Functional renormalization group for interacting fermions - recent developments

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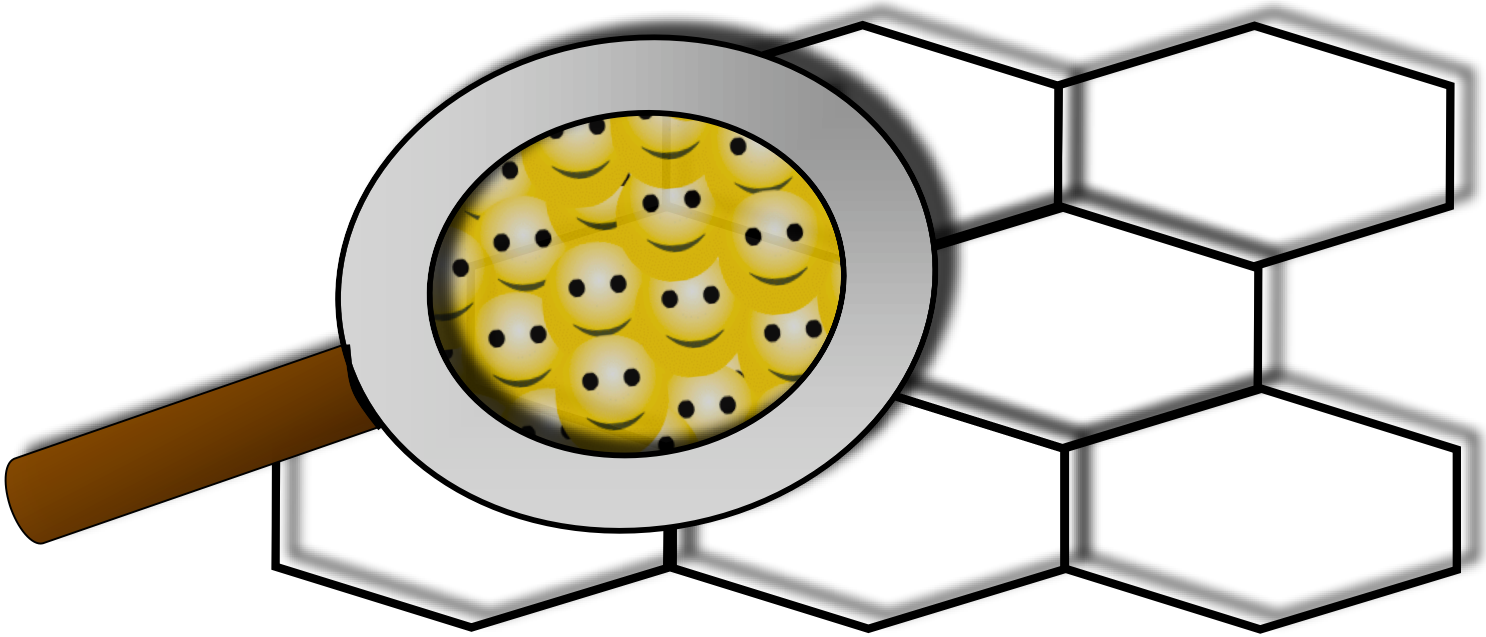
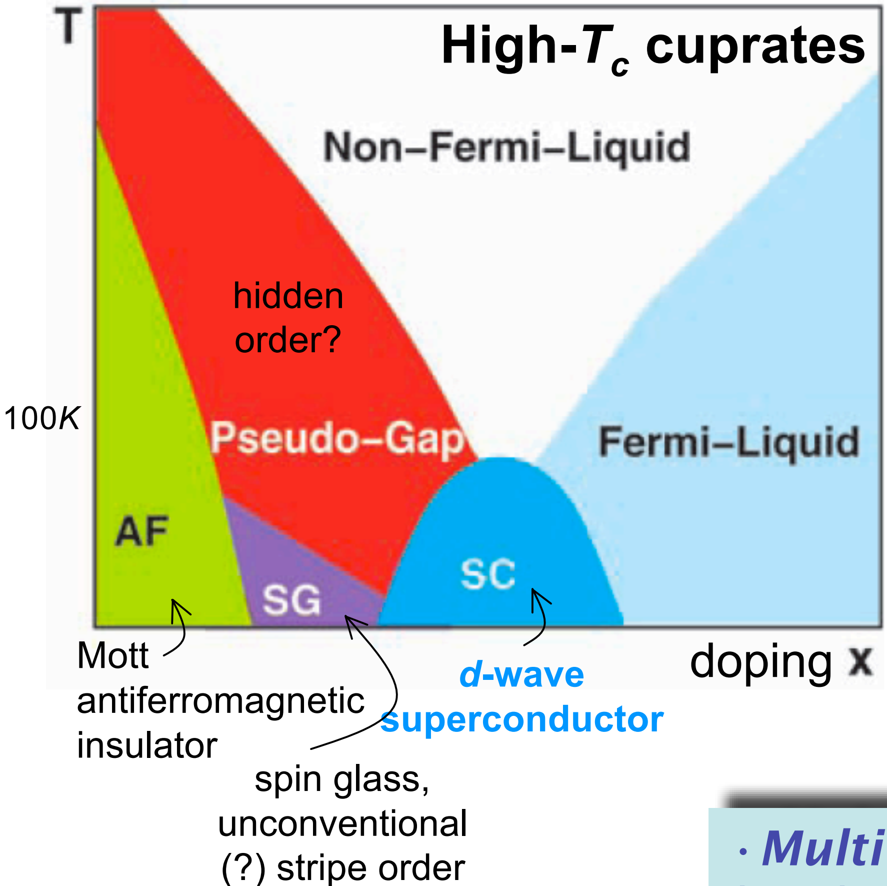
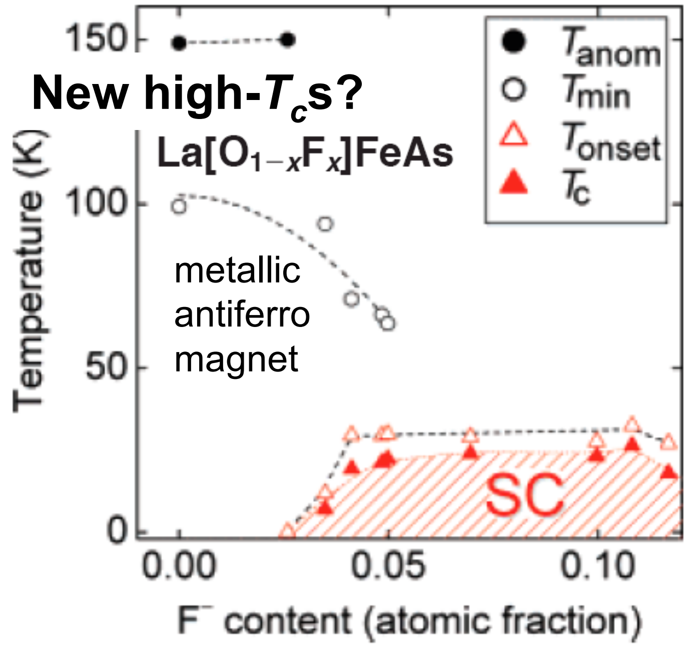


Why functional renormalization group for interacting fermions?

Diversity in correlated electron behavior



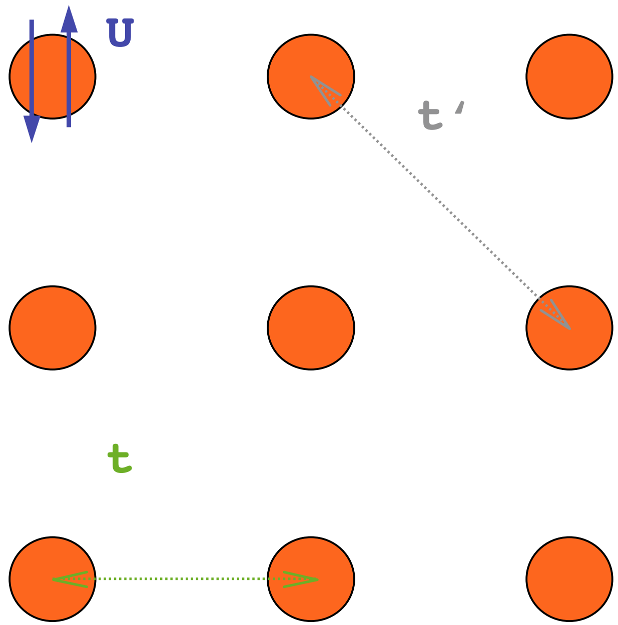


- ***Multitude of infrared fixed points,*** *selected by physics at intermediate energy scales*
- ***Anomalous behavior*** *at intermediate/low energies (critical (?) fluctuations)*

Dagotto 2005,
Kamihara et al., 2008

Standard model for strongly correlated electrons: Hubbard model

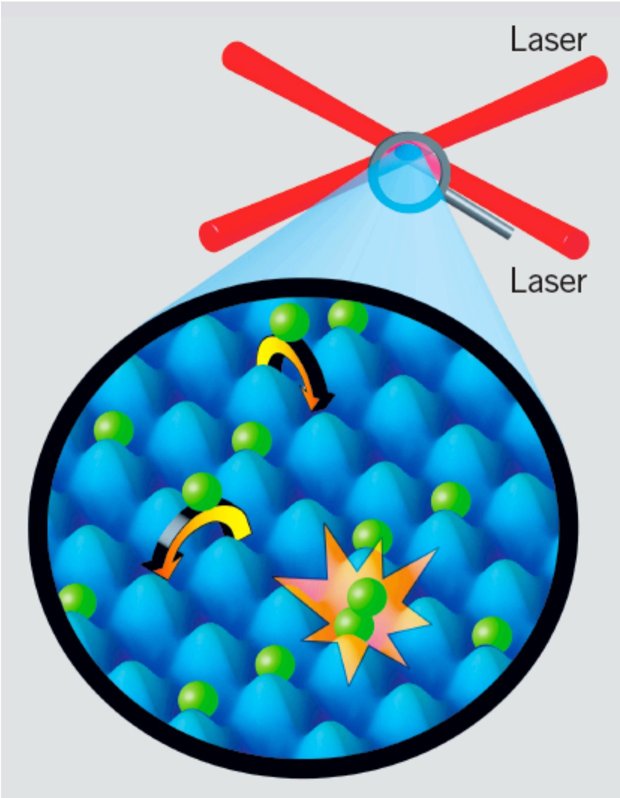
$$H = -t \sum_{\mathrm{nn},s} c_{i,s}^{\dagger} c_{j,s} - t' \sum_{\mathrm{nnn},s} c_{i,s}^{\dagger} c_{j,s} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



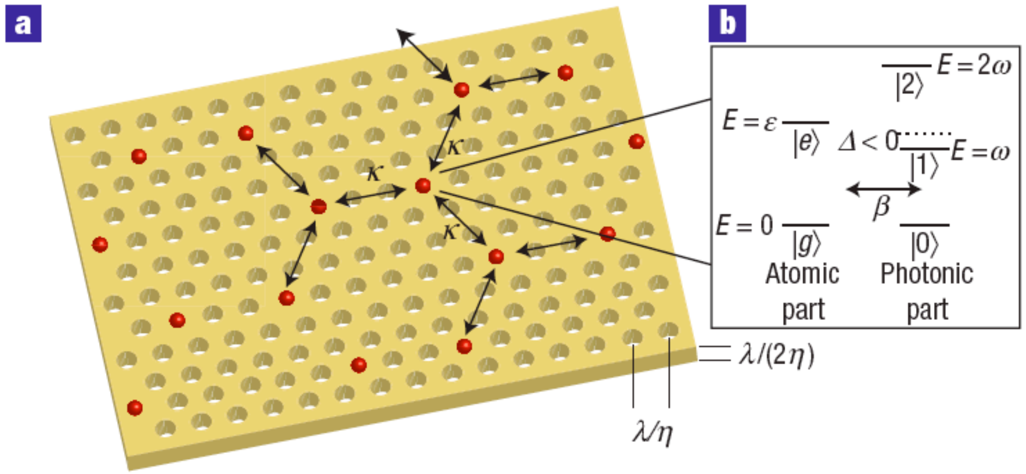
& variants

(different lattices, more bands, more general interactions)

Artificial Hubbard models: new possibilities?



