Optimisation and the Functional RG

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I

generalities

motivation

- **quantum field theory**

  goal: **prediction of physical quantities**

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  unphysical parameters: regularisation, RG scales, 'scheme'

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  physical theory: independent of regularisation scheme

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- **quantum field theory**

  goal: prediction of physical quantities

  unphysical parameters: regularisation, RG scales, 'scheme'

  physical theory: independent of regularisation scheme
  approximations: spurious 'scheme' dependences, e.g.

  perturbative QFT
  Dyson-Schwinger eqs
  lattice
  functional renormalisation group

motivation

- **quantum field theory**

  goal: prediction of physical quantities

  unphysical parameters: regularisation, RG scales, ’scheme’

  physical theory: independent of regularisation scheme
  approximations: spurious scheme dependences

- **perturbative QFT**

  minimum sensitivity condition

motivation

- **quantum field theory**

  goal: prediction of physical quantities

  unphysical parameters: regularisation, RG scales, ’scheme’

  physical theory: independent of regularisation scheme
  approximations: spurious scheme dependences

- **perturbative QFT**

  minimum sensitivity condition

- **functional renormalisation group**

  optimisation $\longleftrightarrow$ stability $\longleftrightarrow$ control of approximations

principle of minimum sensitivity

- **idea** Stevenson ('81)

  physical observable $\mathcal{O}$

  unphysical parameters (RS)

  full theory

  $$\frac{d\mathcal{O}}{d(RS)} \equiv 0$$

principle of minimum sensitivity

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$$\frac{d\mathcal{O}}{d(RS)} \equiv 0$$

'truncated' theory

$$\frac{d\mathcal{O}_{\rm trunc}}{d(RS)} \overset{!}{=} 0 \quad \Rightarrow \quad (\mathrm{RS})_{\rm PMS},\ \mathcal{O}_{\rm PMS}$$

principle of minimum sensitivity

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- **comments**

  existence

  uniqueness

  dependence on truncation

  dependence on observable

  several observables

  non-constructive

principle of minimum sensitivity

- **idea** (Stevenson '81)

  physical observable $\mathcal{O}$

  unphysical parameters (RS)

  full theory

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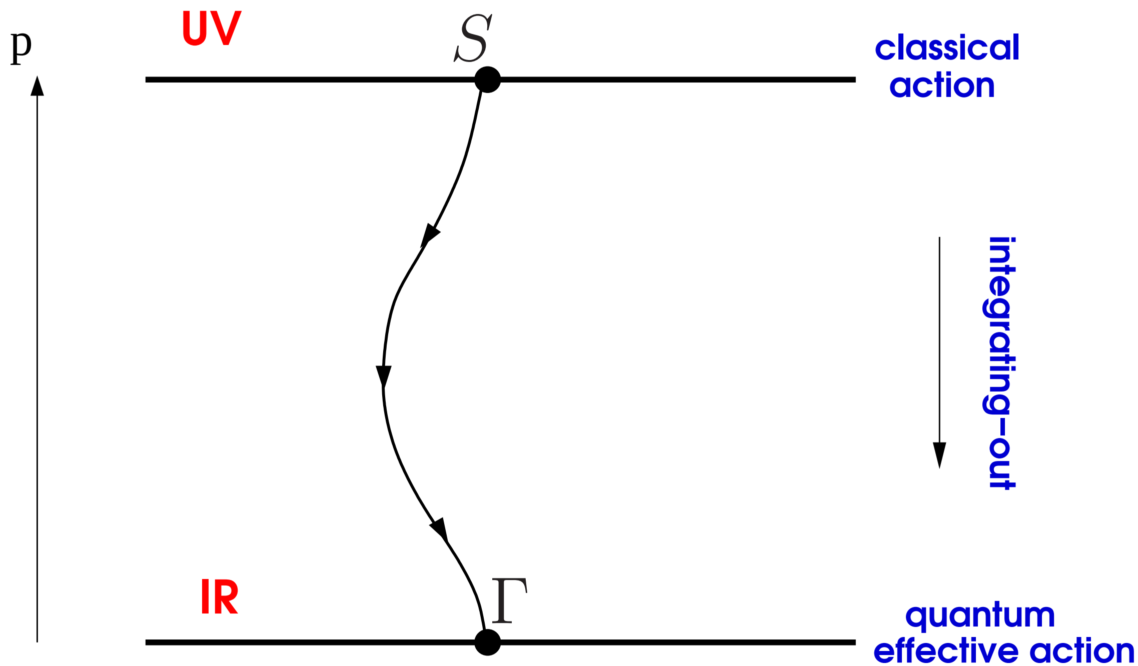
- **flow equation**

  Polchinski flow (Ball et.al. '95)

  functional RG flow (Liao et.al. '99, DL '00, Canet et. al. '02, Pawlowski '05)

functional RG flows

- **integrating-out momentum degrees of freedom**



functional RG flows

- **integrating-out momentum degrees of freedom**

  smooth cutoff in path integral $\Delta S_k[\phi] \sim \int \phi \, R_k \, \phi$

$$Z_k[J] = \int [d\phi] \exp\left(-S[\phi] - \Delta S_k[\phi] + J \cdot \phi\right)$$

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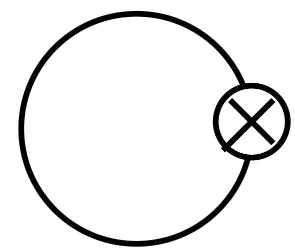
- **exact flow equation** (Wetterich '93)

$$\partial_t \Gamma_k = \frac{1}{2} \, \mathrm{Tr} \left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi \, \delta\phi} + R_k \right)^{-1} \partial_t R_k \right]$$

$= \frac{1}{2}$ 

functional RG flows

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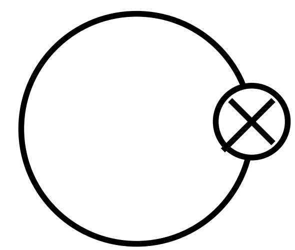

- **limits**

  $\Gamma_{k\to\Lambda} \to S_{\rm cl}$ and $\Gamma_{k\to 0} \to \Gamma$

functional RG flows

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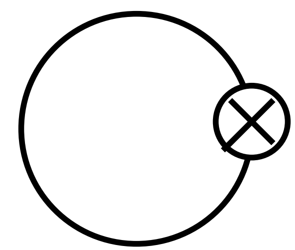
$= \frac{1}{2}$ 

- **properties**

  UV and IR finiteness, locality, amenable to truncations

  $R_k(q^2) \equiv k^2$: Callan-Symanzik equation

where is the physics?

- **integral representation of the flow**

$$\Gamma = \Gamma_\Lambda \; + \; \frac{1}{2} \int_\Lambda^0 \frac{dk}{k} \, \partial_t \Gamma_k[\Gamma_k^{(2)}; R]$$

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initial condition Γ_Λ

where is the physics?

- **integral representation of the flow**

$$\Gamma = \Gamma_\Lambda + \frac{1}{2}\int_\Lambda^0 \frac{dk}{k}\,\partial_t \Gamma_k[\Gamma_k^{(2)}; R]$$

initial condition

relevant physics in flow

where is the physics?

- **integral representation of the flow**

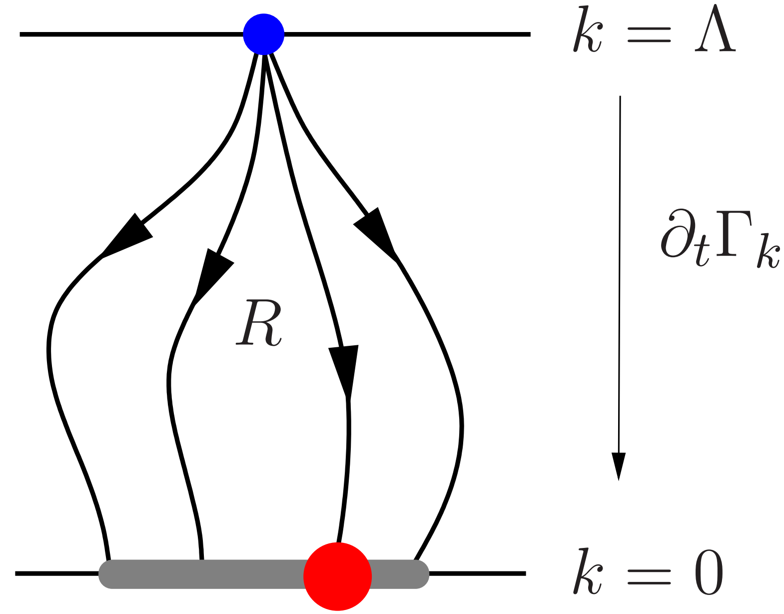
$$\Gamma = \Gamma_\Lambda + \frac{1}{2}\int_\Lambda^0 \frac{dk}{k}\,\partial_t \Gamma_k[\Gamma_k^{(2)}; R]$$

- **approximations**

truncations unavoidable

induce scheme dependence

identify most stable flows



optimisation

- **basic idea** (DL '00, '01)

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$

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- **"mind the gap"**

$$C[\phi_0; R]\,k^2 = \min_{q^2\geq 0}\left[\left.\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi}\right|_{\phi=\phi_0} + R_k(q^2)\right]$$

Corrections $\Gamma_k^{(2)} \to \Gamma_k^{(2)} + \delta\Gamma_k^{(2)}$ **contribute like** $\sim \delta\Gamma_k^{(2)}/C$ **.**

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- **maximise the gap**

$$C_{\rm max} \equiv \max_R \, C \Rightarrow R_{\rm opt}$$

optimisation

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- **comments**

  applicable for any truncational scheme

  optimisation reduces cutoff dependence

  improves physical content

  constructive criterion

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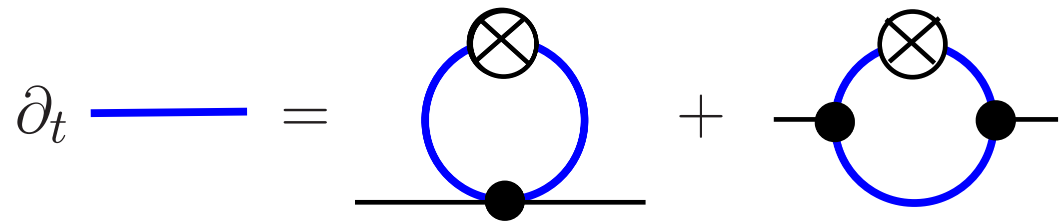
- **functional optimisation** (Pawlowski '05)

  maximise propagator at given gap

  minimise length of RG trajectory

example: scalar field theory

- **propagator flow**



- **leading order**

“mind the gap”

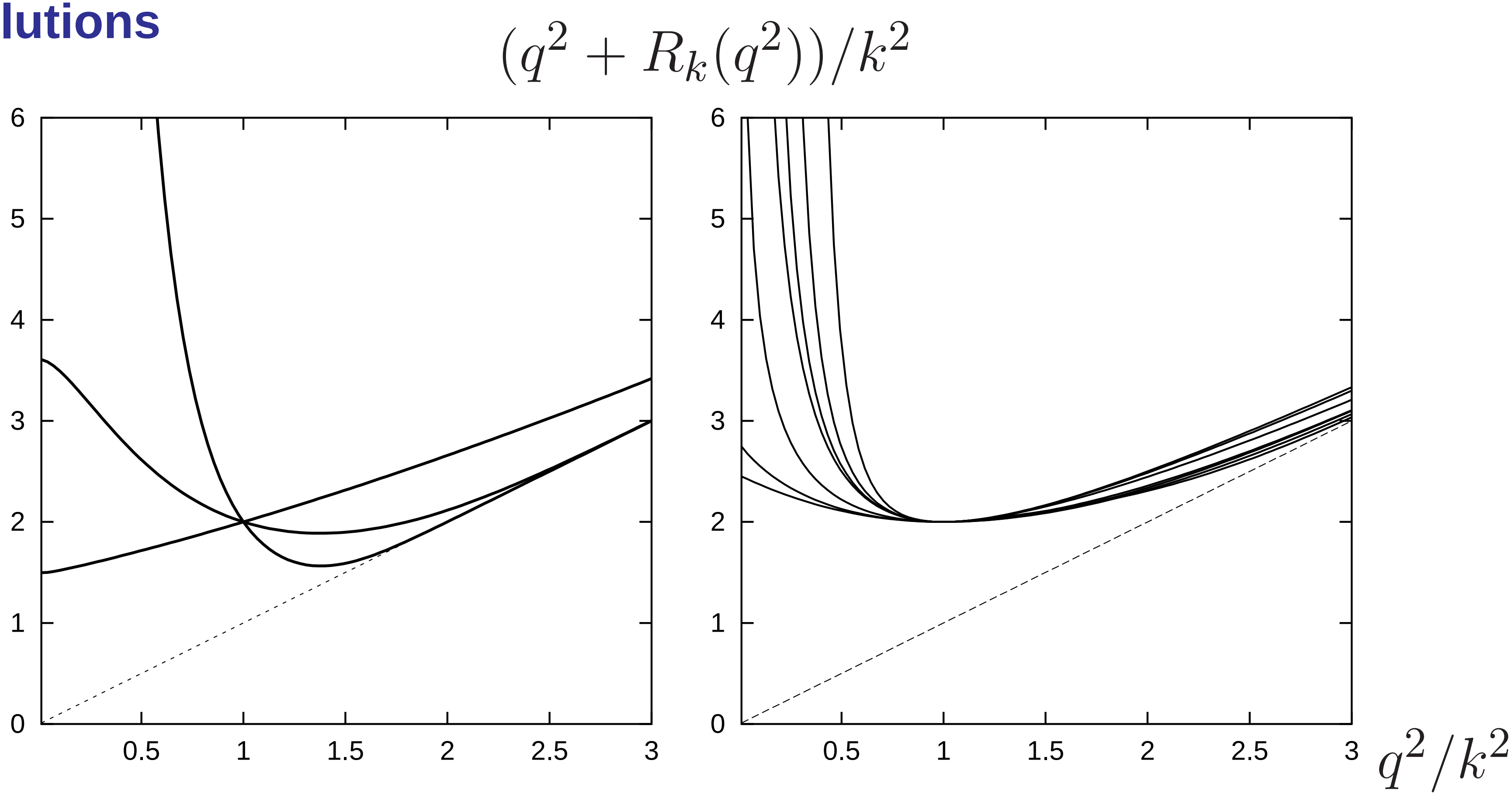
$$\min_R \left[\max_{q^2} \left(\frac{1}{q^2 + R} \right) \right]$$

