Functional RG for transport through quantum dots

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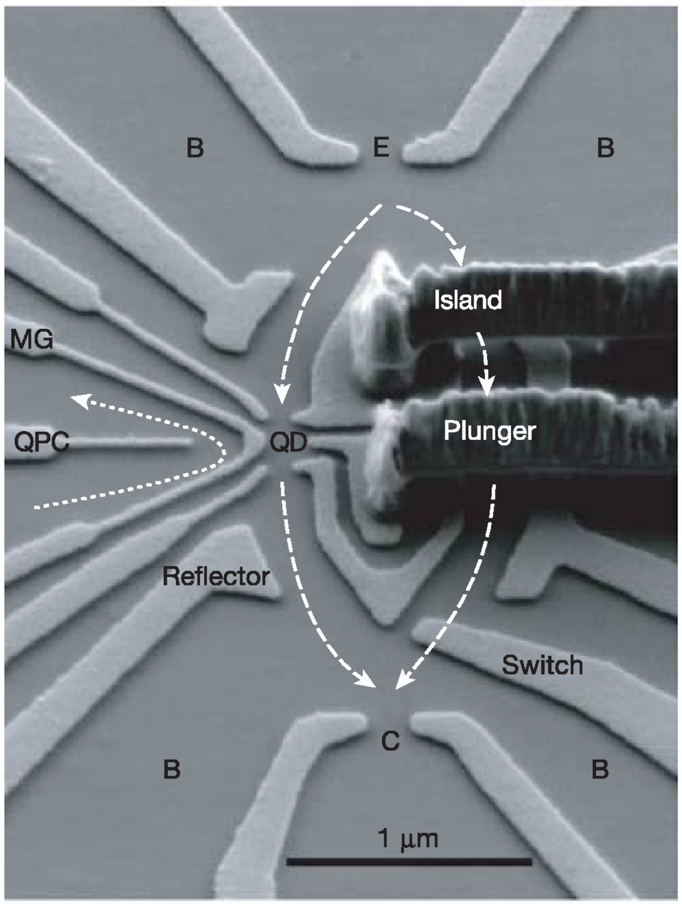
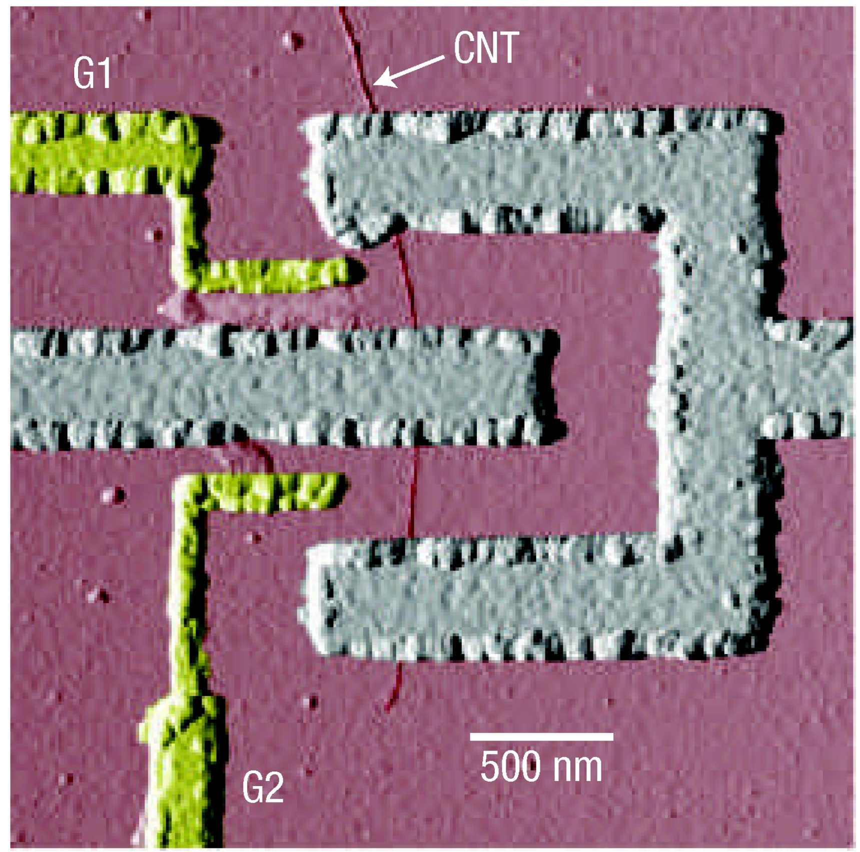
Goal

- use "simple" fRG approximation for correlations in mesoscopic systems
- standard perturbative many-body methods are (quite often) insufficient
- here: two experimentally motivated examples

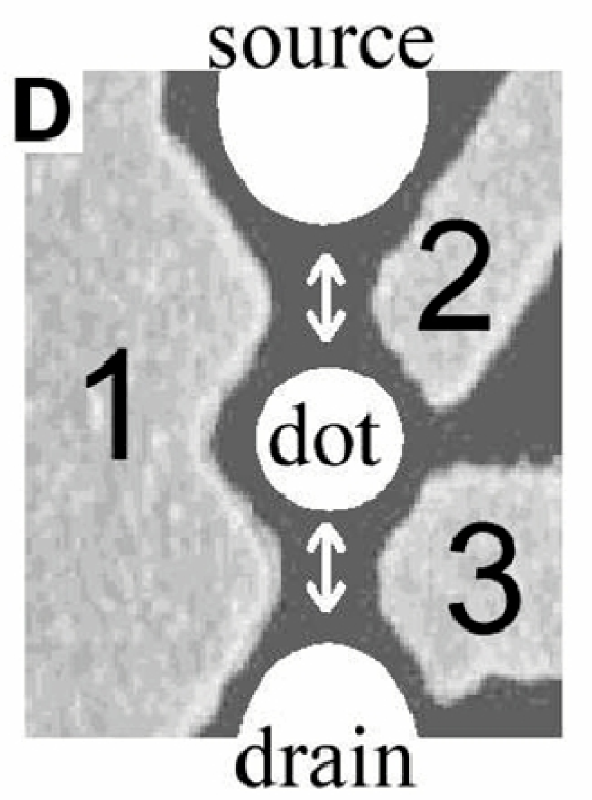


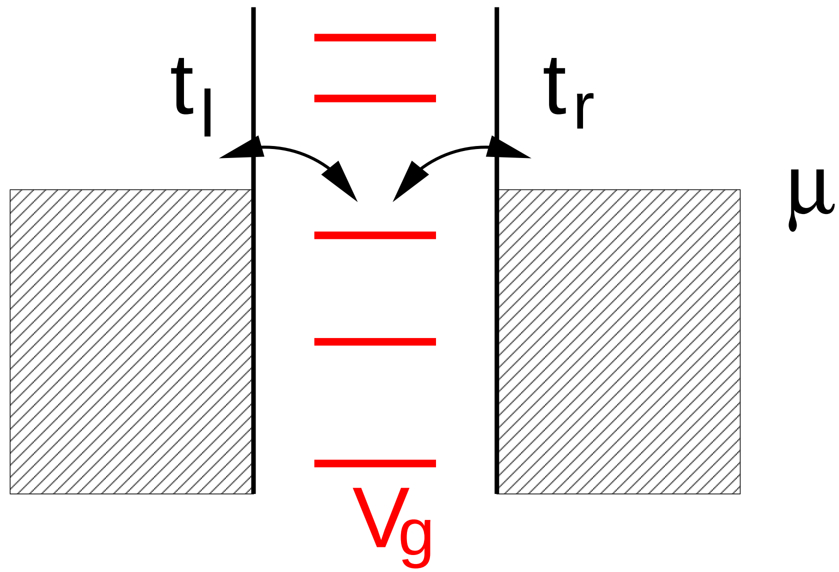
(Avinun-Kalish et al. '05)



(Cleuziou et al. '06)

