Competing Orders in the Hubbard Model at van Hove filling

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July 1, 2008

The 2D (t, t')–Hubbard Model

van Hove filling

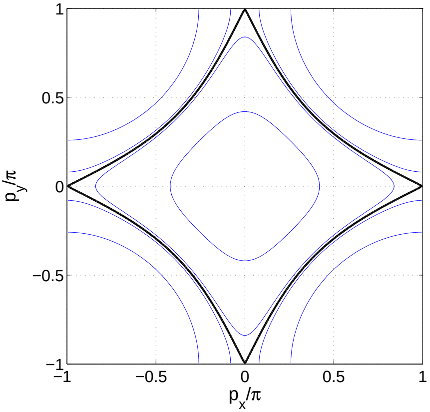


Figure: The Fermi surface at $-t' = 0.3$

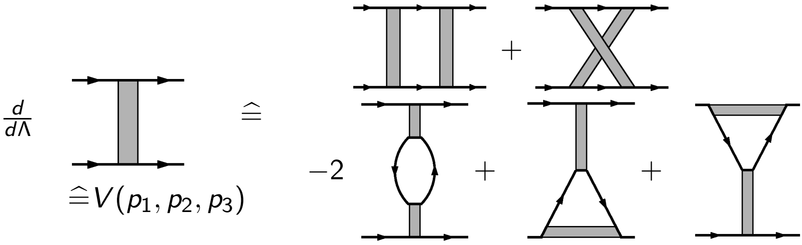
$+$

weak
onsite
repulsion

$\Longrightarrow$ Instabilities of the Landau Fermi Liquid

The One–Loop Truncation

- 1PI RG in the symmetric phase (without selfenergy)



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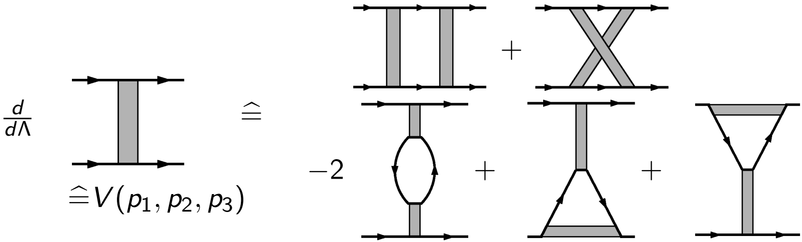


- N–patch momentum discretization
  - neglect frequency dependence
  - cover momentum space by N patches
  - solve system $\sim N^3$ of ODEs

The Temperature RG Flow, $T > 0$

$U = 3t$, van Hove filling

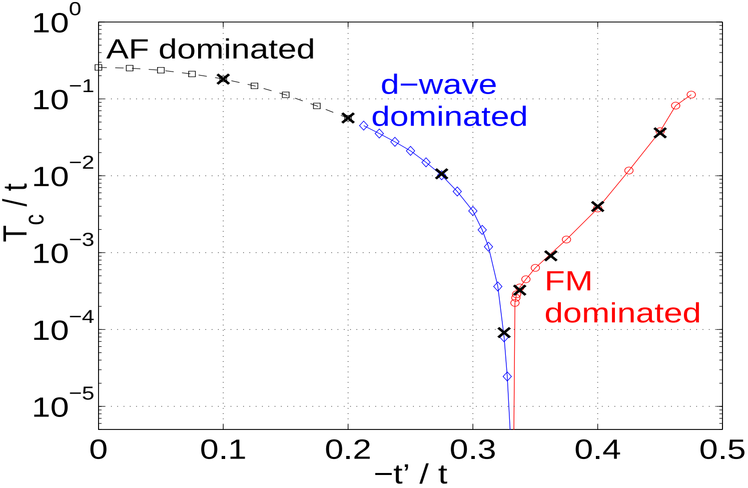
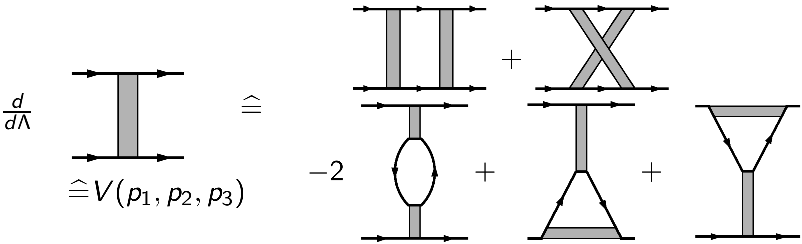


