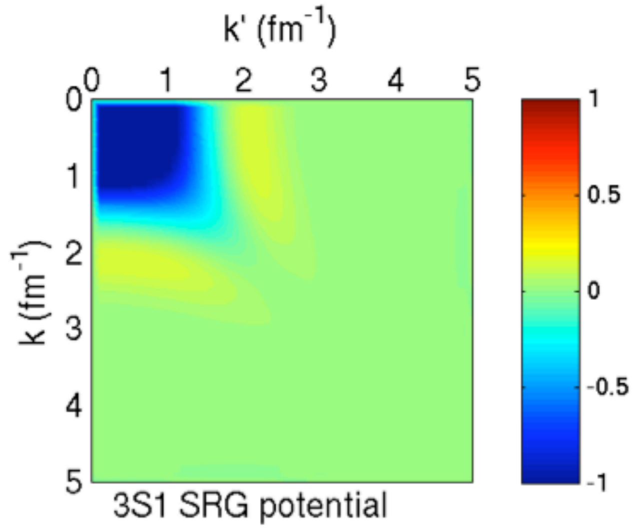


from A. Nogga

Advantages of low-momentum interactions for nuclei

conventional G matrix approach does not solve off-diagonal coupling, renders Bethe-Brueckner-Goldstone expansion necessarily nonperturb.