- **Ultracold atoms on optical lattices** (Bloch et al, ... many groups)
- **Photonic Hubbard models** (2-level systems in microcavities in photonic bandgap materials, Greentree et al. 06)

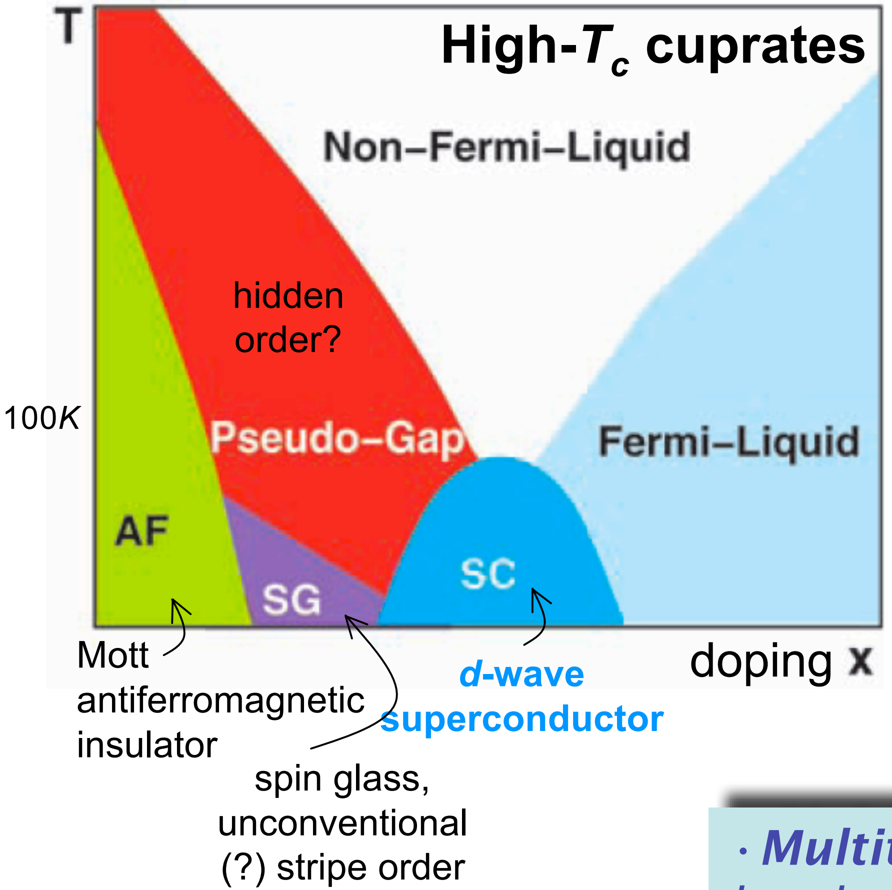


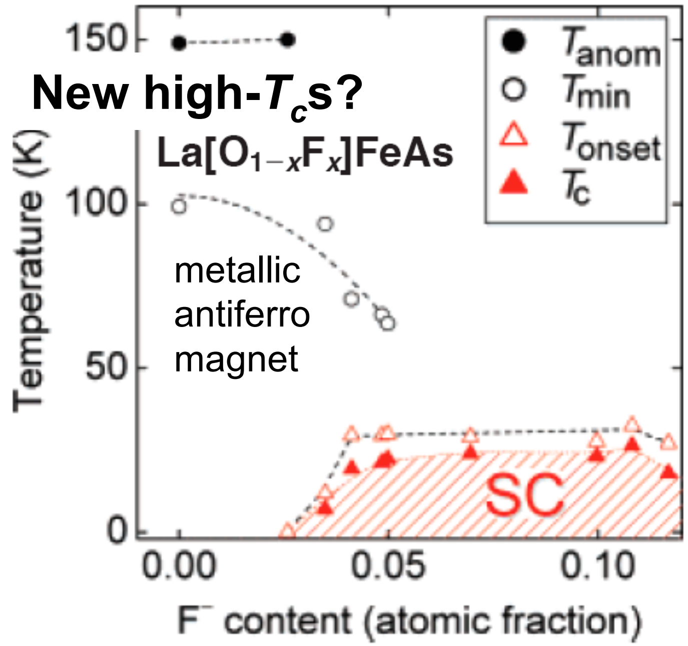
- **Quantum dot arrays** (Byrnes et al., 07) using mesh-gated 2DEGs:

Quantum simulation of Fermi-Hubbard models in semiconductor quantum dot arrays

Tim Byrnes,[1,2] Na Young Kim,[3] Kenichiro Kusudo,[1] and Yoshihisa Yamamoto[1,3]

Diversity in correlated electron behavior





· ***Multitude of infrared fixed points,*** *selected by physics at intermediate energy scales*

· ***Anomalous behavior*** *at intermediate/low energies (critical (?) fluctuations)*

Dagotto 2005,
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Exact renormalization group

Wikipedia: ‘An **exact renormalization group equation** (**ERGE**) is one that takes **irrelevant couplings** into account.’

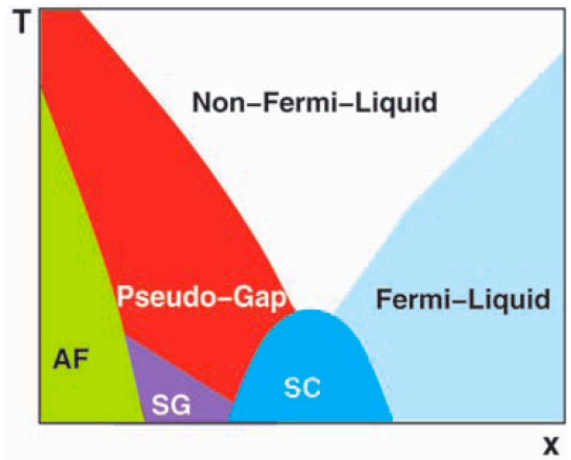
Irrelevant couplings
- *get small near fixed points*
- ***important for selection of fixed point!***

Fermi liquid

charge order

magnetic order

superconductor



Exact renormalization group equation

Start with **generating functional** of theory W_k:

Wilson
Wegner, Houghton
Polchinski
Morris
Salmhofer
Kopietz
...

$$e^{-W_k(\xi,\bar{\xi})} = \int D\psi D\bar{\psi} \exp\left[-S_k(\psi,\bar{\psi}) + \langle\bar{\psi},\xi\rangle + \langle\bar{\xi},\psi\rangle\right] \quad ,$$

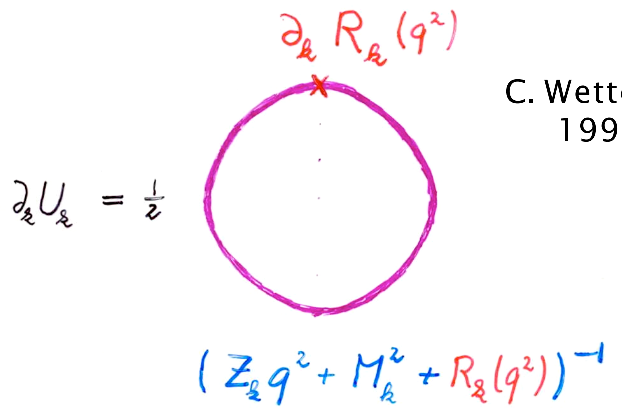
S_k = fermionic action, quadratic part dep. on k

$$U_k(\phi,\bar{\phi}) = W_k(\xi,\bar{\xi}) - \langle\bar{\xi},\phi\rangle - \langle\bar{\phi},\xi\rangle, \quad \frac{\delta W_k}{\delta\bar{\xi}} = \phi$$

generates 1PI vertex functions

k-derivative:

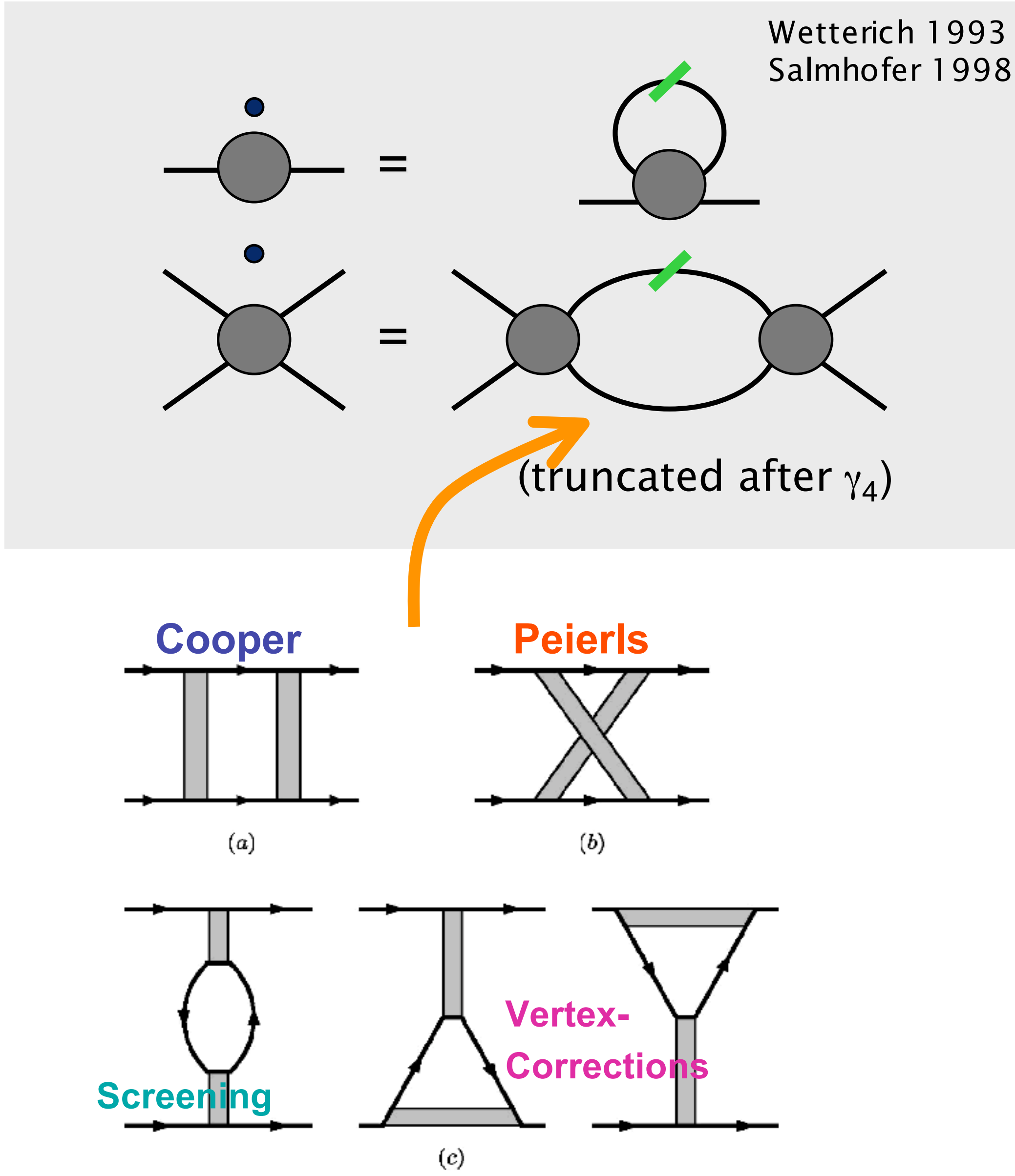
→ exact 1-loop equation for 1PI-generating functional U_k



C. Wetterich
1993

Infinite hierarchy of RG equations ... but unbiased

- Exact flow equation for generating functional when Λ is changed $\rightarrow$ hierarchy of 1-loop equations for 1PI vertices
- Needs truncation, 6pt vertex set to 0 $\rightarrow$ perturbative treatment



Includes all important fluctuation channels on equal footing!

Diversity enters here!

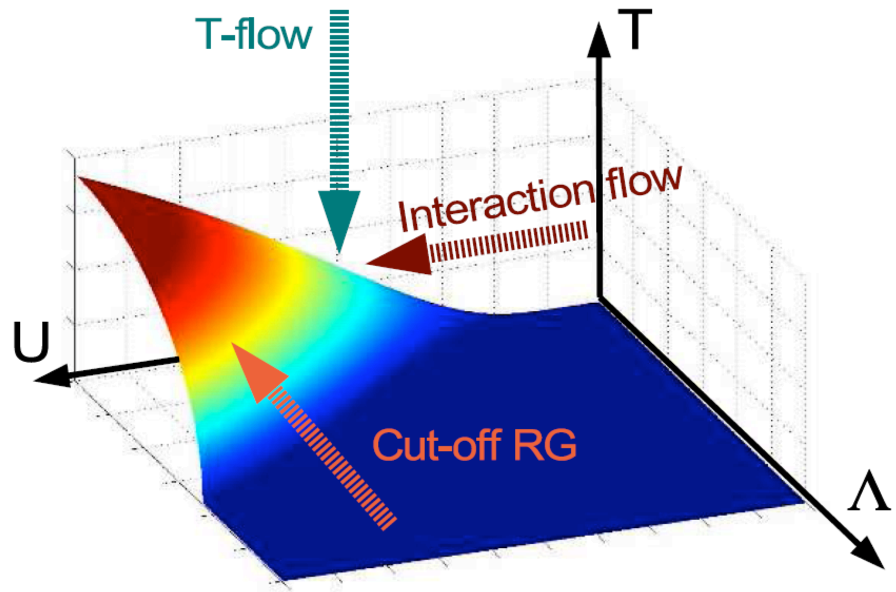
Different flow parameters

$$S = T\sum_n \sum_s \int \frac{d^2k}{(2\pi)^2} \overline{\psi}_s(\vec{k}, i\omega_n)\,(i\omega_n - \varepsilon_{\vec{k}})\,\psi_s(\vec{k}, i\omega_n)$$ = ***Q*, quadratic part contains flow parameter**

$$+\frac{1}{2}T^3 \sum_{n_1,n_2,n_3} \sum_{s,s'} \int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2k_3}{(2\pi)^2} V(\vec{k}_1,\vec{k}_2,\vec{k}_3)\,\overline{\psi}_s(\vec{k}_3, i\omega_{n_3})\overline{\psi}_{s'}(\vec{k}_4, i\omega_{n_4})\psi_{s'}(\vec{k}_2, i\omega_{n_2})\psi_s(\vec{k}_1, i\omega_{n_1})$$

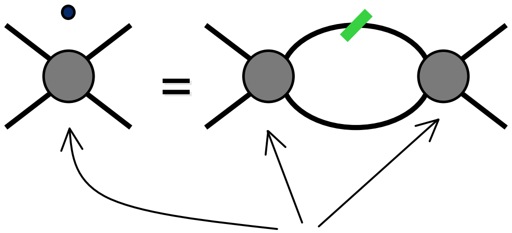
Different choices for **flow parameter *k***:

- Band-energy cutoff Λ,
  Q $\rightarrow$ ***Q*** $\chi^{-1}(\varepsilon/\Lambda)$
  'momentum-shell schemes'
- Frequency cutoff
- Temperature $\rightarrow$ *T*-flow scheme
- Coupling strength *g* $\rightarrow$ interaction flow = flat cutoff
- Combinations

