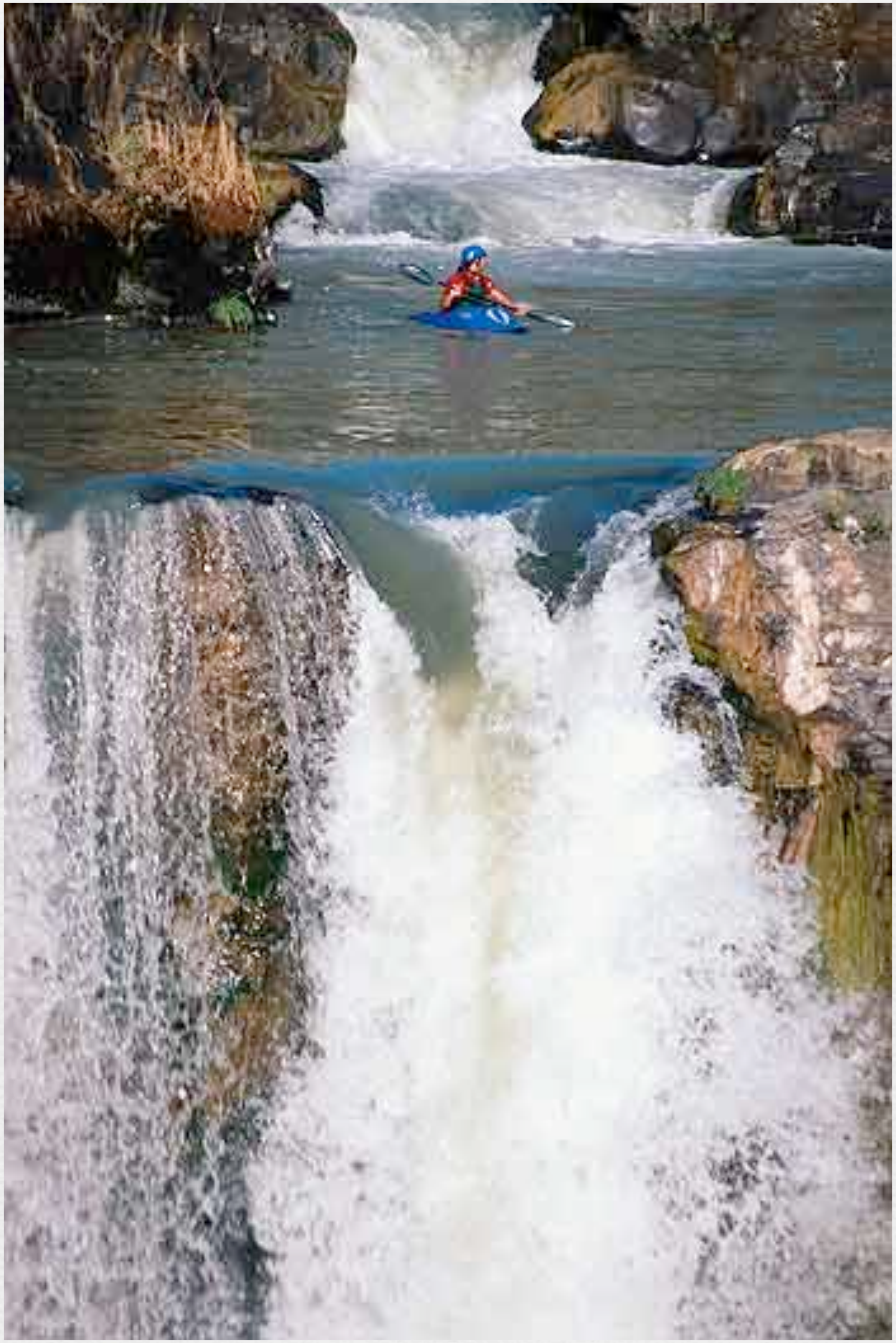
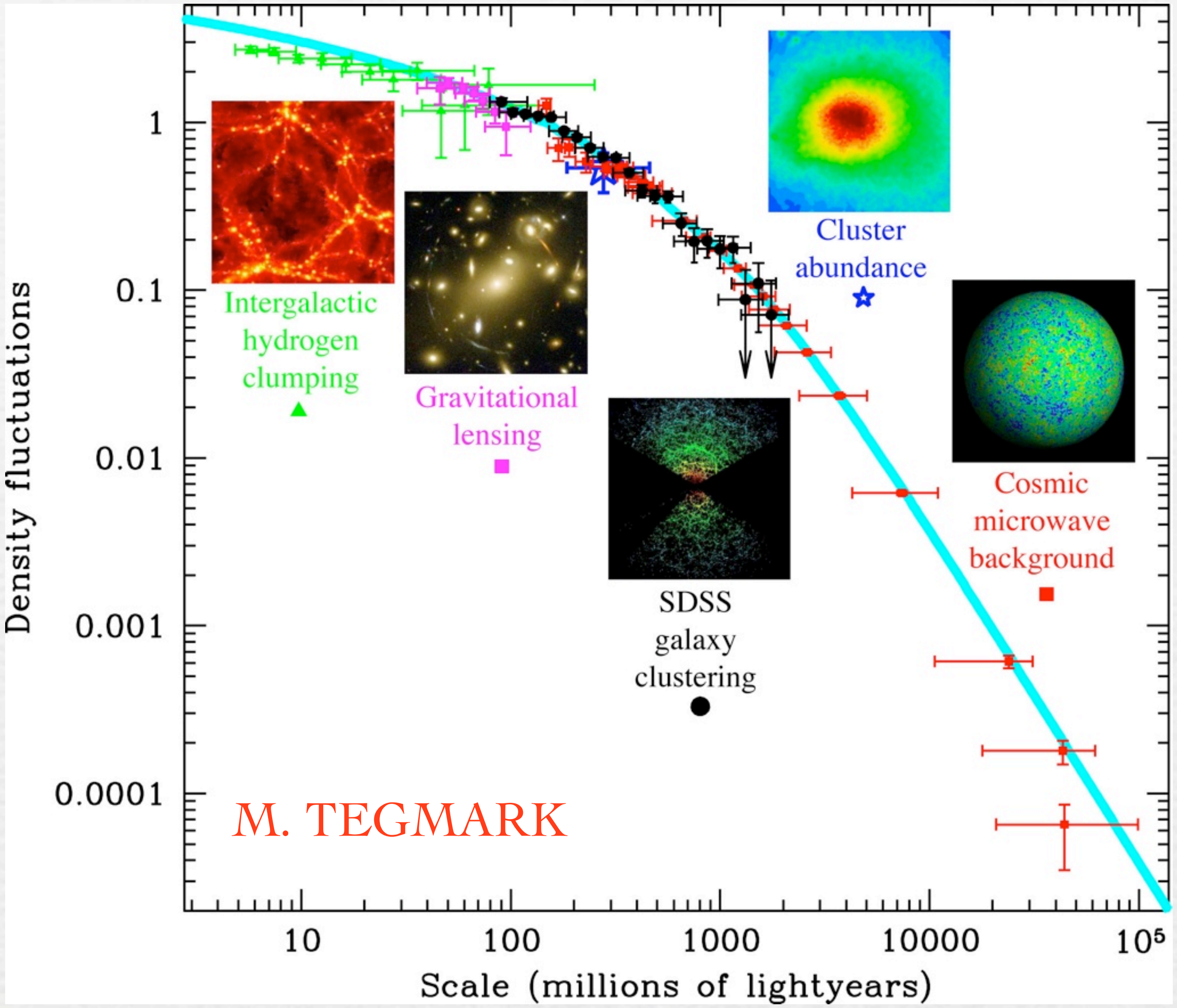
Flowing Towards...

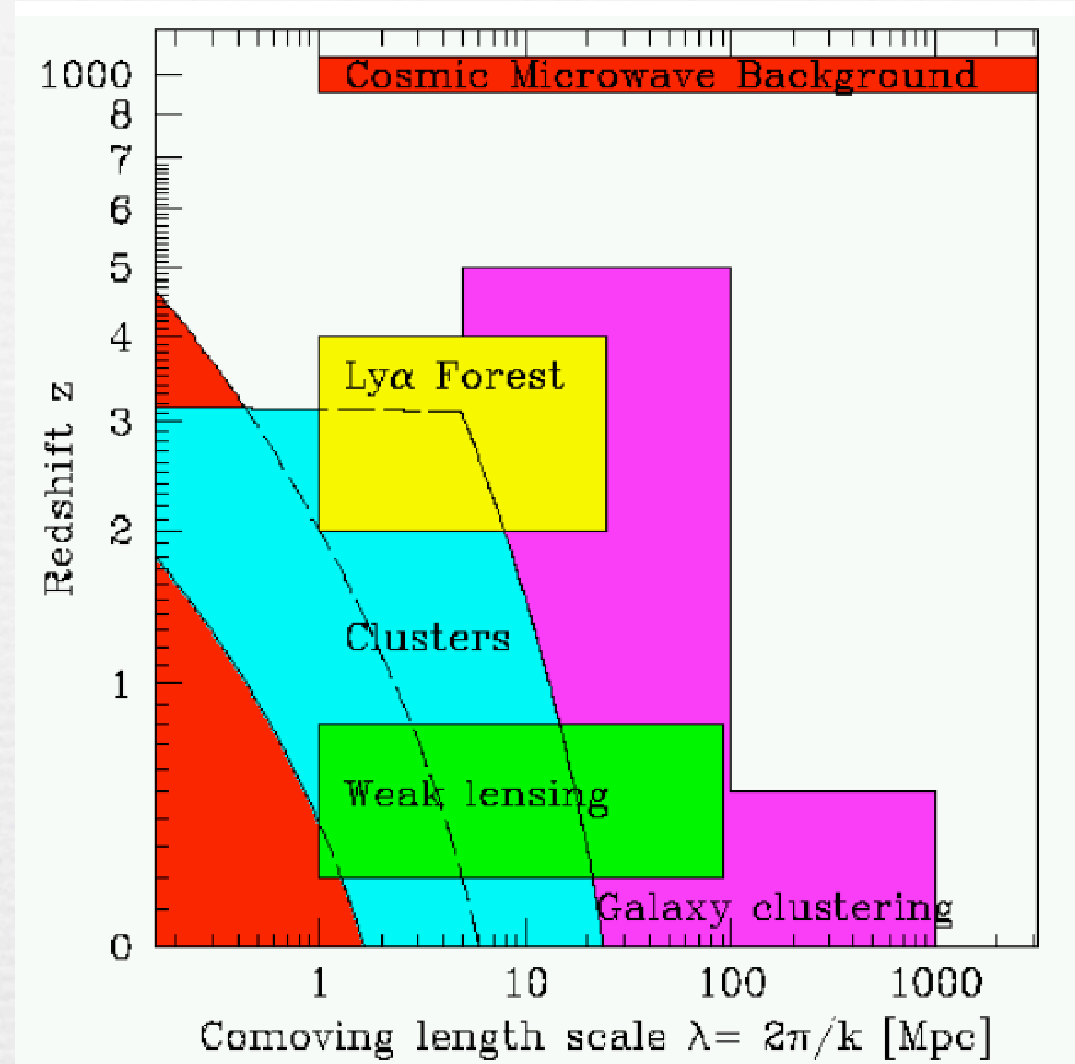


... Cosmological Nonlinearities

M. Pietroni – Infn Padova

Tests of the Perturbed FRW Universe





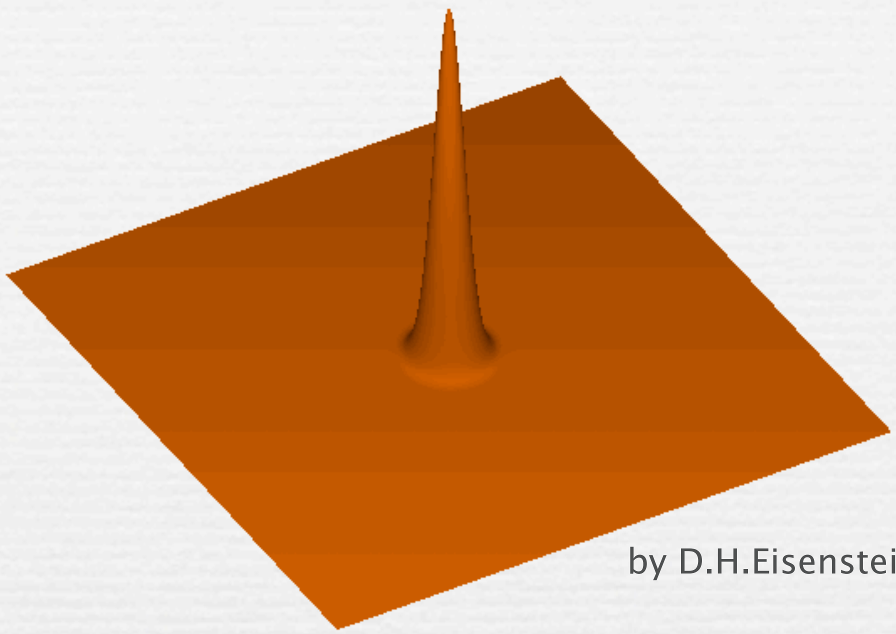
Matter density: $\rho(\mathbf{x},\,\tau) \equiv \bar{\rho}(\tau)[1+\delta(\mathbf{x},\,\tau)]$

Power spectrum: $\langle \delta(\mathbf{k},\,\tau)\,\delta(\mathbf{k}',\,\tau)\rangle = P(k,\,\tau)\delta^{(3)}(\mathbf{k}+\mathbf{k}')$

Clumpiness ~ $4\pi k^3 P(k) \sim a^2\, k^{3+n}$ (large scales, Einstein de Sitter)

perturbations get larger for smaller scales and larger times

A standard ruler: Baryonic Acoustic Oscillations



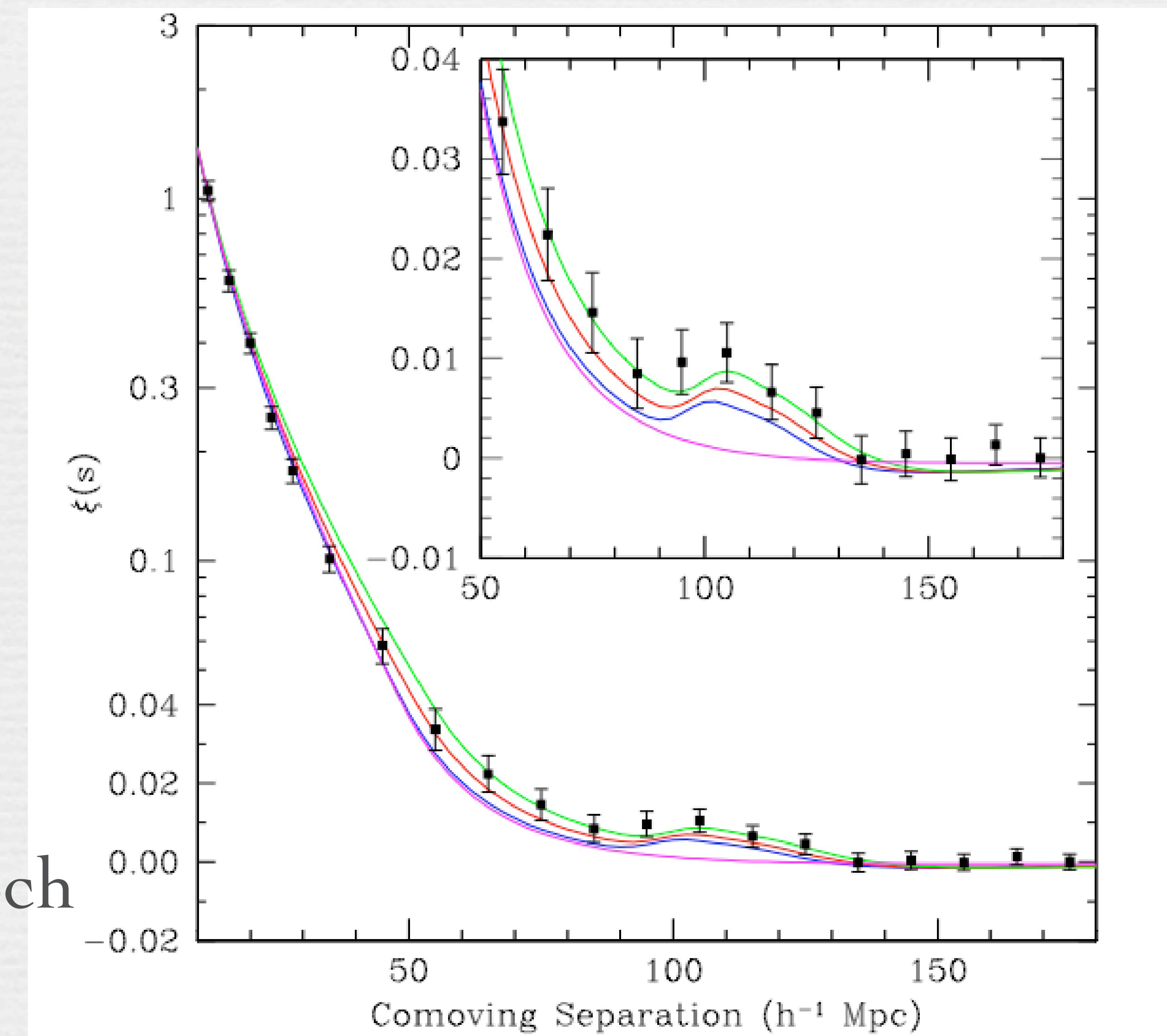
by D.H.Eisenstein

expect an excess probability of finding galaxies ~100 Mpc away from a given one

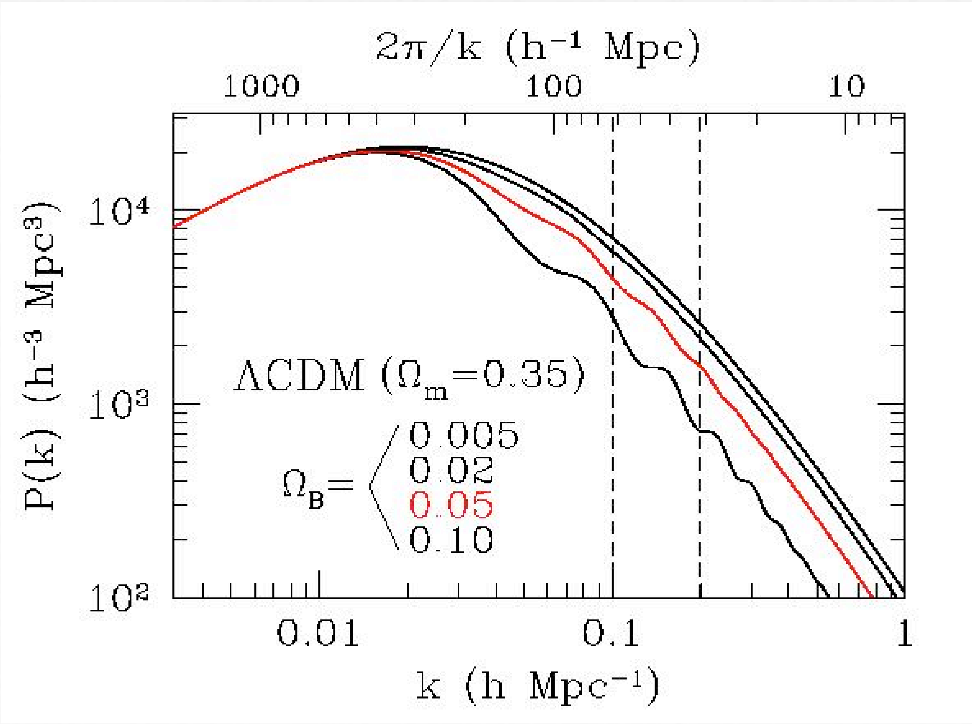
3.5 sigma evidence

Eisenstein et al. '05
Padmanabhan et al. 06
Blake et al. 06

The same scale `seen' in the CMB acoustic peaks, but at a much later epoch



Future surveys

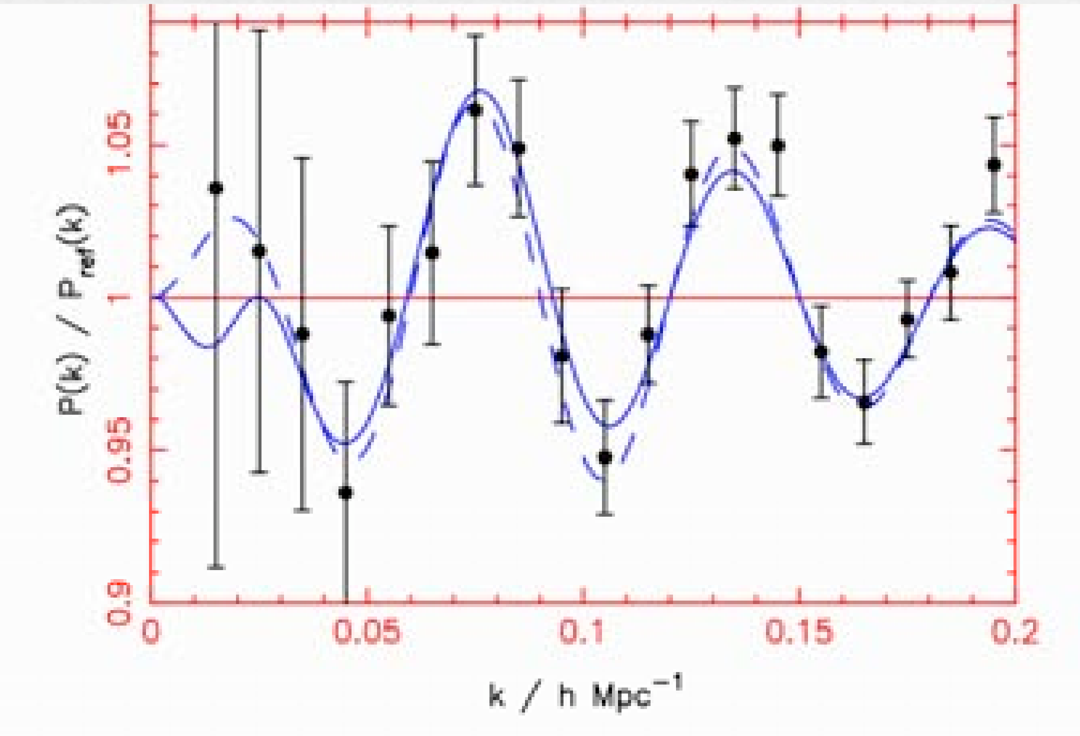


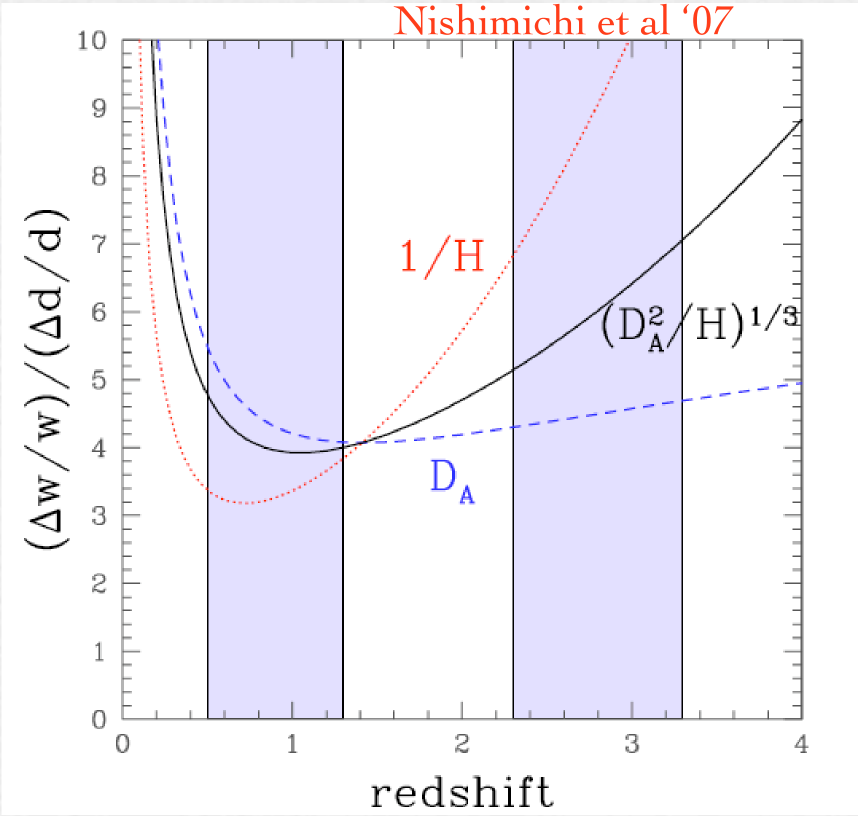
peaks are a small effect.
require large surveys to detect.