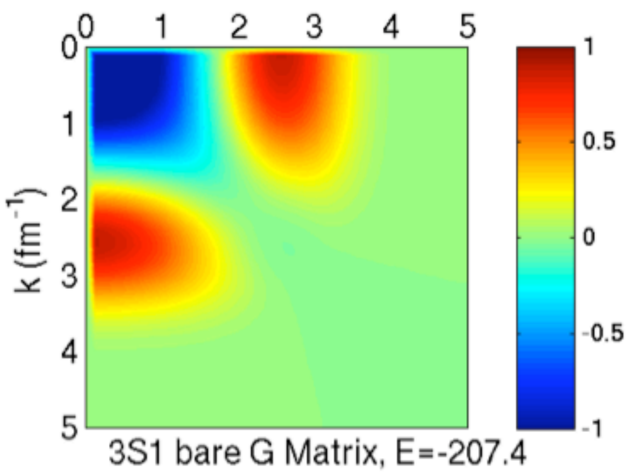


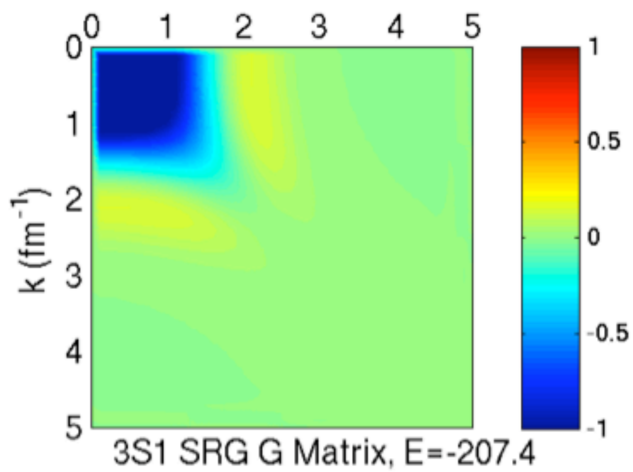


chiral N^{3}LO potential

SRG evolved

G matrix

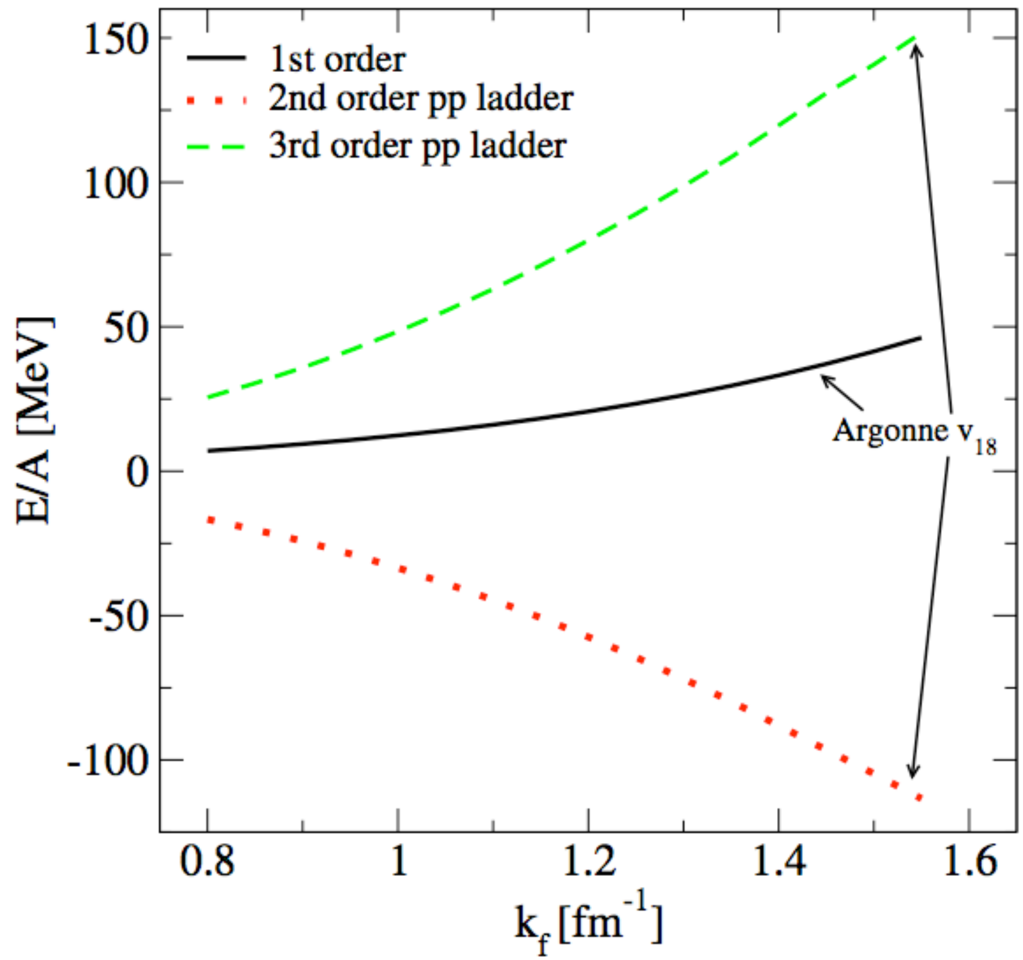




Possibility of perturbative nuclear matter with NN and 3N

start from chiral EFT to given order, soften with RG

nuclear matter converged at ≈ 2nd order,
motivated by Weinberg eigenvalue analysis



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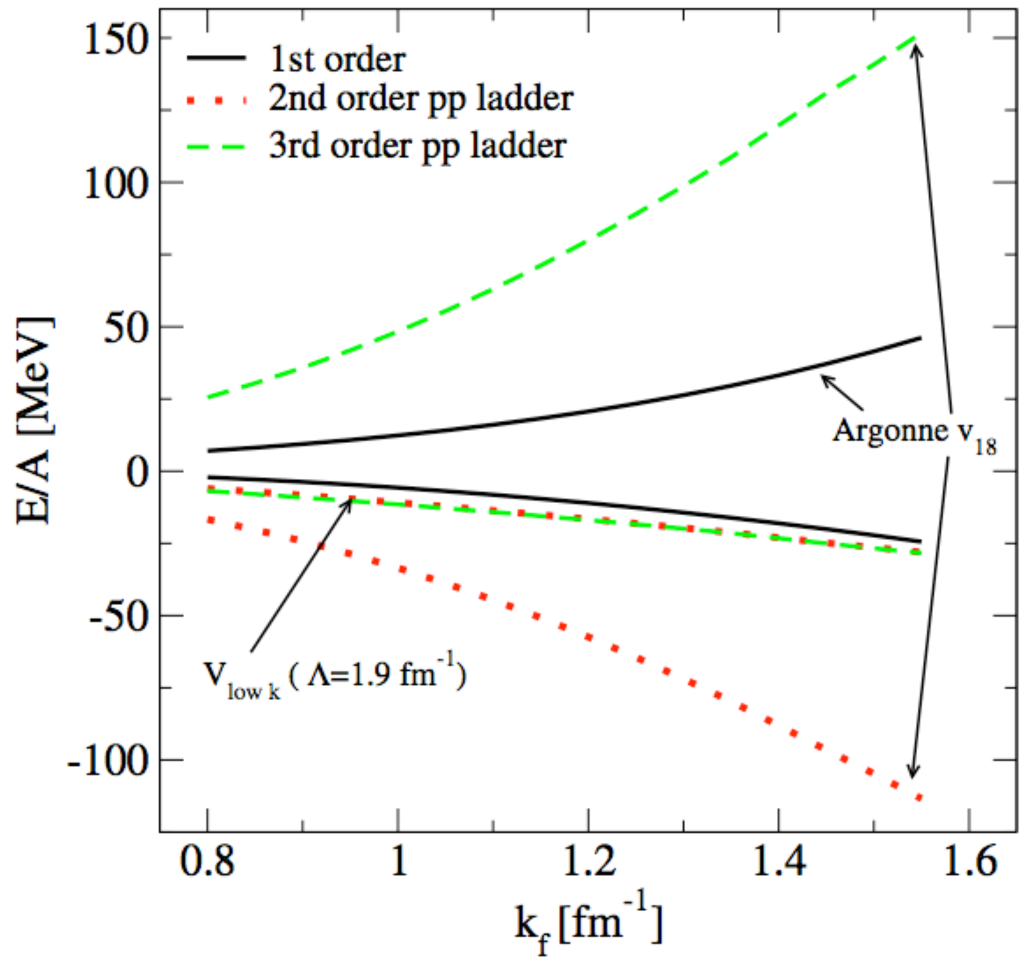


