Effects of retardation in the functional renormalization group approach to interacting fermions

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Effects of retardation in the functional renormalization group approach to interacting fermions

- change critical energy scales
- change the phase diagram

Outline

- system considered: interacting fermions
- frequency dependent interactions, examples
- functional RG for interacting fermions
- simple example: circular Fermi surface in 2D
- another simple example: 1D Holstein-Hubbard model
- 2D square lattice at half-filling
- Concluding remarks

Interacting fermions:

Examples: electrons, quasiparticles, fermionic atoms such as ^{6}Li, ^{40}K.

Generic action:

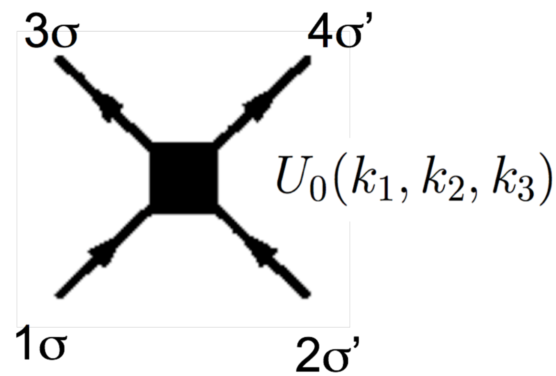
$$S = T \sum_{k,\sigma} \bar{\psi}_{\sigma k}(i\omega_n - \xi_{\mathbf{k}})\psi_{\sigma k} + T^3 \sum_{k_1,k_2,k_3} \sum_{\sigma,\sigma'} U_0(k_1,k_2,k_3)\bar{\psi}_{\sigma k_3}\bar{\psi}_{\sigma' k_4}\psi_{\sigma' k_2}\psi_{\sigma k_1}$$

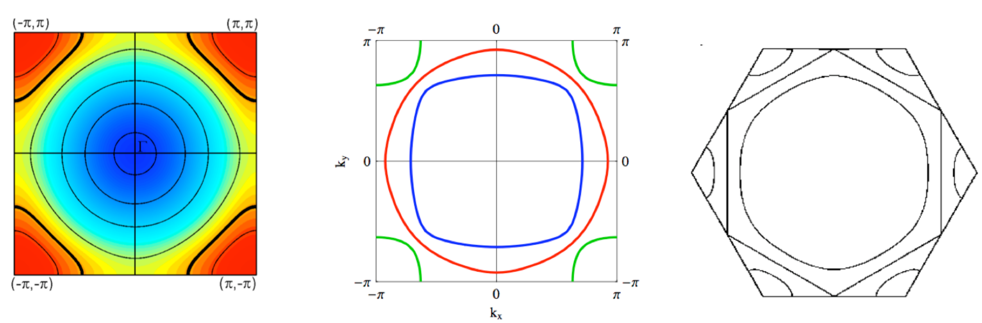
where:

$$k \equiv (\omega_n, \mathbf{k})$$

$$k_1 + k_2 = k_3 + k_4$$

$$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$





Frequency-dependent interactions

$U_0(k_1, k_2, k_3)$

Examples:

Dynamical Coulomb screening

Phonon-mediated interactions

Interactions mediated by BEC fluctuations

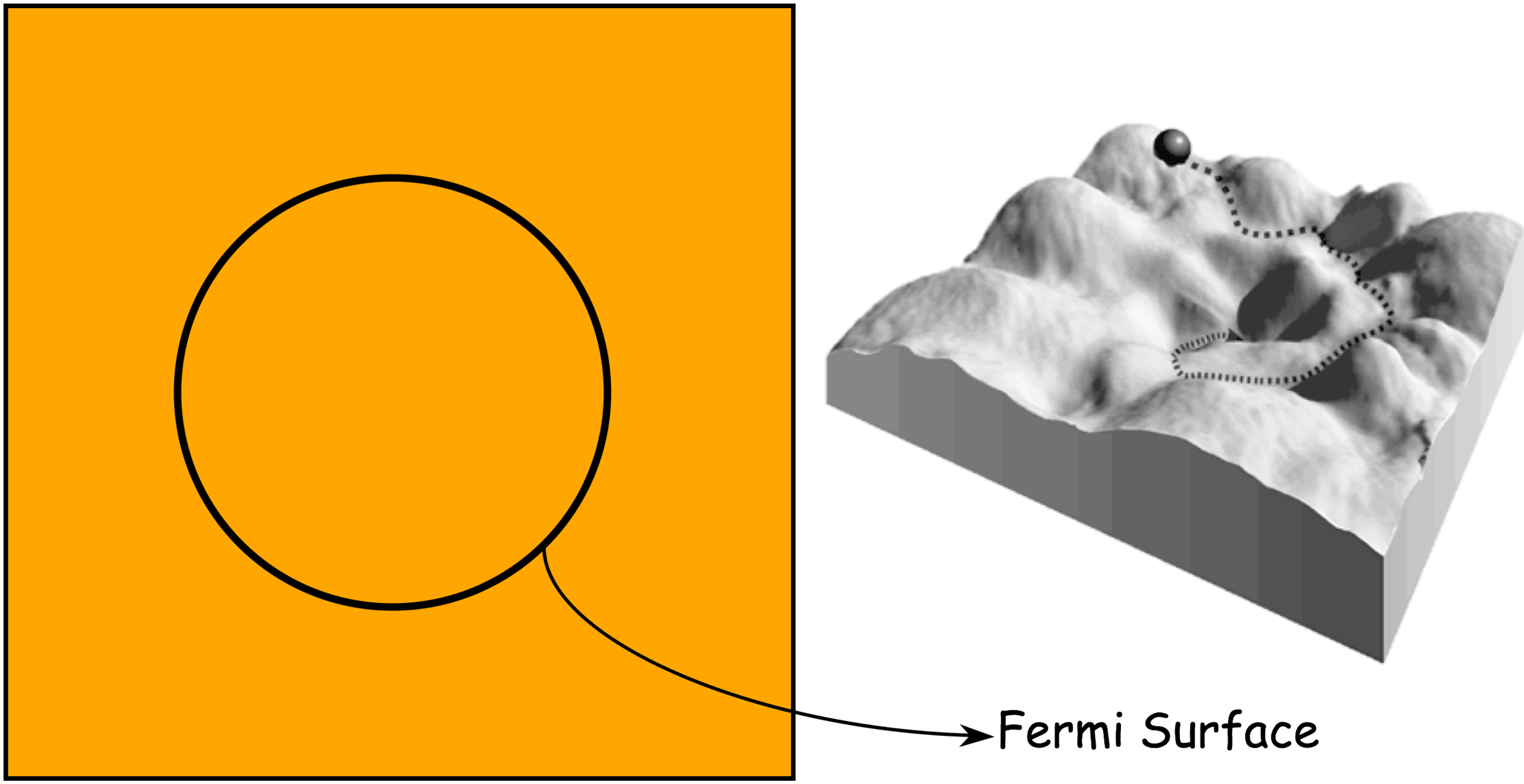
Interactions mediated by any boson-exchange coupling

Retardation effects important when $v_B \lesssim v_F$

$\lambda = 2N(0)g^2/\omega_E$

$$U_0(k_1, k_2, k_3) = u_0(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) - 2g(k_1, k_3)g(k_2, k_4)D(k_1 - k_3)$$

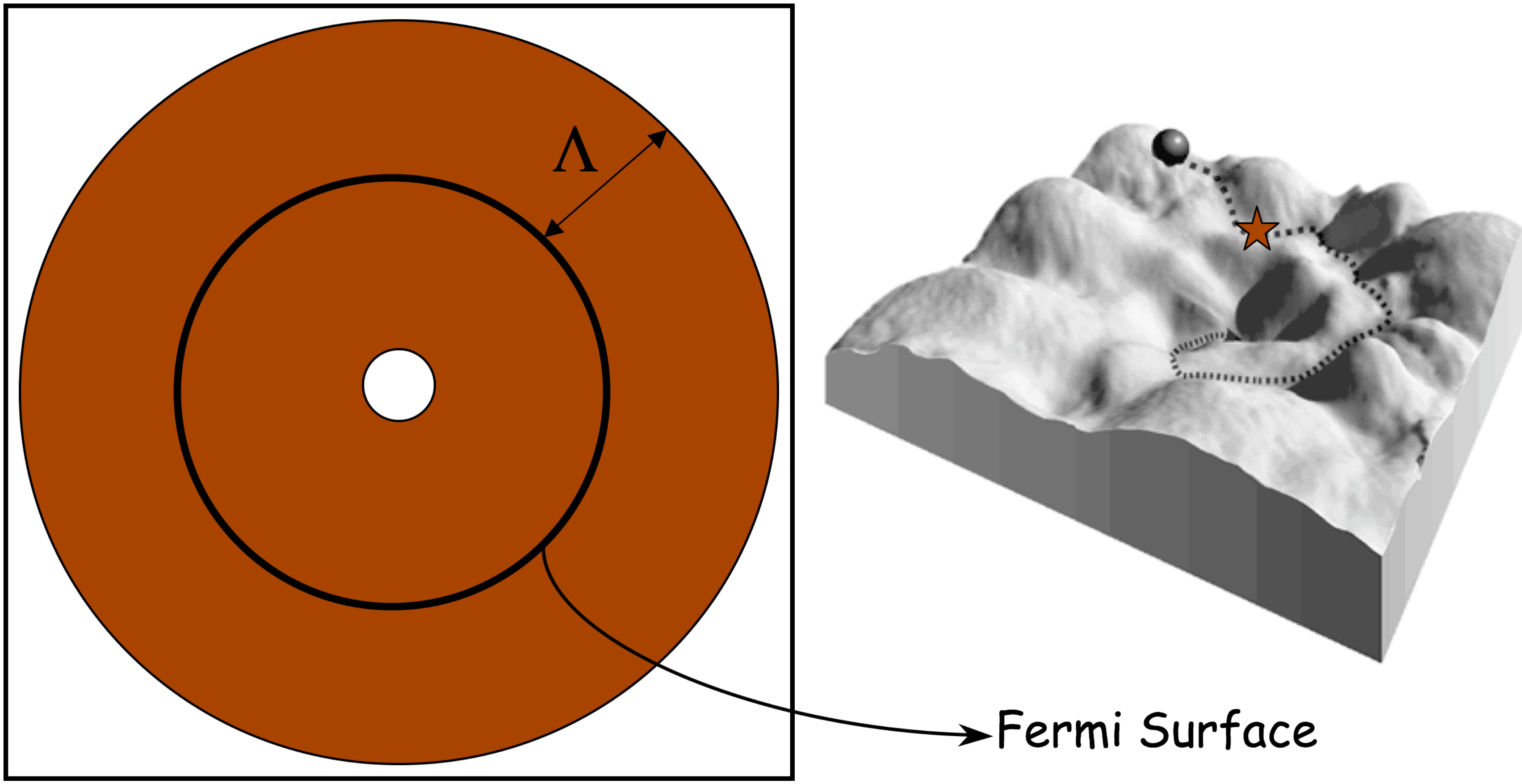
Renormalization-group for interacting electrons:



RG flow equations:

- **give effective action at** $\Lambda < \Lambda_0$
- **track instabilities of the Fermi liquid state (charge/spin order, superconductivity, etc...)**

Renormalization-group for interacting electrons:



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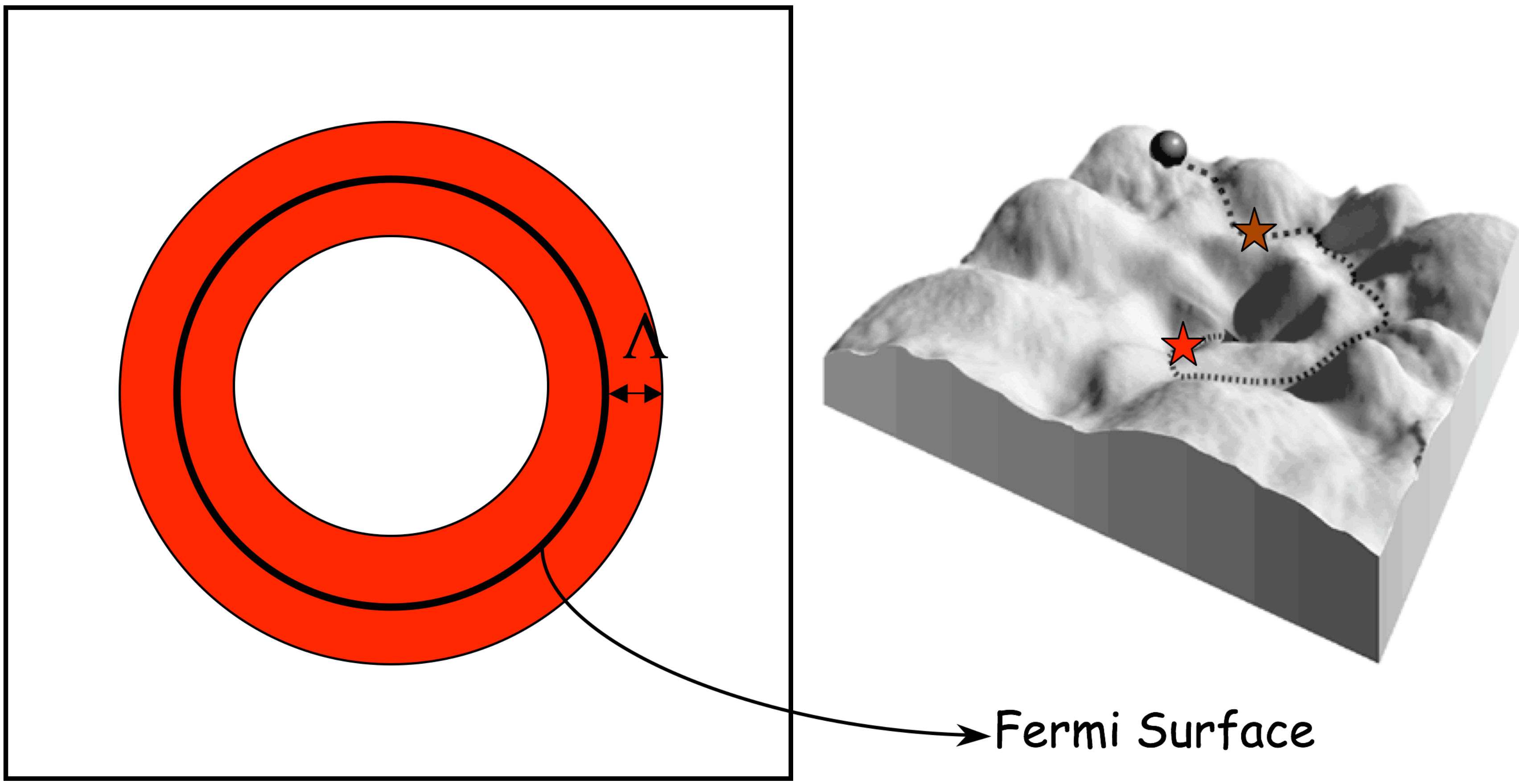
Renormalization-group for interacting electrons:



RG flow equations:

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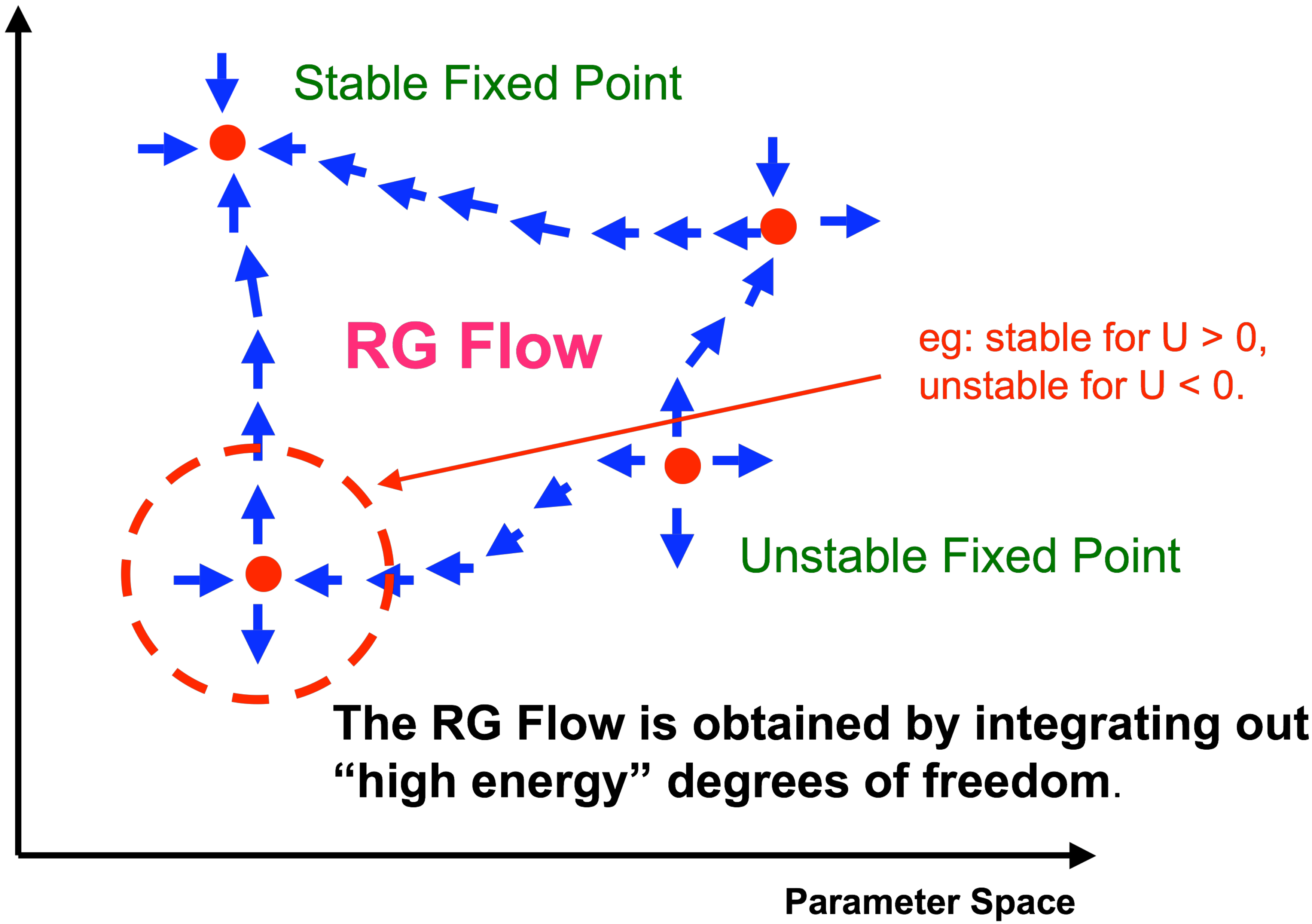
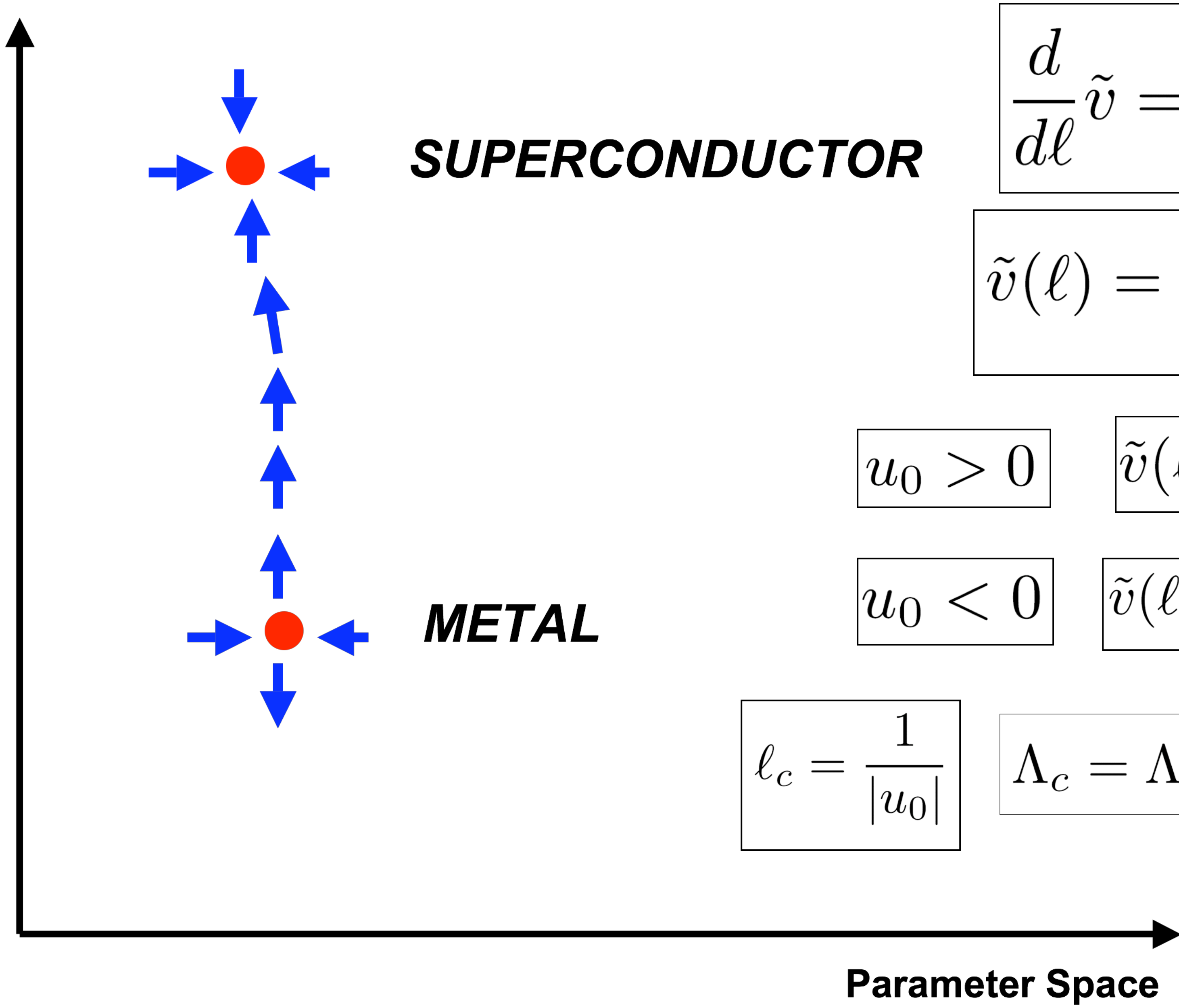

Stable Fixed Point
RG Flow
eg: stable for U > 0,
unstable for U < 0.
Unstable Fixed Point
The RG Flow is obtained by integrating out
“high energy” degrees of freedom.
Parameter Space



$$\frac{d}{d\ell}\tilde{v} = -\tilde{v}^2$$

$$\tilde{v}(\ell) = \frac{u_0}{1 + u_0 \ell}$$

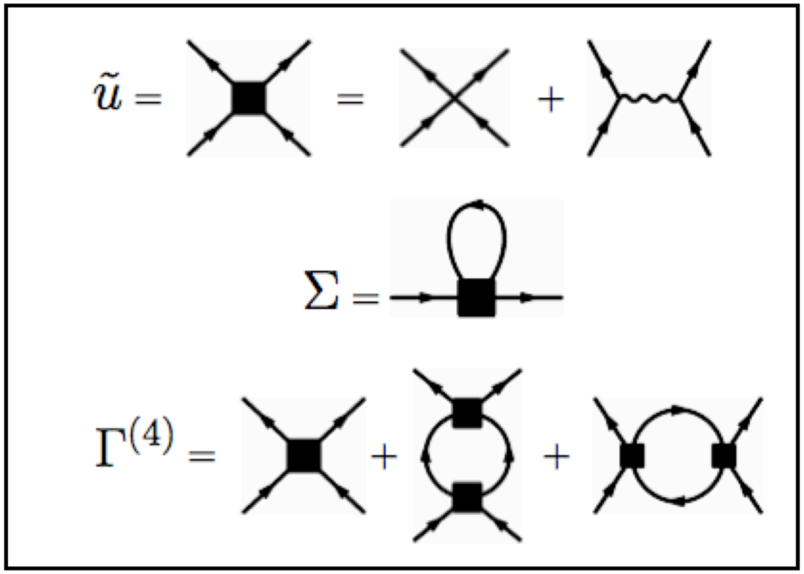
$u_0 > 0$ $\quad$ $\tilde{v}(\ell) \rightarrow 0$

$u_0 < 0$ $\quad$ $\tilde{v}(\ell_c) \rightarrow \infty$

$$\ell_c = \frac{1}{|u_0|}$$

$$\Lambda_c = \Lambda_0 e^{-1/|u_0|}$$

Circular Fermi surface, isotropic interaction, BCS channel:



matrix equation: $U_{ij} = U(\omega_i, \omega_j)$

$$\frac{d\mathbf{U}}{d\ell} = -\mathbf{U} \cdot \mathbf{M} \cdot \mathbf{U}$$

Exact solution:

$$\mathbf{U}(\ell) = [1 + \mathbf{U}(0) \cdot \mathbf{P}(\ell)]^{-1} \mathbf{U}(0)$$

$$\mathbf{P}(\ell) = \int_0^{\ell} d\ell' \mathbf{M}(\ell') \,.$$

Coupling diverges at $\ell = \ell_c$, where:

$$\det [1 + \mathbf{U}(0) \cdot \mathbf{P}(\ell_c)] = 0$$

which is equivalent to:

$$[1 + \mathbf{U}(0) \cdot \mathbf{P}(\ell_c)] \cdot \mathbf{f} = 0$$

→ **gives ELIASHBERG's equations at $T=T_c$**

Weak-intermediate coupling
McMillan, '68

$$T^* \approx 1.13\,\omega_E \exp\{-(1+\lambda)/(\lambda - \mu^*(1+\lambda))\}$$

Strong Coupling
Allen-Dynes, '75

$$T^* \approx 0.16\sqrt{\lambda}\omega_E$$

T* calculated from:

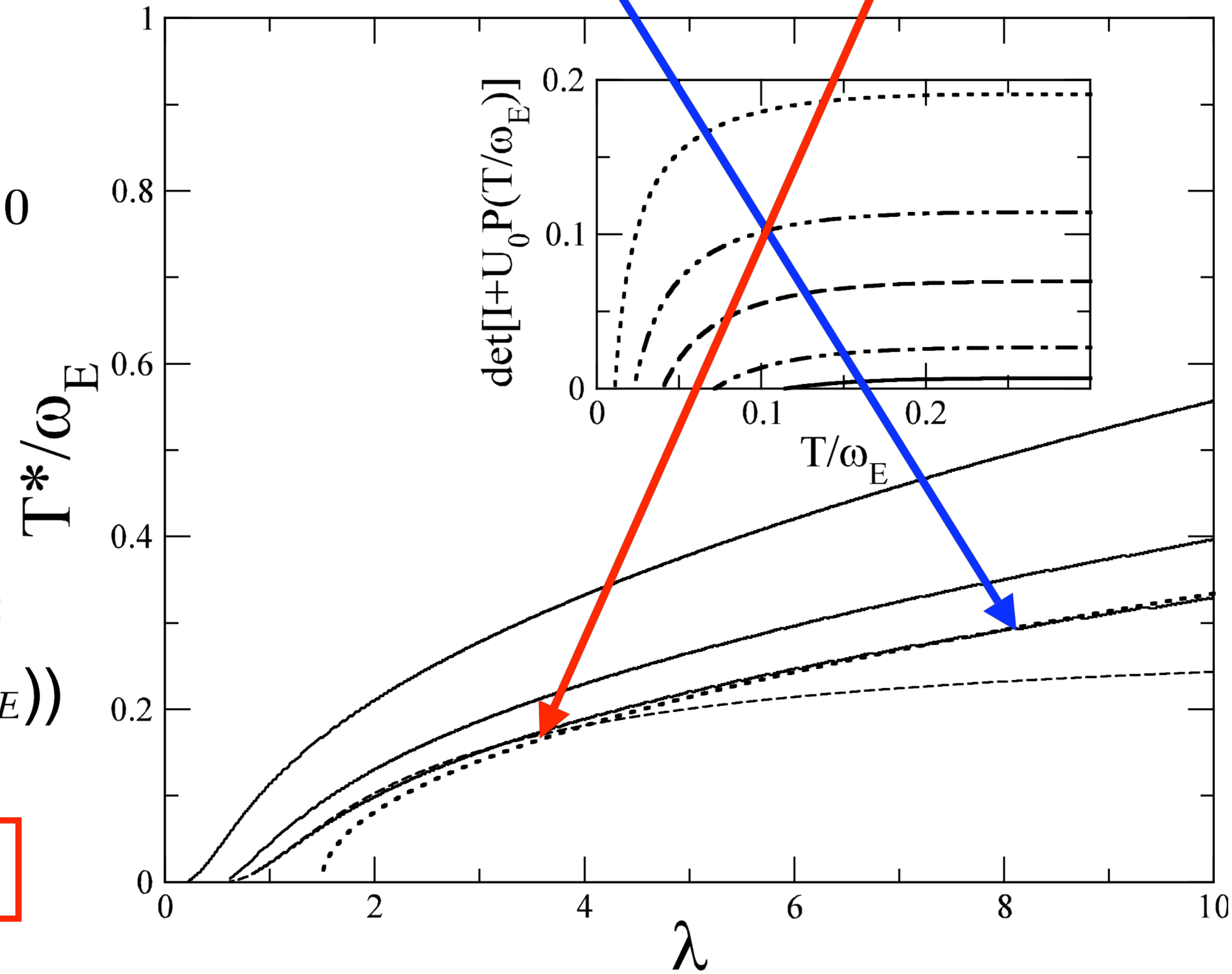
det [**1** + **U**(0). **P**(T*/ω_E)] = 0

$$\lambda = 2N(0)g^2/\omega_E$$

Effective e-e interaction:

$$\mu^* = u_0/(1+u_0\ln(\Lambda_0/\omega_E))$$

SWT, AH Castro Neto, R Shankar, DK Campbell, PRB **72**, 054531(2005)



RG evolution of the couplings in the BCS channel (λ = 0.3, 4.0)

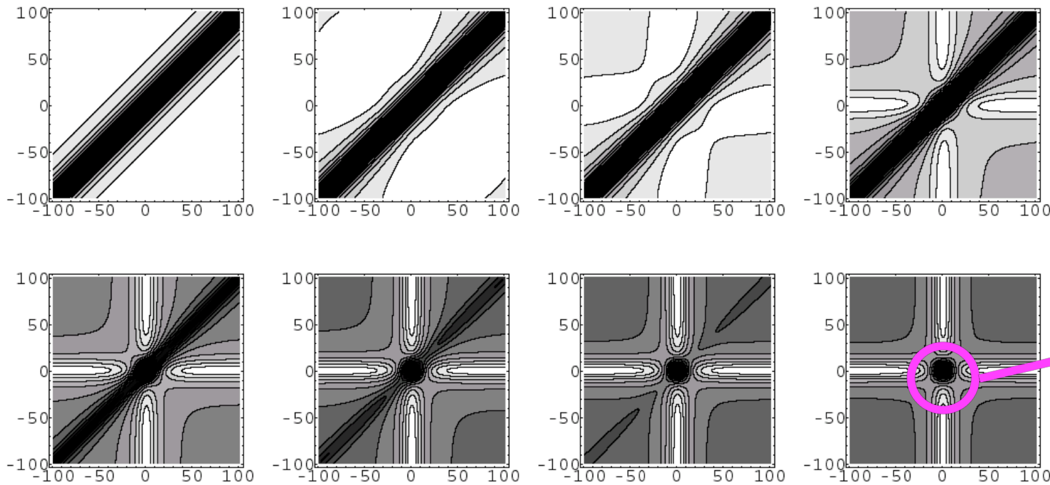


Figure 1: Plots of the NxN matrix U at different RG scales ℓ. Here the number of frequency divisions $N=200$, and the value of the parameters used are $\lambda=0.3$, $\Lambda_0=100$, $\omega_E=10$, $u_0=0.1$. Panels correspond to ℓ = 0, 2.5, 3, 5, 6.5, 6.9, 7.1, and 7.19.

"weak-coupling"
$\lambda < 1$

~ ω_E

"two-step" RG is OK

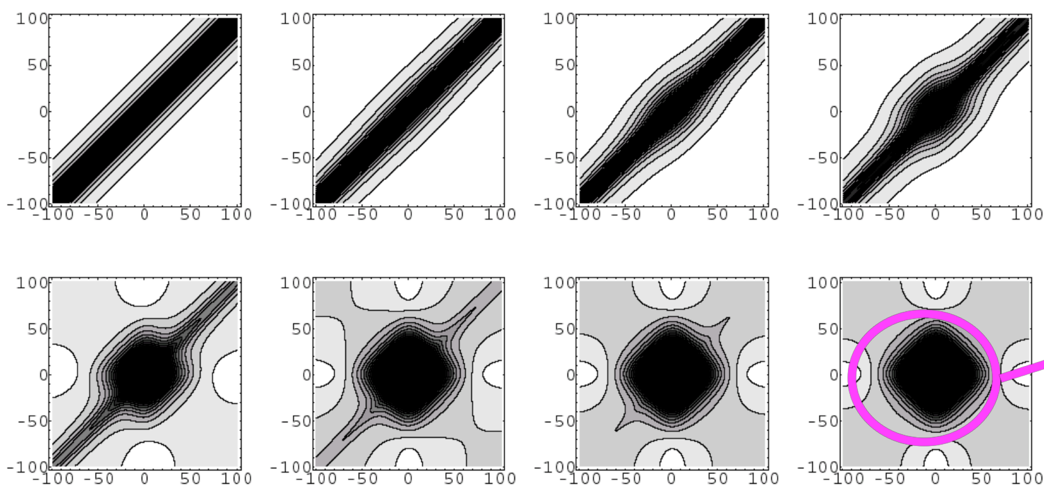


Figure 2: Plots of the NxN matrix U at different RG scales ℓ. Here the number of frequency divisions $N=200$, and the value of the other parameters are $\lambda=4$, $\Lambda_0=100$, $\omega_E=10$, and $u_0=0.1$. Panels correspond to ℓ = 0, 1, 2, 2.5, 3, 3.13, 3.157, 3.172. The scale $2W_c \approx 40$ distinguishes the high and low frequencies close to ℓ_c.

"strong-coupling"
$\lambda > 1$

~ T_c

SWT *et al.*, Philos. Mag. **86**, 2631 (2006)

