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Projected flow equations and competing ordering tendencies in the 2D Hubbard model

Part of a joint project with C. Honerkamp, C. Husemann, W. Metzner and M. Salmhofer

Organization:

- Introduction into fermionic RG flows
- RG equation for the 4-point function
- RG flow of the 2D Hubbard model in a truncated interaction subspace
- Projected RG equations in the limits $V \to \infty$ and $T \to 0$
- First numerical results for the RG flow
- Conclusion and outlook

Introduction into fermionic RG flows

The fermionic RG scheme:

Powerful tool:

- for studying interacting fermion systems.
- in particular for analyzing Fermi surface instabilities.

Example:

Application to the weak-coupling range of the 2-dimensional Hubbard model; result:

RG suggests that in most cases the dominant instabilities lead to states with SB symmetry; e.g.: SC or magn. order.

In this context:

- the onset of SSB is signaled by flow to strong coupling at a small scale $\epsilon = \epsilon_{\text{div}}$, i.e.:

  some components of the 4-pt.-vertex grow larger than the bandwidth $\approx 8\,t$.

- often: $\exists$ several candidates for the strongly coupled state.

  Q: Which one is realized?

  A: The one with highest energy gain;
  in the RG approach: typically associated with the channel dominant at ϵ_{div}.

Evaluation of the RG eqns:

Usually an N-patch discretization scheme is employed, i.e.:

- the 2D BZ is divided into N patches (typically $N = 48$),
- the 4-pt. function is approximated by a constant for all momenta in the same patch,
- RG flow is calculated for the remaining subset of interaction vertices.

Subject of this talk:

Presentation of an alternative approximation scheme to evaluate the RG flow of the Hubbard model.

Basic idea:

Project RG flow onto a restricted interaction space parametrized by only a few (ideally the most important) couplings.

In practice:

1.) Formulate truncation ansatz for the 4-pt.-function containing the terms which presumably dominate the flow.

2.) Construct projectors onto the subspace spanned by the ansatz.

3.) Apply the projectors onto RG eq. for the 4-pt. function.
$\rightarrow$ system of RG eqns. for the couplings considered.

RG equation for the 4-point function

The generating function Γ_s for the 1PI vertices:

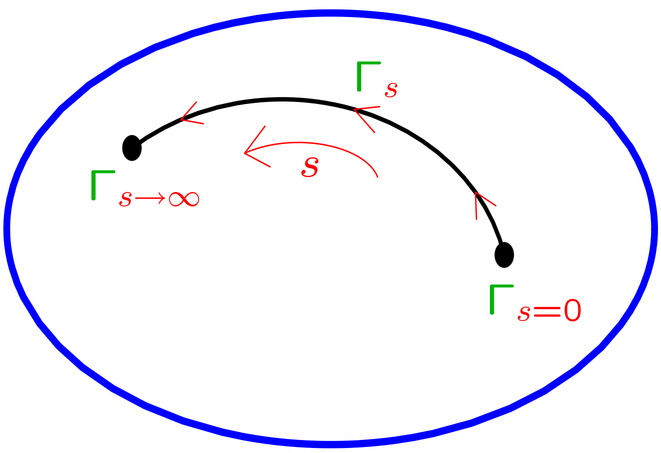
- Starting point of the flow at $s = 0$:

  initial action $\Gamma_{s=0}$ defined at ϵ_0.

  (ϵ_0: fixed energy scale, usually identified with the band-width of the model.)

- In its RG flow with increasing s, degrees of freedom with energy above the scale $\epsilon_s = \epsilon_0 \, \mathrm{e}^{-s}$ are gradually included in Γ_s.

- The full model is recovered as $s \to \infty$ where $\epsilon_s \to 0$.



- Γ_s is the Legendre transform of the generating function $W(Q_s, H)$ defined by

$$e^{-W_s(Q_s,H)} = \int \mathcal{D}\Psi \, e^{-\frac{1}{2}(\Psi,Q_s\Psi)-\mathcal{V}_0(\Psi)+(H,\Psi)}$$

Here:

  - $Q \equiv Q_{s\to\infty}$: free inverse propagator
  - Q_s: scale dependent modification of Q; regulates the successive integrating of the degrees of freedom.
  - we assume $\mathcal{V}_0(\Psi) = \mathcal{V}_0(-\Psi)$.
  - Ψ: column vector for the 4 components of the ordinary fermionic fields $\bar{\psi}_\alpha$, ψ_α, $\alpha = \pm$:

$$\Psi(K) = \left(\bar{\psi}_+(K), \bar{\psi}_-(-K), \psi_+(K), \psi_-(-K)\right)^{\mathsf{T}}$$

  - $K = (k, \omega)$

    ω=fermionic Matsubara frequency,

    k=spatial part of the momentum

  - (f, g): bilinear form; for a system in "box" of finite volume L^d:

$$(f, g) = \beta^{-1} \sum_\omega L^{-d} \sum_k \sum_{j=1}^{4} f_j(k,\omega)\, g_j(k,\omega)$$

The exact RG equation for Γ_s:

$$\dot{\Gamma}_s = \frac{1}{2}(\Psi, \dot{Q}_s \Psi) + \frac{1}{2} \operatorname{Tr} \left[\dot{Q}_s \left(\frac{\delta^2 \Gamma_s}{\delta \Psi^2} \right)^{-1} \right] \tag{1}$$

→nonpolynomial in Ψ.

(Notation: $\dot{} = \frac{\partial}{\partial s}$)

The truncated RG equations for w_s and $\hat{\Sigma}_s$:

- Expand (1) in the fields according to

  $\Gamma_s = K_s + \frac{1}{2}(\Psi, G_s^{-1} \Psi) + \sum_{m \geq 2} \Gamma_s^{(2m)}(\Psi)$

  & compare the homogeneous parts in Ψ

  $\Rightarrow$ system of diff. eqs. for the vertex functions
  ($\rightarrow$ polynomial in Ψ)

- Truncate the system of RGDE by setting $\Gamma_s^{(\geq 6)} = 0$

- Assume that Γ_s is endowed with
  - charge invariance
  - spin rotation invariance
  - translational invariance

Then inserting

$$\Gamma_s^{(4)}[\bar\psi,\psi] = \frac{1}{4}\frac{1}{(\beta L^2)^3}\sum_{\underline{K}}\sum_{\underline{\alpha}} w_s(\underline{K},\underline{\alpha}) \times \bar\psi_{\alpha_1}(K_1)\,\bar\psi_{\alpha_2}(K_2)\,\psi_{\alpha_3}(K_3)\,\psi_{\alpha_4}(K_1+K_2-K_3)$$

with 4-point function

$$w_s(\underline{K},\underline{\alpha}) = -D_{\alpha_1\alpha_2\alpha_3\alpha_4} V_s(K_1,K_2,K_3) + D_{\alpha_2\alpha_1\alpha_3\alpha_4} V_s(K_2,K_1,K_3)$$

$$D_{\alpha_1\alpha_2\alpha_3\alpha_4} = \delta_{\alpha_1,\alpha_4}\,\delta_{\alpha_2,\alpha_3}\ , \quad \underline{K}\equiv(K_1,K_2,K_3)\ , \quad \underline{\alpha}\equiv(\alpha_1,\ldots,\alpha_4)$$

leads to:

$$\dot{\hat\Sigma}_s(K) = \sum_{K'} \hat S_s(-K')\left[V_s(K',K,K') - 2\,V_s(K,K',K')\right]$$

and

$$-\dot w_s(\underline{K},\underline{\alpha}) = -D_{\alpha_1\alpha_2\alpha_3\alpha_4}\mathscr{R}(K_1,K_2,K_3) + D_{\alpha_2\alpha_1\alpha_3\alpha_4}\mathscr{R}(K_2,K_1,K_3)$$

Here:

- $G_s \equiv (Q_s - \Sigma_s)^{-1}$ (G_s=full propagator; Σ_s=selfenergy)
- $S_s = -G_s\dot Q_s G_s$ (single scale propagator)
- $A_{\alpha,\beta}(K) = \delta_{\alpha,\beta}\,\hat A(K)$ for $A = \Sigma_s, G_s, S_s$
- $\mathscr{R}(\underline{K}) = \hat{\mathcal{T}}_{\mathrm{pp}}(\underline{K}) + \hat{\mathcal{T}}^{\mathrm{d}}_{\mathrm{ph}}(\underline{K}) + \hat{\mathcal{T}}^{\mathrm{cr}}_{\mathrm{ph}}(\underline{K})$

Here we set

$$\widehat{\mathcal{T}}_{\text{pp}}(\underline{K}) = -\frac{1}{\beta L^2}\sum_{K'} \mathscr{L}_-(-(K_1+K_2),-K') \times V_s(K_1,K_2,K')\, V_s(K_1+K_2-K',K',K_3)$$

$$\widehat{\mathcal{T}}^{\text{d}}_{\text{ph}}(\underline{K}) = -\frac{1}{\beta L^2}\sum_{K'} \mathscr{L}_+(K_3-K_2,-K') \times \big[-2\, V_s(K',K_2,K_3)\, V_s(K_1,K_2-K_3+K',K') + V_s(K',K_2,K_3)\, V_s(K_2-K_3+K',K_1,K') + V_s(K_2,K',K_3)\, V_s(K_1,K_2-K_3+K',K')\big]$$

$$\widehat{\mathcal{T}}^{\text{cr}}_{\text{ph}}(\underline{K}) = -\frac{1}{\beta L^2}\sum_{K'} \mathscr{L}_+(K_1-K_3,-K') \times V_s(K_2,K',K_1+K_2-K_3) \times V_s(K_3-K_1+K',K_1,K')$$

with

$$\mathscr{L}_\pm(K',K) = \widehat{S}_s(K)\,\widehat{G}_s(K'\pm K) + \widehat{S}_s(K'\pm K)\,\widehat{G}_s(K)$$

Problem:

Even this truncated system of RG equations for $\hat{\Sigma}_s$ and w_s cannot be solved exactly.

