FRG approach to interacting fermions with partial bosonization: from weak to strong coupling

Talk at conference ERG08, Heidelberg, June 30, 2008

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1. FRG for fermions with partial bosonization
2. One dimensional confinement in 2D metals
3. FRG approach to the Anderson impurity model

1.FRG for fermions with partial bosonization

F. Schütz, L. Bartosch, PK, Phys. Rev. B 72, 035107 (2005)
F. Schütz and PK, J. Phys A 39,8205 (2006)

- want: FRG approach for strong coupling regime of interacting fermions
- idea: partial bosonization (Hubbard-Stratonovich transformation)

- problem: HS trafo is not unique
- partial solution: multi-component HS fields for all relevant channels
- input: prejudice about dominant fluctuation channels
- advantage: simple truncation in boson ⟷ non-perturbative in fermion

Non-uniqueness of HS transformations:

- particle-hole charge and longitudinal spin channel:

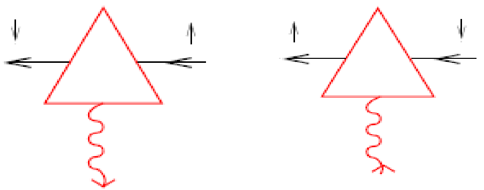
$$U\bar{d}_\uparrow(\tau)d_\uparrow(\tau)\bar{d}_\downarrow(\tau)d_\downarrow(\tau) = \frac{U_n}{2}n^2(\tau) - \frac{U_\parallel}{2}m^2(\tau) \qquad n(\tau) = \sum_\sigma \bar{d}_\sigma(\tau)d_\sigma(\tau)$$

$$U_n = U_\parallel = \frac{U}{2} \qquad m(\tau) = \sum_\sigma \sigma\bar{d}_\sigma(\tau)d_\sigma(\tau)$$

$$\frac{\mathcal{Z}}{\mathcal{Z}_0} = \frac{\int \mathcal{D}[\bar{d},d,\varphi,\eta]e^{-S_0[\bar{d},d]-S_0[\varphi,\eta]-S_1[\bar{d},d,\varphi,\eta]}}{\int \mathcal{D}[\bar{d},d,\varphi,\eta]e^{-S_0[\bar{d},d]-S_0[\varphi,\eta]}}$$

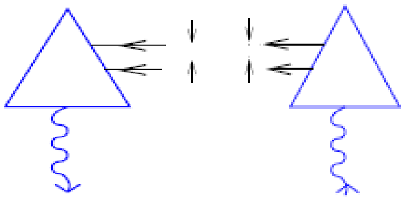
$$S_0[\varphi,\eta] = \frac{1}{2}\int_{\bar{\omega}}\left[U_n^{-1}\varphi_{-\bar{\omega}}\varphi_{\bar{\omega}} + U_\parallel^{-1}\eta_{-\bar{\omega}}\eta_{\bar{\omega}}\right] \qquad S_1[\bar{d},d,\varphi,\eta] = \int_{\bar{\omega}}[i\varphi_{-\bar{\omega}}n_{\bar{\omega}} + \eta_{-\bar{\omega}}m_{\bar{\omega}}]$$

- alternative: particle-hole spin singlet (attractive):



$$U\bar{d}_\uparrow d_\uparrow\bar{d}_\downarrow d_\downarrow = -U(\bar{d}_\uparrow d_\downarrow)(\bar{d}_\downarrow d_\uparrow) = -U\bar{s}(\tau)s(\tau)$$

- alternative: particle-particle spin singlet (repulsive):

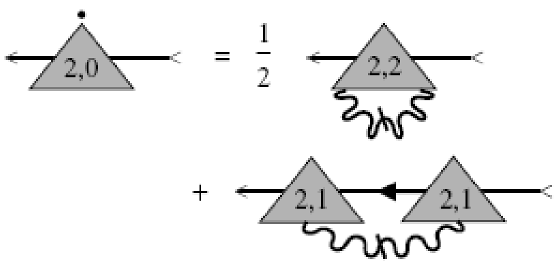


$$U\bar{d}_\uparrow d_\uparrow\bar{d}_\downarrow d_\downarrow = U(\bar{d}_\uparrow\bar{d}_\downarrow)(d_\downarrow d_\uparrow) = U\bar{b}(\tau)b(\tau)$$

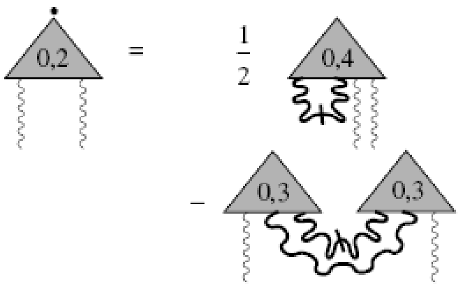
•we have developed a general method for generating the FRG hierarchy of flow equations for any coupled bose-fermi theory (Schütz, Bartosch, PK, 2005)

•RG flow for decoupling in charge channel:

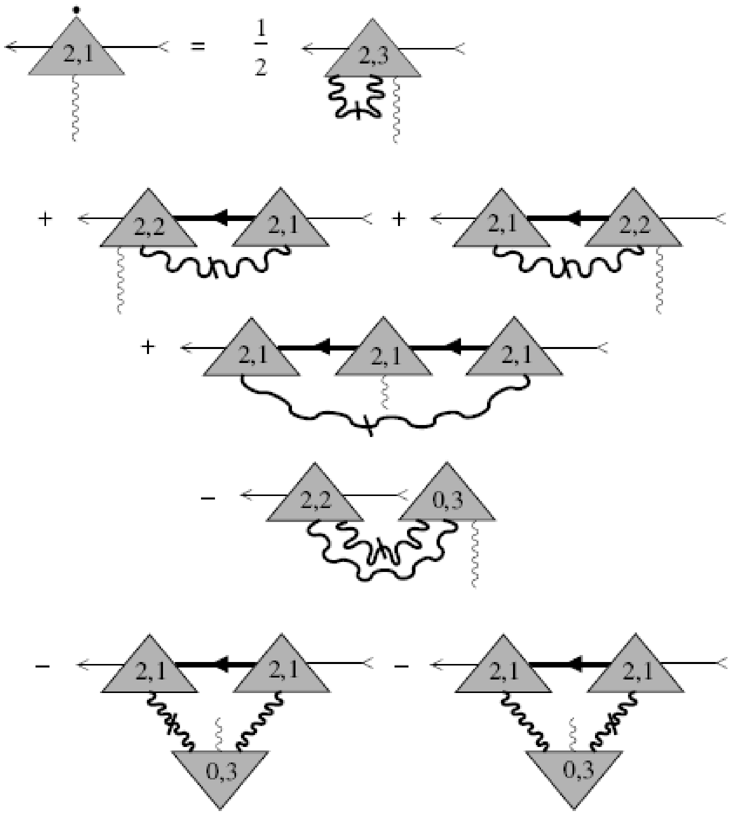
fermionic self-energy:



bosonic self-energy:



three-legged fermion-boson vertex:



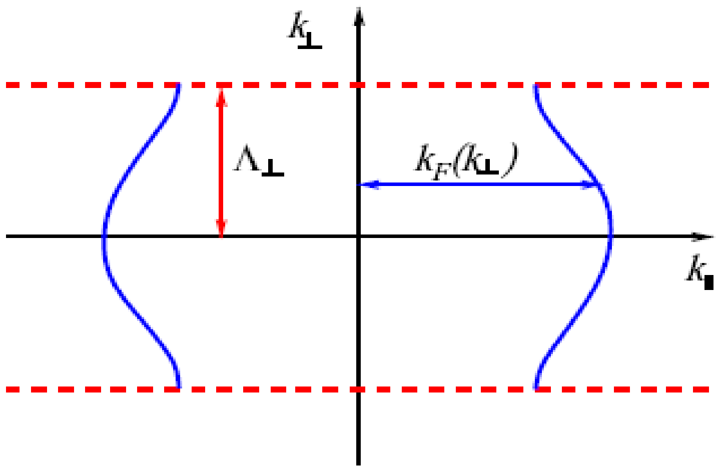
•for 1D fermions with linear dispersion and forward scattering interaction: exact solution of hierarchy possible for cutoff only in boson field

2. One-dimensional confinement in two dimensional metals

S. Ledowski and PK, Phys. Rev B 76, 121403(R), 2007

simple model for anisotropic metal:

$$S[\bar{\psi},\psi] = \sum_{\alpha} \int_K [-i\omega + \alpha v_F \delta k_{\|} + \mu_0(k_\perp)] \bar{\psi}_{K\alpha}\psi_{K\alpha} + \frac{1}{2}\sum_{\alpha\alpha'} \int_{\bar{K}} f_{\alpha\alpha'} \bar{\rho}_{\bar{K}\alpha} \rho_{\bar{K}\alpha'}$$



$$\delta k_{\|} = k_{\|} - \alpha k_F(k_\perp)$$

$$\mu_0(k_\perp) = -\Sigma(\boldsymbol{k}_F, i0)$$

$$\int_K = \int_{k_\perp} \int_{-\Lambda_{\|}}^{\Lambda_{\|}} \frac{d\delta k_{\|}}{2\pi} \int \frac{d\omega}{2\pi}$$

Fermi surface: $k_{\|} = \alpha k_F(k_\perp)$

density-density interaction:
no momentum transfer between sheets:

$$2\pi g_2 = \nu_0 f_{+-} = \nu_0 f_{-+}$$

$$2\pi g_4 = \nu_0 f_{++} = \nu_0 f_{--}$$

$$\nu_0 = \Lambda_\perp (\pi v_F)^{-1} = (a_\perp v_F)^{-1}$$

Question: can strong interactions completely wash out the curvature of Fermi surface (1d confinement)?

First try: second order self-consistent perturtbation theory:

shift of FS due to interactions: $\delta k_F(k_\perp) = k_F(k_\perp) - k_{F,0}(k_\perp)$

satisfies non-linear integral equation:

$$\frac{\delta k_F(k_\perp)}{\Lambda_0} = \left[-g_4 + \frac{g_4^2 + g_2^2}{2}\right] \int_{\bar{k}_\perp} \tilde{\Delta}(k_\perp, \bar{k}_\perp) - g_2^2 \int_{\bar{k}_\perp} \int_{k'_\perp} Y(\tilde{\Delta}(k_\perp, \bar{k}_\perp); \tilde{\Delta}(k'_\perp, \bar{k}_\perp))$$

$$\int_{k_\perp} = \int_{-\Lambda_\perp}^{\Lambda_\perp} \frac{dk_\perp}{2\Lambda_\perp}$$

$$\bar{\Lambda}_{||} = \Lambda_0$$

kernel:

$$Y(\tilde{\Delta}; \tilde{\Delta}') = \frac{\tilde{\Delta} + \tilde{\Delta}'}{4} \ln\left[\frac{4 - (\tilde{\Delta} - \tilde{\Delta}')^2}{(\tilde{\Delta} + \tilde{\Delta}')^2}\right]$$

$$\tilde{\Delta}(k_\perp, \bar{k}_\perp) = [k_F(k_\perp) - k_F(k_\perp + \bar{k}_\perp)]/\Lambda_0$$

numerical solution possible; more instructive: expansion in harmonics.

