

Competing Orders in the Hubbard Model at van Hove filling

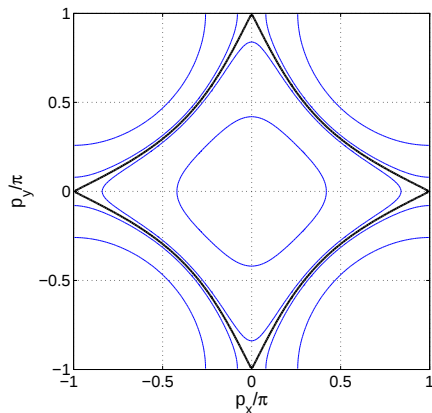
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The 2D (t, t') -Hubbard Model

van Hove filling



+

weak
onsite
repulsion

Figure: The Fermi surface at $-t' = 0.3$

$\Rightarrow$ Instabilities of the Landau Fermi Liquid

The One-Loop Truncation

- 1PI RG in the symmetric phase (without selfenergy)

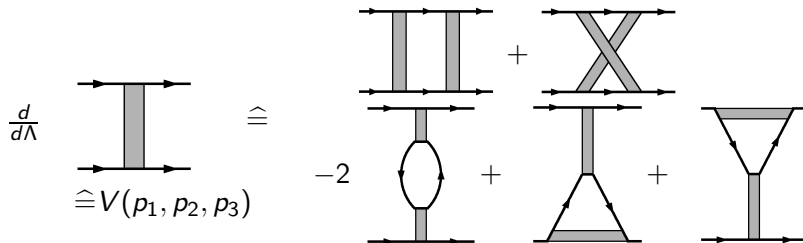
$$\frac{d}{d\Lambda} \hat{\equiv} V(p_1, p_2, p_3) \hat{\equiv} \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \\ \text{Diagram 5} \end{array}$$

The diagram illustrates the one-loop truncation of the 1PI RG equation for the vertex function $V(p_1, p_2, p_3)$. The left side shows the derivative $\frac{d}{d\Lambda}$ of the vertex function, which is equal to the sum of five diagrams representing the one-loop corrections to the vertex function. The diagrams are:

- Diagram 1: A vertical line with two horizontal lines at the top and bottom, representing the tree-level vertex function.
- Diagram 2: A vertical line with two horizontal lines at the top and bottom, and a loop (bubble) in the middle.
- Diagram 3: A vertical line with two horizontal lines at the top and bottom, and a loop (bubble) in the middle, with a factor of -2 in front.
- Diagram 4: A vertical line with two horizontal lines at the top and bottom, and a loop (bubble) in the middle, with a factor of $+$ in front.
- Diagram 5: A vertical line with two horizontal lines at the top and bottom, and a loop (bubble) in the middle, with a factor of $+$ in front.

The One-Loop Truncation

- 1PI RG in the symmetric phase (without selfenergy)



- N -patch momentum discretization
 - neglect frequency dependence
 - cover momentum space by N patches
 - solve system $\sim N^3$ of ODEs

