Fermion-Boson RG for the Ground-state of Fermionic Superfluids

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Key Results

- **Infrared/universal quantities**
  - Exact momentum scaling of longitudinal and Goldstone propagator present any fermionic superfluid

- **Fermion gap reduction/connection to MFT**
  - Included vertex corrections
  - Deviations to MFT transparent

- **Structure of fermion-boson theory in phase with broken symmetry**
  - Careful separation of Goldstone and longitudinal fluctuations coupled to fermions
  - Distinguished gap from order parameter

- **Technical finesse**
  - Frequency cutoff : diagonal for fermions and bosons
  - Smooth flow in the vicinity of critical scale
  - Judicious choice of fermion and boson cutoff

Agenda

- History of Fermionic Superfluids
- Attractive Hubbard Model
- Truncation functional RG
- Frequency Cutoff
- Results
- Summary

(partial) History of Fermionic Superfluids

Theory of Superconductivity*

J. Bardeen, L. N. Cooper,† and J. R. Schrieffer‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

Volume 71, Number 19 PHYSICAL REVIEW LETTERS 8 November 1993

Crossover from BCS to Bose Superconductivity: Transition Temperature and Time-Dependent Ginzburg-Landau Theory

C. A. R. Sá de Melo,[1] Mohit Randeria,[2] and Jan R. Engelbrecht[3]

- 1957, BCS Theory
- 1970's, Kosterlitz-Thouless, Helium
- 1993 Randeria, functional integral
- Since 2005 fRG efforts underway (Manchester, Heidelberg, Würzburg, Stuttgart)

Development of understanding for fermionic superfluids

- 1969 Eagles, 1980 Leggett BCS-BEC Crossover
- 1986 Nozieres, Schmitt-Rink, Crossover for lattice fermions/finite T
- 1997/2004, Castellani, di Castro: IR properties of Bose gas

Infrared Behavior of Interacting Bosons at Zero Temperature

C. Castellani,[1] C. Di Castro,[1] F. Pistolesi,[2,3] and G. C. Strinati[3]
[1]*Dipartimento di Fisica, Università "La Sapienza," Sezione INFM, I-00185 Roma, Italy*
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(Received 22 March 1996)

We exploit the symmetries associated with the stability of the superfluid phase to solve the long-standing problem of interacting bosons in the presence of a condensate at zero temperature.

Ordering, metastability and phase transitions in two-dimensional systems

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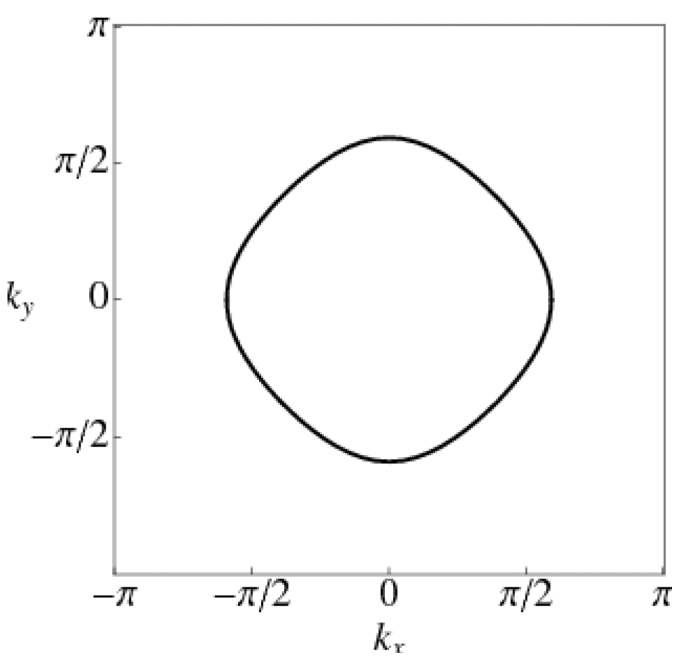
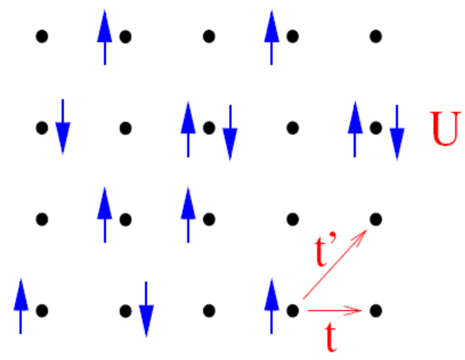
Attractive Hubbard Model as Prototype

