Transport of interacting electrons in 1d: nonperturbative RG approach

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Outline

- Introduction: conductance of electrons in Luttinger liquids

- Barrier in a strongly interacting Luttinger liquid;
  current algebra representation

- Perturbation theory in the interaction g_2 : summing up the principal terms linear in log(T) to all orders in g_2

- Renormalization group approach: nonperturbative β-function;
  d.c. conductance for any Luttinger parameter K and barrier reflection R.

- Demonstration of agreement with known results at R~0 and R~1
  Role of non-universal terms beyond the ladder series.

Introduction: Transport in clean Luttinger liquids

Early results for conductance G of clean infinite spinless Luttinger liquids:

forward scattering, interaction parameters: $g_2, g_4, \quad K = \sqrt{\frac{1+(g_4-g_2)/2\pi v_F}{1+(g_4+g_2)/2\pi v_F}}$

$G = K,$ G in units of $\frac{e^2}{h}$

W. Apel and T. M. Rice, 1982
C. Kane and M.P.A. Fisher, 1992
A. Furukawa and N. Nagaosa, 1993

Later interpreted as four-terminal conductance

Two-terminal conductance: Luttinger liquid attached to ideal leads (T=0)

Proper sequence of limits (1) $\omega \to 0$, (2) $L \to \infty$

$G = 1$

D. Maslov and M. Stone, 1995
I. Safi and H. Schulz, 1995

Screened internal electric field

A. Kawabata, 1996
Y. Oreg, A. Finkelstein, 1995

Introduction: Quantum wire with single barrier I

Landauer conductance in non-interacting limit:

$$G = \int d\varepsilon(-df/d\varepsilon)\left|t(\varepsilon)\right|^2$$

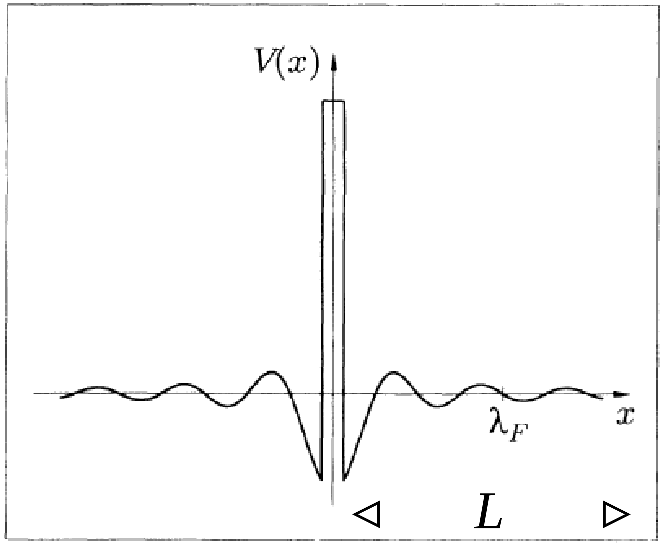
Transmission amplitude $t(\varepsilon)$

Effect of interaction: Friedel oscillations of charge density around impurity lead to dynamically generated extended effective potential

The spatial extent L of the Friedel oscillations is determined by phase relaxing inelastic processes:

$$L = v_F/T, \quad \text{or} \quad L = v_F/\varepsilon$$

Where T is the temperature and
ε is the excitation energy of a fermion



Introduction: Poor man's scaling at weak interaction

Integrating out high momentum states, reducing the band width, one finds a renormalization group equation for the transmission amplitude as a function of the bandwidth D

$$\frac{dt}{d\ln(D_0/D)} = -gt(1-|t|^2), \qquad g = g_2/2\pi v_F$$

The transmission coefficient as a function of energy follows as

:

$$T(\varepsilon) = \frac{T_0(\varepsilon/D_0)^{2g}}{R_0 + T_0(\varepsilon/D_0)^{2g}}, \qquad \varepsilon < D_0$$

Yue, Matveev and Glazman, 1995

Single particle scattering states

Scattering of spinless fermions by potential barrier:

S-matrix: $$S=\begin{pmatrix} t & \tilde{r} \\ r & \tilde{t} \end{pmatrix}=\begin{pmatrix} \cos\theta & i\sin\theta e^{-i\phi} \\ i\sin\theta e^{i\phi} & \cos\theta \end{pmatrix}$$