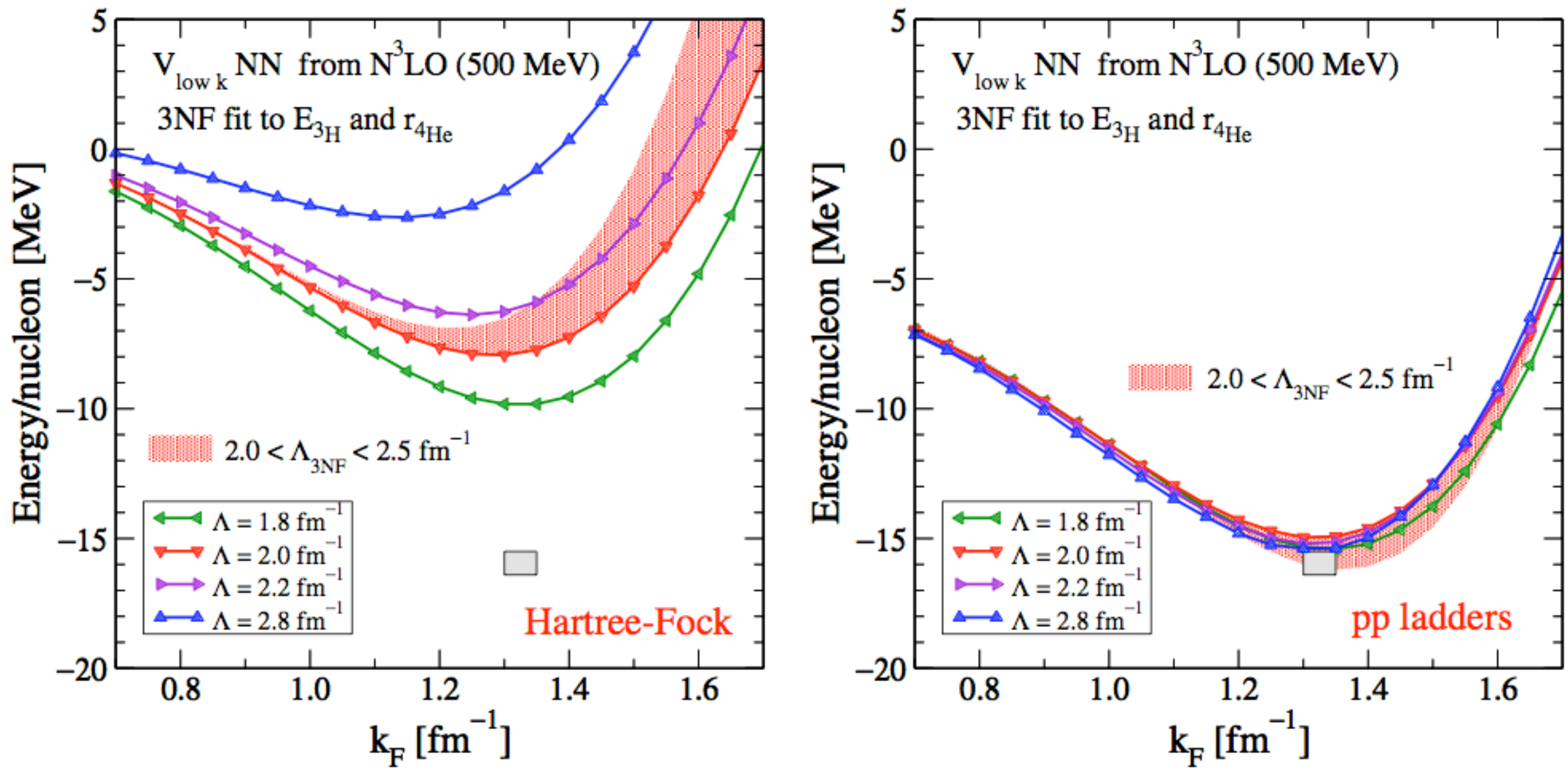
Possibility of perturbative nuclear matter with NN and 3N

start from chiral EFT to given order, soften with RG

nuclear matter converged at ≈ 2nd order,
motivated by Weinberg eigenvalue analysis

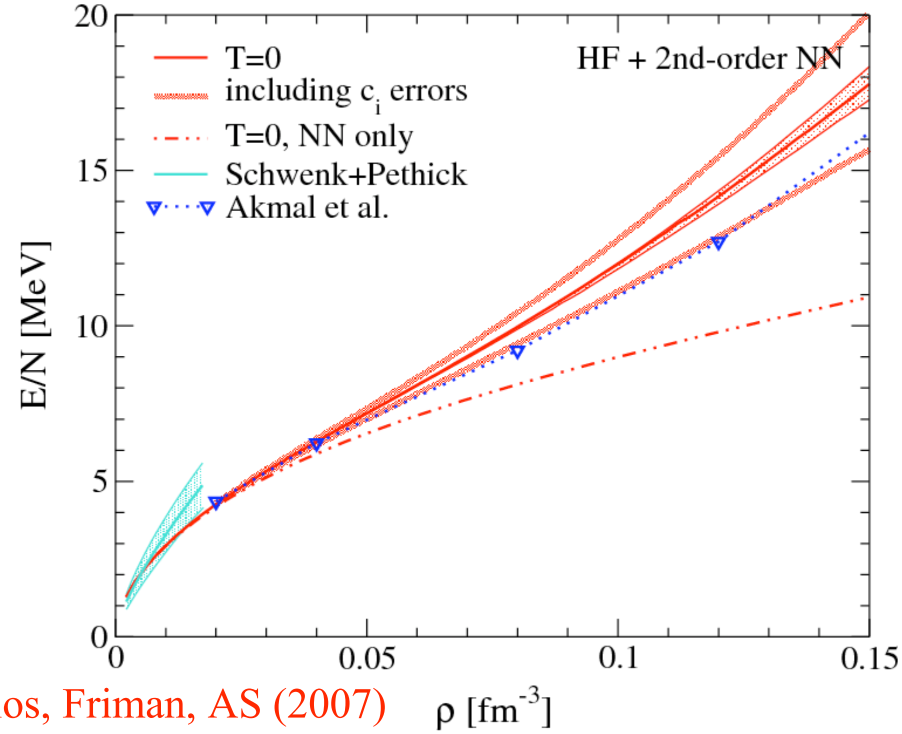
reduced cutoff dependence at low densities, 3N drives saturation

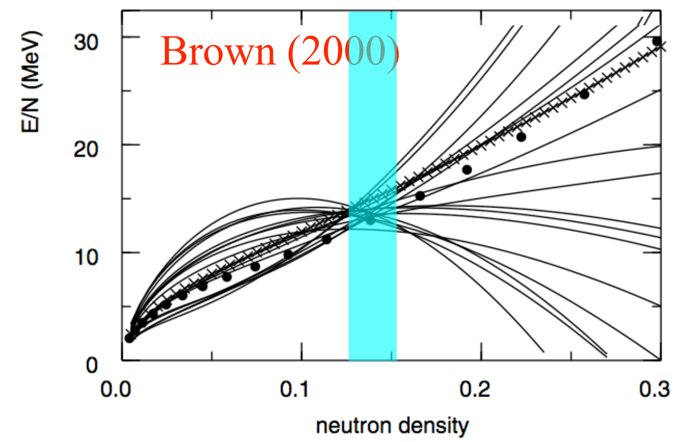
Bogner, AS, Furnstahl, Nogga (2005) + improvements, in prep.



