NON-PERTURBATIVE RENORMALIZATION-GROUP APPROACH TO SUPERFLUIDITY

N. Dupuis - LPTMC Paris VI

K. Sengupta - Saha Institute of Nuclear Physics (India)

Bogoliubov theory: success...

Interacting bosons $H = H_0 + g \int d^d r \, \psi^+(r) \psi^+(r) \psi(r) \psi(r)$

Mean-field theory+Gaussian fluctuations: $\psi(r) = \psi_0 + \psi'(r), \quad n_0 = |\psi_0|^2$

$$S[\psi] = -\frac{1}{2} \sum_{q,\omega} (\psi^*, \psi) \begin{pmatrix} i\omega - \xi_q - 2g n_0 & -g n_0 \\ -g n_0 & -i\omega - \xi_{-q} - 2g n_0 \end{pmatrix} \begin{pmatrix} \psi \\ \psi^* \end{pmatrix}$$

Spectrum

$$E_q = \sqrt{\epsilon_q (\epsilon_q + 2g n_0)} \qquad \epsilon_q = \frac{q^2}{2m}$$

$E_q \sim c|q|$ Bogoliubov sound mode $\Rightarrow$ superfluidity

(Landau)

...and limitations

O(N) Model at low temperature $\langle \vec{\phi}(x) \rangle = \phi_0 \vec{u}$

Effective theory: NlsM

$$\vec{\phi} = \phi_0 \vec{n} \quad \text{with} \quad \vec{n}^2 = 1: \quad S[\vec{n}] \sim \phi_0^2 \int d^d x (\nabla \vec{n})^2$$

$$\vec{n} = (\sigma, \vec{\pi}), \quad \sigma^2 + \vec{\pi}^2 = 1$$

$$G_\perp(q) \sim \frac{1}{q^2} \quad \text{Goldstone mode}$$

$$G_\parallel(q) \sim \frac{1}{q^{4-d}} \quad (\ln q \quad \text{if} \quad d = 4)$$

The coupling between transverse and longitudinal fluctuations drives a singularity in the longitudinal susceptibility.

Same result expected for bosons

$$G_\perp(q,\omega) \sim \frac{1}{\omega^2+c^2q^2}$$ Goldstone mode

$$G_\parallel(q,\omega) \sim \frac{1}{(\omega^2+c^2q^2)^{(3-d)/2}}$$ $(\ln \text{ if } d=3)$

Hugenholtz-Pines (Goldstone) theorem: $\Sigma_{\mathrm{n}}(0,0)-\Sigma_{\mathrm{an}}(0,0)=\mu$

$$G_\parallel(0,0)=\frac{1}{\mu-\Sigma_{\mathrm{n}}(0,0)-\Sigma_{\mathrm{an}}(0,0)}=-\frac{1}{2\Sigma_{\mathrm{an}}(0,0)}$$

The divergence of the longitudinal susceptibility implies $\Sigma_{\mathrm{an}}(0,0)=0$

Nepomnyashchii Y. A. & A. A. (JETP Lett. 1975)

The Bogoliubov result $\Sigma_{\mathrm{an}}(q,\omega)=-g\,n_0$ breaks down in the infrared.

→ Infrared divergencies in the perturbation theory

Beyond Bogoliubov theory

- S. Beliaev (1958): perturbation theory for interacting bosons
- J. Gavoret & P. Nozières (1964): IR divergences
- A. Nepomnyashchy & Y. Nepomnyashchy (1975, 1978, 1979): Bogoliubov theory breaks down in the infrared: $\Sigma_{12}(0,0)=0$
- V. Popov & A.V. Seredniakov (1979): no IR divergence in density-phase representation $\psi=\sqrt{n}e^{i\theta}$
- RG approaches
  - P.B. Weichman (1988)
  - G. Benfatto (1994)
  - F. Pistolesi et al. (2004): RG+Ward identities
- Non-perturbative RG
  - C. Wetterich (PRB 2008)
  - N. Dupuis & K. Sengupta (Europhys. Lett. 2007)

NPRG approach to interacting bosons

Action: $$S=\int_0^\beta d\tau \int_x \left\{ \psi^* \left(\partial_\tau - \mu - \frac{\nabla^2}{2\mathrm{m}} \right) \psi + \frac{g}{2} \psi^* \psi^* \psi \psi \right\}$$

Ansatz: $$\phi(x,\tau)=\langle \psi(x,\tau) \rangle$$

$$\Gamma[\phi]=\int_0^\beta d\tau \int_x \left\{ \phi^* \left(Z Z_1 \partial_\tau - V \partial_\tau^2 - Z \frac{\nabla^2}{2\mathrm{m}} \right) \phi + \frac{\lambda}{2} (n-n_0)^2 \right\}$$

with $n_0 = |\phi^2|$ condensate density

Initial conditions = Bogoliubov theory $Z=Z_1=1, \quad V=0, \quad \lambda=g$

$$n_0=\frac{\mu}{g}$$

NB: a new term $V \phi \partial_\tau^2 \phi$ is generated by the RG (Wetterich, PRB 2008).