$\Rightarrow$ Further approximations are necessary.

Approximation scheme employed here:

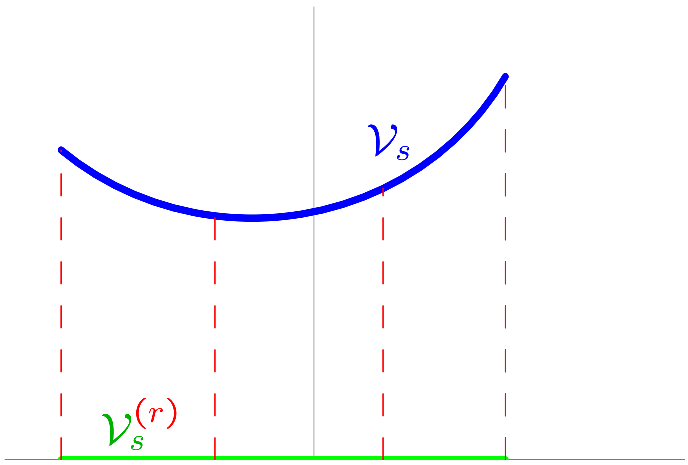
Projection of the RG eqs. onto a truncated interaction space.

This interaction supspace is specified by a corresponding ansatz $\mathcal{V}_s^{(r)}$ for $\mathcal{V}_s \equiv \Gamma_s^{(\mathrm{int})}$ spanning this subspace.

(In our truncation $\Gamma_s^{(\geq 6)} = 0$: $\Gamma_s^{(\mathrm{int})} = \Gamma_s^{(4)}$.)

In order to make calculations feasible:

$\mathcal{V}_s^{(r)}$ is chosen to consist of only a few terms (typically those which are expected to give the leading contributions to the RG flow).



RG flow of the 2D Hubbard model in a truncated interaction space

A.1) The model

In the following: consider the 2-dimensional t-t'-Hubbard model on the torus $\Lambda = \mathbb{Z}^2/L\mathbb{Z}^2$ of sidelength $L \in 2\mathbb{N}$
$\rightarrow$ described by a fermionic action of the form

$$S(\bar{\psi}, \psi) \;=\; \mathcal{A}_0(\bar{\psi}, \psi) + \mathcal{V}_0(\bar{\psi}, \psi)$$

(1) $\mathcal{A}_0$ = quadratic part of the fermionic action, given by the usual Fermi gas kinetic term

$$\begin{aligned} \mathcal{A}_0 &= \frac{1}{\beta L^2} \sum_K \sum_\alpha \left[\bar{\psi}_\alpha(K)\,(\mathrm{i}\omega - e(k))\,\psi_\alpha(K)\right) \\ &= \frac{1}{2}\,(\Psi, Q\,\Psi) \end{aligned}$$

Here:

- $k = (k^x, k^y) \in \Lambda^* \equiv \frac{2\pi}{L}\mathbb{Z}^2/2\pi\mathbb{Z}^2 \Rightarrow K = (k^x, k^y, \omega) \in \Lambda^* \times M_{\mathrm{F}}$
- $Q(K) = \begin{pmatrix} 0_2 & q(K) \\ -q(K) & 0_2 \end{pmatrix}$, $q(K) = i\omega\,\sigma_3 - e(k)\,\mathbb{1}_2$
- $e(k)$ = dispersion relation; for the t-t'-Hubbard model:
  $e(k) = -2t\,[\cos(k^x) + \cos(k^y)] - 4t'\cos(k^x)\cos(k^y) - \mu$
  (μ=chemical potential)

(2) $\mathcal{V}_0$ = usual quartic on-site interaction term:

$$\mathcal{V}_0(\bar\psi,\psi) = \frac{U}{6}\frac{1}{(\beta L^2)^3}\sum_{\underline{K}}\sum_{\underline{\alpha}} \vec\sigma_{\alpha_1\alpha_3}\,\vec\sigma_{\alpha_2\alpha_4}$$
$$\times\bar\psi_{\alpha_1}(K_1)\,\bar\psi_{\alpha_2}(K_2)\,\psi_{\alpha_3}(K_3)\,\psi_{\alpha_4}(K_1+K_2-K_3)$$

Here

- $\underline{K} = (K_1, K_2, K_3)$, $K_i = (k_i, \omega_i)$

  (furthermore: $\underline{k} = (k_1, k_2, k_3)$, $\underline{\omega} = (\omega_1, \omega_2, \omega_3)$)

- $\underline{\alpha} = (\alpha_1, \ldots, \alpha_4)$

- $\vec\sigma_{\alpha\beta} = (\sigma^{(1)}_{\alpha\beta}, \sigma^{(2)}_{\alpha\beta}, \sigma^{(3)}_{\alpha\beta})$;

  $\sigma^{(i)}$, $i = 1, 2, 3$: Pauli matrices

Choose $\mathcal{V}_0$ as initial interaction for the RG flow: $\mathcal{V}_0 \equiv \mathcal{V}_{s=0}$.

A.2) The RG flow

(1) Implementation of the scale dependence in W:

General recipe:
Replace Q by Q_s in the path integral for W such that the RG scale $\epsilon_s = \epsilon_0 \, \mathrm{e}^{-s}$ acts as an infrared cutoff.
(ϵ_0: bandwidth)

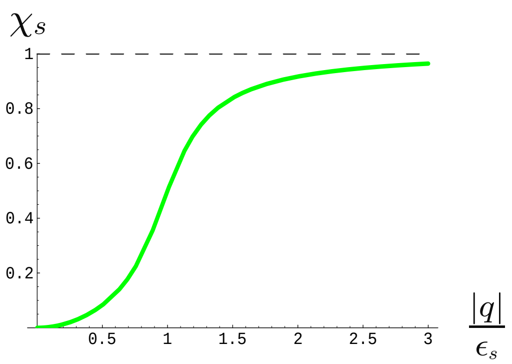
Realization in the present case:

$$q(K) \to q(K)\, \chi_s(K)^{-1} \quad \text{(i.e. } Q(K) \to Q_s(K) = Q(K)\, \chi_s(K)^{-1}\text{)}$$

$\chi_s(K)$ is a positive smooth function with the following properties:

- $\chi_s(K) \to 0$ for $|q(K)|/\epsilon_s \to 0$;
  $\chi_s(K) = 0$ for $q(K) = 0$.
  ($|q(K)| \equiv |i\omega - e(k)| = \sqrt{\omega^2 + e(k)^2}$)

- $\chi_s(K) \to 1$ for $|q(K)|/\epsilon_s \to \infty$,
  in particular for $s \to \infty$ at fixed $q(K)$



In the RG flow both $\mathcal{A}$ and $\mathcal{V}$ become functions of s.

Goal: Derive the corresponding RG equations using a truncation ansatz for $\mathcal{V}_s$.

(2) The ansatz

The quadratic part:

$$\Gamma_s^{(2)}[\bar\psi,\psi] = \frac{1}{\beta L^2}\sum_K \bar\psi(-K)\left[\frac{\mathrm{i}\,\omega - e(k)}{\chi_s(K)} - \Sigma(K)\right]\psi(K)$$

with cutoff function χ_s, cutoff energy scale ϵ_s, self energy Σ and dispersion relation $e(k)$.

Ansatz for $\mathcal{V}_s \equiv \Gamma_s^{(\mathrm{int})}$ specifying the truncated (or restricted) interaction space:

$$\mathcal{V}_s^{(r)}[\bar\psi,\psi] = \frac{1}{4}\frac{1}{(\beta L^2)^3}\sum_{\underline{K}}\sum_{\underline{\alpha}} w_s^{(r)}(\underline{K},\underline{\alpha})$$
$$\times\bar\psi_{\alpha_1}(K_1)\,\bar\psi_{\alpha_2}(K_2)\,\psi_{\alpha_3}(K_3)\,\psi_{\alpha_4}(K_1+K_2-K_3)$$

with truncated 4-point function

$$w_s^{(r)}(\underline{K},\underline{\alpha}) = \Big\{\frac{1}{3}\vec\sigma_{\alpha_1\alpha_3}\,\vec\sigma_{\alpha_2\alpha_4}\,[\mathrm{g_{OS}}(\underline{\omega}) - V_{\mathrm{N.N.}}(k_1-k_3-Q_{\mathrm{AF}},\underline{\omega})]$$
$$-\frac{1}{8}\,\mathrm{g_{SC}}(\underline{\omega})\,\epsilon_{\alpha_1\alpha_2}\,\epsilon_{\alpha_4\alpha_3}\eta_{0,1}^{(4)}(k_1)\,\eta_{0,1}^{(4)}(k_3)\,\delta_{k_1,-k_2}$$
$$+\frac{1}{4}\,\mathrm{g_{DW}}(\underline{\omega})\,\delta_{\alpha_1\alpha_3}\,\delta_{\alpha_2\alpha_4}\eta_{0,1}^{(4)}(k_2)\,\eta_{0,1}^{(4)}(k_3)\,\delta_{k_1,k_3+Q_{\mathrm{AF}}}\Big\} - \Big\{1\leftrightarrow 2\Big\}$$

with $Q_{\mathrm{AF}} = (\pi,\pi)$ and

$$V_{\mathrm{N.N.}}(q,\underline{\omega}) = \frac{1}{2}\left(\mathrm{g_{SS}^{(1)}}(\underline{\omega})\,\eta_{0,1}^{(1)}(q) + \mathrm{g_{SS}^{(4)}}(\underline{\omega})\,\eta_{0,1}^{(4)}(q)\right)$$
$$\eta_{0,1}^{(1)}(q) = \cos(q^x)+\cos(q^y)$$
$$\eta_{0,1}^{(4)}(q) = \cos(q^x)-\cos(q^y)$$