provides guidance to UNEDF http://unedf.org

Neutron matter from NN and 3N





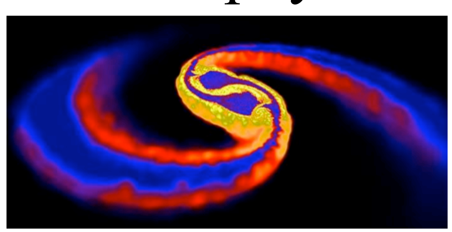
Tolos, Friman, AS (2007)

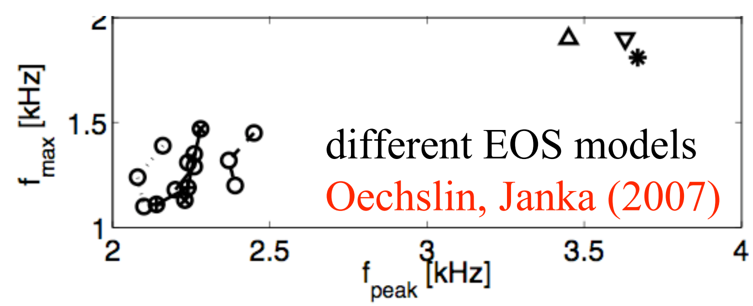
uncertainties from c_i overwhelm errors due to cutoff variation, mainly c_3 for neutron matter

combine with knowledge of basic nuclear properties

important for dense matter in astrophysics

neutron star mergers
→gravitational waves



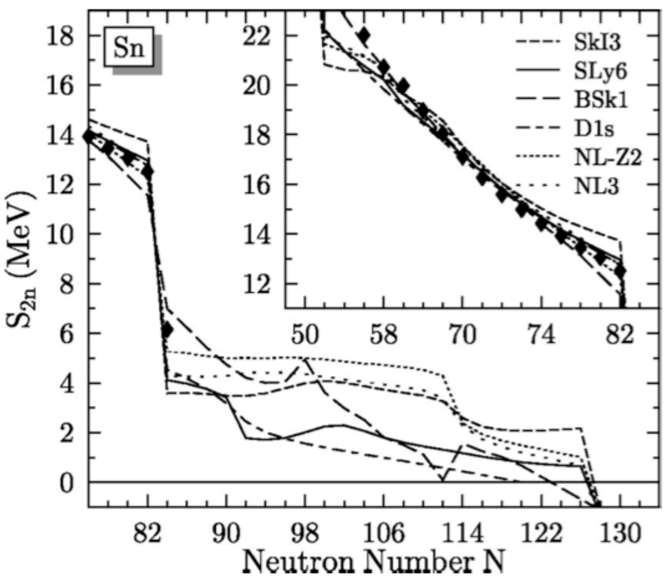


Towards a universal nuclear energy density functional

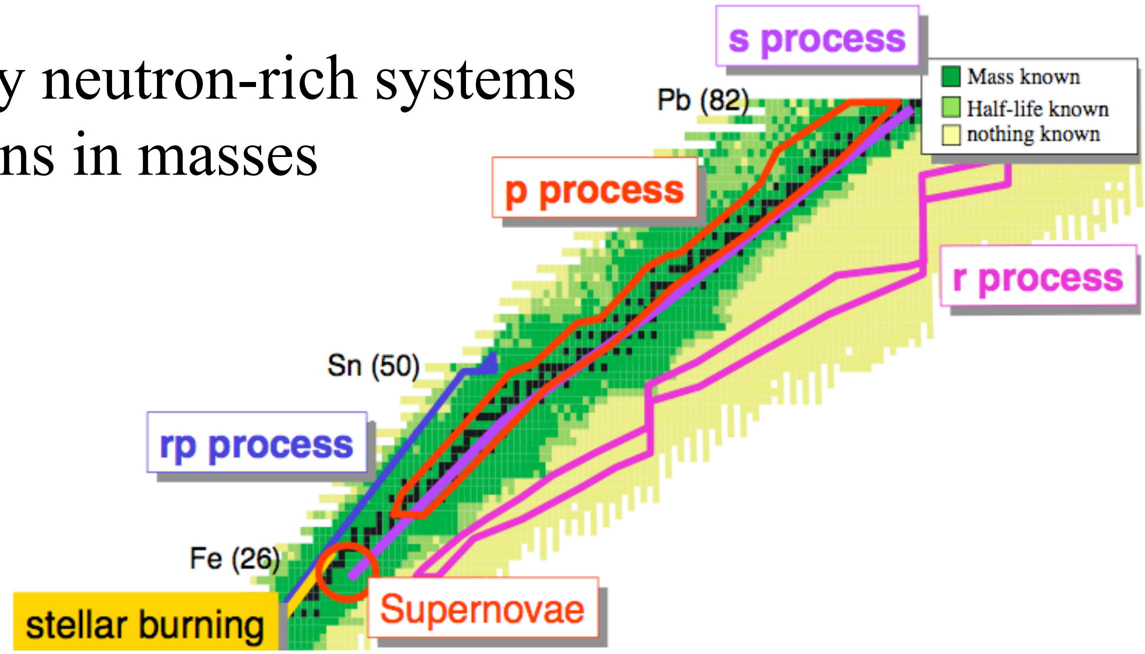
creation of heavy elements in r(-apid neutron capture) process

requires understanding highly neutron-rich systems

need to improve extrapolations in masses



Bender et al. (2003)



