Two-loop functional renormalization-group approach to 1D and 2D Hubbard model

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The Hubbard model

$$H = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c^{+}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

1D

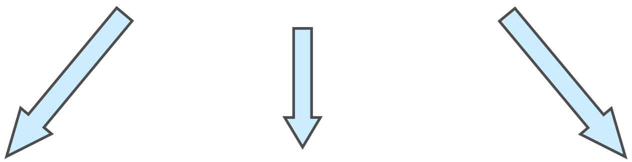
$$\varepsilon_{\mathbf{k}} = -2t\cos k_x - \mu$$

2D

$$\varepsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) + 4t'(\cos k_x \cos k_y + 1) - \mu$$

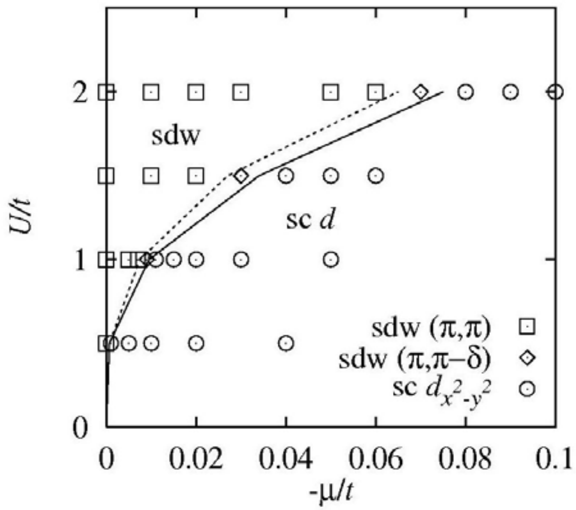
$$t, t' > 0$$

Functional renormalization-group approach

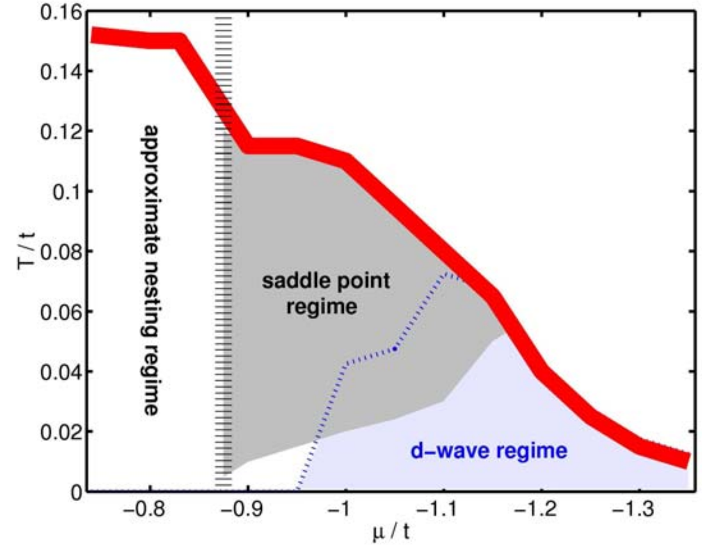


Polchinskii   Wick ordered   1PI

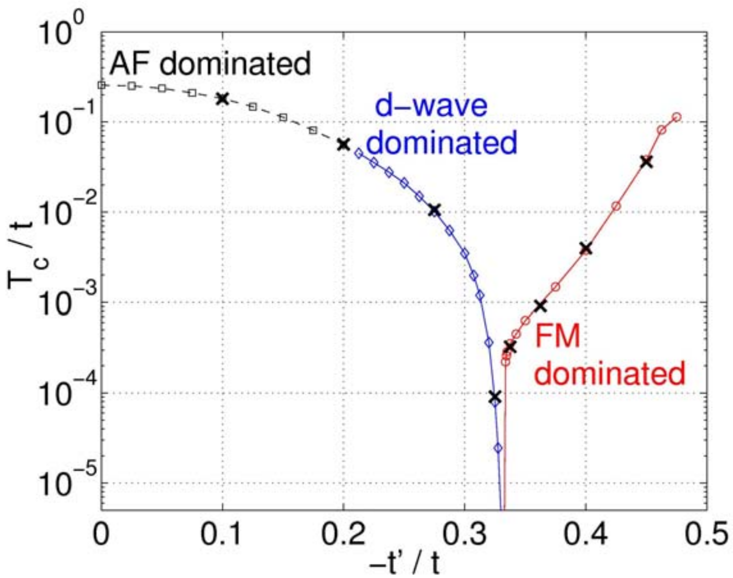
One-loop phase diagrams



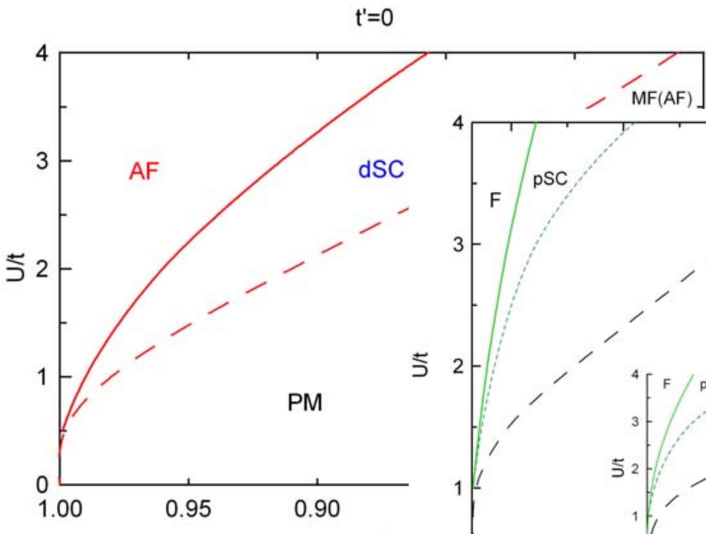
C. Halboth and W. Metzner (2000)