The properties of our ansatz

$$
\begin{aligned}
w_s^{(r)}(\underline{K},\underline{\alpha}) = \Big\{ & \frac{1}{3}\vec{\sigma}_{\alpha_1\alpha_3}\,\vec{\sigma}_{\alpha_2\alpha_4}\,\big[\mathsf{g}_{\mathsf{OS}}(\underline{\omega}) \\
& -\frac{1}{2}\mathsf{g}^{(1)}_{\mathsf{SS}}(\underline{\omega})\,\eta^{(1)}_{0,1}(k_1-k_3-Q_{\mathsf{AF}}) - \frac{1}{2}\mathsf{g}^{(4)}_{\mathsf{SS}}(\underline{\omega})\,\eta^{(4)}_{0,1}(k_1-k_3-Q_{\mathsf{AF}})\big] \\
& -\frac{1}{8}\,\mathsf{g}_{\mathsf{SC}}(\underline{\omega})\,\epsilon_{\alpha_1\alpha_2}\,\epsilon_{\alpha_4\alpha_3}\eta^{(4)}_{0,1}(k_1)\,\eta^{(4)}_{0,1}(k_3)\,\delta_{k_1,-k_2} \\
& +\frac{1}{4}\,\mathsf{g}_{\mathsf{DW}}(\underline{\omega})\,\delta_{\alpha_1\alpha_3}\,\delta_{\alpha_2\alpha_4}\eta^{(4)}_{0,1}(k_2)\,\eta^{(4)}_{0,1}(k_3)\,\delta_{k_1,k_3+Q_{\mathsf{AF}}}\Big\} - \Big\{1 \leftrightarrow 2\Big\}
\end{aligned}
$$

are:

(i) It contains the usual on-site term, two spin-spin-interaction terms, a d-wave SC term, and a d-DW term.

⇒ Competing ordering tendencies of these types of interaction can be investigated with this ansatz.

(ii) Apart from the on-site term only N.N. interactions are considered.

(iii) The g's depend both on ω and on s: $\mathsf{g}_{\mathsf{XY}}(\underline{\omega}) \equiv \mathsf{g}_{\mathsf{XY}}(\underline{\omega}, s)$.

(iv) Initial conditions:

- $\mathcal{V}_{s=0} = \mathcal{V}_0$

$$
\Rightarrow \begin{cases} \mathsf{g}_{\mathsf{OS}}(\underline{\omega}, s=0) = U \\ \mathsf{g}^{(1)}_{\mathsf{SS}}(\underline{\omega}, s=0) = \mathsf{g}^{(4)}_{\mathsf{SS}}(\underline{\omega}, s=0) = 0 \\ \mathsf{g}_{\mathsf{SC}}(\underline{\omega}, s=0) = \mathsf{g}_{\mathsf{DW}}(\underline{\omega}, s=0) = 0 \end{cases}
$$

B) Construction of the projector

For this purpose:

Introduce an inner product on the space of 4-point functions $w_s(\underline{K},\underline{\alpha})$ according to

$$\langle w_s^{(1)} | w_s^{(2)} \rangle \equiv \frac{1}{(\beta L^2)^3} \sum_{\underline{K}} \sum_{\underline{\alpha}} \overline{w_s^{(1)}(\underline{K},\underline{\alpha})}\, w_s^{(2)}(\underline{K},\underline{\alpha})$$

for arbitrary 4-point functions $w^{(1)}$, $w^{(2)}$.

The projector onto $w_s^{(r)}$:

Goal: construct projector P such that

$$(P\, w_s)(\underline{K},\underline{\alpha}) = w_s^{(r)}(\underline{K},\underline{\alpha}) \quad (P^2 = P = P^\dagger)$$

for any arbitrary 4-point-function w_s.

$\longrightarrow$ achieved by choosing $P(\underline{K},\underline{\alpha};\underline{K}',\underline{\alpha}') \equiv \langle \underline{K},\underline{\alpha} | P | \underline{K}',\underline{\alpha}' \rangle$ as

$$P(\underline{K},\underline{\alpha};\underline{K}',\underline{\alpha}') = \sum_{n=1}^{9} \int_{[0,\beta)^3} \mathrm{d}^3\tau\, u_{\underline{\tau}}^{(n)}(\underline{K},\underline{\alpha})\, \overline{u_{\underline{\tau}}^{(n)}(\underline{K}',\underline{\alpha}')}$$

$u^{(n)}_{\underline{\tau}}(\underline{K},\underline{\alpha})$: follow via a Gram-Schmidt procedure from

$$\tilde{u}^{(\mathrm{OS})}_{\pm;\underline{\tau}}(\underline{K},\underline{\alpha}) = \frac{1}{\sqrt{12\,(2\mp 1)}}\left(\vec{\sigma}_{\alpha_1\alpha_3}\vec{\sigma}_{\alpha_2\alpha_4} \pm \vec{\sigma}_{\alpha_1\alpha_4}\vec{\sigma}_{\alpha_2\alpha_3}\right)\, e^{-\mathrm{i}\underline{\omega}\cdot\underline{\tau}}$$

$$\tilde{u}^{(\mathrm{SS})}_{1;i;\underline{\tau}}(\underline{K},\underline{\alpha}) = \frac{1}{\sqrt{12}}\,\vec{\sigma}_{\alpha_1\alpha_3}\vec{\sigma}_{\alpha_2\alpha_4}\,\eta^{(i)}_{0,1}(k_1-k_3)\,e^{-\mathrm{i}\underline{\omega}\cdot\underline{\tau}}\,, \quad i\in\{1,4\}$$

$$\tilde{u}^{(\mathrm{SS})}_{2;i;\underline{\tau}}(\underline{K},\underline{\alpha}) = \frac{1}{\sqrt{12}}\,\vec{\sigma}_{\alpha_1\alpha_4}\vec{\sigma}_{\alpha_2\alpha_3}\,\eta^{(i)}_{0,1}(k_2-k_3)\,e^{-\mathrm{i}\underline{\omega}\cdot\underline{\tau}}\,, \quad i\in\{1,4\}$$

$$\tilde{u}^{(\mathrm{dSC})}_{\underline{\tau}}(\underline{K},\underline{\alpha}) = \frac{L}{2}\,\epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_4\alpha_3}\,\delta_{k_1,-k_2}\,\eta^{(4)}_{0,1}(k_1)\,\eta^{(4)}_{0,1}(k_3)\,e^{-\mathrm{i}\underline{\omega}\cdot\underline{\tau}}$$

$$\tilde{u}^{(\mathrm{dDW})}_{\pm;\underline{\tau}}(\underline{K},\underline{\alpha}) = \frac{L^2}{\sqrt{8\,L^2\pm 9}}\Big[\delta_{\alpha_1\alpha_3}\,\delta_{\alpha_2\alpha_4}\,\eta^{(4)}_{0,1}(k_2)\,\eta^{(4)}_{0,1}(k_3)\,\delta_{k_1,k_3+Q_{\mathrm{AF}}} \pm\delta_{\alpha_1\alpha_4}\,\delta_{\alpha_2\alpha_3}\,\eta^{(4)}_{0,1}(k_1)\,\eta^{(4)}_{0,1}(k_3)\,\delta_{k_2,k_3+Q_{\mathrm{AF}}}\Big]\,e^{-\mathrm{i}\underline{\omega}\cdot\underline{\tau}}$$

with

$$\underline{\omega}\cdot\underline{\tau} = \omega_1\,\tau_1 + \omega_2\,\tau_2 - \omega_3\,\tau_3$$

Inner products:

$$
\begin{aligned}
\langle \tilde{u}^{(\mathrm{OS})}_{s;\underline{\tau}} | \tilde{u}^{(\mathrm{OS})}_{s';\underline{\tau}'} \rangle &= \delta_{s,s'}\, \delta(\underline{\tau} - \underline{\tau}') \\
\langle \tilde{u}^{(\mathrm{SS})}_{\mu;i;\underline{\tau}} | \tilde{u}^{(\mathrm{SS})}_{\nu;i';\underline{\tau}'} \rangle &= \delta_{\mu,\nu}\, \delta_{i,i'}\, \delta(\underline{\tau} - \underline{\tau}') \\
\langle \tilde{u}^{(\mathrm{dSC})}_{\underline{\tau}} | \tilde{u}^{(\mathrm{dSC})}_{\underline{\tau}'} \rangle &= \delta(\underline{\tau} - \underline{\tau}') \\
\langle \tilde{u}^{(\mathrm{dDW})}_{s;\underline{\tau}} | \tilde{u}^{(\mathrm{dDW})}_{s';\underline{\tau}'} \rangle &= \delta_{s,s'}\, \delta(\underline{\tau} - \underline{\tau}')
\end{aligned}
$$