The Temperature RG Flow, $T > 0$

$U = 3t$, van Hove filling

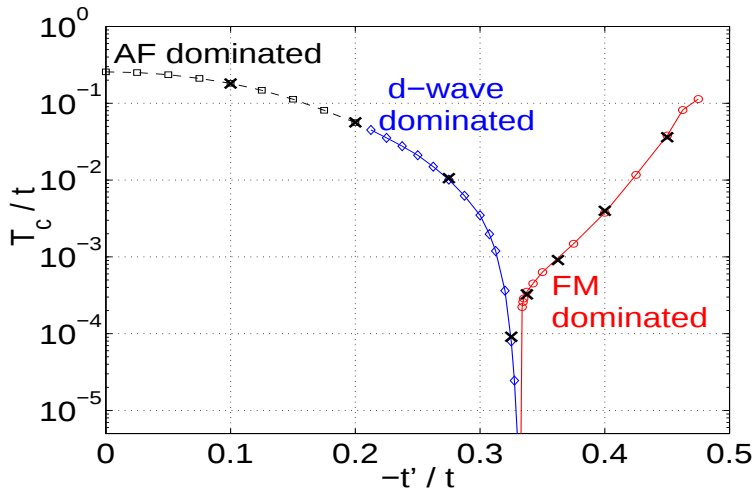
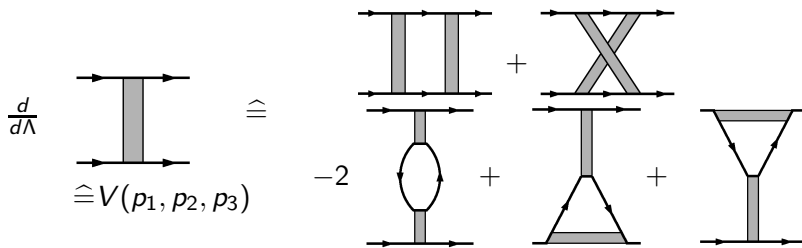


Figure: Honerkamp and Salmhofer, Phys. Rev. B 64 (2001) 184516

Parametrization of the Vertex Function



Observation: The leading weak coupling instabilities are mainly determined by the *singular* momentum and frequency structure of the flow equation.

Definition of Channels in the Flow Equation

$$\begin{aligned}
 \dot{\Phi}_{\text{SC}}^{\Lambda}(p_1, p_3, p_1 + p_2) &\hat{=} - \text{[Diagram: Two vertical shaded bars with horizontal lines above and below, and a central vertical line with an arrow pointing down]} \\
 \dot{\Phi}_{\text{K}}^{\Lambda}(p_1, p_2, p_2 - p_3) &\hat{=} 4 \text{ [Diagram: A central vertical shaded bar with a horizontal line above and below, and a central oval with an arrow pointing down]} \\
 &\quad , \quad \dot{\Phi}_{\text{M}}^{\Lambda}(p_1, p_2, p_3 - p_1) \hat{=} \text{[Diagram: A central vertical shaded bar with a horizontal line above and below, and a central triangle with an arrow pointing down]} \\
 &\quad -2 \text{ [Diagram: A central vertical shaded bar with a horizontal line above and below, and a central triangle with an arrow pointing up]} \\
 &\quad -2 \text{ [Diagram: A central vertical shaded bar with a horizontal line above and below, and a central triangle with an arrow pointing down]} \\
 &\quad + \text{[Diagram: Two crossed shaded bars with horizontal lines above and below, and a central vertical line with an arrow pointing down]}
 \end{aligned}$$

$$\text{with } \Phi_{\text{SC}}^{\Lambda_0} = \Phi_{\text{M}}^{\Lambda_0} = \Phi_{\text{K}}^{\Lambda_0} = 0$$

Definition of Channels in the Flow Equation

$$\begin{aligned}
 \dot{\Phi}_{\text{SC}}^{\Lambda}(p_1, p_3, p_1 + p_2) &\hat{=} - \text{[Diagram: Two vertical lines with horizontal arrows pointing right, connected by a horizontal line at the top and bottom.]} \\
 \dot{\Phi}_{\text{M}}^{\Lambda}(p_1, p_2, p_3 - p_1) &\hat{=} \text{[Diagram: Two horizontal lines with arrows pointing right, connected by a diagonal line from the top-left to the bottom-right.]} \\
 \dot{\Phi}_{\text{K}}^{\Lambda}(p_1, p_2, p_2 - p_3) &\hat{=} 4 \text{ [Diagram: A vertical line with a horizontal line at the top and bottom, and a loop in the middle.]} \\
 &\quad -2 \text{ [Diagram: A horizontal line with a vertical line at the top and bottom, and a triangle in the middle.]} \\
 &\quad -2 \text{ [Diagram: A horizontal line with a vertical line at the top and bottom, and an inverted triangle in the middle.]} \\
 &\quad + \text{[Diagram: Two horizontal lines with arrows pointing right, connected by a diagonal line from the top-left to the bottom-right, and a vertical line at the bottom-right.]}
 \end{aligned}$$

$$\text{with } \Phi_{\text{SC}}^{\Lambda_0} = \Phi_{\text{M}}^{\Lambda_0} = \Phi_{\text{K}}^{\Lambda_0} = 0$$