Consider first a simpler setup





Hamiltonian $(s = l/r)$

$$H^{\text{dot}} = (\varepsilon - U/2) \sum_{\sigma} d_{\sigma}^{\dagger} d_{\sigma} + U d_{\uparrow}^{\dagger} d_{\uparrow} d_{\downarrow}^{\dagger} d_{\downarrow}$$

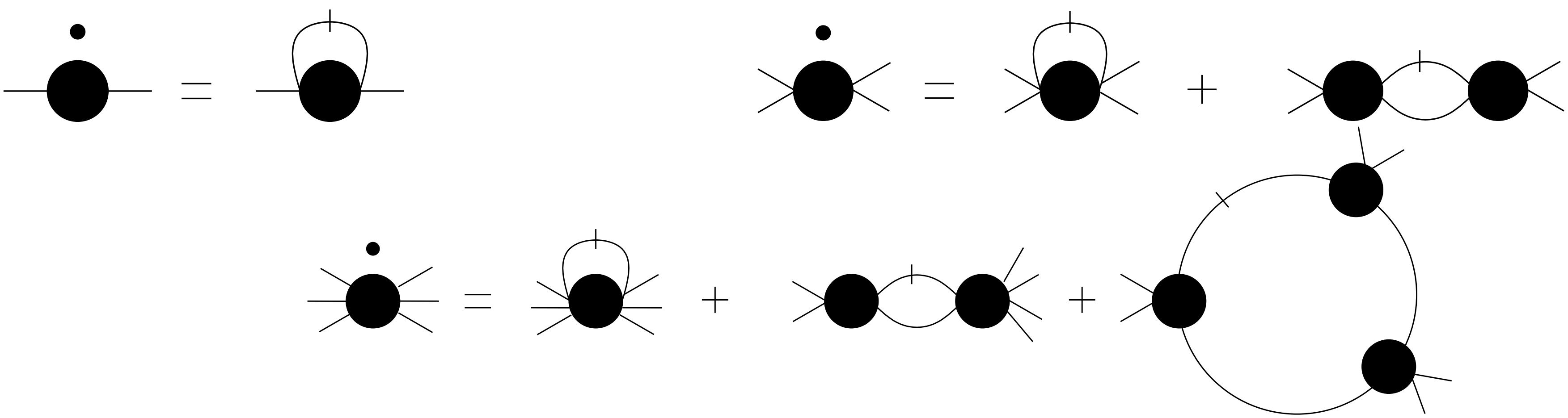
$$H_s^{\text{leads}} = \sum_{k,\sigma} \varepsilon_{s,k} c_{s,k,\sigma}^{\dagger} c_{s,k,\sigma}$$

$$H_s^{\text{coup}} = -t \sum_{\sigma} c_{s,\sigma}^{\dagger} d_{\sigma} + \text{H.c.}$$

Use a (truncated) functional RG

(Wegner & Houghton '73, Polchinski '84, Wetterich '93, Salmhofer '98, . . . , V.M. et al. '02)

- quantities considered: irreducible m-particle vertices $\rightarrow$ full propagator $\mathcal{G}$
- introduce infrared cutoff Λ in $\mathcal{G}^0$ (here energy cutoff $\Lambda \in]\infty, 0]$)
- exact hierarchy of flow-equations:



- with $\mathcal{S}^\Lambda = \mathcal{G}^\Lambda \left(\partial_\Lambda \left[\mathcal{G}^{0,\Lambda} \right]^{-1} \right) \mathcal{G}^\Lambda$, $\mathcal{G}^\Lambda = \left[\left[\mathcal{G}^{0,\Lambda} \right]^{-1} - \Sigma^\Lambda \right]^{-1}$
- keep flow of frequency independent part of two-particle vertex
- results in: flow equation for a frequency independent self-energy
- more than RG enhanced perturbation theory?

Single-level dot (neglect flow of “interaction”)

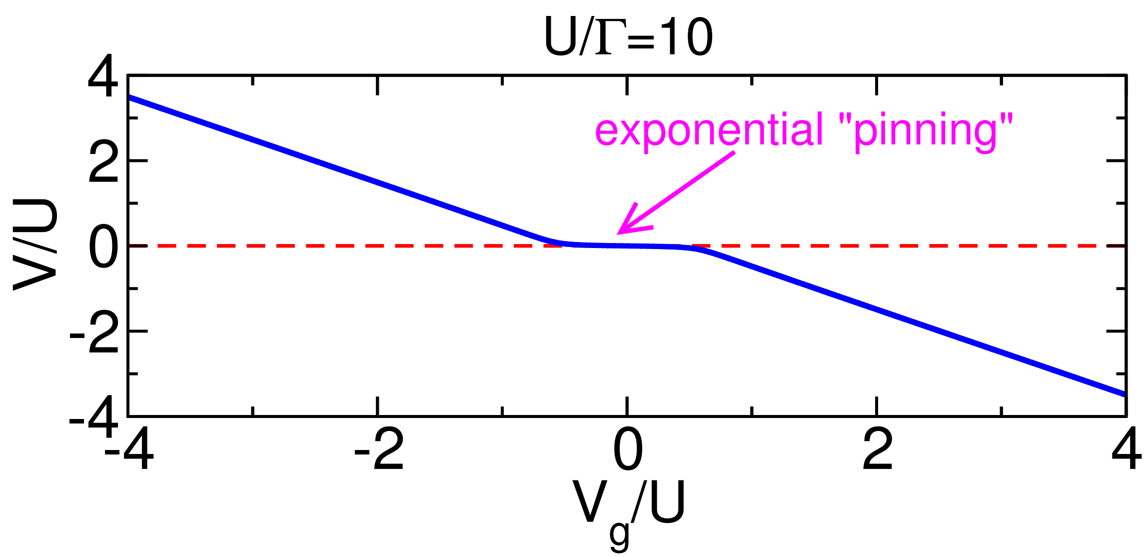
- free propagator:

$$\mathcal{G}^0_{\text{dot}}(i\omega) = \frac{1}{i\omega + V_g + i\Gamma\,\text{sign}(\omega)}\,, \quad \rho^0_{\text{dot}}(\omega) = \frac{1}{\pi}\frac{\Gamma}{(\omega+V_g)^2+\Gamma^2}$$

- flow equation for effective level position $V^\Lambda = -V_g + \Sigma^\Lambda_{\text{dot}}$:

$$\partial_\Lambda V^\Lambda = -\frac{U}{2\pi}\sum_{\omega=\pm\Lambda}\mathcal{G}^\Lambda_{\text{dot}}(i\omega) = -\frac{UV^\Lambda/\pi}{(\Lambda+\Gamma)^2+(V^\Lambda)^2}\,, \quad V^{\Lambda=\infty} = -V_g$$

- spectral function: as $\rho^0_{\text{dot}}(\omega)$ with $-V_g \to V = V^{\Lambda=0} \Rightarrow G(V_g)$ from $V(V_g, U)$

