Renormalization group for phases with broken discrete symmetry near quantum critical points

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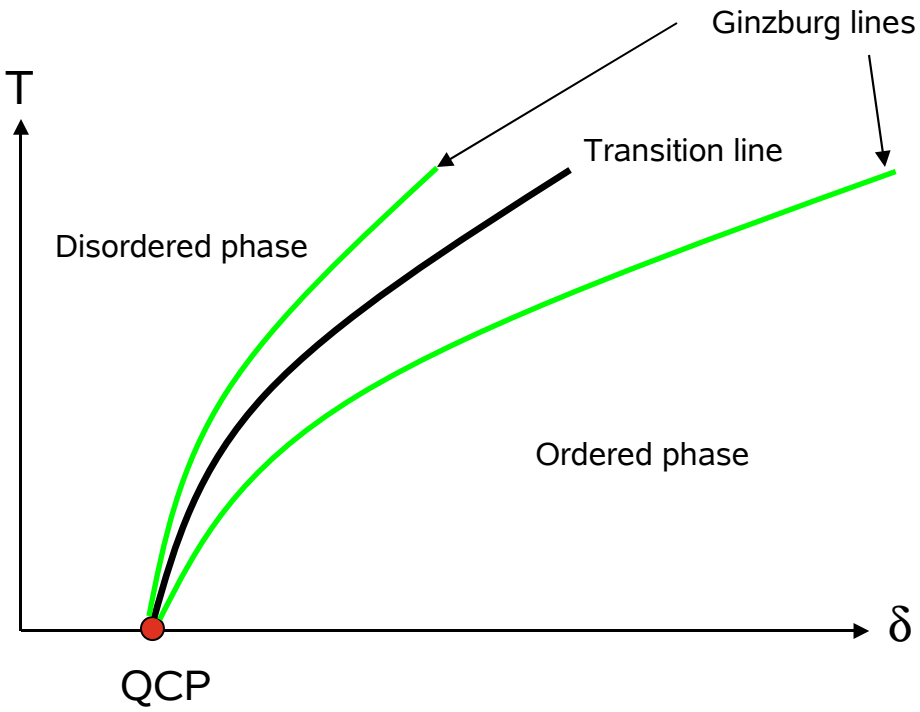
Quantum phase transitions



$$d \geq d_C = 4 - z \qquad (z = 2,3)$$

Questions:

- What is the shape of the T_C line near QCP?
- What is the critical region´s size ($T_G(\delta)$)?

The Hertz action

$$Z = \int D(\Psi^*, \Psi) e^{-\int_0^\beta d\tau \left[\sum_l \Psi^* (\partial_\tau - \mu) \Psi + H(\Psi^*, \Psi) \right]}$$

1) S – H transformation,

2) integrate out the fermions

$$Z(T, \delta) = Z_0 \int D\varphi(\vec{r}, \tau) e^{-S[\varphi]}$$

$$S[\varphi] = \int_0^\beta d\tau \int d^d r L\left[\varphi(\vec{r}, \tau)\right]$$

L – W expansion

$$S = \frac{1}{2} \sum_{\vec{p}, \omega_n} \left(m^2 + p^2 + \frac{|\omega_n|}{p^{z-2}} \right) \varphi(\vec{p}, i\omega_n) \varphi(-\vec{p}, -i\omega_n) + u \int_0^\beta d\tau \int d^d r \left[\varphi(\vec{r}, \tau)\right]^4$$

Perturbative RG treatment (Hertz–Millis theory)

- Using Wilsonian RG derive flow equations for the parameters T, m^2, u
- Linearize around the Gaussian fixed point $T = m^2 = u = 0$
- Investigate the behaviour of interaction coupling under scaling $\longrightarrow$ Ginzburg criterion

$T_C(\delta)$ is estimated by the Ginzburg temperature $T_G(\delta)$ in the *symmetric* phase

$$T_G \sim (\delta - \delta_0)^{\Psi}$$

for *d=2* additional multiplicative log corrections occur

$$\Psi = \frac{z}{d+z-2}$$

Alternative:

Calculate $T_C(\delta)$ directly from the condition $\xi(\delta, T_C) = \infty$

Remarks:

- The flow should start in the phase with broken symmetry
- The non-Gaussian fluctuation regime must be accessed

Non-perturbative RG approach

1PI scheme:

$$\partial_\Lambda \Gamma^\Lambda[\varphi] = \frac{1}{2} Tr \frac{\partial_\Lambda R^\Lambda}{\Gamma^{(2)}[\varphi] + R^\Lambda}$$

$\xrightarrow{\varphi = const}$ $\partial_\Lambda U = \ldots$

$\xrightarrow{\frac{\delta^2}{\delta\varphi^2}}$ $\partial_\Lambda \Gamma^{(2)}(\vec{p}, \omega_n) = \ldots$

Parametrization of $\Gamma^{(2)}(p, \omega_n)$

$$\Gamma^{(2)}(\vec{p}, \omega_n) = Z_p \vec{p}^{\,2} + Z_\omega \frac{|\omega_n|}{|\vec{p}|^{z-2}} + R^\Lambda(\vec{p})$$

Choice of cutoff function

$$R^\Lambda(p) = Z_p(\Lambda^2 - p^2)\theta(\Lambda^2 - p^2)$$

Effective potential:

$$U[\varphi] = \frac{u}{4!}\int_0^\beta d\tau \int d^d r \left(\varphi^2 - \varphi_0{}^2\right)^2 = \int_0^\beta d\tau \int d^d r \left[u \frac{\varphi'^4}{4!} + \sqrt{3\delta u}\, \frac{\varphi'^3}{3!} + \delta \frac{\varphi'^2}{2!} \right]$$

$$\begin{cases} \varphi = \varphi_0 + \varphi' \\ \delta = \frac{u\varphi_0{}^2}{3} \end{cases}$$

Remarks:

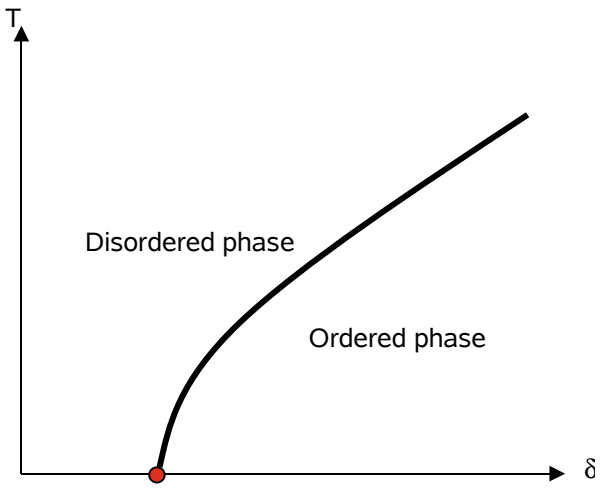
- fermions integrated out before
- not necessarily suitable for symmetry-broken phases with gaps in fermionic spectrum

Anomalous exponent

$$\eta = -\frac{\Lambda}{Z_p}\partial_\Lambda Z_p$$

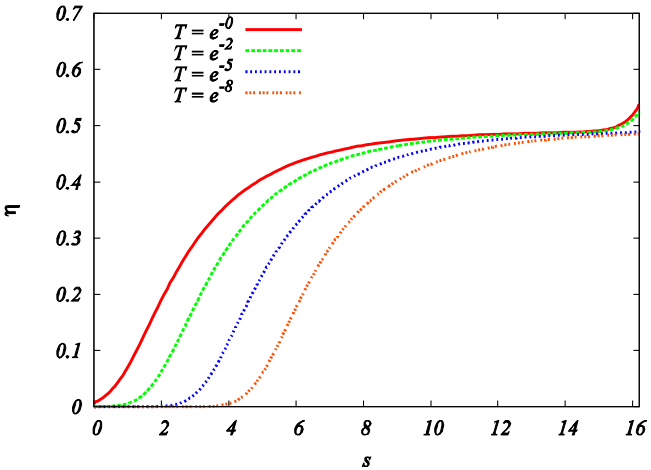
Transition line

- Fix *u, T*
- Choose δ_{UV}
- Run the flow
  - if $\phi_0 \longrightarrow 0$ as the cutoff is removed, δ_{UV} corresponds to the disordered phase
  - otherwise δ_{UV} corresponds to the ordered phase