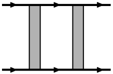
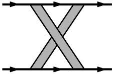
Figure: Honerkamp and Salmhofer, Phys. Rev. B 64 (2001) 184516

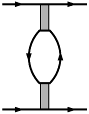
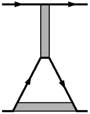
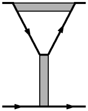
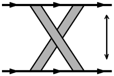
Parametrization of the Vertex Function



Observation: The leading weak coupling instabilities are mainly determined by the *singular* momentum and frequency structure of the flow equation.

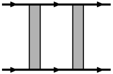
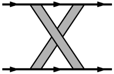
Definition of Channels in the Flow Equation

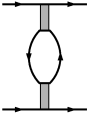
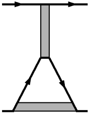
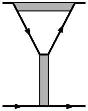
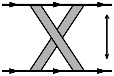
$\dot{\Phi}^{\Lambda}_{\mathrm{SC}}(p_1, p_3, p_1+p_2) \;\widehat{=}\; -$  , $\dot{\Phi}^{\Lambda}_{\mathrm{M}}(p_1, p_2, p_3-p_1) \;\widehat{=}\;$ 

$\dot{\Phi}^{\Lambda}_{\mathrm{K}}(p_1, p_2, p_2-p_3) \;\widehat{=}\; 4$  -2  -2  $+$ 

with $\Phi^{\Lambda_0}_{\mathrm{SC}} = \Phi^{\Lambda_0}_{\mathrm{M}} = \Phi^{\Lambda_0}_{\mathrm{K}} = 0$

Definition of Channels in the Flow Equation

$\dot\Phi^{\Lambda}_{\mathrm{SC}}(p_1, p_3, p_1 + p_2) \;\widehat{=}\; -$  , $\dot\Phi^{\Lambda}_{\mathrm{M}}(p_1, p_2, p_3 - p_1) \;\widehat{=}$ 

$\dot\Phi^{\Lambda}_{\mathrm{K}}(p_1, p_2, p_2 - p_3) \;\widehat{=}\; 4$  -2  -2  $+$ 

with $\Phi^{\Lambda_0}_{\mathrm{SC}} = \Phi^{\Lambda_0}_{\mathrm{M}} = \Phi^{\Lambda_0}_{\mathrm{K}} = 0$

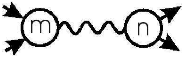
Vertex Function

$$V(p_1, p_2, p_3) = U - \Phi^{\Lambda}_{\mathrm{SC}}(p_1, p_3, p_1 + p_2) + \Phi^{\Lambda}_{\mathrm{M}}(p_1, p_2, p_3 - p_1) + \frac{1}{2}\Phi^{\Lambda}_{\mathrm{M}}(p_1, p_2, p_2 - p_3) - \frac{1}{2}\Phi^{\Lambda}_{\mathrm{K}}(p_1, p_2, p_2 - p_3)$$

Decomposition of the Superconducting Channel

$$\Phi^{\Lambda}_{\mathrm{sc}}(q,q',l) = \sum_{mn} D_{mn}(l) f_m(\frac{l}{2}-q) f_n(\frac{l}{2}-q') + R_{\mathrm{sc}}(q,q',l)$$

$$\widehat{=} \ \sum_{mn}$$



$$+ R_{\mathrm{sc}}(q,q',l)$$

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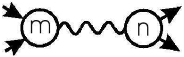
$$\widehat{=} \ \sum_{mn}$$  $$+ R_{\mathrm{SC}}(q,q',l)$$

If the Fermi Surface is *curved* and *regular*,

- particle–hole graphs are subleading
- largest (positive) $D^{\Lambda_0}_{nn}$ determines the instability
- particle–hole graphs induce $d_{x^2-y^2}$–wave superconductivity