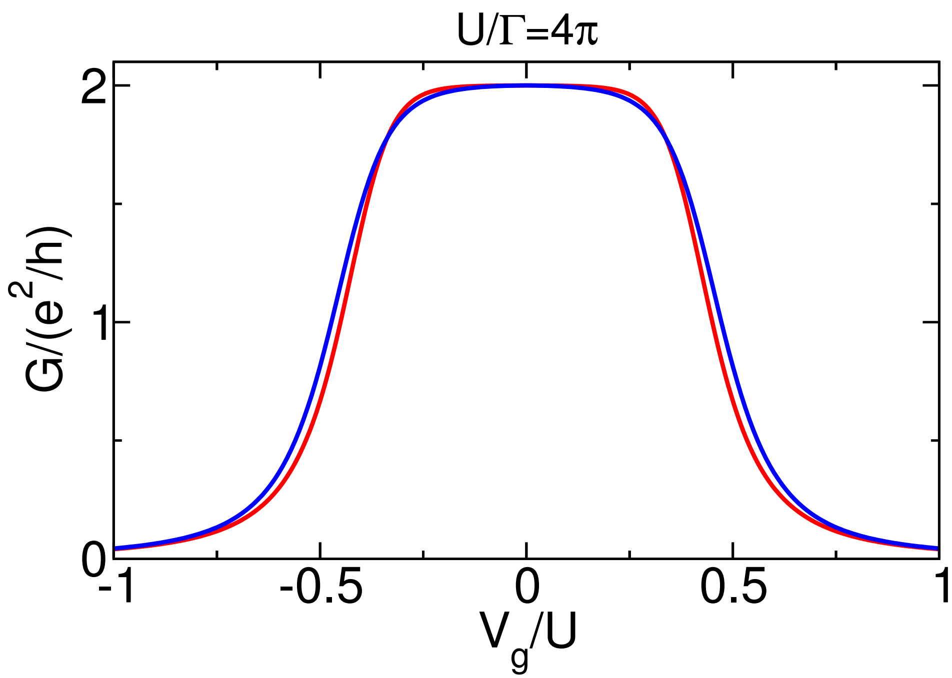


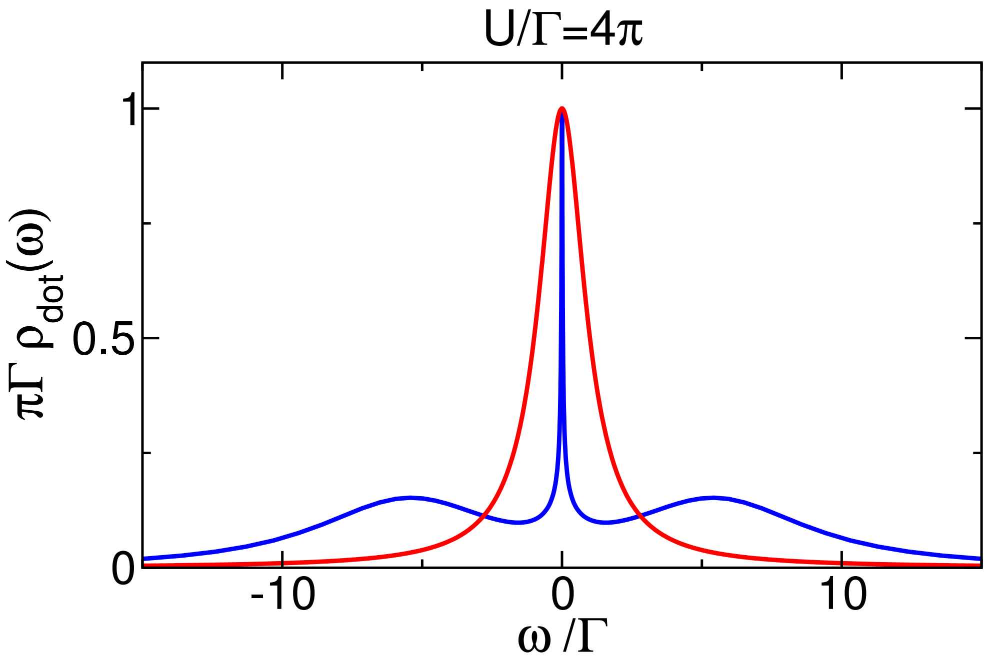
solution ($v = V\pi/U\,, \ldots$):

$$\frac{vJ_1(v)-\gamma J_0(v)}{vY_1(v)-\gamma Y_0(v)} = \frac{J_0(v_g)}{Y_0(v_g)}$$

(Andergassen, Enss & V.M. ’06)

Results (with flow of static "interaction")





(Karrasch, Enss & V.M. '06)

shortcomings of static approximation:

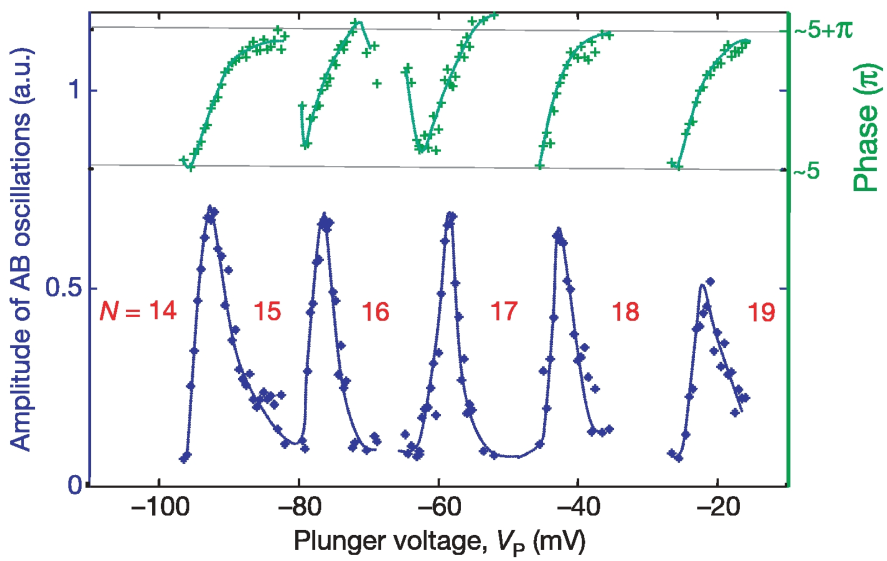
- fails for dynamical properties
- fails for finite temperatures

improved truncations:

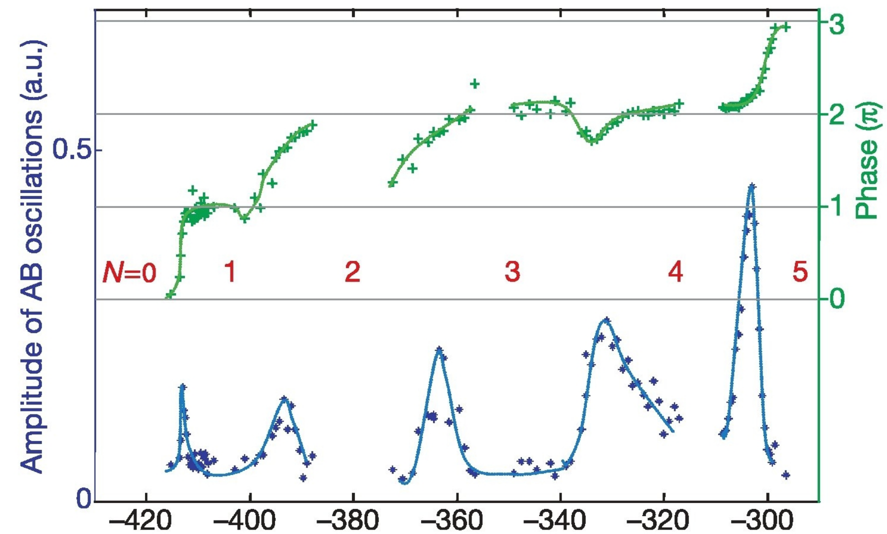
see the talks by C. Karrasch and M. Pletyukhov

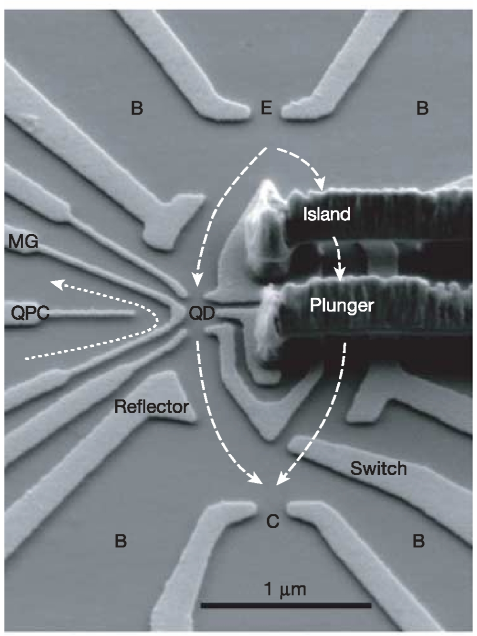
First experiment: transmission phase of quantum dot