Vertex Function

$$\begin{aligned}
 V(p_1, p_2, p_3) = & U - \Phi_{\text{SC}}^{\Lambda}(p_1, p_3, p_1 + p_2) + \Phi_{\text{M}}^{\Lambda}(p_1, p_2, p_3 - p_1) \\
 & + \frac{1}{2} \Phi_{\text{M}}^{\Lambda}(p_1, p_2, p_2 - p_3) - \frac{1}{2} \Phi_{\text{K}}^{\Lambda}(p_1, p_2, p_2 - p_3)
 \end{aligned}$$

Decomposition of the Superconducting Channel

$$\begin{aligned}\Phi_{\text{SC}}^{\Lambda}(q, q', l) &= \sum_{mn} D_{mn}(l) f_m\left(\frac{l}{2} - q\right) f_n\left(\frac{l}{2} - q'\right) + R_{\text{SC}}(q, q', l) \\ &\hat{=} \sum_{mn} \text{diagram} + R_{\text{SC}}(q, q', l)\end{aligned}$$

The diagram represents a Feynman diagram for the superconducting channel. It consists of two vertices, labeled 'm' and 'n', connected by a wavy line. Each vertex has two external lines with arrows indicating momentum flow: one incoming and one outgoing for each vertex.

Decomposition of the Superconducting Channel

$$\Phi_{\text{SC}}^{\Lambda}(q, q', l) = \sum_{mn} D_{mn}(l) f_m\left(\frac{l}{2} - q\right) f_n\left(\frac{l}{2} - q'\right) + R_{\text{SC}}(q, q', l)$$
$$\hat{=} \sum_{mn} \text{Diagram} + R_{\text{SC}}(q, q', l)$$

The diagram shows two circles labeled 'm' and 'n' connected by a wavy line. Each circle has two external arrows: the 'm' circle has an incoming arrow from the left and an outgoing arrow to the right; the 'n' circle has an incoming arrow from the right and an outgoing arrow to the left.

If the Fermi Surface is *curved* and *regular*,

- particle-hole graphs are subleading
- largest (positive) $D_{nn}^{\Lambda_0}$ determines the instability
- particle-hole graphs induce $d_{x^2-y^2}$ -wave superconductivity

Magnetic and Forward Scattering Channel

$$\Phi_M^\Lambda(q, q', l) = \sum_{mn} M_{mn}(l) f_m(q + \frac{l}{2}) f_n(q' - \frac{l}{2}) + R_M(q, q', l)$$

$$\hat{=} \sum_{mn} \text{[Diagram: Two triangles connected by a horizontal line. The left triangle is labeled 'm' and the right triangle is labeled 'n'. Both triangles have two external lines with arrows pointing outwards.] } + R_M(q, q', l)$$

$$\Phi_K^\Lambda(q, q', l) = \sum_{mn} K_{mn}(l) f_m(q + \frac{l}{2}) f_n(q' - \frac{l}{2}) + R_K(q, q', l)$$

$$\hat{=} \sum_{mn} \text{[Diagram: Two squares connected by a horizontal line. The left square is labeled 'm' and the right square is labeled 'n'. Both squares have two external lines with arrows pointing outwards.] } + R_K(q, q', l)$$

... neglect remainders R_{SC} , R_M , and R_K ...