Single particle scattering states for right (left) moving particles (k>0) :

$$\chi_{1k}(x)=[e^{ikx}+re^{-ikx}]\theta(-x)+te^{ikx}\theta(x)$$
$$\chi_{2k}(x)=[e^{-ikx}+\tilde{r}e^{ikx}]\theta(x)+\tilde{t}e^{-ikx}\theta(-x)$$

Neglect k-dependence of t, r in the following

Fermion operators in scattering state representation

Creation ops. for right (left) moving fermions in scattering states k : $c_{k1}^\dagger, c_{k2}^\dagger$

Creation ops. for R, L fermions at position x:

$$\psi_1^\dagger(x) = \int_0^\infty \frac{dk}{2\pi} e^{ikx} c_{1k}^\dagger$$

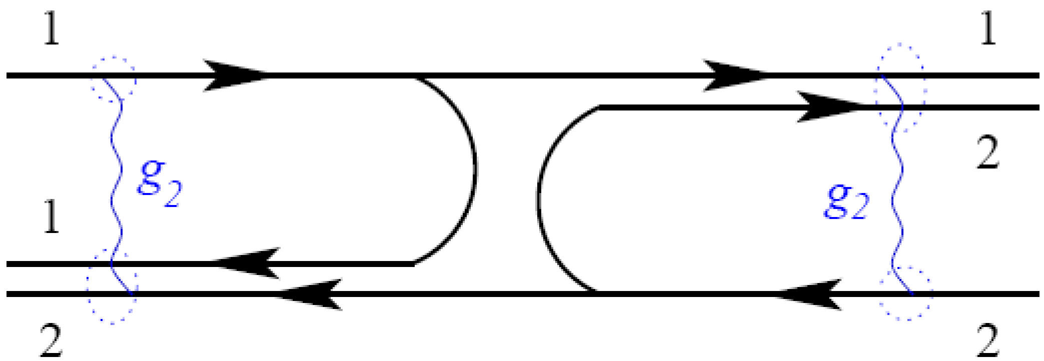
$$\psi_2^\dagger(x) = \int_0^\infty \frac{dk}{2\pi} e^{-ikx} c_{1k}^\dagger$$

Creation operators for fermions at position x::

$$\psi^\dagger(x) = \int_0^\infty \frac{dk}{2\pi} \{\chi_{1k}(x) c_{1k}^\dagger + \chi_{2k}(x) c_{2k}^\dagger\}$$

$$\psi^\dagger(x) = \theta(-x)[\psi_1^\dagger(x) + r\psi_1^\dagger(-x) + \tilde{t}\psi_2^\dagger(x)] + \theta(x)[\psi_2^\dagger(x) + \tilde{r}\psi_2^\dagger(-x) + t\psi_1^\dagger(x)]$$

Interaction in scattering state representation



Current algebra representation

Definition of current operators: *Affleck, 1990*

$$J_\mu(x) = \tfrac{1}{2}\Psi^\dagger(x)\underline{\tau}^\mu\Psi(x), \qquad \Psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(-x) \end{pmatrix}, \quad \mu=0,..,3; \quad \text{Pauli matrices } \underline{\tau}^\mu$$

Current operators obey Kac-Moody algebra.

Scattering by the potential barrier does not affect the isocharge component J_0

but rotates the isospin vector $\vec{J} \to (\hat{R}\vec{J}) = \tilde{\vec{J}}$, where $\hat{R}$ is a rotation matrix