- "universal" regime



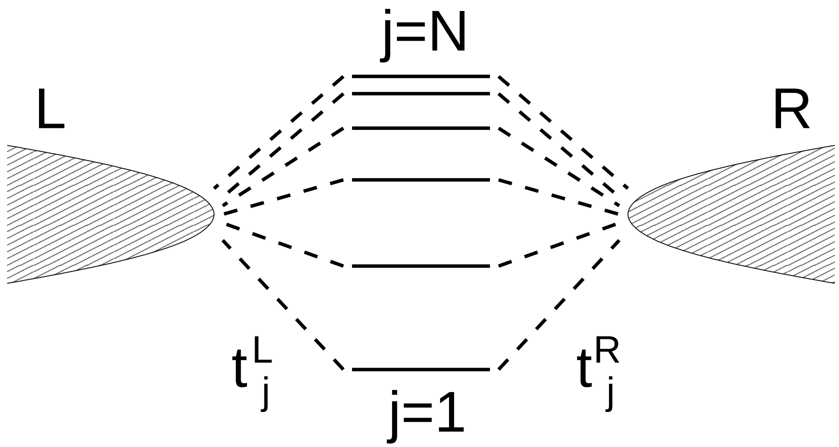
- "mesoscopic" regime





- experiments: Heiblums group '95-'05
- status of theory: Hackenbroich '01

Theory: multi-level quantum dot without correlations



- "mesoscopic" regime: large δ
- "universal" regime: small δ

"mesoscopic" regime with $\delta_j \gg \Gamma_j^l$ at $U = 0$

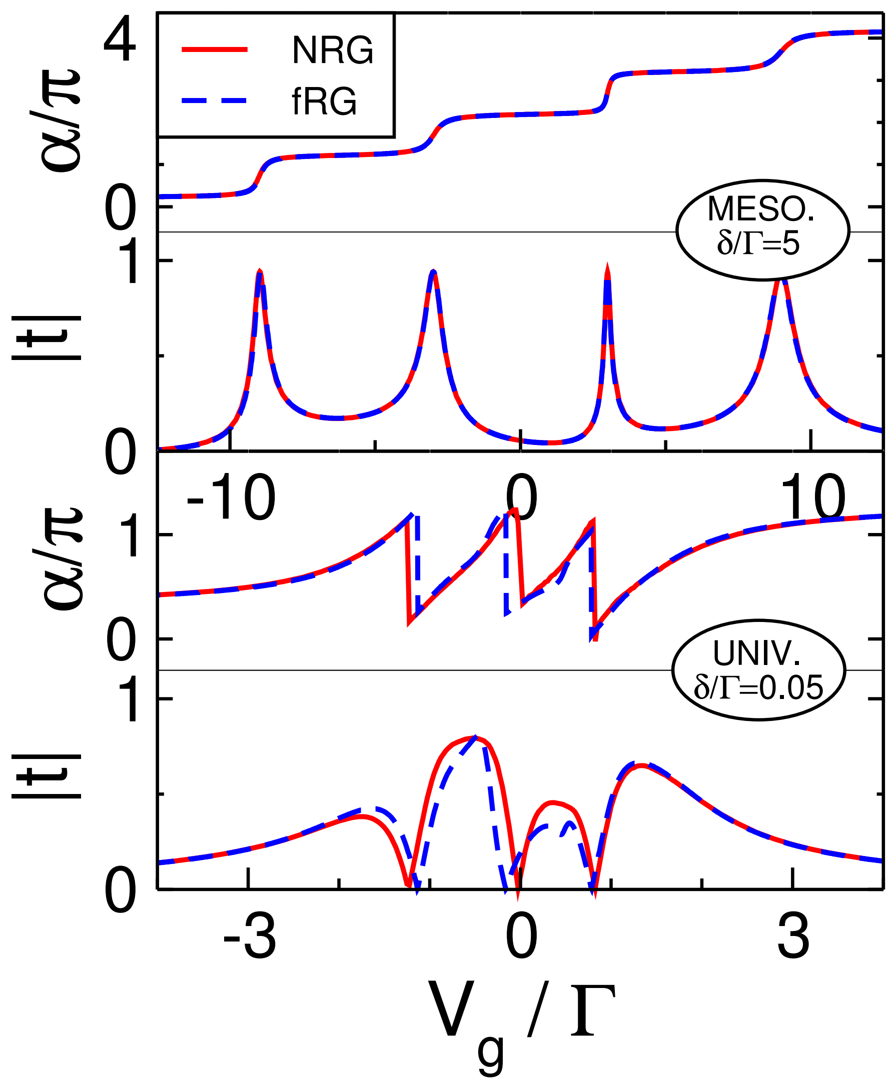
- Breit-Wigner transmission resonances with separation δ_j
- important quantity: $s_j = \mathrm{sgn}(t_j^L t_j^R t_{j+1}^L t_{j+1}^R)$
- s_j given by parity of involved states $\Rightarrow$ s_j is "random" in experiment
- π phase lapse and $|t| = 0$, if $s_j = +1$
- phase grows continuously and $|t| > 0$, if $s_j = -1$
- resulting picture consistent with experiments in "mesoscopic" regime

$U = 0$ with small δ_j: no "universal" behavior of phase

Comparison fRG—NRG with correlations (but without spin)

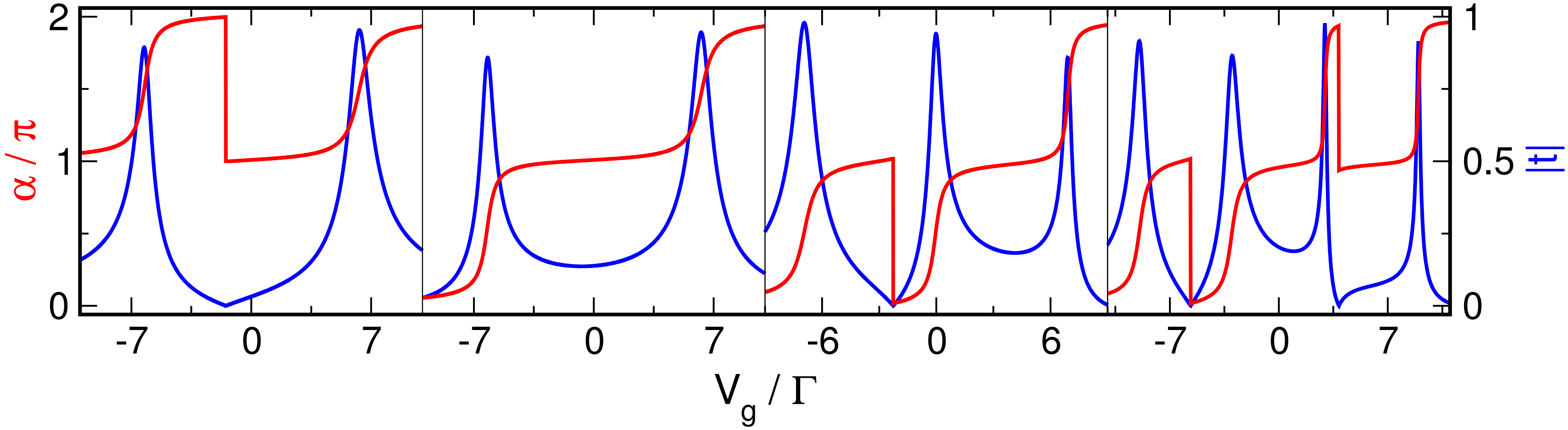
- generalized Landauer-Büttiker: $T = 0$ transmission from propagator