approximate solution of perturbative integral equation :

- bare FS: $k_{F,0}(k_\perp) = \bar{k}_F + t_0 \cos(k_\perp a_\perp)$

nearest neighbor hopping:

$t_0 = 2t_\perp / v_F \ll \Lambda_0$

- renormalized FS: $k_F(k_\perp) \doteq \bar{k}_F + t\cos(k_\perp a_\perp) + \ldots$

- self-consistency condition: $t/t_0 = [1 + R(t)]^{-1}$ $\quad R(t) \approx \frac{g_4}{2} - \frac{g_4^2}{4} + \frac{g_2^2}{2} \ln\left(\frac{\Lambda_0}{|t|}\right)$

logarithmic correction dominates for small interchain hopping

- at weak coupling FS never becomes flat!
- can renormalzed hopping vanish at strong coupling?
- Anderson 1993: YES!
- use FRG with partial bosonization to study strong coupling regime

Strong coupling FRG for the two-dimensional FS:

S. Ledowski and PK, J. Phys. Cond. Mat. (2003), PRB 2005, 2007.

- FS can be defined as RG fixed point by fine tuning initial conditions of relevant couplings: $r_l(k_\perp) = Z_l(k_\perp)[\Sigma_\Lambda(\boldsymbol{k}_F, i0, \alpha) + \mu_0(k_\perp)]/v_F\Lambda$

- exact RG flow equation: $\partial_l r_l(k_\perp) = [1 - \eta_l(k_\perp)] r_l(k_\perp) + \dot{\Gamma}_l(k_\perp)$

flowing anomalous dimension: $\eta_l(k_\perp) = -\partial_l \ln Z_l(k_\perp)$

exact integral equation for FS shift:

$$\frac{\delta k_F(k_\perp)}{\Lambda_0} = r_0(k_\perp) = -\int_0^\infty dl e^{-l + \int_0^l dt \eta_t(k_\perp)} \dot{\Gamma}_l(k_\perp)$$

- here: approximation for inhomogeneity on rhs:

boson propagator — vertex corrections

$$\dot{\Gamma}_l(k_\perp) = -\int_{\bar{k}_\perp} \int \frac{d\bar{q} d\bar{\epsilon}}{(2\pi)^2} \frac{\delta(|\bar{q}| - 1)[\mathbf{F}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp)]_{\alpha\alpha} e^{i\bar{\epsilon}0}}{i\bar{\epsilon} - \alpha\bar{q} - \tilde{\Delta}_l(k_\perp, \bar{k}_\perp)} \gamma_l(k_\perp, \bar{k}_\perp) \gamma_l(k_\perp + \bar{k}_\perp, -\bar{k}_\perp)$$

$$\tilde{\Delta}_l(k_\perp, \bar{k}_\perp) = \tilde{k}_{F,l}(k_\perp) - \tilde{k}_{F,l}(k_\perp + \bar{k}_\perp) \qquad \tilde{k}_{F,l}(k_\perp) = k_F(k_\perp)/\Lambda - r_l(k_\perp)$$

•adiabatic approximation for propagator of density fluctuations:

$$[\mathbf{F}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp)]^{-1}_{\alpha\alpha'} = [\nu_0 \mathbf{f}]^{-1}_{\alpha\alpha'} + \delta_{\alpha\alpha'} \tilde{\Pi}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp, \alpha)$$

$$\tilde{\Pi}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp, \alpha) = \frac{1}{2\pi} \int_{k_\perp} \frac{\tilde{\Delta}_l(k_\perp, \bar{k}_\perp) + \alpha \bar{q}}{\tilde{\Delta}_l(k_\perp, \bar{k}_\perp) + \alpha \bar{q} - i\bar{\epsilon}} \gamma_l(k_\perp, \bar{k}_\perp) \gamma_l(k_\perp + \bar{k}_\perp, -\bar{k}_\perp)$$

•flowing anomalous dimension:

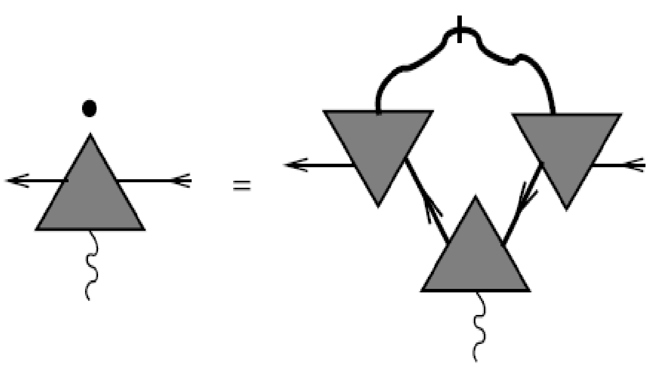
$$\eta_l(k_\perp) = - \int_{\bar{k}_\perp} \int \frac{d\bar{q} d\bar{\epsilon}}{(2\pi)^2} \frac{\delta(|\bar{q}| - 1)[\mathbf{F}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp)]_{\alpha\alpha}}{[i\bar{\epsilon} - \alpha\bar{q} - \tilde{\Delta}_l(k_\perp, \bar{k}_\perp)]^2} \gamma_l(k_\perp, \bar{k}_\perp) \gamma_l(k_\perp + \bar{k}_\perp, -\bar{k}_\perp)$$