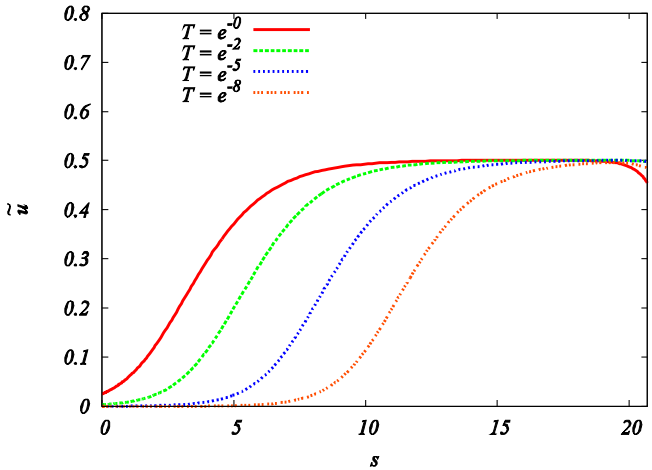


Results

d=2, z=3



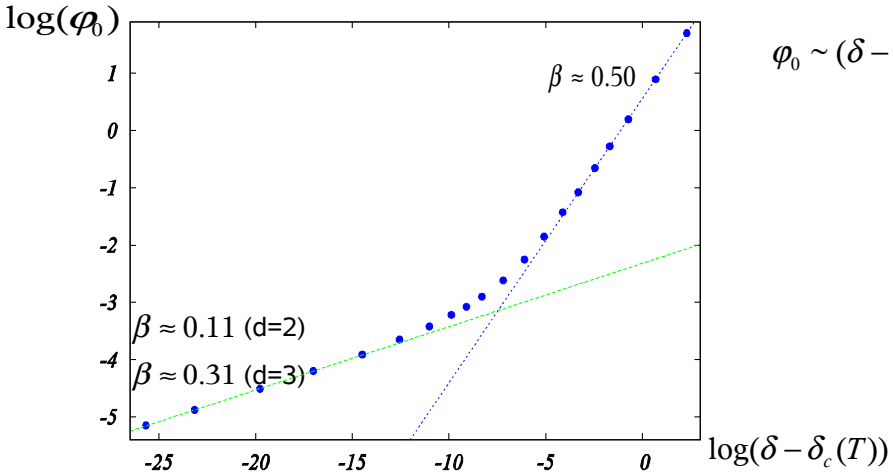
d=3, z=3



$\Lambda = \exp(-s)$

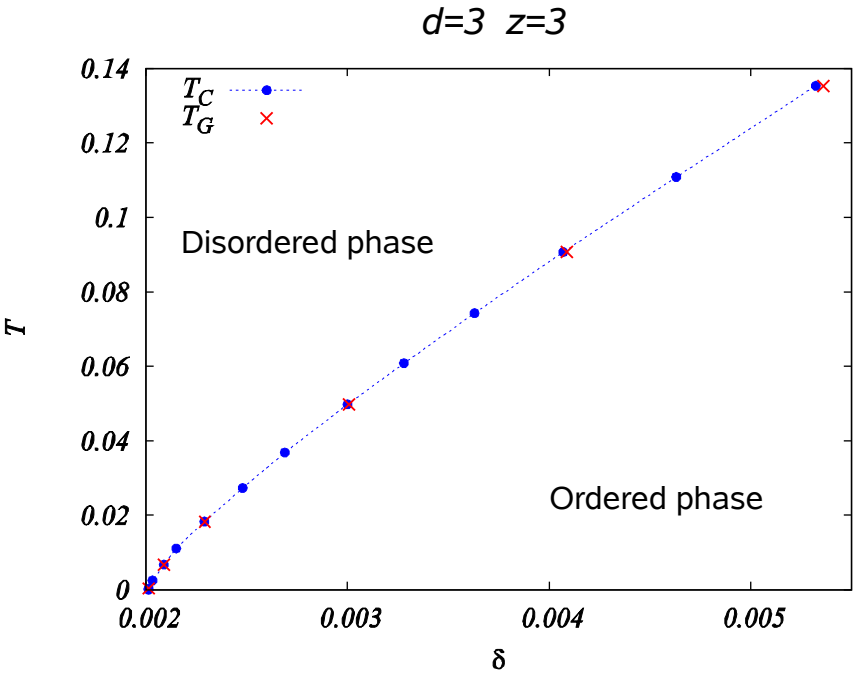