Ex.: BAO from WFMOS

(2M galaxies at 0.5<z<1)3)

P(k)/Pref(k)





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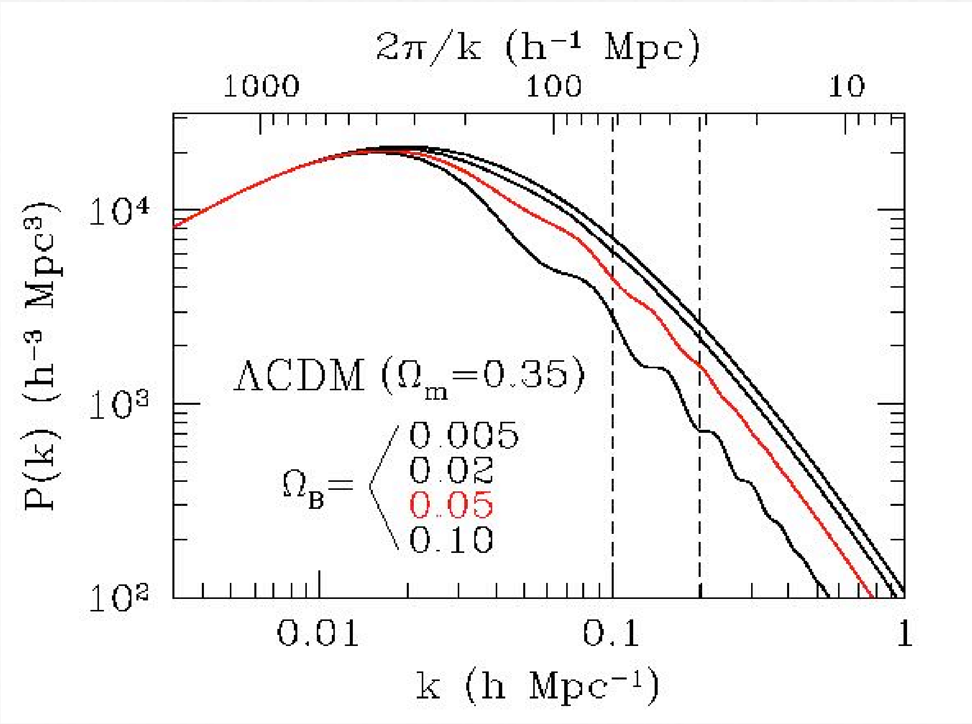

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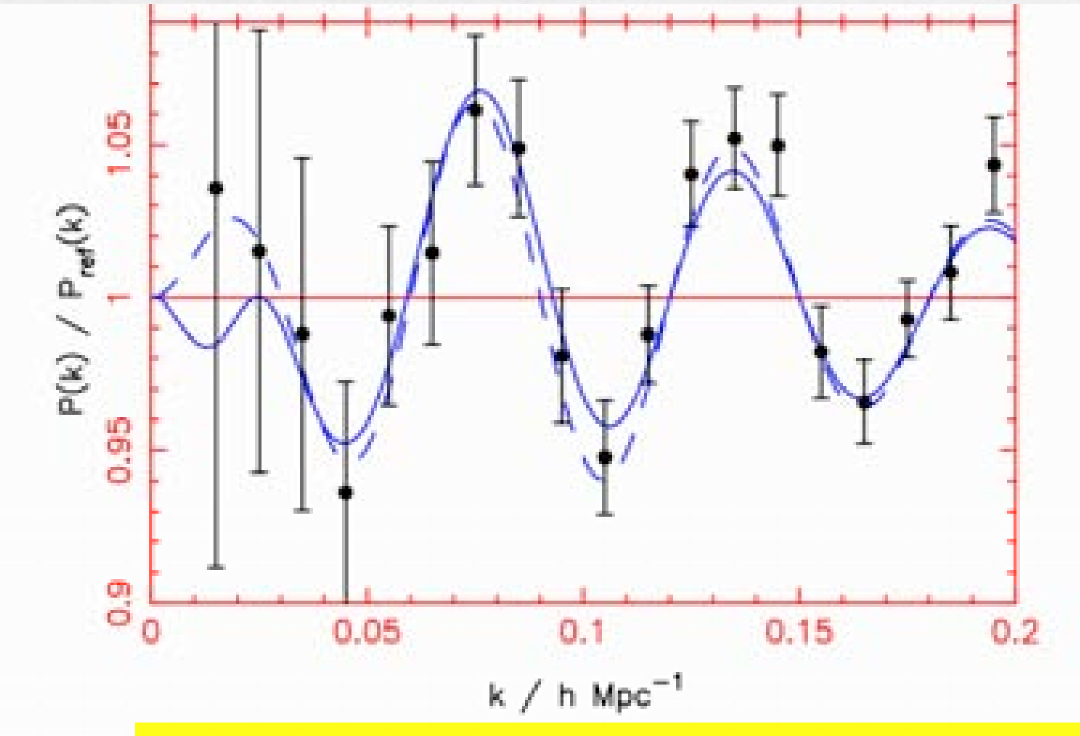

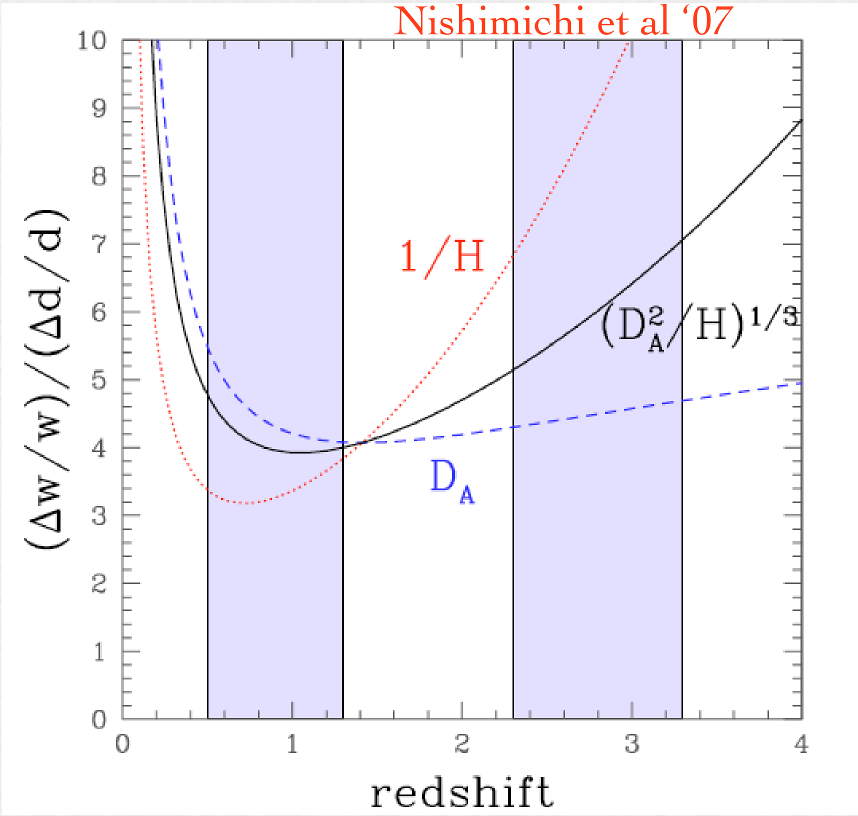

Goal: predict the LSS power spectrum to % accuracy

Goals

- Improve Pert. Theory towards lower z and higher k
- Study the effect of non-linearities on baryonic acoustic oscillations

Cold dust cosmology: equations of motion

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$

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In Fourier space, (defining $\theta(\mathbf{x},\tau) \equiv \nabla\cdot\mathbf{v}(\mathbf{x},\tau)$),

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) + \int d^3\mathbf{k_1}d^3\mathbf{k_2}\,\delta_D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2})\alpha(\mathbf{k_1},\mathbf{k_2})\theta(\mathbf{k_1},\tau)\delta(\mathbf{k_2},\tau) = 0$$

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mode-mode coupling controlled by:

$$\alpha(\mathbf{k_1},\mathbf{k_2}) \equiv \frac{(\mathbf{k_1}+\mathbf{k_2})\cdot\mathbf{k_1}}{k_1^2}$$

$$\beta(\mathbf{k_1},\mathbf{k_2}) \equiv \frac{|\mathbf{k_1}+\mathbf{k_2}|^2(\mathbf{k_1}\cdot\mathbf{k_2})}{2k_1^2k_2^2}$$

linear approximation: $\alpha(\mathbf{k_1}, \mathbf{k_2}) = \beta(\mathbf{k_1}, \mathbf{k_2}) = 0$

no mode-mode coupling

$$\frac{\partial\, \delta(\mathbf{k},\tau)}{\partial\, \tau} + \theta(\mathbf{k},\tau) = 0$$

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$$\Omega_M = 1 \rightarrow \mathcal{H} \sim a^{-1/2}$$

$$\downarrow$$

$$\begin{aligned} \delta(\mathbf{k},\, \tau) &= \delta(\mathbf{k},\, \tau_i) \left(\frac{a(\tau)}{a(\tau_i)} \right)^m \\ -\frac{\theta(\mathbf{k},\, \tau)}{\mathcal{H}} &= m\, \delta(\mathbf{k},\, \tau) \end{aligned} \qquad m = \begin{cases} 1 & \text{growing mode} \\ -\frac{3}{2} & \text{decaying mode} \end{cases}$$

Compact Perturbation Theory

Crocce, Scoccimarro '05

Consider again the (non-linear) continuity and Euler equations

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0 \,, \qquad \frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \phi \,,$$

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define $\left(\begin{array}{c} \varphi_1(\eta,\mathbf{k}) \\ \varphi_2(\eta,\mathbf{k}) \end{array}\right) \equiv e^{-\eta}\left(\begin{array}{c} \delta(\eta\,,\mathbf{k}) \\ -\theta(\eta\,,\mathbf{k})/\mathcal{H} \end{array}\right)$ with $\eta = \log\dfrac{a(\tau)}{a(\tau_i)}$ $\quad \mathbf{\Omega} = \left(\begin{array}{cc} 1 & -1 \\ -3/2 & 3/2 \end{array}\right)$

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then we can write (we assume an EdS model):

$$\left(\delta_{ab}\partial_\eta + \Omega_{ab}\right)\varphi_b(\eta,\mathbf{k}) = e^{\eta}\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

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and the only non-zero components of the mode-mode coupling are

$$\gamma_{121}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \gamma_{112}(\mathbf{k_1},\,\mathbf{k_3},\,\mathbf{k_2}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\frac{\alpha(\mathbf{k_2},\,\mathbf{k_3})}{2}$$

$$\gamma_{222}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\beta(\mathbf{k_2},\,\mathbf{k_3})$$

An action principle

Matarrese, M.P., '07

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The field equation can be derived by varying the action

$$S = \int d\eta_a d\eta_b \, \chi_a \, g^{-1}_{ab} \, \varphi_b - \int d\eta \, e^{\eta} \, \gamma_{abc} \, \chi_a \, \varphi_b \, \varphi_c$$

w.r.t. the auxiliary field $\chi_a(\eta, \mathbf{k})$.

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$g_{ab}(\eta_1, \eta_2)$ is the retarded propagator: $(\delta_{ab}\partial_\eta + \Omega_{ab}) \, g_{bc}(\eta, \eta') = \delta_{ac} \, \delta_D(\eta - \eta')$

so that $\varphi^0_a(\eta, \mathbf{k}) = g_{ab}(\eta, \eta')\varphi^0_b(\eta', \mathbf{k})$ is the solution of the ***linear*** equation

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Explicitly, one finds: $\mathbf{g}(\eta_1, \eta_2) = \begin{cases} \mathbf{B} + \mathbf{A}\, e^{-5/2(\eta_1 - \eta_2)} & \eta_1 > \eta_2 \\ 0 & \eta_1 < \eta_2 \end{cases}$

$$\mathbf{B} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix}$$

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growing mode

Initial conditions: $\varphi^0_b(\eta', \mathbf{k}) \propto u_b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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The probability of the final configuration $\varphi_a(\eta_f)$, given the initial condition $\varphi_a(\eta_i)$, is

$$P[\varphi(\eta_f);\varphi(\eta_i)] = \delta\left[\varphi(\eta_f) - \overline{\varphi}[\eta_f;\varphi(\eta_i)]\right]$$

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fixed extrema

$$S[\varphi,\,\chi] = \int_{\eta_i}^{\eta_f} d\eta\,\chi_a\left[(\delta_{ab}\partial_\eta + \Omega_{ab})\varphi_b - e^\eta\gamma_{abc}\varphi_b\varphi_c\right]$$

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The generating functional ***at fixed initial conditions*** is

$$Z[J,\Lambda;\varphi(\eta_i)] = \int \mathcal{D}\varphi(\eta_f)\,\mathcal{D}''\varphi\,\mathcal{D}\chi\; e^{iS[\varphi,\,\chi]+i\int_{\eta_i}^{\eta_f} d\eta(J_a\varphi_a+\Lambda_b\chi_b)}$$

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$$W[\varphi(\eta_i);K's] = \exp\left\{-\frac{1}{2}\varphi_a(\eta_i;\mathbf{k_a})K_{ab}(\mathbf{k_a},\mathbf{k_b})\varphi_b(\eta_i;\mathbf{k_b}) + \cdots\right\}$$

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Putting all together...

$$Z[\mathbf{J},\,\mathbf{\Lambda}] = \int \mathcal{D}\varphi\,\mathcal{D}\chi \exp\left\{\int d\eta_1 d\eta_2 \left[-\frac{1}{2}\chi\,\mathbf{g}^{-1}\mathbf{P^L}\mathbf{g^T}^{-1}\chi + i\,\chi\,\mathbf{g}^{-1}\varphi\right] - i\int d\eta\left[\mathbf{e}^{\eta}\gamma\,\chi\varphi\varphi - \mathbf{J}\varphi - \mathbf{\Lambda}\chi\right]\right\}$$

where the initial conditions are encoded in the linear power spectrum: $P^L_{ab}(\eta,\,\eta';\,\mathbf{k}) \equiv \left(\mathbf{g}(\eta)\mathbf{P^0}(\mathbf{k})\mathbf{g}^T(\eta')\right)_{ab}$

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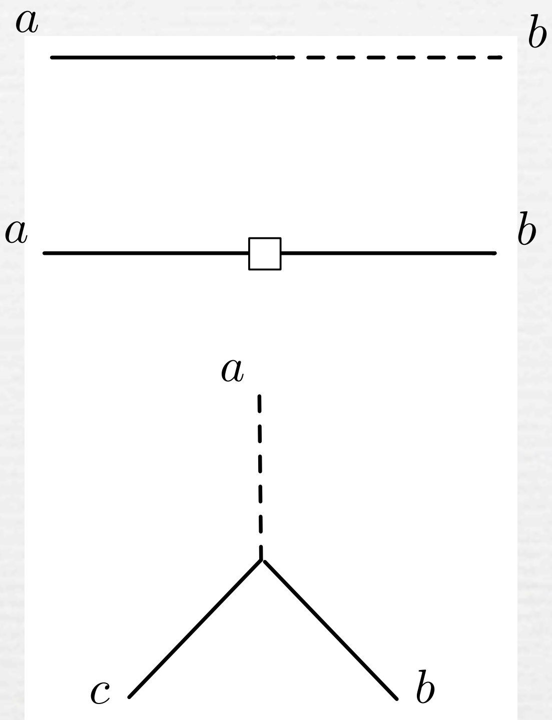
Derivatives of Z w.r.t. the sources J and Λ give all the n-point correlation functions (power spectrum, bispectrum, ...) and the full non-linear propagator

Perturbation Theory: Feynman Rules

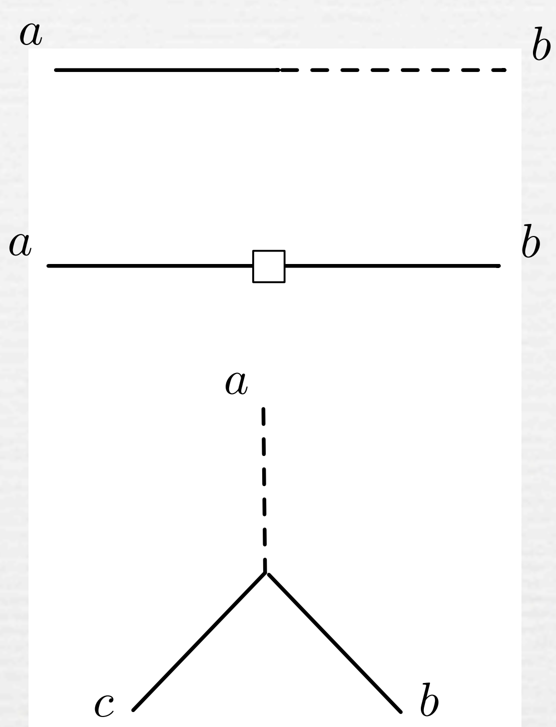


propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$

interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

Perturbation Theory: Feynman Rules

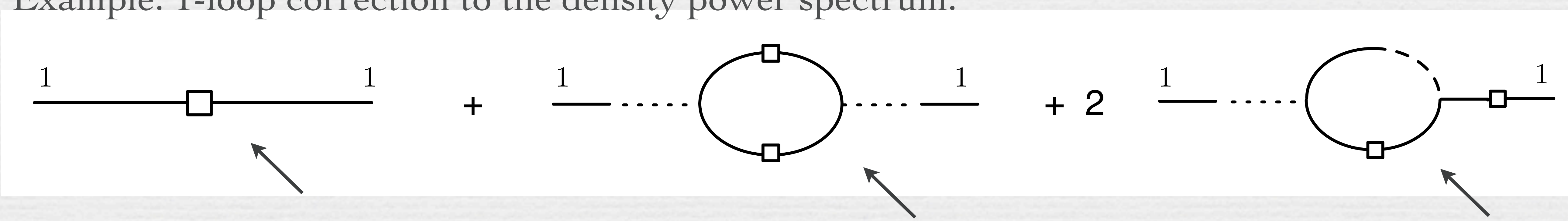


propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$

interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

Example: 1-loop correction to the density power spectrum:

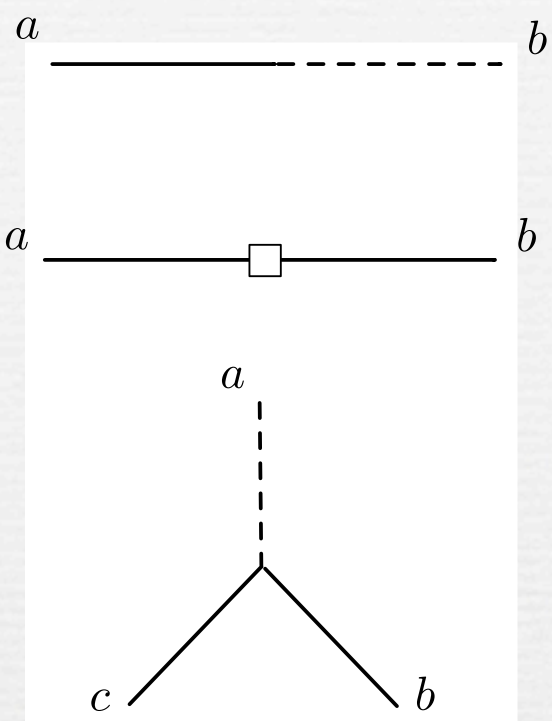


Linear Power spectrum

“P22”

“P13”

Perturbation Theory: Feynman Rules



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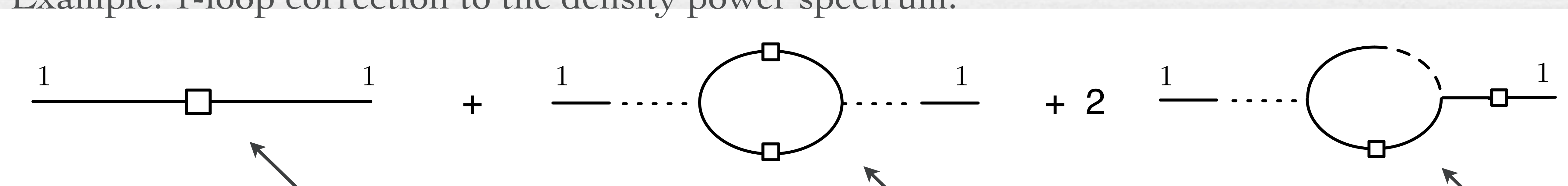