- Attractively interacting lattice fermions
- Superfluid ground state for average fermion density per lattice site 0< n < 2
- Consider quarter-filling (away from van-Hove filling)

$$\Gamma_0[\psi,\bar\psi] = -\int_{k\sigma} \bar\psi_{k\sigma}(ik_0 - \xi_{\mathbf{k}})\,\psi_{k\sigma} + \int_{k,k',q} U\,\bar\psi_{-k+\frac{q}{2}\downarrow}\bar\psi_{k+\frac{q}{2}\uparrow}\psi_{k'+\frac{q}{2}\uparrow}\psi_{-k'+\frac{q}{2}\downarrow}$$





- Experimentally realized in 3d optical lattice with cold fermion atoms (Zurich, Boston)

Objectives:

- Compute effect of quantum fluctuations on:
  - -Fermionic gap, vertex (non-universal)
  - -Infrared behavior (universal)

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Truncation functional RG

- Replacing the four-fermion interaction with Hubbard-Stratonovich field in the BCS pairing channel:

$$\Gamma_0[\psi,\bar\psi,\phi] = -\int_{k\sigma}\bar\psi_{k\sigma}(ik_0-\xi_{\mathbf{k}})\,\psi_{k\sigma} - \int_q \phi_q^*\frac{1}{U}\phi_q + \int_{k,q}\left(\bar\psi_{-k+\frac{q}{2}\downarrow}\bar\psi_{k+\frac{q}{2}\uparrow}\,\phi_q + \psi_{k+\frac{q}{2}\uparrow}\psi_{-k+\frac{q}{2}\downarrow}\,\phi_q^*\right)$$

Baier, Bick, Wetterich 2004

- Exact flow equation in superfield formalism:

$$\frac{d}{d\Lambda}\Gamma^\Lambda[\mathcal{S},\bar{\mathcal{S}}] = \mathrm{Str}\,\frac{\dot{\mathbf{R}}^\Lambda}{\mathbf{\Gamma}^{(2)\,\Lambda}[\mathcal{S},\bar{\mathcal{S}}]+\mathbf{R}^\Lambda}$$

$$\mathcal{S}_b=\Phi\,,\quad \mathcal{S}_f=\Psi \qquad \Psi_k=\begin{pmatrix}\psi_{k\uparrow}\\ \bar\psi_{-k\downarrow}\end{pmatrix},\quad \bar\Psi_k=\left(\bar\psi_{k\uparrow},\psi_{-k\downarrow}\right)$$

$$\Phi_q=\begin{pmatrix}\phi_q\\ \phi^*_{-q}\end{pmatrix},\quad \bar\Phi_q=\left(\phi_q^*,\phi_{-q}\right)$$

Truncation inspired by MFT in symmetric (left) and superfluid (right) regime

$$\Gamma_{\bar\psi\psi} = -\int_{k\sigma}\bar\psi_{k\sigma}(ik_0-\xi_{\mathbf{k}})\,\psi_{k\sigma}$$

$$\Gamma_{\phi^*\phi} = \frac{1}{2}\int_q \phi_q^*(m_b^2+Z_b q_0^2 + A_b\omega_{\mathbf{q}}^2)\,\phi_q$$

$$\Gamma_{\psi^2\phi^*} = g\int_{k,q}\left(\bar\psi_{-k+\frac{q}{2}\downarrow}\bar\psi_{k+\frac{q}{2}\uparrow}\,\phi_q + \psi_{k+\frac{q}{2}\uparrow}\psi_{-k+\frac{q}{2}\downarrow}\,\phi_q^*\right)$$

$$\Gamma_{|\phi|^4} = \frac{\lambda}{8}\int_{q,q',p}\phi^*_{q+p}\phi^*_{q'-p}\phi_{q'}\phi_q$$