masses and ground state properties from density functional theory

based on densities

not wave functions

from Kohn's 1998 Nobel Prize lecture

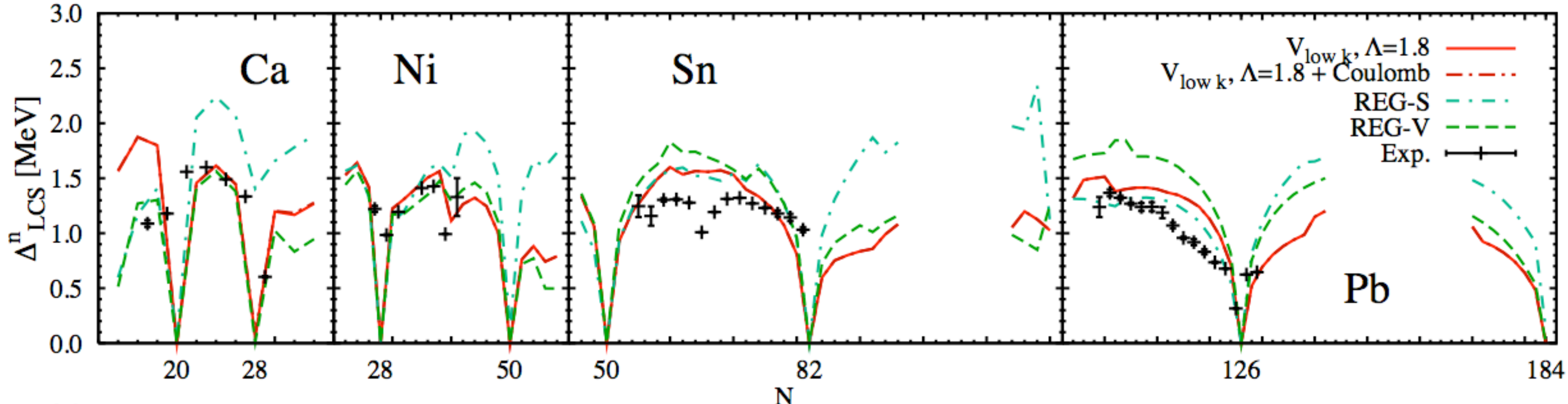
I begin with a provocative statement. *In general the many-electron wavefunction* $\Psi(r_1,\dots,r_N)$ *for a system of N electrons is not a legitimate scientific concept, when* $N \geq N_0$*, where* $N_0 \approx 10^3$.

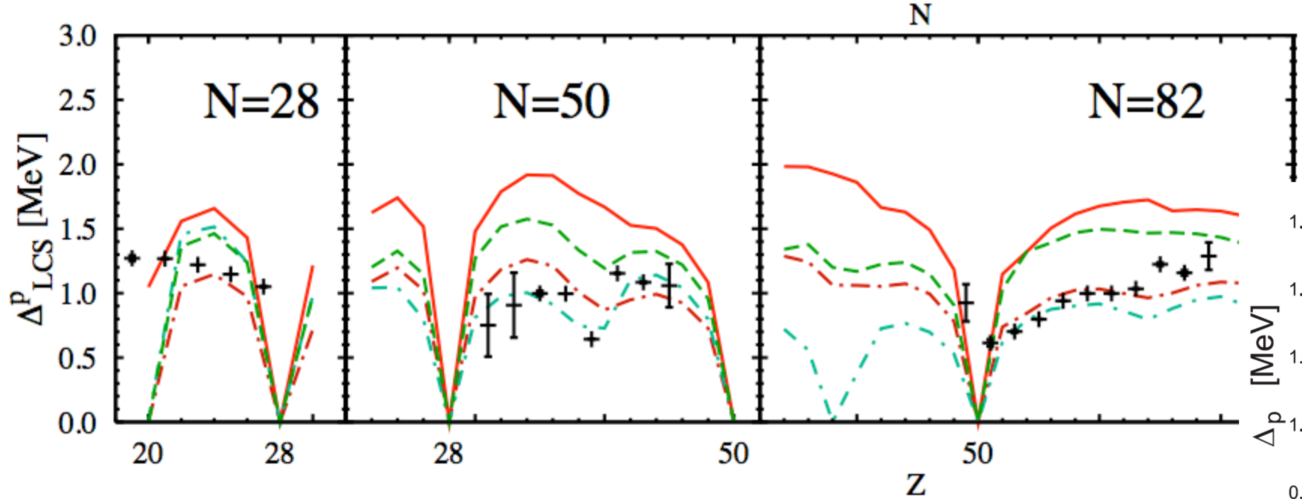
I will use two criteria for defining "legitimacy": a) That Ψ can be calculated with sufficient accuracy and b) can be recorded with sufficient accuracy.

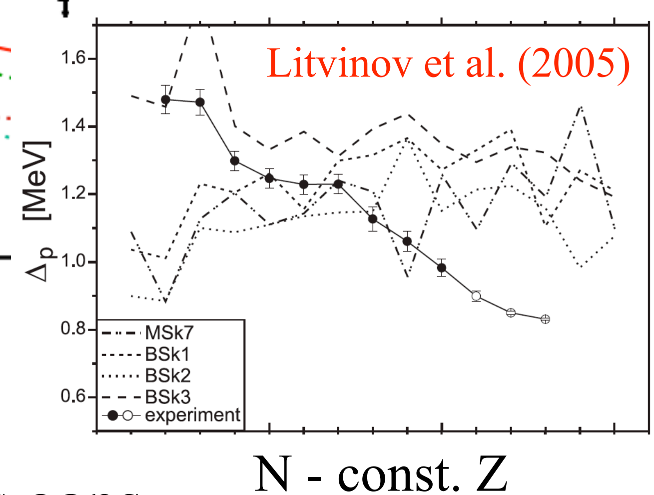
Nuclear masses and pairing

first microscopic pairing functional from low-momentum interactions

Lesinski, Duguet, arXiv:0711.4386 and in prep.