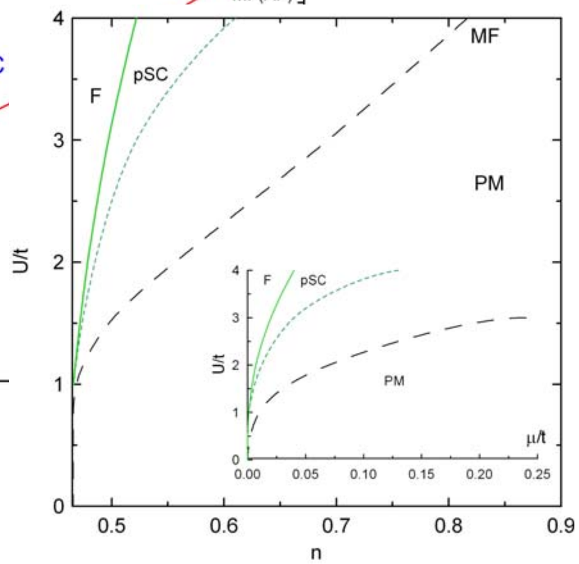


C. Honerkamp, M. Salmhofer, N. Furukawa, and T. M. Rice (2001)



C. Honerkamp and M. Salmhofer (2001)

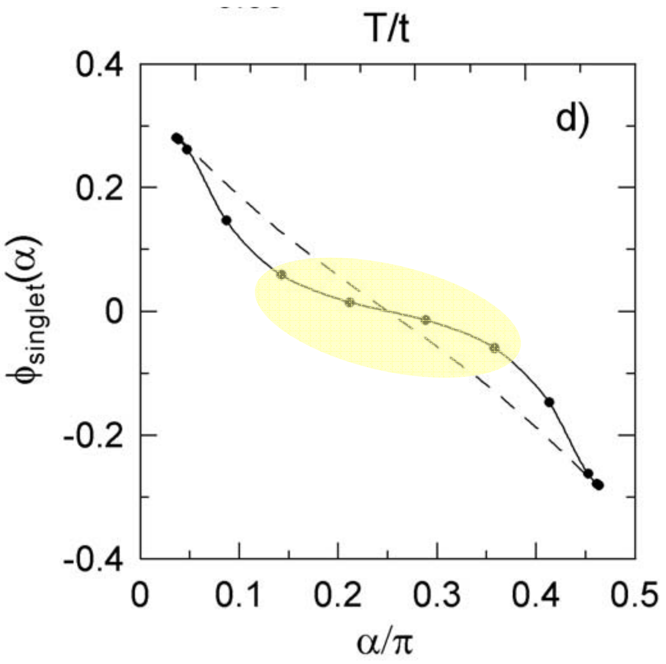




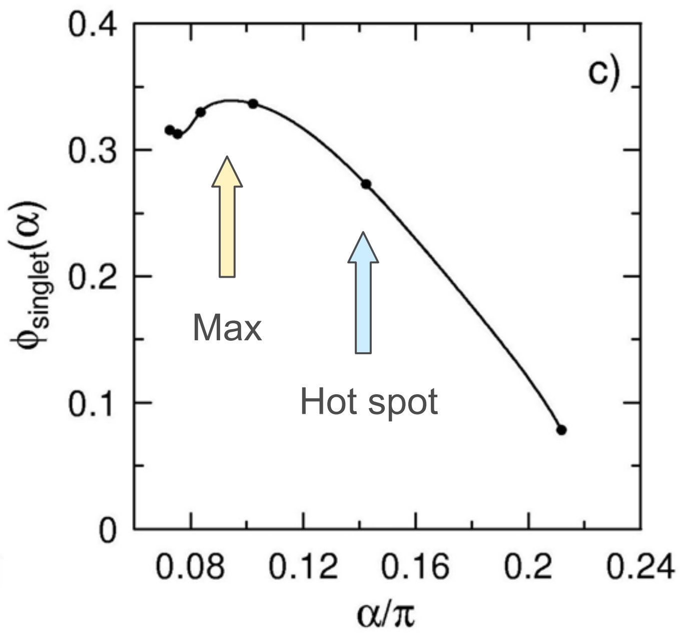
A. A. Katanin and A. P. Kampf (2003)

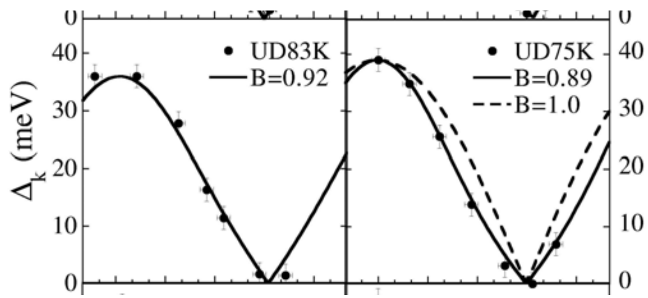
Angular dependence of the order parameter

Hole-doped sc, A. A. Katanin and A. P. Kampf, Phys. Rev. 2005

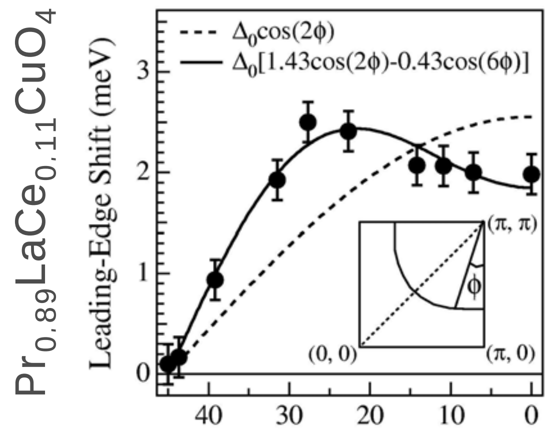


Electron-doped sc, A. A. Katanin, Phys. Rev. 2006





J. Mesot et al., Phys. Rev. Lett. **83**, 840 (1999).



H. Matsui et al., Phys. Rev. Lett. **95**, 017003 (2005).

One-loop equations

