Effective Field Theory with a Variable Ultraviolet Cutoff

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Publications

1. N. Tetradis, arXiv:0805.1840 [hep-th]
2. A. Strumia and N. Tetradis, arXiv:0805.1615 [hep-ph]

Outline

- Speculations
- Exact renormalization group
- Implications
- Perturbative renormalization group
- Experimental constraints
- Conclusions

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- A. G. Cohen, D. B. Kaplan and A. E. Nelson, "Effective field theory, black holes, and the cosmological constant," Phys. Rev. Lett. **82** (1999) 4971 [arXiv:hep-th/9803132].
- For an effective field theory with ultraviolet cutoff Λ in a box of volume k^{-3} the entropy scales $\sim \Lambda^3/k^3$.
- The thermodynamics of black holes suggests that the maximum entropy must scale with the area, i.e. $S \lesssim M_{\rm Pl}^2/k^2$.
- Reconciling these facts is possible if $\Lambda \lesssim M_{\rm Pl}^{2/3} k^{1/3}$.
- Assume that the total (vacuum) energy within the volume k^{-3} does not lead to a Schwazschild radius for the system larger than its size k^{-1}.
- If the energy density scales $\sim \Lambda^4$, we must impose that $\Lambda^4/k^3 \lesssim M_{\rm Pl}^2/k$, or $\Lambda \lesssim M_{\rm Pl}^{1/2} k^{1/2}$.
- The saturation of the stronger bound for an infrared cutoff of the order of the Hubble scale results in $\Lambda \sim 10^{-3}$ eV.

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Theories with many degrees of freedom

- Links between various energy scales of a theory can often be established on general grounds.
- G. Dvali, “Black Holes and Large N Species Solution to the Hierarchy Problem,” arXiv:0706.2050 [hep-th].
- A connection between the number N of particle species of a theory, the scale Λ that sets their masses, and $M_{\rm Pl}$: $N\Lambda^2 \lesssim M_{\rm Pl}^2$.
- If this bound is saturated, there must be a direct link between Λ and $M_{\rm Pl}$.
- Example: compact extra dimensions, with N the number of Kaluza-Klein gravtion modes with masses smaller than Λ.
- $N \sim (\Lambda/k)^n$, with $1/k$ is the compactification radius and n the number of extra dimensions. The bound is saturated, with $\Lambda^{2+n}/k^n \sim M_{\rm Pl}^2$.
- A change in k (of possible dynamical origin) would result in the variation of either Λ or $M_{\rm Pl}$.

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Intuitive approach

- Toy model of a scalar field ϕ with a Z_2 symmetry $\phi \leftrightarrow -\phi$.
- One-loop effective potential ($\rho = \phi^2/2$):

$$U_k^{(1)}(\rho) = V(\rho) + \frac{1}{2(2\pi)^d} \int_k^{\Lambda(k)} d^d q \ln\left(q^2 + V'(\rho) + 2\rho V''(\rho)\right),$$

- Tree-level potential: $V = U_k$ for $k = \Lambda = \Lambda_0$.
- Renormalization-group improved potential:

$$\begin{aligned}\frac{\partial U_k(\rho)}{\partial \ln k} = & - 2v_d \left[k^d \ln k^2 - \frac{d\ln\Lambda}{d\ln k}\Lambda^d \ln \Lambda^2\right] \\ & - 2v_d \left[k^d \ln\left(1 + \frac{U_k' + 2\rho U_k''}{k^2}\right) \right. \\ & \left. - \frac{d\ln\Lambda}{d\ln k}\Lambda^d \ln\left(1 + \frac{U_k' + 2\rho U_k''}{\Lambda^2}\right)\right]\end{aligned}$$

- If $\Lambda(k) = k^\delta \Lambda_0^{1-\delta}$, then $d(\ln\Lambda)/d(\ln k) = \delta$.