The Boson Propagator Flow

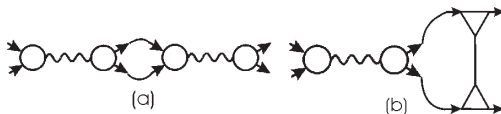
$$\begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \text{---} \end{array} \begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \text{---} \end{array} = P_{mn}^{\text{SC}} \left[\begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} - \text{---} \text{---} - \frac{3}{2} \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} + \frac{1}{2} \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \right]^2$$

The diagram shows the flow of the boson propagator. On the left, a wavy line connects two vertices, each with two external lines. The vertices are labeled 'm' and 'n'. This is equal to the product of P_{mn}^{SC} and a bracketed sum of diagrams. The bracketed sum contains: a wavy line between two vertices, a tadpole diagram, a triangle diagram with a coefficient of $-\frac{3}{2}$, and a square diagram with a coefficient of $+\frac{1}{2}$. The entire bracketed sum is squared.

The Boson Propagator Flow

$$\begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \text{---} \end{array} \begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \text{---} \end{array} = P_{mn}^{\text{SC}} \left[\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} - \text{---} \text{---} - \frac{3}{2} \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} + \frac{1}{2} \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \right]^2$$

two examples how the square is taken:



The Boson Propagator Flow

$$\text{Diagram with wavy line and dots} = P_{mn}^{\text{SC}} \left[\text{Diagram 1} - \text{Diagram 2} - \frac{3}{2} \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} \right]^2$$

$$\text{Diagram with solid line and dots} = -P_{mn}^{\text{M}} \left[\text{Diagram 5} + \text{Diagram 6} - \text{Diagram 7} + \frac{1}{2} \text{Diagram 8} - \frac{1}{2} \text{Diagram 9} \right]^2$$

$$\text{Diagram with solid line and dots} = -P_{mn}^{\text{K}} \left[\text{Diagram 10} - \text{Diagram 11} + \text{Diagram 12} - \frac{3}{2} \text{Diagram 13} - \frac{1}{2} \text{Diagram 14} \right]^2$$

Numerical Implementation

- minimal set of formfactors:

$$f_s(p) = 1$$

$$f_1(p) = \cos p_x - \cos p_y$$

- neglect frequency dependence of the boson propagators
- discretize momentum dependence with patches
 - most important around $(0,0)$ and (π, π)
 - different broadness of particle–particle and particle–hole bubble

Numerical Implementation

- minimal set of formfactors:

$$f_s(p) = 1$$

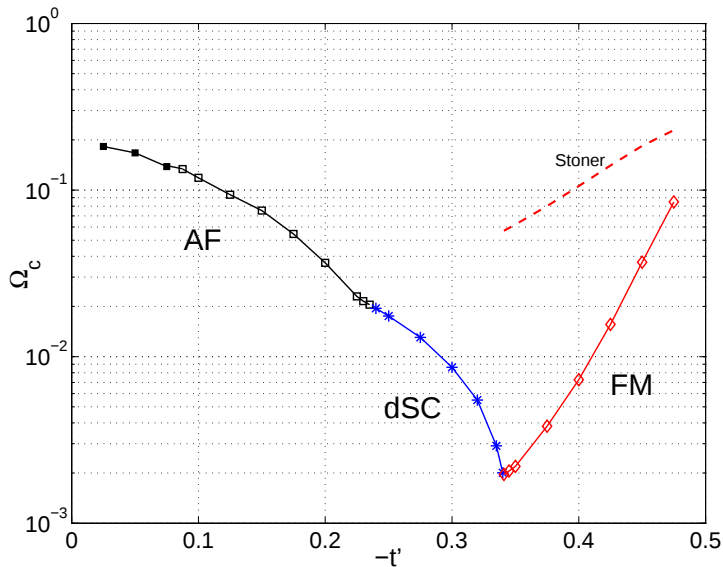
$$f_1(p) = \cos p_x - \cos p_y$$

- neglect frequency dependence of the boson propagators
- discretize momentum dependence with patches
 - most important around $(0, 0)$ and (π, π)
 - different broadness of particle–particle and particle–hole bubble
- regularization (choice of cut-off):