•flowing vertex correction:

$$\partial_l \gamma_l(k_\perp, \bar{k}_\perp) = -\frac{1}{2}[\eta_l(k_\perp) + \eta_l(k_\perp + \bar{k}_\perp)] \gamma_l(k_\perp, \bar{k}_\perp)$$

$$- \int_{\bar{k}'_\perp} \int \frac{d\bar{q} d\bar{\epsilon}}{(2\pi)^2} \delta(|\bar{q}| - 1)[\mathbf{F}_l(\bar{q}, i\bar{\epsilon}, \bar{k}'_\perp)]_{\alpha\alpha}$$

$$\times \frac{\gamma_l(k_\perp + \bar{k}'_\perp, \bar{k}_\perp) \gamma_l(k_\perp, \bar{k}'_\perp) \gamma_l(k_\perp + \bar{k}_\perp + \bar{k}'_\perp, -\bar{k}'_\perp)}{[i\bar{\epsilon} - \alpha\bar{q} - \tilde{\Delta}_l(k_\perp, \bar{k}'_\perp)][i\bar{\epsilon} - \alpha\bar{q} - \tilde{\Delta}_l(k_\perp + \bar{k}_\perp, \bar{k}'_\perp)]}$$



Result 1: RG flow of vertex correction:

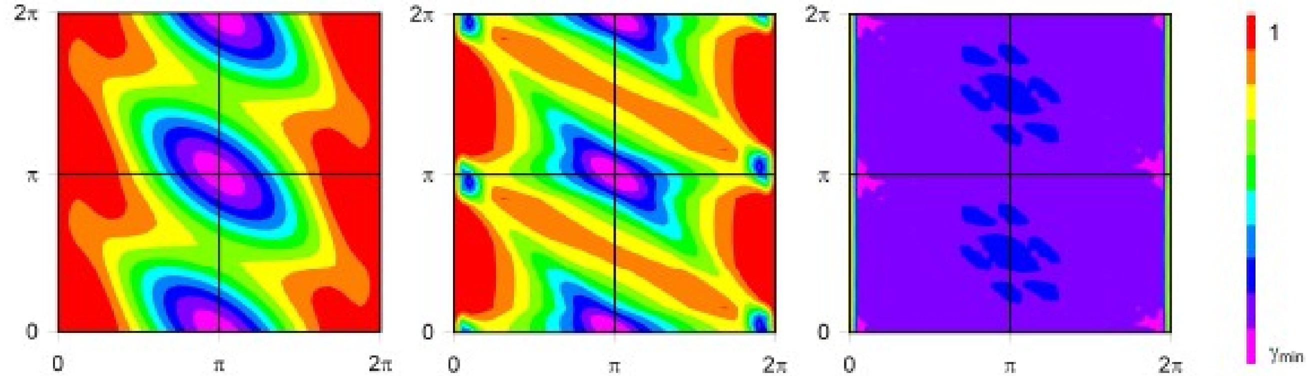
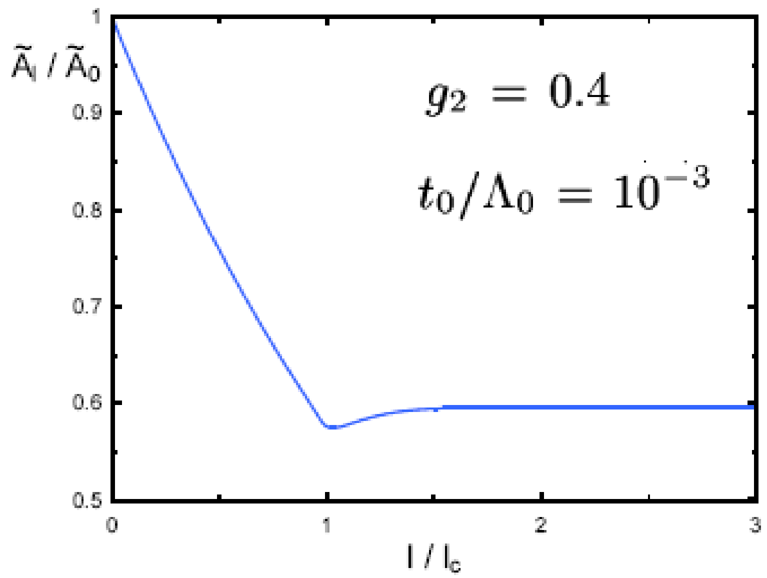


FIG. 3: (Color online) Evolution of the vertex $\gamma_l(k_\perp, \bar{k}_\perp)$ for a harmonic bare FS with amplitude $t_0/\Lambda_0 = 10^{-3}$ and bare coupling $g_2 = 0.4$ for different values of the flow parameter l. The abscissa indicates $\bar{k}_\perp$ and the ordinate $k_\perp$. From left to right $l = \frac{1}{2}l_c, l_c, 3l_c$ and $\gamma_{\min} = 0.999989, 0.981, 0.85$. To evaluate the flow we have expanded in Eqs. (7) and (11) up to g_2^2.