$$\frac{d\Sigma_\Lambda}{d\Lambda} = V_\Lambda \circ S_\Lambda \qquad S_\Lambda = -G_\Lambda^2 (G_\Lambda^0)^{-1}$$

$$\frac{dV_\Lambda}{d\Lambda} = V_\Lambda \circ (G_\Lambda \circ S_\Lambda + S_\Lambda \circ G_\Lambda) \circ V_\Lambda$$

Solve by iterations:

- keeps correct initial conditions
- generates expansion in U

$$\Sigma_\Lambda^{(1)} = U \mathrm{Tr}(G_\Lambda^0) \quad \longleftarrow \quad \Sigma_\Lambda^{(0)} = 0 \qquad V_\Lambda^{(0)} = U$$

$$V_\Lambda^{(1)} = U + U^2 (G_\Lambda^0 \circ G_\Lambda^0)$$

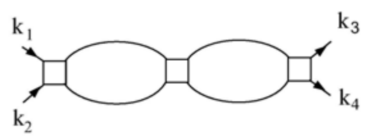
$$\Downarrow$$

$$\Sigma_\Lambda^{(2),1-\text{loop}} = U \mathrm{Tr}(G_\Lambda^{(1)}) + U^2 G_\Lambda^0 \circ G_\Lambda^0 \circ G_\Lambda^0$$