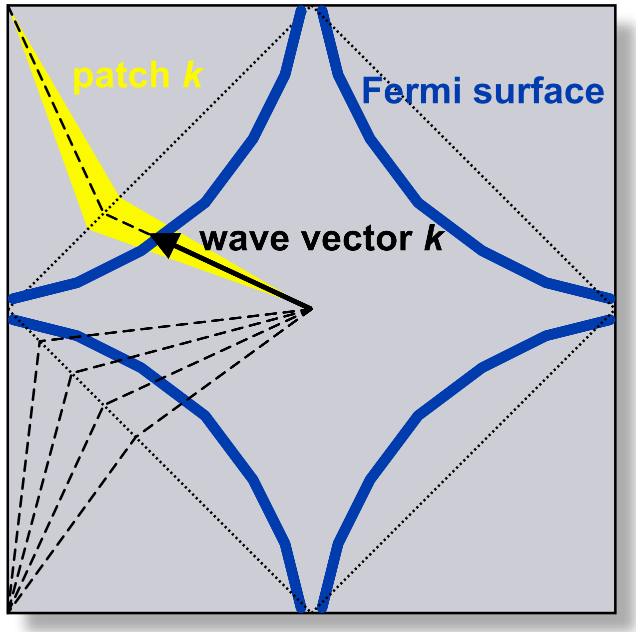


Start where interactions and selfenergy are known, flow to physical point

Implementation for two-dimensional models



- Coupling function $\boldsymbol{V(k_1,k_2,k_3)}$ with incoming wavevectors $\boldsymbol{k_1}$, $\boldsymbol{k_2}$ and outgoing $\boldsymbol{k_3}$
- Discretize: approximate $\mathbf{V}(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3})$ as **constant** for $\boldsymbol{k_1}$, $\boldsymbol{k_2}$ and $\boldsymbol{k_3}$ **in same patch**
- Up to 144 patches (good angular resolution), consistent with other discretizations
- Frequency dependence can be taken into account (→ S.W. Tsai)



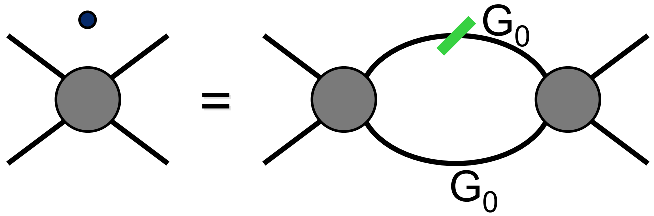
Zanchi and Schulz 1997

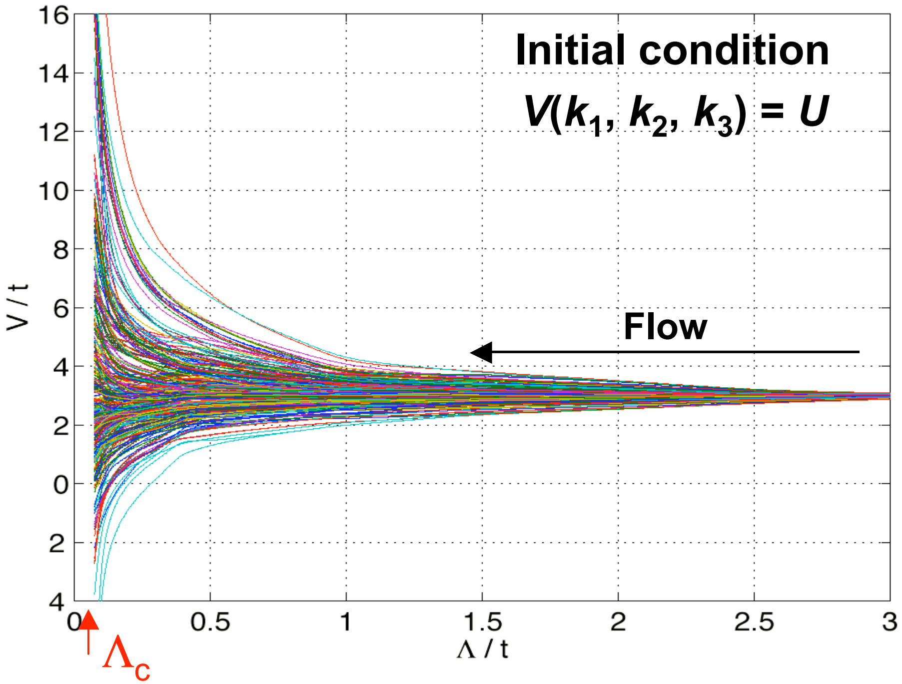
***Functional RG**: flow of coupling functions, derived from RG eqn for generating functional*

The basic picture: Flows to strong coupling

Flow to strong coupling

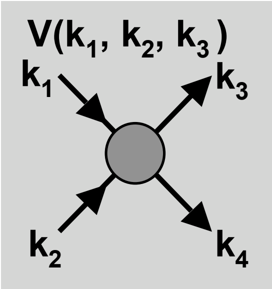
Flows without self-energy feedback:
Analysis of flow to strong coupling





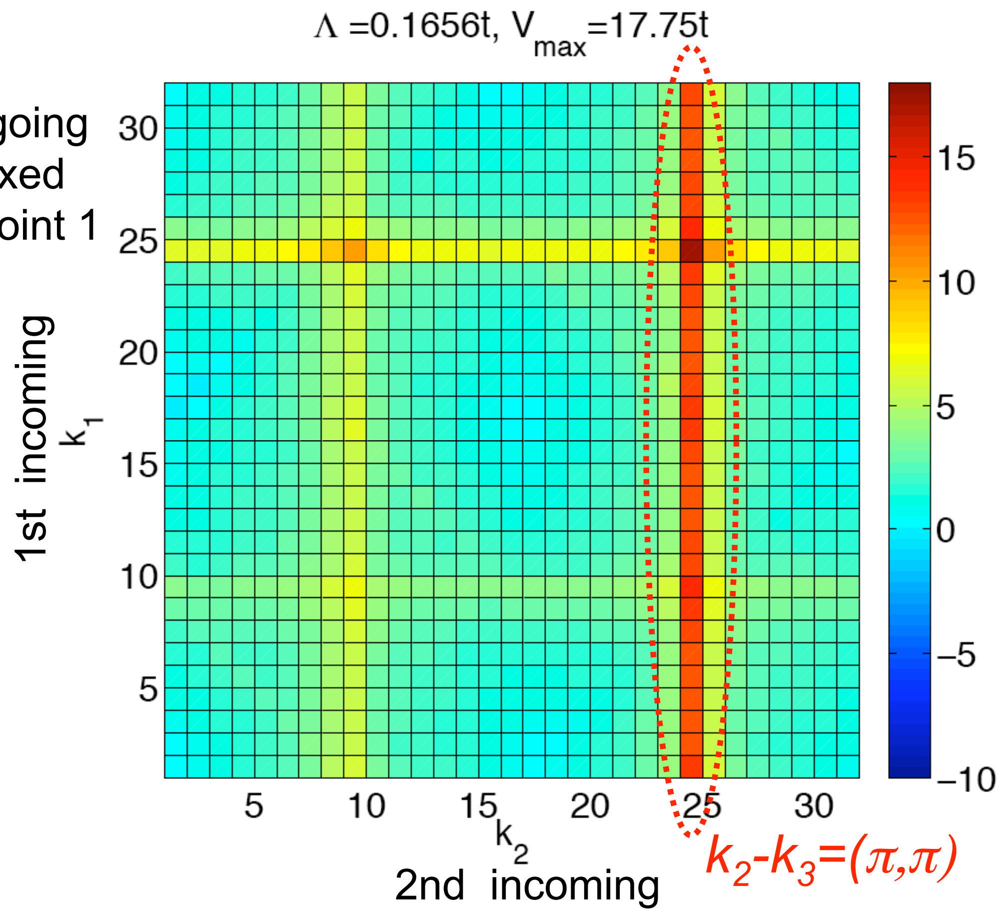
Leading low-energy correlations?

Energy scales?

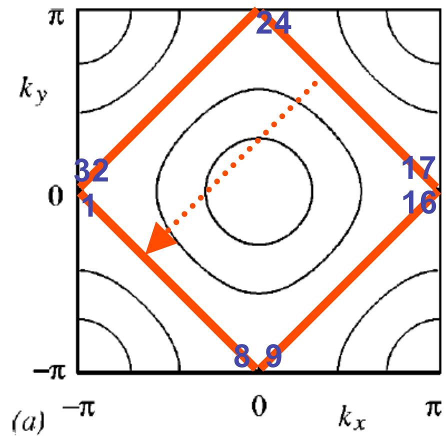


Emergent collective behavior: Spin-density wave

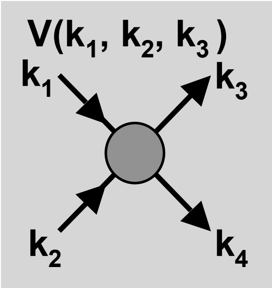
1st outgoing k_3 fixed at point 1



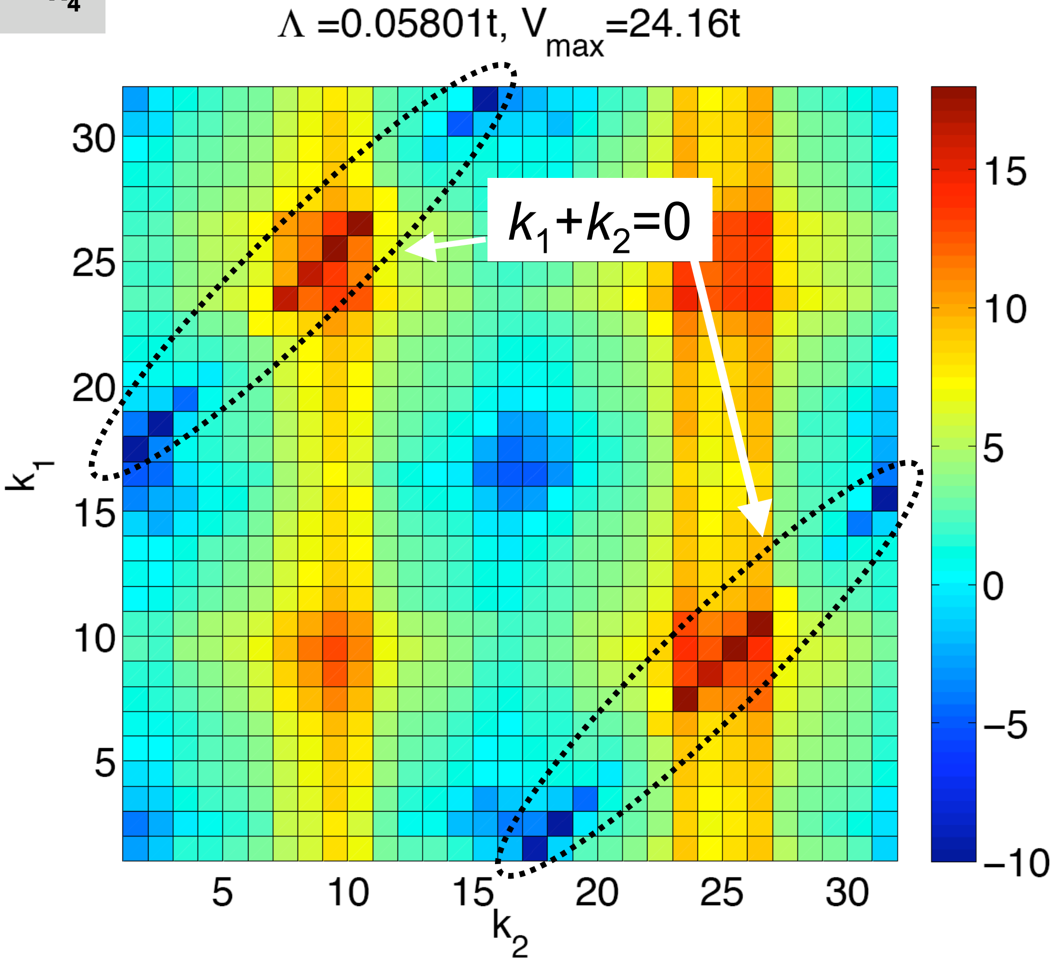
- Fully nested Fermi surface,
- U=2t, t'=0, T=0.001t



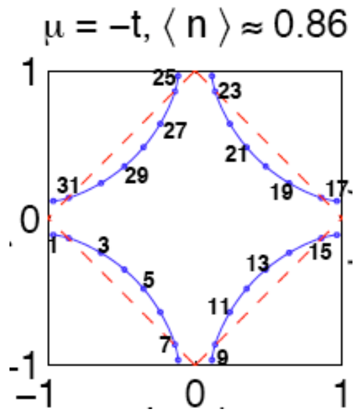
Interpretation: antiferromagnetic spin-density wave