$\eta \approx 0.48$

($\eta \approx 0.08$ for *d=3*)

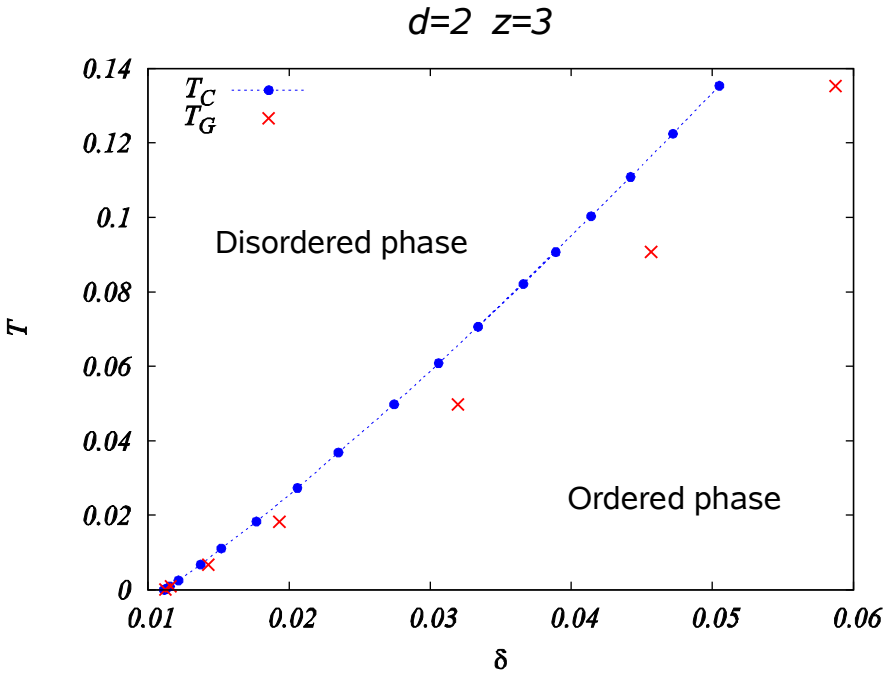


$\varphi_0 \sim (\delta - \delta_C)^\beta$

Phase diagrams (*z*=3)