- vertex becomes approximately independent of fermionic momentum

- effective momentum-dependent interaction $g_l(\bar{k}_\perp) = g_2 \gamma_l^2(\bar{k}_\perp)$

Result 2: RG flow of effective interchain hopping for intermediate coupling



- interchain-hopping strongly reduced: FS tends to become flat
- at scale $l_c \approx -\ln(2t_0/\Lambda_0)$ effective cutoff comparable with bare hopping:

$$\Lambda_c = \Lambda_0 e^{-l_c} = 2t_0$$

- vertex becomes independent of fermionic momentum

Result 3: confinement transition at strong coupling:

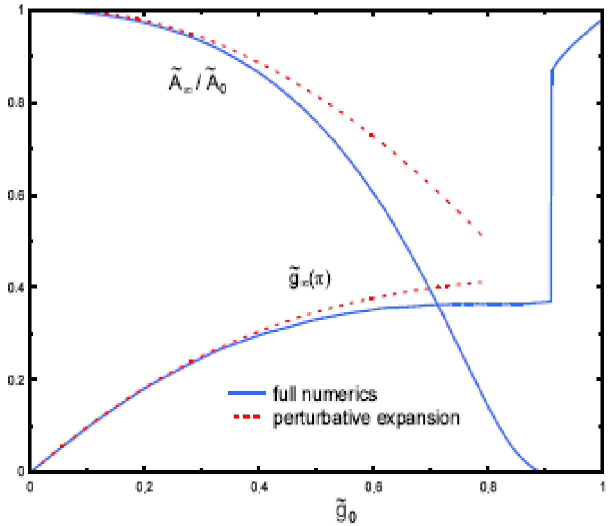
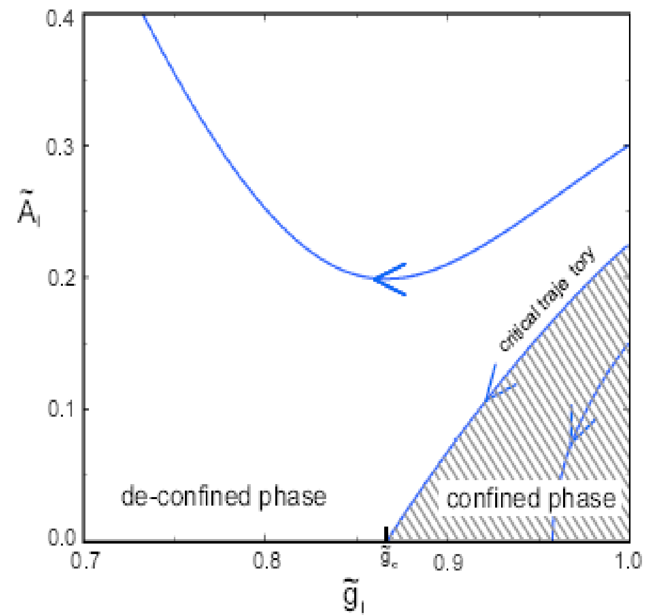


FIG. 6: (Color online) RG flow of $\tilde{t}_l$ and u_l as a function of the bare interaction g_2 for $\tilde{t}_0 = 0.1$ as obtained from the numerical solution of Eqs. (7) and (11).



- qualitative description via simplified flow equations:

- effective interchain hopping: $$\partial_l \tilde{t}_l = \left[1 - 2 \int_{\bar{k}_\perp} \sin^2(\bar{k}_\perp a_\perp/2) \frac{1 - \sqrt{1 - g_l^2(\bar{k}_\perp)}}{\sqrt{1 - g_l^2(\bar{k}_\perp)}} \right] \tilde{t}_l$$

- effective interaction: $$\partial_l g_l(\bar{k}_\perp) = - \frac{4 \sin^2(\bar{k}_\perp a_\perp/2) g_l(\bar{k}_\perp) u_l^2 \tilde{t}_l^2}{\sqrt{1 - u_l^2}[1 + \sqrt{1 - u_l^2}]^3} \qquad u_l = g_l(\pi/a_\perp)$$

3. FRG approach to the Anderson impurity model

L. Bartosch, H. Freire, J.J. Cardenas, PK, in preparation

- FRG based on purely fermionic degrees of freedom fails at strong coupling (Hedden, Karrasch et al, 2004/08)
- idea: treat important interactions to all orders via suitable HS fields:
- what is essential to get coupling Fermi liquid fixed point? local moments and transverse spin fluctuations!

→ FRG with partial bosonization in spin-singlet particle-hole channel

$$s(\tau) = \bar{d}_{\downarrow}(\tau) d_{\uparrow}(\tau)$$

$$S_1[\bar{d}, d] = U \int_0^{\beta} d\tau \bar{d}_{\uparrow}(\tau) d_{\uparrow}(\tau) \bar{d}_{\downarrow}(\tau) d_{\downarrow}(\tau)$$

$$U \bar{d}_{\uparrow} d_{\uparrow} \bar{d}_{\downarrow} d_{\downarrow} = -U (\bar{d}_{\uparrow} d_{\downarrow})(\bar{d}_{\downarrow} d_{\uparrow}) = -U \bar{s}(\tau) s(\tau)$$