- **normalisation**

$$R_k(q^2 = k^2) = c\, k^2$$

optimisation condition

- **solutions**



- **convex hull**

$$R_{\rm opt} = (a\,k^2 - q^2)\theta(a\,k^2 - q^2)$$

II

applications

local potential approximation

- **functional RG study**

  Ising universality class (d=3)

$$\partial_t u = -3u + \rho\, u' + \int_0^\infty dy\, y^{5/2}\, r'(y)\, \frac{1}{(y + y\, r(y) + u' + 2\rho\, u'')}$$

momentum cutoff $R = q^2 \cdot r(y),\ y = q^2/k^2$

scaling solution $\partial_t u'_* = 0$

scaling exponents $\partial_t (u'_* + \delta u'_n) = \omega_n\, \delta u'_n$

analyse $\{\omega_n(R)\}$

tomography of the Ising model

(DL '07)

$$r_{\rm mod} = 1/(\exp[c(y + (b-1)y^b)/b] - 1),\ c = \ln 2$$

$$r_{{\rm opt},n} = b(1/y - 1)^n \theta(1-y),\ n = 1$$

$$r_{\rm mexp} = b/((b+1)^y - 1)$$

$(r_{\rm PT})$ proper time flow

$$r_{\rm exp} = 1/(\exp cy^b - 1)$$

$$r_{\rm mix} = \exp[-b(\sqrt{y} - 1/\sqrt{y})]$$

$$r_{\rm mix,opt} = \exp[-\frac{1}{b}(y^b - y^{-b})]$$

$$r_{\rm power} = y^{-b}$$

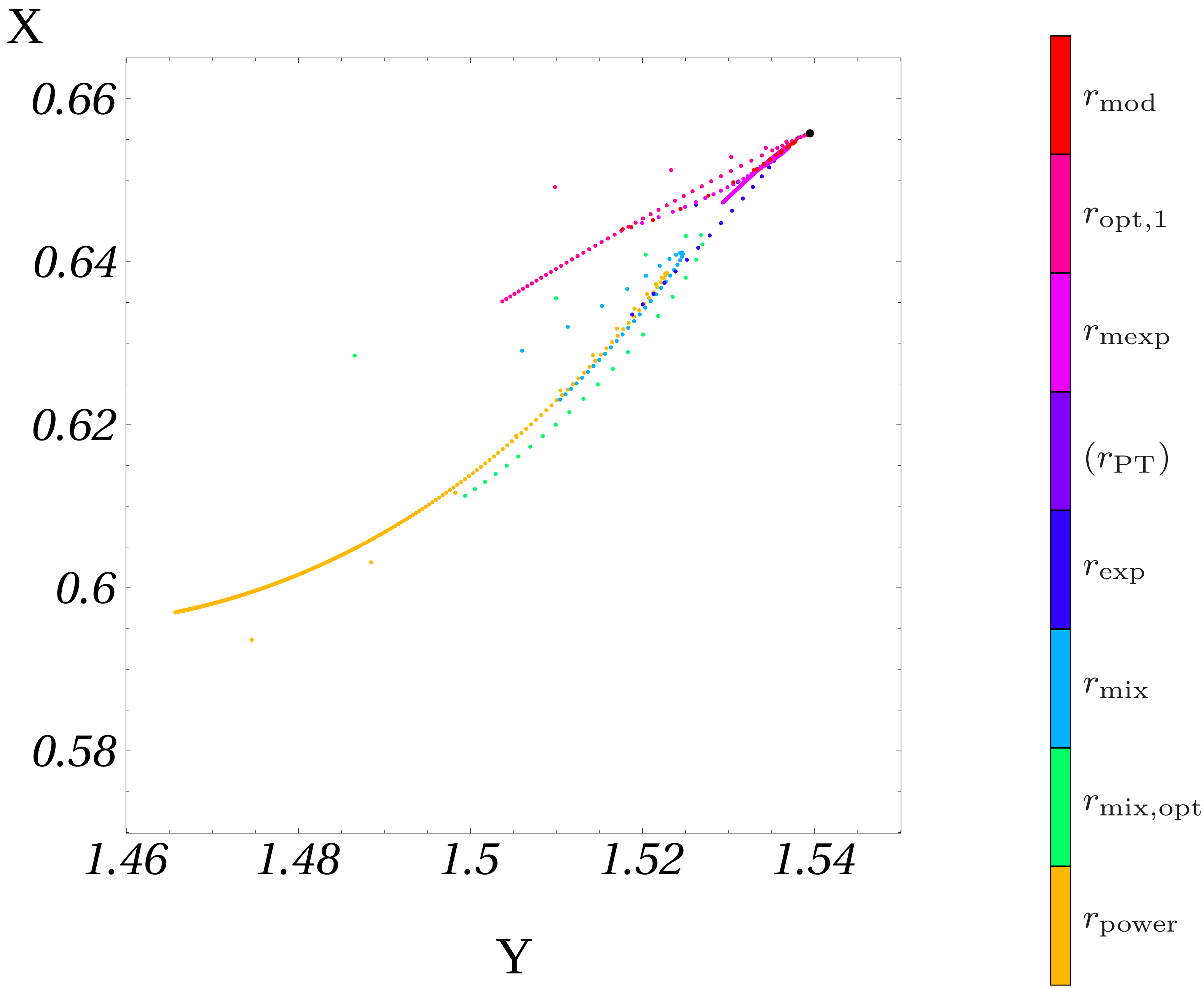
tomography of the Ising model

(DL '07)

X = ω

Y = $|\omega_0|$

$(\nu = |\omega_0|^{-1})$

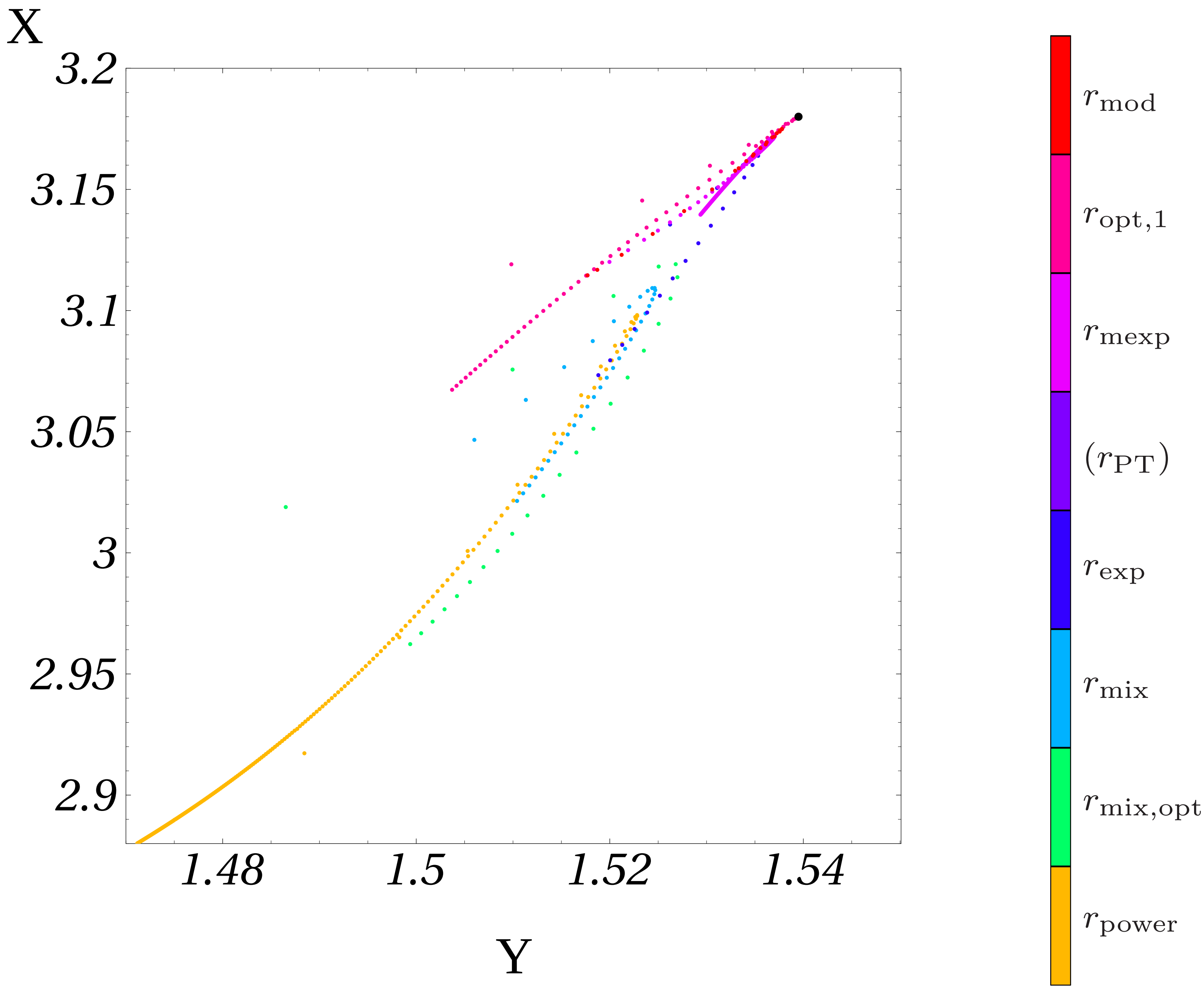


tomography of the Ising model

(DL '07)