Circular Fermi surface, but anisotropic boson-exchange couplings

$\lambda = 0.4$ $\lambda = 4.0$

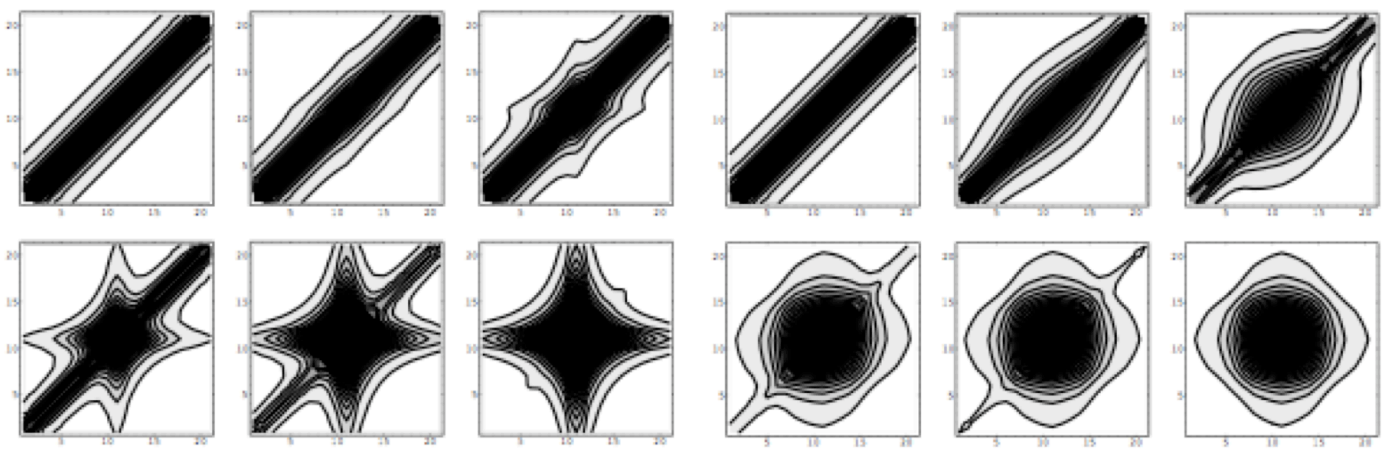


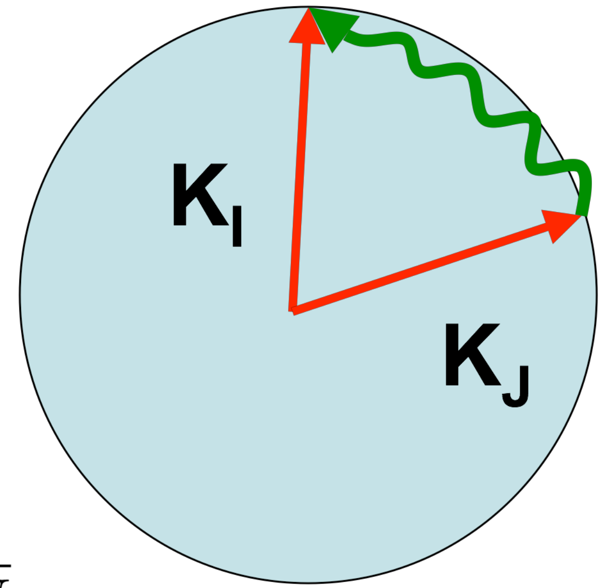
FIG. 2: Same as Fig.[1] but for the $d_{x^2-y^2}$-channel, $\gamma = 2$. The six panels on the left side are the evolution of the matrix elements at weak coupling, $\lambda = 0.4$ and the panels on the right side are the same but for the strong coupling regime, $\lambda = 4.0$

Generalized Eliashberg equations.

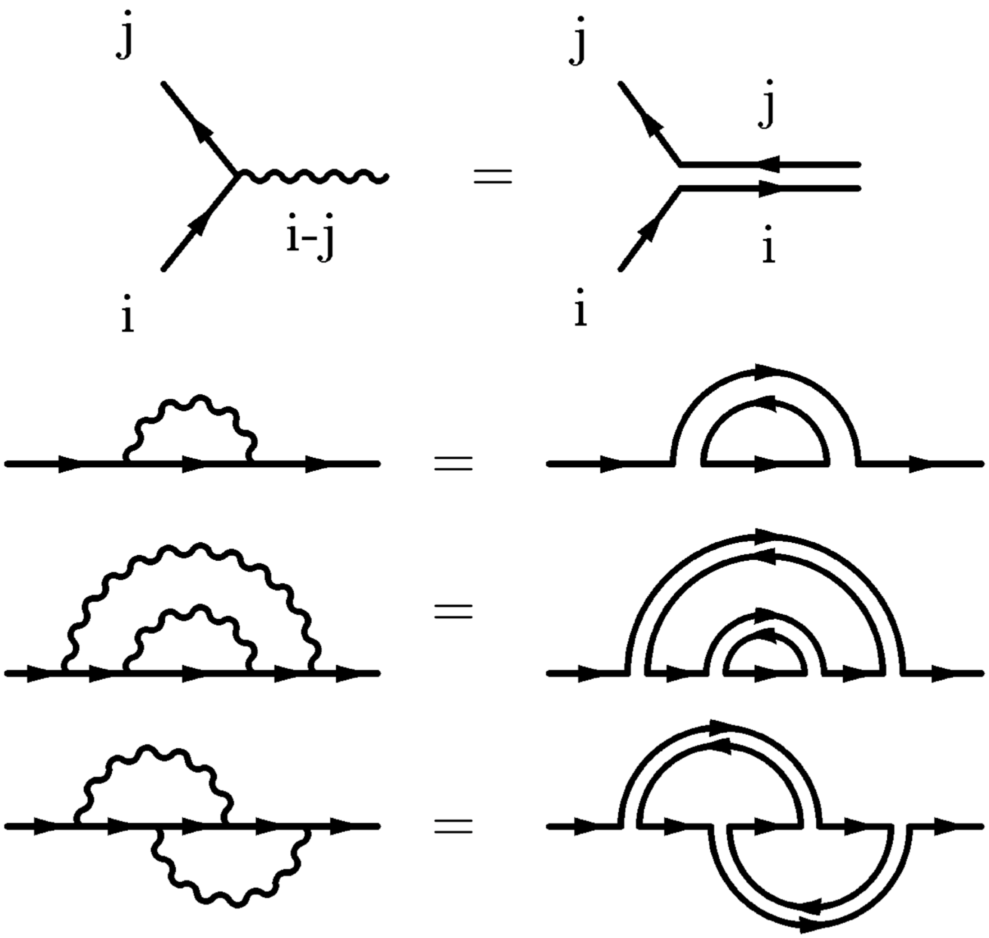
R. Roldan and SWT, cond-mat/0702673

Migdal's theorem ('58)

$$u \sim g^2 \sim 1/N$$



t'Hooft 1974



$\propto \; g/\sqrt{N}$

$\propto \; g^2$

$\propto \; g^4$

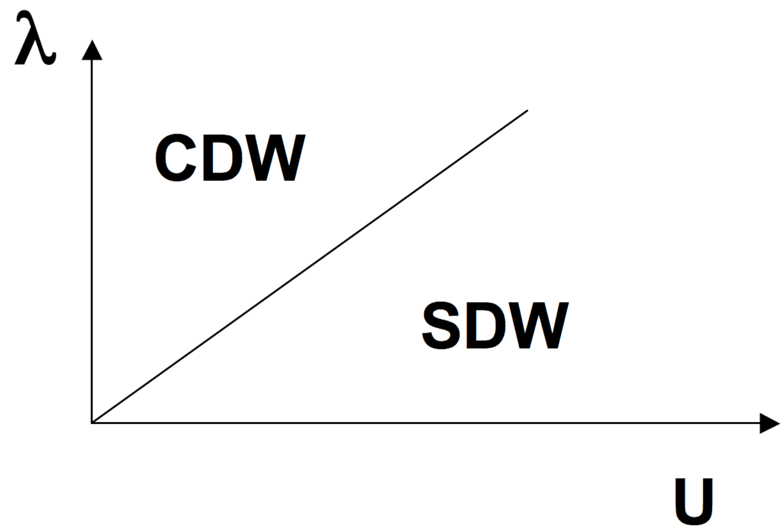
$\propto \; g^4/N^2$

Self-energy

No e-ph vertex corrections!

1D Holstein-Hubbard model:

$$H = -t\sum_{i,\sigma}(c^\dagger_{i+1,\sigma}c_{i,\sigma} + H.c.) + U\sum_i n_{i,\uparrow}n_{i,\downarrow} + g_{ep}\sum_{i,\sigma}(a^\dagger_i + a_i)n_{i,\sigma} + \omega_0\sum_i b^\dagger_i b_i,$$



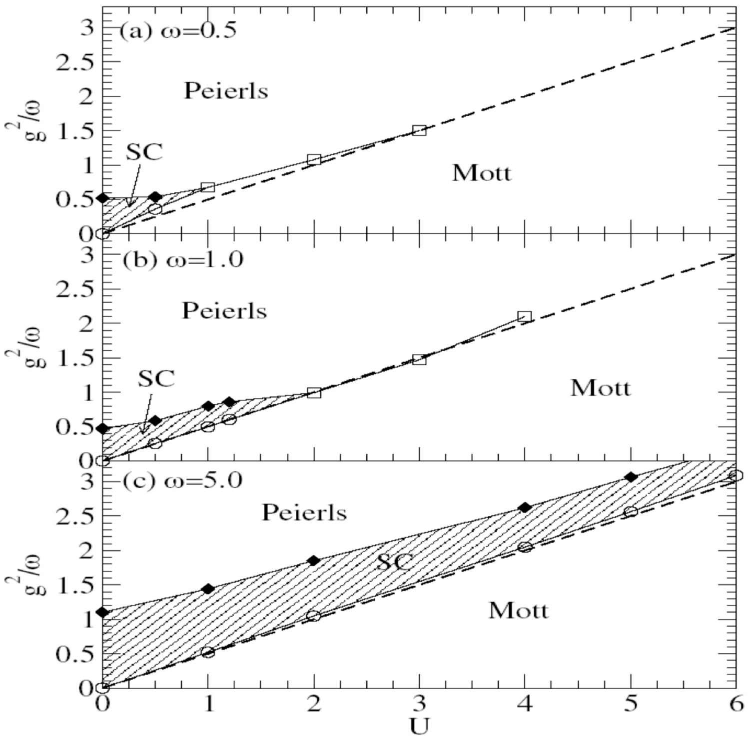
$\lambda = 2g_{ep}^2 / \omega_0$

$U_{eff} \equiv U - \lambda$

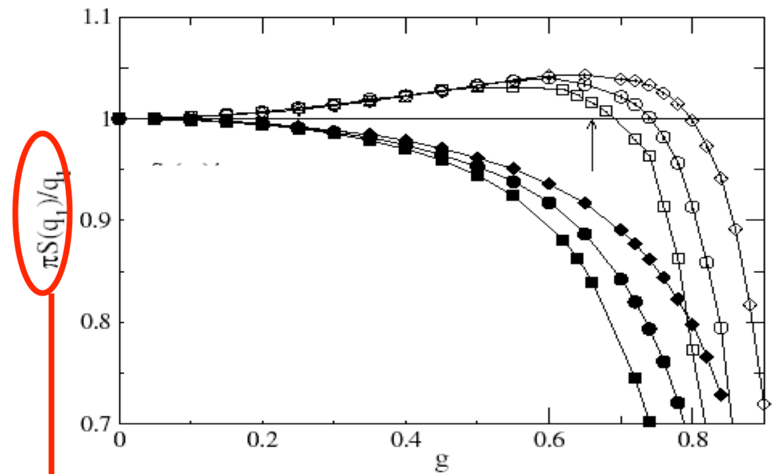
J. E. Hirsch and E. Fradkin, PRB 27, 4302 (1983)

also: H. Fehske, *et al.,* PRB 69, 165115 (2004);
I. P. Bindloss, PRB 71, 205113 (2005)