$$N = 4 \text{ levels}, \; U/\Gamma = 1, \; s = \{- - -\}$$

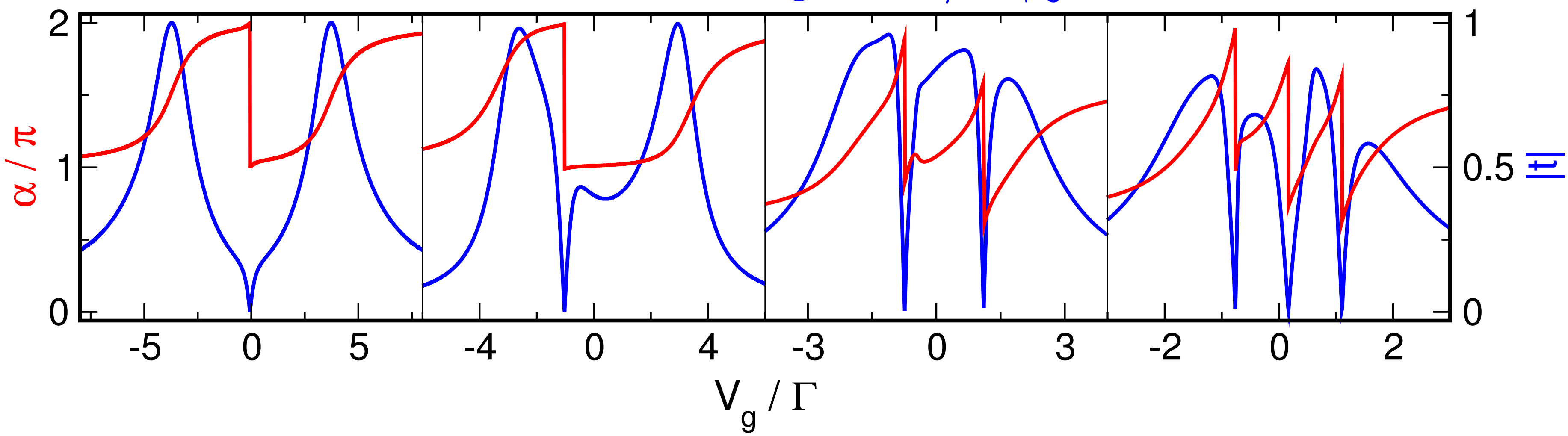


Theory: multi-level quantum dot with correlations

"mesoscopic" regime $\delta/\Gamma \gtrsim 1$



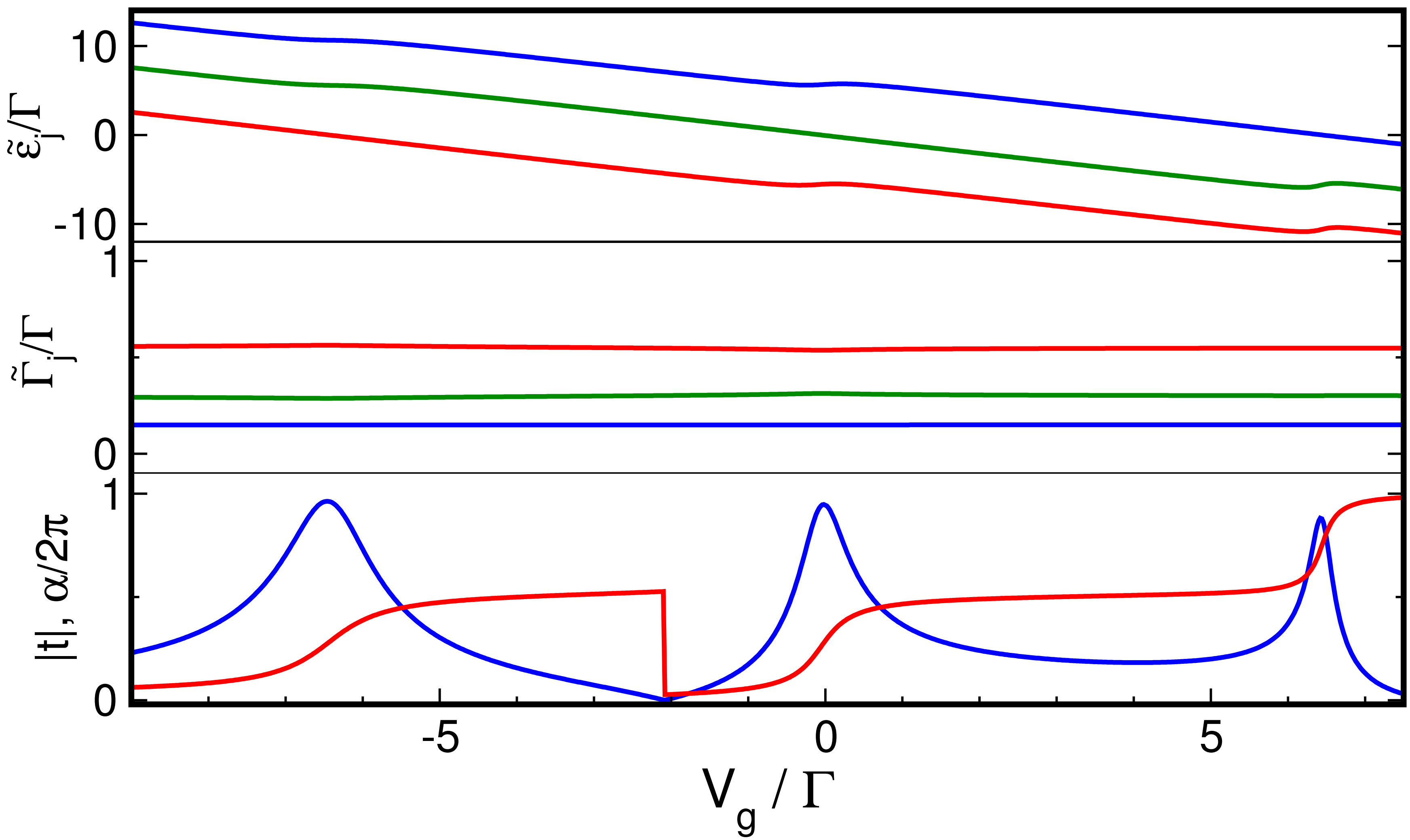
"universal" regime $\delta/\Gamma \lesssim 1$



N=2 U/Γ=8 s={+} N=2 U/Γ=8 s={-} N=3 U/Γ=2 s={+-} N=4 U/Γ=1 s={+-+}

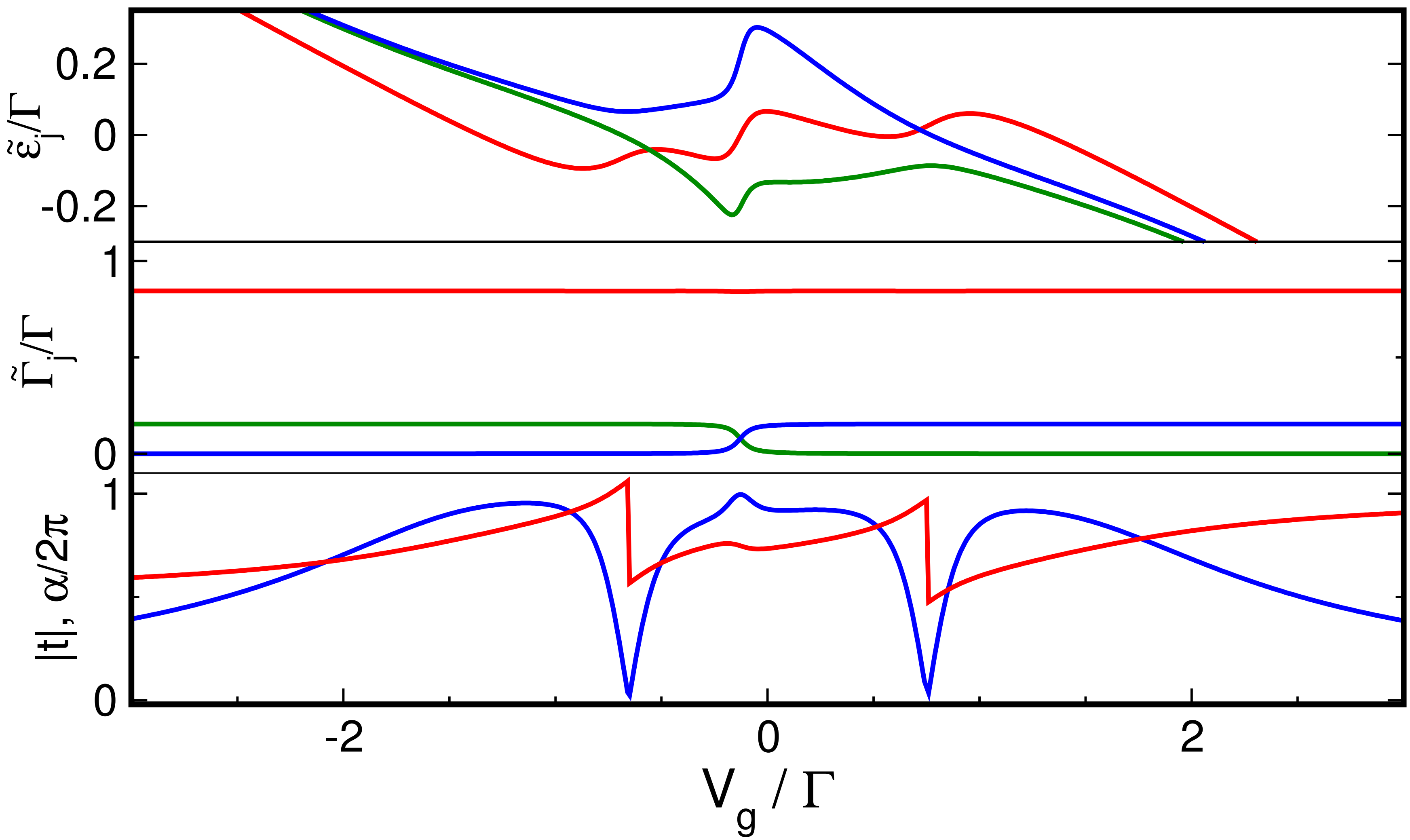
consistent with experiments in both regimes