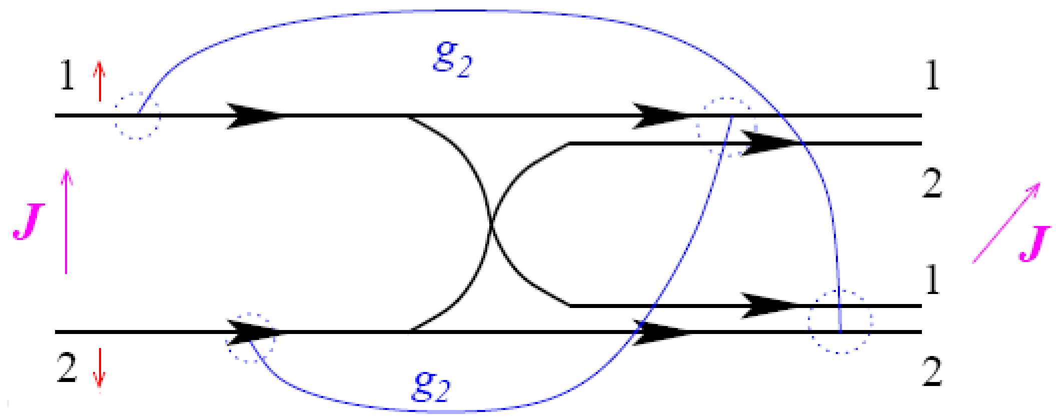
Comps. of R: $R_{33} = |t|^2 - |r|^2 = \cos(2\theta), \quad R_{32} = -\sin(2\theta)\cos(\phi), \quad R_{31} = \sin(2\theta)\sin(\phi)$

$$H_0 = 2\pi v_F \int_0^\infty dx[J_0^2(x) + J_0^2(-x) + J_3^2(-x) + \tilde{J}_3^2(x)]$$

$$H_1 = 2g_2 \int_0^\infty dx[J_0(-x)J_0(x) - J_3(-x)\tilde{J}_3(x)]$$

Interaction in chiral (current algebra) representation

"Nonlocal" interaction:



In this representation the potential barrier may be viewed as a local magnetic field rotating the isospin vector of a wave packet, when it passes through the field.

Physical currents and conductance

Electron density:

$$\rho(x) = [\rho_{iR}(x) + \rho_{oL}(x)]\theta(-x) + [\rho_{oR}(x) + \rho_{iL}(x)]\theta(x)$$
$$= J_0(-x) + J_0(x) + sign(x)[-J_3(-|x|) + \tilde{J}_3(|x|)]$$
$$= \rho_c(x) + \rho_s(x)$$

Electron current obtained from continuity equation:

$$\partial_t \rho(x) = -\partial_x j(x) = -i[\rho(x), H]$$

$$j(x) = v_F[J_0(x) - J_0(-x) + J_3(-|x|) + \tilde{J}_3(|x|)] = j_c + j_s$$

Applied voltage: $V(x,t) = \frac{1}{2}V(t)sign(x)$, couples only to isospin component

Linear response two-terminal conductance (in units of e^2/h) :

$$G(x,t) = -2\pi i\theta(t)\left\langle\left[j_s(x,t), \int_0^\infty dy\,\rho_s(y,0)\right]\right\rangle$$

Perturbation theory in g_2 : Feynman diagrams

Diagram rules for nth order contributions in (energy-position)-representation:

(1) Draw n vertical wavy lines representing interaction ($-2g_2$) , the ith line

connecting the upper point $-x_i$ with vertex $\frac{1}{2}\tau^{3}_{\alpha\beta}$

and the lower point x_i with vertex $\frac{1}{2}R_{3\mu}\tau^{\mu}_{\alpha\beta}$

(2) Connect all points with two propagator lines entering and leaving the point:

$$G_{\alpha\beta}(x,\omega_n) = -\delta_{\alpha\beta}\tfrac{i}{v_F}\,sign(\omega_n)\theta(\omega_n x)e^{-\omega_n x/v_F}$$

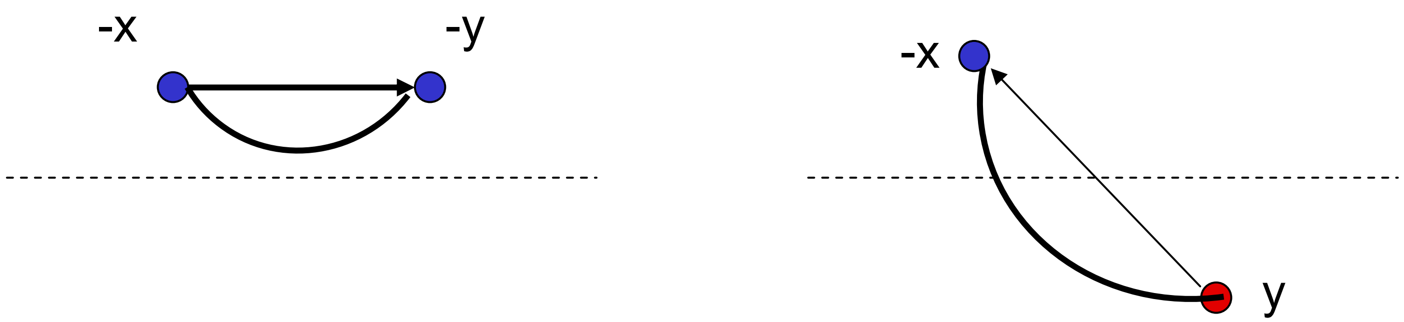
$$\omega_n = (2n+1)\pi T, \quad \text{T: temperature}$$