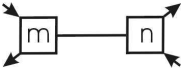
Magnetic and Forward Scattering Channel

$$\Phi_{\mathrm{M}}^{\Lambda}(q,q',l) = \sum_{mn} M_{mn}(l) f_m(q+\frac{l}{2}) f_n(q'-\frac{l}{2}) + R_{\mathrm{M}}(q,q',l)$$

$$\widehat{=} \ \sum_{mn}$$ 

$$+ R_{\mathrm{M}}(q,q',l)$$

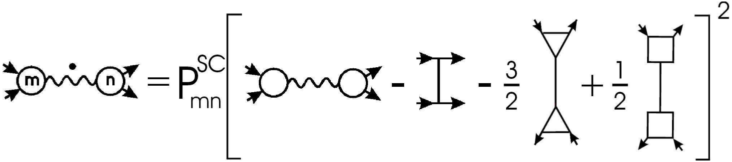
$$\Phi_{\mathrm{K}}^{\Lambda}(q,q',l) = \sum_{mn} K_{mn}(l) f_m(q+\frac{l}{2}) f_n(q'-\frac{l}{2}) + R_{\mathrm{K}}(q,q',l)$$

$$\widehat{=} \ \sum_{mn}$$ 

$$+ R_{\mathrm{K}}(q,q',l)$$

... neglect remainders R_{SC}, R_{M}, and R_{K} ...

The Boson Propagator Flow



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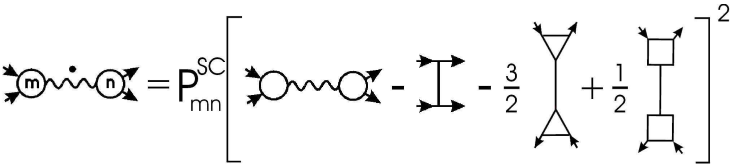
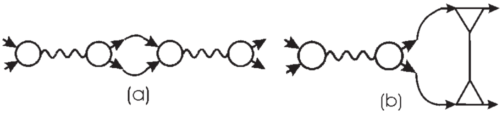


two examples how the square is taken:



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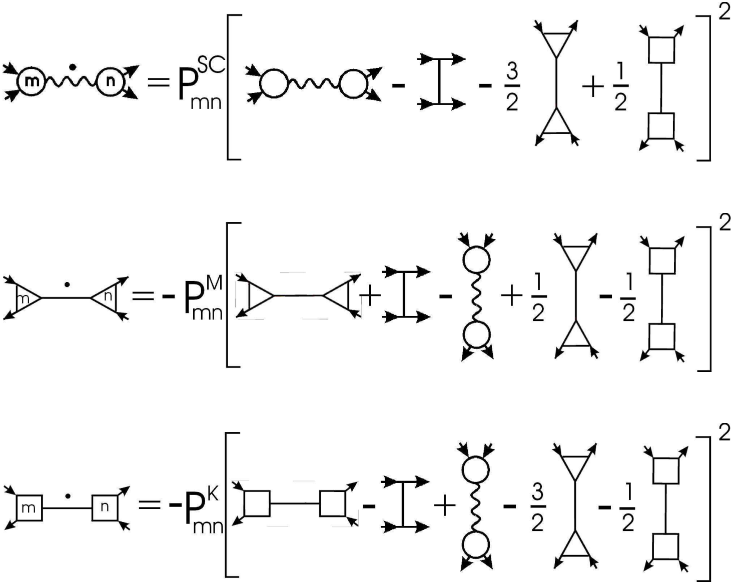

Numerical Implementation

- minimal set of formfactors:

$$f_s(p) = 1$$
$$f_1(p) = \cos p_x - \cos p_y$$

- neglect frequency dependence of the boson propagators
- discretize momentum dependence with patches
  - most important around $(0, 0)$ and (π, π)
  - different broadness of particle–particle and particle–hole bubble