"Mesoscopic" regime: renormalized single-particle levels



renormalized levels ≈ bare levels ⇒ as for $U = 0$

“Universal” regime: renormalized single-particle levels

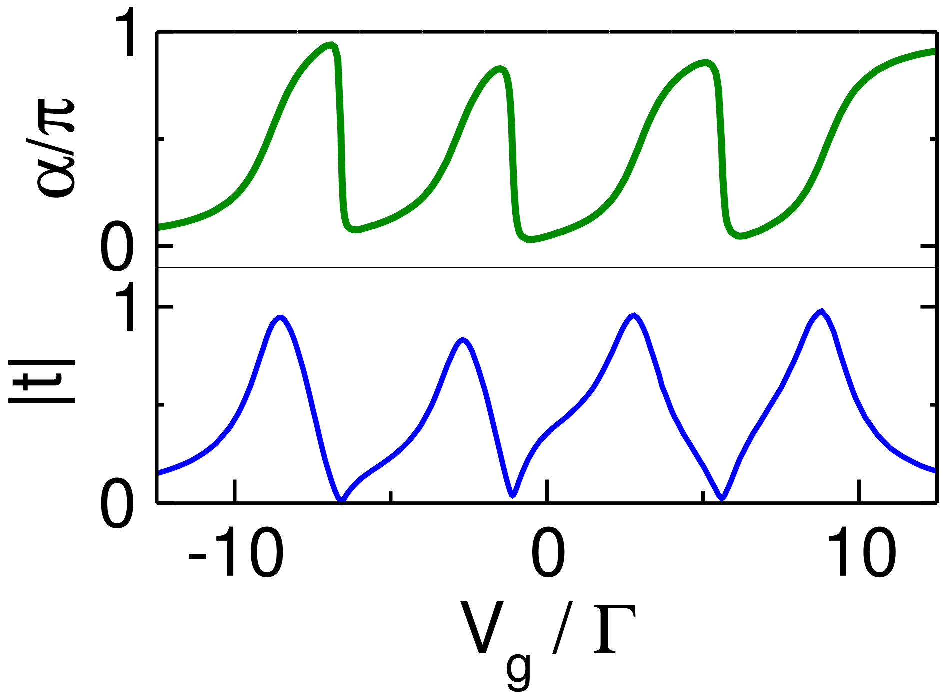


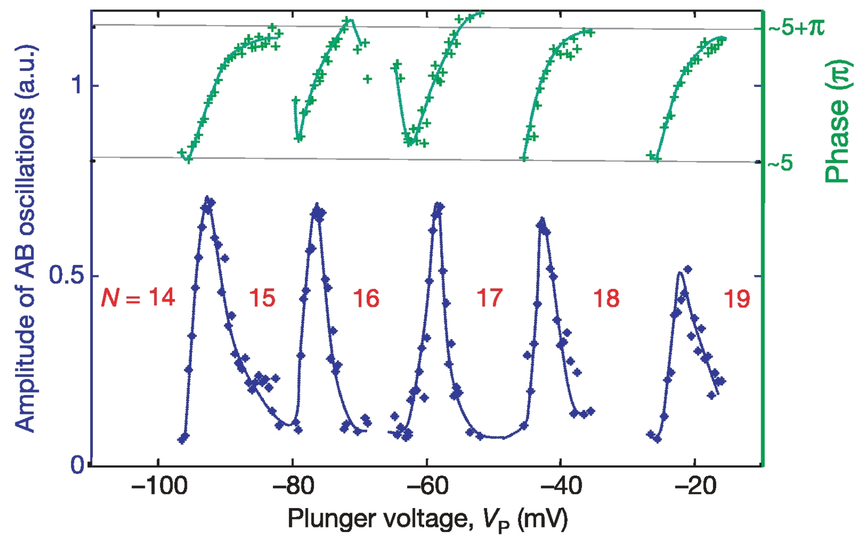
strong renormalization ⇒ Fano effect

(Karrasch, Hecht, Weichselbaum, Oreg, von Delft & V.M. ’07)

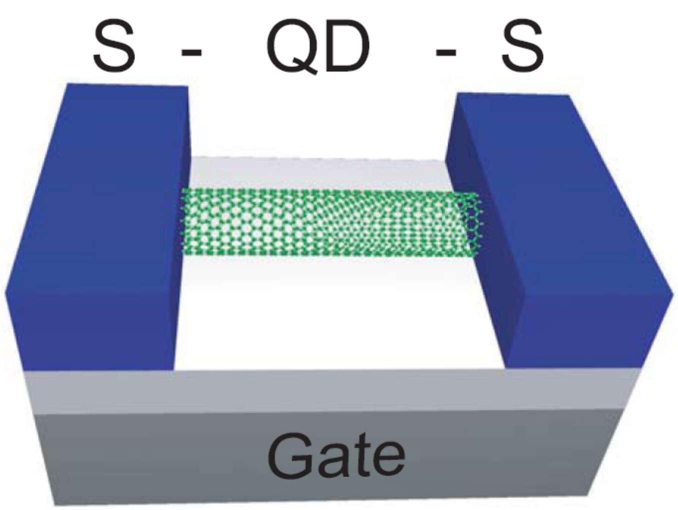
Comparison theory—experiment in "universal" regime

$N = 4$ levels, $U/\Gamma = 6$, $s = \{- - +\}$, $\delta/\Gamma = 0.1$, $T/\Gamma = 0.04$, NRG

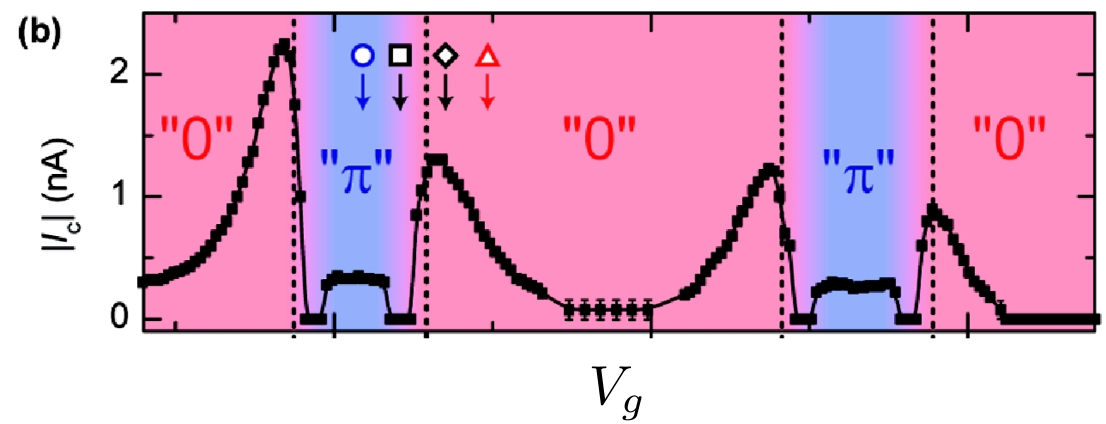




Second experiment: Josephson current through quantum dot



(Jørgensen et al. '07)