Linear Power spectrum

"P22"

"P13"

All known results in cosmological perturbation theory are expressible in terms of diagrams in which <u>only a trilinear fundamental interaction</u> appears

The exact (not the Exact®!) Renormalization Group

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Inspired by applications of Wilsonian RG to field theory: the RG parameter is momentum

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then, plug it into the generating functional: $Z[\mathbf{J},\,\mathbf{\Lambda}] \longrightarrow Z_\lambda[\mathbf{J},\,\mathbf{\Lambda}]$

$$Z_\lambda[\mathbf{J},\,\mathbf{\Lambda}] = \int \mathcal{D}\varphi\,\mathcal{D}\chi\,\exp\left\{\int d\eta_1 d\eta_2 \left[-\frac{1}{2}\,\chi\,\mathbf{g}^{-1}\mathbf{P}^{\mathbf{L}}_\lambda\mathbf{g}^{\mathbf{T}^{-1}}\chi + i\,\chi\,\mathbf{g}^{-1}\,\varphi\right] - i\int d\eta\,[\mathbf{e}^\eta\gamma\,\chi\varphi\varphi - \mathbf{J}\varphi - \mathbf{\Lambda}\chi]\right\}$$

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The evolution from $\lambda = 0$ to $\lambda = \infty$ can be described non-perturbatively by RG equations:

$$\boxed{\frac{\partial}{\partial\lambda} Z_\lambda = \frac{1}{2}\int d^3q\,\delta(\lambda - q)\,P^0_{ab}(q)\,\frac{\delta^2 Z_\lambda}{\delta\Lambda_b(q)\delta\Lambda_a(-q)}}$$

with $Z_{\lambda=0} = Z_{\mathrm{lin.\,th.}}$

The propagator

$$G_{\lambda,ab}(k;\,\eta_a,\,\eta_b) = -\frac{\delta^2 W_\lambda[J,\,\Lambda]}{\delta J_a(\mathbf{k},\,\eta_a)\delta\Lambda_b(-\mathbf{k},\,\eta_b)}$$

$$(W_\lambda = -i\log Z_\lambda)$$

$$\boxed{\frac{\partial}{\partial\lambda}\,\frac{\delta^2 W_\lambda}{\delta J_a\,\delta\Lambda_b} = \frac{1}{2}\int d^3q\,\delta(\lambda-q)\,P^0_{cd}(q)\,\frac{\delta^4 W_\lambda}{\delta J_a\,\delta\Lambda_b\,\delta\Lambda_c\,\delta\Lambda_d}}$$

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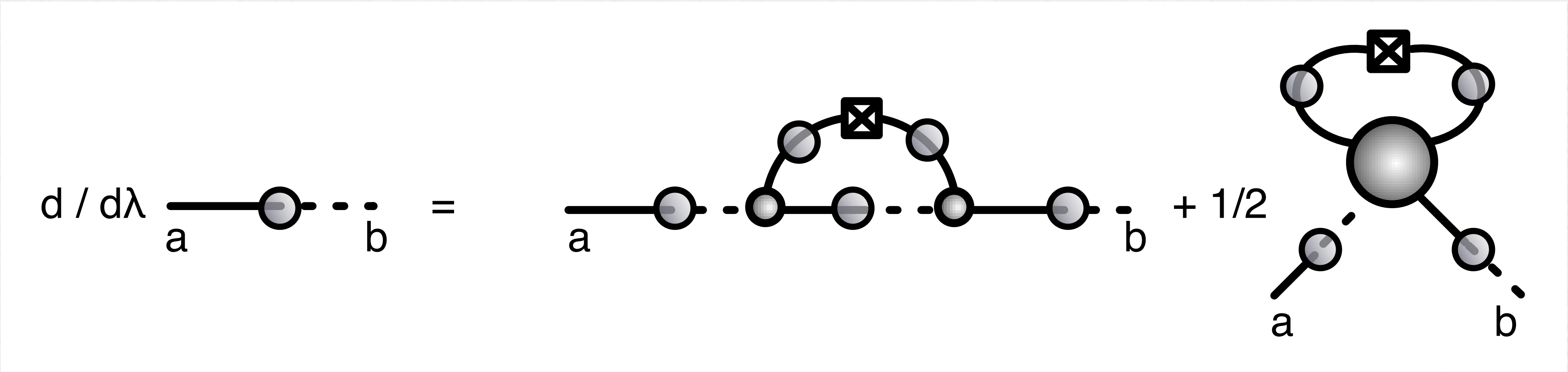
in pictures...



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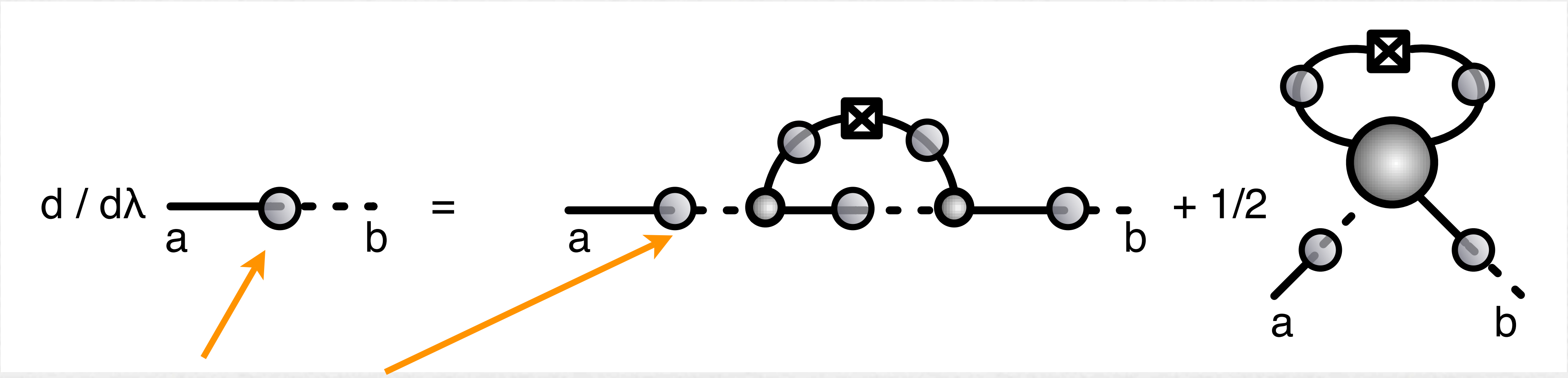

full non-linear propagators

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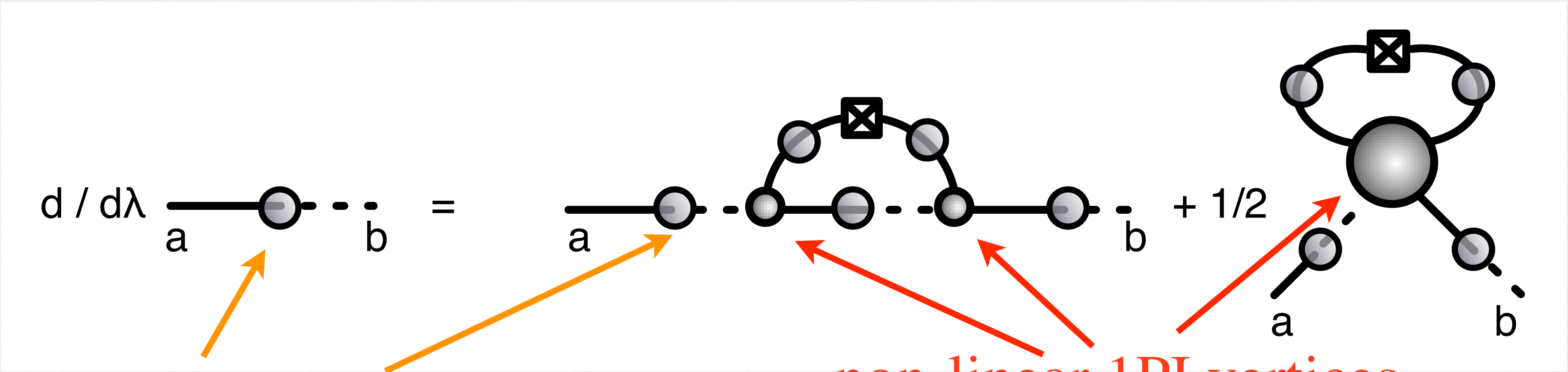

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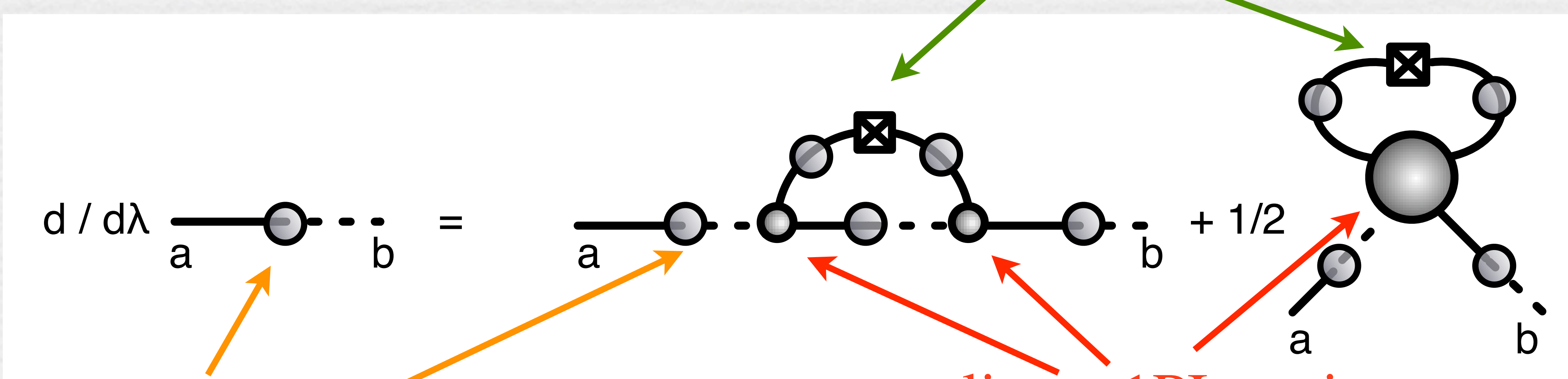
RG Kernel

in pictures...



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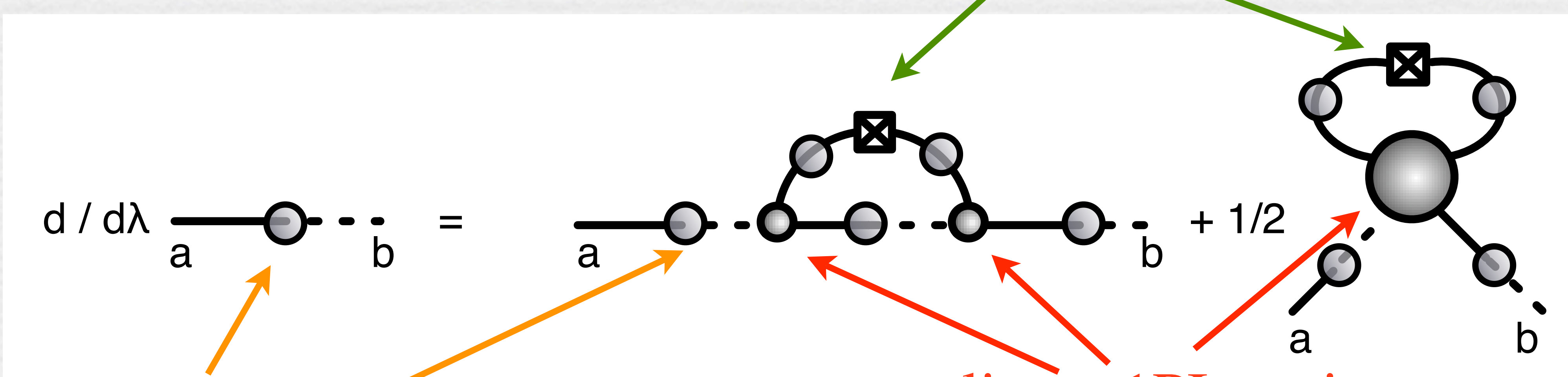

full non-linear propagators

non-linear 1PI vertices

[formally 1-loop, but exact](#)

[infinite tower of RGE's](#)

I: The propagator in Perturbation Theory

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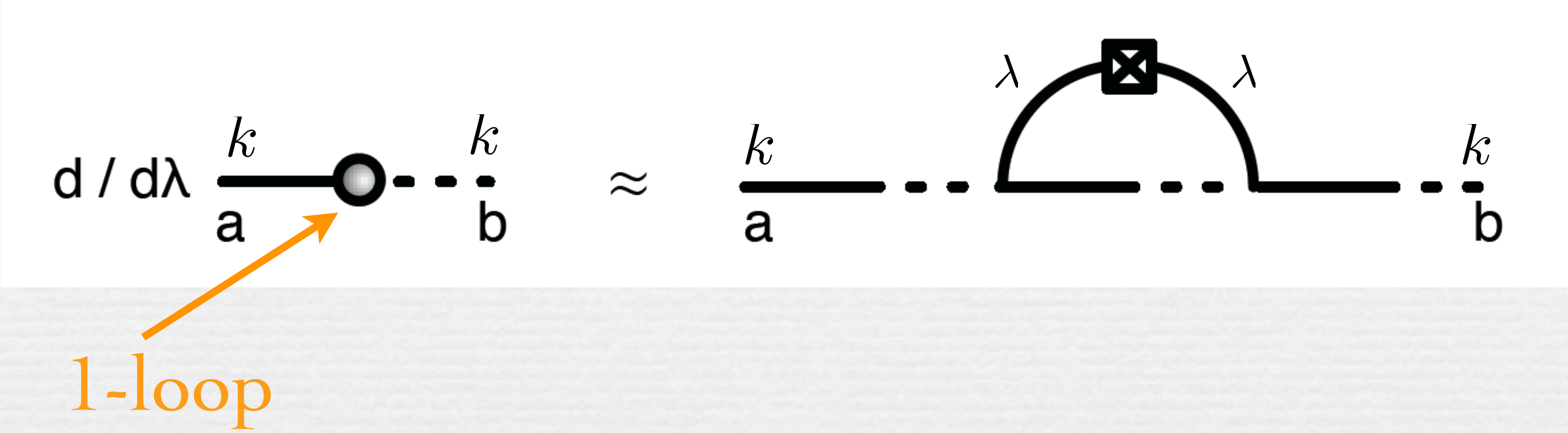
1-loop PT is recovered by taking tree-level propagators and vertices at the RHS:



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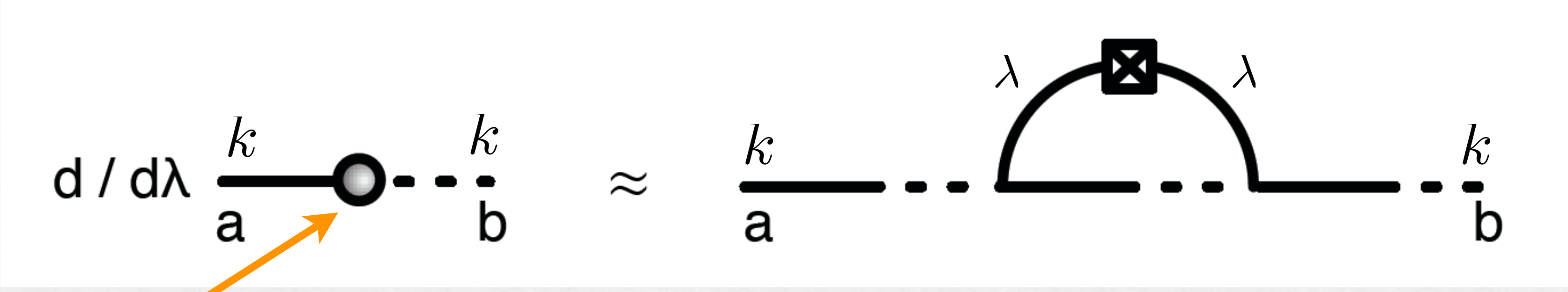

1-loop

in the k>>λ limit: $\partial_\lambda G_{ab\,,\lambda}(k;\,\eta_a,\,\eta_b) = -g_{ab}(\eta_a,\,\eta_b)\,\frac{k^2}{3}\,\frac{\left(e^{\eta_a}-e^{\eta_b}\right)^2}{2}\int d^3q\,\delta(\lambda-q)\,\frac{P^0(q)}{q^2}$

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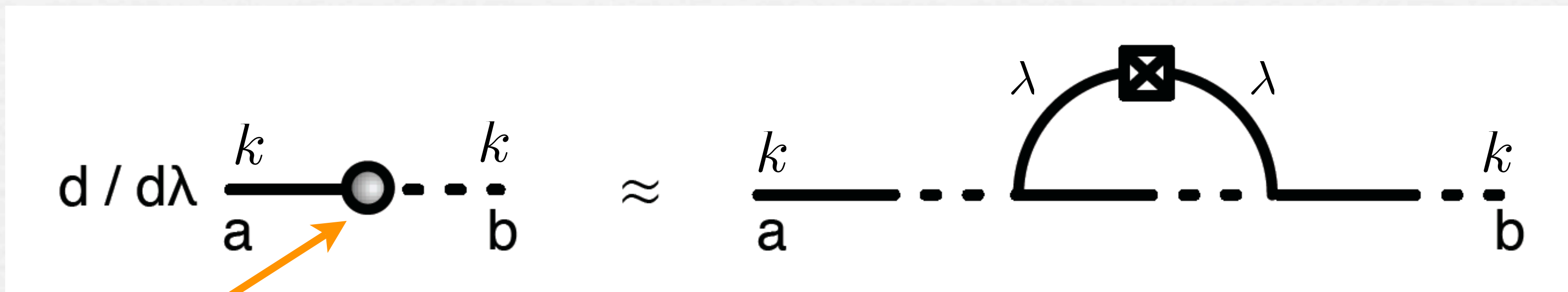

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$$G_{ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)\left[1 - k^2\sigma^2\,\frac{(e^{\eta_a}-e^{\eta_b})^2}{2}\right] + O(k^4\sigma^4)$$

2-loop

$$\left(\sigma^2 \equiv \frac{1}{3}\int d^3q\,\frac{P^0(q)}{q^2}\right)$$

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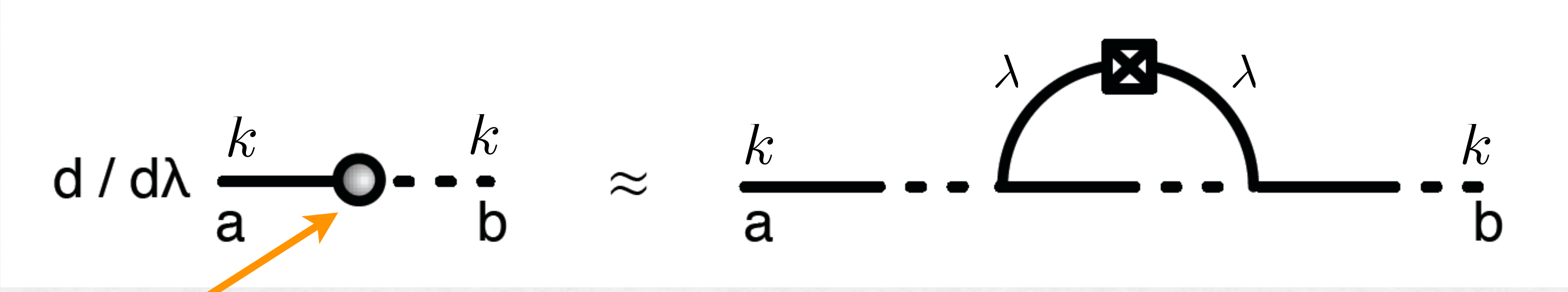

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$\left(\sigma\,e^{\eta_a}\right)^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

in the BAO range!