$X = \omega_2$

$Y = |\omega_0|$

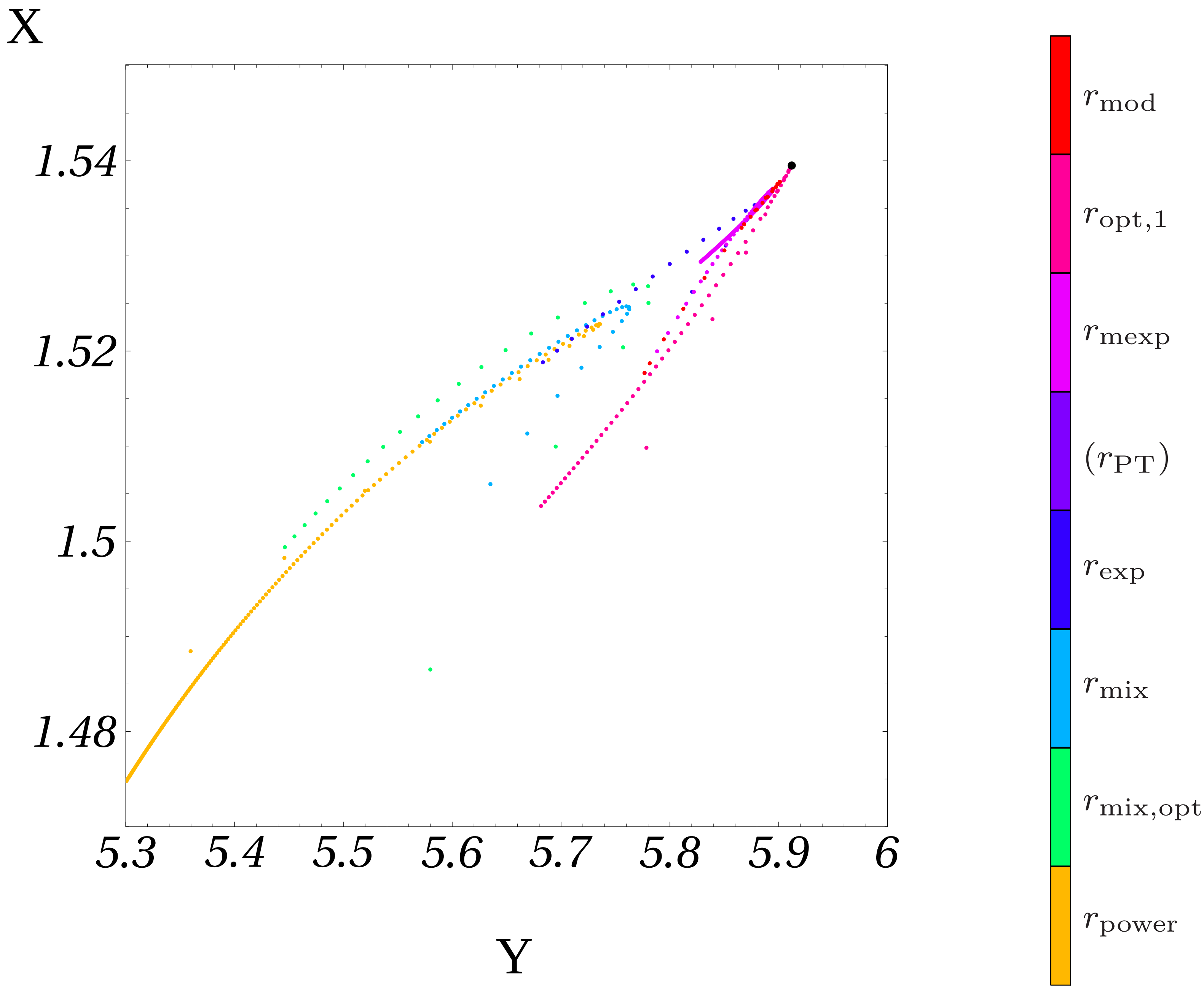


tomography of the Ising model

(DL '07)

$X = |\omega_0|$

$Y = \omega_3$

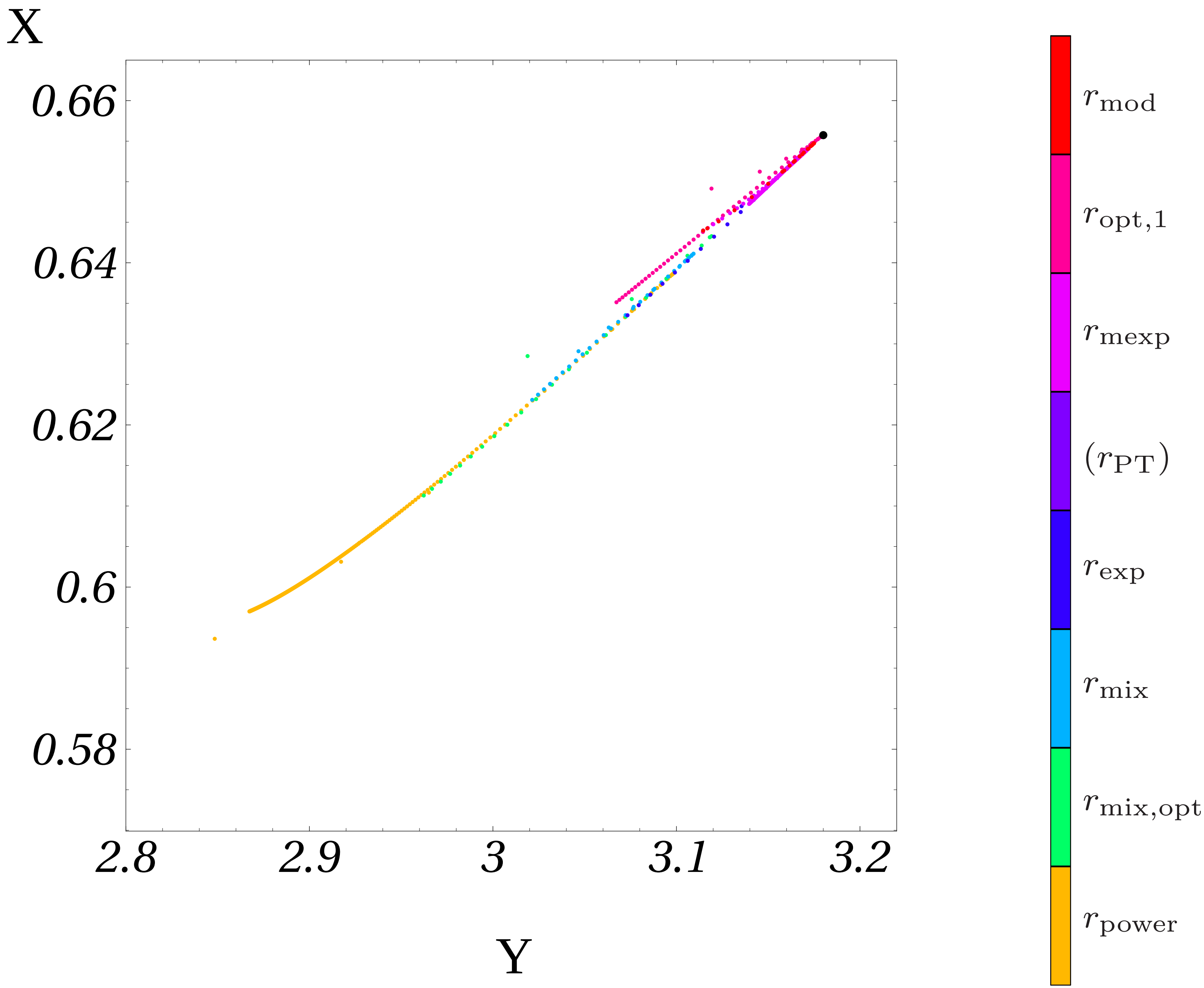


tomography of the Ising model

(DL '07)

$X = \omega$

$Y = \omega_2$



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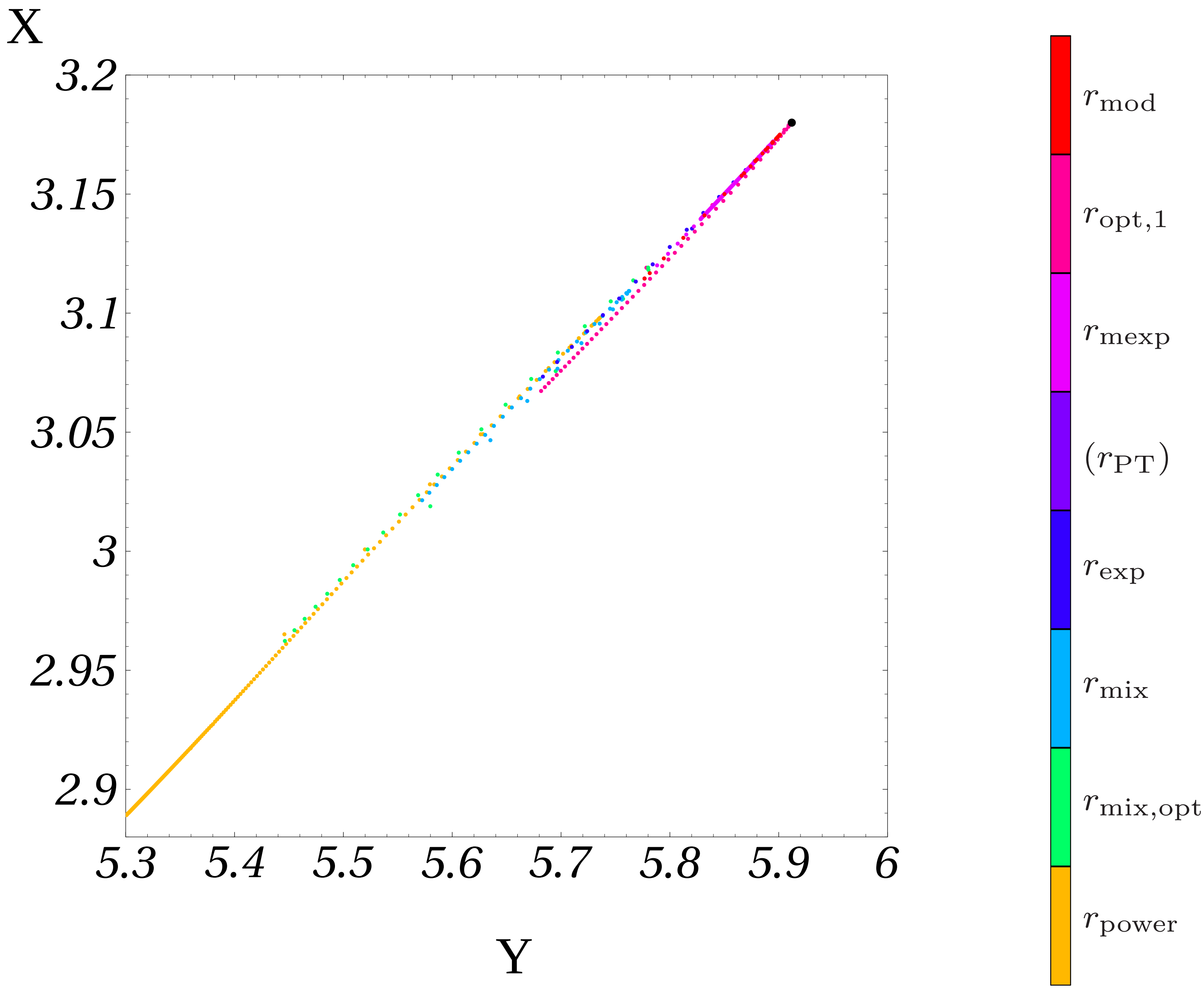