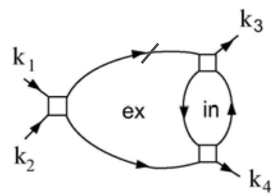
2-nd order perturbation theory

$$V_\Lambda^{(2),1-\text{loop}} = U + U^2 \int_\Lambda^{\Lambda_0} d\Lambda' [S_{\Lambda'}^{(1)} \circ G_{\Lambda'}^{(1)} + G_{\Lambda'}^{(1)} \circ S_{\Lambda'}^{(1)}]$$

$$+ U^3 [G_\Lambda^0 \circ G_\Lambda^0 \circ G_\Lambda^0 \circ G_\Lambda^0]_{\text{ladder}}$$



$$+ U^3 \int_\Lambda^{\Lambda_0} d\Lambda' [(G_{\Lambda'}^0 \circ G_{\Lambda'}^0)_{\text{in}} \circ \frac{d}{d\Lambda'} (G_{\Lambda'}^0 \circ G_{\Lambda'}^0)_{\text{ex}}]_{\text{non-ladder}}$$



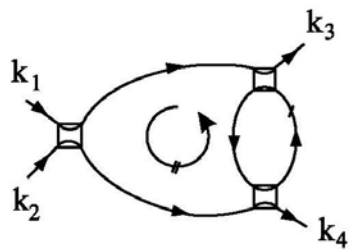
Two-loop equations

$$\frac{dV_\Lambda}{d\Lambda} = V_\Lambda \circ (G_\Lambda \circ S_\Lambda + S_\Lambda \circ G_\Lambda) \circ V_\Lambda + V_\Lambda^{(6)} \circ S_\Lambda$$

$$\frac{dV_\Lambda^{(6)}}{d\Lambda} = V_\Lambda \circ G_\Lambda \circ V_\Lambda \circ G_\Lambda \circ V_\Lambda \circ S_\Lambda + V_\Lambda^{(6)} \circ S_\Lambda \circ V_\Lambda \circ G_\Lambda$$

$$\frac{d\Sigma_\Lambda}{d\Lambda} = V_\Lambda \circ S_\Lambda$$

$$\frac{dV_\Lambda}{d\Lambda} = V_\Lambda \circ (G_\Lambda \circ S_\Lambda + S_\Lambda \circ G_\Lambda) \circ V_\Lambda + S_\Lambda \circ \int_\Lambda^{\Lambda_0} d\Lambda' V_{\Lambda'} \circ G_{\Lambda'} \circ V_{\Lambda'} \circ G_{\Lambda'} \circ V_{\Lambda'} \circ S_{\Lambda'}$$



(4 pp + 12 ph)

Two-loop equations: projections

$$V_\Lambda = \bar{V}_\Lambda + \delta V_\Lambda$$

$$\delta V_\Lambda = \int_\Lambda^{\Lambda_0} d\Lambda' [V_{\Lambda'} \circ G_{\Lambda'} \circ S_{\Lambda'} \circ V_{\Lambda'} - \hat{P}(V_{\Lambda'} \circ G_{\Lambda'} \circ S_{\Lambda'} \circ V_{\Lambda'})]$$

$$\frac{d\Sigma_\Lambda}{d\Lambda} = \bar{V}_\Lambda \circ S_\Lambda + S_\Lambda \circ \int_\Lambda^{\Lambda_0} d\Lambda' [\bar{V}_{\Lambda'} \circ G_{\Lambda'} \circ S_{\Lambda'} \circ \bar{V}_{\Lambda'} - \hat{P}(\bar{V}_{\Lambda'} \circ G_{\Lambda'} \circ S_{\Lambda'} \circ \bar{V}_{\Lambda'})]$$

$$\frac{d\bar{V}_\Lambda}{d\Lambda} = \hat{P}\{\bar{V}_\Lambda \circ (G_\Lambda \circ S_\Lambda) \circ \bar{V}_\Lambda$$

one loop

$$+\bar{V}_\Lambda \circ (G_\Lambda \circ S_\Lambda) \circ (1-\hat{P}) \int_\Lambda^{\Lambda_0} d\Lambda' \bar{V}_{\Lambda'} \circ G_{\Lambda'} \circ S_{\Lambda'} \circ \bar{V}_{\Lambda'}$$

corrects a vertex projection

$$+S_\Lambda \circ \int_\Lambda^{\Lambda_0} d\Lambda' \bar{V}_{\Lambda'} \circ G_{\Lambda'} \circ \bar{V}_{\Lambda'} \circ G_{\Lambda'} \circ \bar{V}_{\Lambda'} \circ S_{\Lambda'} \}$$

two-loop correction

Two-loop equations: local version etc.