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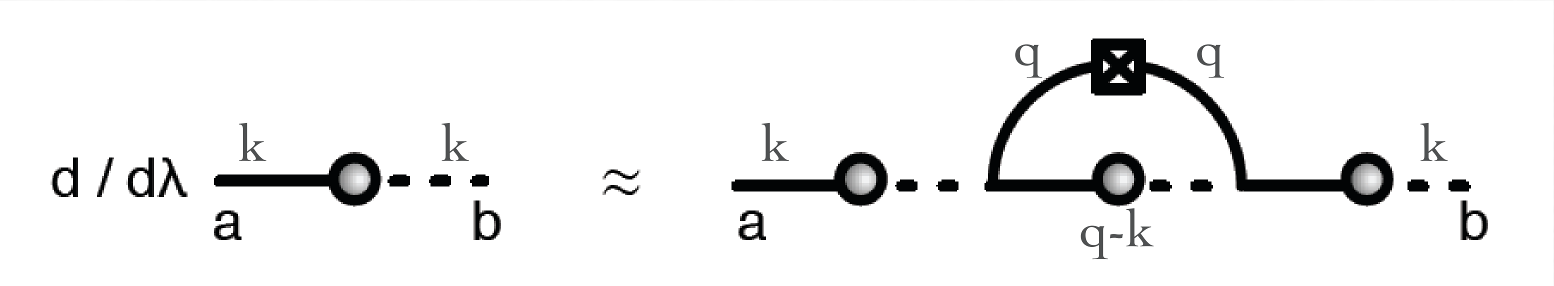
take running 'hard' (k>>λ) propagators and tree-level vertices at the RHS:



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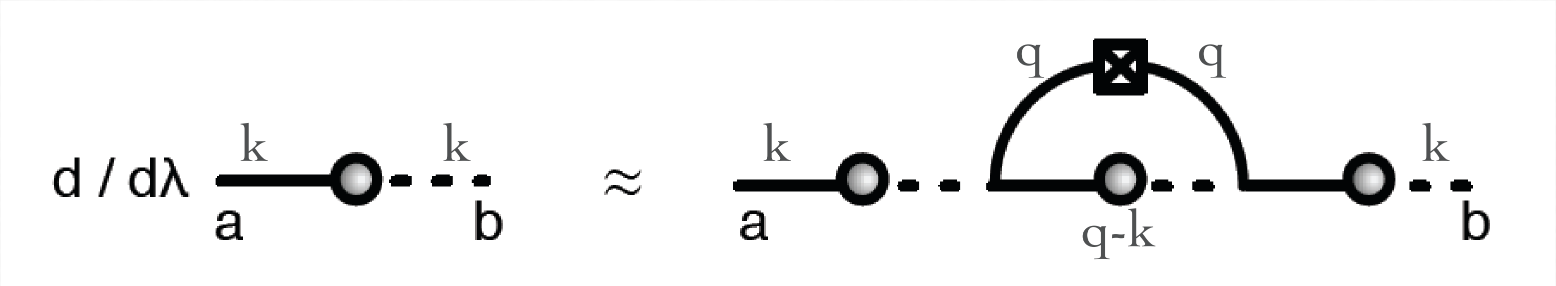

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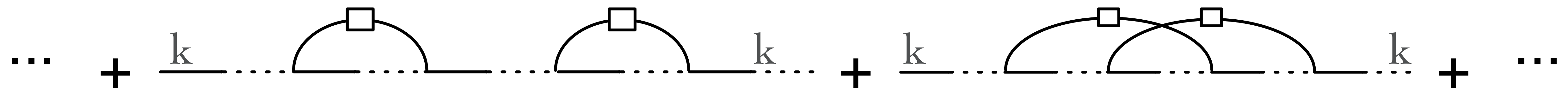
resummed propagator

this result was first obtained (Crocce Scoccimarro, '06) by resumming the infinite series of chain diagrams:



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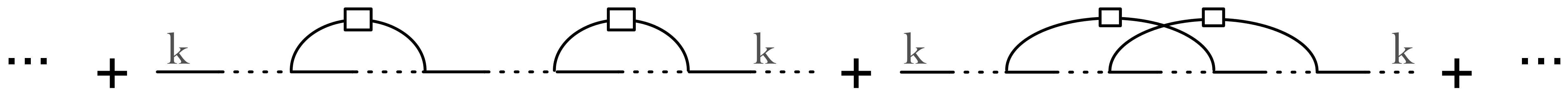

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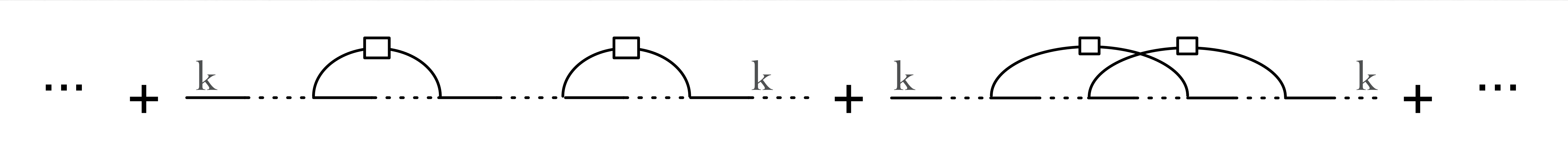

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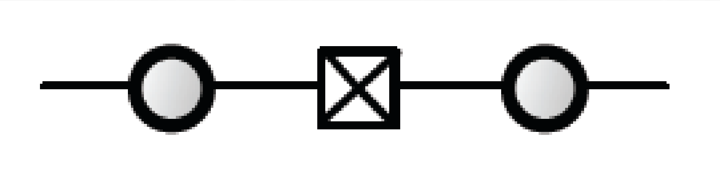
`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

damping in the BAO range!

A self-generated UV cutoff

A self-generated UV cutoff

Inserting the approximated expression for the propagators in the RG kernel, we get:



$$\delta(\lambda - q)\, P^0_{cd}(q)\, \exp\left[-\frac{q^2\sigma^2_\lambda}{2}\left((e^{\eta_c}-1)^2 + (e^{\eta_d}-1)^2\right)\right]$$

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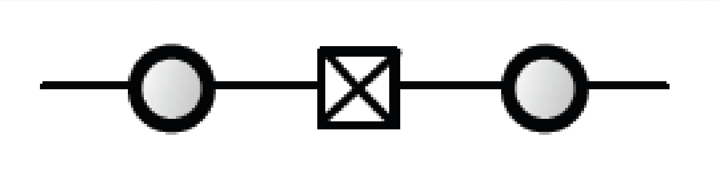

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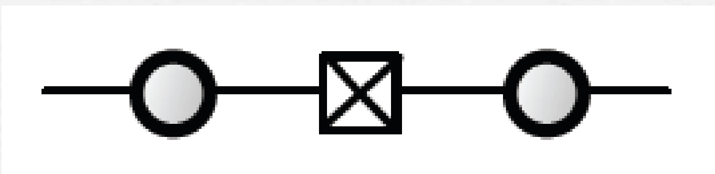

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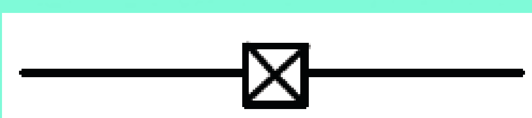
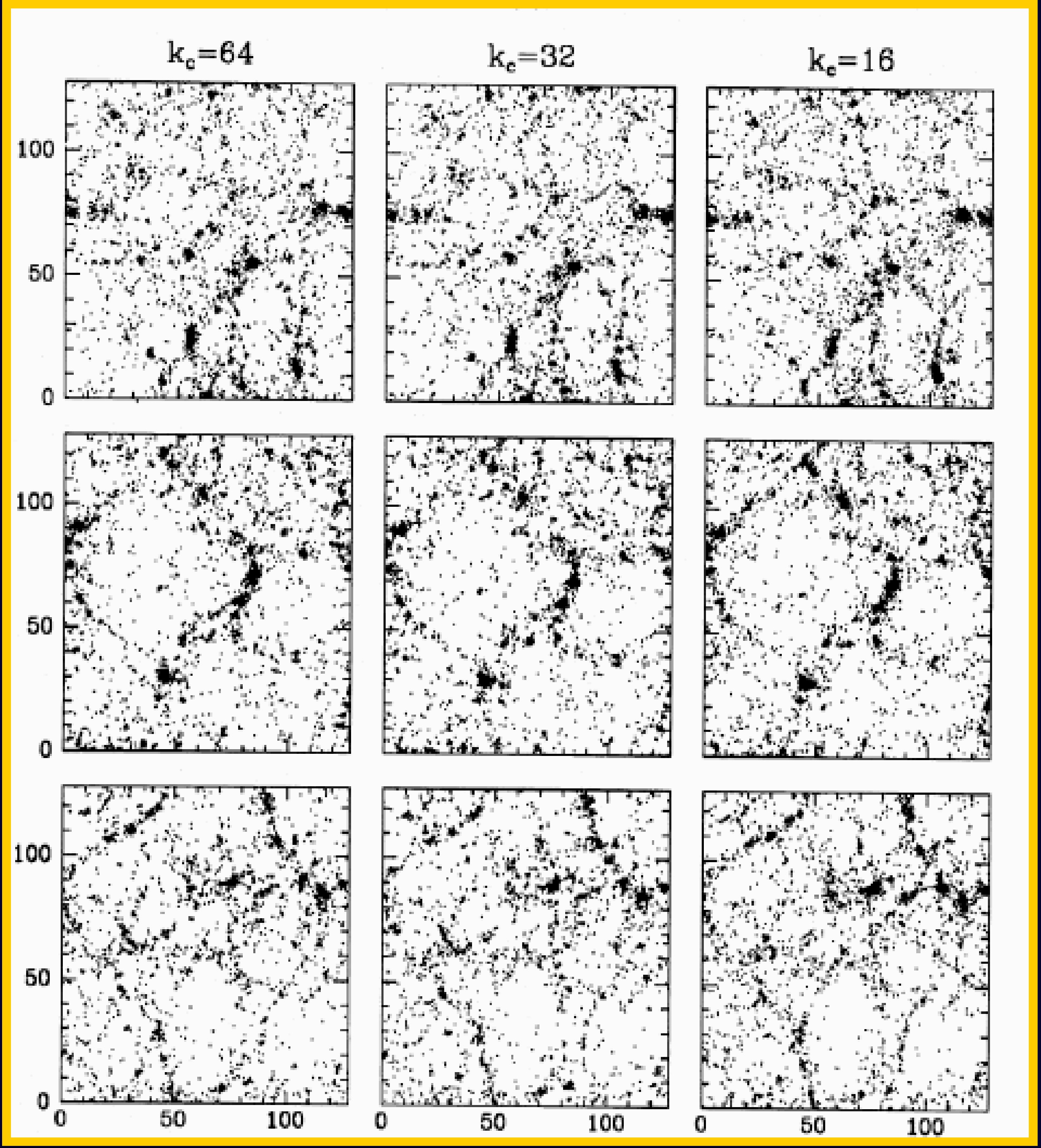
The UV is much better behaved than one would guess from `usual' perturbation theory ( $\rightarrow \delta(\lambda - q) P^0_{cd}(q)$) !!

Self-generated UV cutoff

- Same result obtained by Crocce & Scoccimarro 2005 and already noticed in N-body simulations
- The high-frequency modes of the initial (linear) power-spectrum are truncated down to the present-day non-linearity scale. The final (z=0) outputs are left almost unchanged.



Little, Weinberg & Park 1991

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with $$P^{I}_{ab}(k;\eta_a,\eta_b) = G_{ac}(k;\eta_a,0)G_{bd}(k;\eta_b,0)P^{0}_{cd}(k)$$

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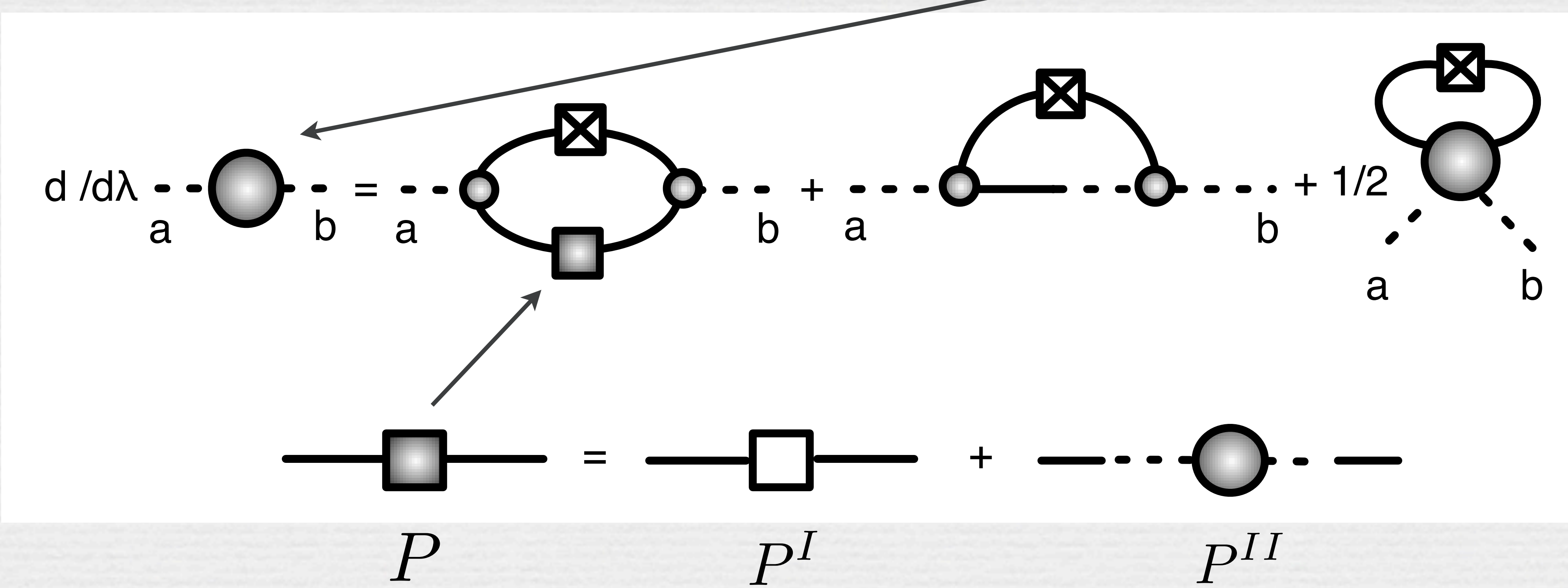
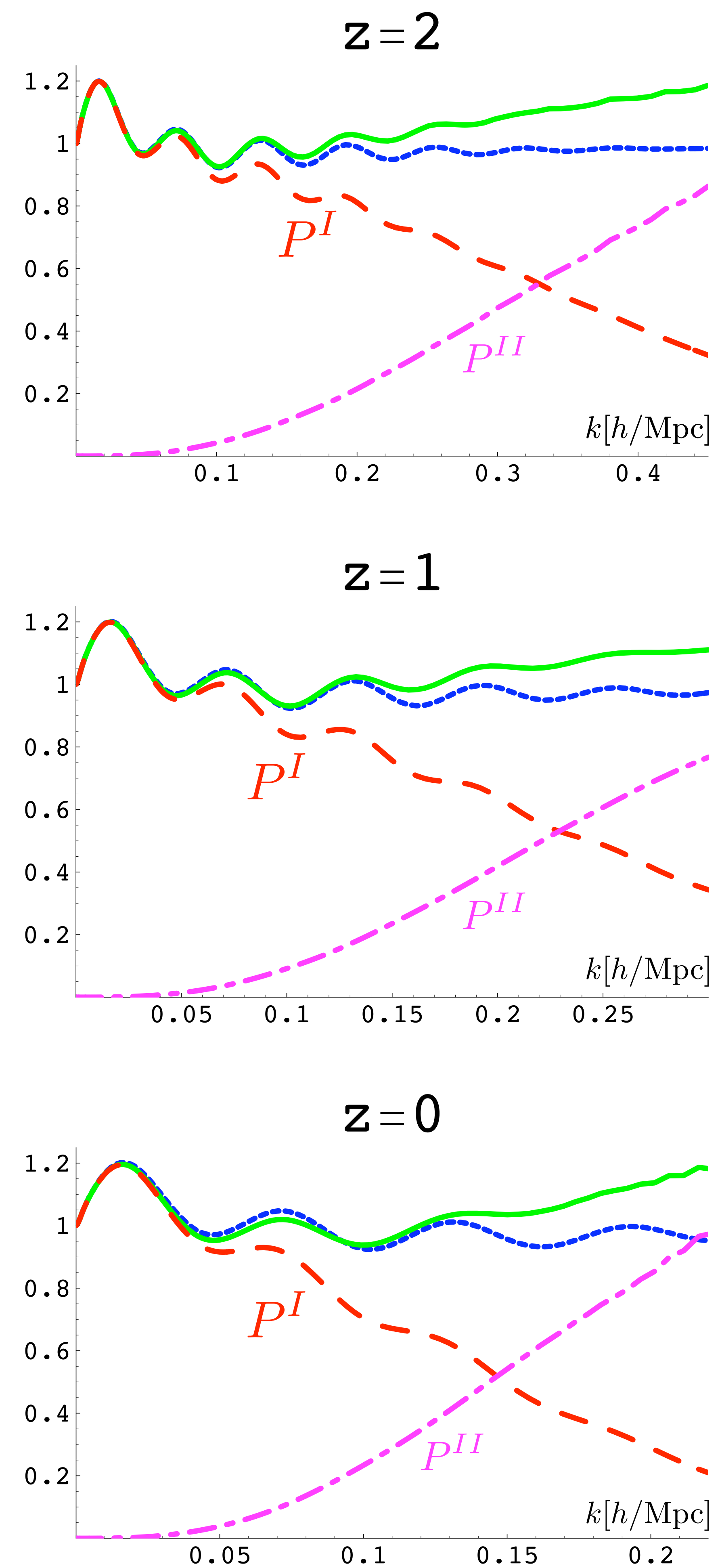


again, tree-level vertices...

$$\partial_\lambda \Phi_{ab,\,\lambda}(k;\, s_1,\, s_2) = 4\, e^{s_1+s_2} \int d^3\mathbf{q}\, \delta(\lambda - q) P^I_{dc,\lambda}(q;\, s_1,\, s_2) \times$$
$$P_{fe,\lambda}(|\mathbf{q}-\mathbf{k}|;\, s_1,\, s_2)\, \gamma_{adf}(\mathbf{k}, -\mathbf{q}, -\mathbf{k}+\mathbf{q}) \gamma_{bce}(-\mathbf{k}, \mathbf{q}, \mathbf{k}-\mathbf{q})$$



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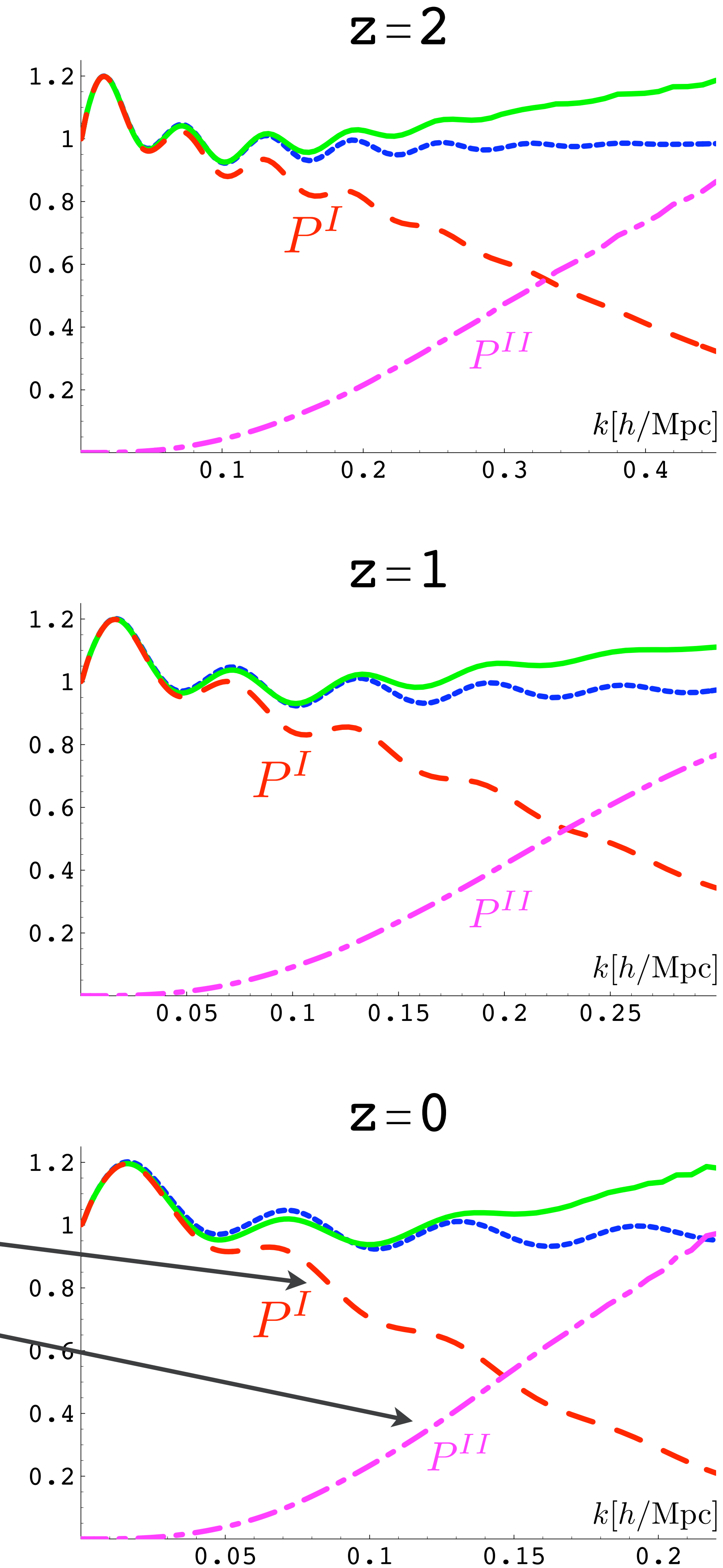
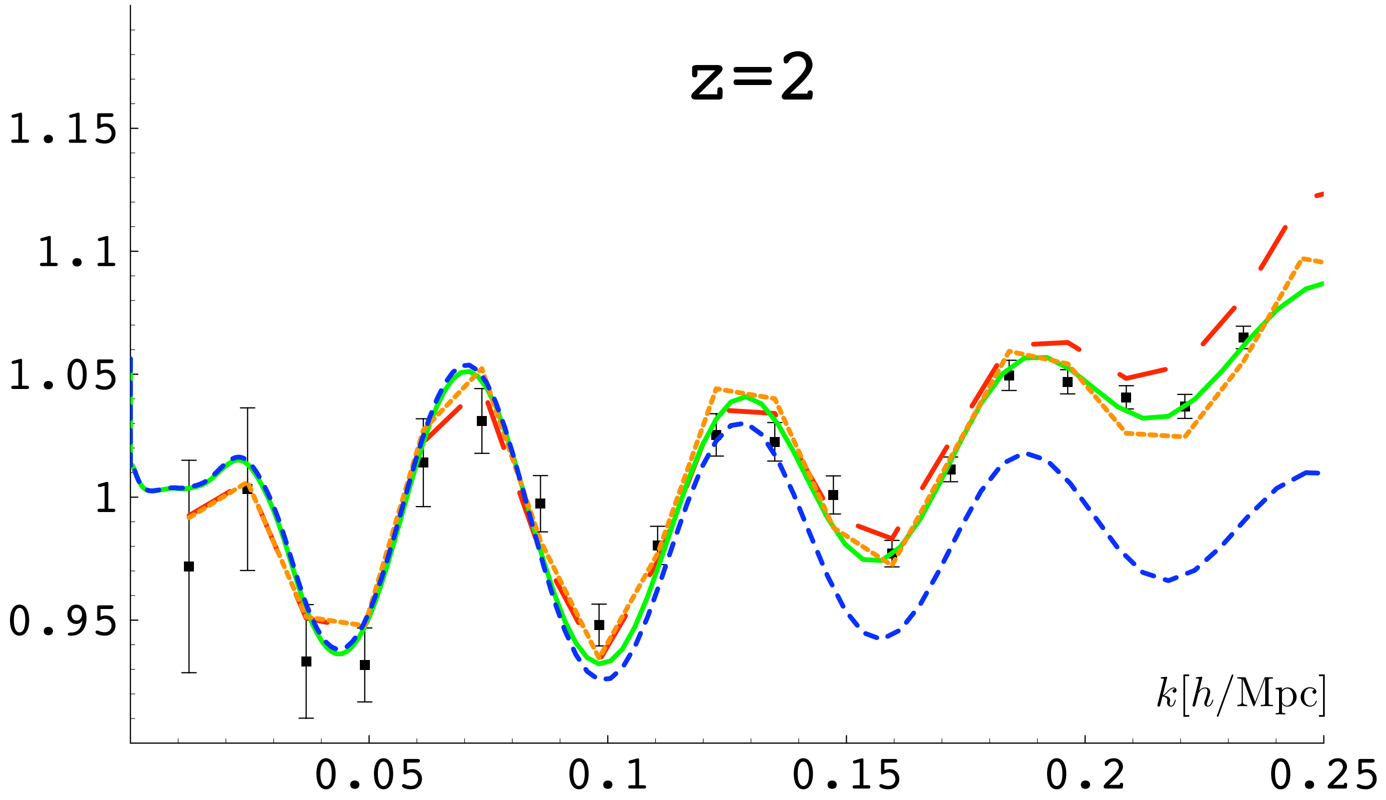