and

$$
\begin{aligned}
\langle \tilde{u}^{(\mathrm{dSC})}_{\underline{\tau}} | \tilde{u}^{(\mathrm{OS})}_{s;\underline{\tau}'} \rangle &= 0 \\
\langle \tilde{u}^{(\mathrm{dSC})}_{\underline{\tau}} | \tilde{u}^{(\mathrm{SS})}_{\mu;i;\underline{\tau}'} \rangle &= -(-1)^{\mu}\, \frac{\sqrt{3}}{4\,L}\, \delta_{i,1}\, \delta(\underline{\tau} - \underline{\tau}') \\
\langle \tilde{u}^{(\mathrm{dSC})}_{\underline{\tau}} | \tilde{u}^{(\mathrm{dDW})}_{s;\underline{\tau}'} \rangle &= -\frac{9}{2\,L}\, \frac{1}{\sqrt{8\,L^2 - 9}}\, \delta_{s,-}\, \delta(\underline{\tau} - \underline{\tau}') \\
\langle \tilde{u}^{(\mathrm{SS})}_{\mu;i;\underline{\tau}} | \tilde{u}^{(\mathrm{OS})}_{s;\underline{\tau}'} \rangle &= 0 \\
\langle \tilde{u}^{(\mathrm{SS})}_{\mu;i;\underline{\tau}} | \tilde{u}^{(\mathrm{dDW})}_{s;\underline{\tau}'} \rangle &= \frac{\sqrt{3}}{2}\, \frac{s^{\mu}}{\sqrt{8\,L^2 + 9\,s}}\, \delta_{i,1}\, \delta(\underline{\tau} - \underline{\tau}') \\
\langle \tilde{u}^{(\mathrm{dDW})}_{s;\underline{\tau}} | \tilde{u}^{(\mathrm{OS})}_{s';\underline{\tau}'} \rangle &= 0
\end{aligned}
$$

$\rightarrow$ $\tilde{u}$'s not pairwise orthogonal

C) Derivation of the RG eqns. for the couplings g

Steps to be performed:

- Insert ansatz $w_s^{(r)}$ into

$$-\dot{w}_s(\underline{K},\underline{\alpha}) = -D_{\alpha_1\alpha_2\alpha_3\alpha_4}\mathscr{R}(K_1,K_2,K_3) + D_{\alpha_2\alpha_1\alpha_3\alpha_4}\mathscr{R}(K_2,K_1,K_3)$$

- Apply P onto both sides of the resulting equation.

- Evaluate (hundreds of) k-sums on the projected RHS.

- Compare the coefficients of the u's.

This leads to

$$\begin{aligned}
-\dot{g}_{OS} &= \bar{\beta}_{OS}(g_{OS}, g_{SS}^{(1)}, g_{SS}^{(4)}, g_{SC}, g_{DW}) \\
-\dot{g}_{SS}^{(1)} &= \bar{\beta}_{SS1}(g_{OS}, g_{SS}^{(1)}, g_{SS}^{(4)}, g_{SC}, g_{DW}) \\
-\dot{g}_{SS}^{(4)} &= \bar{\beta}_{SS4}(g_{OS}, g_{SS}^{(1)}, g_{SS}^{(4)}, g_{SC}, g_{DW}) \\
-\dot{g}_{SC} &= \bar{\beta}_{SC}(g_{OS}, g_{SS}^{(1)}, g_{SS}^{(4)}, g_{SC}, g_{DW}) \\
-\dot{g}_{DW} &= \bar{\beta}_{DW}(g_{OS}, g_{SS}^{(1)}, g_{SS}^{(4)}, g_{SC}, g_{DW})
\end{aligned}$$

The $\bar{\beta}$'s:

- contain a lot of terms, each of them involving sums over one or two momenta (→ very lengthy expressions).

- any term is bilinear in the g's.

Projected RG equations in the limits $V \to \infty$ and $T \to 0$

After

- taking the limit $L^2 \equiv V \to \infty$
- neglecting the frequency dependence of the couplings g
- neglecting self-energy effects (i.e. setting $\Sigma \equiv 0$)

the flow eqns. boil down to

$$
\begin{aligned}
-\dot{g}_{OS} &= \beta^{(1)}_{OS}\, g^2_{OS} + \beta^{(2)}_{OS}\, g_{OS}\, g^{(1)}_{SS} + \beta^{(3)}_{OS} \left(g^{(1)}_{SS}\right)^2 + \beta^{(4)}_{OS} \left(g^{(4)}_{SS}\right)^2 \\
-\dot{g}^{(1)}_{SS} &= \beta^{(1)}_{SS}\, g^2_{OS} + \beta^{(2)}_{SS}\, g_{OS}\, g^{(1)}_{SS} + \beta^{(3)}_{SS} \left(g^{(1)}_{SS}\right)^2 + \beta^{(4)}_{SS} \left(g^{(4)}_{SS}\right)^2 \\
-\dot{g}^{(4)}_{SS} &= \beta^{(5)}_{SS}\, g_{OS}\, g^{(4)}_{SS} + \beta^{(6)}_{SS}\, g^{(1)}_{SS}\, g^{(4)}_{SS} \\
-\dot{g}_{SC} &= \beta^{(1)}_{SC}\, g^2_{SC} + \beta^{(2)}_{SC}\, g^2_{OS} + \beta^{(3)}_{SC}\, g_{SC}\, g^{(1)}_{SS} \\
&\quad + \beta^{(4)}_{SC} \left(g^{(1)}_{SS}\right)^2 + \beta^{(5)}_{SC} \left(g^{(4)}_{SS}\right)^2 \\
-\dot{g}_{DW} &= \beta^{(1)}_{DW}\, g^2_{DW} + \beta^{(2)}_{DW}\, g^2_{OS} + \beta^{(3)}_{DW}\, g_{DW}\, g^{(1)}_{SS} \\
&\quad + \beta^{(4)}_{DW} \left(g^{(1)}_{SS}\right)^2 + \beta^{(5)}_{DW} \left(g^{(4)}_{SS}\right)^2
\end{aligned}
$$

The β's are sums of different types of momentum integrals, denoted with $\mathscr{F}^{(\alpha;\lambda;\gamma)}_{\nu;\pm}$, $\mathscr{E}^{(\alpha;\beta)}_{p;\pm}$; for example:

$$
\begin{aligned}
\beta^{(4)}_{SC} &= -\frac{1}{4}\,\mathscr{E}^{(1;2)}_{0;-} + \frac{1}{6}\,\mathscr{F}^{(0;0;1)}_{0;-} + \frac{5}{96}\,\mathscr{F}^{(0;0;1)}_{1;-} \\
&\quad + \frac{1}{24}\left[-\mathscr{F}^{(1;1;2)}_{0;+} + \mathscr{F}^{(1;1;0)}_{0;+} + \mathscr{F}^{(0;1;-2)}_{0;+} - 6\,\mathscr{F}^{(0;0;1)}_{0;+}\right]
\end{aligned}
$$

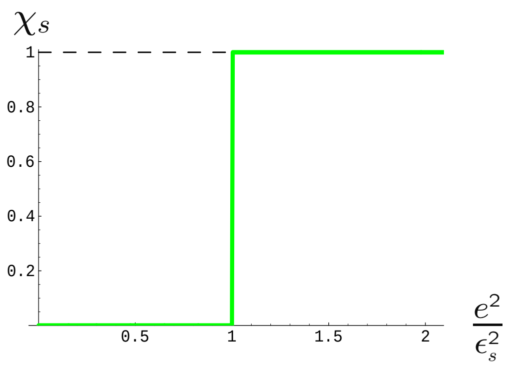
Next steps:

- Use appropriate cutoff of the form

  $\chi_s(k) = \chi(\frac{e(k)^2}{\epsilon_s^2})$

  and take the sharp cutoff limit (s.c.l.):

  $\chi_s(k) \stackrel{\text{s.c.l.}}{\longrightarrow} \Theta\left(\frac{e(k)^2}{\epsilon_s^2} - 1\right)$



  Main advantages of this cutoff:

  - $\chi_s = \chi_s(k)$, i.e. χ_s independent of ω

    $\Rightarrow \sum_\omega$ can be performed for arbitrary s

  - in the s.c.l.: $\chi_s^2 = \chi_s$, $\dot{\chi}_s = 2\epsilon_s\,\delta(e(\mathbf{k})^2 - \epsilon_s^2)$

    $\Rightarrow$ number of k-integrals is reduced by one

- Consider the limiting case $T \to 0$ (i.e. $\beta \to \infty$)

Consequence:

Expressions for $\mathscr{F}_{\nu;\pm}^{(\alpha;\lambda;\gamma)}$, $\mathscr{E}_{p;\pm}^{(\alpha;\beta)}$ boil down considerably.

$\Rightarrow$ Numerical solutions to the RG equations for the g's can be found for various choices of the free parameters

$U \equiv g_{OS}(\epsilon_0)$, $\quad \delta \equiv \frac{\mu - 4t'}{4t}$, $\quad \theta \equiv -2t'/t$ with $0 < \theta < 1$

without too much effort.