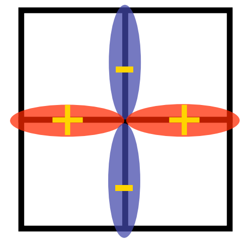


Emergent collective behavior: *d*-wave pairing on square lattice



- Imperfectly nested 'high-T_c' Fermi surface
- $t'=-0.3t$, $U=3t$, $T=0.001t$

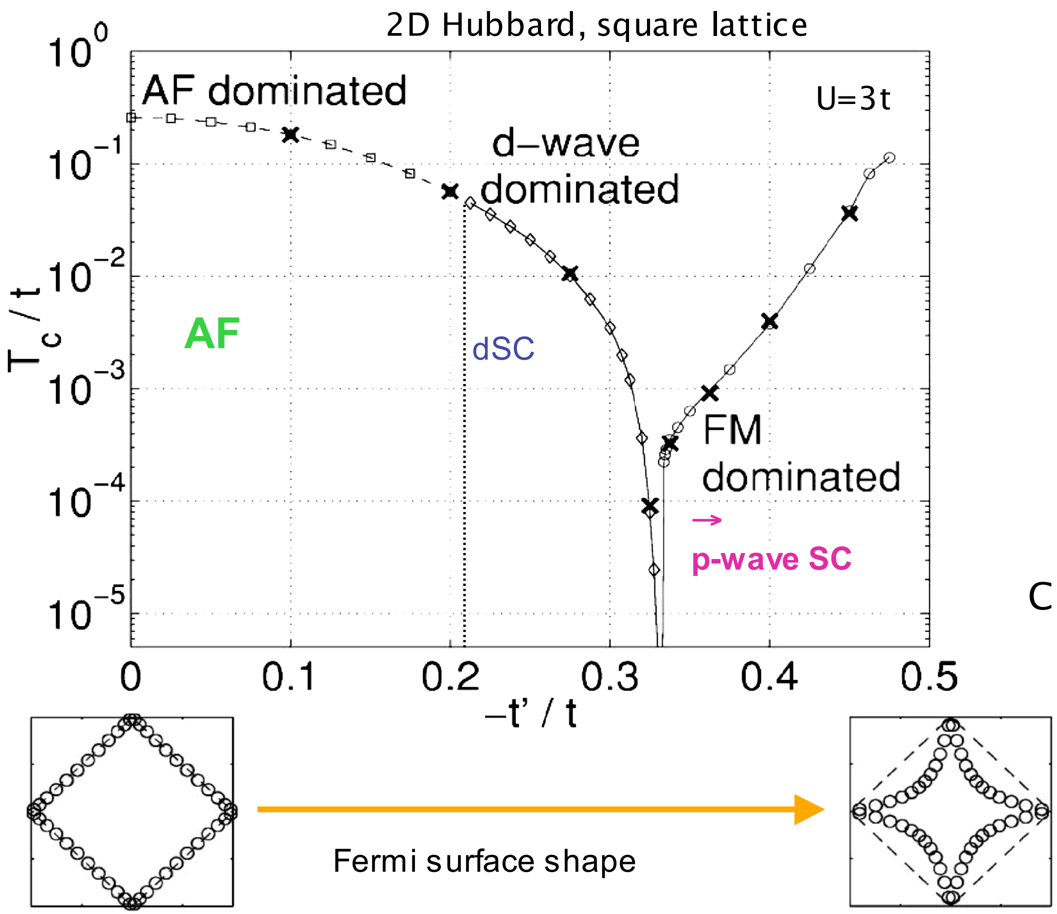




Zanchi, Schulz, 1997
Halboth, Metzner 2000
CH et al. 2001, Tsai, Marston 2001

$d_{x^2-y^2}$-wave Cooper pairing instability!

Competing orders near van Hove filling



CH& Salmhofer, PRL 2001

AF spin-fluctuations induce $d_{x^2-y^2}$-wave Cooper pairing instability
Triplet pairing near ferromagnetic instability

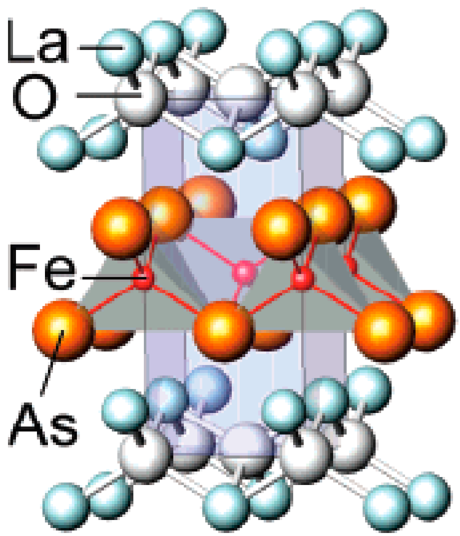
New testing grounds?

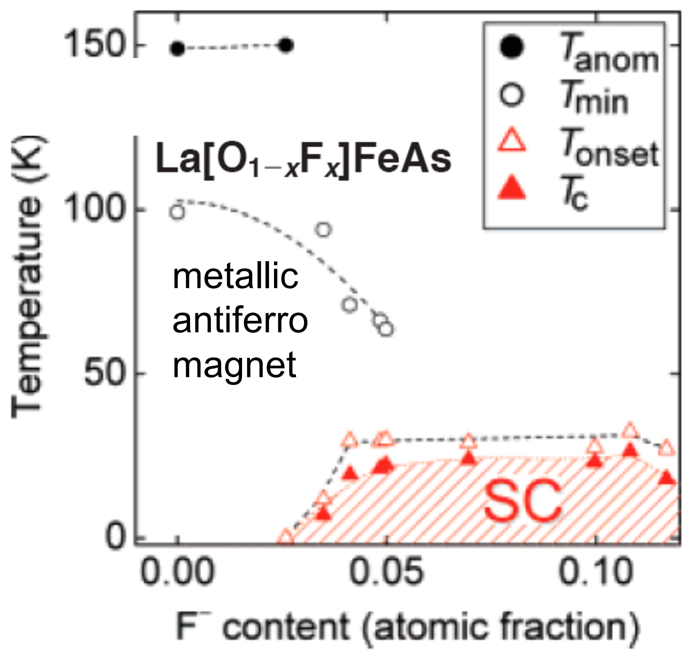
Iron-Based Layered Superconductor La[$O_{1-x}F_x$]FeAs (x = 0.05–0.12) with T_c = 26 K

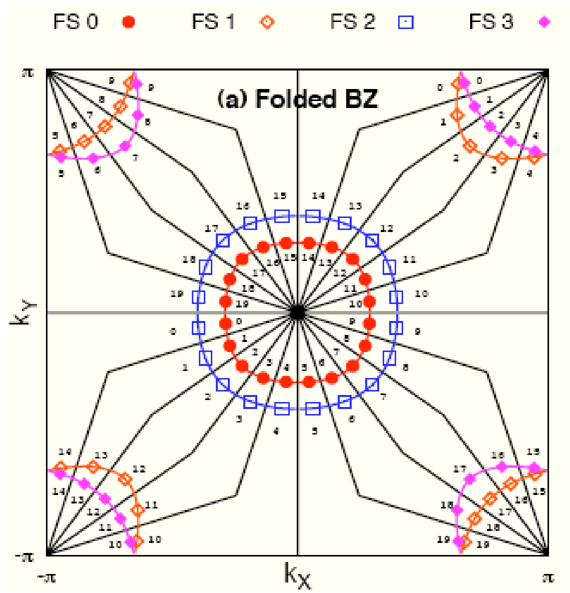
Yoichi Kamihara,*,† Takumi Watanabe,‡ Masahiro Hirano,†,§ and Hideo Hosono†,‡,§

ERATO-SORST, JST, Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, Materials and Structures Laboratory, Tokyo Institute of Technology, Mail Box R3-1, and Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan

Received January 9, 2008; E-mail: hosono@msl.titech.ac.jp



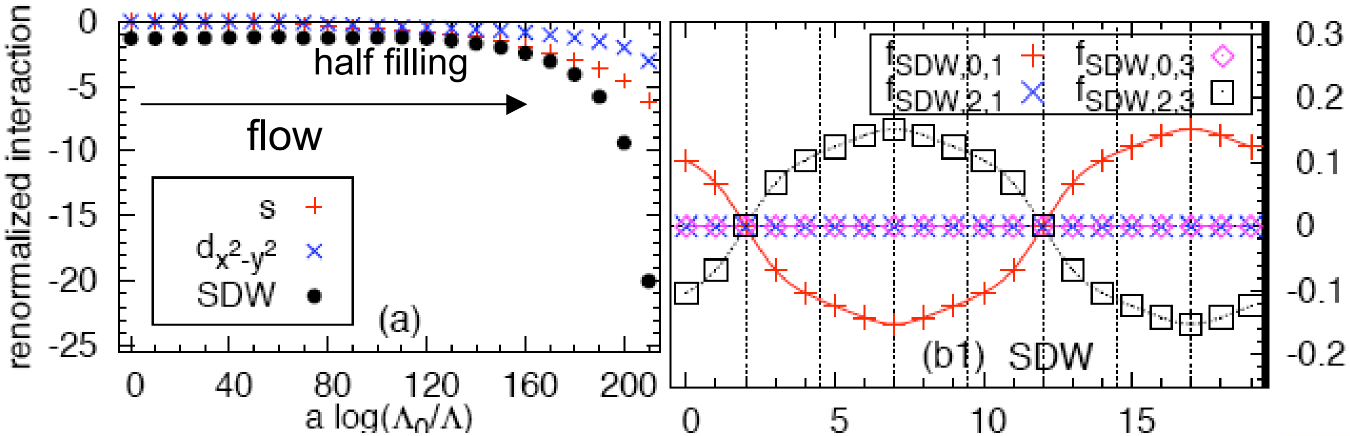




- Layered oxypnictides: (RE)$O_{1-x}F_x$FeAs: T_c up to 55K
- Minimal model: 2 bands (=4 Fermi surfaces in folded zone)

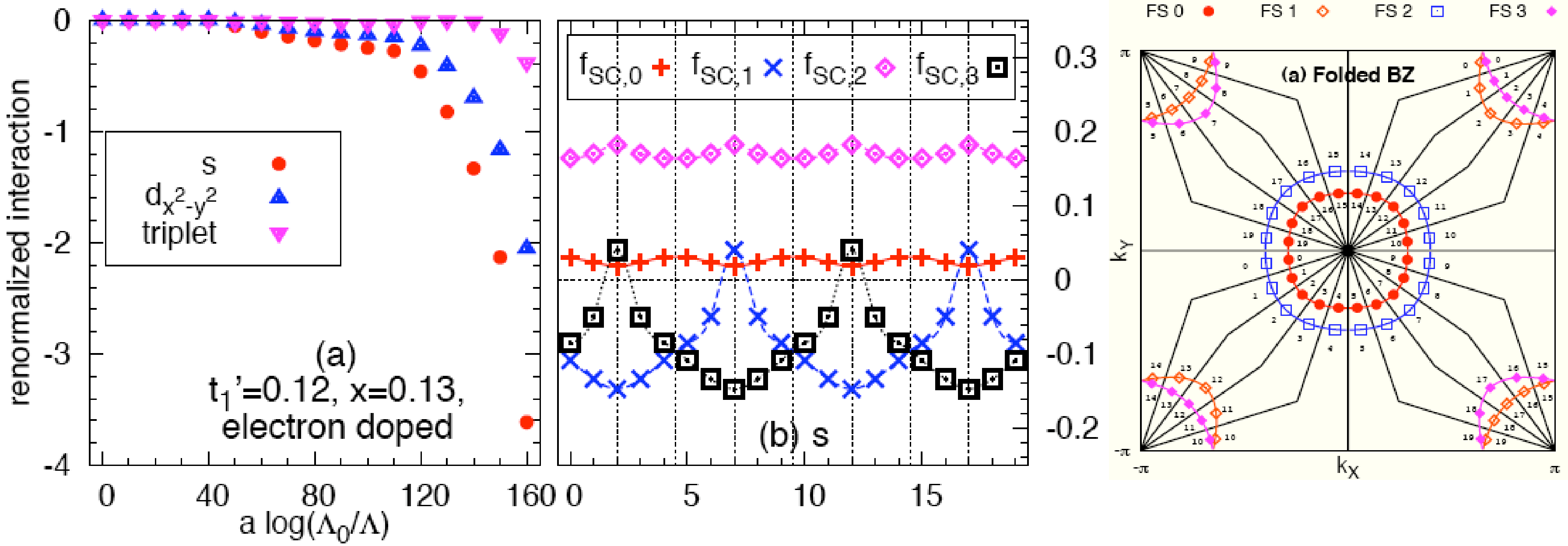
Pairing in oxypnictides

Wang, Zhai, Ran,
Vishwanath, DH Lee
0805.3343



Nodal AF spin density wave

for zero doping



Unconventional *s*-wave pairing at reasonable scales

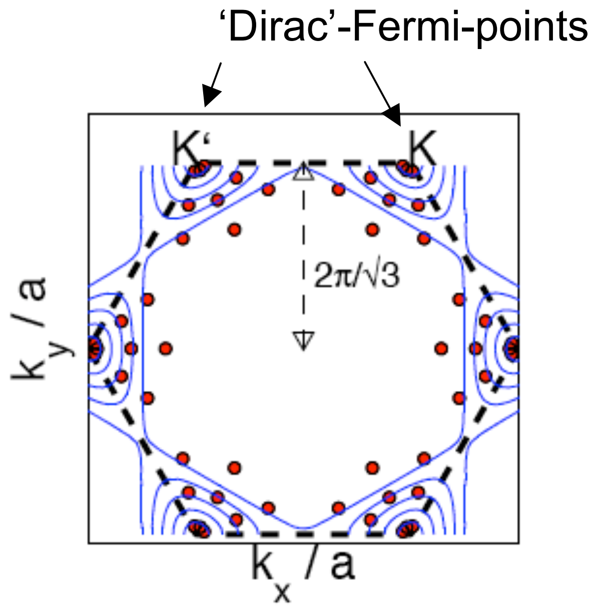
Graphene:
Many-body effects on the honeycomb lattice?
Cooper pairing?