(3) Integrate over internal position variables from a to L

(4) In each fermion loop take trace of product of vertex matrices

(5) Take limit of external frequency $\Omega_m \to 0$

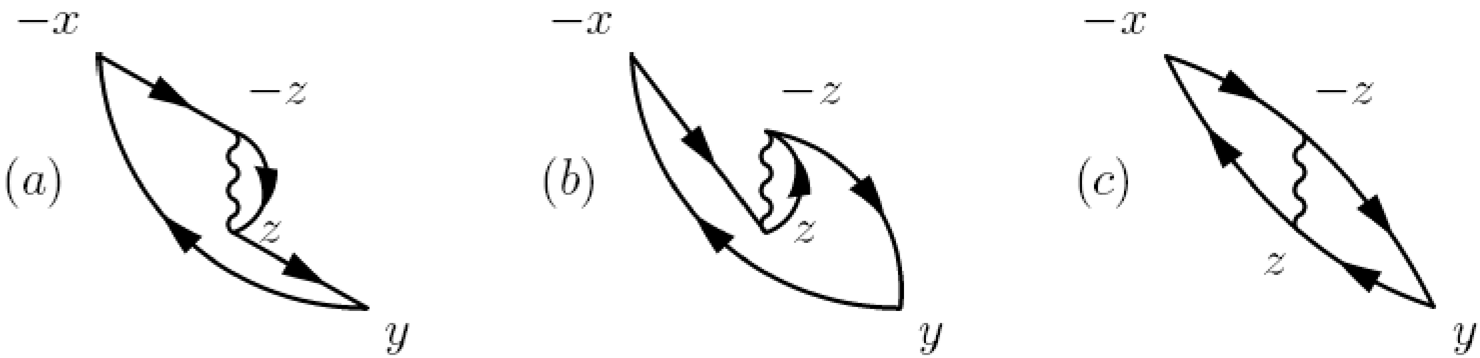
Conductance in 0 th order



$$G = \frac{1}{2}[1 + \cos(2\theta)] = \cos^2\theta = |t|^2$$

Define : $Y = \cos(2\theta)$

Conductance in 1st order



Logarithmic correction: $\int_a^L \frac{dz}{z} = \ln(\frac{L}{a}) \approx \ln(\frac{T_0}{T}), \quad L = v_F/T, \quad a = v_F/T_0$

$$G^{(1)} = -\frac{g_2}{4\pi}\sin^2(2\theta)\ln(\frac{T_0}{T})$$

Agrees with *Yue, Matveev, and Glazman, 1995*

Summation of linear log-terms

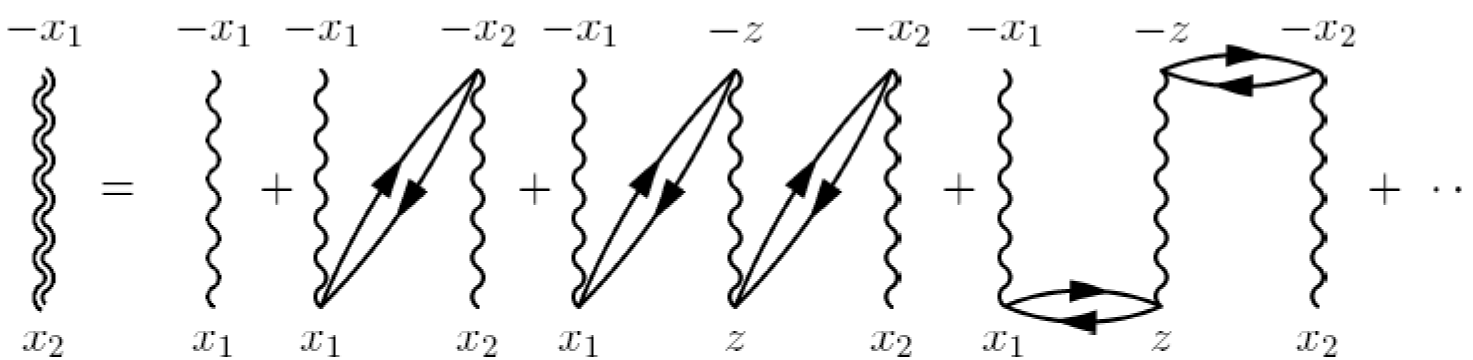
Each diagram of n th order has a leading scale dependent contribution