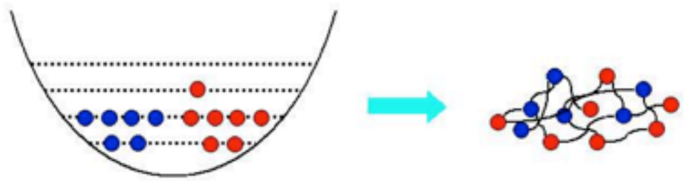
contributions from higher pw's and 3N?

can study beyond BCS contributions to pairings gaps

Density functional RG for nuclei

Braun, Polonyi, AS, in prep.

Density Functional: $\Gamma_\lambda[\rho] = \ln \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \, \mathrm{e}^{-S_\lambda[\psi^\dagger,\psi] + \int \frac{\delta \Gamma_\lambda}{\delta \rho} \cdot (\psi^\dagger \psi)}$



$\lambda = 0$ $\lambda = 1$

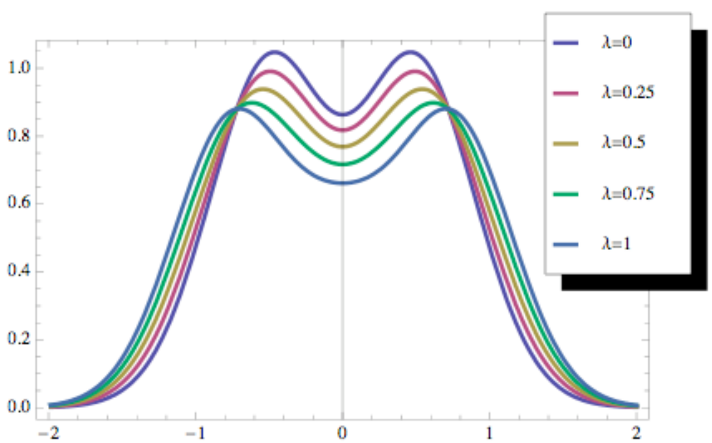
start from mean-field (background potential U_λ) and include many-body correlations

$$\partial_\lambda \Gamma_\lambda[\rho] = \frac{1}{2} V_{\mathrm{low\,k}}$$

$\Gamma_\lambda[\rho]$ is the density functional

$$S_\lambda[\psi^\dagger,\psi] = \int \psi^\dagger \left[\partial_t - \frac{1}{2m}\Delta + (1-\lambda)U_\lambda\right]\psi + \frac{1}{2}\int \psi^\dagger \psi \lambda V_{\mathrm{low\,k}} \psi^\dagger \psi$$

+ (λV_{3N} will be included later)



Example: RG flow of the ground-state density for 1d model ("smeared-out" δ-function interaction)

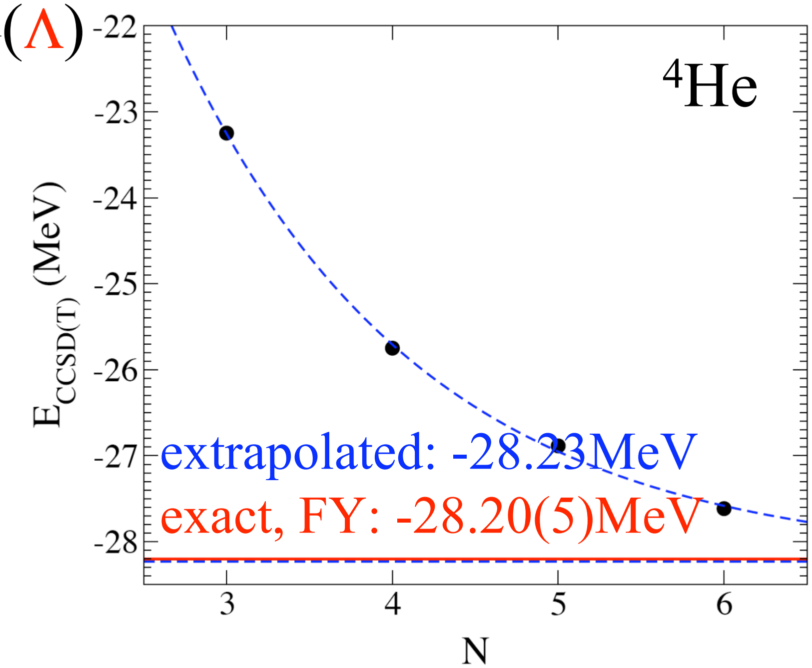
density basis expansion scales favorably to large systems

gs energy and density from microscopic nuclear interactions

results for ^{16}O in prep.