tomography of the Ising model

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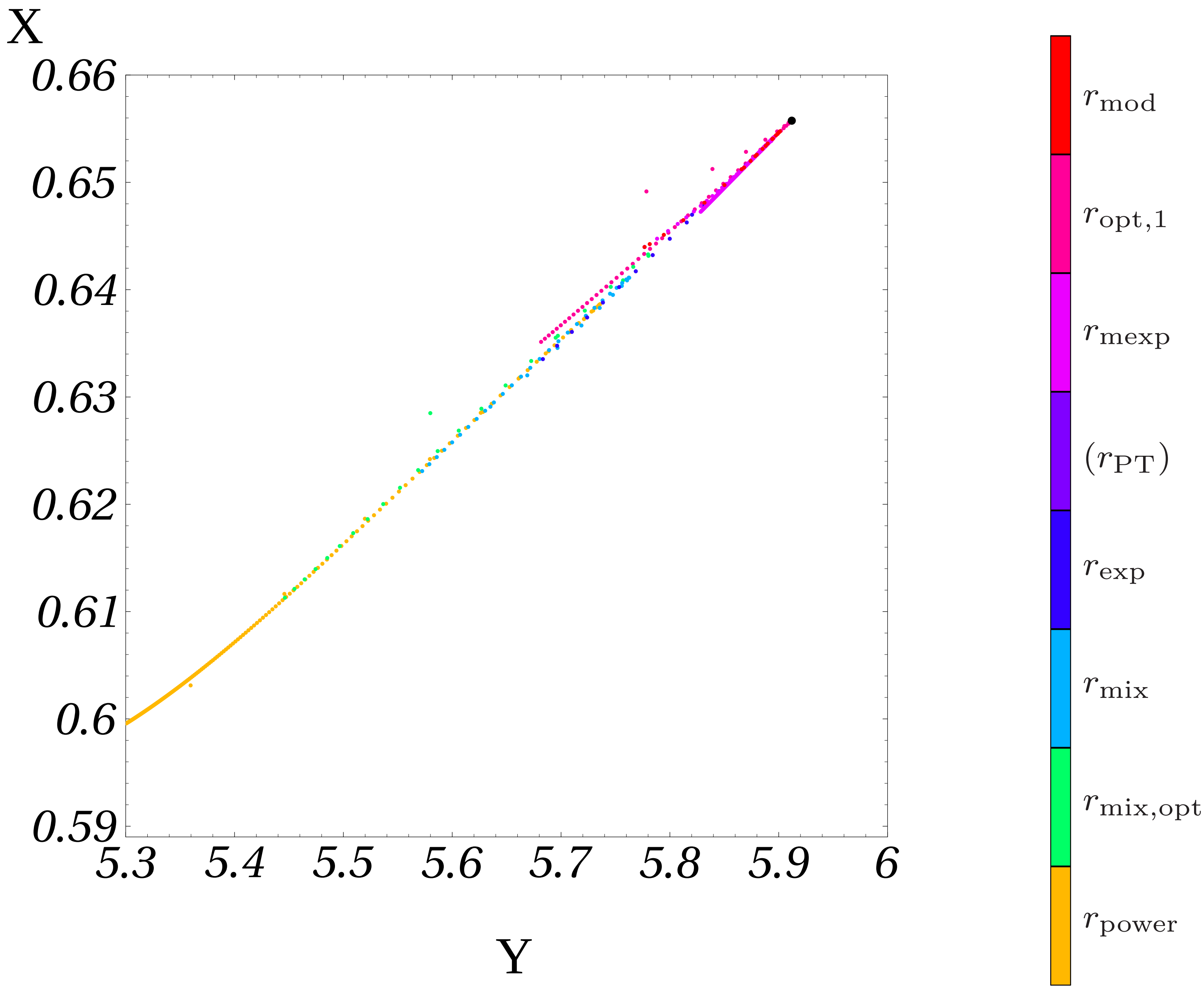
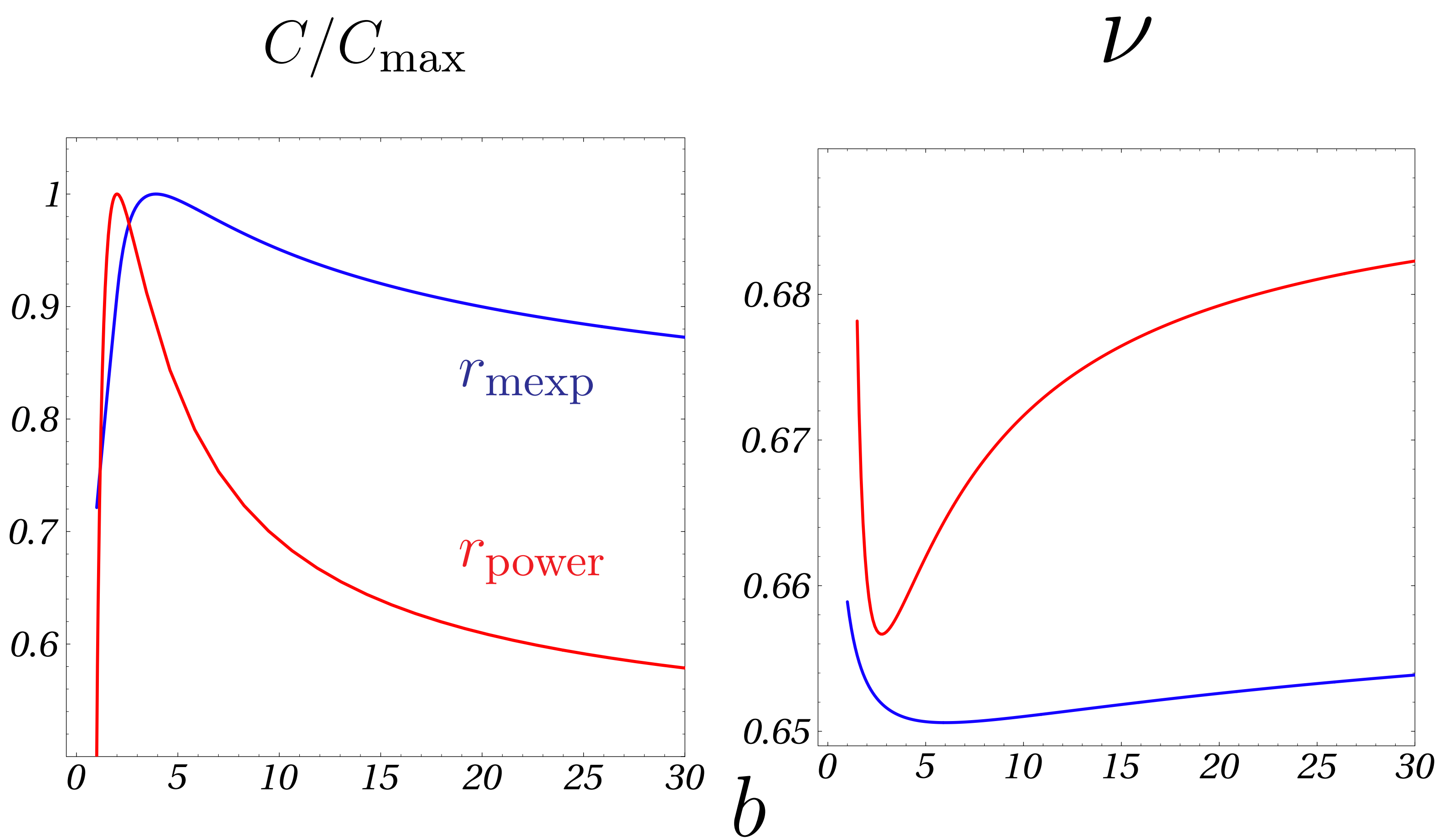
$X = \omega$

$Y = \omega_3$



local potential approximation



local potential approximation

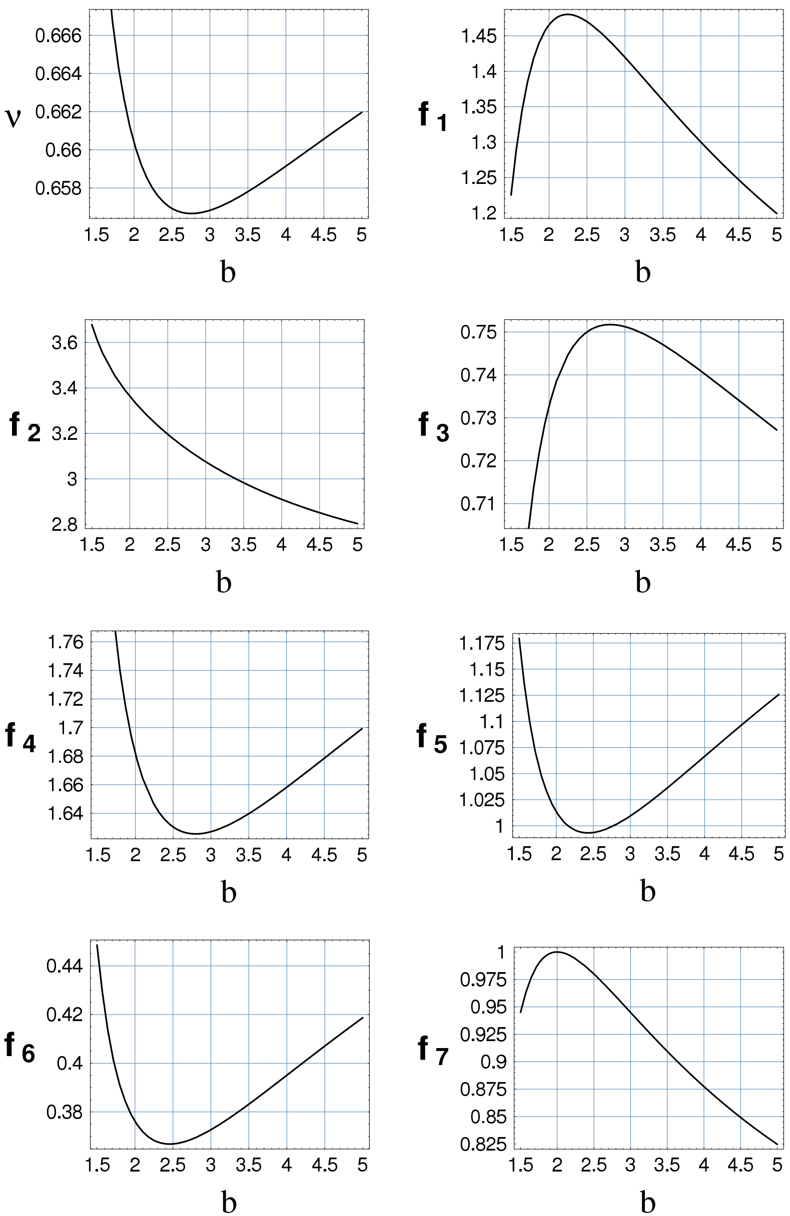
	cutoff	$b_{\rm opt}$	$\nu_{\rm opt}$	$\nu_{\rm opt}/\nu_{\rm global}$
$r_{\theta,b} = b(1/y-1)\theta(1-y)$	$r_{\theta,b}$	1	0.64956	1
$r_{\rm mod} = 1/(\exp[c(y+(b-1)y^b)/b]-1),\ c=\ln 2$	$r_{\rm mod}$	1.92	0.65055	1.0015
$r_{\rm mexp} = b/((b+1)^y-1)$	$r_{\rm mexp}$	3.92	0.65096	1.0022
$r_{\rm exp} = 1/(\exp cy^b - 1)$	$r_{\rm exp}$	1.44	0.65132	1.0027
$r_{{\rm mix},a} = \exp[-b(y^a - y^{-a})]$	$r_{\rm mix\,2}$	2	0.65506	1.0085
	$r_{\rm mix\,1}$	2	0.65496	1.0083
	$r_{\rm mix\,\frac{1}{2}}$	2	0.65702	1.0115
$r_{\rm power} = y^{-b}$	$r_{\rm power}$	2	0.66039	1.0167

comparison: PMS vs optimisation

cutoff	$b_{\rm opt}$	$b_{\rm PMS}$	$C_{\rm PMS}/C_{\rm opt}$	$\nu_{\rm PMS}$	$\nu_{\rm opt}/\nu_{\rm PMS}$	$\nu_{\rm PMS}/\nu_{\rm global}$	$\nu_{\rm opt}/\nu_{\rm global}$
$r_{\theta,b}$	1	1	1	0.649562	1	1	1
	1	∞	$\frac{1}{2}$	0.6895	0.9421	1.0615	1
	1	$\frac{1}{2}$	1	1	0.6496	1.5395	1
$r_{\rm mod}$	1.92	2.15	0.987	0.6503	1.0004	1.0011	1.0015
	1.92	∞	$\frac{1}{2}$	0.6895	0.9431	1.0615	1.0015
	1.92	0	$\frac{1}{2}$	1.	0.6506	1.5395	1.0015
$r_{\rm mexp}$	3.92	5.99	0.986	0.6506	1.0006	1.0016	1.0022
	3.92	∞	$\frac{1}{2}$	0.6895	0.9441	1.0615	1.0022
	3.92	0	$\frac{1}{2}$	1.	0.6510	1.5395	1.0022
$r_{\rm exp}$	1.44	1.52	0.998	0.6512	1.0001	1.0026	1.0027
	1.44	∞	$\frac{1}{2}$	0.6895	0.9421	1.0615	1.0027
	1.44	0	$\frac{1}{2}$	1	0.6513	1.5395	1.0027
$r_{\rm power}$	2	2.76	0.963	0.6567	1.0057	1.0109	1.0167
	2	∞	$\frac{1}{2}$	0.6895	0.9578	1.0615	1.0167
	2	1	$\frac{1}{2}$	1.	0.6604	1.5395	1.0167

optimisation: beyond leading order

with M. Strickland ('08)



- **field-dependent inverse propagator**

$$\left(q^2 + R_k(q^2) + U_k''(\phi; R)\right) / k^2$$

$$\begin{aligned}
f_1 &: \quad C + m_s^2(R) \\
f_2 &: \quad C + m_0^2(R) \\
f_3 &: \quad 1 + m_s^2(R)/C \\
f_4 &: \quad 1 + m_0^2(R)/C \\
f_5 &: \quad \rho_0(R)/C \\
f_6 &: \quad \rho_1(R)/C \\
f_7 &: \quad C
\end{aligned}$$