Anderson impurity model:

$$\hat{H} = \sum_{\boldsymbol{k}\sigma} [\epsilon_{\boldsymbol{k}} - \sigma h] \hat{c}^{\dagger}_{\boldsymbol{k}\sigma} \hat{c}_{\boldsymbol{k}\sigma}$$ non-interacting conduction electrons

$$+ \sum_{\sigma} [E_d - \sigma h] \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow}$$ interacting d-electrons

$$+ \sum_{\boldsymbol{k}\sigma} (V^*_{\boldsymbol{k}} \hat{d}^{\dagger}_{\sigma} \hat{c}_{\boldsymbol{k}\sigma} + V_{\boldsymbol{k}} \hat{c}^{\dagger}_{\boldsymbol{k}\sigma} \hat{d}_{\sigma}) ,$$ hybridisation

•effective action of d-electons after integration over conduction electrons:

$$S[\bar{d}, d] = - \int_{\omega} \sum_{\sigma} [i\omega - \xi_0^{\sigma} - \Delta^{\sigma}(i\omega)] \, \bar{d}_{\omega\sigma} d_{\omega\sigma} + U \int_0^{\beta} d\tau \bar{d}_{\uparrow}(\tau) d_{\uparrow}(\tau) \bar{d}_{\downarrow}(\tau) d_{\downarrow}(\tau)$$

$$\xi_0^{\sigma} = E_d - \mu - \sigma h \qquad \Delta^{\sigma}(i\omega) = \sum_{\boldsymbol{k}} \frac{|V_{\boldsymbol{k}}|^2}{i\omega - \epsilon_{\boldsymbol{k}} + \mu + \sigma h}$$

•local moment regime: $E_d \ll \mu \ll E_d + U$

•low energy spin susceptibility is the same as of Kondo model

•saturation below exponentially small low energy scale $T_K \propto \exp[-\pi^2 g_0/8]$

•strong coupling fixed point: Fermi liquid

$$g_0 = \frac{U}{\pi \Delta}$$

dynamic spin susceptibility in ladder approximation:

$$\Pi^{\rm ph}_{\rm LA}(i\bar{\omega}) = \frac{\Pi^{\rm ph}_0(i\bar{\omega})}{1 - U\Pi^{\rm ph}_0(i\bar{\omega})} = \int_0^\infty d\omega\, S^{\rm ph}_{\rm LA}(\omega)\frac{2\omega}{\omega^2 + \bar{\omega}^2}$$

- low energy behavior:

$$S^{\rm ph}_{\rm LA}(\omega) \approx \frac{\pi^{-2}\omega}{\omega_*^2 + \omega^2}$$

$$\omega_* = \Delta(1 - g_0)$$

- unphyscial magnetic instability

  for $g_0 \to 1$

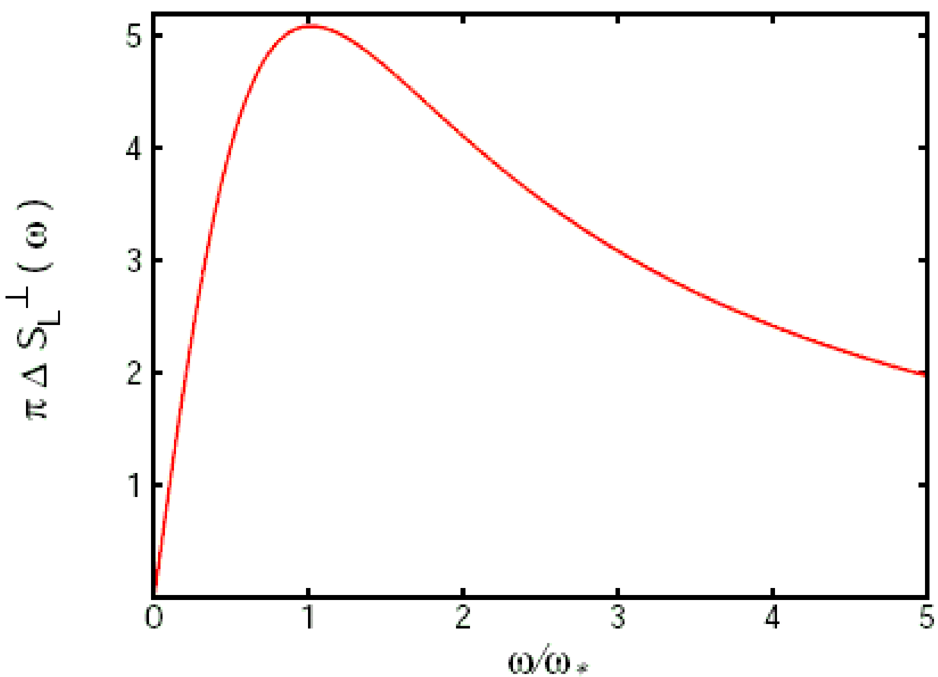


FIG. 2: Graph of the spin-singlet particle-hole structure factor $S^{\rm ph}_{\rm LA}(\omega)$ in ladder approximation for $g_0 = U/(\pi\Delta) = 0.9$. For $0 < 1 - g_0 \ll 1$ the energy scale of the peak position is $\omega_* \approx \Delta(1 - g_0)$ and the width of the peak is of order Δ.

quasiparticle residue in ladder approximation:

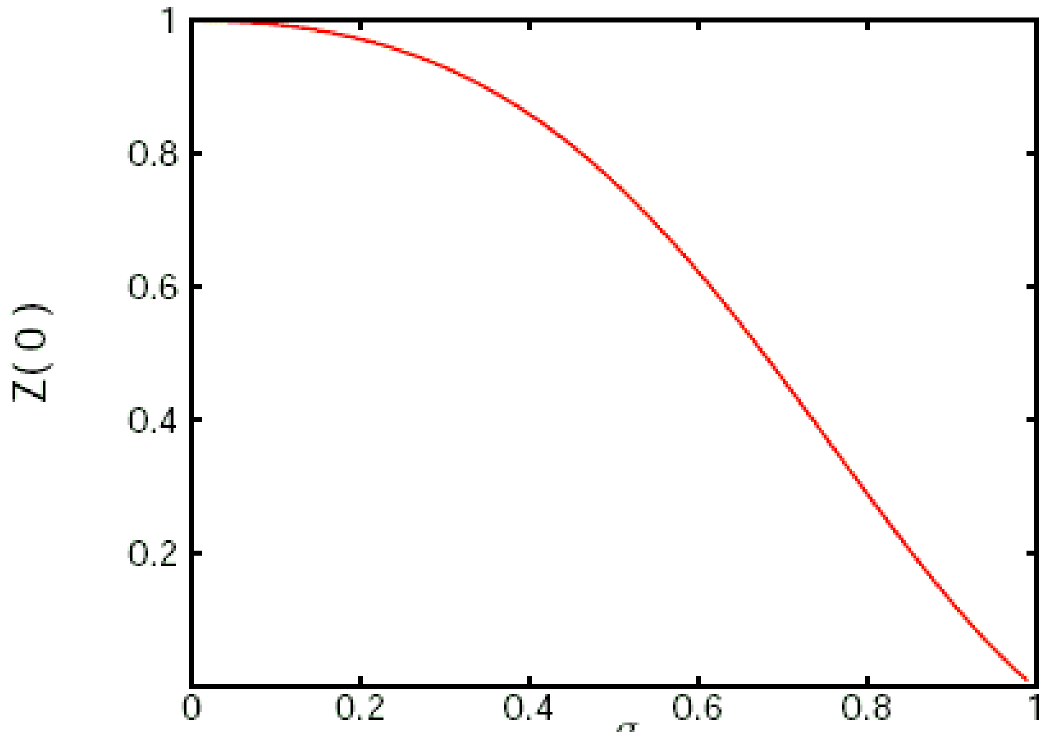


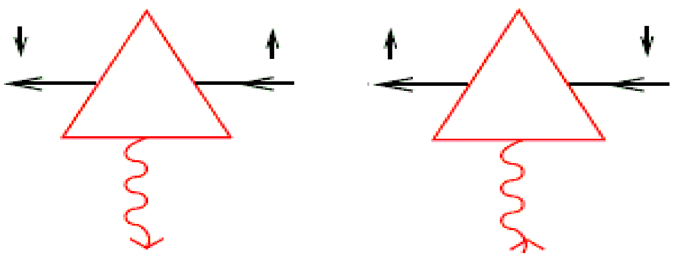
FIG. 6: (Color online) Graph of the quasi-particle residue $Z(0)$ as a function of $g_0 = U/(\pi\Delta)$.

FRG with partial bosonization in spin-flip channel:

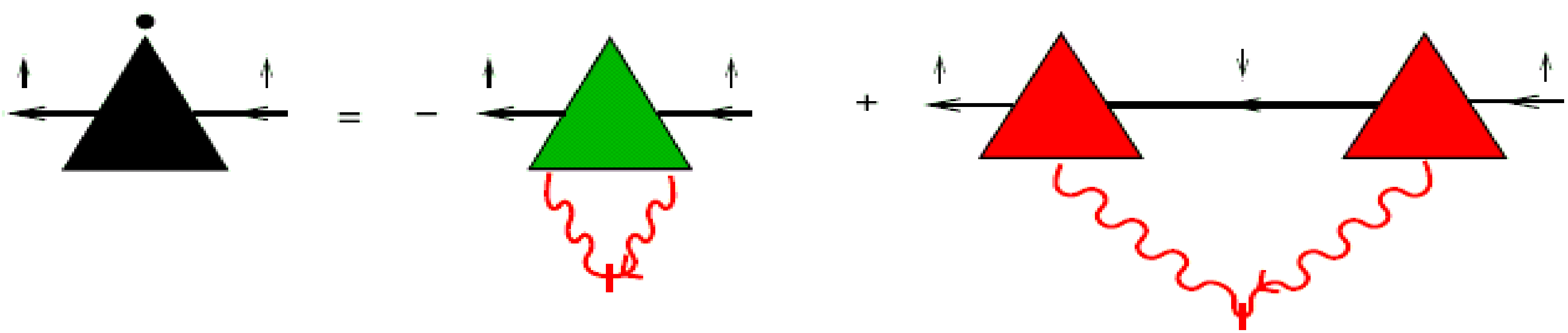
$$U\bar{d}_\uparrow d_\uparrow \bar{d}_\downarrow d_\downarrow \;=\; -U\bar{s}(\tau)s(\tau) \qquad s(\tau) = \bar{d}_\downarrow(\tau)d_\uparrow(\tau)$$

- after HS trafo: $S_0[\Phi] \;=\; -\int_\omega \sum_\sigma \left[G_0^\sigma(i\omega)\right]^{-1} \bar{d}_{\omega\sigma} d_{\omega\sigma} + \int_{\bar{\omega}} U^{-1} \bar{\chi}_{\bar{\omega}} \chi_{\bar{\omega}},$

$$S_1[\Phi] \;=\; \int_{\bar{\omega}} [\bar{s}_{\bar{\omega}} \chi_{\bar{\omega}} + s_{\bar{\omega}} \bar{\chi}_{\bar{\omega}}]$$