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Exact flow equation

- Consider a theory of a real scalar field χ, in d dimensions, with a Z_2-symmetric action $S[\chi]$.
- Add a regulating piece

$$\Delta S = \frac{1}{2}\int d^d q \hat{R}_k(q)\chi^*(q)\chi(q),$$

where $\chi(q)$ are the Fourier modes of the scalar field.
- The function $\hat{R}_k$ cuts off modes with characteristic momenta outside the interval $k^2 \lesssim q^2 \lesssim \Lambda^2(k)$.
- Legendre transform, remove the regulating piece. The resulting cutoff-dependent effective action obeys the usual exact flow equation ($t = \ln k$)

$$\frac{\partial \Gamma_k[\phi]}{\partial t} = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)}[\phi] + \hat{R}_k\right)^{-1}\frac{\partial \hat{R}_k}{\partial \ln k}\right].$$

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Evolution equation for the potential

- Use the derivative expansion

$$\Gamma_k = \int d^d x \left[U_k(\rho) + \frac{1}{2} Z_k(\rho) \partial^\mu \phi \partial_\mu \phi + ... \right]$$

- In the lowest order, with $U_k(\rho)$, $Z_k = 1$, we have

$$\begin{aligned} \frac{\partial U_k(\rho)}{\partial \ln k} &= \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} \frac{\partial \hat{R}_k(q)}{\partial \ln k} \frac{1}{q^2 + \hat{R}_k(q) + U_k'(\rho) + 2\rho U_k''(\rho)} \\ &= 2 v_d k^d \hat{l}_0^d \left(\frac{U_k'(\rho) + 2\rho U_k''(\rho)}{k^2} \right), \end{aligned}$$

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$$v_d^{-1} = 2^{d+1} \pi^{\frac{d}{2}} \Gamma\left(\frac{d}{2}\right).$$

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- The threshold function

$$\hat{l}_0^d(w) = \frac{1}{2} v_d^{-1} k^{-d} \int \frac{d^d q}{(2\pi)^d} \frac{\partial \hat{R}_k(q)}{\partial \ln k} \frac{1}{q^2 + \hat{R}_k(q) + k^2 w}$$

is a generalization of a similar function defined in the formulation with constant Λ.

- There are also "higher" threshold functions: $\hat{l}_1^d = -\partial \hat{l}_0^d(w)/\partial w$ and $\hat{l}_{n+1}^d = -(1/n)\partial \hat{l}_n^d(w)/\partial w$ for $n \geq 1$.
- The dimensionless ratio $\hat{R}_k(q)/q^2$ is a function of q^2/k^2 and $q^2/\Lambda^2(k)$. This means that the k-derivative of $\hat{R}_k(q)/q^2$ produces terms proportional to its derivatives with respect to q^2/k^2 or q^2/Λ^2. As a result, the momentum integral above receives contributions mainly from the regions around $q = k$ and $q = \Lambda$.

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- The threshold function

$$\hat{l}_0^d(w) = \frac{1}{2} v_d^{-1} k^{-d} \int \frac{d^d q}{(2\pi)^d} \frac{\partial \hat{R}_k(q)}{\partial \ln k} \frac{1}{q^2 + \hat{R}_k(q) + k^2 w}$$

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Sharp cutoff

- Write the flow equation as

$$\frac{\partial U_k'(\rho)}{\partial \ln k} = -2v_d\, k^{d-2}\, \left(3U_k'' + 2\rho U_k'''\right)\, \hat{l}_1^d \left(\frac{U_k'(\rho) + 2\rho U_k''(\rho)}{k^2}\right).$$

  The integral in the definition of $\hat{l}_1^d$ has better convergence properties than the one in $\hat{l}_0^d$, so that the choice of a cutoff function is easier.

- Example of cutoff functions:

$$\hat{R}_k(q) = q^2 \left[\frac{1}{\exp\left(-a\left(q^2/\Lambda^2(k)\right)^b\right) - \exp\left(-a\left(q^2/k^2\right)^b\right)} - 1\right],$$

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is the standard form of the threshold function for constant Λ in the sharp-cutoff limit.
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Fixed points

- Use dimensionless variables

$$\tilde{\rho} = k^{2-d}\rho, \qquad u_k(\tilde{\rho}) = k^{-d}U_k(\rho).$$

- The evolution equation for the potential takes the form

$$\frac{\partial u_k'}{\partial \ln k} = - \quad 2u_k' + (d-2)\tilde{\rho}u_k'' - 2v_d\frac{3u_k'' + 2\tilde{\rho}u_k'''}{1+u_k'}$$
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- The presence of the second term destabilizes most of the fixed-point solutions.