More recently a third phase has been proposed:



R. T. Clay and R. P. Hardikar, PRL 95, 096401 (2005)



Kρ

From Tomonaga-Luttinger liquid theory:

$O^{CDW} \sim x^{-K\rho}$ $\qquad$ $O^{SC} \sim x^{-1/K\rho}$

also: C. Wu, *et al.*, PRB 52, 15683 (1995)

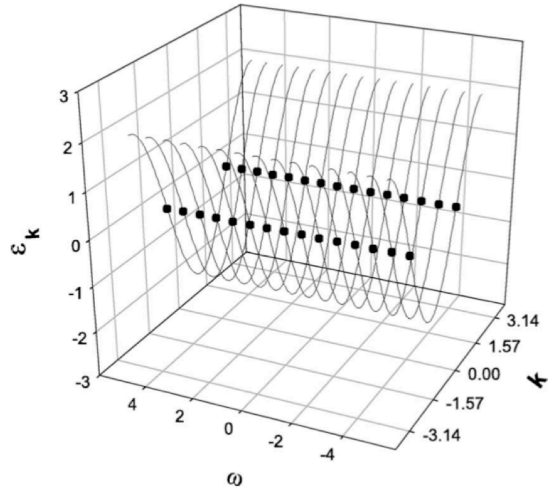
E. Jeckelmann, *et al.*, PRB 60, 7950 (1999)

Y. Takada and A. Chatterjee, PRB 67, 0811102 (2003)

Y. Takada, J. Phys. Soc. Jpn., 65, 1544 (1996)

Functional RG analysis:

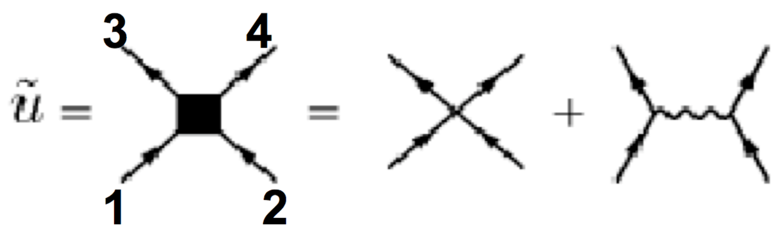
K.-M. Tam *et al.*, PRB **75**, 161103 (2007)



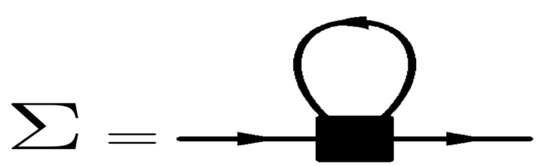
$$g_1 \rightarrow g_1(\omega_1, \omega_2, \omega_3, \omega_4)$$
$$g_2 \rightarrow g_2(\omega_1, \omega_2, \omega_3, \omega_4)$$
$$g_3 \rightarrow g_3(\omega_1, \omega_2, \omega_3, \omega_4)$$
$$g_4 \rightarrow g_4(\omega_1, \omega_2, \omega_3, \omega_4)$$

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

Initial conditions:

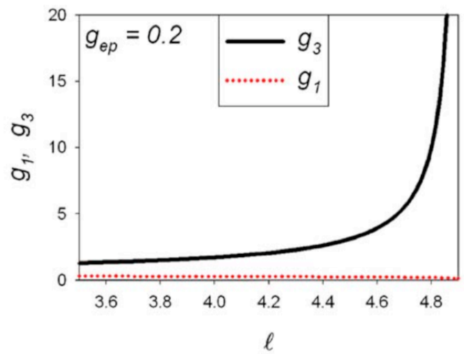


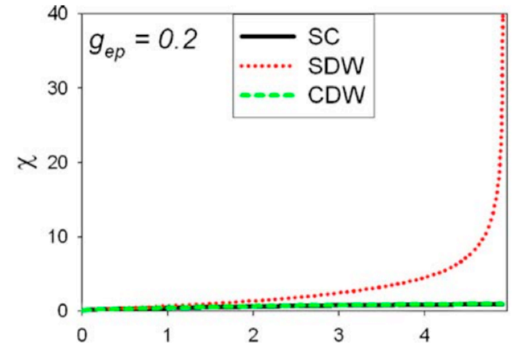
$$g_i(\omega_1, \omega_2, \omega_3, \omega_4) = U - \frac{2g_{ep}^2}{\omega_0}\left(\frac{\omega_0^2}{\omega_0^2 + (\omega_1 - \omega_3)^2}\right)$$



RG flows of susceptibilities and couplings ($\omega_0 = 1$):

$U_{eff} > 0$



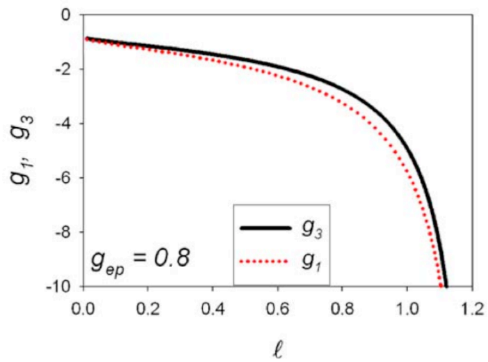


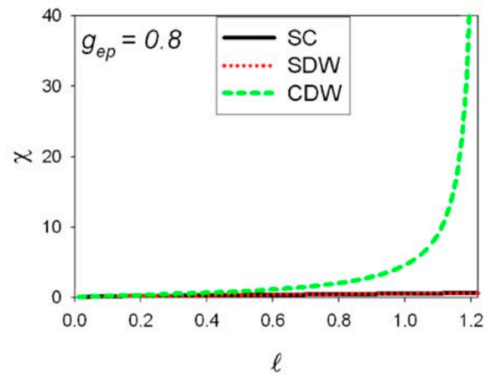
SDW

$\Delta_C \neq 0$

$\Delta_S = 0$

$U_{eff} < 0$





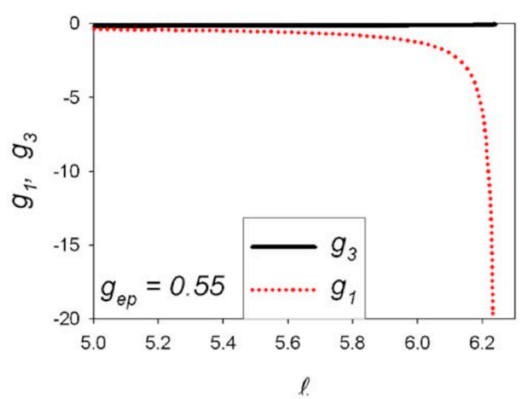
CDW

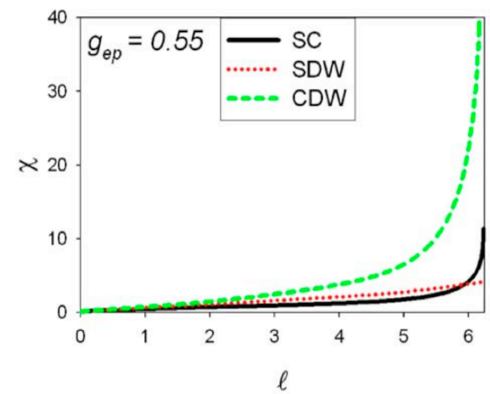
$\Delta_C \neq 0$

$\Delta_S \neq 0$

"Intermediate" region:

$U_{eff} = -\delta$





CDW

$\Delta_C = ?$

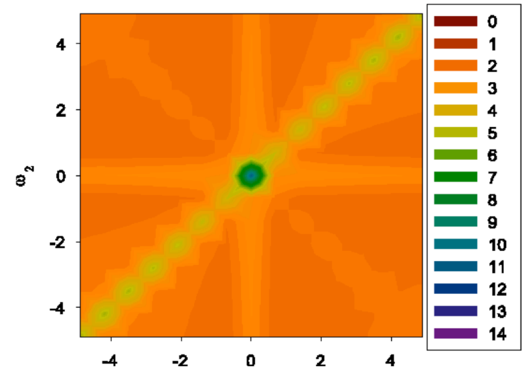
$\Delta_S \neq 0$

Frequency structure of $g_3(\omega_1,\omega_2,\omega_1,\omega_2)$

K.-M. Tam *et al.*, PRB **75**, 161103 (2007)