$$\bar{V}_{\Lambda'} \to \bar{V}_{\Lambda}$$

$$\frac{d\Sigma_\Lambda}{d\Lambda} = \bar{V}_\Lambda \circ S_\Lambda + S_\Lambda \circ \bar{V}_\Lambda \circ \left[(1-\hat{P})(G_\Lambda \circ G_\Lambda) \right] \circ \bar{V}_\Lambda$$

$$\frac{d\bar{V}_\Lambda}{d\Lambda} = \hat{P} \left\{ \bar{V}_\Lambda \circ \frac{d}{d\Lambda}(G_\Lambda \circ G_\Lambda) \circ \bar{V}_\Lambda \right.$$

$$+ \bar{V}_\Lambda \circ (G_\Lambda \circ S_\Lambda) \circ \bar{V}_\Lambda \circ \left[(1-\hat{P})(G_\Lambda \circ G_\Lambda) \right] \circ \bar{V}_\Lambda$$

$$\left. + S_\Lambda \circ \bar{V}_\Lambda \circ G_\Lambda \circ \bar{V}_\Lambda \circ G_\Lambda \circ \bar{V}_\Lambda \circ G_\Lambda \right\}$$

$$G_\Lambda(\mathbf{k}, i\nu_n) = \frac{\theta(|\varepsilon_\mathbf{k}| - \Lambda)}{i\nu_n - \varepsilon_\mathbf{k} - \theta(|\varepsilon_\mathbf{k}| - \Lambda)\Sigma_\Lambda(\mathbf{k}, i\nu_n)} \Rightarrow$$

$$G_\Lambda(\mathbf{k}, i\nu_n) = \frac{Z^\Lambda_{\mathbf{k}_F} \theta(|\varepsilon_\mathbf{k}| - \Lambda)}{i\nu_n - \varepsilon_\mathbf{k}}$$

1D Hubbard model: fRG and field-theory RG

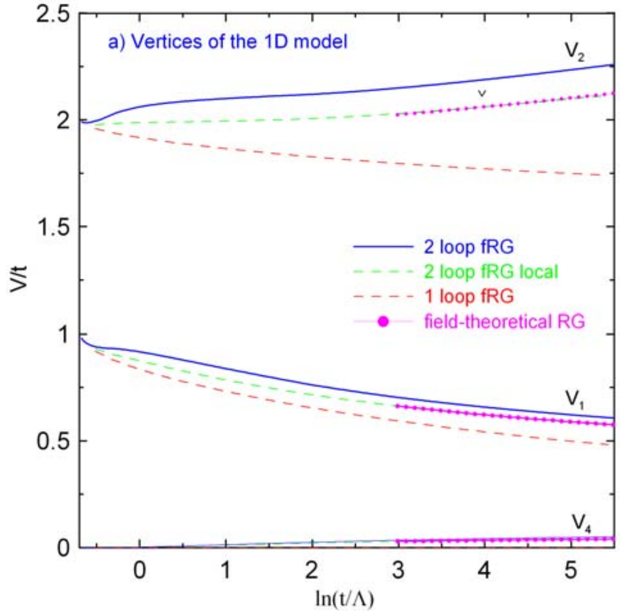
$$dg_1/dl = \frac{1}{\pi v_F} g_1^2 + \frac{1}{2\pi^2 v_F^2} g_1^2 (g_1 + g_4)$$

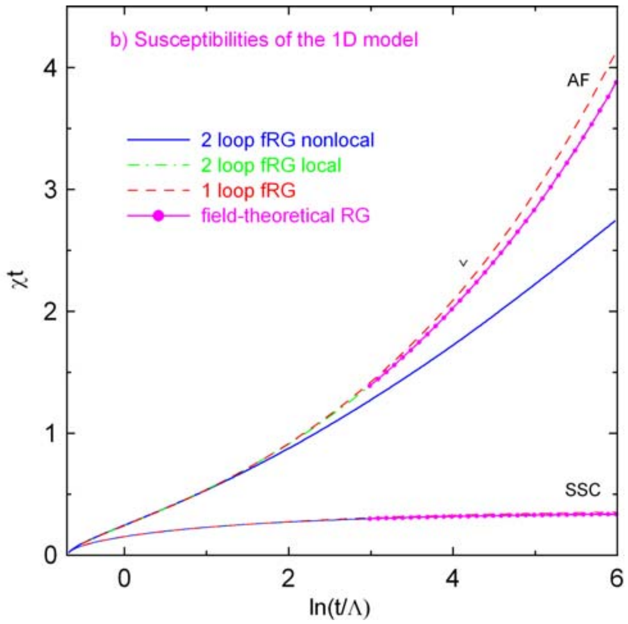
$$dg_2'/dl = \frac{1}{\pi v_F} g_3^2 + \frac{1}{2\pi^2 v_F^2} g_3^2 (g_1 - 2g_2 - g_4)$$