$$\chi_\Omega(p) = \frac{p_0^2}{p_0^2 + \Omega^2}$$

- initial condition at Ω_0 obtained by perturbation theory

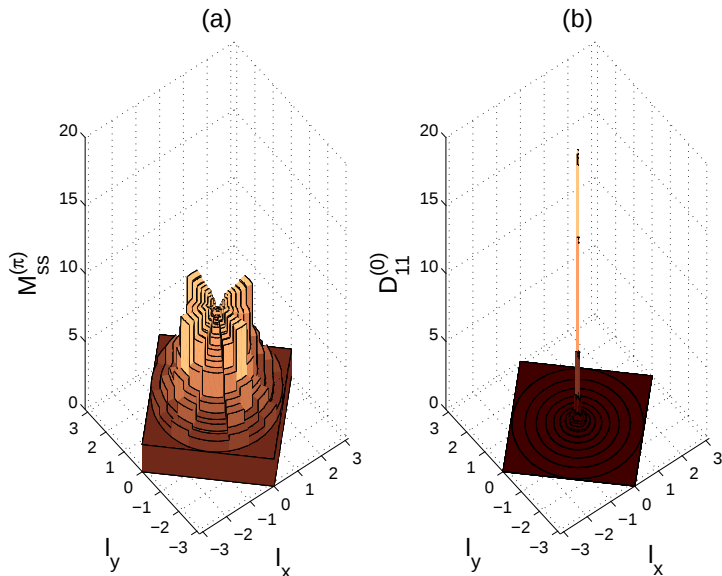
Instabilities at van Hove filling, $U = 3t$, and $T = 0$



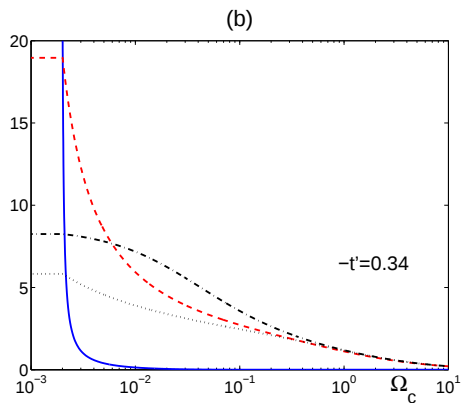
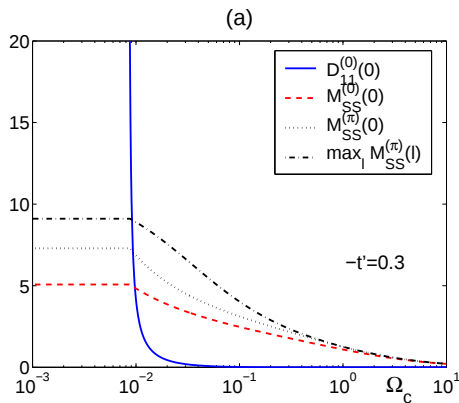
Conclusion

- essential structure of the one-loop RG is preserved
- less computing cost ($\sim N$ ODEs)
- ambiguity of introducing boson fields above Ω_c is reduced
- ahead: analytic momentum and frequency parametrization of the boson propagators

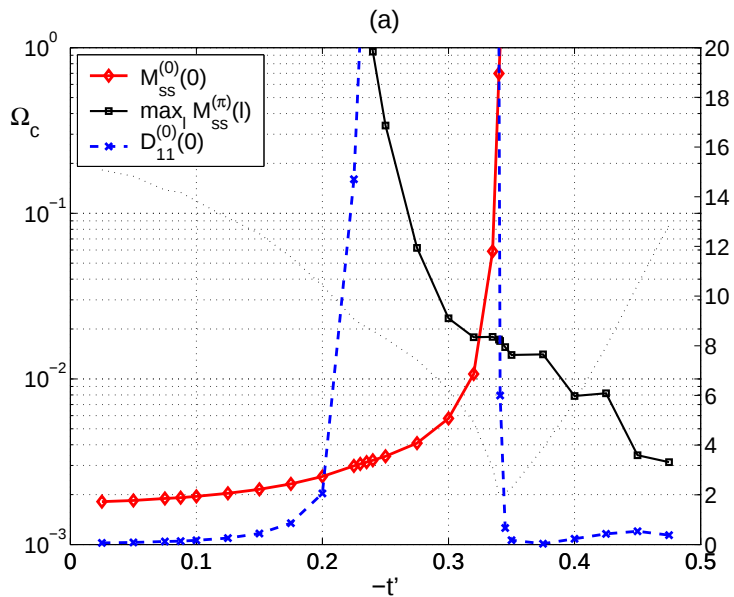
Momentum Dependence of the Boson Propagators