$$\Gamma_{\psi\psi} = \int_k\left(\Delta\,\bar\psi_{-k\downarrow}\bar\psi_{k\uparrow} + \Delta^*\psi_{k\uparrow}\psi_{-k\downarrow}\right)$$

$$U^{\mathrm{loc}}[\phi] = \frac{\lambda}{8}\int\left(|\phi|^2-|\alpha|^2\right)^2$$

$$\Gamma_{\sigma\sigma} = \frac{1}{2}\int_q \sigma_{-q}(m_\sigma^2+Z_\sigma q_0^2+A_\sigma\omega_{\mathbf{q}}^2)\,\sigma_q$$

$$\Gamma_{\pi\pi} = \frac{1}{2}\int_q \pi_{-q}(Z_\pi q_0^2+A_\pi\omega_{\mathbf{q}}^2)\,\pi_q$$

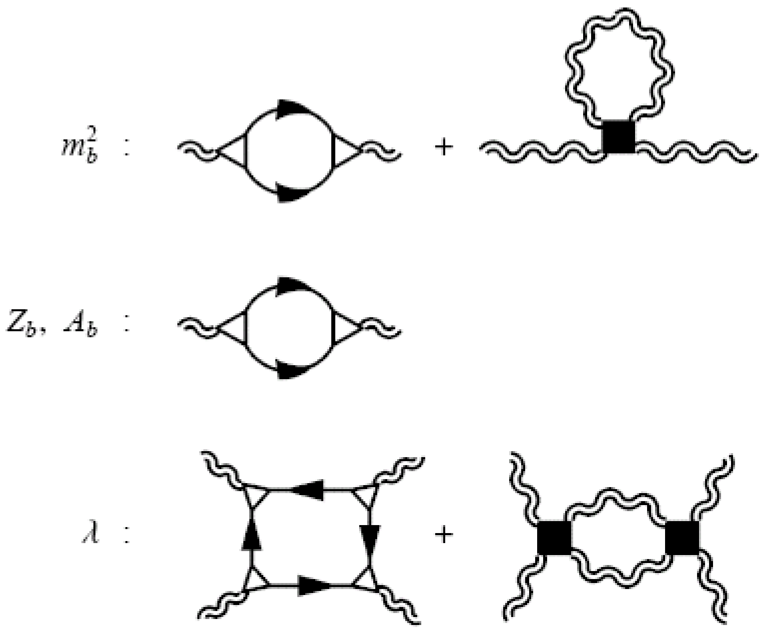
$$\Gamma_{\psi^2\sigma} = g_\sigma\int_{k,q}\left(\bar\psi_{-k+\frac{q}{2}\downarrow}\bar\psi_{k+\frac{q}{2}\uparrow}\,\sigma_q + \psi_{k+\frac{q}{2}\uparrow}\psi_{-k+\frac{q}{2}\downarrow}\,\sigma_{-q}\right),$$

$$\Gamma_{\psi^2\pi} = ig_\pi\int_{k,q}\left(\bar\psi_{-k+\frac{q}{2}\downarrow}\bar\psi_{k+\frac{q}{2}\uparrow}\,\pi_q - \psi_{k+\frac{q}{2}\uparrow}\psi_{-k+\frac{q}{2}\downarrow}\,\pi_{-q}\right)$$

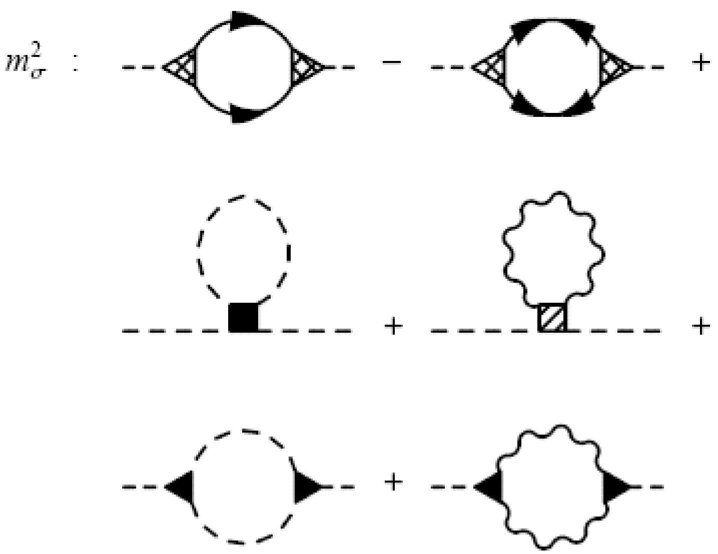
Truncation functional RG

- Leads to truncated flow equations:

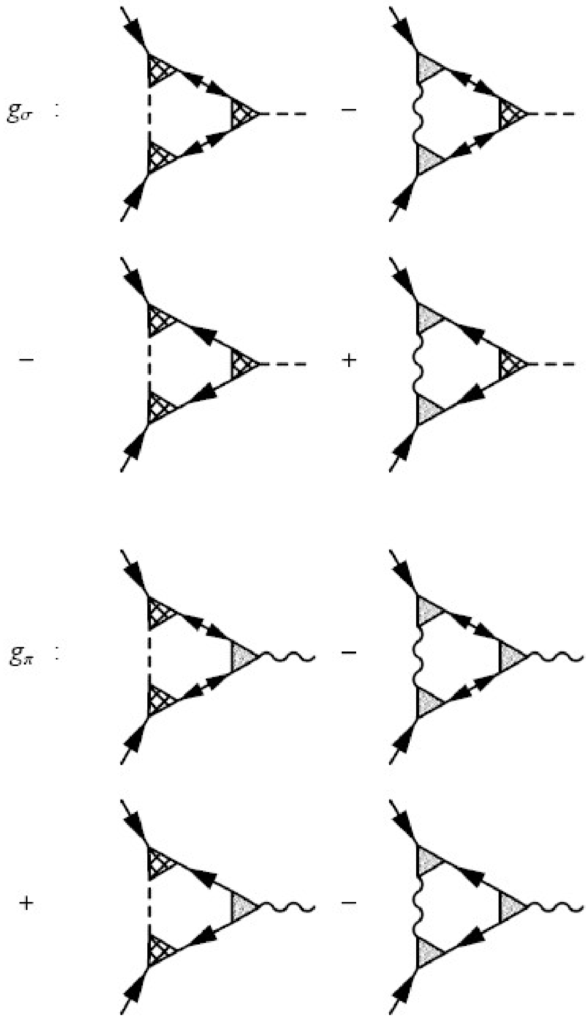
SYM:



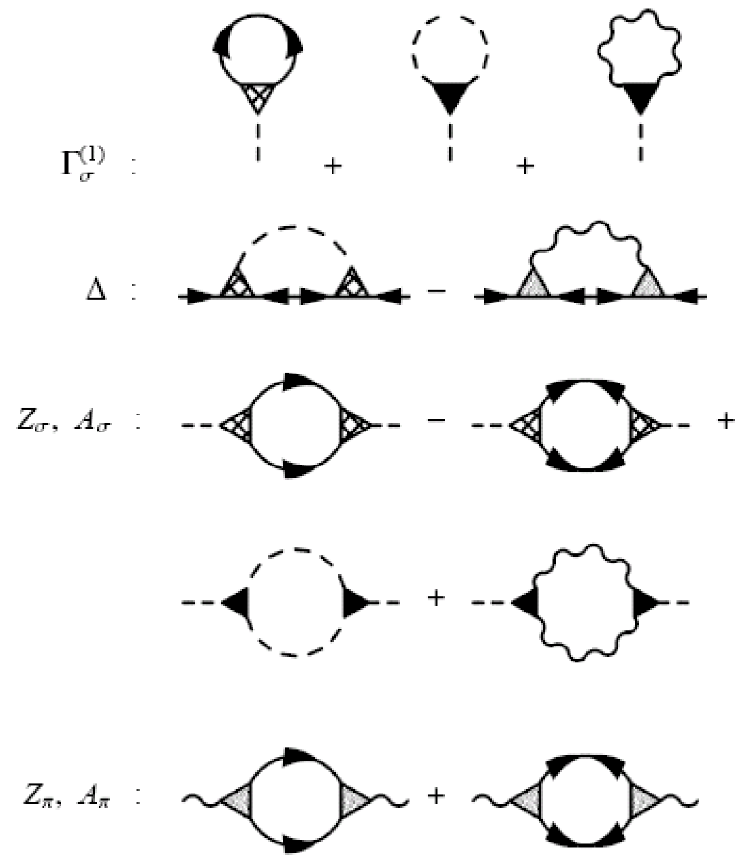
SSB:



Truncation functional RG



SSB:



- Total number of variables in SYM: **4**
- Total number of variables in SSB: **9**

13 flow equations total

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Frequency Cutoff

- Implement frequency cutoff for both: fermions and bosons.

$$\mathcal{R}^\Lambda = \frac{1}{2}\int_q \bar{\Phi}_q\, \mathbf{R}_b^\Lambda(q)\, \Phi_q + \int_k \bar{\Psi}_k\, \mathbf{R}_f^\Lambda(k)\, \Psi_k$$
$$\mathbf{R}_s^\Lambda(k) = [\mathbf{G}_{s0}(k)]^{-1} - [\chi_s^\Lambda(k_0)\, \mathbf{G}_{s0}(k)]^{-1}$$
$$\chi_s^\Lambda(k_0) = \Theta(|k_0| - \Lambda_s)$$

Göttingen, Aachen

- „Diagonal" for both particle species; point singularity at origin of frequency axis
- Equations free of regulator:

$$n \int dk_0\, \mathbf{G}'_{sR}(k_0)\, \mathbf{A}\, [\mathbf{G}_{sR}(k_0)\, \mathbf{A}]^{n-1} = \Lambda'_s \sum_{k_0=\pm\Lambda_s} [\mathbf{G}_s(k_0)\, \mathbf{A}]^n$$

- Judicious choice of relative cutoff scales:

SYM: $\Lambda_f(\Lambda) = \Lambda_b(\Lambda) = \Lambda$

SSB:
$$\Lambda_f(\Lambda) = \frac{\Lambda^2}{\Lambda_c}$$
$$\Lambda_b(\Lambda) = \Lambda$$

- Consistent treatment of IR sector by equating:

$$\max G_f(\Lambda_f(\Lambda), \mathbf{k}) \sim \max G_b(\Lambda_b(\Lambda), \mathbf{k})$$

→

- 2d-integration to be performed numerically
- Results robust under cutoff changes.