The coefficients $\mathscr{E}$ and $\mathscr{F}$ in the s.c.l. for $T=0$ & $\delta=0$

$$\mathscr{F}^{(\alpha;\lambda;\gamma)}_{\nu;\pm} = \mp 8t\,\mathbb{1}\left(\frac{\epsilon_s}{4t}\leq 1\mp\theta\right)\hat{j}^{(\alpha;\lambda;\gamma)}_{\nu}(\tfrac{\epsilon_s}{4t})\int_{\frac{\epsilon_s}{4t}}^{1\mp\theta}\mathrm{d}X'\,\frac{\frac{\epsilon_s}{4t}\,\hat{j}^{(\alpha;\lambda;\gamma)}_{\nu}(\mp X')}{\frac{\epsilon_s}{4t}+X'}$$

$$\mp 8t\,\mathbb{1}\left(\frac{\epsilon_s}{4t}\leq 1-\theta\right)\hat{j}^{(\alpha;\lambda;\gamma)}_{\nu}(-\tfrac{\epsilon_s}{4t})\int_{\frac{\epsilon_s}{4t}}^{1\pm\theta}\mathrm{d}X'\,\frac{\frac{\epsilon_s}{4t}\,\hat{j}^{(\alpha;\lambda;\gamma)}_{\nu}(\pm X')}{\frac{\epsilon_s}{4t}+X'}$$

$$\mathscr{E}^{(\alpha;\lambda)}_{0;-} = \frac{1}{2}\left[\hat{j}^{(\alpha;\lambda;0)}_{0}(\tfrac{\epsilon_s}{4t})\,\mathbb{1}\left(\frac{\epsilon_s}{4t}\leq 1+\theta\right)+\hat{j}^{(\alpha;\lambda;0)}_{0}(-\tfrac{\epsilon_s}{4t})\,\mathbb{1}\left(\frac{\epsilon_s}{4t}\leq 1-\theta\right)\right]$$

$$\mathscr{E}^{(\alpha;\lambda)}_{Q_{\mathrm{AF}};+} = \mathbb{1}\left(\frac{\epsilon_s}{4t}\leq 1-\theta\right)\frac{\epsilon_s}{4t}\,\hat{y}^{(\alpha;\lambda)}(-\tfrac{\epsilon_s}{4t})$$

with

$$\hat{j}^{(\alpha;\lambda;\gamma)}_{\nu}(X) = \frac{1}{\pi^2\,|t|}\int_{r_\zeta(|X|)}^{1}\frac{\mathrm{d}\rho}{\sqrt{1-\rho^2}}\,\frac{\Phi^{(\alpha;\lambda;\gamma)}_{\nu}(\rho,\Sigma(\rho,X))}{F(\rho,X)\,\sqrt{1-\Sigma^2(\rho,X)}}$$

$$\hat{y}^{(\alpha;\lambda)}(X) = \frac{1}{\pi^2 t}\int_{r_\zeta(|X|)}^{1}\frac{d\rho}{\sqrt{1-\rho^2}}\,\frac{\Phi^{(\alpha;\lambda;0)}_{0}(\rho,\Sigma(\rho,X))}{\rho\,\Sigma(\rho,X)\,F(\rho,X)\,\sqrt{1-\Sigma^2(\rho,X)}}$$

and

$$\Phi^{(\alpha;\lambda;\gamma)}_{\nu}(\rho,\Sigma) = [1-\nu+2\,\nu\,\rho\,\Sigma]\left[4\alpha\,\rho^2\,\Sigma^2-2\lambda\,(\rho^2+\Sigma^2-1)-\gamma\right]$$

$$F(\rho,X) = \sqrt{(1-\theta^2)\,\rho^2+2\theta\,X}$$

$$\Sigma(\rho,X) = \frac{1}{\theta}\,(\rho-F(\rho,X))$$

$$\zeta\equiv\mathrm{sgn}(X)\,,\quad r_\pm(|X|)\equiv\sqrt{\frac{|X|}{1\pm\theta}}$$

First numerical results for the RG flow

So far the projected flow eqns. were solved numerically

- for $T = 0$
- in the case $\delta = 0$ (i.e. at van Hove filling)
- for $\theta \in \{0.1, 0.5, 0.8\}$
- with the initial conditions $U = n\,t$ with $n \in \{1, 2, \ldots, 10\}$

Main Results:

- ordinary flow to strong coupling; at least g_{SC} diverges at nonzero values of ϵ_s.

  Usual procedure in this situation: analyze the behavior of the g's in the ϵ-regime where the largest coupling g_L exceeds the bandwidth, e.g. where $|\mathrm{g}_L| \lesssim 20\,t$.

- In the parameter regions

  (a) $\theta = 0.1$, $1 \leq U \lesssim 4\,t$

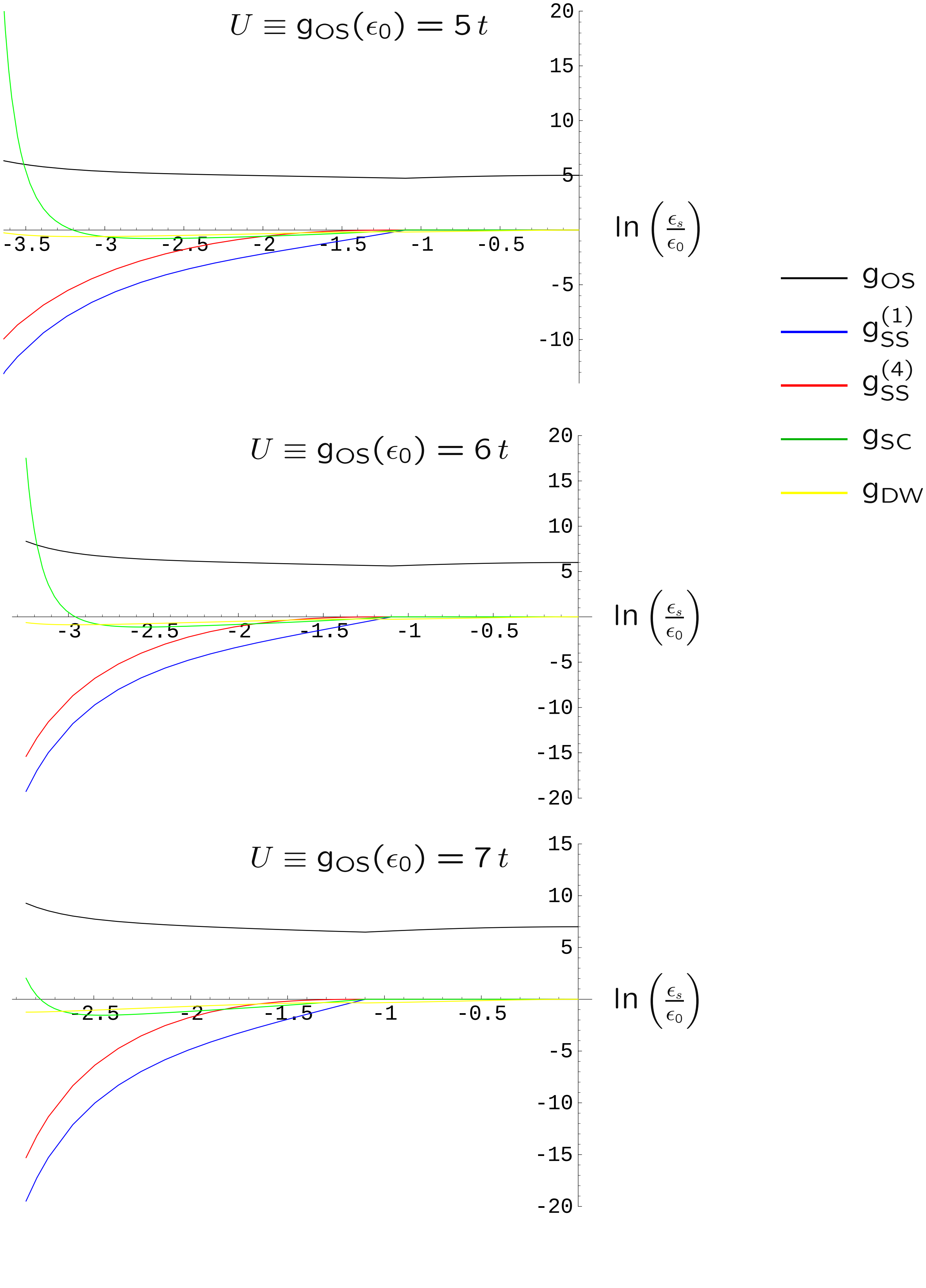
  (b) $\theta = 0.5$, $1 \leq U \lesssim 5\,t$

  (c) $\theta = 0.8$, $1 \leq U \lesssim 8\,t$

  g_{SC} grows "large" (i.e. exceeds $\approx 20\,t$) first.

- In the other parameter regions (with larger U-values) $\mathrm{g}_{\mathrm{SS}}^{(1)}$ seems to grow comparably strong or even stronger.

$\theta = 0.5$



$U \equiv g_{OS}(\epsilon_0) = 5\,t$
$U \equiv g_{OS}(\epsilon_0) = 6\,t$
$U \equiv g_{OS}(\epsilon_0) = 7\,t$
$\ln\left(\frac{\epsilon_s}{\epsilon_0}\right)$
20
15
10
5
-5
-10
-15
-20
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
g_{OS}
$g_{SS}^{(1)}$
$g_{SS}^{(4)}$
g_{SC}
g_{DW}

Conclusion and outlook

- The fermionic RG approach was applied to study the competition of different ordering tendencies in the two-dim. t-t'-Hubbard model.

- In contrast to earlier analyses in this framework the RG eqns. for the couplings respected were derived by applying appropriate projectors onto the RG eqn. for the 4-point-vertex.

- The solutions exhibit the notorious flow to strong coupling, in (at least qualitative) agreement with earlier results.

- Next steps:

  – More detailed analysis of the nature of the strong coupling state by studying the RG flow of the susceptibilities.

  – Investigate the flow away from the van Hove filling, i.e. for $\delta \neq 0$.

  – One could also study the RG flow in the case of nonzero temperatures $T > 0$.

- This formalism applies to any other ansatz for the 4-point function containing different or additional types of interactions.