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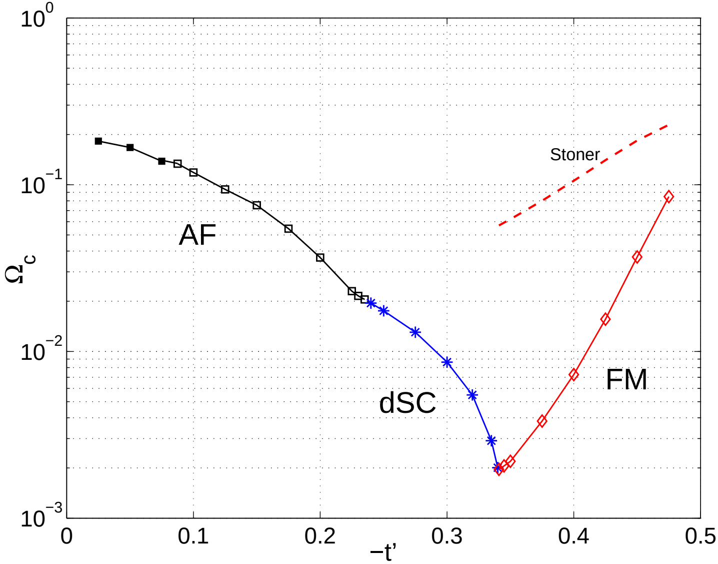
- regularization (choice of cut–off):

$$\chi_\Omega(p) = \frac{p_0^2}{p_0^2 + \Omega^2}$$

- initial condition at Ω_0 obtained by perturbation theory

Instabilities at van Hove filling, $U = 3t$, and $T = 0$



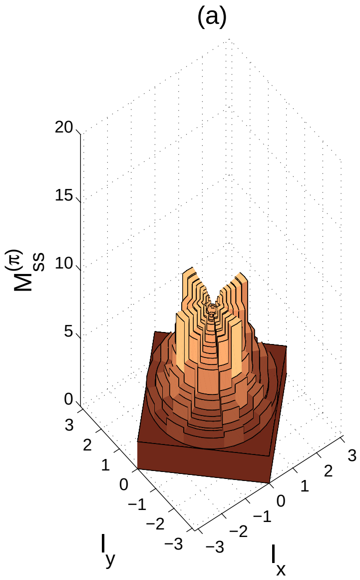
Conclusion

- essential structure of the one–loop RG is preserved
- less computing cost ($\sim N$ ODEs)

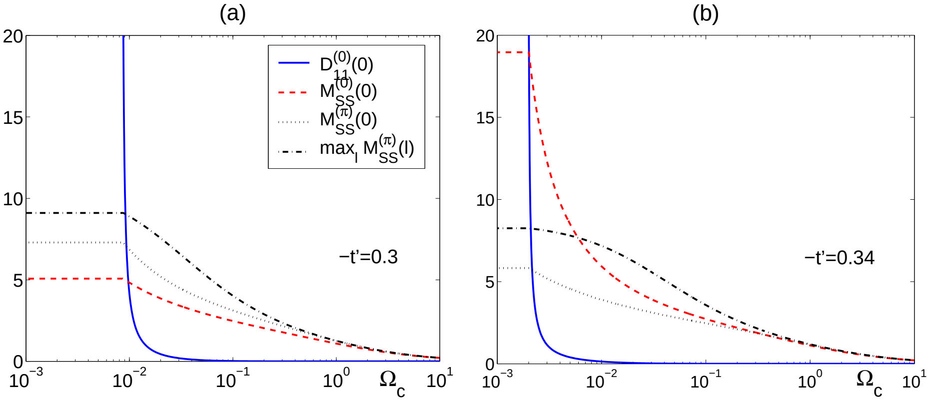
- ambiguity of introducing boson fields above Ω_c is reduced

- ahead: analytic momentum and frequency parametrization of the boson propagators