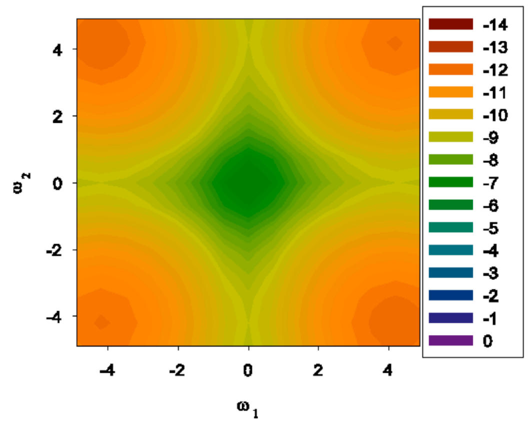
g_{ep} = 0.2 (SDW)

$U_{eff} < 0$



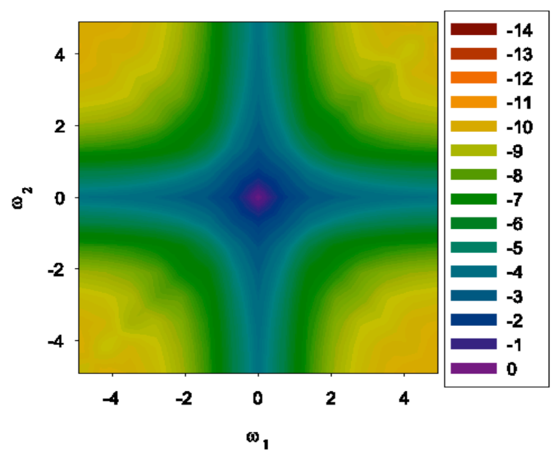
g_{ep} = 0.8 (CDW)

$U_{eff} > 0$



g_{ep} = 0.55 (CDW)

$U_{eff} \approx 0$



"dynamical umklapp"

How to conciliate with Kρ > 1:

- Kρ > 1 does not mean SC is dominant!

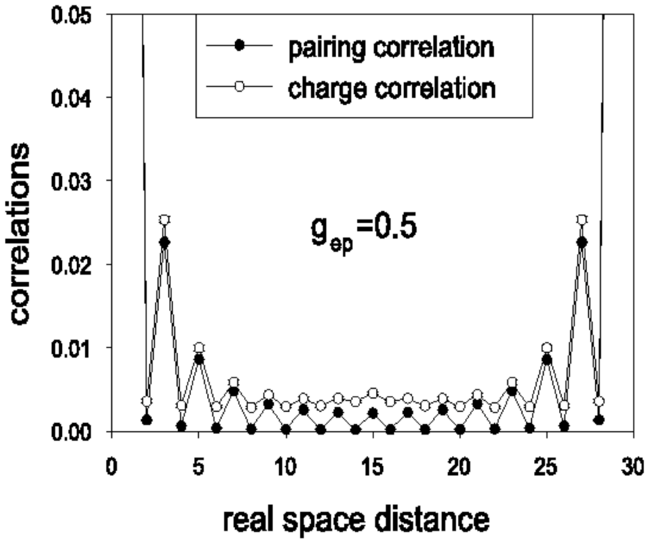
$$O^{CDW}(x) \propto x^{-\alpha K_\rho} \equiv x^{-K_{CDW}}$$

$$O^{SC}(x) \propto x^{-\beta / K_\rho} \equiv x^{-K_{SC}}$$

D. Loss and T. Martin, PRB 50, 12160 (1994)
M. Tezuka, *et al.*, PRL 96, 226401 (2005)

Ladder systems:
K.-M. Tam *et al.*, PRB 75, 195119 (2007)

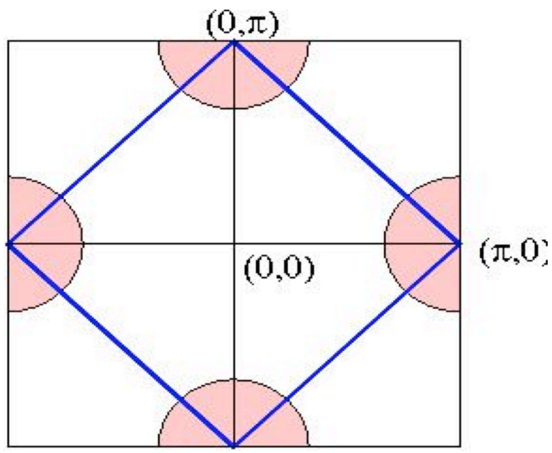
Direct calculation of susceptibilities (Determinantal Quantum Monte-Carlo):

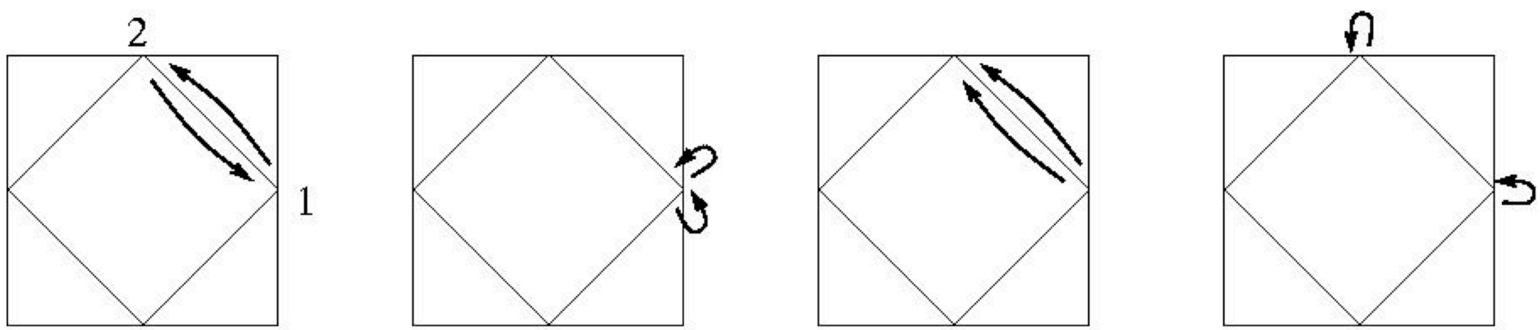


K.-M. Tam *et al.* (unpublished)

Two-patch model for van Hove problem:

H. Schulz, Europhys. Lett. (1987)





$g_1 = u(1221)$ $\quad g_2 = u(1111)$ $\quad g_3 = u(2211)$ $\quad g_4 = u(1212)$

phonon coupling: $\lambda = 2N(0)g^2/\omega_E$

Fermions with spin:

van Hove problem without phonons has been extensively studied, *e. g.,*
J. Gonzalez, F. Guinea, and M. A. H. Vozmediano 1997, N. Furukawa, T. M. Rice and M. Salmhofer 1998, C. Honerkamp, M. Salmhofer, N. Furukawa, and T. M. Rice 2001, B. Binz, D. Baeriswyl, and B. Doucot, 2002, …

- What is the interplay between effects of nesting and phonons?
- Are phonons always pair-breaking in the d-wave superconducting channel?
- Can phonons and AF fluctuations cooperate to enhance T_c for d-wave superconducivity ?

Allow for anisotropic phonons, calculate flow of susceptibilities:

$u_0 = 0.5$, $\omega_E = 1.0$

$\lambda_0 < \lambda_\pi$ $\lambda_0 > \lambda_\pi$

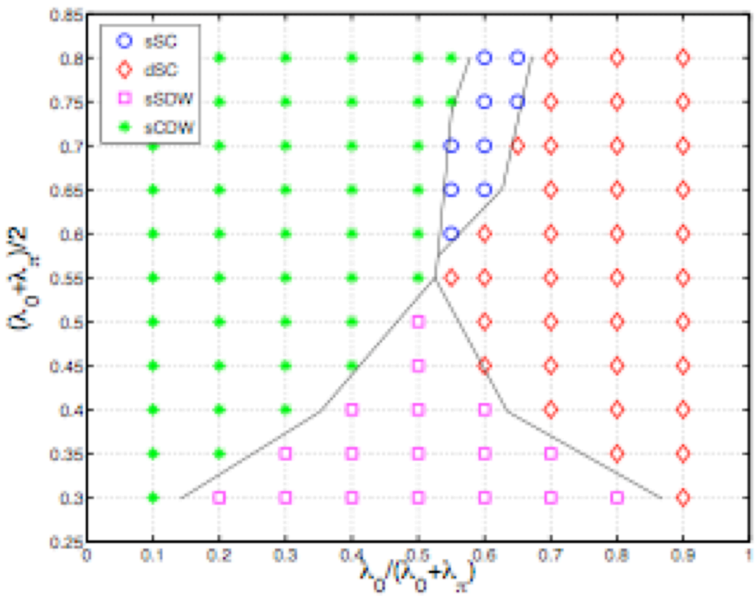


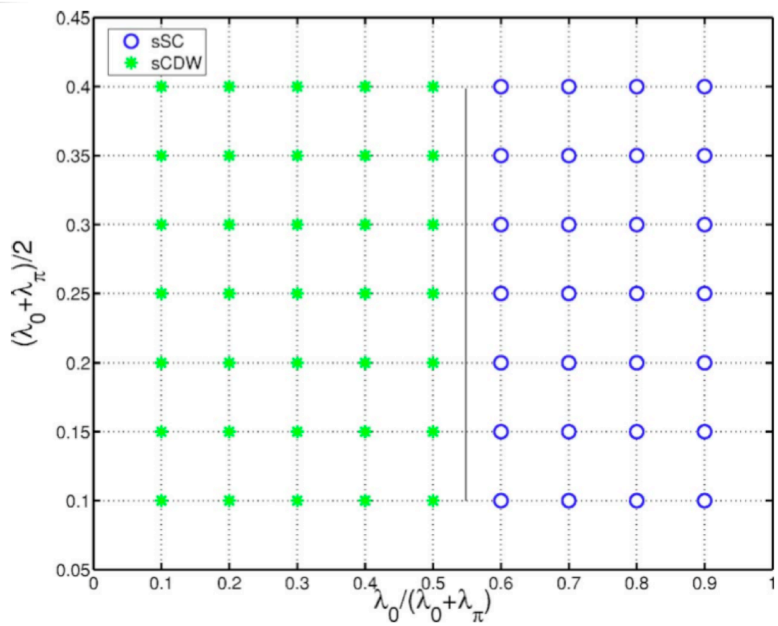
FIG. 1: (Color online) Phase diagram for Einstein phonons of frequency $\omega_E = 1.0$. Four phases involving antiferromagnetism (sSDW) (purple squares), charge density wave (sCDW) (green stars) and s-wave (sSC) (blue circles) and d-wave (dSC) (red rhombs) superconductivity compete in the vicinity where the average phononic strength $\bar{\lambda}$ approaches the bare on-site repulsion $u_0 = 0.5$. The lines distinguishing the different domains are guides to the eye.