Supplementary material

The "full" β-functions - an example

$$\bar{\beta}_{\mathrm{SC}} = -\frac{6\,L^2}{16\,L^2+9}\,\Big(\Phi^{(\mathrm{SS})}_{\mathrm{pp;1}}(\underline{\omega}) + \Phi^{(\mathrm{SS})}_{\mathrm{cph;1}}(\underline{\omega}) + \Phi^{(\mathrm{SS})}_{\mathrm{dph;1}}(\underline{\omega})$$

$$+\Phi^{(\mathrm{SS})}_{\mathrm{pp;1}}(\omega_2,\omega_1,\omega_3) + \Phi^{(\mathrm{SS})}_{\mathrm{cph;1}}(\omega_2,\omega_1,\omega_3) + \Phi^{(\mathrm{SS})}_{\mathrm{dph;1}}(\omega_2,\omega_1,\omega_3)\Big)$$

$$+\frac{2\,L^2\,(16\,L^2-21)}{(16\,L^2+9)(4\,L^2-9)}\,\Big(\Phi^{(\mathrm{SC})}_{\mathrm{pp}}(\underline{\omega}) + \Phi^{(\mathrm{SC})}_{\mathrm{cph}}(\underline{\omega}) + \Phi^{(\mathrm{SC})}_{\mathrm{dph}}(\underline{\omega})$$

$$+\Phi^{(\mathrm{SC})}_{\mathrm{pp}}(\omega_2,\omega_1,\omega_3) + \Phi^{(\mathrm{SC})}_{\mathrm{cph}}(\omega_2,\omega_1,\omega_3) + \Phi^{(\mathrm{SC})}_{\mathrm{dph}}(\omega_2,\omega_1,\omega_3)\Big)$$

$$+\frac{15\,L^2}{(16\,L^2+9)(4\,L^2-9)}\,\Big(\Phi^{(\mathrm{DW})}_{\mathrm{pp;-}}(\underline{\omega}) + \Phi^{(\mathrm{DW})}_{\mathrm{cph;-}}(\underline{\omega}) + \Phi^{(\mathrm{DW})}_{\mathrm{dph;-}}(\underline{\omega})$$

$$+\Phi^{(\mathrm{DW})}_{\mathrm{pp;-}}(\omega_2,\omega_1,\omega_3) + \Phi^{(\mathrm{DW})}_{\mathrm{cph;-}}(\omega_2,\omega_1,\omega_3) + \Phi^{(\mathrm{DW})}_{\mathrm{dph;-}}(\omega_2,\omega_1,\omega_3)\Big)$$

For instance,

$$\Phi^{(\mathrm{SS})}_{\mathrm{cph;1}}(\underline{\omega}) = \frac{1}{\beta}\sum_{\omega'}\Big[-2\,B_1(\underline{\omega},\omega')\,h^{(1;1)}_{0,1;0,0;+}(\omega_3-\omega_1,\omega')$$

$$-2\sum_{i'\in\{1,4\}} B_2^{(i')}(\underline{\omega},\omega')\,h^{(i',1;1)}_{0,1;0,0;+}(\omega_3-\omega_1,\omega')$$

$$-2\sum_{i',i''\in\{1,4\}} B_3^{(i',i'')}(\underline{\omega},\omega')\,h^{(i',i'',1;1)}_{0,1;0,0;+}(\omega_3-\omega_1,\omega')$$

$$+\frac{1}{L^2}\,B_9(\underline{\omega},\omega')\sum_{j=1}^{4}\chi_j\,h^{(j,1;5-j,4)}_{0,1;0,1;+}(\omega_3-\omega_1,\omega')$$

$$+\frac{2}{L^2}\sum_{i'\in\{1,4\}} B_7^{(i')}(\underline{\omega},\omega')\sum_{j=1}^{4}\chi_j\,h^{(i',j,1;5-j,4)}_{0,1;0,1;+}(\omega_3-\omega_1,\omega')$$

$$-\frac{1}{L^4}\,B_{10}(\underline{\omega},\omega')\sum_{j,l=1}^{4}\chi_j\,\chi_l\,h^{(j,l,1;5-j,5-l,4,4)}_{0,1;0,1;+}(\omega_3-\omega_1,\omega')\Big]$$

$$h^{(\underline{i};\underline{j})}_{J_1;J_2;\pm}(\omega,\omega') = \frac{1}{L^4}\sum_{k,k'}\mathscr{L}_{\pm}(K,K')\left(\prod_{\mu=1}^{m}\eta^{(i_\mu)}_{J_1}(k)\right)\left(\prod_{\nu=1}^{n}\eta^{(j_\nu)}_{J_2}(k')\right)$$

$$\underline{i} = (i_1,\ldots,i_m)\,,\quad \underline{j} = (j_1,\ldots,j_n)\,,\quad \chi_j = 1+(j-4)(j-1)$$

B_i: quadratic in the couplings, e.g:

$$B_3^{(i,i')}(\underline{\omega},\omega') = \tfrac{1}{3}\,\mathrm{g}^{(i)}_{\mathrm{SS}}(\omega_2,\omega',\omega_1+\omega_2-\omega_3)\,\mathrm{g}^{(i')}_{\mathrm{SS}}(\omega_3-\omega_1+\omega',\omega_1,\omega')$$

The shape functions

The "shape functions"

$$\begin{aligned}
\eta^{(1)}_{0,0} &= 1 \\
\eta^{(1)}_{0,1}(q) &= \cos(q^x) + \cos(q^y) \\
\eta^{(2)}_{0,1}(q) &= \sin(q^x) + \sin(q^y) \\
\eta^{(3)}_{0,1}(q) &= \sin(q^x) - \sin(q^y) \\
\eta^{(4)}_{0,1}(q) &= \cos(q^x) - \cos(q^y)
\end{aligned}$$

and their generalizations $\eta^{(i)}_{m,n}$ with $m+n \geq 2$ $(m \leq n)$ satisfy

$$\frac{1}{L^2} \sum_k \eta^{(i)}_J(k)\, \eta^{(i')}_{J'}(k) = \delta_{J,J'}\, \delta_{i,i'}$$

and are in a one to one correspondence with the Fourier basis $\mathcal{B} = \{\mathrm{e}^{\mathrm{i}k\cdot x} : x \in \Lambda\}$.

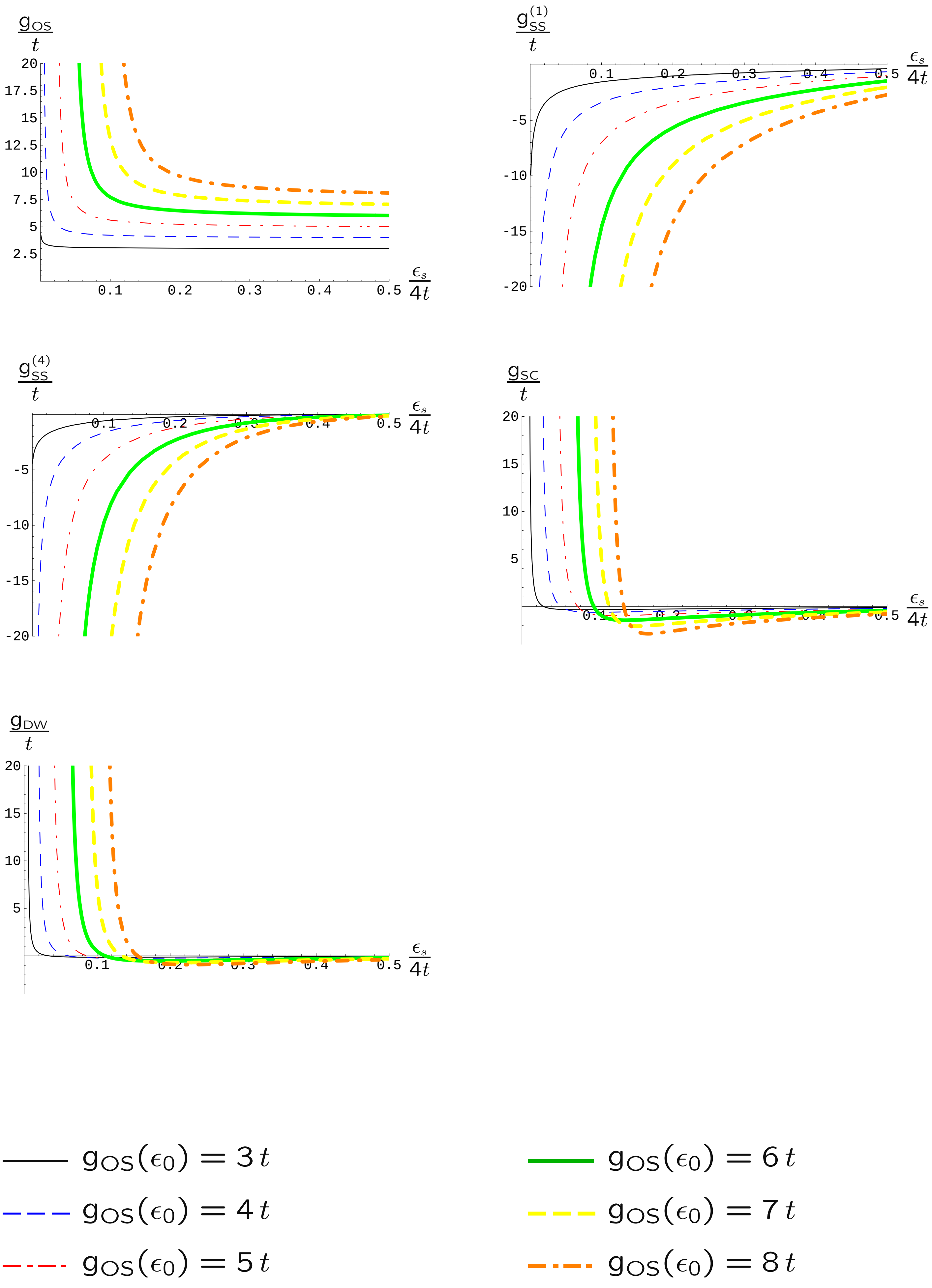
Consequences:

(i) Any $w_s(\underline{K}, \underline{\alpha})$ has an expansion in terms of η's.