- For $d = 3$ the second term dominates as $k \to 0$ because Λ/k diverges. This implies that the Wilson-Fisher fixed point disappears. The same conclusion can be reached for the fixed points of the $O(N)$-symmetric theory.
- The only fixed point that survives in all dimensions is the Gaussian fixed point $u_k' = 0$.
- For $d = 2$ fixed points are possible for $k \to 0$. They would correspond to solutions of

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Logarithimic running in 4d

- Parametrize the potential as

$$U_k(\rho) = m^2(k)\,\rho + \frac{1}{4}\lambda(k)\,\rho^2 + \frac{1}{6}\sigma(k)\,\rho^3 + \frac{1}{24}\nu(k)\,\rho^4 \ldots$$

- In d dimensions, the β-functions for the first two generalized couplings are

$$\begin{aligned}
\frac{dm^2}{d\ln k} &= -6v_d k^{d-2}\lambda\left[\left(1+\frac{m^2}{k^2}\right)^{-1} - \frac{d\ln\Lambda}{d\ln k}\left(\frac{\Lambda}{k}\right)^{d-2}\left(1+\frac{m^2}{\Lambda^2}\right)^{-1}\right] \\
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- Consider the standard "renormalizable" theory with $m^2(\Lambda_0) = m_w^2$, $\lambda(\Lambda_0) = \lambda_w$, $\sigma(\Lambda_0) = \nu(\Lambda_0) = ... = 0$.
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$$m^2(k) = \left[m_w^2 - \frac{3\lambda_w}{32\pi^2}(1-2\delta)\Lambda_0^2\right] + \frac{3\lambda_w}{32\pi^2}\left(k^2 - 2\delta\Lambda_0 k\right).$$

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Assumptions

- Consider the possibility that the ultraviolet cutoff Λ depends on the typical energy scale of the process E.
- We could allow for an infrared cutoff $\ell(E)$, but it is irrelevant for $\ell \ll E$.
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- The Lagrangian is

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Constant ultraviolet cutoff

- The renormalized and bare Green's functions are related through

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with Λ_0 some fixed reference value of the ultraviolet cutoff.
- The quantities Z and λ depend on the renormalization scale M. Under a variation $M \to M + \delta M$ we have $\lambda \to \lambda + \delta\lambda$, $\phi \to (1 + \delta H)\phi$, with $H = \ln(Z^{-1/2})$.
- The connected Green's functions satisfy $G^{(n)} \to (1 + n\delta H)G^{(n)}$. If they are viewed as functions of M and λ, their variation is $\delta G^{(n)} = (\partial G^{(n)}/\partial M)\,\delta M + (\partial G^{(n)}/\partial\lambda)\,\delta\lambda$. This gives the Callan-Symanzik equation

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- It is instructive to study the running of the quartic coupling through an explicit calculation.
- At one loop, the (Fourier transformed) four-point function is

$$G^{(4)} = -i\left[3\lambda + 9\lambda^2\left(V(s) + V(t) + V(u)\right) + \delta_\lambda\right] \prod_{i=1,\ldots,4} \frac{1}{p_i^2}.$$

Impose as a renormalization condition that the corrections cancel at the symmetric point $s = t = u = -M^2$. This means that $\delta_\lambda = -27\lambda^2 V(-M^2)$.
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Energy-dependent ultraviolet cutoff

- Now assume that $\Lambda = \Lambda(|s|^{1/2})$. The counterterm is

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$$\beta(\lambda) = \frac{9\lambda^2}{16\pi^2}\left(1 - \frac{\partial \ln \Lambda(M)}{\partial \ln M}\right). \qquad (1)$$