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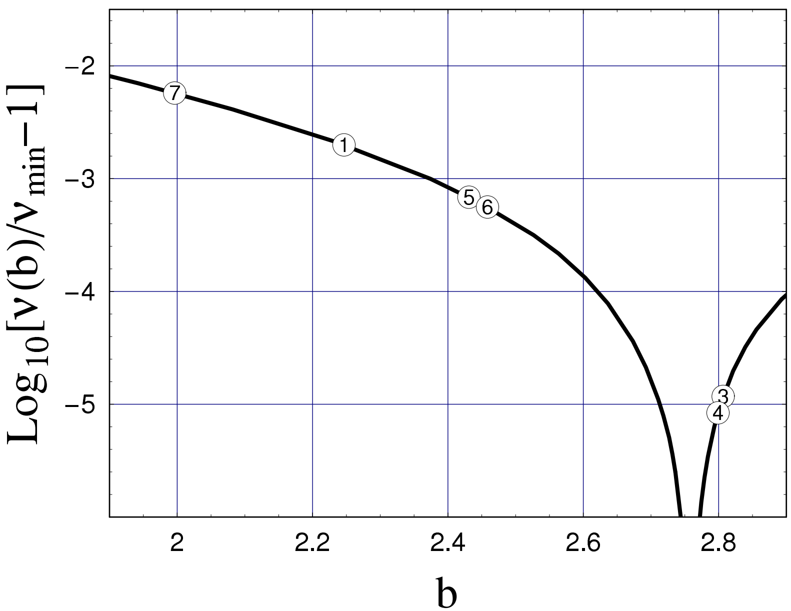

- $N = 1,\ r_{\text{power}}$

- **implicit scheme dependence**

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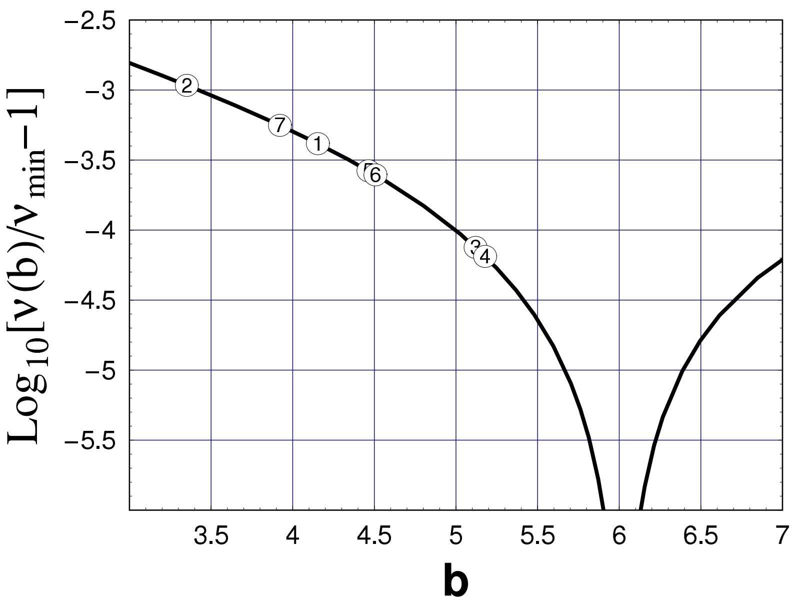

- $N = 1,\ r_{\rm mexp}$

- **implicit scheme dependence**

optimisation: beyond leading order

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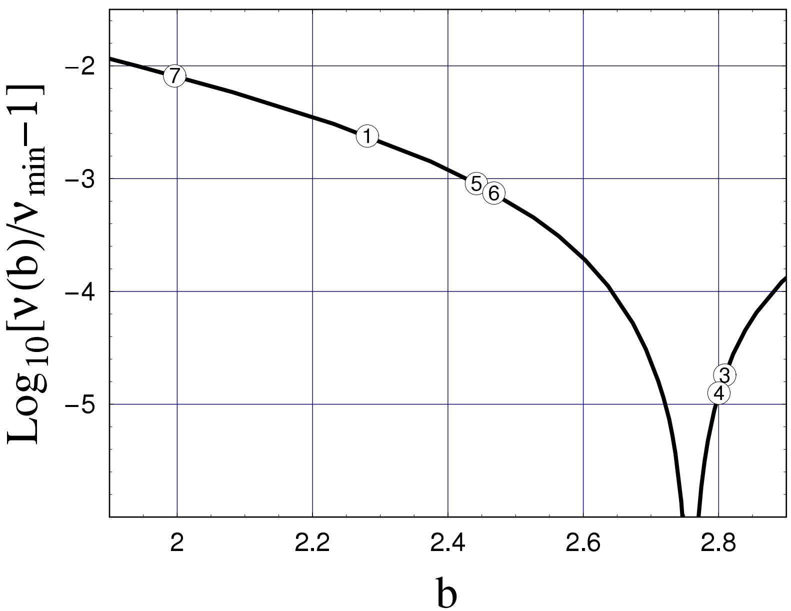
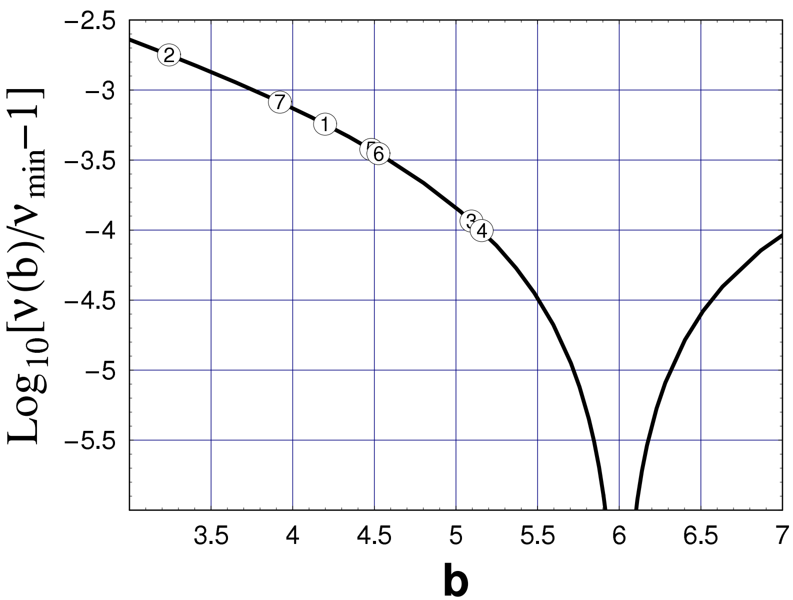


- $N = 2,\ r_{\rm power}$

- **implicit scheme dependence**

optimisation: beyond leading order

with M. Strickland ('08)



- $N = 2,\ r_{\mathrm{mexp}}$

- **implicit scheme dependence**

  improvement for generic regulator

  no effect for $R = (q^2 - k^2)\theta(k^2 - q^2)$

summary

- **Ising-like universality class**

summary

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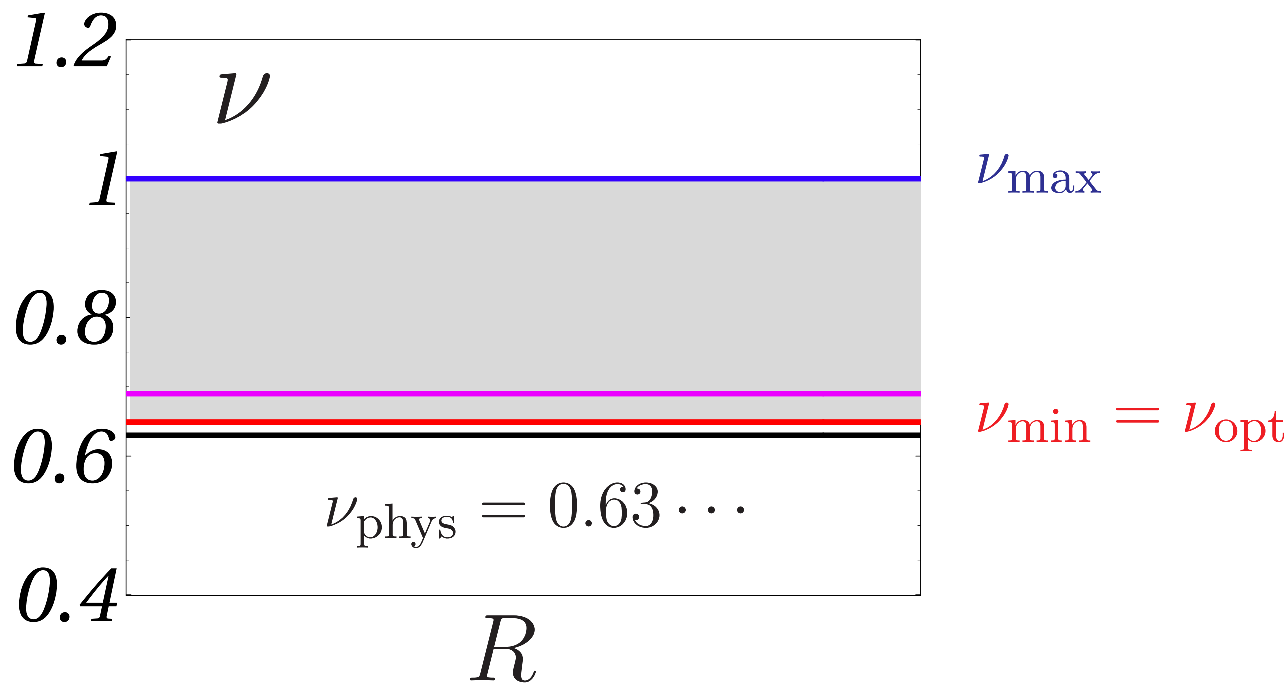


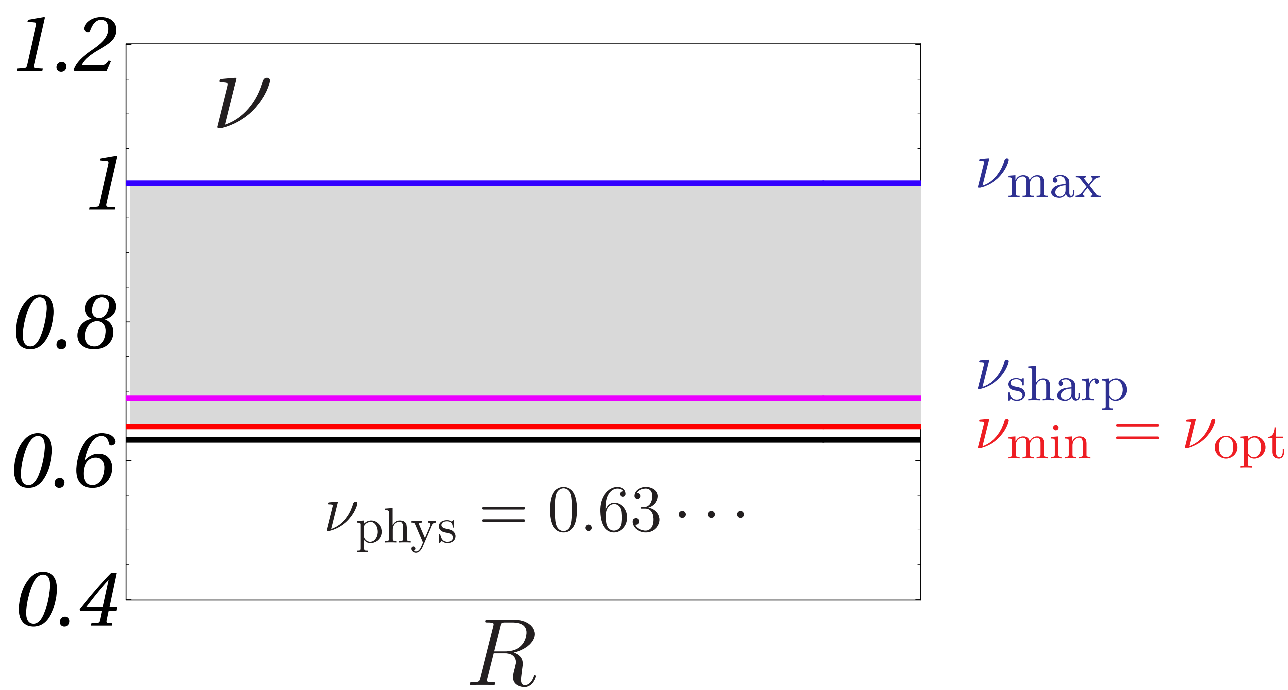
summary

- **Ising-like universality class**

- **optimisation entails ’global’ PMS**

- **’local’ PMS has multiple solutions**

- **sharp cutoff always solves ’local’ PMS**



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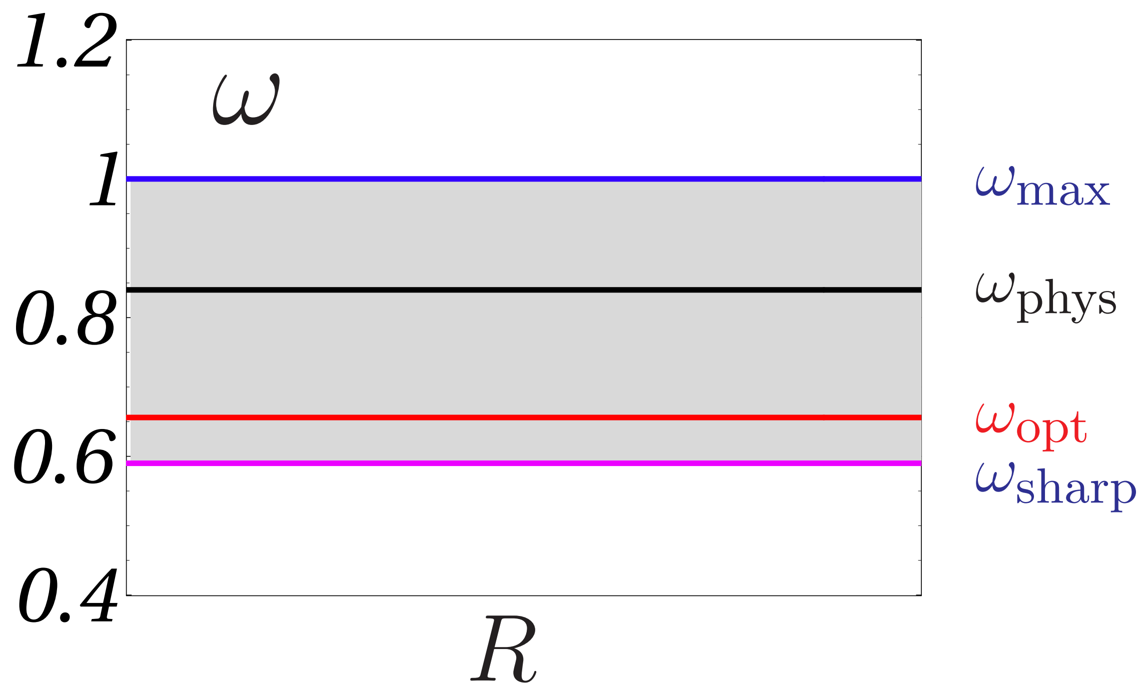