(ii) Any projector acting on w_s can be constructed from the η's.

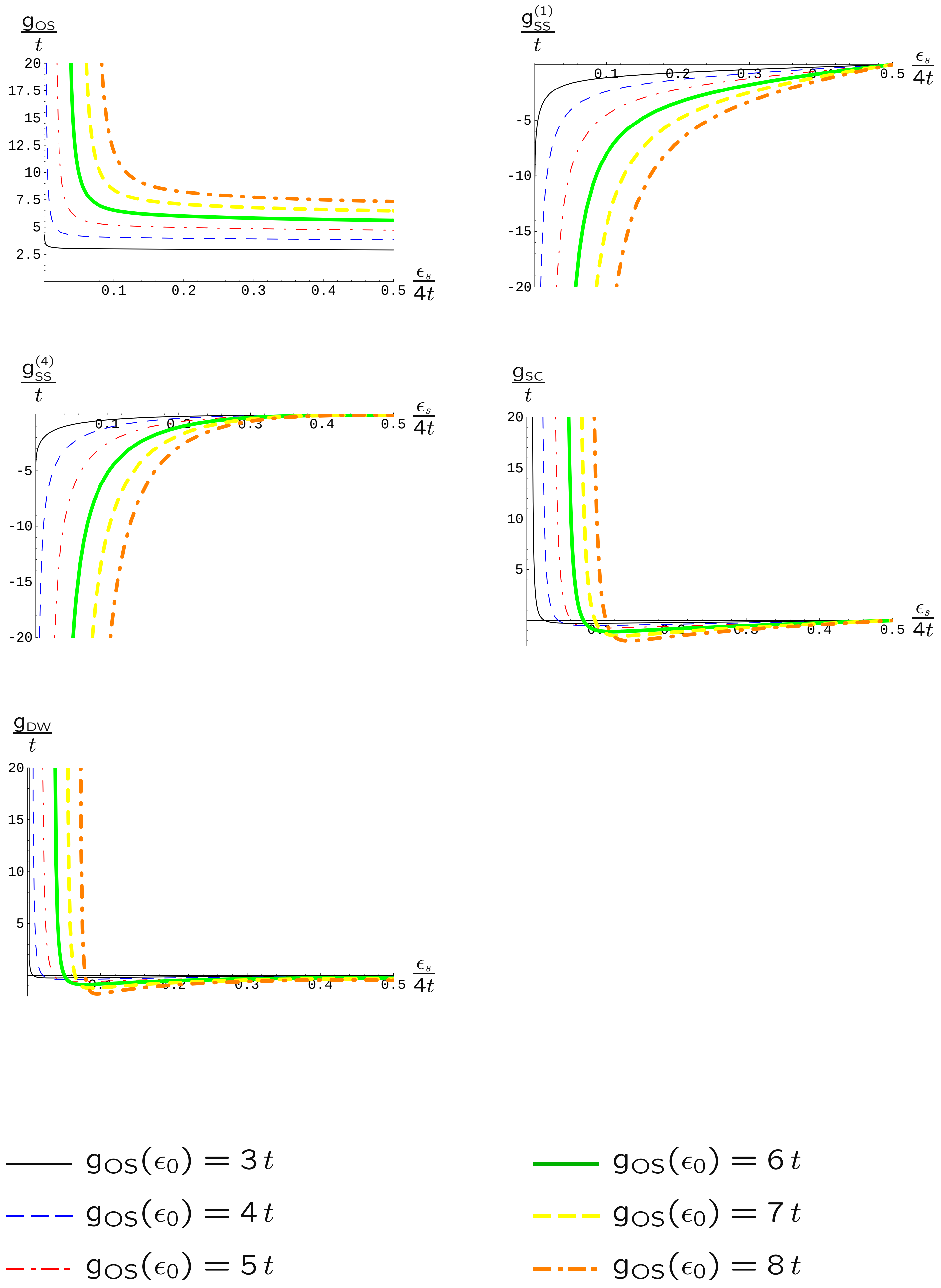
$\rightarrow$ particularly convenient since any of the interesting interaction terms is typically characterized by a certain η, which then specifies the symmetry inherent to the interaction (e.g. SC gap symmetries, etc.)

$\theta = 0.1$

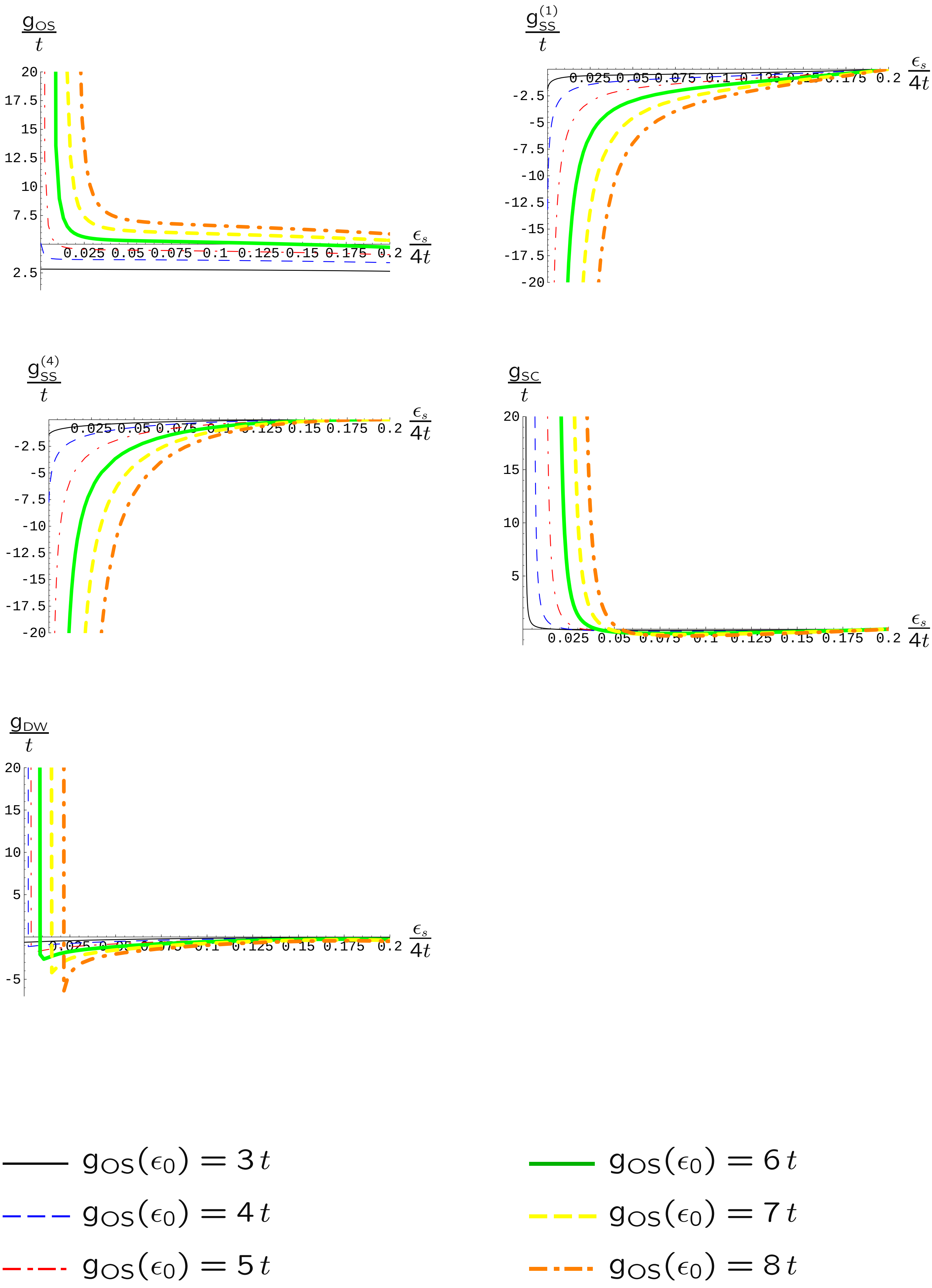


$\frac{g_{OS}}{t}$
$\frac{g_{SS}^{(1)}}{t}$
$\frac{g_{SS}^{(4)}}{t}$
$\frac{g_{SC}}{t}$
$\frac{g_{DW}}{t}$
$\frac{\epsilon_s}{4t}$
$g_{OS}(\epsilon_0) = 3\,t$
$g_{OS}(\epsilon_0) = 4\,t$
$g_{OS}(\epsilon_0) = 5\,t$
$g_{OS}(\epsilon_0) = 6\,t$
$g_{OS}(\epsilon_0) = 7\,t$
$g_{OS}(\epsilon_0) = 8\,t$