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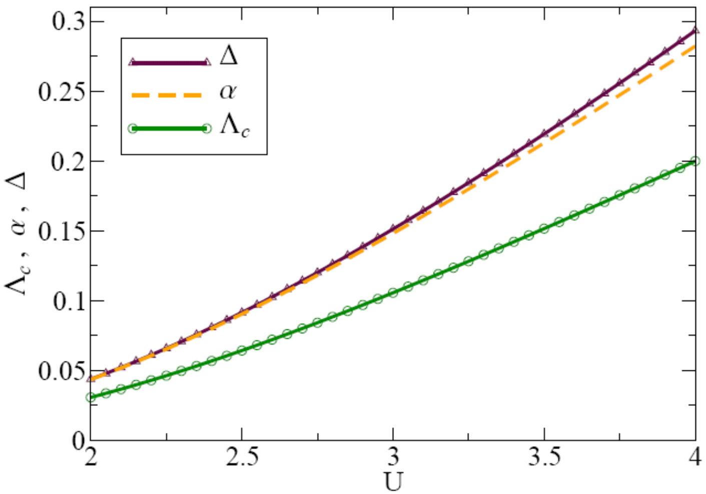
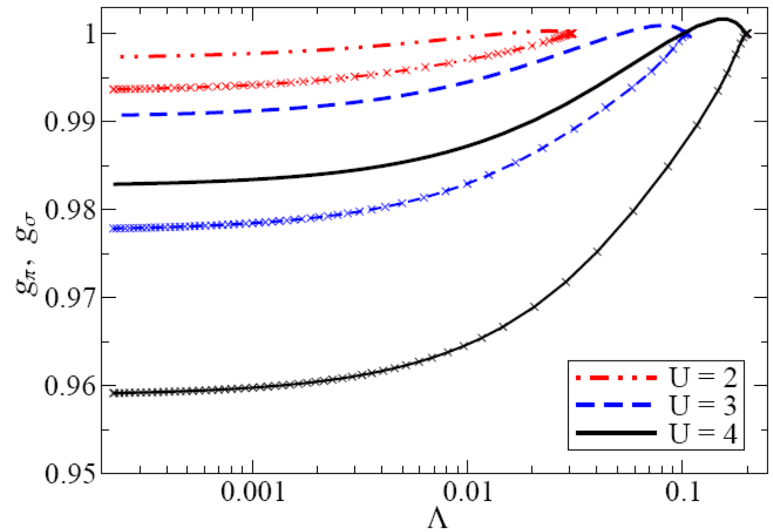
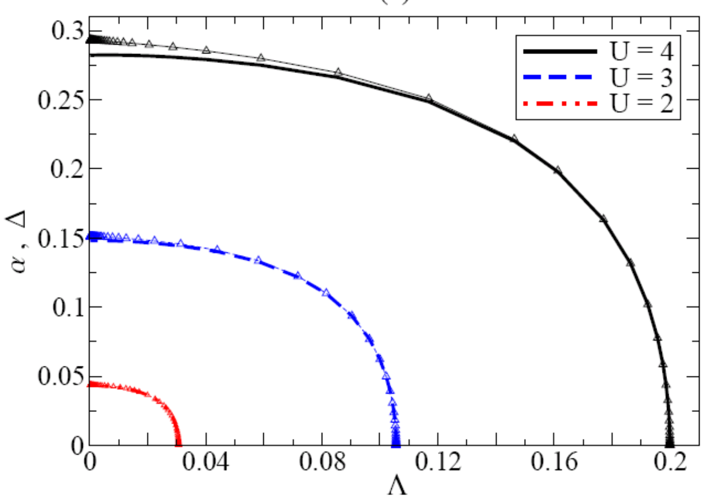
(a)



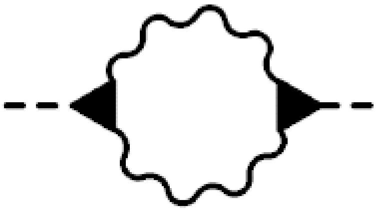


(b)



- Gap and order parameter split when increasing attraction
- Vertex renormalization relatively weak
- Gap is reduced by approx. Factor 4 compared to MFT.

- Functional RG flow delivers exact IR scaling: $m_\sigma^2,\ Z_\sigma,\ A_\sigma$:



$$
\begin{aligned}
m_\sigma^2 &= \lambda\alpha^2 \\
\partial_\Lambda \lambda &= \lambda^2 \int_{q|\Lambda} G_\pi^2(q)
\end{aligned}
$$

$$
\frac{d\tilde{\lambda}}{d\log\Lambda} = -\tilde{\lambda} + \frac{\tilde{\lambda}^2}{4\pi^2 A_\pi Z_\pi}
$$

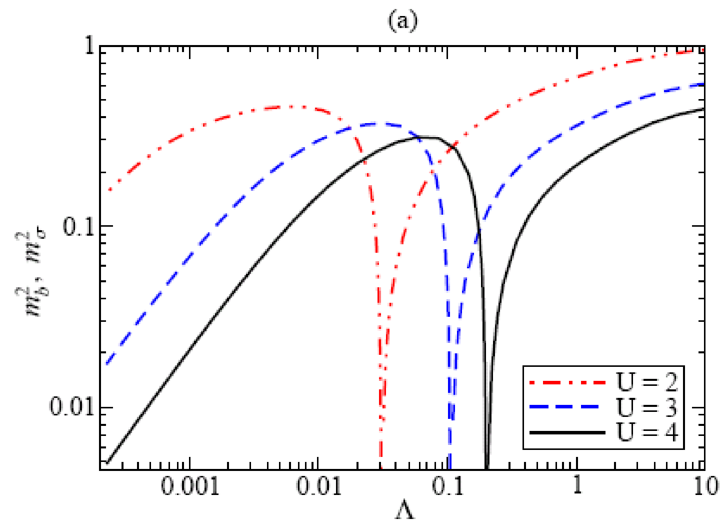
$$
\lambda \to 4\pi^2 A_\pi Z_\pi \Lambda \quad \text{for} \quad \Lambda \to 0
$$