Hamiltonian $(s = l/r)$

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$$H_s^{\text{leads}} = \sum_{k,\sigma} \varepsilon_{s,k} c_{s,k,\sigma}^{\dagger} c_{s,k,\sigma} - \Delta \sum_{k} e^{i\phi_s} c_{s,k,\uparrow}^{\dagger} c_{s,-k,\downarrow}^{\dagger} + \text{H.c.}$$

$$H_s^{\text{coup}} = -t \sum_{\sigma} c_{s,\sigma}^{\dagger} d_{\sigma} + \text{H.c.}$$

Basic physics and basic relations

energy scales

(Abrikosov & Gorkov '61, Shiba & Soda '69, . . .)

superconducting gap $\Delta \leftrightarrow$ Kondo temperatur $T_K \sim \exp\left[-\pi U/(8\Gamma)\right]$

- here: particle-hole symmetric point $\varepsilon = 0$, phase difference $\phi = 0$
- $T_K \gg \Delta$: screening active, Cooper pairs broken, Kondo singlet ground state
- $T_K \ll \Delta$: screening disturbed, energy gap at μ, free magnetic moment
- quantum phase transition in between

important relations

- approximate Green function

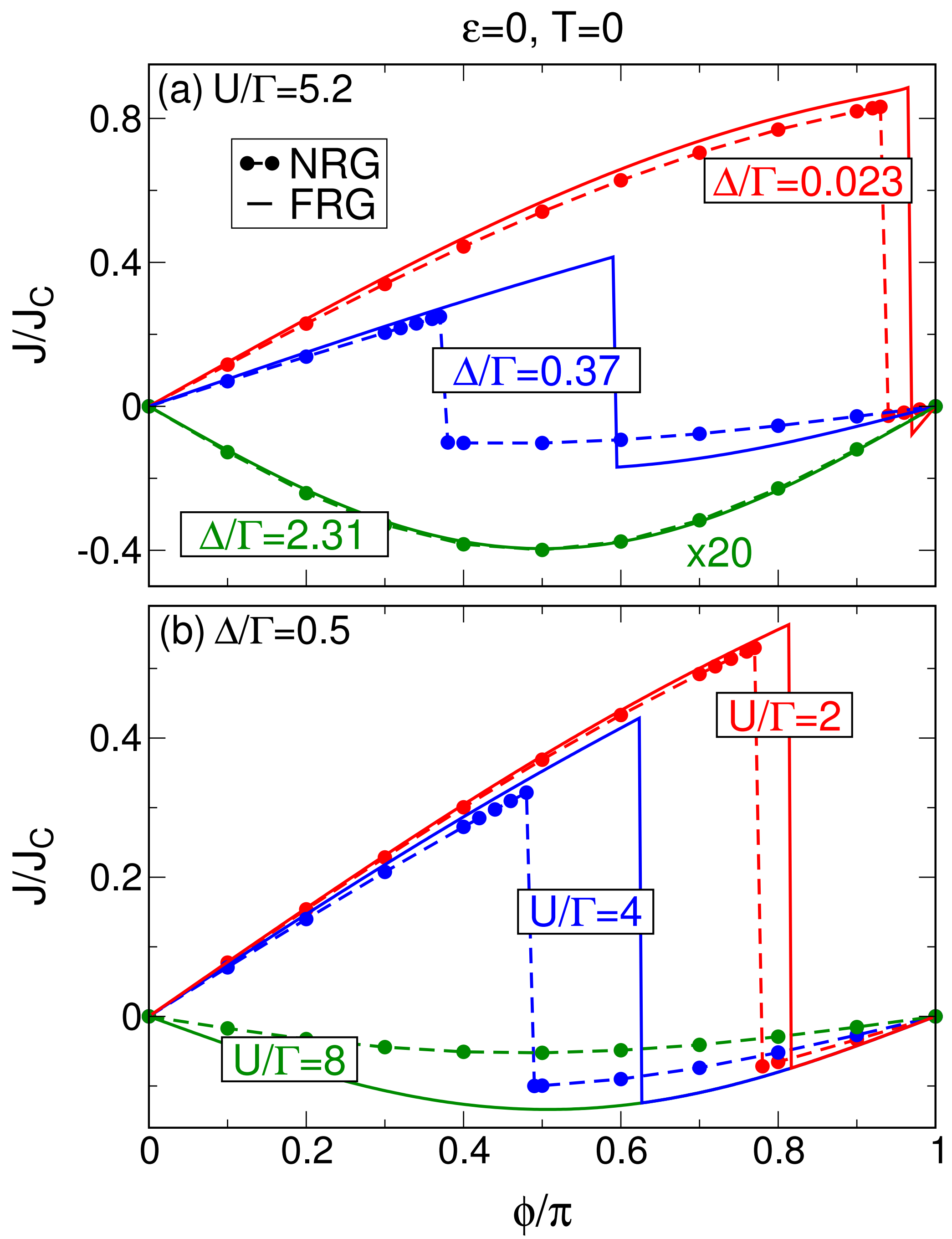
$$\mathcal{G}^\Lambda(i\omega) = -\frac{1}{\text{det}} \begin{pmatrix} i\tilde{\omega} + \epsilon + \Sigma^\Lambda & \Sigma^\Lambda_\Delta - \tilde{\Delta} \\ \Sigma^\Lambda_\Delta - \tilde{\Delta} & i\tilde{\omega} - \epsilon - \Sigma^\Lambda \end{pmatrix}$$

$$\tilde{\omega} = \omega + \omega\Gamma/\sqrt{\omega^2 + \Delta^2}\,, \qquad \tilde{\Delta} = \Gamma\Delta\cos(\phi/2)/\sqrt{\omega^2 + \Delta^2}$$

- Josephson current: $J = \langle \partial_t N \rangle = it\big\langle c^\dagger_\uparrow d_\uparrow + c^\dagger_\downarrow d_\downarrow - \text{H.c.}\big\rangle$
- computed from self-energy

$$J = T \sum_{i\omega} \left[\frac{\Gamma^2\Delta^2 \sin(\phi)}{(\omega^2 + \Delta^2)\,\text{det}} - \frac{2\Gamma\Delta \text{Im}\left[e^{-i\phi/2}\Sigma_\Delta\right]}{\sqrt{\omega^2 + \Delta^2}\,\text{det}} \right]$$

Results



(Karrasch, Oguri & V.M. '08)