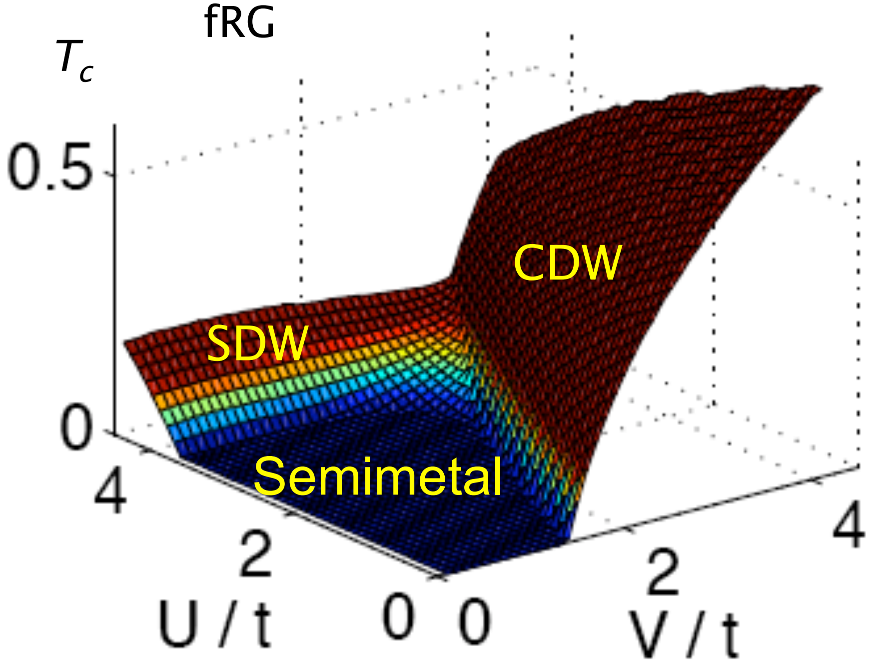
Ordering tendencies on honeycomb lattice: half filling

Short-range interactions *U,V*

$$H = -t \sum_{\langle i,j \rangle, s} \left(c^\dagger_{i,s} c_{j,s} + c^\dagger_{j,s} c_{i,s} \right) + U \sum_i n_{i,\uparrow} n_{i,\downarrow} + V \sum_{\langle i,j \rangle,\, s,s'} n_{i,s} n_{j,s'}$$

graphene:
$t \approx 2.5$eV



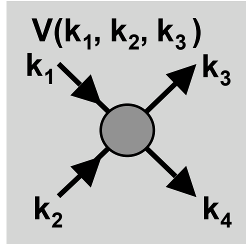


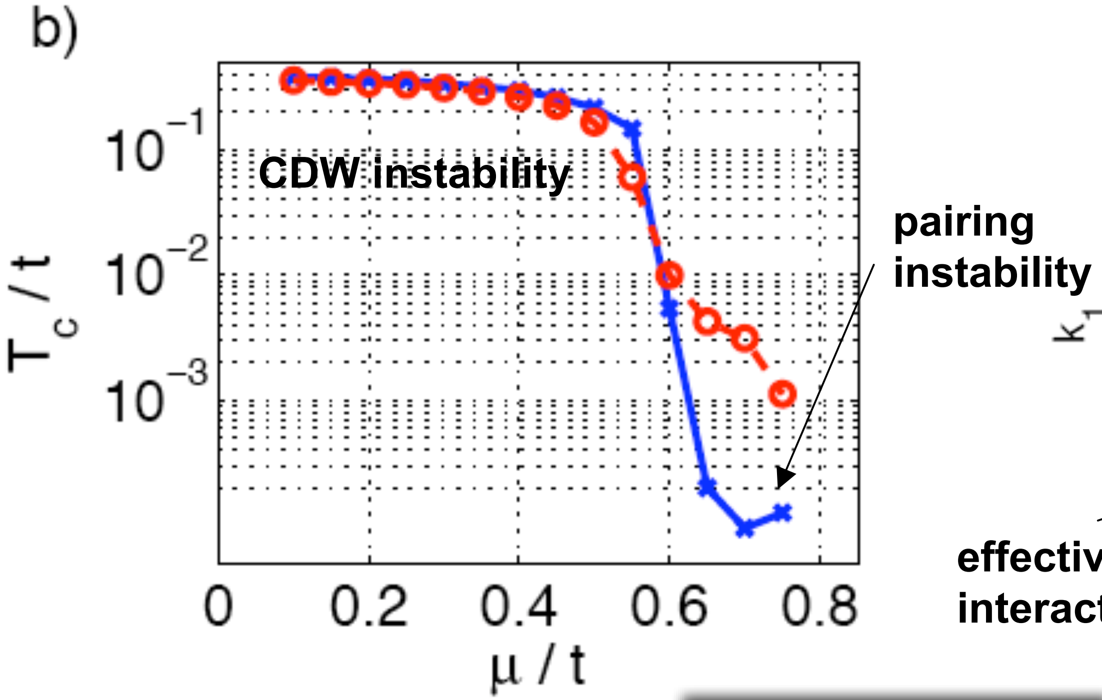
CH 2008, cf. Herbut 2006, Sorella&Tosatti 1992

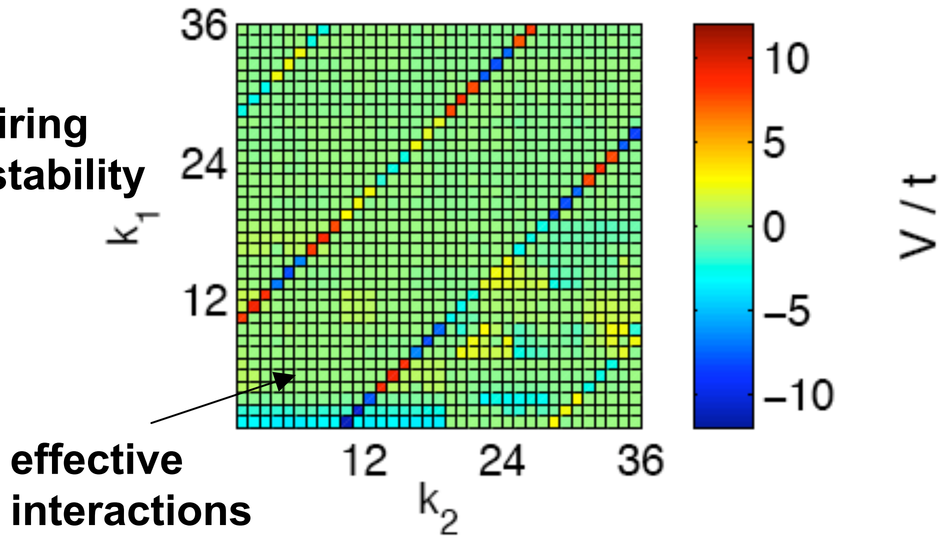
- Nonzero critical *U, V* required for flow to strong coupling
- Dominant *U*: AF spin density wave instability
- Dominant *V*: charge density wave

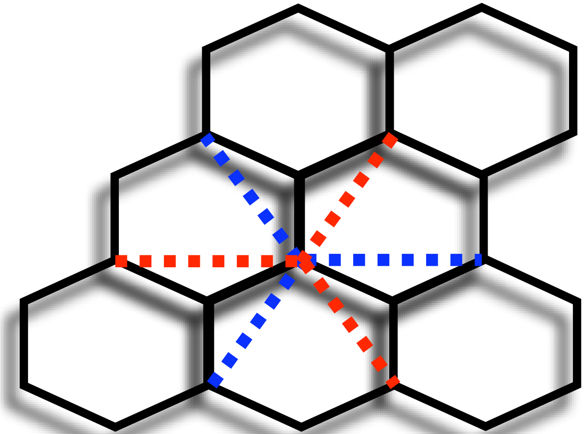
Dope away from half filling! ... pairing?

Doping the CDW regime



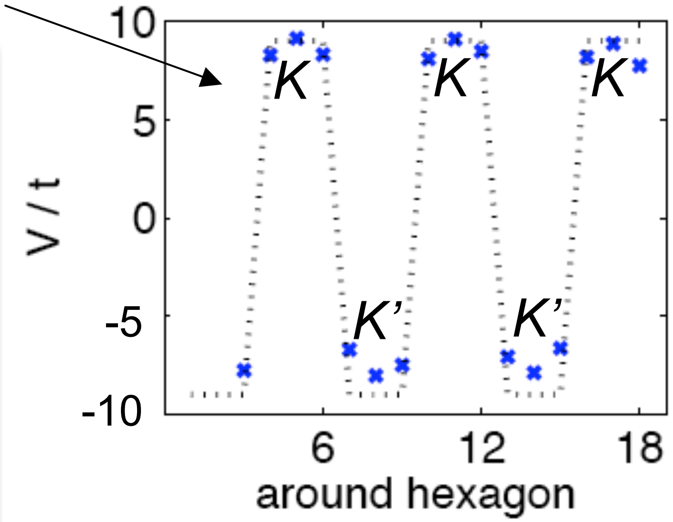






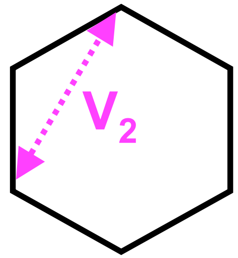
Pairing instability, ∝cos(3ϕ) around hexagon →

***f*-wave triplet pairing (2nd nearest neighbors)**

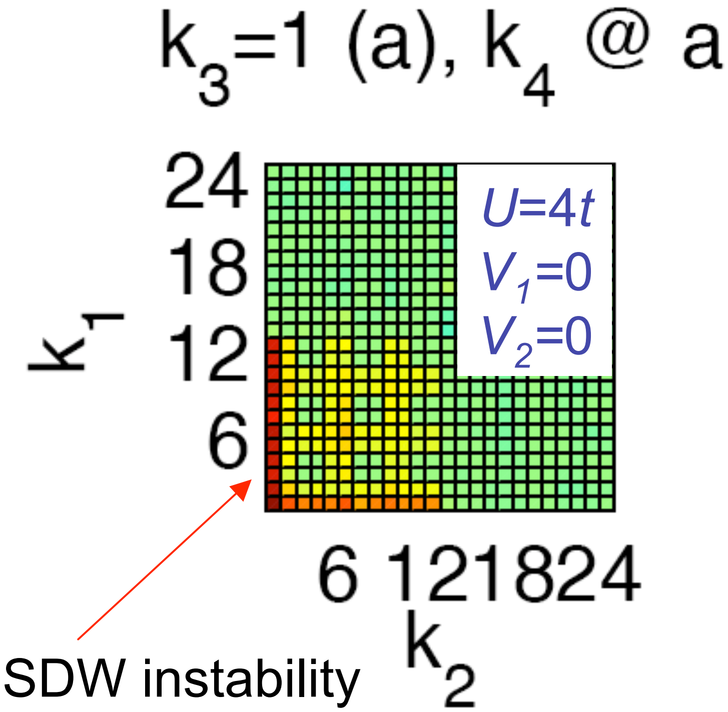


CH, PRL 2008

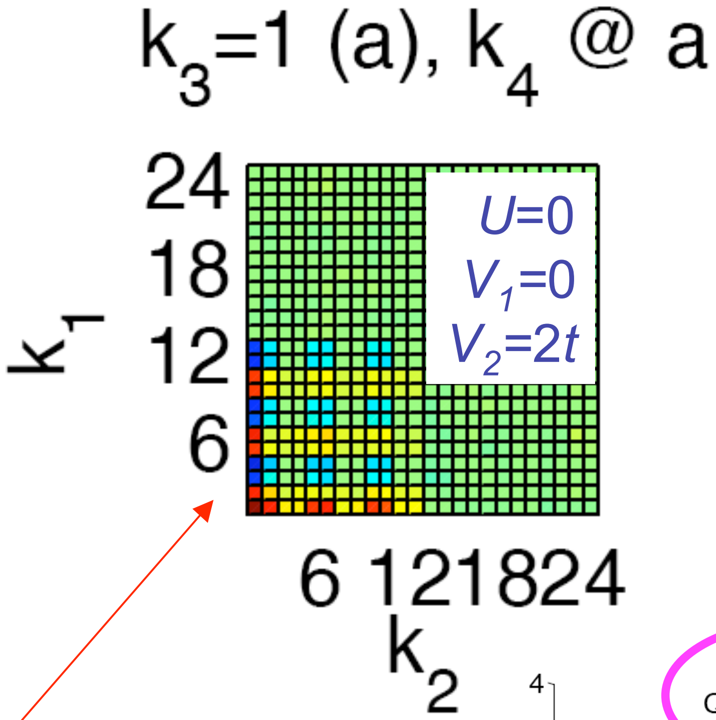
2nd nearest neighbor repulsion

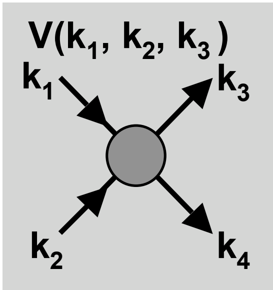


- Novel spin instability for **dominant next-nearest-neighbor repulsion V_2 at half filling**

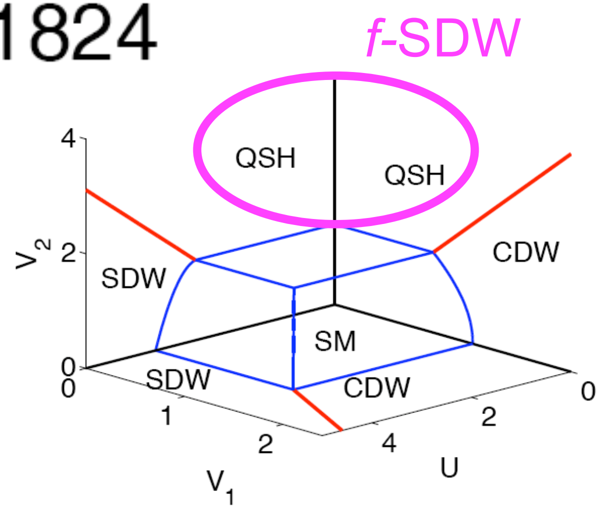


SDW instability





same wavevectors as SDW, but
odd formfactor $\propto \cos(3\phi)$
→ ***f*-SDW instability**
→ nontrivial order parameter



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Quantum Spin Hall Effect in Graphene

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(Received 29 November 2004; published 23 November 2005)

We study the effects of spin orbit interactions on the low energy electronic structure of a single plane of graphene. We find that in an experimentally accessible low temperature regime the symmetry allowed spin orbit potential converts graphene from an ideal two-dimensional semimetallic state to a quantum spin Hall insulator. This novel electronic state of matter is gapped in the bulk and supports the transport of spin and charge in gapless edge states that propagate at the sample boundaries. The edge states are nonchiral, but