$$g_{1,3}^{\ell=0}(\omega_1,\omega_2,\omega_3) = u_0 - \lambda_\pi \frac{\omega_E^2}{\omega_E^2 + (\omega_1 - \omega_3)^2}$$

$$g_{2,4}^{\ell=0}(\omega_1,\omega_2,\omega_3) = u_0 - \lambda_0 \frac{\omega_E^2}{\omega_E^2 + (\omega_1 - \omega_3)^2}$$

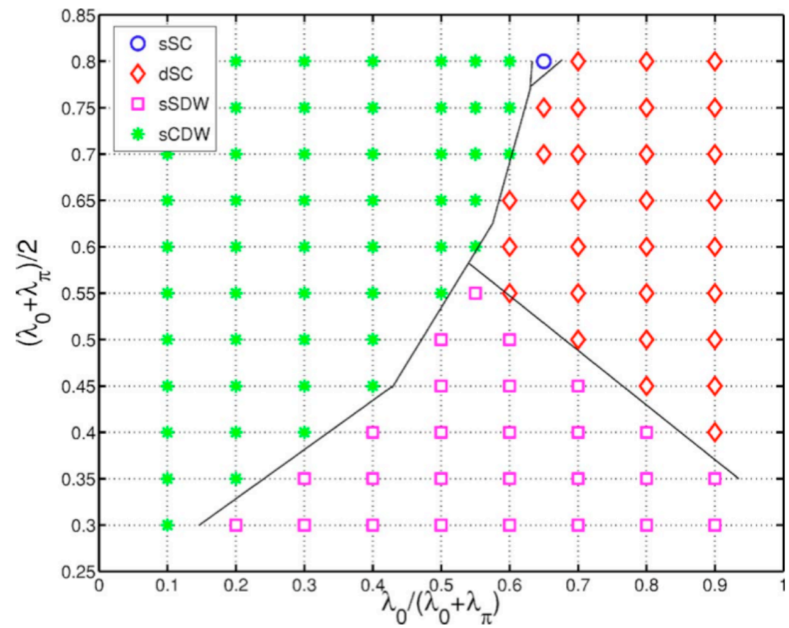
F. D. Klironomos and SWT, PRB **74**, 205109 (2006)

$u_0 = 0, \omega_E = 1.0$



Need repulsive component for d-wave SC to develop.

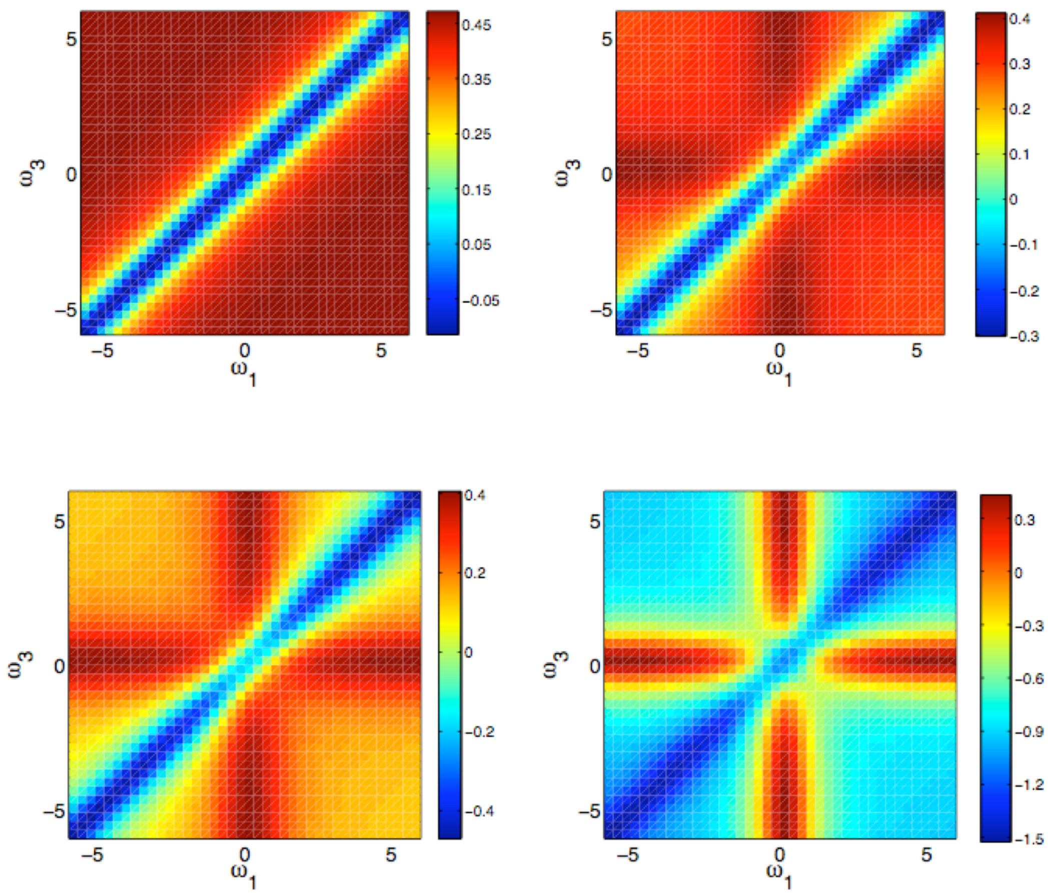
$u_0 = 0.5, \omega_E = 0.1$



Density-wave phases regions increase when ω_E is decreased.

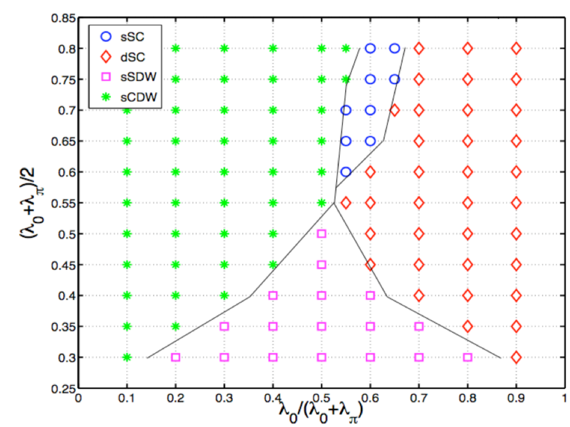
F. D. Klironomos and SWT, PRB **74**, 205109 (2006)

RG evolution of $g_2(\omega_1, -\omega_1, \omega_3, -\omega_3)$ for λ_0=0.6, λ_π=0.4, and ω_E=1.0.



$$g_{1,3}^{\ell=0}(\omega_1,\omega_2,\omega_3) = u_0 - \lambda_\pi \frac{\omega_E^2}{\omega_E^2 + (\omega_1 - \omega_3)^2}$$

$$g_{2,4}^{\ell=0}(\omega_1,\omega_2,\omega_3) = u_0 - \lambda_0 \frac{\omega_E^2}{\omega_E^2 + (\omega_1 - \omega_3)^2}$$



F. D. Klironomos and SWT, PRB **74**, 205109 (2006)

There can be dominant BCS pairings even at half-filling due to a separation of scales: a given coupling may have a different sign at low and high frequencies

Summary:

- Functional RG for interacting fermions with frequency-dependent interactions.
- Multiple energy scales.
- Applications:
- *2D Circular Fermi surface*
- *1D Holstein-Hubbard model*
- *2D square lattice at half-filling*

Fermion-Boson mixtures of cold atoms (poster):

- *fermionic atoms + BEC of bosonic atoms*
- *on-site repulsion + long-range attraction + lattice geometry*
- *square lattice, triangular lattice*
- *L. Mathey et al., PRL 2006; PRB 2007, Klironomos et al., PRL 2007.*

Collaborators:

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Rafael Roldán *(CNRS, U. Paris-SudParis)*
Maria Pilar López-Sancho *(ICMM-CSIS, Madrid)*
Ludwig Mathey *(NIST)*
Ka-Ming Tam *(U. Waterloo)*

Antonio H. Castro Neto *(BU)*
Ramamurti Shankar *(Yale)*
David K. Campbell *(BU)*