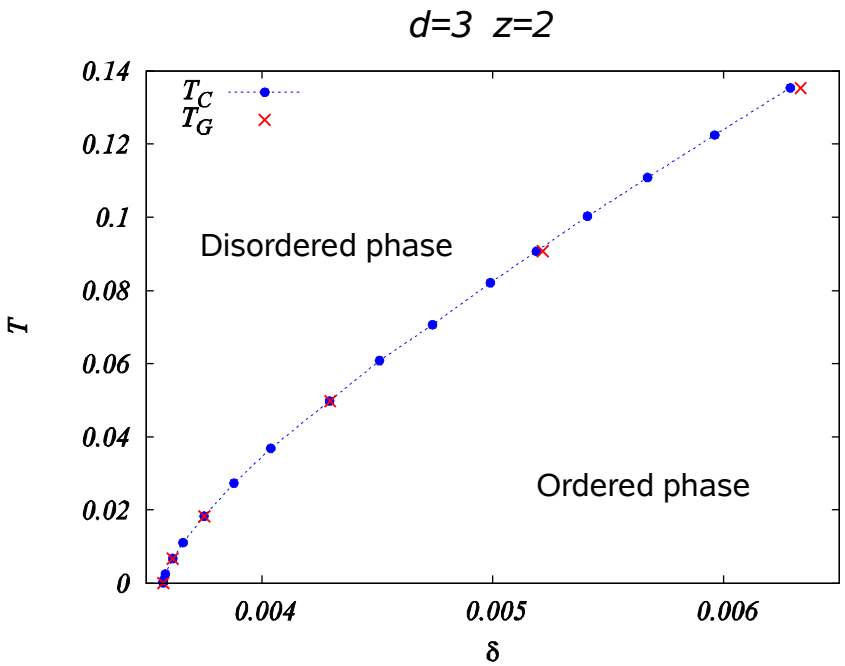


$$T_C \propto (\delta - \delta_0)^{0.75}$$

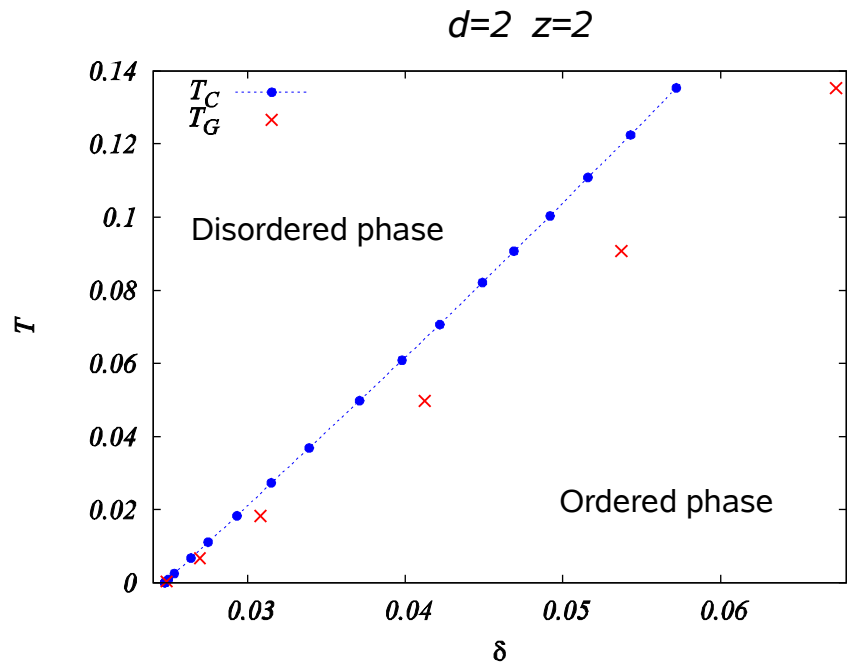


$$(\delta - \delta_0) \propto T_C \log T_C$$

Phase diagrams (*z*=2)



$$T_C \propto (\delta - \delta_0)^{0.66}$$



$$(\delta - \delta_0) \propto T_C \log(\log T_C) / \log(T_C)$$

For all cases $T_C(\delta)$ agrees with the estimate by Millis: $T_G(\delta) \propto (\delta - \delta_0)^{\psi}$

where $\psi = \dfrac{z}{d+z-2}$ (valid for $d > 2$)

For $d=3$ T_C and T_G almost coincide, while a sizable window between T_C and T_G opens in $d=2$

Conclusions

- An extension of Hertz-Millis theory to phases with broken discrete symmetry, accessing the non-Gaussian regime

- Ginzburg criterion yields an accurate estimate of the T_C – line in $d = 3$ and a qualitatively correct estimate in $d=2$.

- Sizable window between T_C and T_G in $d=2$ even for discrete symmetry breaking.