$\theta = 0.5$

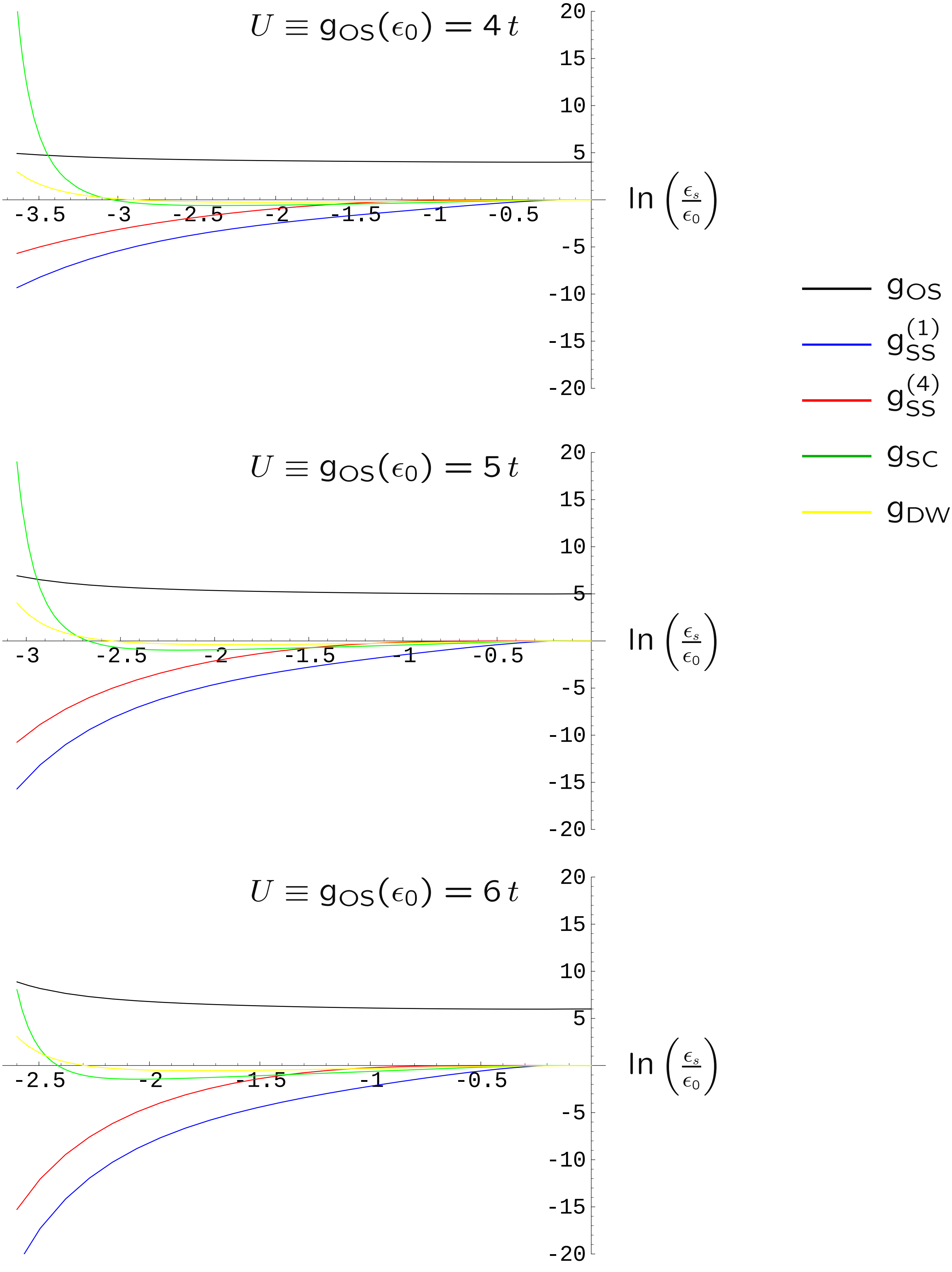

g_{OS}/t
$g_{SS}^{(1)}/t$
$g_{SS}^{(4)}/t$
g_{SC}/t
g_{DW}/t
$\epsilon_s/4t$
20
17.5
15
12.5
10
7.5
5
2.5
-5
-10
-15
-20
0.1
0.2
0.3
0.4
0.5
$g_{OS}(\epsilon_0) = 3\,t$
$g_{OS}(\epsilon_0) = 4\,t$
$g_{OS}(\epsilon_0) = 5\,t$
$g_{OS}(\epsilon_0) = 6\,t$
$g_{OS}(\epsilon_0) = 7\,t$
$g_{OS}(\epsilon_0) = 8\,t$

$\theta = 0.8$



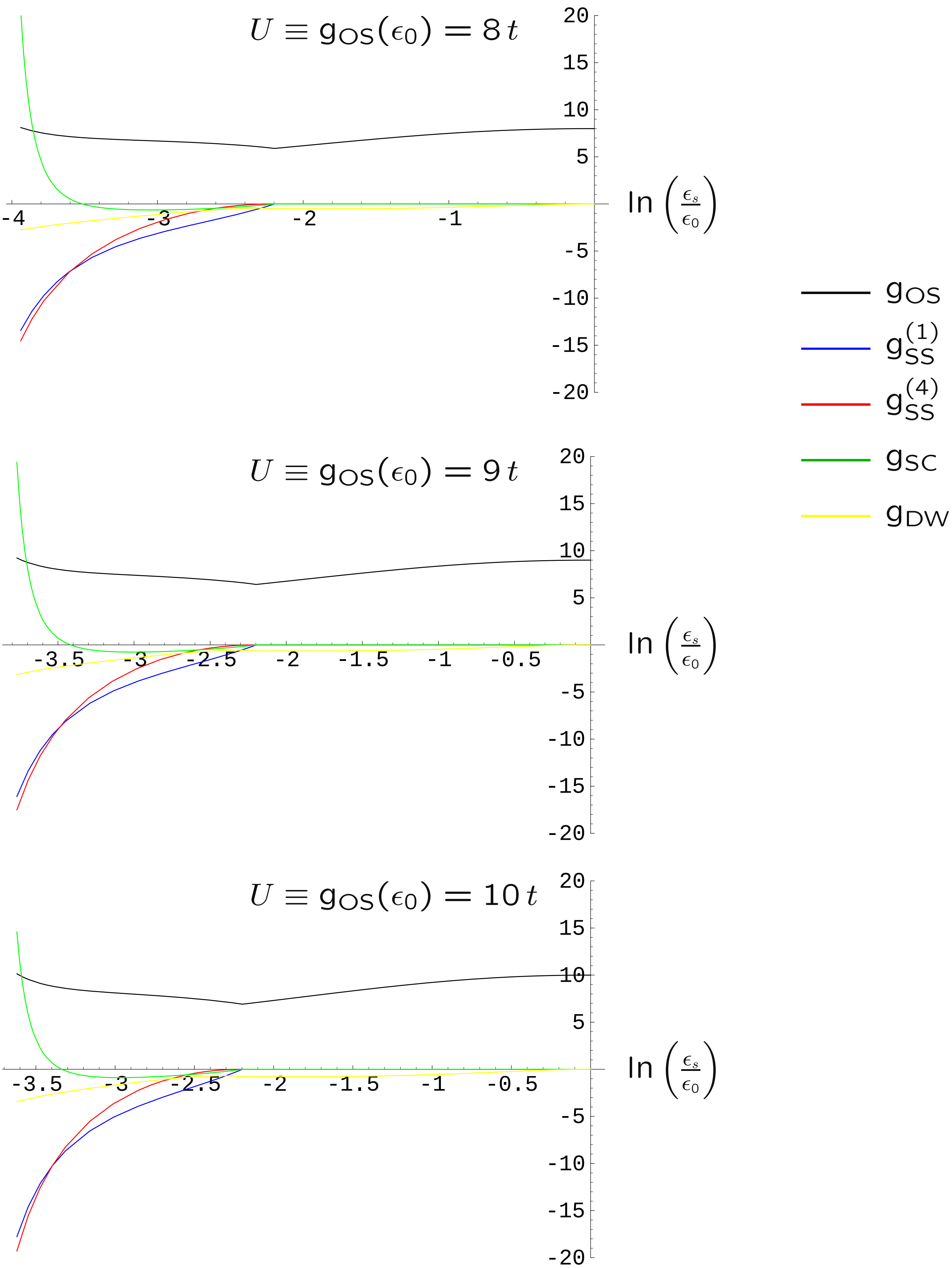
$\frac{g_{OS}}{t}$
$\frac{g_{SS}^{(1)}}{t}$
$\frac{g_{SS}^{(4)}}{t}$
$\frac{g_{SC}}{t}$
$\frac{g_{DW}}{t}$
$\frac{\epsilon_s}{4t}$
$g_{OS}(\epsilon_0) = 3\,t$
$g_{OS}(\epsilon_0) = 4\,t$
$g_{OS}(\epsilon_0) = 5\,t$
$g_{OS}(\epsilon_0) = 6\,t$
$g_{OS}(\epsilon_0) = 7\,t$
$g_{OS}(\epsilon_0) = 8\,t$

$\theta = 0.1$



$U \equiv g_{OS}(\epsilon_0) = 4\,t$
$U \equiv g_{OS}(\epsilon_0) = 5\,t$
$U \equiv g_{OS}(\epsilon_0) = 6\,t$
$\ln\left(\frac{\epsilon_s}{\epsilon_0}\right)$
20
15
10
5
-5
-10
-15
-20
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
g_{OS}
$g_{SS}^{(1)}$
$g_{SS}^{(4)}$
g_{SC}
g_{DW}

$\theta = 0.8$



$U \equiv g_{OS}(\epsilon_0) = 8\,t$
$U \equiv g_{OS}(\epsilon_0) = 9\,t$
$U \equiv g_{OS}(\epsilon_0) = 10\,t$
$\ln\left(\frac{\epsilon_s}{\epsilon_0}\right)$
20
15
10
5
-5
-10
-15
-20
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
g_{OS}
$g_{SS}^{(1)}$
$g_{SS}^{(4)}$
g_{SC}
g_{DW}