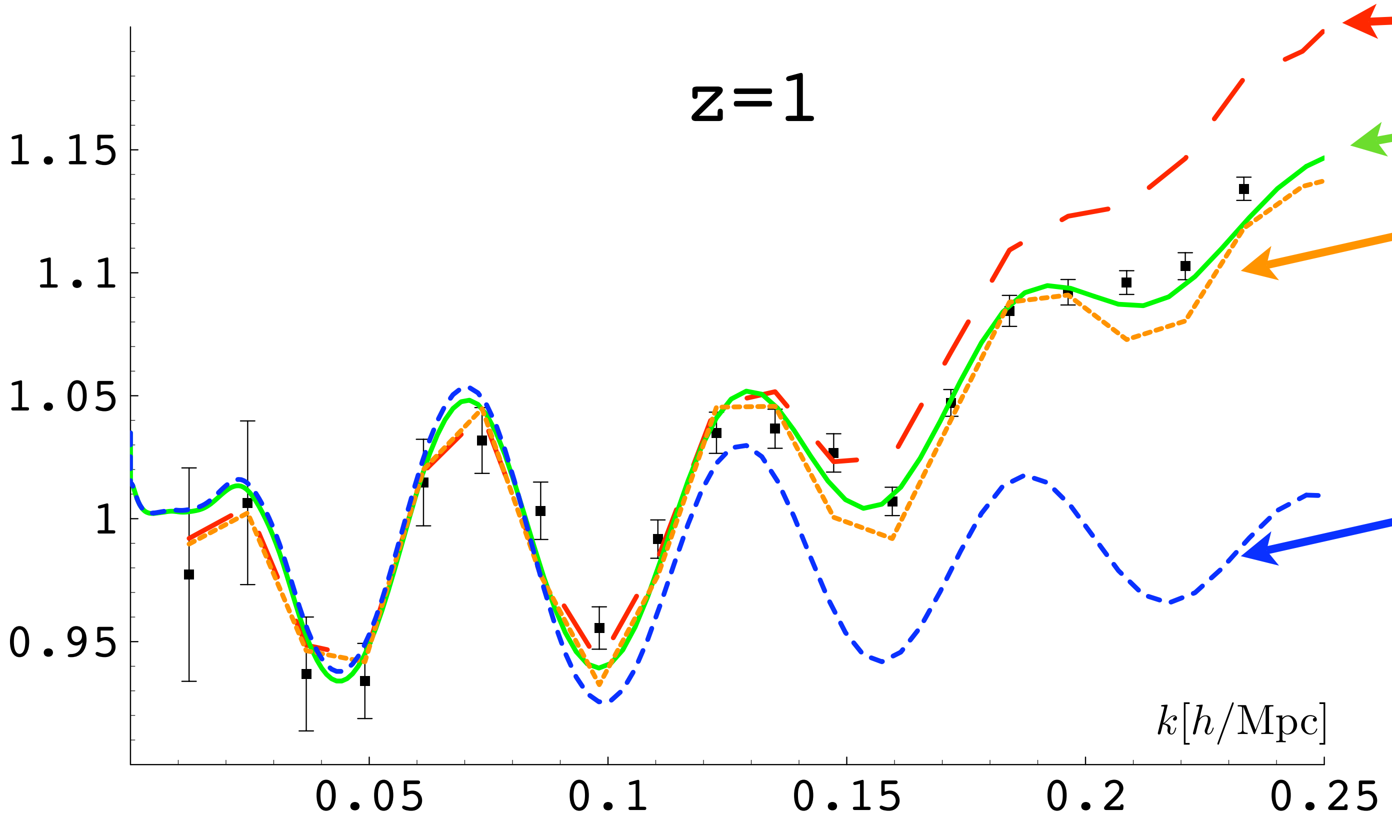
accuracy of linear theory up to k~0.12 h/Mpc is fortuitous: cancellation between two large non-linear effects

comparison with other approaches



ΛCDM model

$\Omega_\Lambda = 0.73$, $\Omega_b = 0.043$ h=0.7



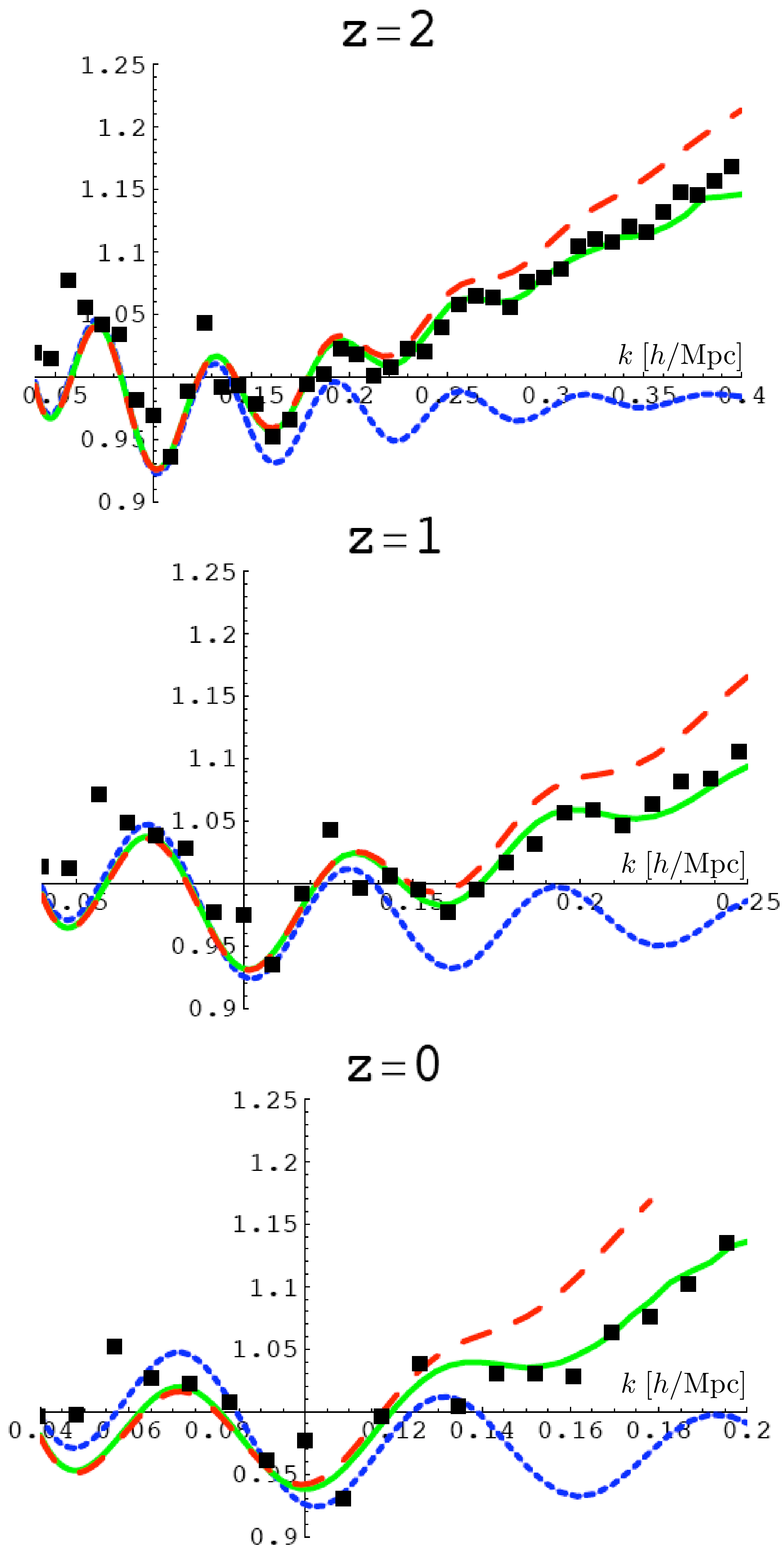
1-loop PT

RG (Matarrese, M.P.)

Halo model

Linear theory

data from N-body simulations of Jeong and Komatsu, astro-ph/0604075



ΛCDM model

$\Omega_\Lambda = 0.7$, $\Omega_b = 0.046$ $h = 0.72$

data from N-body simulations of
Huff et al, Astrop. Phys. 26, 351 (2007)

More General Cosmologies

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,,$$

$$\frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}(1+A(\vec{x},\,\tau))\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

$$\nabla^2\phi = 4\pi G\,(1+B(\vec{x},\,\tau))\,\rho\,a^2\,\delta$$

More General Cosmologies

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,,$$

deviation from geodesic
(e.g. DM-scalar field interaction)

$$\frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}(1 + A(\vec{x},\,\tau))\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

$$\nabla^2\phi = 4\pi G\,(1 + B(\vec{x},\,\tau))\,\rho\,a^2\,\delta$$

More General Cosmologies

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$$(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta,\,\mathbf{k}))\,\varphi_b(\eta,\mathbf{k}) = e^\eta\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

$$(\eta = \log a)$$

$$\mathbf{\Omega}_{ab} = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(1+B(\eta,\,\mathbf{k})) & 2+\frac{\mathcal{H}'}{\mathcal{H}}+A(\eta,\,\mathbf{k}) \end{pmatrix}$$

Time-RG

(also for cosmologies with $D^{\pm} = D^{\pm}(k,\, z)$)

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta, \mathbf{k}) = e^\eta \gamma_{abc}(\mathbf{k},\, -\mathbf{k_1},\, -\mathbf{k_2})\,\varphi_b(\eta, \mathbf{k_1})\,\varphi_c(\eta, \mathbf{k_2})$$

$$\partial_\eta \varphi = -\Omega\,\varphi + e^\eta \gamma\,\varphi\,\varphi$$

$$\partial_\eta \langle \varphi\,\varphi \rangle = -\sum \Omega\,\langle \varphi\,\varphi \rangle + \sum e^\eta \gamma\,\langle \varphi\,\varphi\,\varphi \rangle$$

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$$\ldots$$

infinite tower of equations

Advantages

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Works also for cosmologies with $\Omega_{ab} = \Omega_{ab}(k,\, \eta)$

not only for $\Omega_{ab} = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$

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Power spectrum ($\langle\varphi\,\varphi\rangle$) and bispectrum ($\langle\varphi\,\varphi\,\varphi\rangle$) from a single run!

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Power spectrum ($\langle \varphi\, \varphi \rangle$) and bispectrum ($\langle \varphi\, \varphi\, \varphi \rangle$) from a single run!

Systematic approximation scheme more straightforward

Approximation

$$
\begin{aligned}
&\langle \varphi_a(\mathbf{k},\ \eta)\varphi_b(\mathbf{q},\ \eta)\rangle \equiv \delta_D(\mathbf{k}+\mathbf{q})P_{ab}(\mathbf{k}\,,\eta)\ ,\\
&\langle \varphi_a(\mathbf{k},\ \eta)\varphi_b(\mathbf{q},\ \eta)\varphi_c(\mathbf{p},\ \eta)\rangle \equiv \delta_D(\mathbf{k}+\mathbf{q}+\mathbf{p})B_{abc}(\mathbf{k},\ \mathbf{q},\ \mathbf{p};\ \eta)\ ,\\
&\langle \varphi_a(\mathbf{k},\ \eta)\varphi_b(\mathbf{q},\ \eta)\varphi_c(\mathbf{p},\ \eta)\varphi_d(\mathbf{r},\ \eta)\rangle \equiv\\
&\qquad [\delta_D(\mathbf{k}+\mathbf{q})\,\delta_D(\mathbf{p}+\mathbf{r})P_{ab}(\mathbf{k}\,,\eta)P_{cd}(\mathbf{p}\,,\eta)\\
&\qquad +\delta_D(\mathbf{k}+\mathbf{p})\,\delta_D(\mathbf{q}+\mathbf{r})P_{ac}(\mathbf{k}\,,\eta)P_{bd}(\mathbf{q}\,,\eta)\\
&\qquad +\delta_D(\mathbf{k}+\mathbf{r})\,\delta_D(\mathbf{q}+\mathbf{p})P_{ad}(\mathbf{k}\,,\eta)P_{bc}(\mathbf{q}\,,\eta)\\
&\qquad +\delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q}+\mathbf{r})\,Q_{abcd}(\mathbf{k}\,,\mathbf{q}\,,\mathbf{p}\,,\mathbf{r}\,,\eta)]\ ,
\end{aligned}
$$

Only approximation: $Q_{abcd} = 0$

Equations to solve:

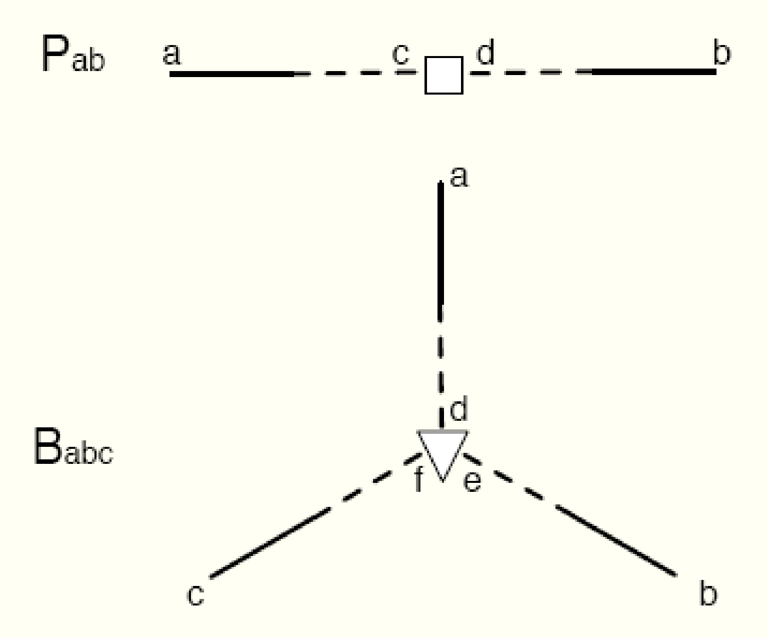
$$\partial_\eta P_{ab}(\mathbf{k},\eta) = -\Omega_{ac}(\mathbf{k},\eta)P_{cb}(\mathbf{k},\eta) - \Omega_{bc}(\mathbf{k},\eta)P_{ac}(\mathbf{k},\eta)$$
$$+ e^\eta \int d^3q \, [\gamma_{acd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}) \, B_{bcd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$+B_{acd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta) \, \gamma_{bcd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k})] \, ,$$

$$\partial_\eta B_{abc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta) = -\Omega_{ad}(\mathbf{k},\eta)B_{dbc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$- \Omega_{bd}(-\mathbf{q},\eta)B_{adc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$- \Omega_{cd}(\mathbf{q}-\mathbf{k},\eta)B_{abd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$+ 2e^\eta \, [\gamma_{ade}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k})P_{db}(\mathbf{q},\eta)P_{ec}(\mathbf{k}-\mathbf{q},\eta)$$
$$+ \gamma_{bde}(-\mathbf{q}, \mathbf{q}-\mathbf{k}, \mathbf{k})P_{dc}(\mathbf{k}-\mathbf{q},\eta)P_{ea}(\mathbf{k},\eta)$$
$$+ \gamma_{cde}(\mathbf{q}-\mathbf{k}, \mathbf{k}, -\mathbf{q})P_{da}(\mathbf{k},\eta)P_{eb}(\mathbf{q},\eta)] \, .$$

initial conditions given at $\eta = 0$, corresponding to $z = z_{in}$

Iterative solution: step 1

$O(\gamma^0)$: linear PT



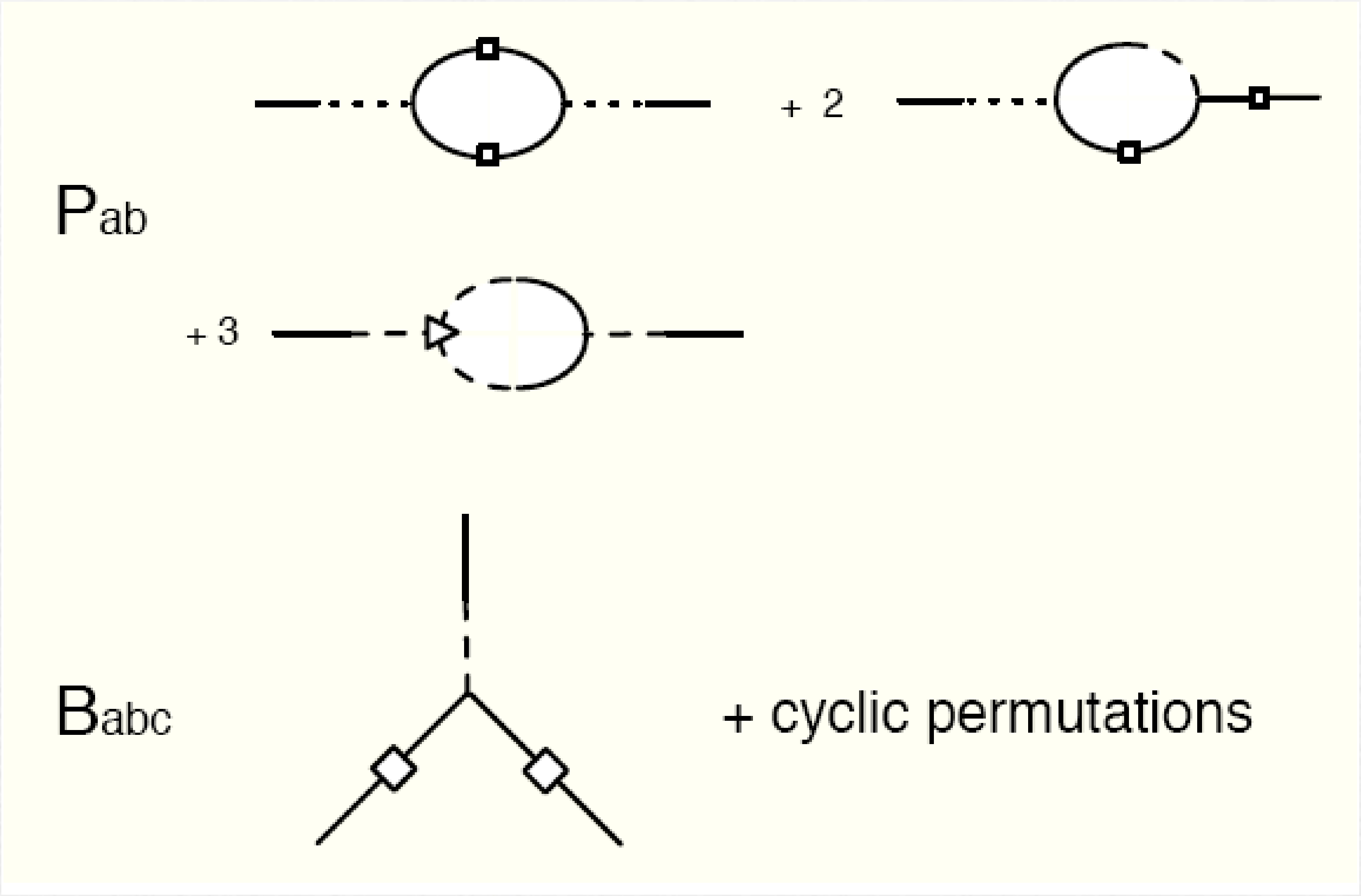
$$P^L_{ab}(\mathbf{k}\,,\eta) = g_{ac}(\mathbf{k}\,,\eta,0)\, g_{bd}(\mathbf{k}\,,\eta,0) P_{cd}(\mathbf{k}\,,\eta=0)\,,$$

$$B^L_{abc}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k};\, \eta) =$$
$$g_{ad}(\mathbf{k}\,,\eta,0) g_{be}(-\mathbf{q}\,,\eta,0) g_{cf}(\mathbf{q}-\mathbf{k}\,,\eta,0) B_{def}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k};\, \eta=0)$$