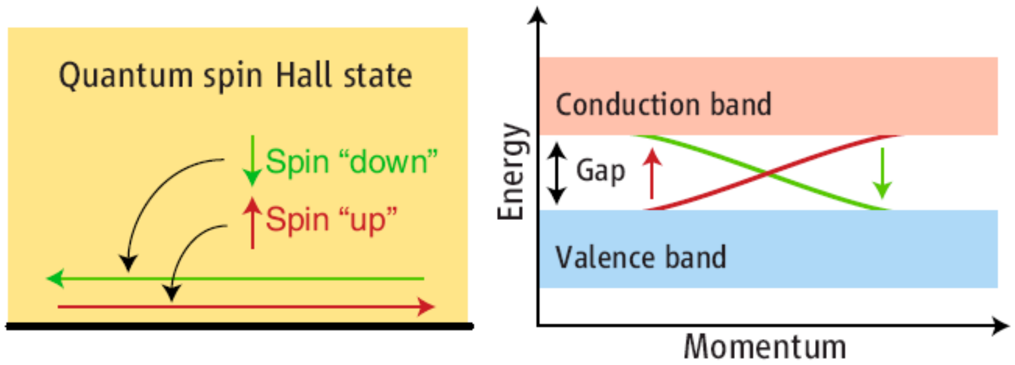
Mass term for Dirac fermions

$$\mathcal{H}_{SO} = \Delta_{so}\psi^\dagger \sigma_z \tau_z s_z \psi$$

sublattice

spin

K, K' points



fSDW-meanfield generates same term from interactions

Edges of fSDW-state carry spin current

= 'non-trivial' Mott QSH insulator

without spin: Haldane PRL 1988

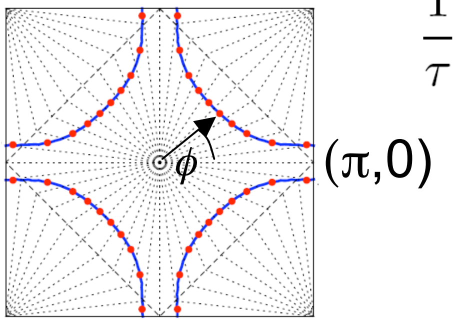
Raghu, Qi, CH, Zhang PRL 2008

Observables beyond ground state correlations?

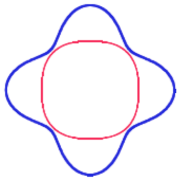
Quasiparticle scattering rate in overdoped cuprates

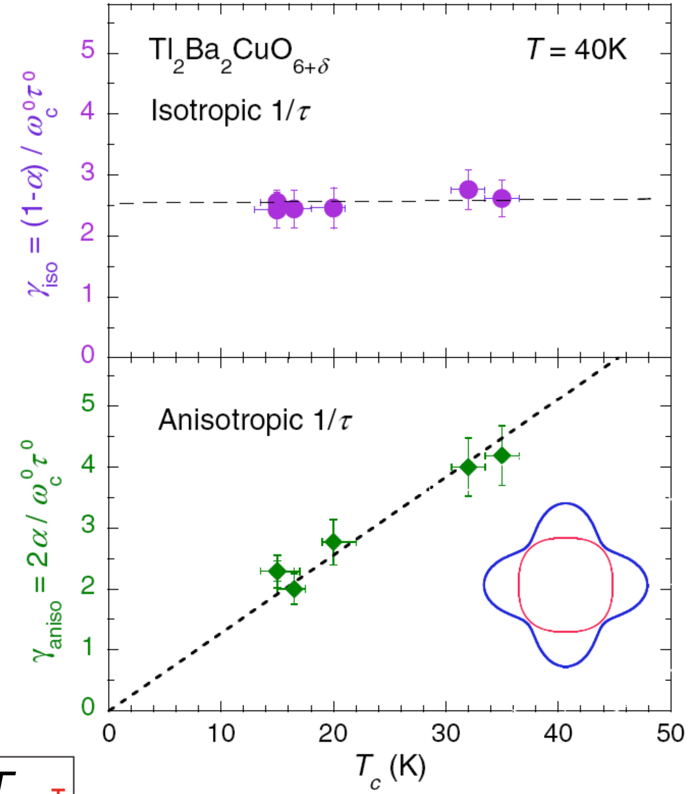
Angle-dependent magnetotransport (ADMR, N. Hussey group, Tl2201):

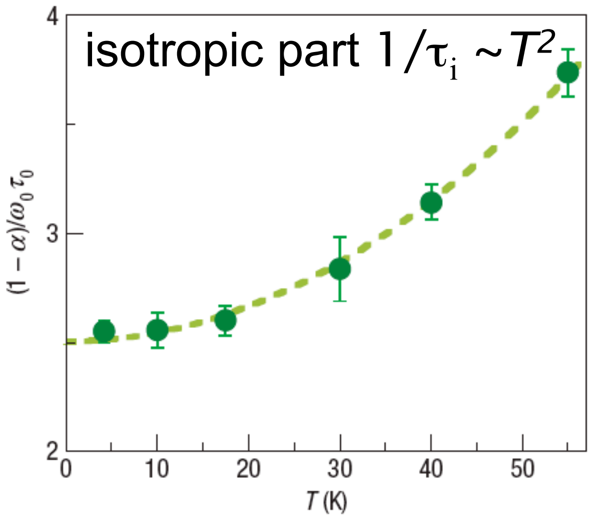
Inplane **transport scattering rate** $1/\tau$ of quasiparticles **anisotropic**, enhanced near $(\pi,0)$

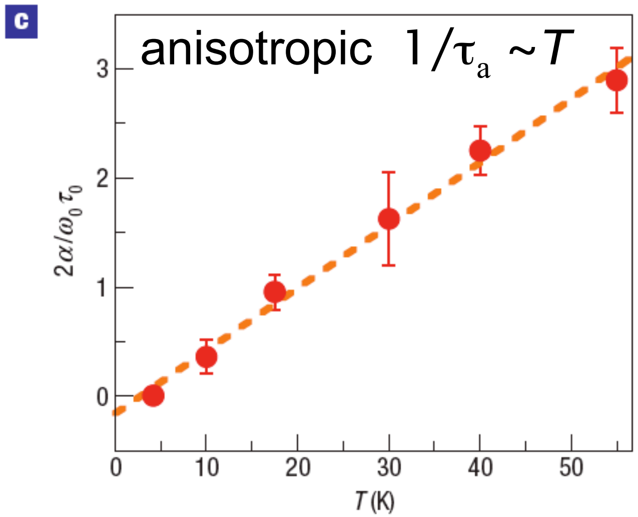


$$\frac{1}{\tau}(\phi,T) = \frac{1}{\tau_i}(T) + \frac{1}{\tau_a}(\phi,T)$$









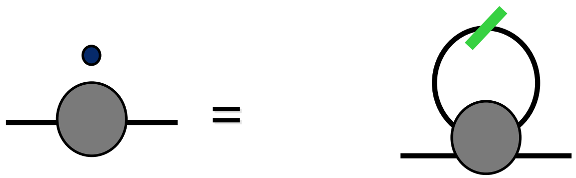
Overdoped cuprates are Fermi liquids:

Functional RG should reproduce experimental trends!

Quasiparticle scattering rate from fRG

Ossadnik,
CH, Rice,
Sigrist 2008
CH 2001

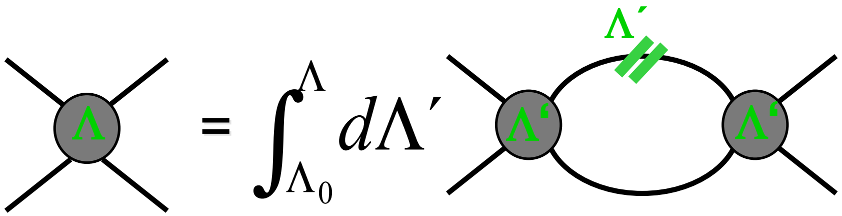
Flow of self-energy $\Sigma(k_F,\omega=0)$



Problem: In our approximation coupling function is frequency-independent & real self-energy real

$\rightarrow 1/\tau = 0$!

Solution: Reconstruct 2-loop frequency dependence of interaction



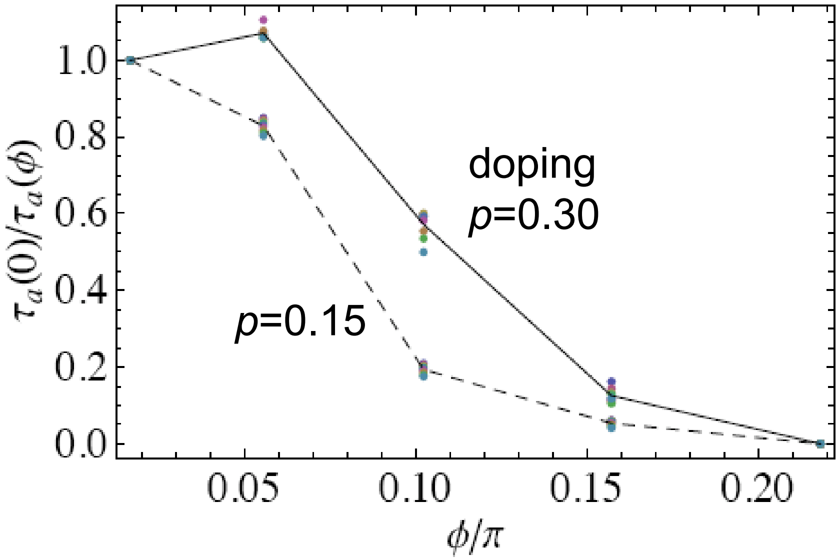
Insert solution of RG equation for coupling function

$\rightarrow$ Two-loop equation for self-energy

$\rightarrow$ Im $\Sigma(k_F,\omega=0) \neq 0$ (e.g. T^2 for spherical Fermi surface)

Quasiparticle scattering rate from fRG

Ossadnik,
CH, Rice,
Sigrist 2008

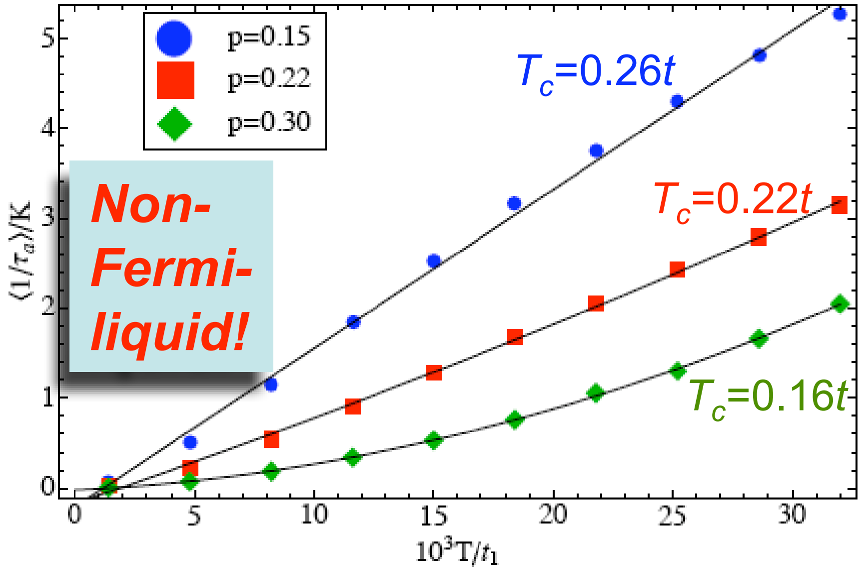


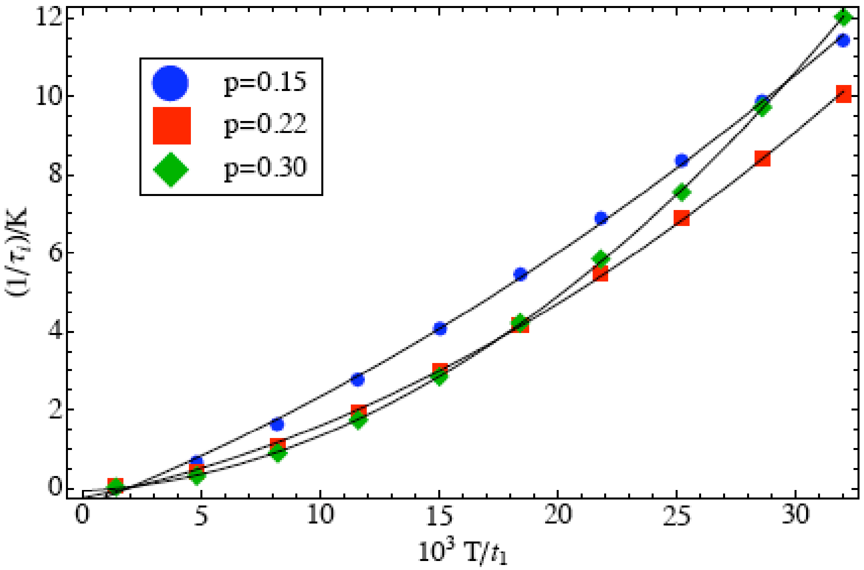
fRG with Cooper instability suppressed:

· anisotropic part $1/\tau_a \sim T$, grows with T_c

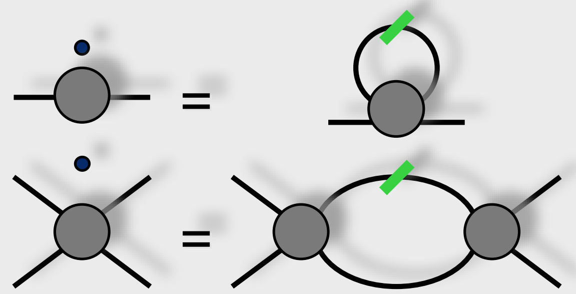
· isotropic part $1/\tau_i \sim T^2$, independent of T_c

Rise of T_c correlated with breakdown of Fermi liquid!