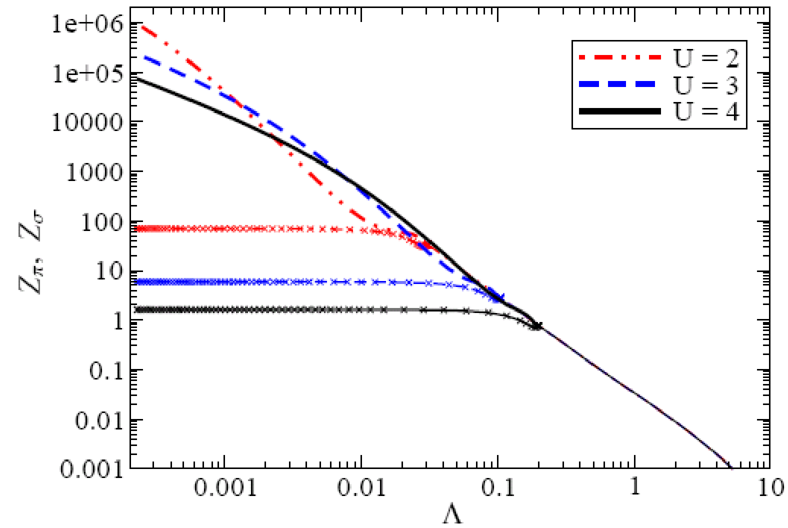
$$
\begin{aligned}
G_\sigma(sq) &\propto s^{-1} \quad \text{for } d = 2 \\
G_\sigma(sq) &\propto \log s \quad \text{for } d = 3
\end{aligned}
$$

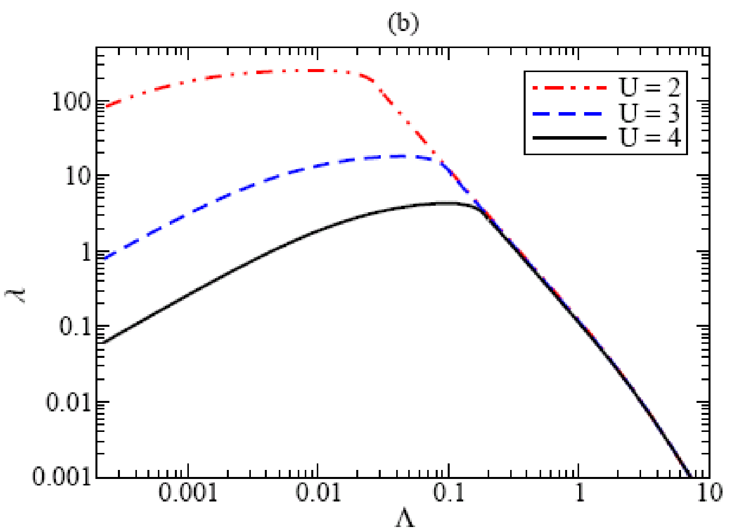
Exact results (proof in Castellani, et. al. PRB **69**, 024513, 2004)

- Goldstone propagator remains unrenormalized as a result of Ward identities
- So far not emergent from functional RG.
- Numerical results confirm analytical calculation.

Numerical Results







- At critical scale, „boson splits into longitudinal and Goldstone mode“
- Flow continuous across critical scale
- IR scaling confirms analytical calculation

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Infrared/Universal quantities

+ Exact momentum scaling of longitudinal and Goldstone propagator present in any fermionic superfluid

Fermion gap reduction/connection to MFT

+ Deviations to MFT transparent
+ Quantum fluctuations incorporated
+ Vertex corrections relatively weak
+ Distinguished gap from order parameter

Technical finesse

+ Utilized T=0 frequency cutoff
+ Consistent choice of relative cutoff scale between fermions and bosons

Outlook

+ Treat small T in Kosterlitz-Thouless
+ BEC-BCS crossover for lattice fermions
+ Goldstone mode with real frequencies (Kopietz, et. al.)

• To be published in Phys. Rev. B, arXiv:0804.3994

Thank you for your attention!

??? Questions ???

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