Towards 3N interactions in medium-mass nuclei

based on low-momentum $V_{\text{low k}}(\Lambda) + V_{3N}(\Lambda)$

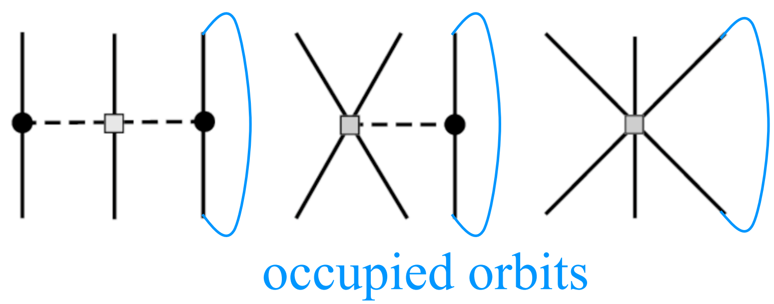


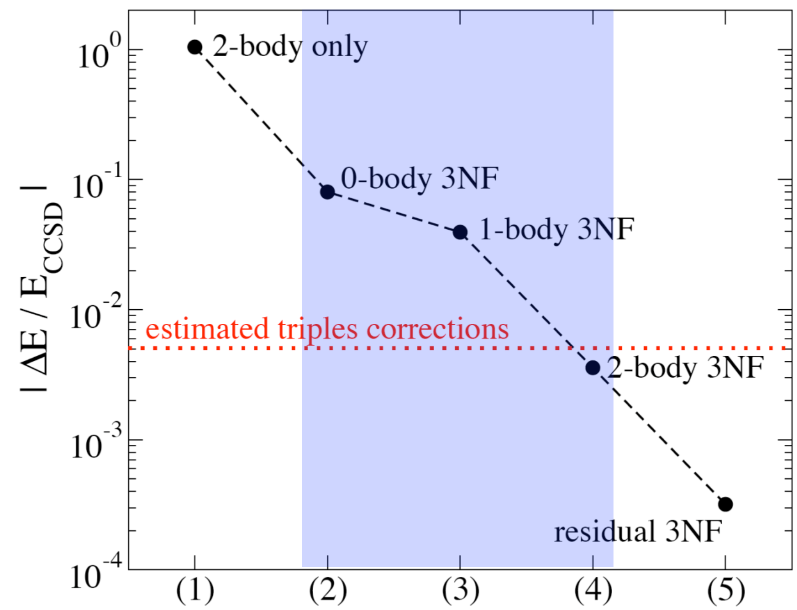
Hagen et al. (2007)

developed coupled-cluster theory with 3N interactions, first benchmark for ^{4}He