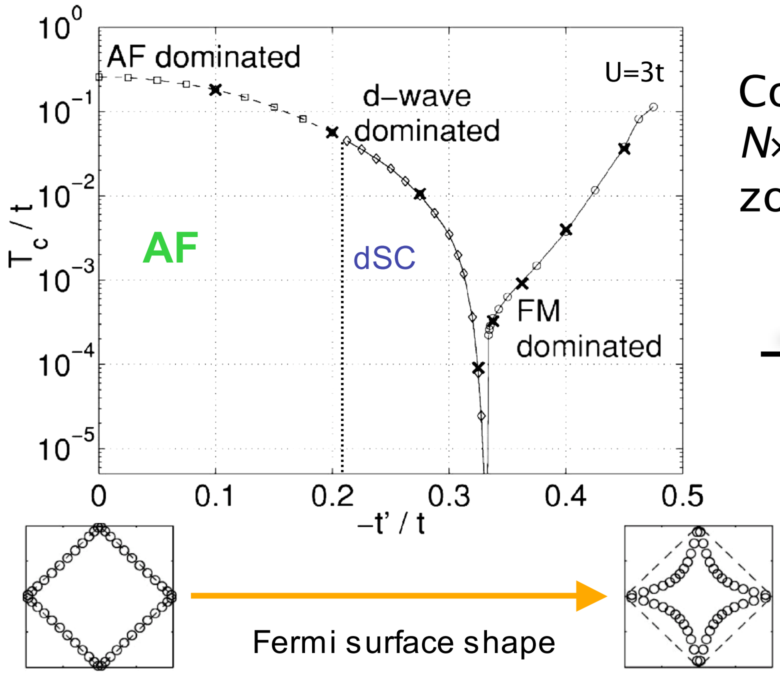
Self-energy flows



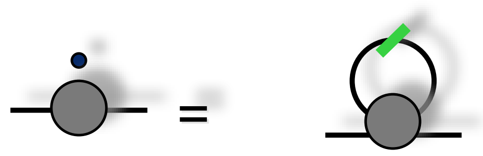
(truncated after γ_4)

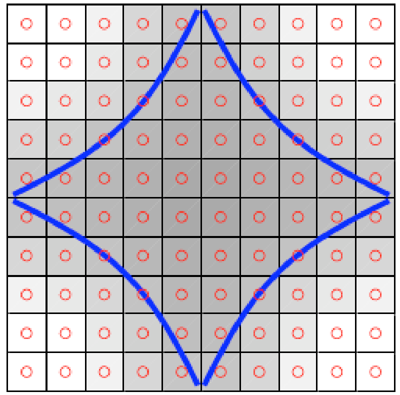
Re Σ: Renormalized dispersion

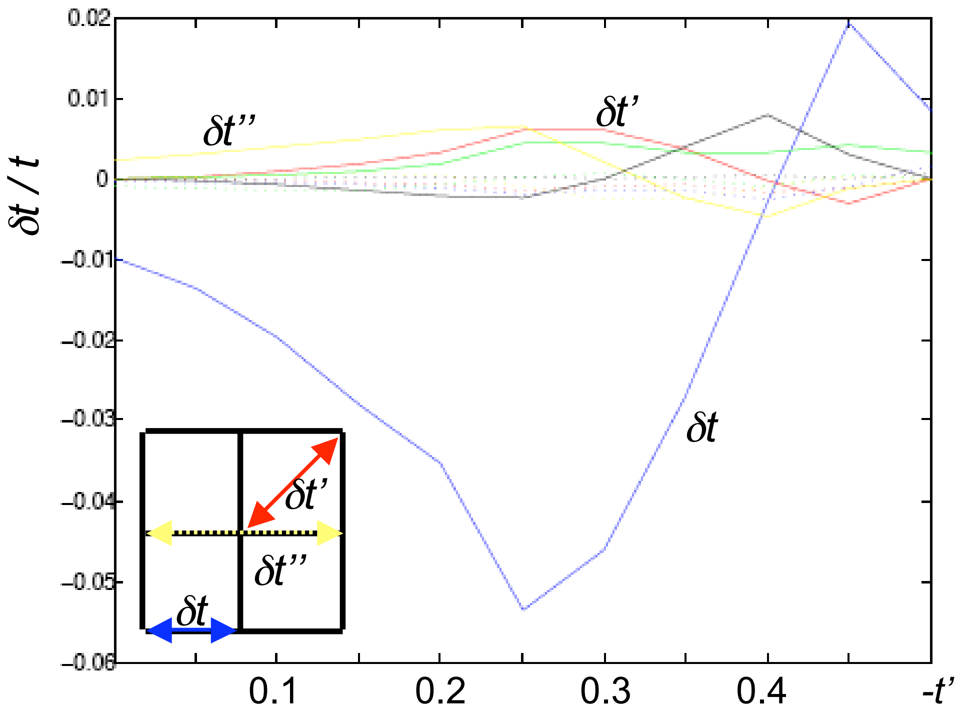
Übelacker, Ortloff, CH 2008



Compute Re $\Sigma(k,\omega=0)$ on $N\times N$ points in Brillouin zone (flat cutoff flow)





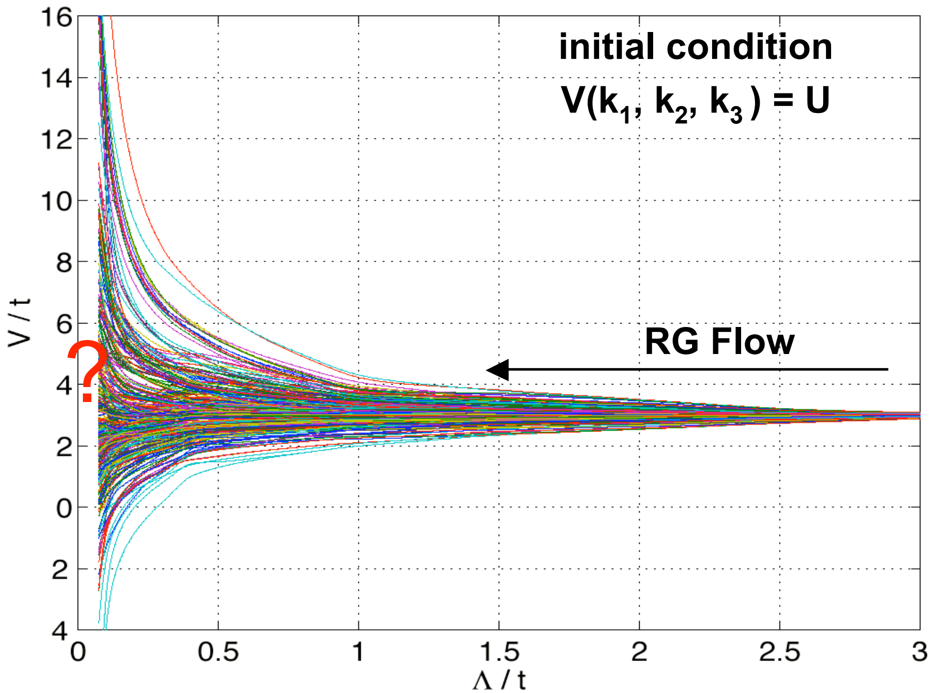


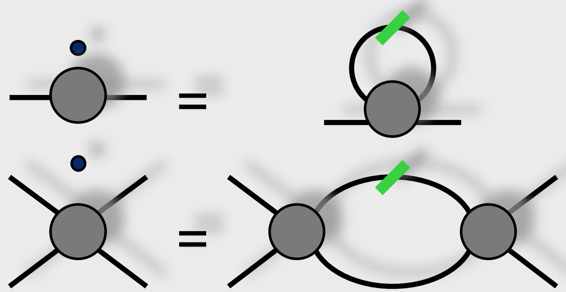
→ Renormalized hopping amplitudes from Re $\Sigma(k,\omega=0)$ near instability at van Hove filling (flat cutoff flow)

→ **Instabilities unchanged** (cf. A. Katanin, 2-loop, 2008)

Into the strong coupling regime ?

Flow shows instability toward symmetry breaking, but no controlled access into low-energy state





(truncated after γ_4)

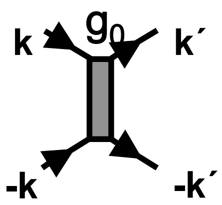
→ *Include flow of* ***self-energy!***

→ *Allow for* ***symmetry breaking***

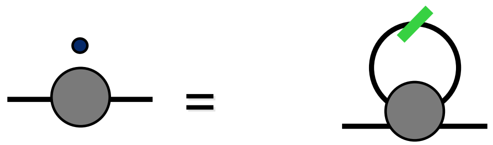
Here: *<u>Fermionic approach</u> (vs. bosonization schemes)*

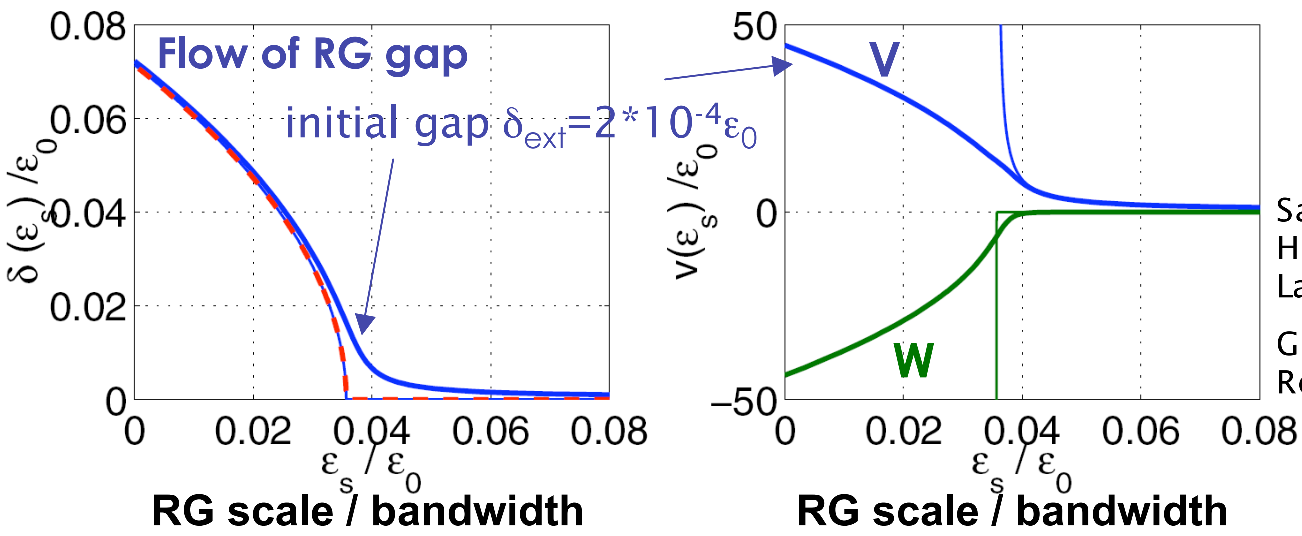
Flow into symmetry breaking regime

- Toy model: reduced BCS Hamiltonian
- Include small symmetry-breaking field into initial condition for self-energy (anomalous self-energy Δ_{ext})
- Divergence regularized by initial Δ_{ext}, self-energy Δ_Λ flows to correct value



$$\mathbb{G}_0(\xi,\omega_n) = \frac{1}{\omega_n^2+\xi^2+\Delta_{\mathrm{ext}}^2}\begin{pmatrix} -i\omega_n-\xi & -\Delta_{\mathrm{ext}} \\ -\Delta_{\mathrm{ext}} & -i\omega_n+\xi \end{pmatrix}$$





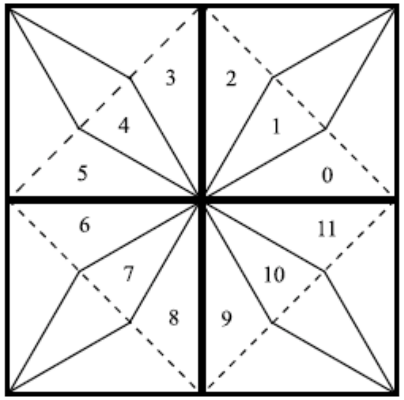
Salmhofer, Honerkamp, Metzner & Lauscher 2004