$$[\ln(\frac{T_0}{T})]^m, \quad m \leq n$$

Principal diagrams with linear logarithmic dependence are those with the **maximum number of loops;** they are independent of the cutoff scheme

The sum of these diagrams is obtained from a ladder summation:

$\bar{L}(x_1, x_2; \omega_n) =$



Conductance up to linear log-terms

Substituting L in place of the bare interaction into the first order diagrams, one finds the conductance

$$G^{(L)} = \frac{1}{4}(1-Y^2)T^2 \sum_{\varepsilon,\omega} \int_0^\infty dx_1 dx_2 dy \bar{L}(x_1, x_2; \omega) \mathbf{G}(-x + x_1, \varepsilon)$$

$$\times \mathbf{G}(-x_1 - x_2, \varepsilon - \omega) \mathbf{G}(x_2 - y, \varepsilon) \mathbf{G}(y + x, \varepsilon + \Omega)$$

Taking the limits $\Omega \to 0$ and $x >> v_F / T$

$$G^{(L)} = -\frac{g(1-Y^2)}{1+\sqrt{1-g^2}+gY} \ln(\frac{T_0}{T})$$

where we defined: $g = \frac{g_2}{2\pi v_F}$ and put v_F=1 .

Renormalization group approach

In perturbation theory the n-th order contribution is a polynomial in

$\Lambda = \ln(\frac{L}{a}) \simeq \ln(\frac{\varepsilon_0}{\varepsilon}) \simeq \ln(\frac{T_0}{T})$ of degree n

If the theory is renormalizable, all terms of higher powers in Λ should be generated by a renormalization group equation for the scaled conductance $G(\Lambda)$

We will use $Y(\Lambda) = 2G(\Lambda) - 1$ instead.

The beta-function is given by the prefactor of $\ln(T_0 / T)$ in the perturbation expansion of G:

$$\frac{dY}{d\Lambda} = -\frac{2g(1-Y^2)}{1+\sqrt{1-g^2}+gY} = \beta_L(Y)$$

Renormalization group equation

The RG-equation may be expressed in the symmetric form:

$$\frac{dY}{d\Lambda} = 2\left\{\frac{1}{(1+Y)(K^{-1}-1)} - \frac{1}{(1-Y)(K-1)}\right\}^{-1}$$

where $K = \sqrt{\dfrac{1-g}{1+g}}$ is the Luttinger parameter.

The RG-equation is invariant under $K^{-1} \leftrightarrow K, \quad Y \leftrightarrow -Y$

Solution of the renormalization group equation

The RG-equation may be integrated from $Y(T_0) = |t|^2 - |r|^2$ to $Y(T)$ to give

$$\frac{G^K}{1-G} = \left(\frac{T}{T_0}\right)^{2(1-K)} \frac{|t|^{2K}}{|r|^2}$$

Limiting cases:

High T: $$G(T) = |t|^2 + \frac{2(1-K)|t|^2|r|^2}{K|r|^2 + |t|^2} \ln\frac{T}{T_0}, \quad T < T_0$$

Low T:

Repulsive int., K<1 $$G = \left(\frac{T}{T_0}\right)^{2(K^{-1}-1)} \frac{|t|^2}{|r|^{2K^{-1}}}$$

Attractive int., K>1 $$G = 1 - \left(\frac{T}{T_0}\right)^{2(K-1)} \frac{|r|^2}{|t|^{2K}}$$

Agrees with Kane a. Fisher (cases |t|→1,0)
except that G→1 for K>1

Check of renormalizability

The renormalizability of the theory may be checked by comparing the terms with higher powers of Λ generated by expanding the solution of the RG equation with perturbation theory.