Results show that 0-, 1- and 2-body parts of 3N interaction dominate

2-body part



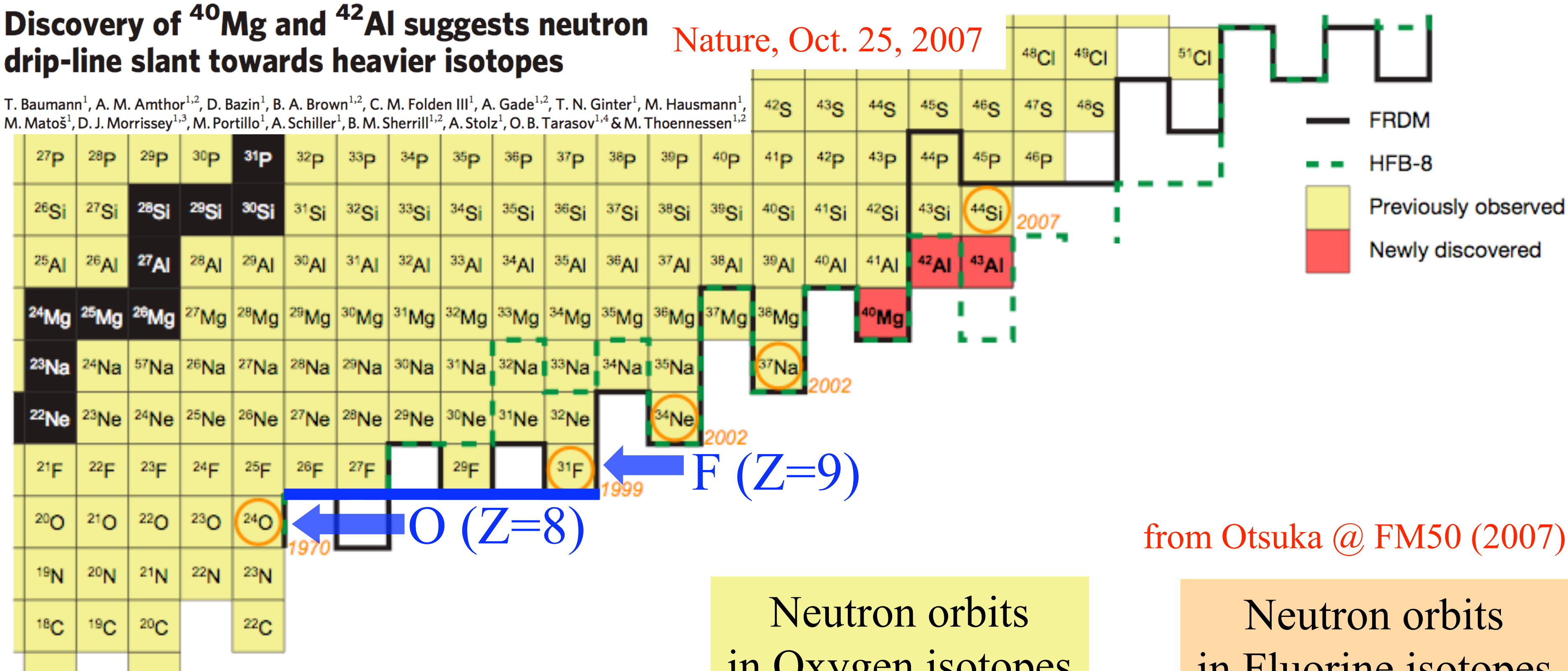


residual 3N interaction can be neglected

very promising

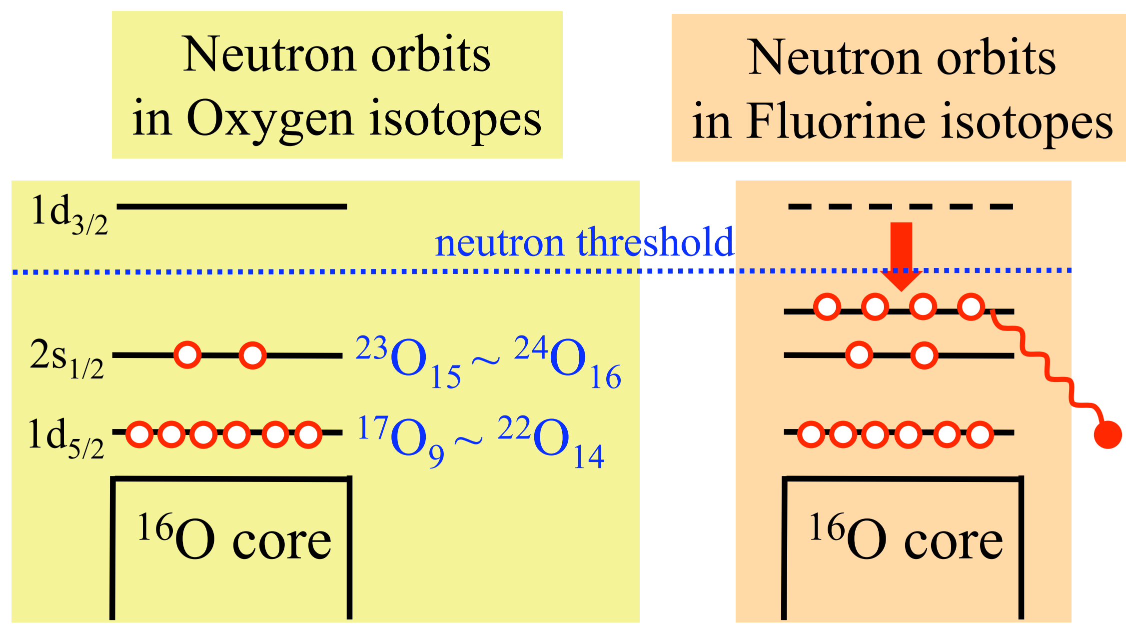
can include via normal-ordered RG

Location of the neutron drip line: Why so near in Oxygen?…



neutron $d_{3/2}$ - proton $d_{5/2}$ interaction pulls down $d_{3/2}$ neutrons in Fluorine

Why do $d_{5/2}$ neutrons not pull down $d_{3/2}$ in oxygen?

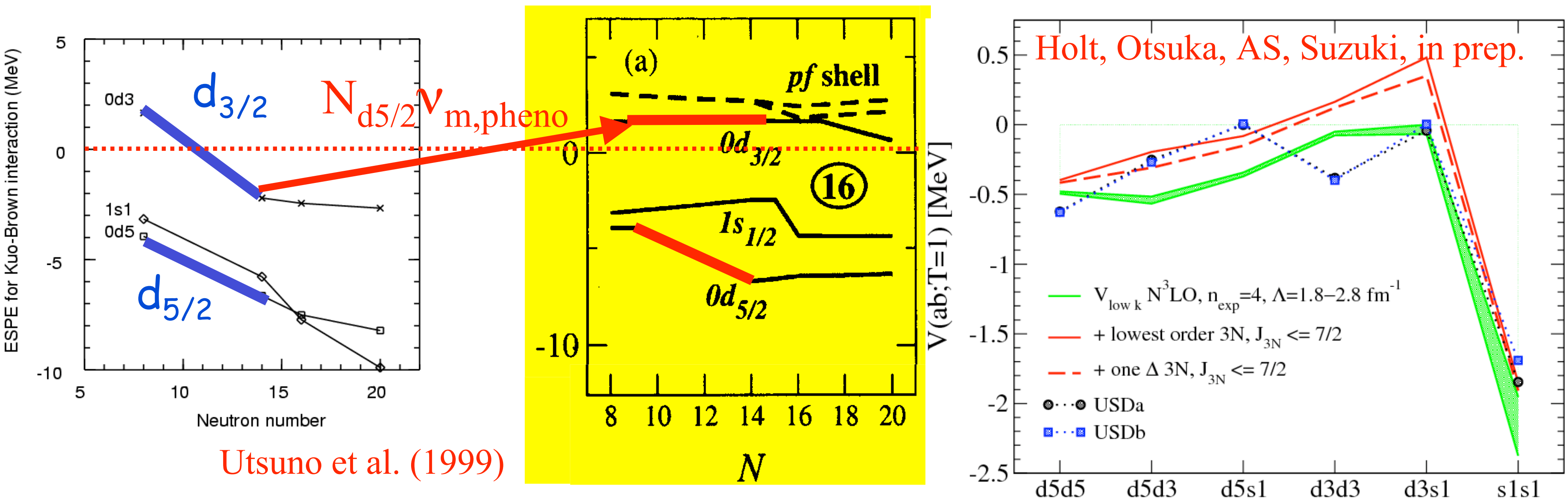


Monopole interaction and drip lines

Monopole part of nuclear forces $\mathcal{V}^T_{st} = \dfrac{\sum_J \mathcal{V}^{JT}_{stst}(2J+1)[1-(-)^{J+T}\delta_{st}]}{\sum_J (2J+1)[1-(-)^{J+T}\delta_{st}]}$

determines interaction of s with t orbit → change in $d_{3/2}$ by $N_{d5/2}\, \nu_m$

⇒ small changes in monopoles enhanced by number of neutrons



microscopic results based only on NN interactions require phenomenological repulsive contribution to T=1 monopoles

→ neutron $d_{3/2}$ remains high, dripline at N=16 for Oxygen

first results indicate that $\nu_{m,pheno}$ due to 3N interactions

Thanks to collaborators

S. Bacca, S. Baroni, J. Braun, J.D. Holt, M. Ku

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A. Nogga

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C.J. Pethick

J. Polonyi, A.P. Zuker

P. Maris, J.P. Vary

M. Hjorth-Jensen

T. Otsuka T. Suzuki

Summary

Exciting era with advances on many fronts

For the first time, approaches from light to heavy nuclei and for astrophysics based on the same interactions

Three-nucleon interactions are a frontier

Major investments in new facilities worldwide

Exciting intersections with problems in many related areas