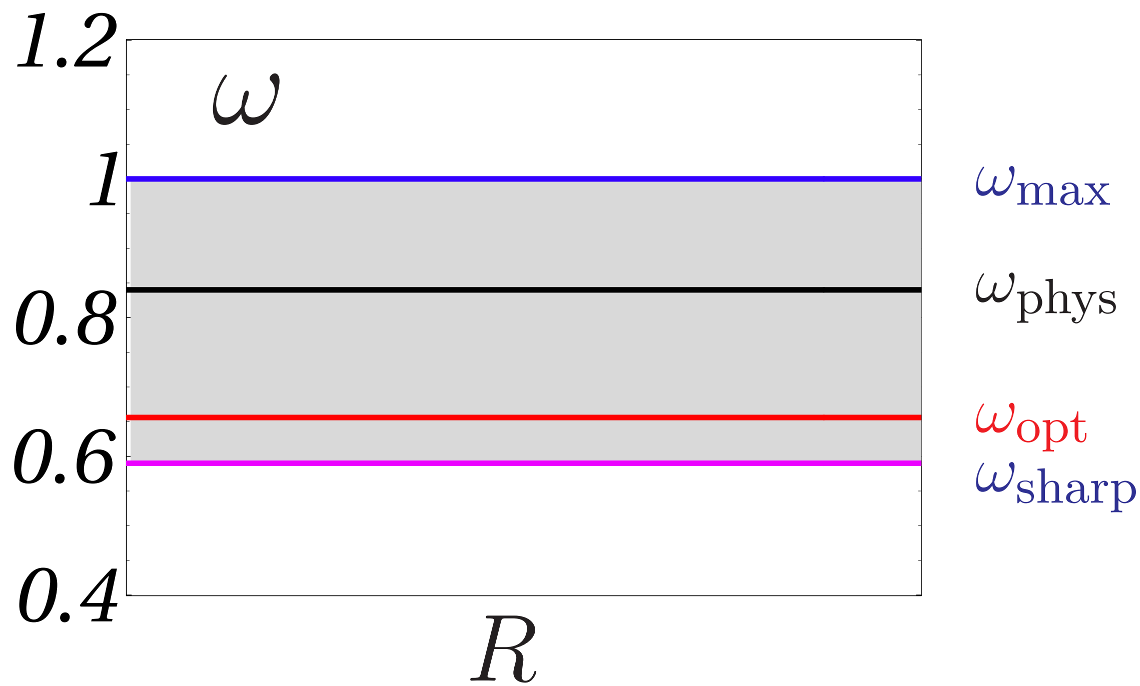
summary

- **Ising-like universality class**

- **generic behaviour**

- **'global' PMS fails**

- **optimisation provides 'measure' for reliability of truncation**



fixed points of quantum gravity

- **effective action ansatz**

$$\Gamma_k = \frac{1}{16\pi G_k} \int \sqrt{g}\,(\Lambda_k + R + \cdots) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

fixed points of quantum gravity

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up to now: $\sqrt{g},\ \sqrt{g}R,\ \sqrt{g}R^2, \cdots,\ \sqrt{g}R^8,\ \sqrt{g}f(R)$**, matter fields**

mainly four dimensions

Reuter (1996), Souma (1999), Lauscher, Reuter ('01), Reuter, Saueressig ('01),
DL ('03), Percacci, Perini ('03), Bonnano, Reuter ('04), Bonanno ('05),
Lauscher, Reuter ('05), Percacci ('05), Fischer, DL ('06), Codello, Percacci ('06)
Codello, Percacci, Rahmede ('07,'08), Machado, Saueressig ('07)
Reuter, Weyer ('08)

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- **wilsonian RG flow**

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[g_{\mu\nu}] = \frac{1}{2}\,\mathrm{Tr}\,\left[\left(\Gamma_k^{(2)}[g_{\mu\nu}] + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right]$$

fixed points of quantum gravity

- **effective action ansatz**

$$\Gamma_k = \frac{1}{16\pi G_k}\int\sqrt{g}\,(\Lambda_k + R + \cdots) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

- **wilsonian RG flow**

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[g_{\mu\nu}] = \frac{1}{2}\,\mathrm{Tr}\,\left[\left(\Gamma_k^{(2)}[g_{\mu\nu}] + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right]$$

- **scalar propagator**

$$G(q) = \frac{1}{q^2 + R_k(q^2) - 2\Lambda_k}$$

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- **scalar propagator**

$$G(q) = \frac{1}{q^2 + R_k(q^2) - 2\Lambda_k}$$

- **optimisation**

improve stability/convergence through adequate momentum cutoffs

fixed points of quantum gravity

with P. Fischer ('06) (and in prep. '08)

- **Einstein-Hilbert theory**

fixed points of quantum gravity

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- **Einstein-Hilbert theory**

  ansatz

$$\Gamma_k = \frac{1}{16\pi G_{N,k}} \int \sqrt{g}\,(R + \Lambda_k)$$

  search for UV fixed point $g_\star,\ \lambda_\star$ **in** D **dimensions**

fixed points of quantum gravity

with P. Fischer ('06) (and in prep. '08)