The expansion gives in 2nd and 3rd order:

$$G^{(2,2)} = -\frac{g^2}{8} Y(1-Y^2)\Lambda^2$$

$$G^{(3,3)} = \frac{g^3}{(3!)8}(1-Y^2)(1-3Y^2)\Lambda^3$$

$$G^{(3,2)} = -\frac{g^3}{32}(1-Y^2)(1-5Y^2)\Lambda^2$$

By using computer algebra to evaluate Feynman diagrams up to third order (more than 4000 diagrams) one finds agreement with the RG result,
except for additional terms ~ $(1-Y^2)^2 g^3\Lambda$ within the hard cutoff (T=0) scheme

β-function beyond ladder series

Feynman graphs, leading to lowest order linear logarithmic contribution in third order in g, beyond ladder series.

$$\frac{dY}{d\Lambda} = -\frac{2g(1-Y^2)}{1+\sqrt{1-g^2}+gY} + c_3 g^3 (1-Y^2)^2$$

Our evaluation at T=0, Λ = ln(L/a) : $c_3 = \pi^2/12$

Exact solution in S. Lukyanov, Ph. Werner (2007) :

$L=\infty$, Λ = ln($v_F/\pi a T$) (?) => $c_3 = 1/4$

β-function is not universal, but depends on cutoff scheme

Summary

- Calculated d.c. linear conductance G of Luttinger liquid with barrier in perturbation theory in g_2 as a function of length of interacting region and of temperature T

- Analyzed scale dependent terms of G(T): powers of $\Lambda = \ln \frac{T_0}{T}$, and summed up the principal terms linear in Λ

- Assuming (and checking to third order) renormalizability of the theory extracted the beta-function of the renormalization group equation for G

- Integrated the RG-equation to give G(T) for any g_2 or equivalently, Luttinger liquid parameter K, and any (narrow) potential barrier

- Comparing with exact solutions known from Thermodynamic Bethe Ansatz, clarified the meaning of the obtained solution as a robust, “universal”, part of Beta-function independent of the RG cutoff scheme

Thank you

Luttinger Hamiltonian

Define partial densities for incoming and outgoing particles:

$$\rho_{iR}(x) = \psi_1^\dagger(x)\psi_1(x), \qquad x<0$$
$$\rho_{iL}(x) = \psi_2^\dagger(x)\psi_2(x), \qquad x>0$$

$$\rho_{oR}(x) = [t\psi_1^\dagger(x) + \tilde{r}\psi_2^\dagger(-x)][t^*\psi_1(x) + \tilde{r}^*\psi_2(-x)], \qquad x>0$$
$$\rho_{oL}(x) = [r\psi_1^\dagger(-x) + \tilde{t}\psi_2^\dagger(x)][r^*\psi_1(-x) + \tilde{t}^*\psi_2(x)], \qquad x<0$$

Hamiltonian (free H in bosonized form):

$$H = H_0 + H_1$$

$$H_0 = \pi v_F \int_0^\infty dx[\rho_{iR}^2(-x) + \rho_{iL}^2(x) + \rho_{oR}^2(x) + \rho_{oL}^2(-x)]$$

$$H_1 = g_2 \int_0^\infty dx[\rho_{iR}(-x)\rho_{oL}(-x) + \rho_{oR}(x)\rho_{iL}(x)]$$

Hard cutoff scheme: a<x<L

Nonuniversality of β-function

Usual reasoning: higher order coefficients in beta-function depend on the cutoff scheme in (third loop and higher order)

In our problem, we used a hard cutoff scheme at T=0 and a soft cutoff at $T \neq 0$
Λ_0 = ln(L/a) vs. $\Lambda = \ln[\coth(2\pi aT)/\coth(2\pi LT)]$

1st diagram is linear in Λ => universal
2nd and 3rd diagrams contain both Λ^3 and Λ :
at finite T the coefficient in front of Λ is different !

Overall result : $\frac{\pi^2}{12}\Lambda_0$ is replaced by 0

"Non-universality" for observable quantity:
dependence on the problem setup!