Iterative solution: step 2

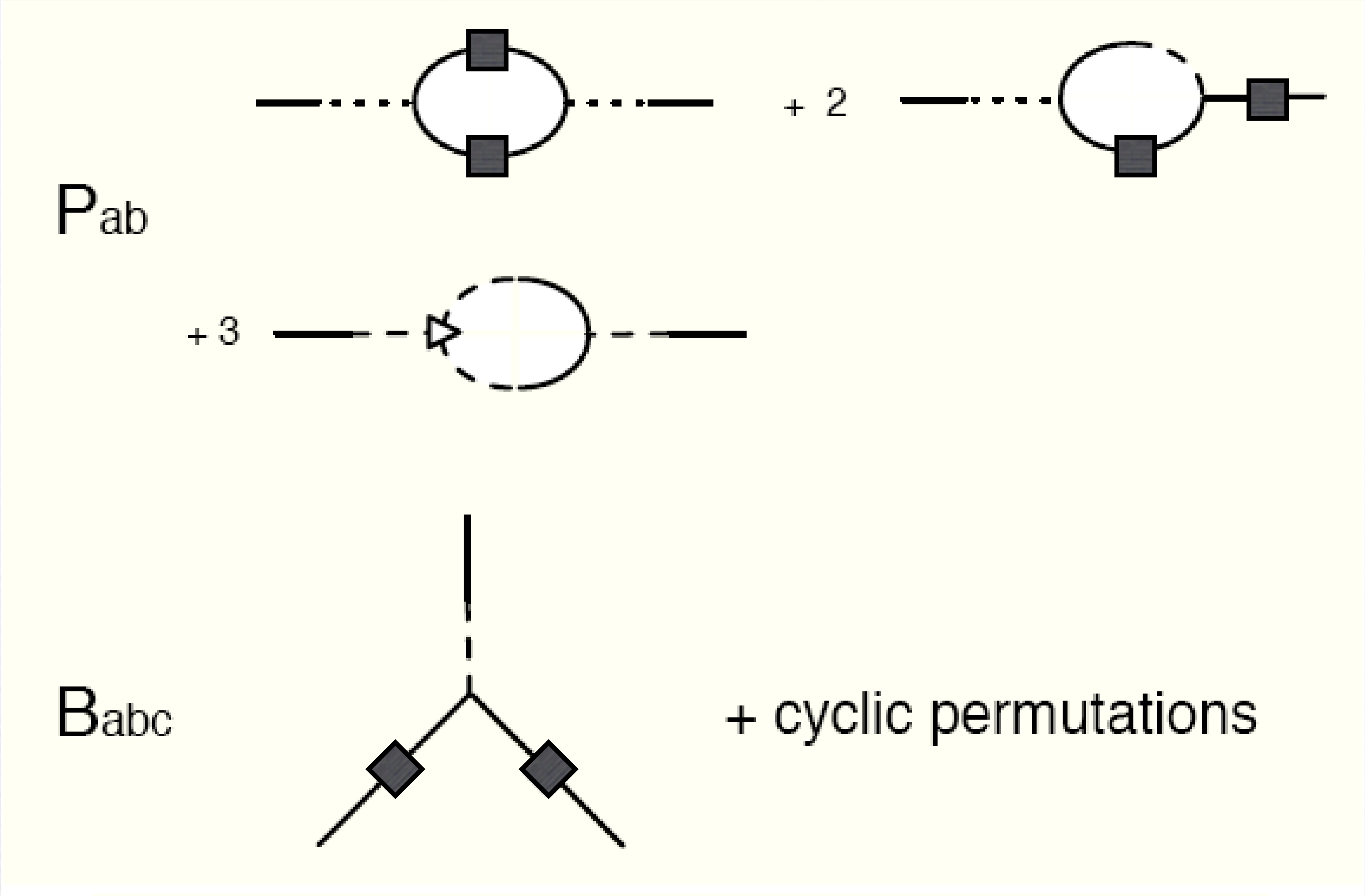
1-loop corrections for P_{ab} :

$O(\gamma)$ corrections for B_{abc} :



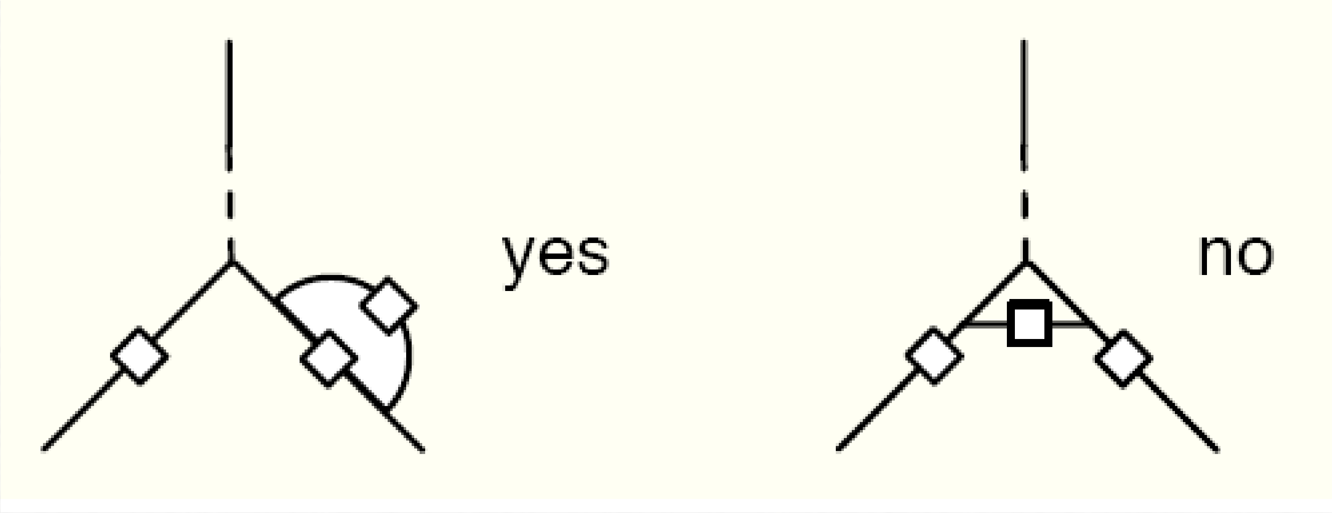
Iterative solution: step n>2

n-1-loop corrections for P_{ab}, <u>but not all!!</u>

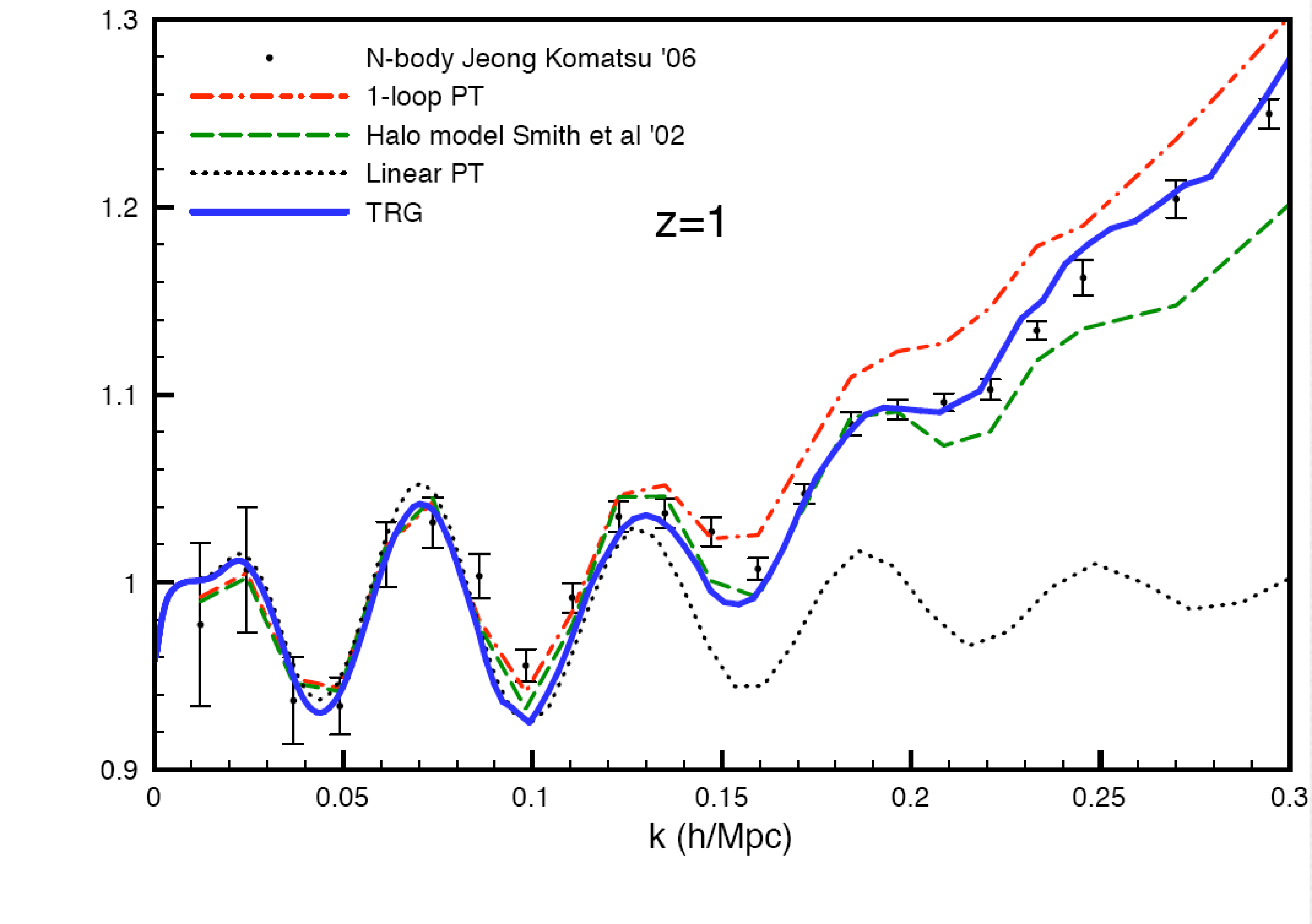


$O(\gamma^{2n-3})$ corrections for B_{abc} <u>but not all!</u>

e.g.: for n=3



Full equation: numerical results



initial conditions:

$$P_{ab}(\mathbf{k},0) = P_{\rm Lin}(\mathbf{k}, z_{in}) u_a u_b$$

$$B_{abc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}, 0) = 0$$

To do:

1) getting closer to the real world: bias + redshift space;

2) neutrino mass limit;

3) initial non-gaussianity;

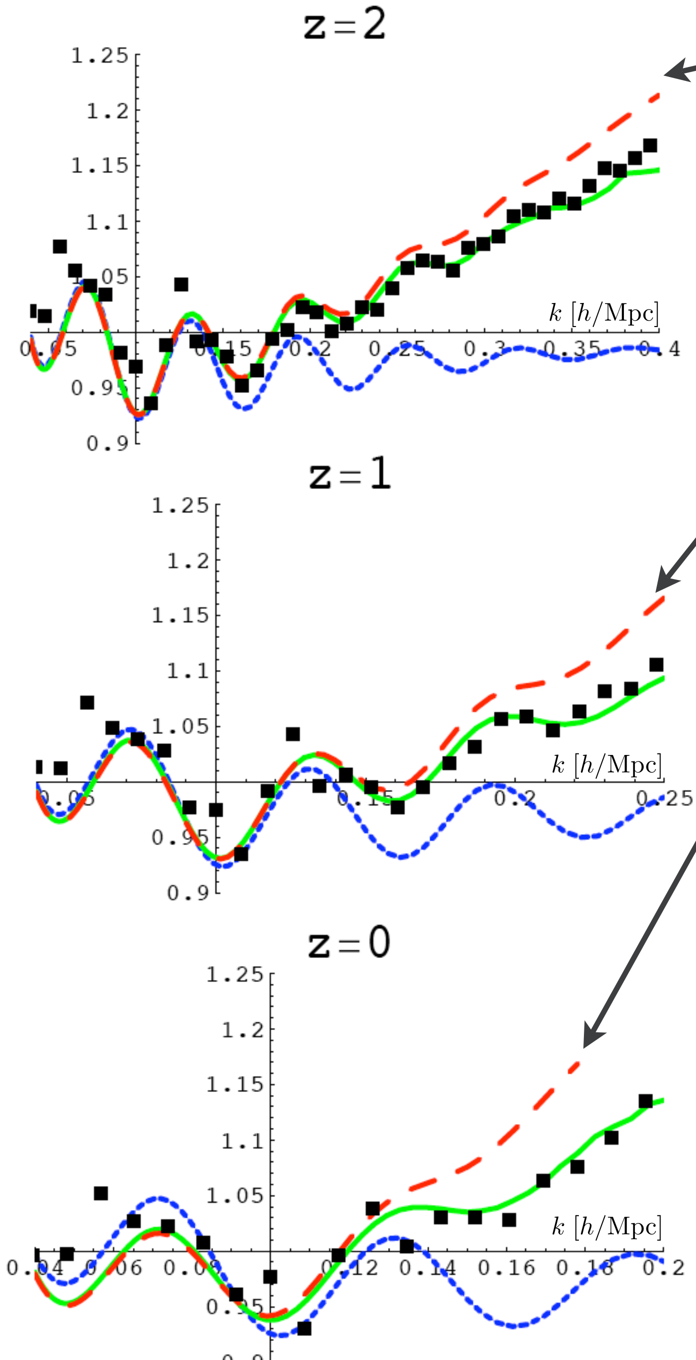
4) including the trispectrum in the running;

5) go to lensing scales;

6) ...

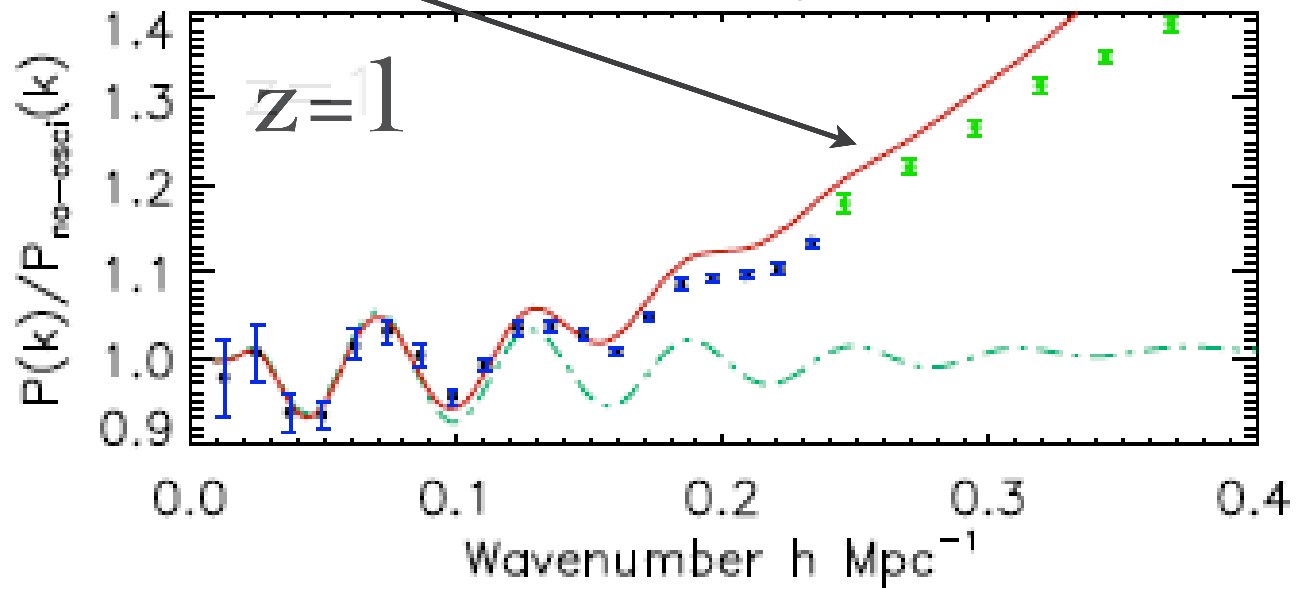
Present Status: Pert. Theory

1-loop PT



data from N-body simulations of
Huff et al, Astrop. Phys. 26, 351 (2007)

Jeong Komatsu, '06



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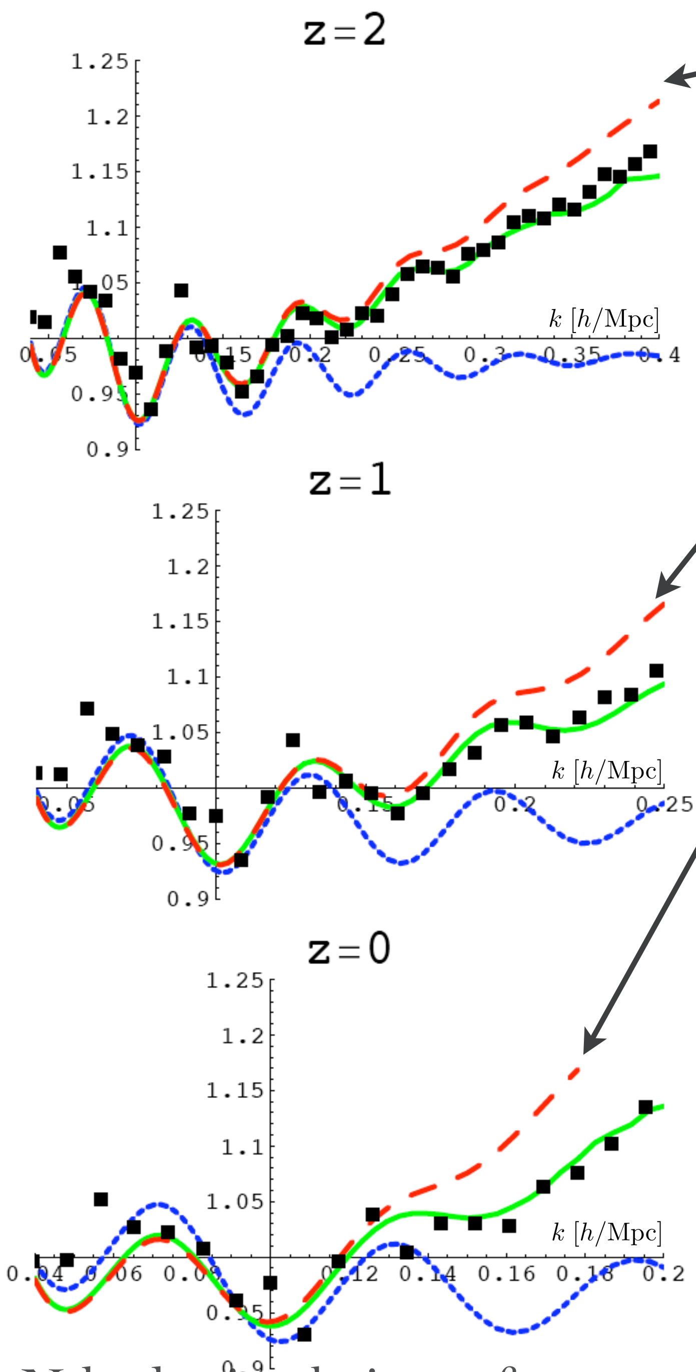

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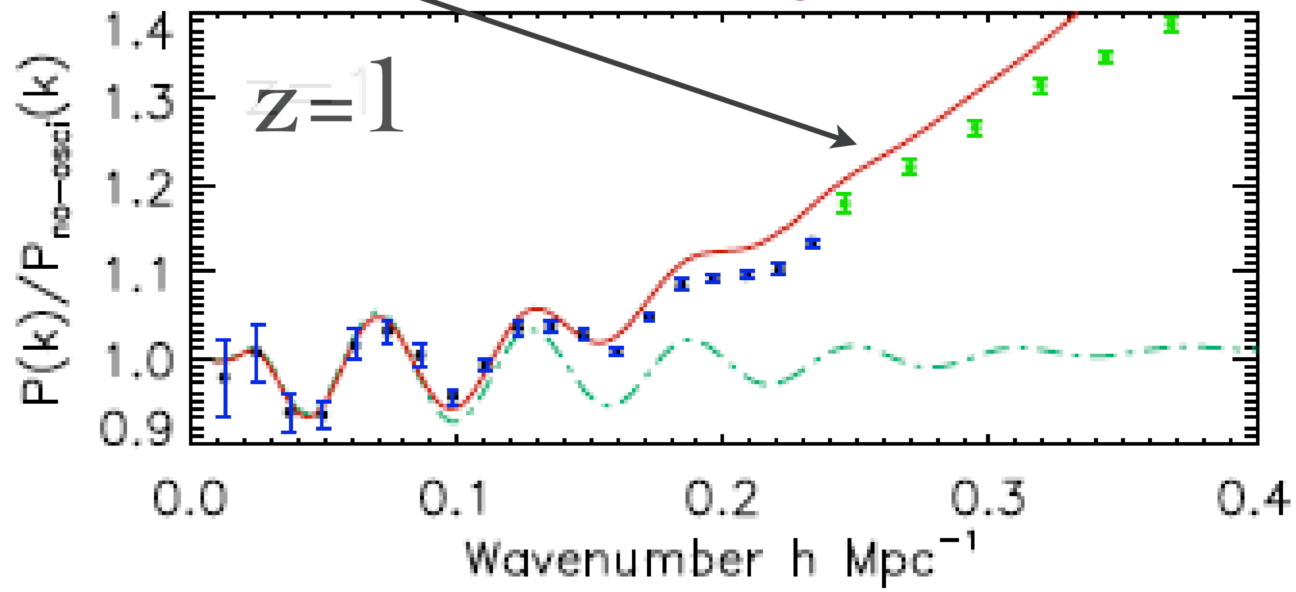
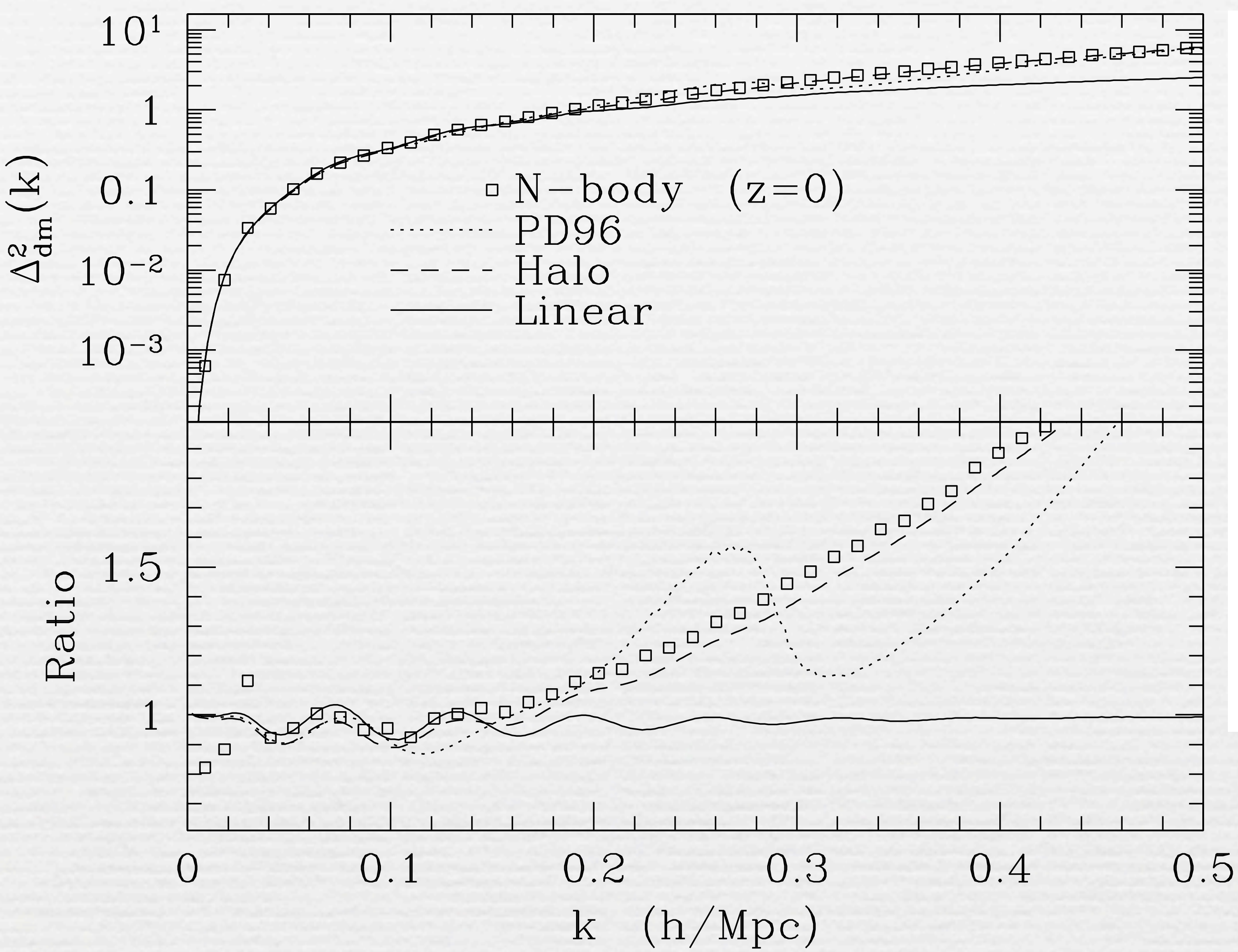