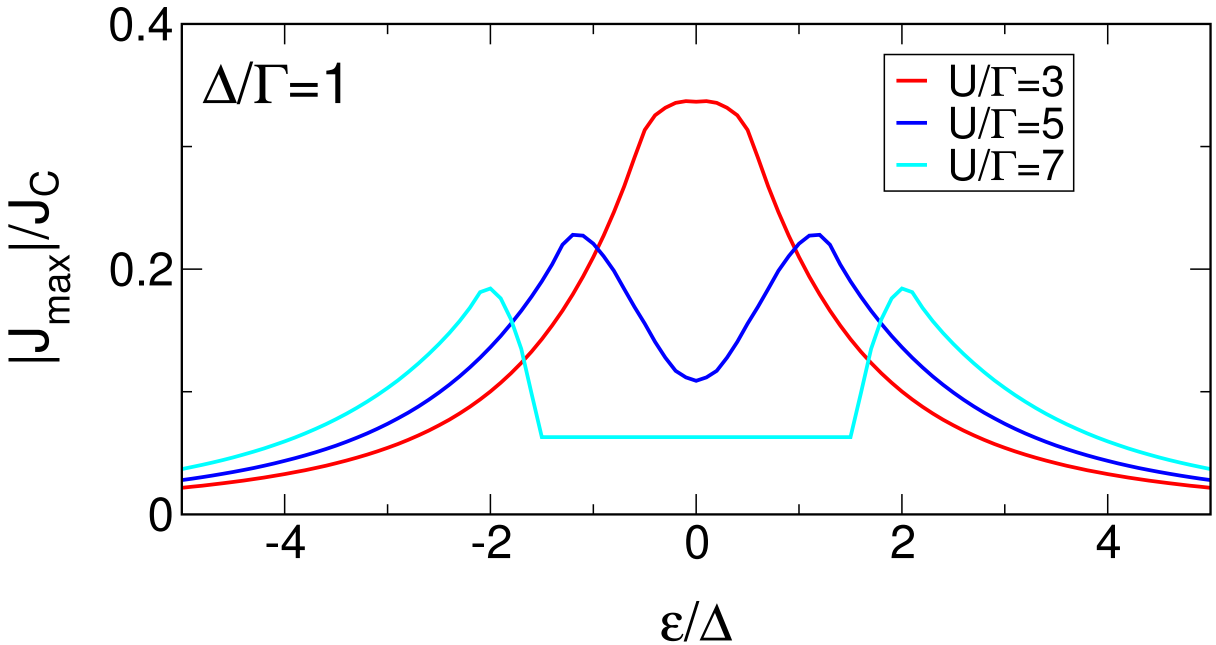
singlet phase: $J > 0$

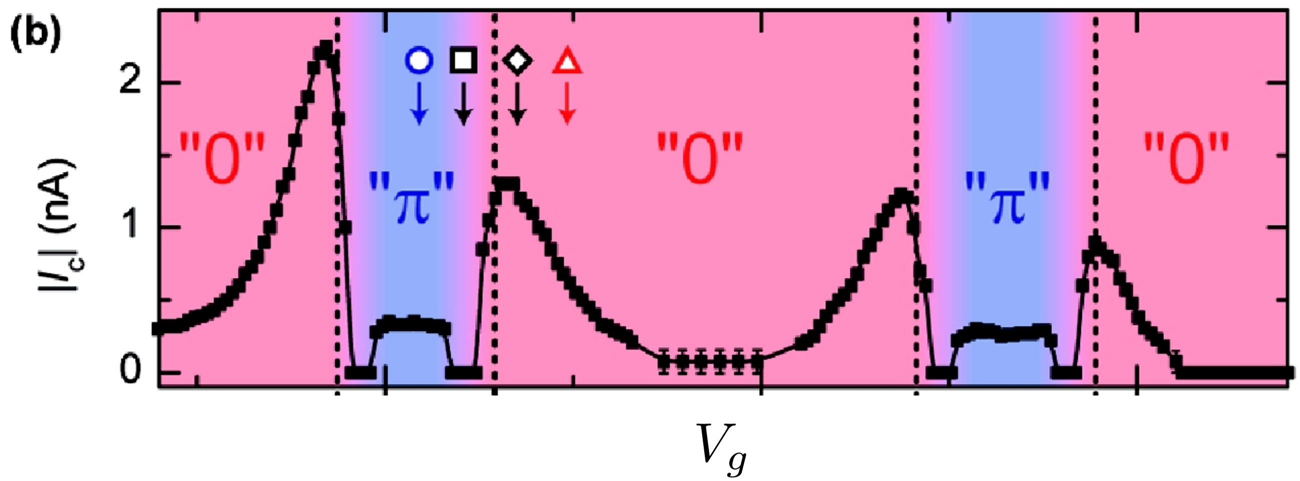
doublet phase: $J < 0$

$U = 0$ critical current:
$J_c = e\Delta/\hbar$

NRG data only at $\varepsilon = 0$

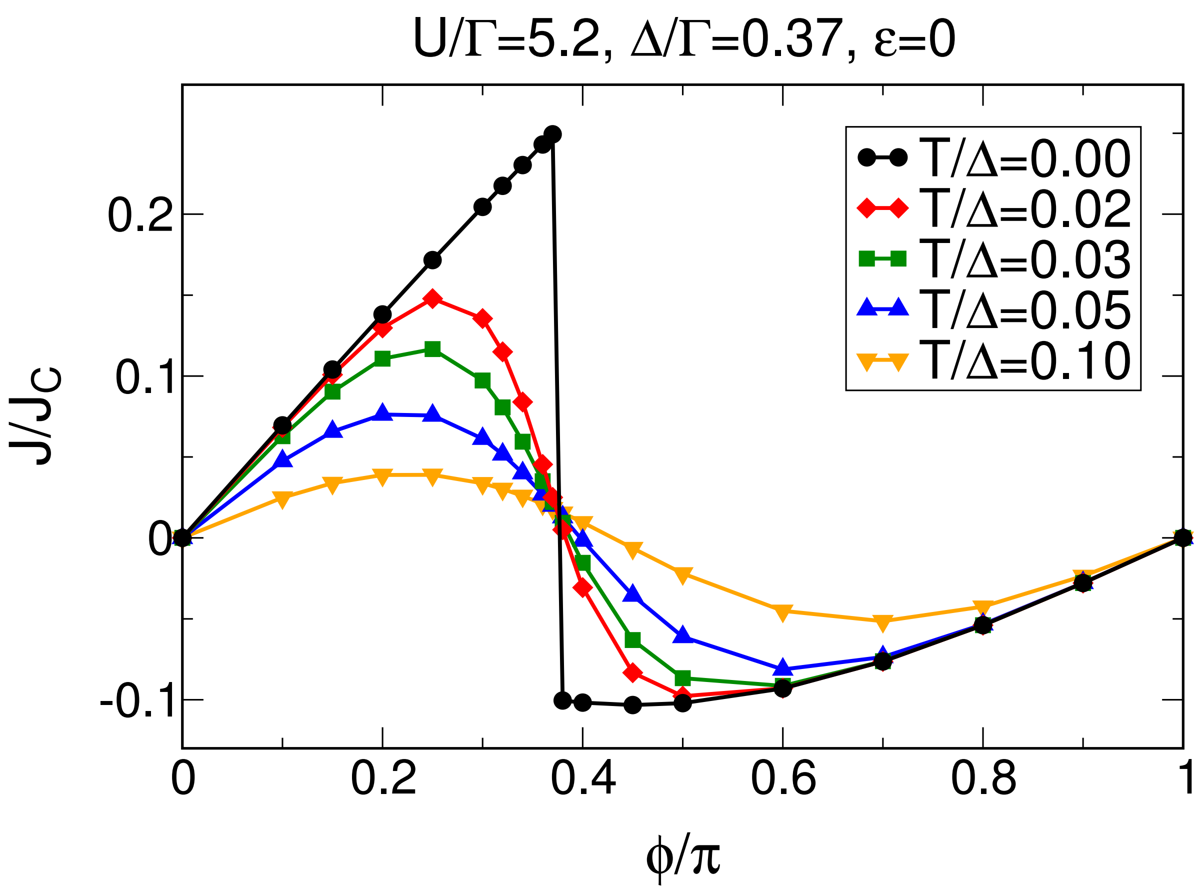
Comparison of critical current to experiments





Temperature dependence of Josephson current

- static fRG approximation not reliable
- use NRG for this (at least at particle-hole symmetric point $\varepsilon = 0$)
- compare to earlier numerical studies (Choi et al. '04, Siano & Egger '04)



(Karrasch, Oguri & V.M. '08)

Summary and outlook

- "simple" truncated fRG is usefull tool for correlation effects in mesoscopic systems
- improved schemes for T and ω dependence

see the talks by C. Karrasch and M. Pletyukhov

Many thanks to. . .

Christoph Karrasch

Akira Oguri, Theresa Hecht, Jan von Delft