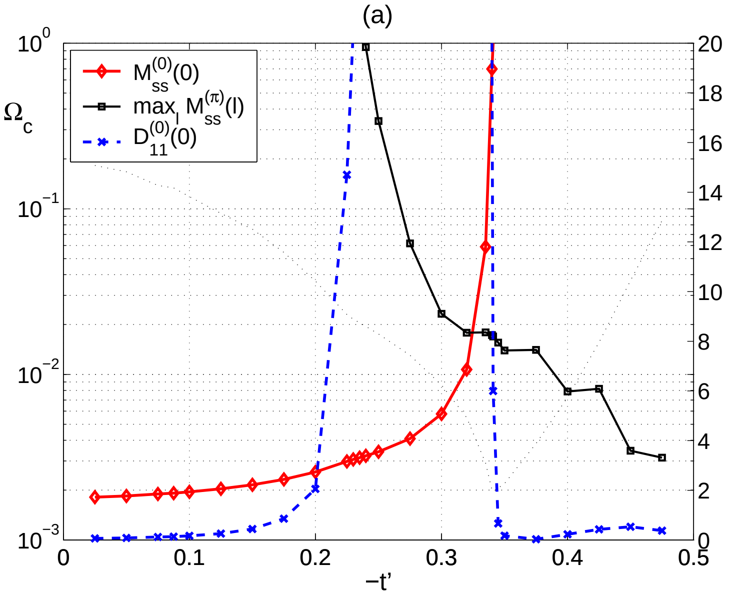
Momentum Dependence of the Boson Propagators



Flow of the boson propagators (Maxima)



Overview of the Maxima at Ω_c



Different Channels in the Interaction

$$\mathcal{V}[\Psi] = \frac{U}{2}\int \mathrm{d}p_1 \ldots \mathrm{d}p_3 \sum_{\sigma,\sigma'} \overline{\Psi}_\sigma(p_1)\overline{\Psi}_{\sigma'}(p_2)\Psi_{\sigma'}(p_3)\Psi_\sigma(p_4)$$

$$-\tfrac{1}{4}\int \mathrm{d}q\mathrm{d}q'\mathrm{d}l\ \Phi^\Lambda_{\mathrm{SC}}(q,q',l)\sum_{J=0}^{3}\Big(\overline{\Psi}(q)\sigma^{(J)}\overline{\Psi}(l-q)\Big)\Big(\Psi(q')\sigma^{(J)}\Psi(l-q')\Big)$$

$$-\tfrac{1}{4}\int \mathrm{d}q\mathrm{d}q'\mathrm{d}l\ \Phi^\Lambda_{\mathrm{M}}(q,q',l)\sum_{j=1}^{3}\Big(\overline{\Psi}(q)\sigma^{(j)}\Psi(q+l)\Big)\Big(\overline{\Psi}(q')\sigma^{(j)}\Psi(q'-l)\Big)$$

$$-\tfrac{1}{4}\int \mathrm{d}q\mathrm{d}q'\mathrm{d}l\ \Phi^\Lambda_{\mathrm{K}}(q,q',l)\Big(\overline{\Psi}(q)\Psi(q+l)\Big)\Big(\overline{\Psi}(q')\Psi(q'-l)\Big)$$

with $\Phi^{\Lambda_0}_{\mathrm{SC}} = \Phi^{\Lambda_0}_{\mathrm{M}} = \Phi^{\Lambda_0}_{\mathrm{K}} = 0$

Definition of three Channels

Vertex Function

$$V(p_1, p_2, p_3) = U - \Phi^{\Lambda}_{\mathrm{SC}}(p_1, p_3, p_1 + p_2) + \Phi^{\Lambda}_{\mathrm{M}}(p_1, p_2, p_3 - p_1)$$
$$+ \frac{1}{2}\Phi^{\Lambda}_{\mathrm{M}}(p_1, p_2, p_2 - p_3) - \frac{1}{2}\Phi^{\Lambda}_{\mathrm{K}}(p_1, p_2, p_2 - p_3)$$

Assigning the Graphs

$$\dot{\Phi}^{\Lambda}_{\mathrm{SC}}(p_1, p_3, p_1 + p_2) = -\mathcal{T}_{\mathrm{pp}}(p_1, p_2, p_3)$$
$$\dot{\Phi}^{\Lambda}_{\mathrm{M}}(p_1, p_2, p_3 - p_1) = \mathcal{T}^{\mathrm{cr}}_{\mathrm{ph}}(p_1, p_2, p_3)$$
$$\dot{\Phi}^{\Lambda}_{\mathrm{K}}(p_1, p_2, p_2 - p_3) = -2\mathcal{T}^{\mathrm{d}}_{\mathrm{ph}}(p_1, p_2, p_3) + \mathcal{T}^{\mathrm{cr}}_{\mathrm{ph}}(p_1, p_2, p_1 + p_2 - p_3)$$