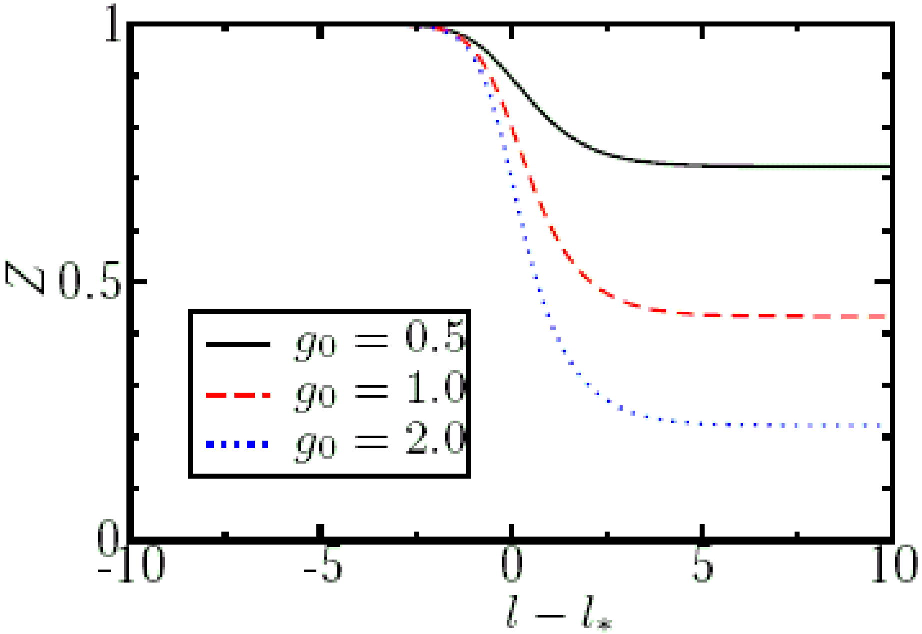


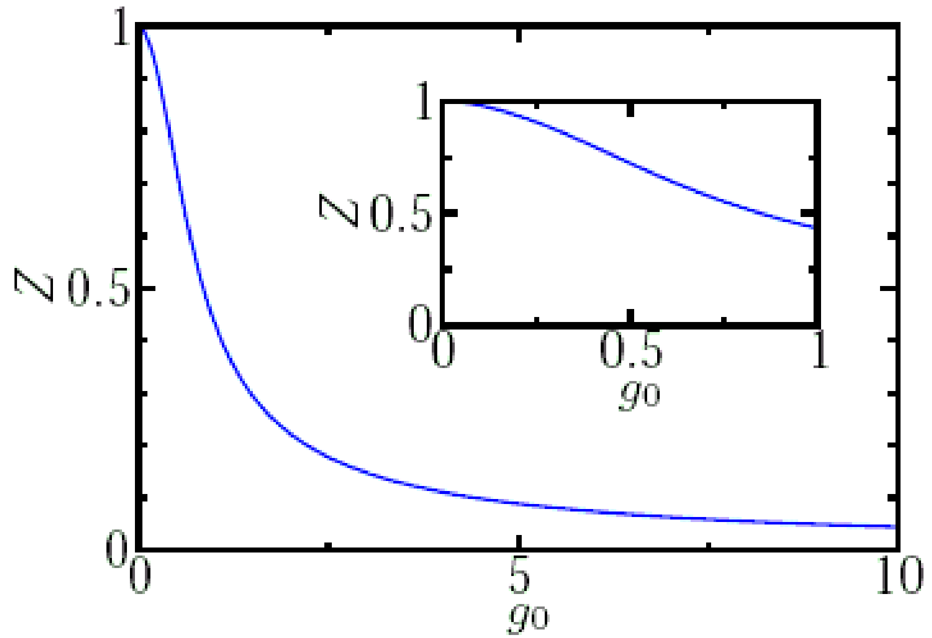
- FRG flow equation for fermionic self-energy (cutoff only in HS-field):



- close flow in bosonic sector via Dyson-Schwinger equation:

Results for wave-function renormalization:





- no magnetic instability, Fermi liquid for all couplings $Z \sim \frac{0.445}{g_0}, \quad g_0 \gg 1.$

- probably have to add longitudinal spin fluctuations to get Kondo scale
(work in progress...)

Summary, Conclusions

- FRG for fermions with partial bosonization
  - general formalism to generate FRG flow equations for vertices of coupled Fermi-Bose theories (Schuetz, Bartosch PK, 2005)
  - good method to deal with symmetry breaking (see also Metzner group)
- Application to 1D confinement in 2D metals:
  - calculate renormalized Fermi surface using FRG
  - confinement possible at strong coupling (Ledowski, PK, 2007)
- Application to Anderson impurity model:
  - HS trafo in spin-singlet particle-hole channel: no magnetic instability
  - work in progess: add longitudinal spin channel (Bartosch, Freire, Cardenas, PK, 2008)

forthcoming textbook on FRG:
(to appear by Springer, early 2009)

Peter Kopietz, Lorenz Bartosch, and Florian Schütz

Lectures on the Renormalization Group

SPIN Springer's internal project number, if known

– From the foundations to the functional renormalization group–

June 25, 2008

Springer

Part I The foundations

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Part II The functional renormalization group