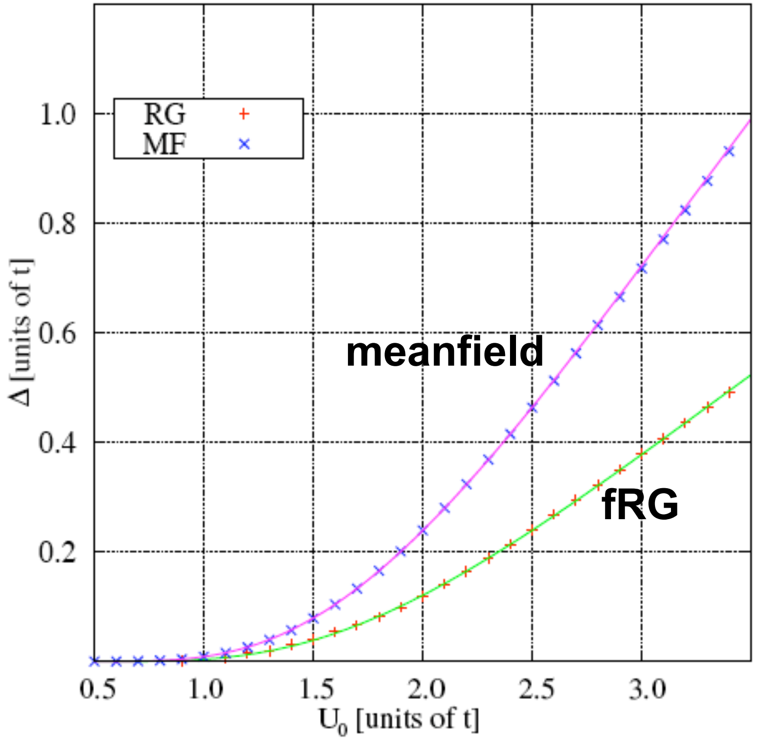
Gersch, Honerkamp, Rohe, Metzner 2005

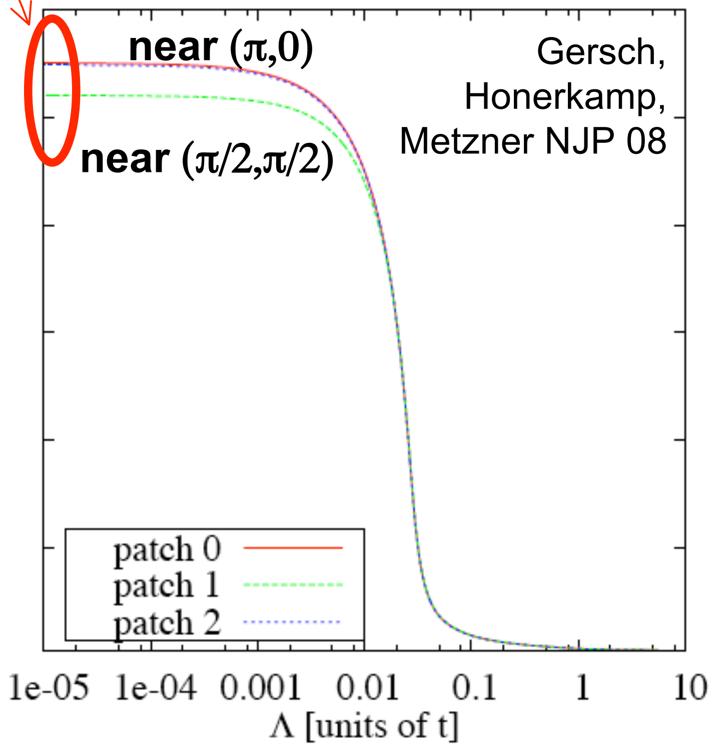
modified RG eqns, Katanin 2004

Application to attractive Hubbard model: beyond meanfield

- Attractive Hubbard on square lattice away from half filling
- Gap magnitude suppressed w.r.t. meanfield result
- Gap function anisotropic



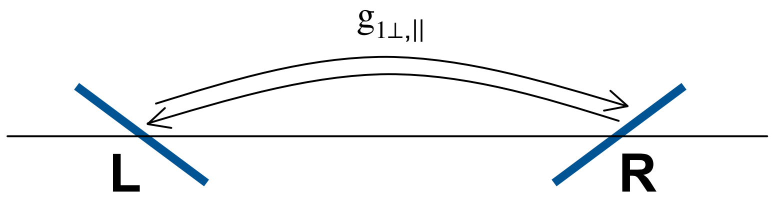




Competing order in 1D model

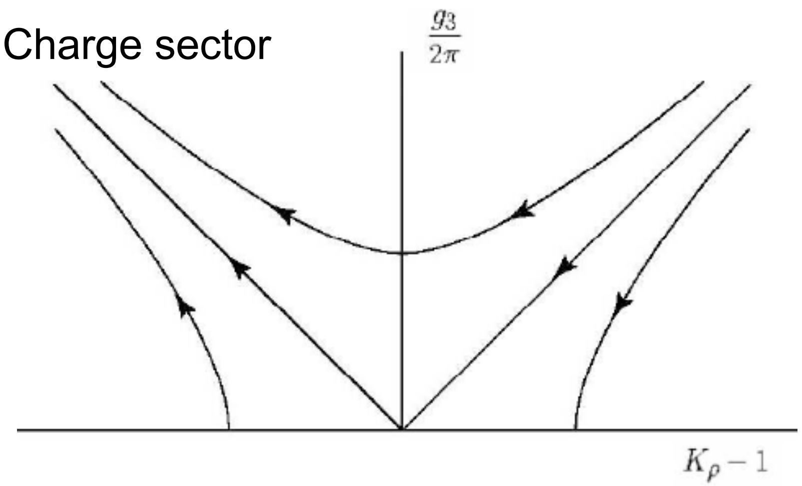
$$H_F = H_0 + v_F \int \mathrm{d}x g_{1\perp} \sum_{\sigma} L_{\sigma}^{\dagger} R_{-\sigma}^{\dagger} L_{-\sigma} R_{\sigma} - g_{2\perp} \sum_{\sigma} L_{\sigma}^{\dagger} R_{-\sigma}^{\dagger} L_{\sigma} R_{-\sigma}$$
$$- g_{\|} \sum_{\sigma} L_{\sigma}^{\dagger} R_{\sigma}^{\dagger} L_{\sigma} R_{\sigma} + g_3 \sum_{\sigma} \left(L_{\sigma}^{\dagger} L_{-\sigma}^{\dagger} R_{\sigma} R_{-\sigma} + R_{\sigma}^{\dagger} R_{-\sigma}^{\dagger} L_{\sigma} L_{-\sigma} \right)$$

Half-filled band, attractive interactions

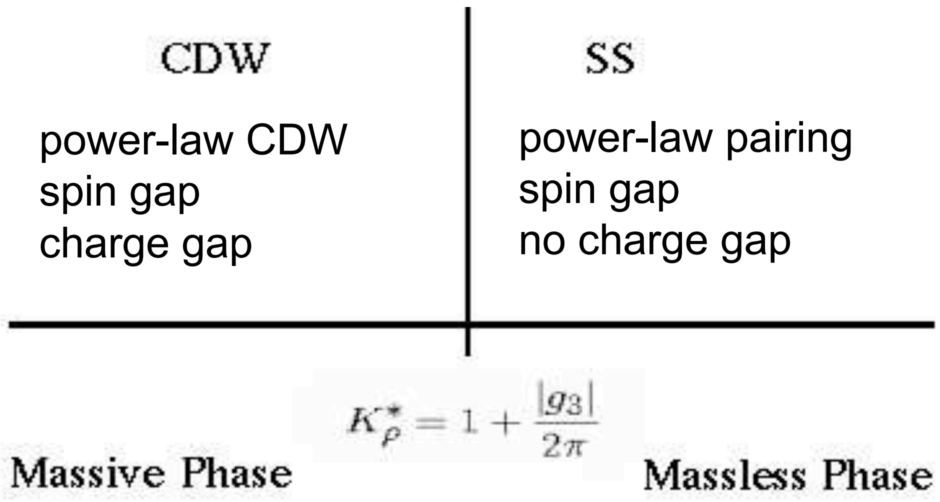


Phase diagram from bosonization

Charge sector



$$K_\rho = \sqrt{\frac{\pi - (g_{\|} + g_{2\perp})}{\pi + (g_{\|} + g_{2\perp})}}$$



Competing (quasi-) long range order

- Extended Nambu formalism for superconducting and CDW order

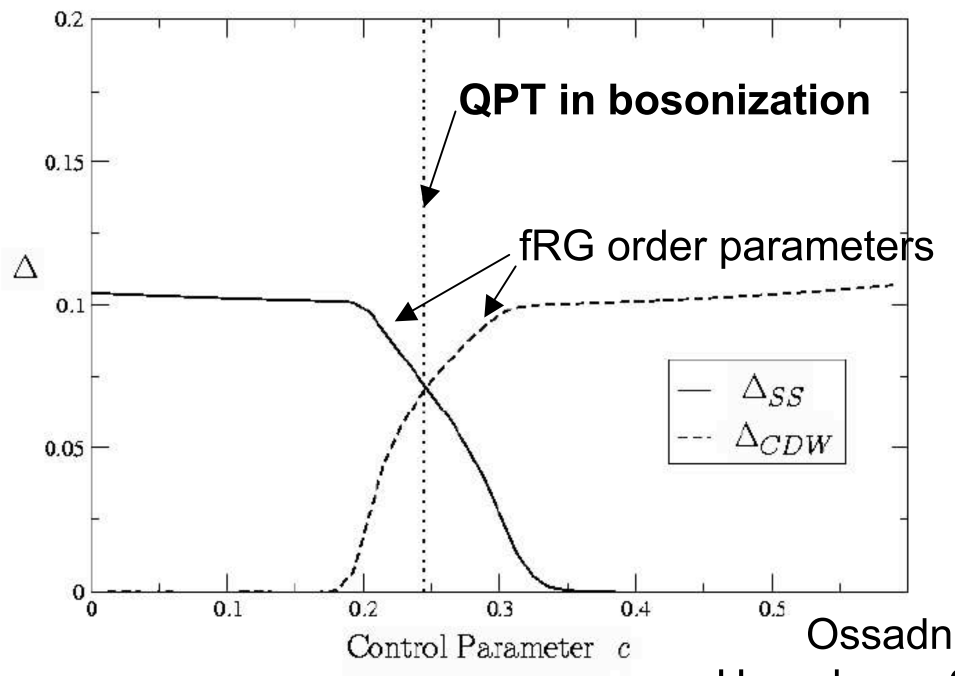
$$\Psi_k = \begin{pmatrix} R_+ \\ \bar{L}_- \\ L_+ \\ \bar{R}_- \end{pmatrix} \qquad Q = \begin{pmatrix} Q_k^R & \Delta_{SS} & \Delta_{CDW} & 0 \\ \Delta_{SS} & \tilde{Q}_{-k}^L & 0 & -\Delta_{CDW} \\ \Delta_{CDW} & 0 & Q_k^L & \Delta_{SS} \\ 0 & -\Delta_{CDW} & \Delta_{SS} & \tilde{Q}_{-k}^R \end{pmatrix}$$

$$Q_k^R = Q_{-k}^L = i\omega - v_F k$$

fRG describes quantum phase transition well (U<0):

$$\begin{aligned} g_1 &= U, \\ g_2 &= (1-c)U, \\ g_{\|} &= -cU, \end{aligned}$$

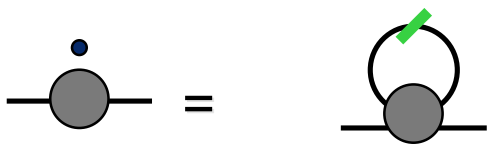
$$K_\rho = \sqrt{\frac{2\pi - U(1-2c)}{2\pi + U(1-2c)}}$$

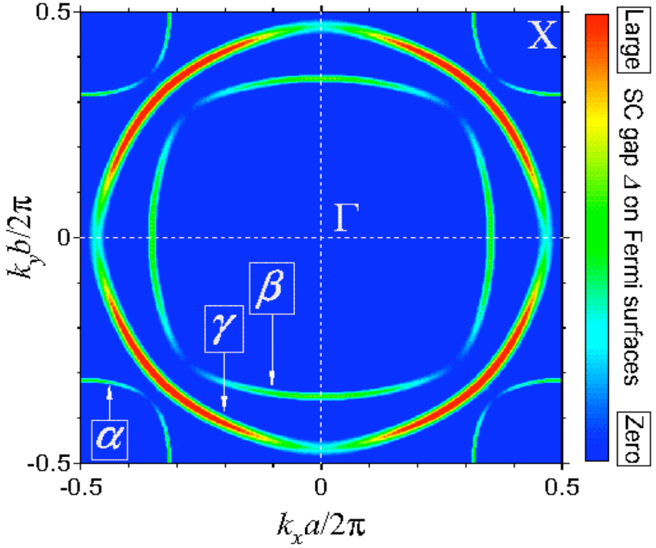


Ossadnik, Honerkamp 08

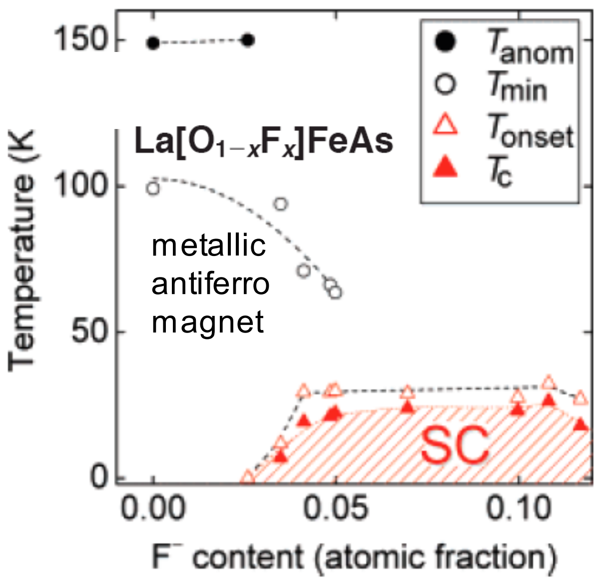
Outlook: Low temperature spectra from fRG

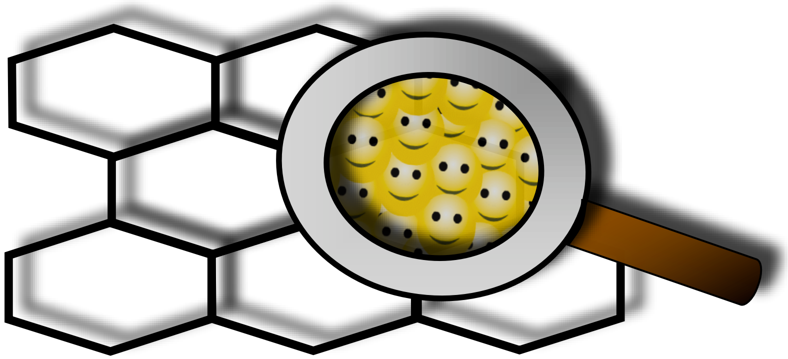
- Use fRG to get gap function **$\Delta(k)$** of superconducting state **around Fermi surface**!





- **Competing order** in 2D systems





... thank you for listening!

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Matthias Ossadnik (now ETH Zürich)

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