- **Einstein-Hilbert theory**

  ansatz

$$\Gamma_k = \frac{1}{16\pi G_{N,k}} \int \sqrt{g}\,(R + \Lambda_k)$$

  search for UV fixed point $g_\star,\ \lambda_\star$ **in** D **dimensions**

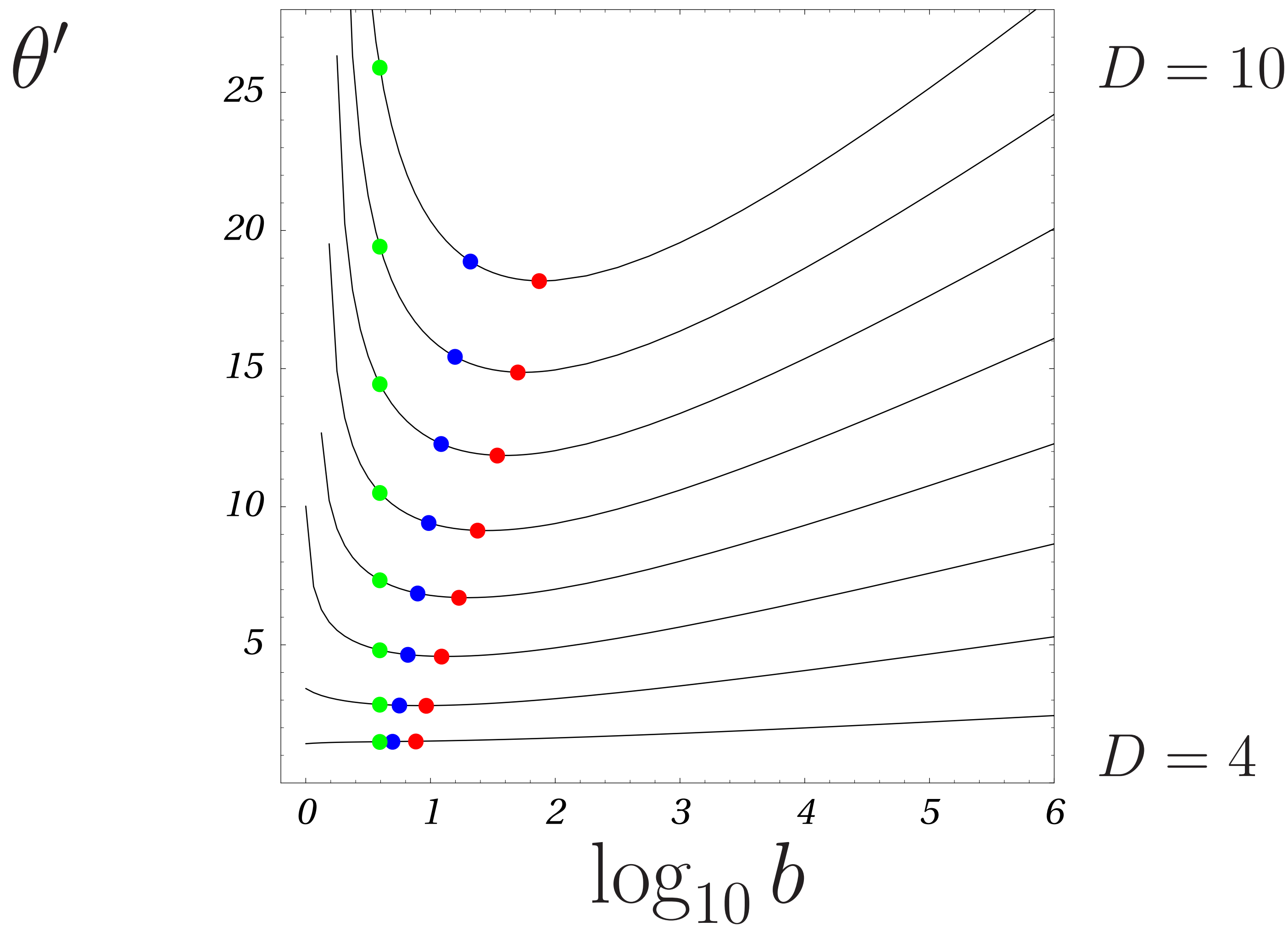
- **physical observables**

  universal eigenvalues $\theta = \theta' + i\theta''$

  universal ratio of couplings $\tau = \lambda_\star (g_\star)^{\frac{2}{D-2}}$

results

- **scaling exponents**



results

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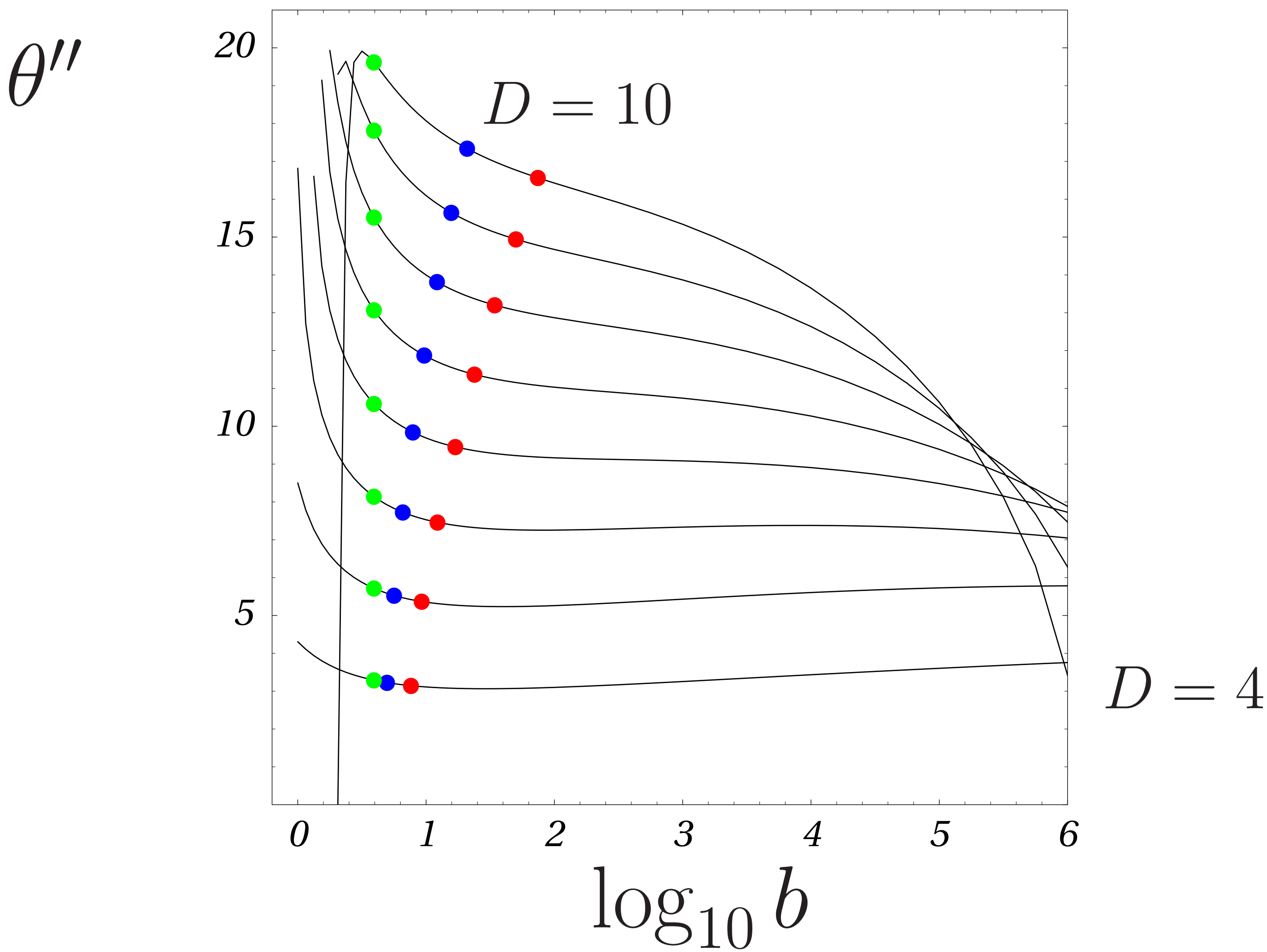
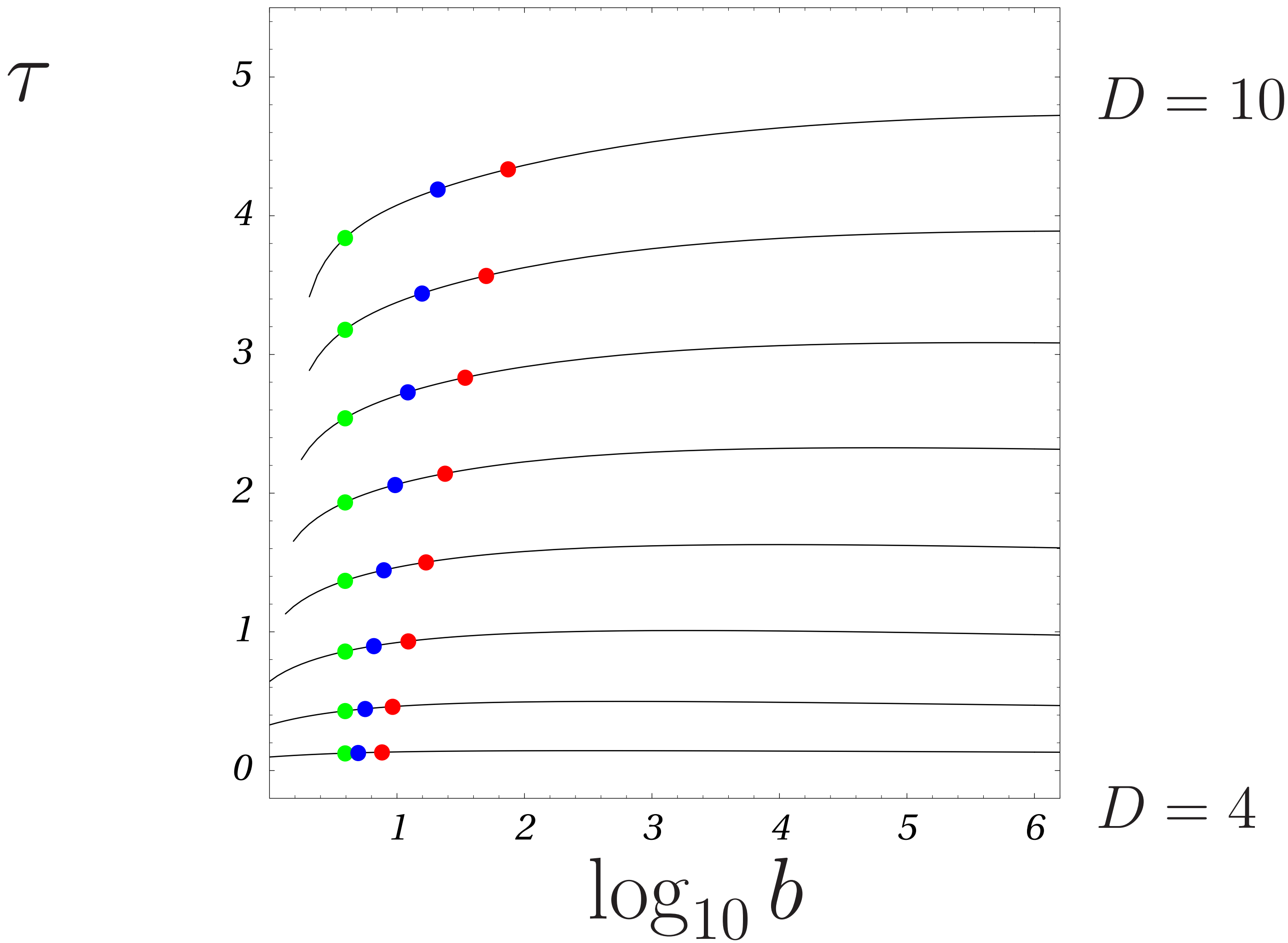


results

- **scaling exponents**



results

- **weak cutoff sensitivity**

θ'	pow	mexp	exp	mod	opt
$D = 4$	1.63	1.51	1.53	1.51	1.48
5	3.19	2.80	2.83	2.77	2.69
6	5.31	4.58	4.60	4.50	4.33
7	7.83	6.71	6.68	6.54	6.27
8	10.7	9.14	9.03	8.86	8.46
9	13.9	11.9	11.6	11.4	10.9
10	17.4	14.9	14.5	14.2	13.5
11	21.3	18.2	17.6	17.3	16.4

results

- **weak cutoff sensitivity**

θ''	pow	mexp	exp	mod	opt
$D = 4$	3.38	3.14	3.13	3.10	3.04
5	5.73	5.37	5.33	5.27	5.15
6	7.85	7.46	7.37	7.31	7.14
7	9.79	9.46	9.32	9.26	9.05
8	11.6	11.4	11.2	11.1	10.9
9	13.2	13.2	13.0	13.0	12.7
10	14.5	14.9	14.7	14.7	14.5
11	15.7	16.6	16.4	16.4	16.2

results

- **weak cutoff sensitivity**

τ	pow	mexp	exp	mod	opt
$D = 4$	0.132	0.132	0.134	0.135	0.137
5	0.463	0.461	0.468	0.469	0.478
6	0.943	0.933	0.946	0.946	0.963
7	1.528	1.502	1.521	1.521	1.544
8	2.186	2.142	2.165	2.162	2.192
9	2.900	2.834	2.858	2.853	2.888
10	3.655	3.568	3.591	3.585	3.623
11	4.445	4.336	4.356	4.348	4.389

results

- **quantum gravity fixed points**

  stable fixed point solution in $D \geq 4$

  strengthens $D = 4$ result

  implicit dependences of varying size

  optimisation: huge reduction of cutoff dependence

  no implicit cutoff dependence for R_{opt}

  essential for extended truncations

  e.g. Codello, Percacci, Rahmede ’08, Machado, Saueressig ’08, Reuter, Weyer ’08

propagator flows in Landau gauge

Pawlowski, DL, Nedelko, Smekal
PRL (2004)[hep-th/0312324], hep-th/0410241, hep-th/0412326

propagator flows in Landau gauge

- **signatures of confinement** Kugo, Ojima '79 / Gribov '78, Zwanziger '94

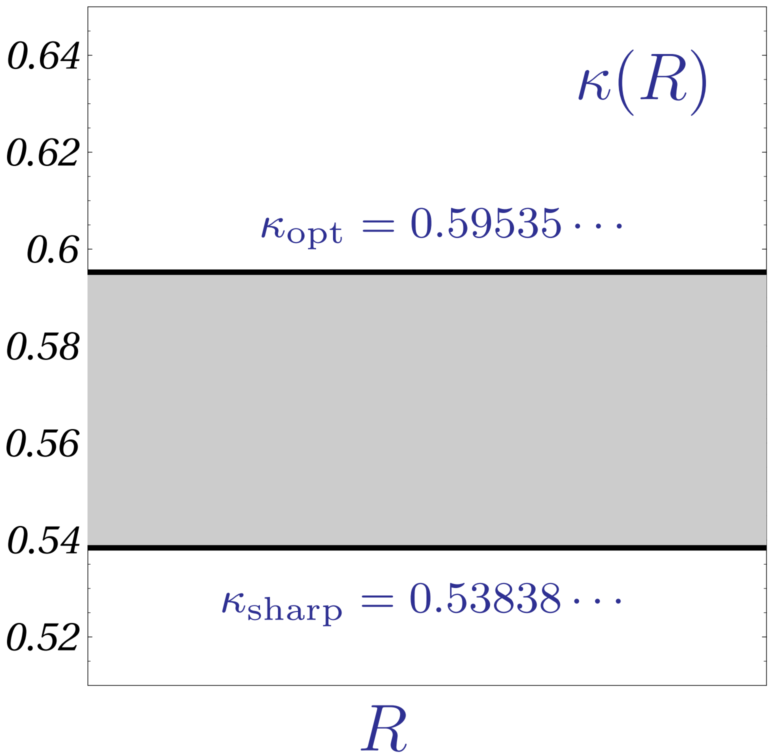
$$\langle A(p)A(-p)\rangle\Big|_{p^2\ll\Lambda^2_{\rm QCD}} \sim p^{-2(1-2\kappa)}$$

$$\langle C(p)C(-p)\rangle\Big|_{p^2\ll\Lambda^2_{\rm QCD}} \sim p^{-2(1+\kappa)}$$