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$$\lambda = \lambda_1 + \frac{9\lambda_1^2}{16\pi^2}\left(\ln\frac{M}{M_1} - \ln\frac{\Lambda(M)}{\Lambda(M_1)}\right).$$

Energy-dependent ultraviolet cutoff

- Now assume that $\Lambda = \Lambda(|s|^{1/2})$. The counterterm is

$$\delta_\lambda = \frac{9\lambda^2}{32\pi^2}\left[2 + \ln\frac{\Lambda^2(M)}{M^2}\right].$$

- Then,

$$G^{(4)} = -i\left[3\lambda + \frac{9\lambda^2}{32\pi^2}\left(\ln\frac{|s|}{M^2} + \ln\frac{|t|}{M^2} + \ln\frac{|u|}{M^2} - 3\ln\frac{\Lambda^2(|s|^{1/2})}{\Lambda^2(M)}\right)\right]\prod_i \frac{1}{p_i^2}.$$

- At this order and we have $\gamma = 0$. The β-function is

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- In order to make contact with the exact renormalization group approach, choose a reference scale $M_1 = \Lambda_0$ (where Λ_0 should be identified with $M_{\rm Pl}$). Impose $\Lambda(\Lambda_0) = \Lambda_0$. An example is $\Lambda(M) = \Lambda_0^{1-\delta} M^\delta$ with constant δ. The β-function agrees with the one derived through the exact renormalization group.
- The running coupling becomes

$$\lambda(M) = \lambda_w + \frac{9\lambda_w^2}{16\pi^2} \ln \frac{M}{\Lambda(M)} = \lambda_w + (1-\delta)\frac{9\lambda_w^2}{16\pi^2} \ln \frac{M}{\Lambda_0}.$$

The bare coupling of the Wilsonian approach must be identified with λ_w in the above expressions. We have $\lambda(\Lambda_0) = \lambda_w$.
- The bare coupling in the Lagrangian is

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Running of the electromagnetic coupling

- Running of $\alpha_{\rm em}$ from m_e to m_μ
- Anomalous magnetic moments of the electron and muon: $g_i = 2 + \alpha_{\rm em}(m_i)/\pi + \cdots$.
- Infer the electromagnetic coupling $\alpha_{\rm em}(\mu)$ at the scales $\mu = m_e$ and m_μ.
- Assume the anomalous running

$$\frac{1}{\alpha_{\rm em}(m_e)} - \frac{1}{\alpha_{\rm em}(m_\mu)} = \frac{1-\delta}{3\pi} \ln \frac{m_\mu}{m_e} + \cdots \qquad (3)$$

- The most precise determination of δ:

$$\delta = -(0.047 \pm 0.018). \qquad (4)$$

The central value of δ is about 3σ below zero, because, for $\delta = 0$, g_μ is about 3σ above the SM prediction.
- Usual new-physics interpretation: new particles with heavy mass M, which give $\Delta g_\mu \sim \alpha_2 m_\mu^2/M^2$.

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- Precision tests at the Z pole offer another precision determination of the electromagnetic coupling.
- A global fit within the SM gives

$$\frac{1}{\alpha_{\rm em}(M_Z)} = 128.92 + 0.23 \ln \frac{m_h}{M_Z} \pm 0.06. \tag{5}$$

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Running of the strong coupling

Running of α_s from m_τ to m_Z

- Another sensitive probe to δ comes from the running of the strong coupling α_s. The strong coupling constant has been measured at various scales, and the two most precise determinations are at m_τ and M_Z.
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Conclusions

- The notion of a variable ultraviolet cutoff, linked to the infrared one, does not pose a conceptual problem.
- It can implemented within both the exact and the perturbative renormalization group.
- It leads to a modification of the flow equations.
- In four dimensions the strongest observable consequence concerns the logarithimic running of couplings.
- In the simplest implementation, it is strongly constrained by data.
- The framework is interesting, because it provides a window to high energy scales.
  For $\delta = 1/2$, $k \sim 1$ GeV implies $\Lambda \sim 10^{10}$ GeV.
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