$$dg_3/dl = \frac{1}{\pi v_F} g_2' g_3 + \frac{1}{4\pi^2 v_F^2} g_3 [(g_2')^2 + g_3^2 - 2g_2' g_4]$$

$$dg_4/dl = \frac{3}{4\pi^2 v_F^2} (g_2' g_3^2 - g_1^3)$$

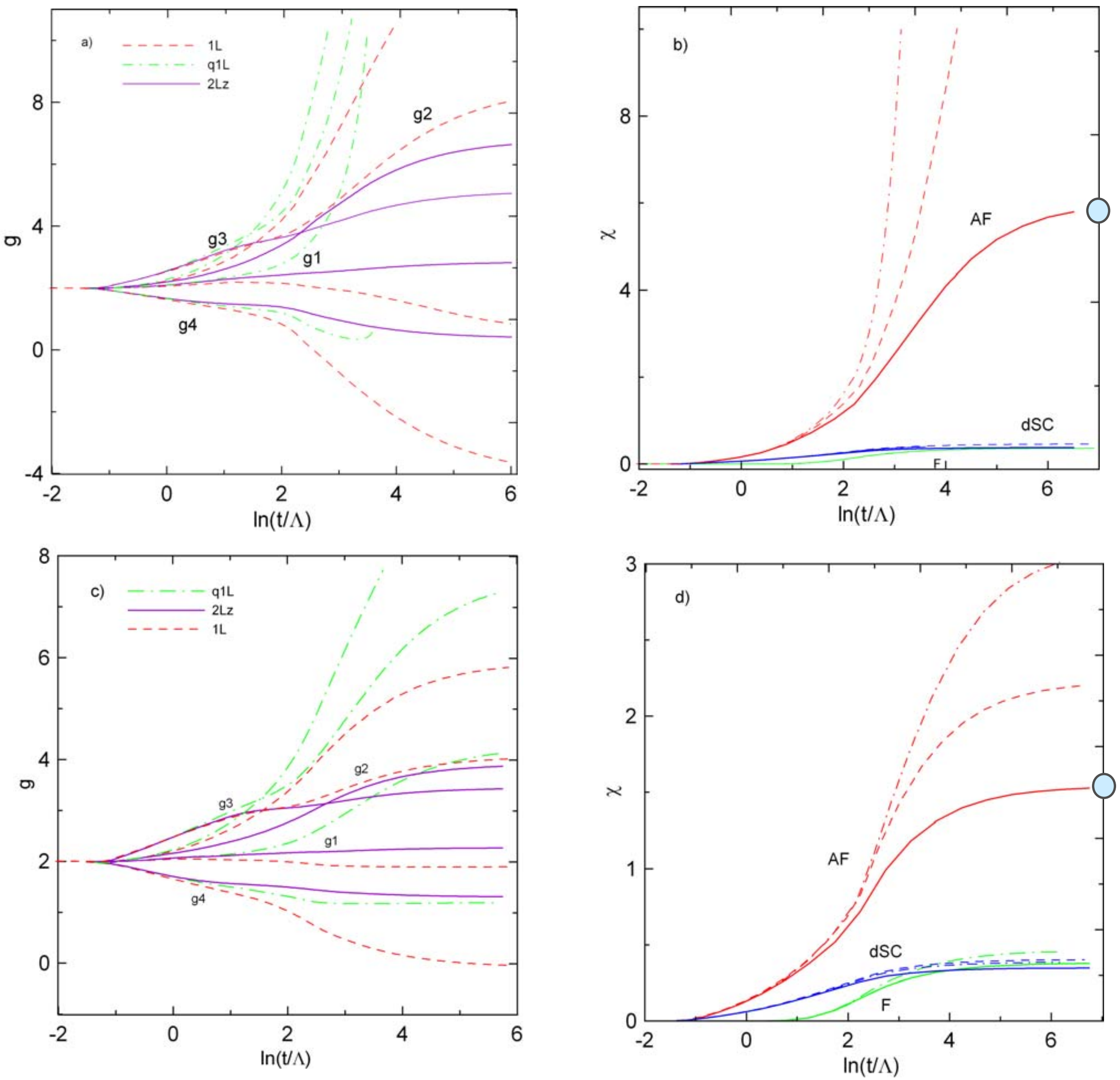
$$d \ln Z/dl = \frac{1}{4\pi^2 v_F^2} (g_1^2 - g_1 g_2 + g_2^2 + g_3^2)$$