Flow of the boson propagators (Maxima)



Overview of the Maxima at Ω_c



Different Channels in the Interaction

$$\begin{aligned}
 \mathcal{V}[\Psi] = & \frac{U}{2} \int \mathrm{d}p_1 \dots \mathrm{d}p_3 \sum_{\sigma, \sigma'} \bar{\Psi}_{\sigma}(p_1) \bar{\Psi}_{\sigma'}(p_2) \Psi_{\sigma'}(p_3) \Psi_{\sigma}(p_4) \\
 & - \frac{1}{4} \int \mathrm{d}q \mathrm{d}q' \mathrm{d}l \Phi_{\text{SC}}^{\Lambda}(q, q', l) \sum_{j=0}^3 \left(\bar{\Psi}(q) \sigma^{(j)} \bar{\Psi}(l - q) \right) \left(\Psi(q') \sigma^{(j)} \Psi(l - q') \right) \\
 & - \frac{1}{4} \int \mathrm{d}q \mathrm{d}q' \mathrm{d}l \Phi_{\text{M}}^{\Lambda}(q, q', l) \sum_{j=1}^3 \left(\bar{\Psi}(q) \sigma^{(j)} \Psi(q + l) \right) \left(\bar{\Psi}(q') \sigma^{(j)} \Psi(q' - l) \right) \\
 & - \frac{1}{4} \int \mathrm{d}q \mathrm{d}q' \mathrm{d}l \Phi_{\text{K}}^{\Lambda}(q, q', l) \left(\bar{\Psi}(q) \Psi(q + l) \right) \left(\bar{\Psi}(q') \Psi(q' - l) \right)
 \end{aligned}$$

with $\Phi_{\text{SC}}^{\Lambda_0} = \Phi_{\text{M}}^{\Lambda_0} = \Phi_{\text{K}}^{\Lambda_0} = 0$

Definition of three Channels

Vertex Function

$$V(p_1, p_2, p_3) = U - \Phi_{\text{SC}}^{\Lambda}(p_1, p_3, p_1 + p_2) + \Phi_{\text{M}}^{\Lambda}(p_1, p_2, p_3 - p_1) \\ + \frac{1}{2}\Phi_{\text{M}}^{\Lambda}(p_1, p_2, p_2 - p_3) - \frac{1}{2}\Phi_{\text{K}}^{\Lambda}(p_1, p_2, p_2 - p_3)$$

Assigning the Graphs

$$\dot{\Phi}_{\text{SC}}^{\Lambda}(p_1, p_3, p_1 + p_2) = -\mathcal{T}_{\text{pp}}(p_1, p_2, p_3)$$

$$\dot{\Phi}_{\text{M}}^{\Lambda}(p_1, p_2, p_3 - p_1) = \mathcal{T}_{\text{ph}}^{\text{cr}}(p_1, p_2, p_3)$$

$$\dot{\Phi}_{\text{K}}^{\Lambda}(p_1, p_2, p_2 - p_3) = -2\mathcal{T}_{\text{ph}}^{\text{d}}(p_1, p_2, p_3) + \mathcal{T}_{\text{ph}}^{\text{cr}}(p_1, p_2, p_1 + p_2 - p_3)$$