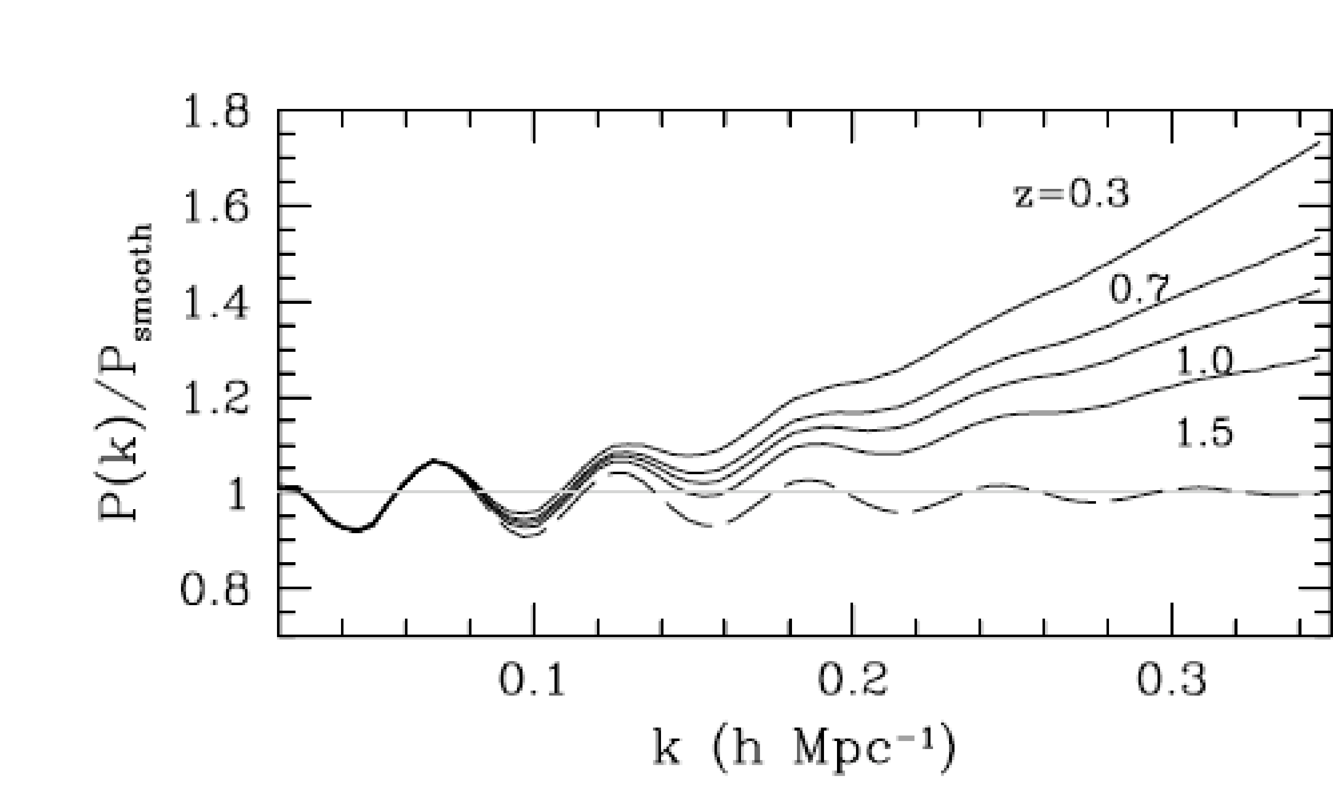
Non-linearities becomes more and more

relevant in the DE-sensitive range 0<z<1

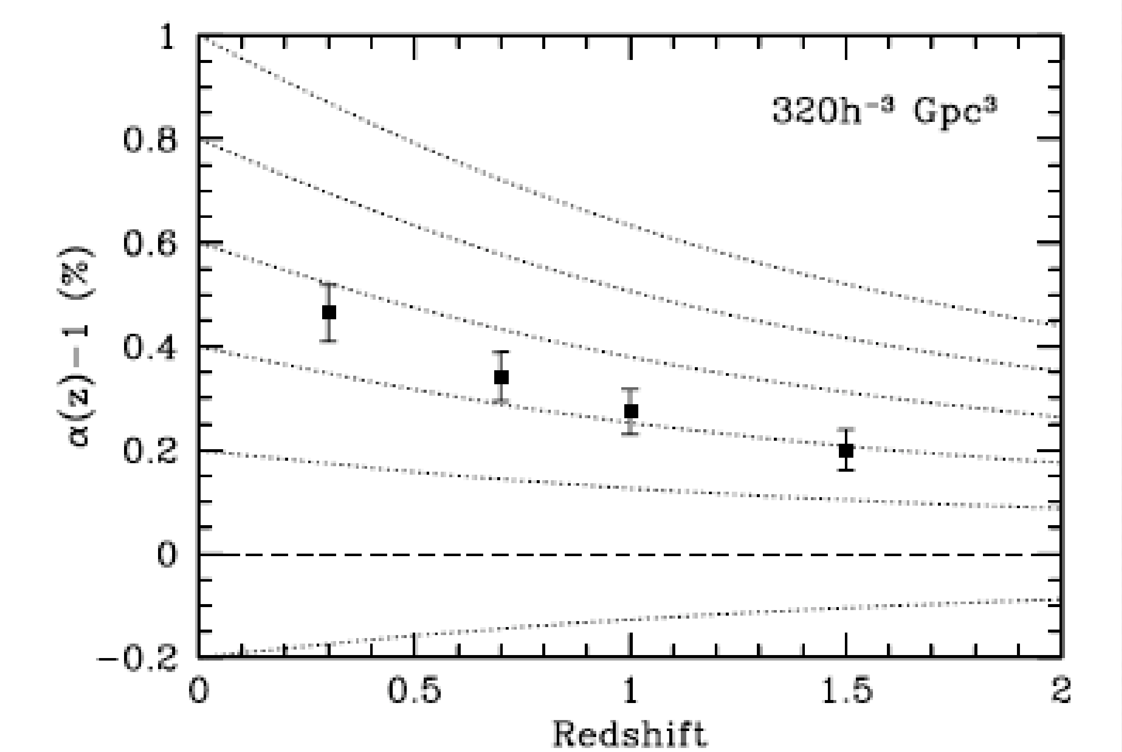
Standard Approach: N-body simulations+fitting functions



Huff et al, '06



Seo et al, '08



5-10% discrepancies between

fitting functions and simulations

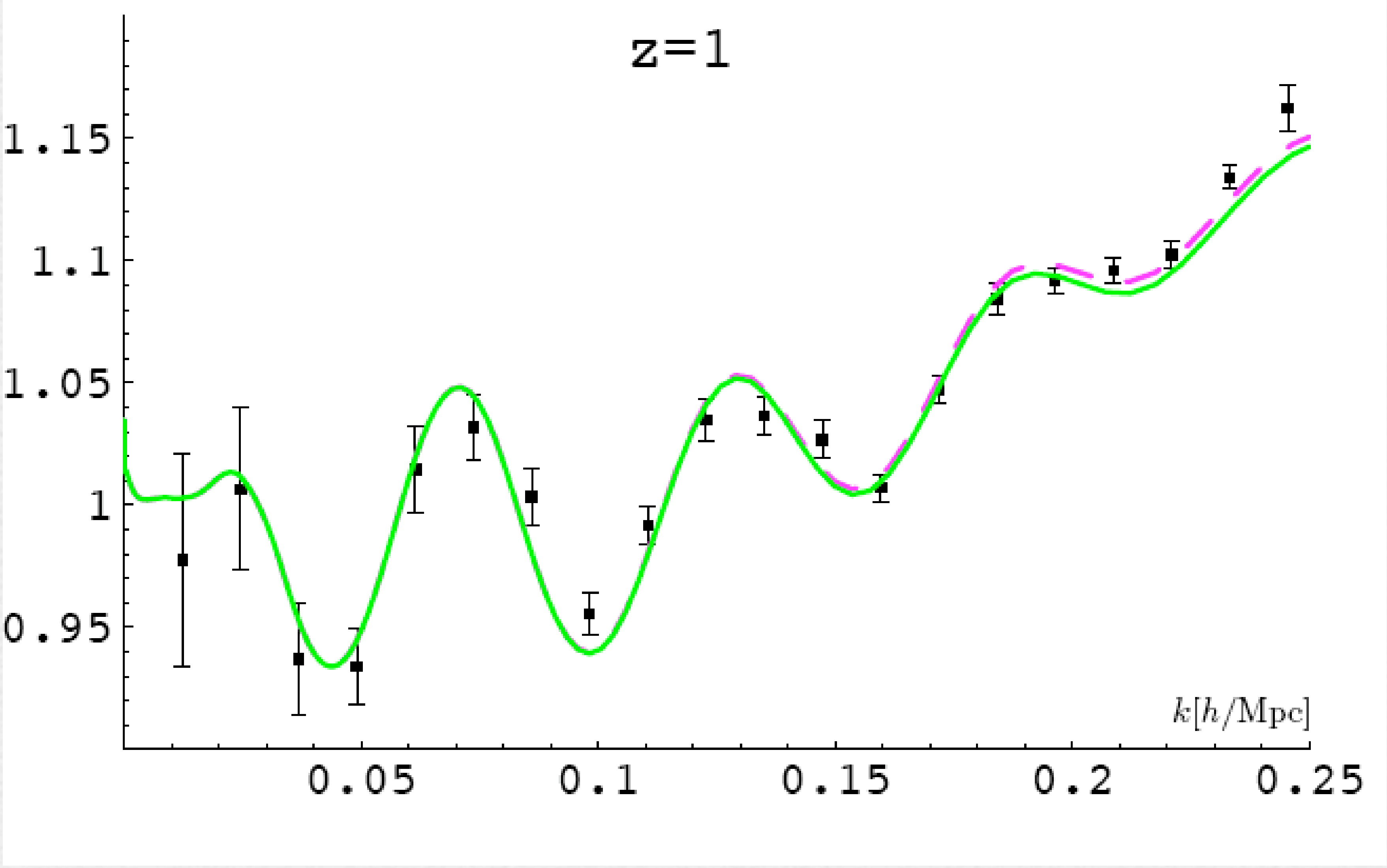
peak displacement by nonlinearities

Modeling the time-dependence

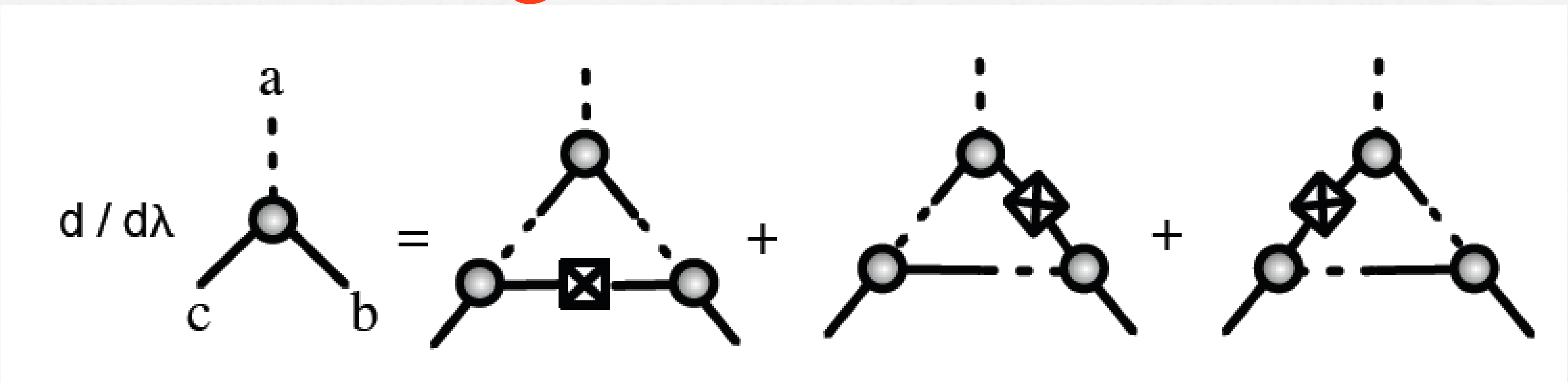
$$\bar{P}_{\lambda}^{II}(q;\, \eta_a,\, \eta_b) = \bar{P}_{\lambda}^{II}(q;\, s_1,\, s_2)$$

vs.

$$\bar{P}_{\lambda}^{II}(q;\, \eta_a,\, \eta_b) = \bar{P}_{\lambda}^{II}(q;\, s_1,\, s_2)\, e^{-q^2\sigma_v^2\, \frac{(e^{\eta_a}-e^{s_1})^2+(e^{\eta_b}-e^{s_2})^2}{2}}$$



Making the vertices run



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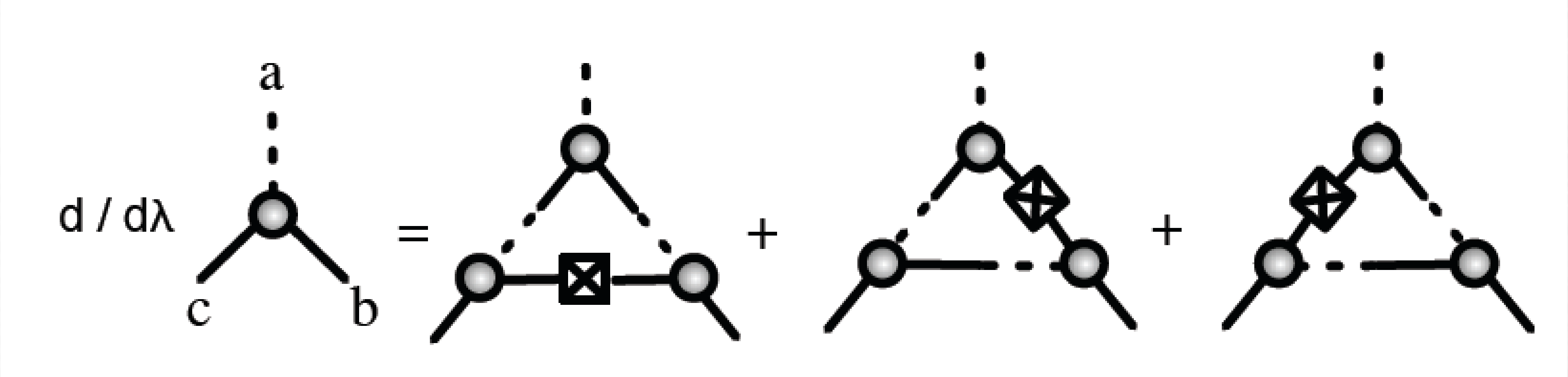

if $\eta_a = \eta_b = \eta_c$ the 1-loop correction is proportional to the tree-level:

$$\Delta\gamma_{abc}(k_a,\, k_b,\, k_c) = g(k_a,\, k_b,\, k_c)\, \gamma_{abc}(k_a,\, k_b,\, k_c)$$

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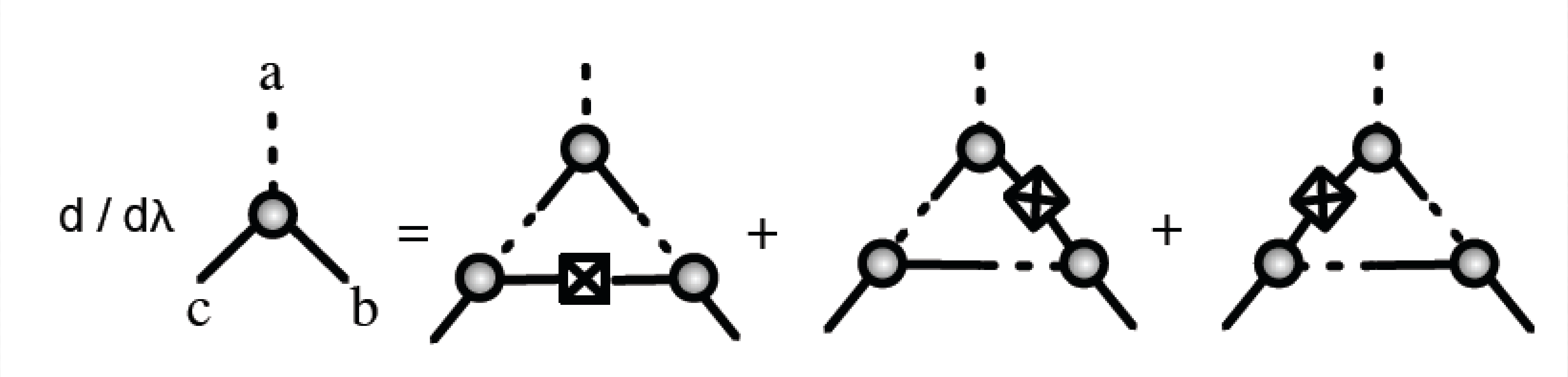

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in the k>>q limit $$g(k,\, -q,\, q-k) = \left[1 + \frac{(e^{\eta}-1)^2}{2}\sigma^2 k^2\right]^{-1/2}$$

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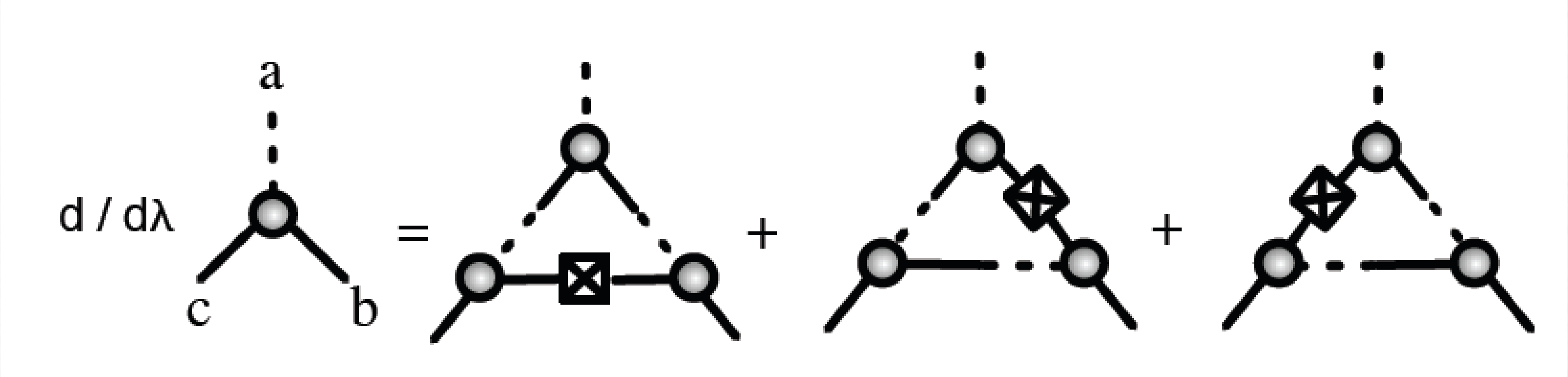

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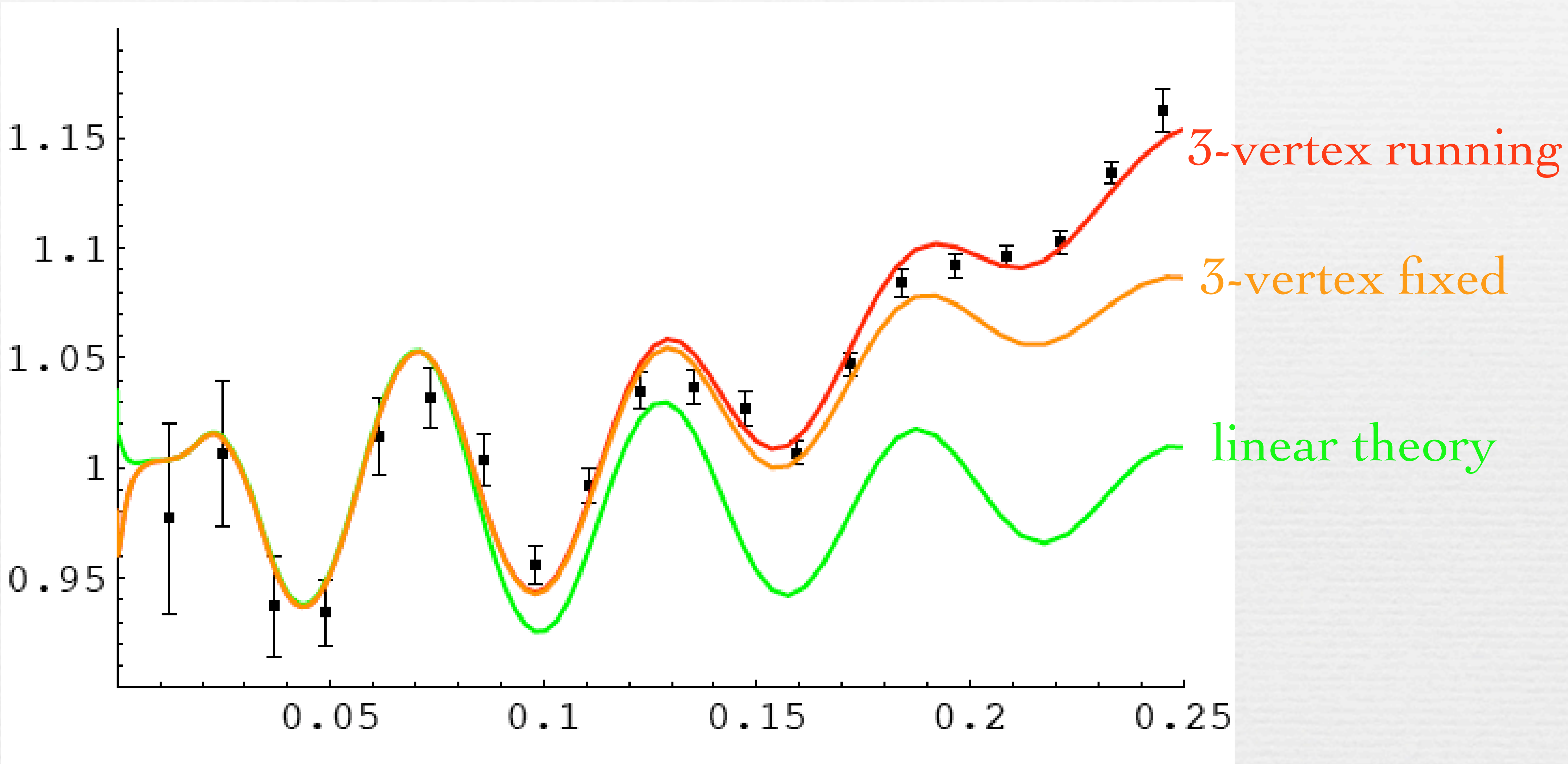
$g \to 0$ for $k \gg 1/(\sigma e^\eta)$: "asymptotic freedom"!