2D Hubbard model: vertices and susceptibilities

$U = 2.5t;$
$t' = 0.1t$
$T = 0.1t$

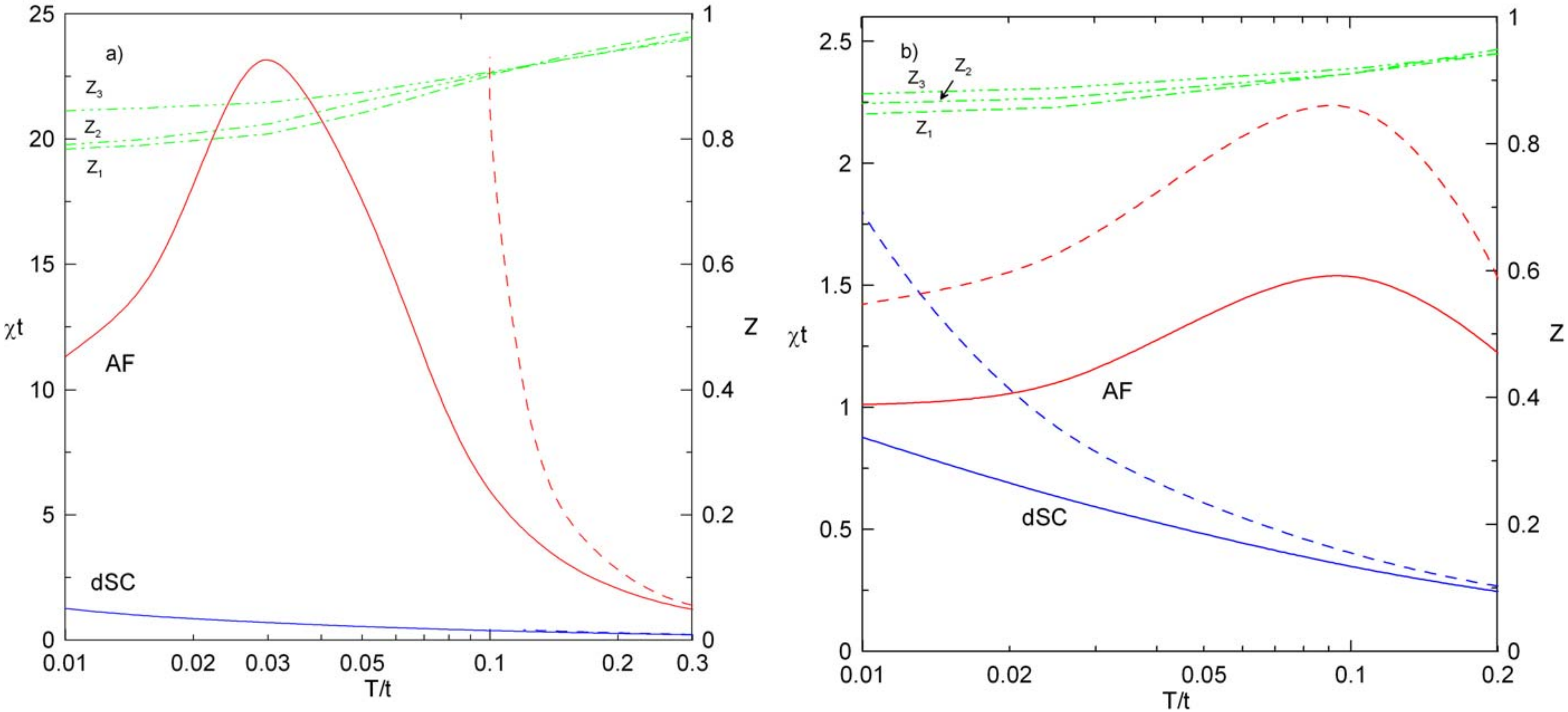


$\mu = 0.1t$

$\mu = -0.1t$

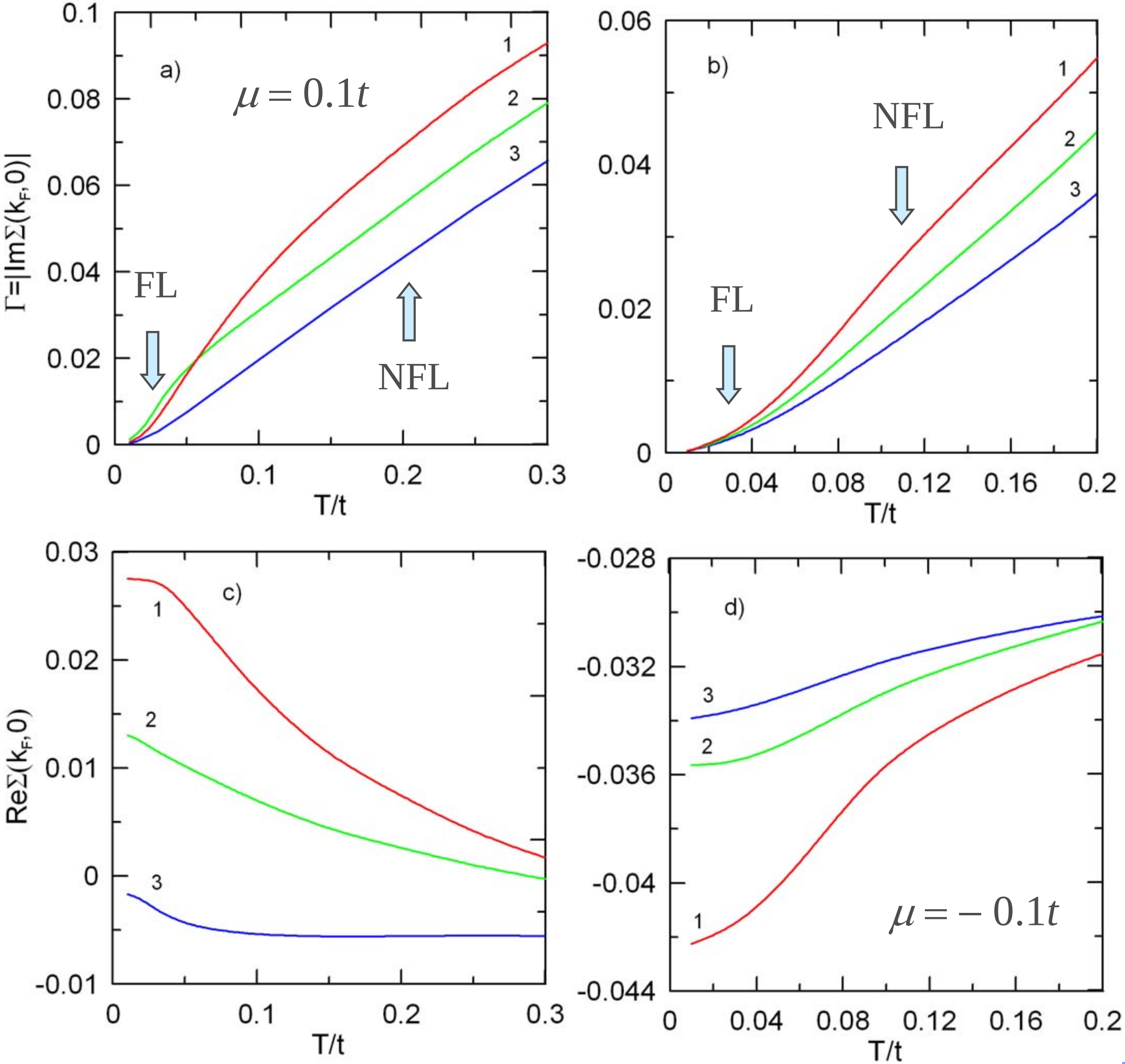
Temperature dependence of susceptibilities

$U = 2.5t$

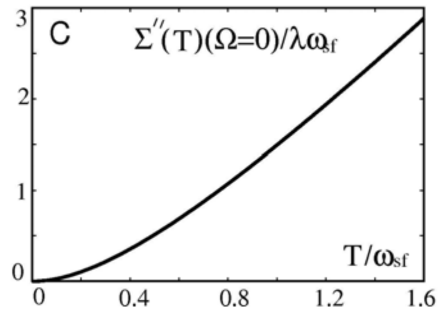


$\mu = 0.1t$ $\qquad$ $\mu = -0.1t$

Scattering rate and FS shifts



From spin-fermion theory:



(R. Haslinger, Ar. Abanov, and A. Chubukov, 2001)

Summary

- The two-loop corrections to the flow of vertices and susceptibilities in the 1D and 2D Hubbard model are investigated
- The results of the two loop approach are closer to the results of one loop approach with vertex projection; the divergence of vertices and susceptibilities is however suppressed
- The quasiparticle residues remain finite in the paramagnetic state
- The quasiparticle damping shows a T^2 dependence at low T and $T^{1-\alpha}$ dependence at higher T, $\alpha \geq 0$

Possible future extensions and perespectives

- Detail investigations of ImΣ(0,T)
- Self-consistent treatment of scattering rates
- Two-loop temperature flow

Thank you!