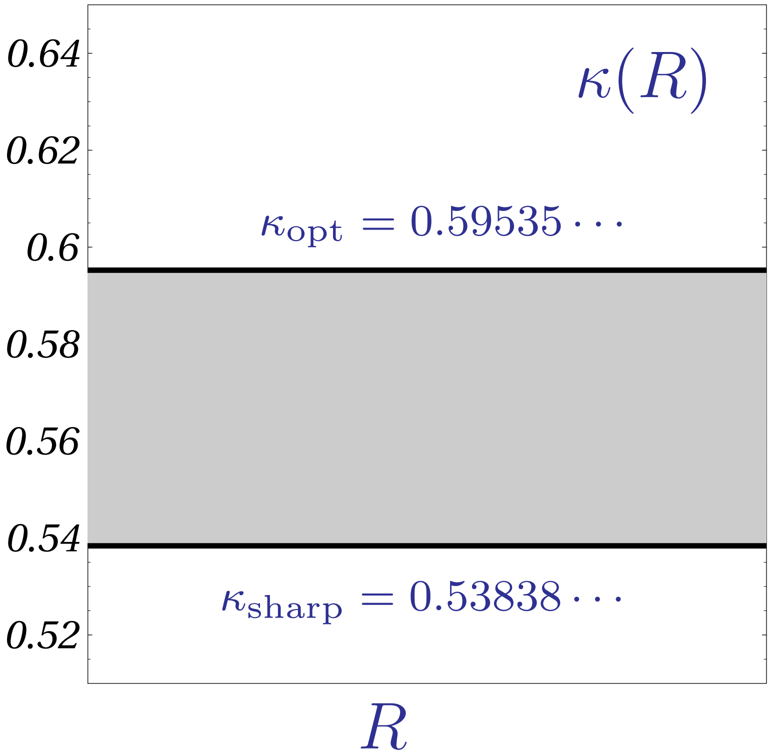
confined gluons: $\kappa > 0$

mass-like gluons: $\kappa = 1/2$

propagator flows in Landau gauge



propagator flows in Landau gauge

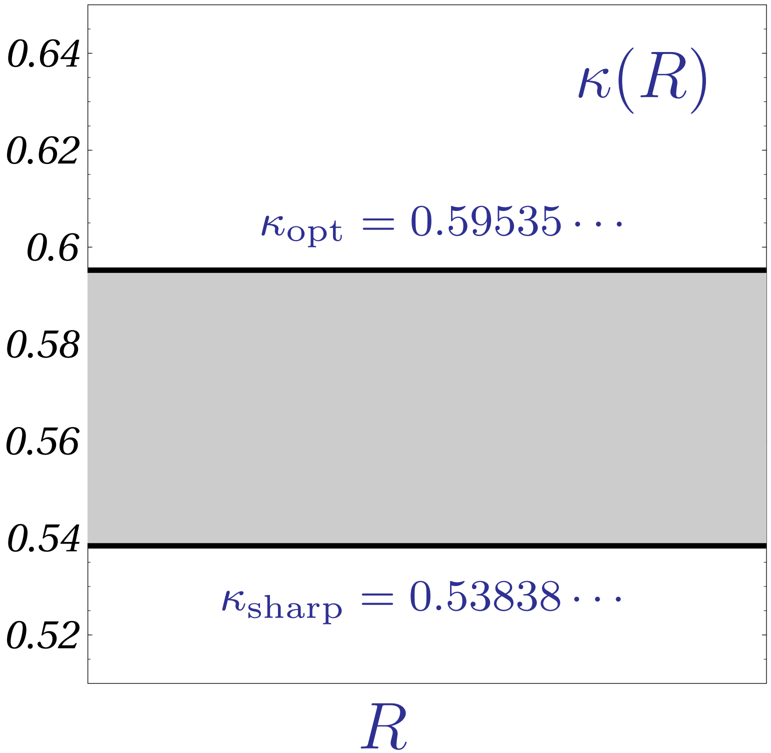


optimisation

- **LO is cutoff independent**

$$\kappa(R) = \kappa_{\rm opt}$$

propagator flows in Landau gauge

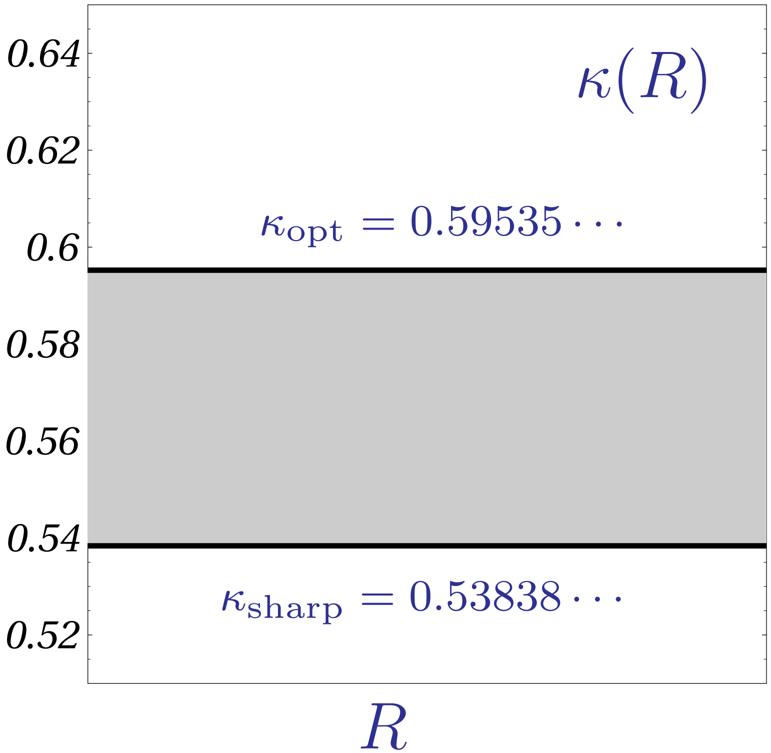


optimisation

- LO is cutoff independent
- beyond LO: global extrema

$$\kappa_{\rm opt} = {\rm extr}_R\,\kappa(R)$$

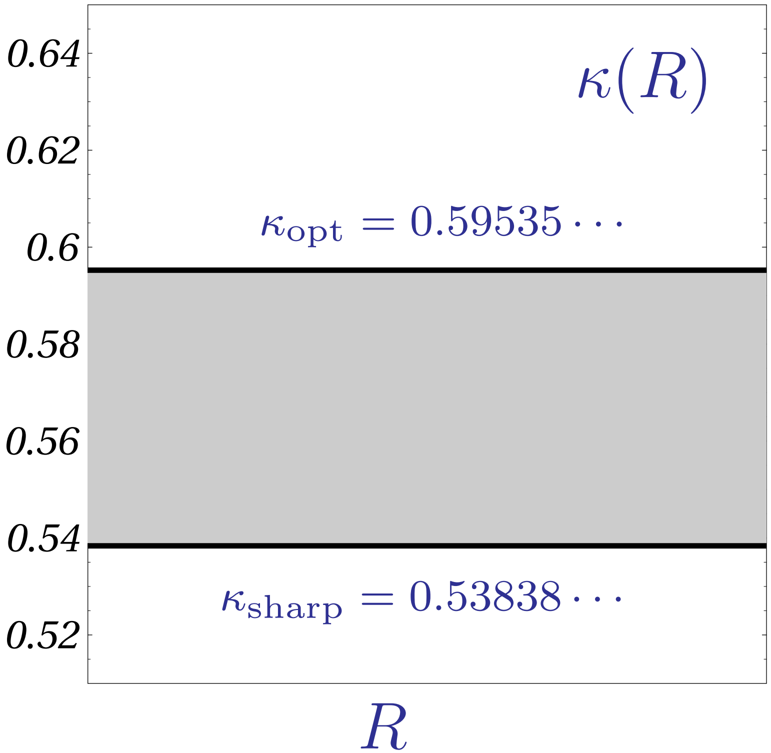
propagator flows in Landau gauge



optimisation

- LO is cutoff independent
- beyond LO: global extrema
- optimisation leads to $\kappa_{\rm opt}$

propagator flows in Landau gauge



optimisation

- LO is cutoff independent
- beyond LO: global extrema
- optimisation leads to κ_{opt}

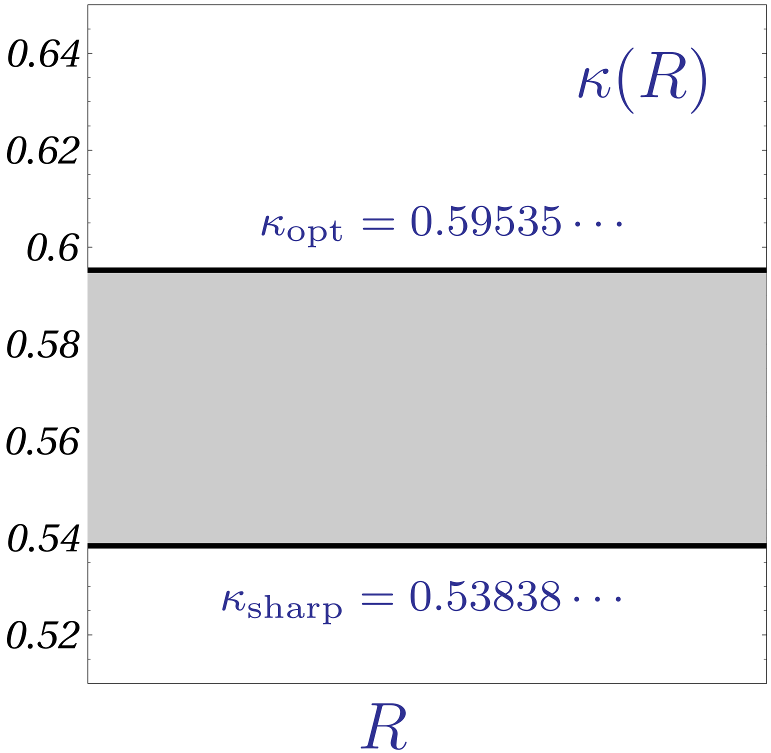
consistency

- DS result Lerche, Smekal '02

$$\kappa_{\mathrm{DS}} = 0.59535$$

$$\alpha_s = 2.9717$$

propagator flows in Landau gauge



optimisation

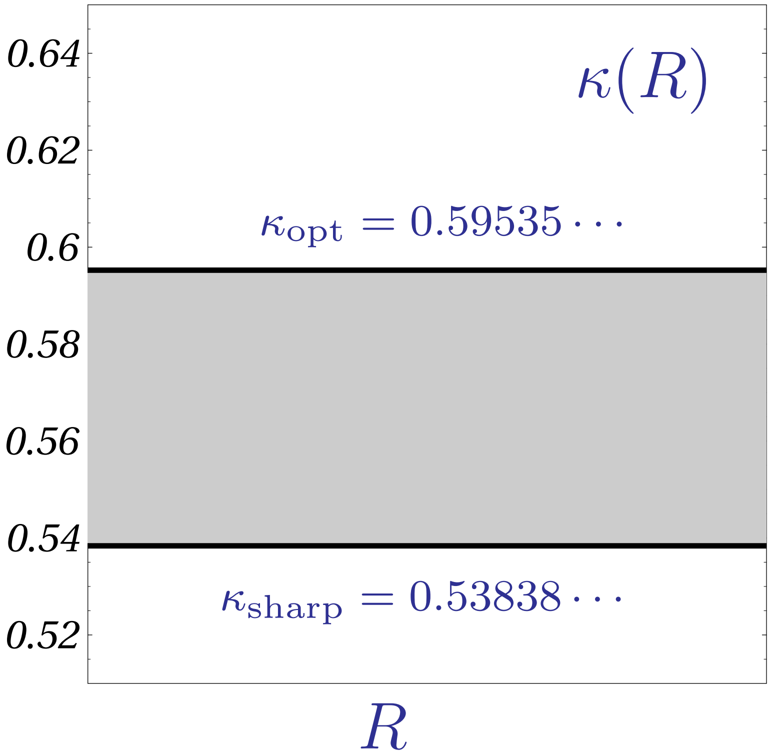
- LO is cutoff independent
- beyond LO: global extrema
- optimisation leads to $\kappa_{\rm opt}$

consistency

- $\kappa_{\rm opt}$ coincides with DS result
- IR running Fischer, Gies '04

$$\kappa = 0.51 - 0.52$$

propagator flows in Landau gauge



optimisation

- LO is cutoff independent
- beyond LO: global extrema
- optimisation leads to $\kappa_{\rm opt}$

consistency

- $\kappa_{\rm opt}$ coincides with DS result
- IR running

lower bound

- sharp cutoff $\Leftrightarrow$ finite volume scaling

conclusions and outlook

- **optimisation**

  minimum sensitivity for the flow

  cutoff dependence beneficial

  stable flows within truncations

  strongly reduced cutoff dependence, error control

  constructive procedure, analytical flows

  maximises physical content / truncation

conclusions and outlook

- **optimisation**

  minimum sensitivity for the flow

  - **cutoff dependence beneficial**
  - **stable flows within truncations**
  - **strongly reduced cutoff dependence, error control**
  - **constructive procedure, analytical flows**

  maximises physical content / truncation

- **scalar theories**

  LPA as benchmark test, very well understood

  - **clear link between optimisation, flow stability, convergence**
  - **optimisation entails 'global' PMS, whenever applicable**
  - **pattern persist to higher order**

conclusions and outlook

- **optimisation**

  minimum sensitivity for the flow

  cutoff dependence beneficial

  stable flows within truncations

  strongly **reduced** cutoff dependence, error control

  constructive procedure, analytical flows

  maximises physical content / truncation

- **scalar theories**

- **quantum gravity**

  fixed point searches: optimisation improves result

  stable valley of RG flows

  crucial for higher order computations

conclusions and outlook

- **optimisation**

  minimum sensitivity for the flow

  cutoff dependence beneficial

  stable flows within truncations

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  constructive procedure, analytical flows

  maximises physical content / truncation

- **scalar theories**
- **quantum gravity**
- **infrared QCD**

  scaling analysis: optimisation crucial

  extensions: mid-momentum regime $\longrightarrow$ talk by Pawlowski

conclusions and outlook

- **optimisation**

  minimum sensitivity for the flow

  cutoff dependence beneficial

  stable flows within truncations

  strongly **reduced** cutoff dependence, error control

  constructive procedure, analytical flows

  maximises physical content / truncation

- **scalar theories**
- **quantum gravity**
- **infrared QCD**
- **more to come ...**