Local-time truncations



Dark Matter Hydrodynamics

The DM particle distribution function, $f(\mathbf{x}, \mathbf{p}, \tau)$, obeys the Vlasov equation:

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla \phi \cdot \nabla_{\mathbf{p}} f = 0$$

where $p = am \frac{d\mathbf{x}}{d\tau}$ and $\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta$

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Taking moments, i.e.,

$$\int d^3\mathbf{p}\, f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau) \equiv \overline{\rho}\,[1 + \delta(\mathbf{x}, \tau)]$$

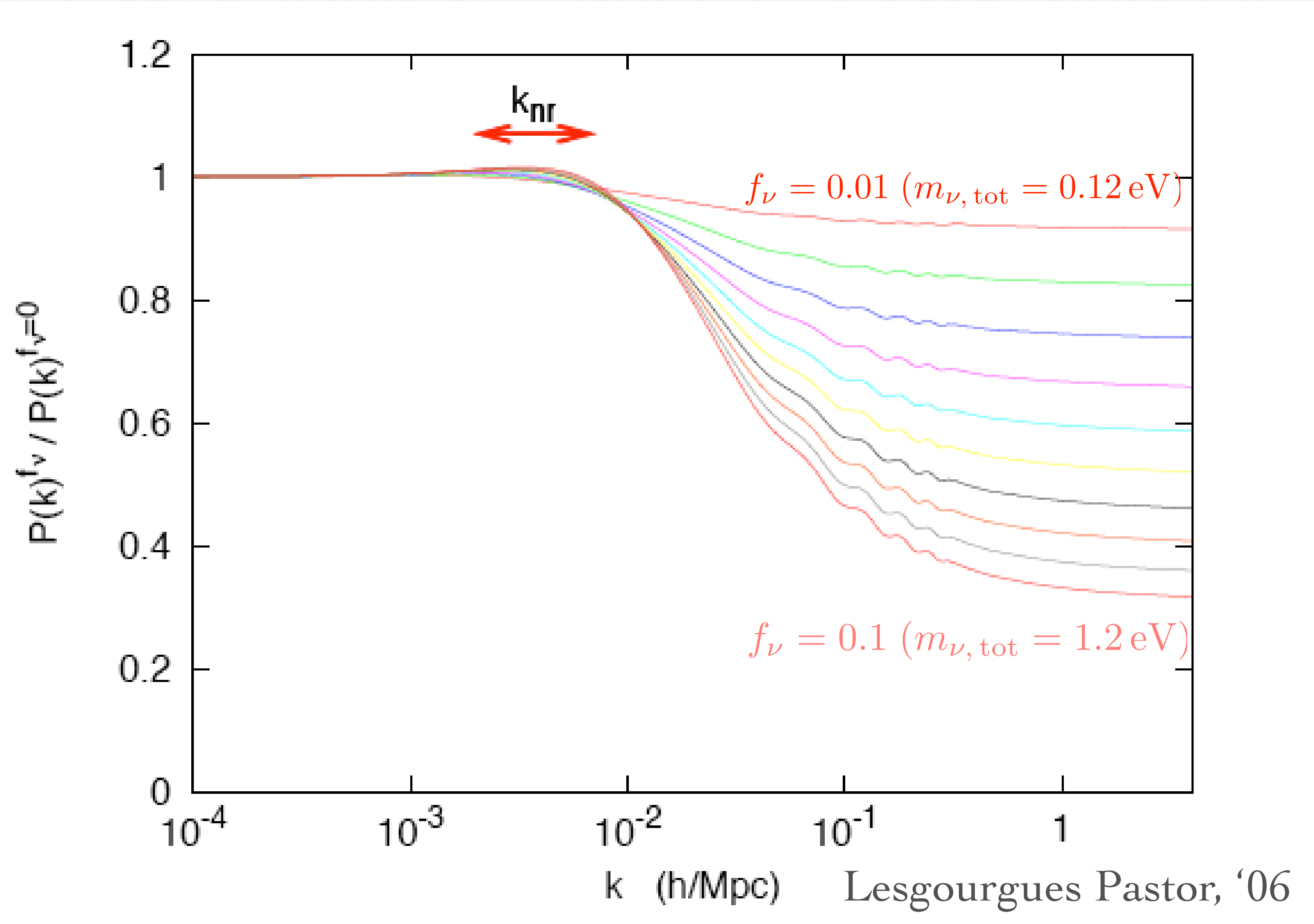
$$\int d^3\mathbf{p}\, \frac{p_i}{am} f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau) v_i(\mathbf{x}, \tau)$$

$$\int d^3\mathbf{p}\, \frac{p_i\, p_j}{a^2 m^2} f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau) v_i(\mathbf{x}, \tau) v_j(\mathbf{x}, \tau) + \sigma_{ij}(\mathbf{x}, \tau)$$

$$\cdots$$

and neglecting σ_{ij} and higher moments (single stream approximation), one gets...

Massive Neutrinos and the Power Spectrum



$$f_\nu = \frac{\Omega_\nu}{\Omega_m} = \frac{m_{\nu,\,\mathrm{tot}}}{94.1\,\Omega_m\,h^2\,\mathrm{eV}}$$

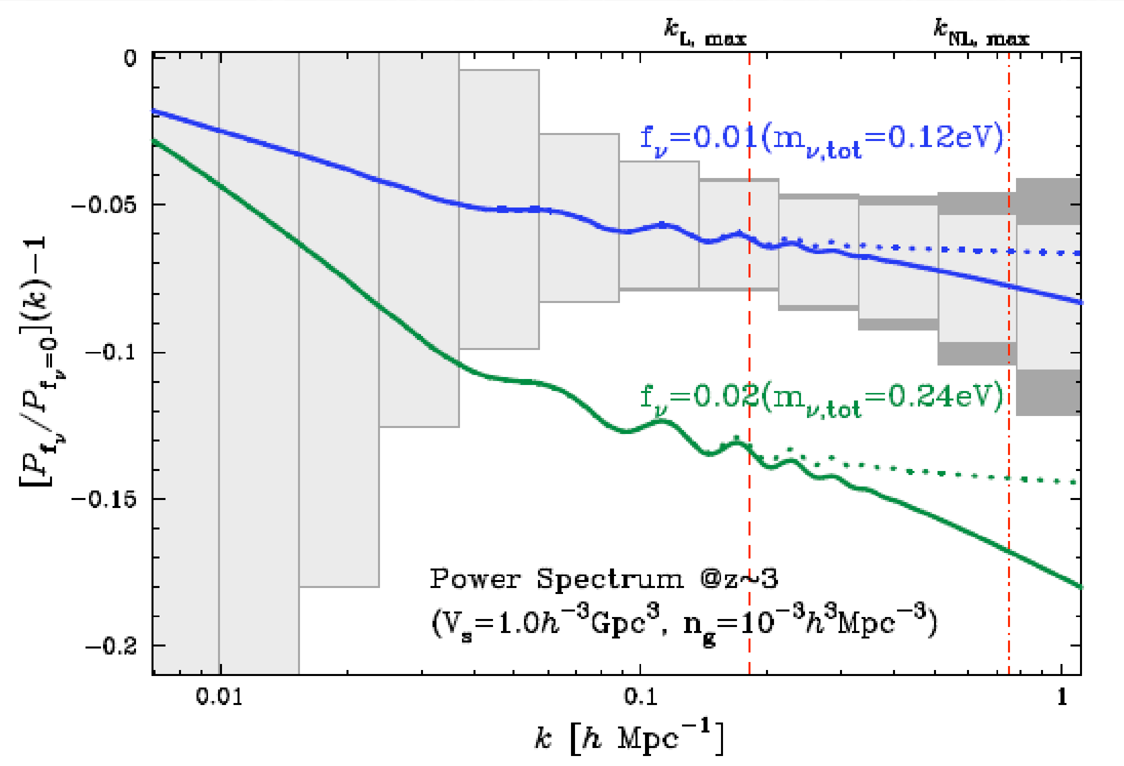
$$k_{nr} \simeq 0.018\,\Omega_m^{1/2}\left(\frac{m_\nu}{1\mathrm{eV}}\right)\,h\,\mathrm{Mpc}^{-1}$$

$$\frac{\Delta P}{P^{f_\nu=0}} \sim -8 f_\nu$$

The linear growth factor
is scale dependent: $\delta_m(k \ll k_{nr}) \sim a\,,\quad \delta_m(k \gg k_{nr}) \sim a^{1-3/5 f_\nu}$

Non-linear effects on neutrino mass bounds

(Saito, Takada, Taruya,'08)



$$P_m = (1-f_\nu)^2 P_{c\,c} + 2 f_\nu (1-f_\nu)\, P^L_{c\,\nu} + f^2_\nu\, P^L_{\nu\,\nu}$$

1-loop

linear

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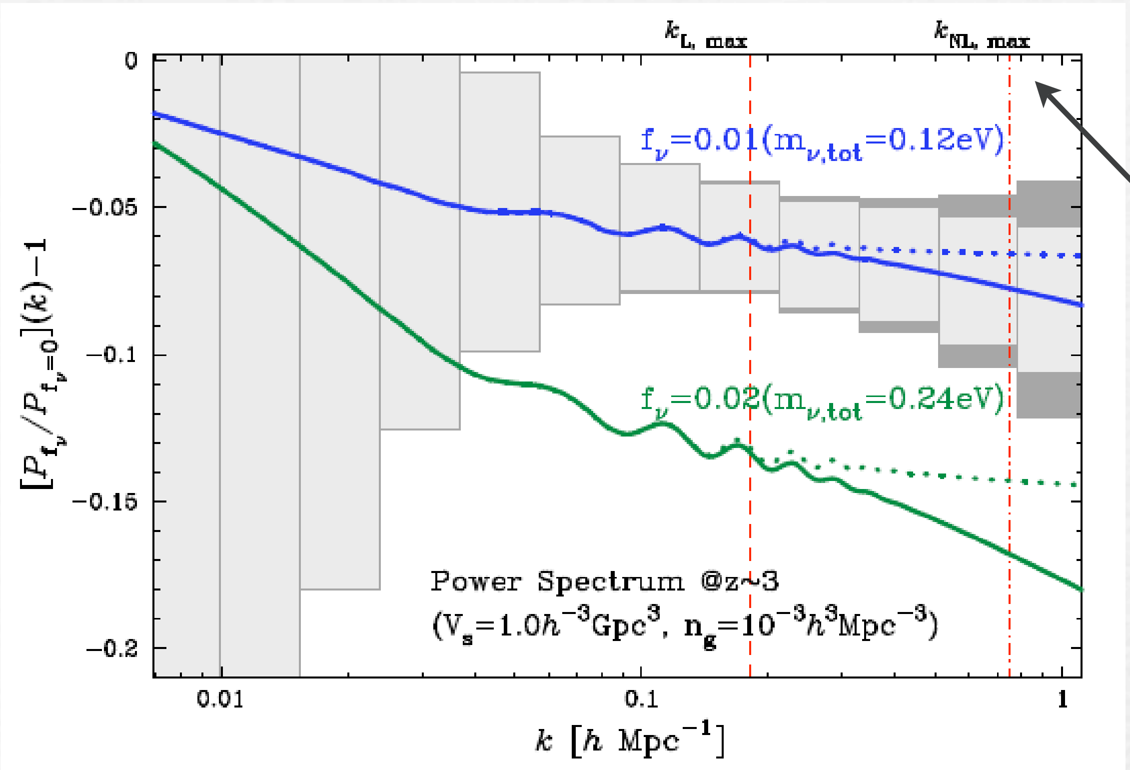

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$$\Delta^2(k_{NL}, z) = 4\pi k_{NL}^3 P_m(k_{NL}, z) \simeq 0.4$$

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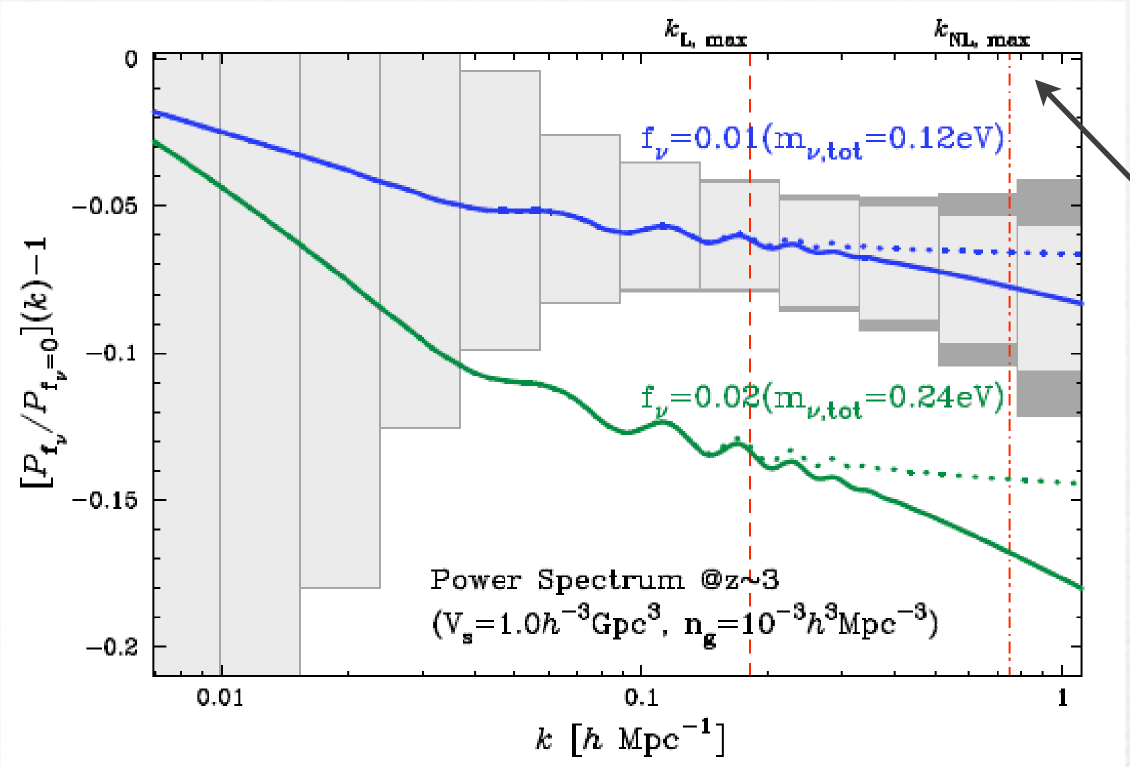

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1-loop

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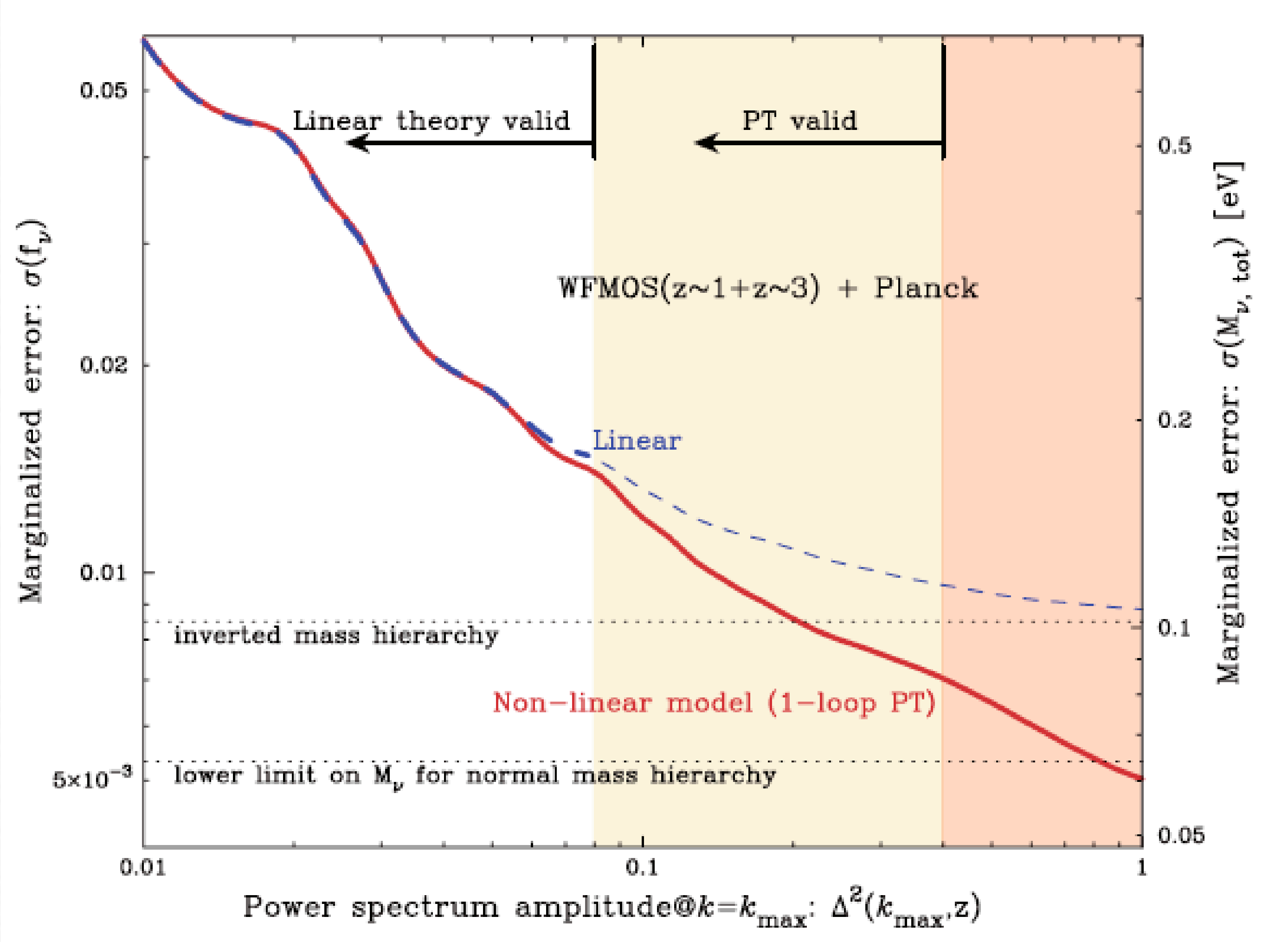

the neutrino mass bound
could go down to ~ 0.09 eV !

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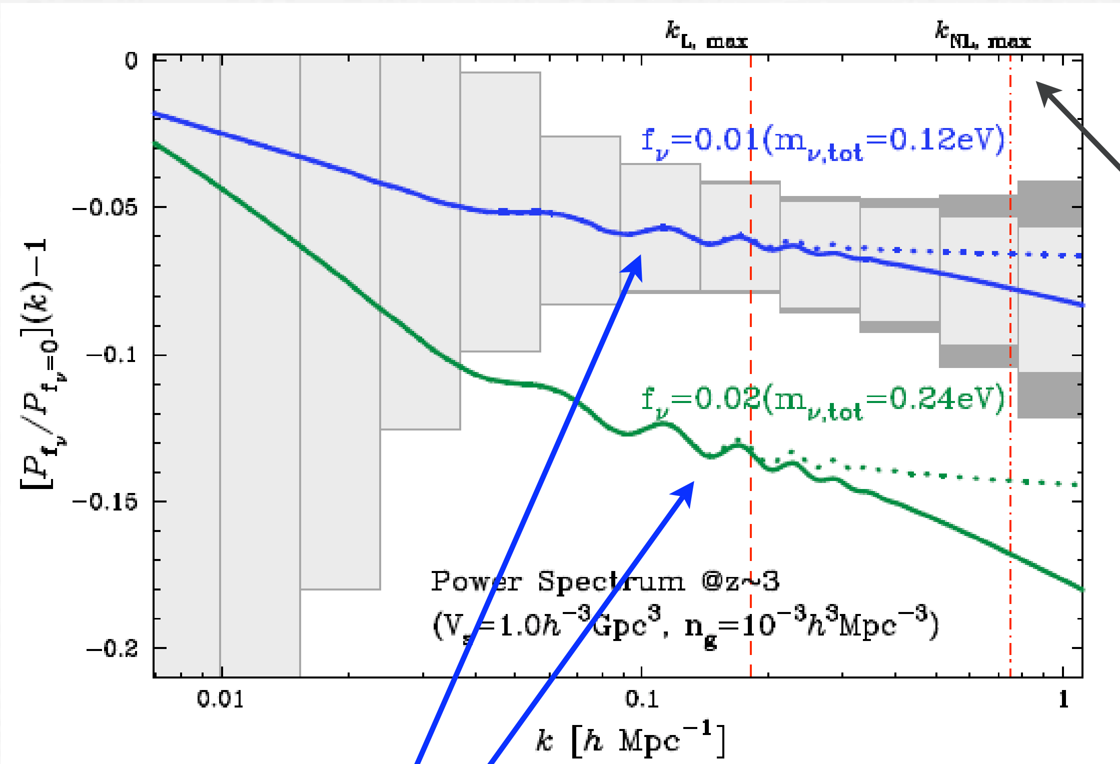

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BAO shifting by neutrinos!

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