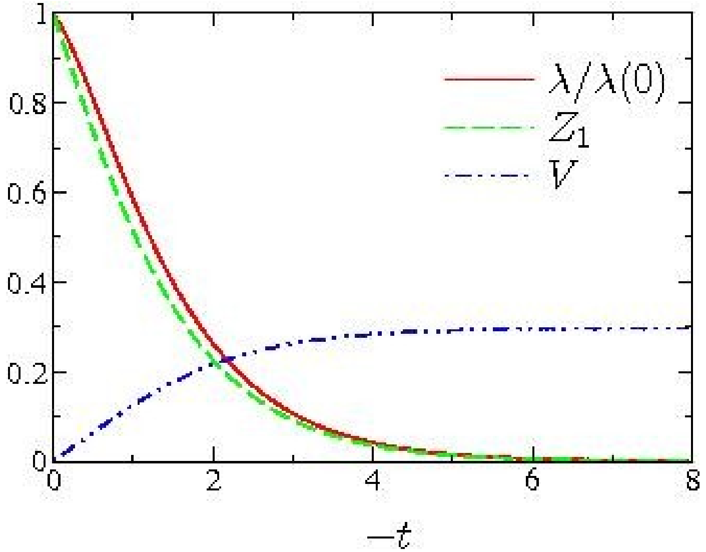
Inverse propagator in basis $(\phi_{1,}\phi_2)=(\Re\phi,\Im\phi)$ with $\bar{\phi}=(\sqrt{n_0},0)$

$$G^{-1}(q,\omega)=\Gamma^{(2)}(q,\omega)=\begin{pmatrix} V\omega^2+Z\epsilon(q)+2\lambda n_0 & ZZ_1\omega \\ -ZZ_1\omega & V\omega^2+Z\epsilon(q) \end{pmatrix}$$

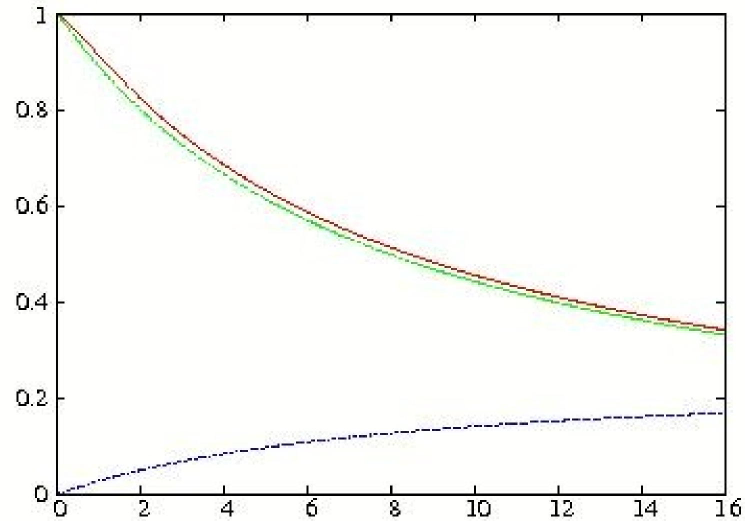
Superfluid density: $n_s=Zn_0$

Goldstone mode velocity: $c=\left(\frac{Z/2\mathrm{m}}{V+(ZZ_1)^2/(2\lambda n_0)}\right)^{1/2}$

d=2



d=3



ND, K. Sengupta (2007)

for $k \to 0$: $\lambda \to 0$, $\Sigma_{an}(0,0) = \lambda n_0 \to 0$

Emergence of SO(d+1) symmetry: $Z_1 \to 0, \quad V \to V^*$

(C. Wetterich)

$$\lim_{k\to 0} \Gamma[\phi] = \int d^{d+1}\tilde{x} \left\{ \tilde{\phi}^* \left(\partial_{\tilde{\tau}}^2 + \nabla_{\tilde{r}}^2 \right) \tilde{\phi} + \frac{\tilde{\lambda}'}{2} \left(\tilde{n} - \tilde{n}_0 \right)^2 \right\}$$

$$d=2: \quad \lambda, Z_1 \sim k$$

$$d=3: \quad \lambda, Z_1 \sim 1/\ln k$$

Bogoliubov theory remains valid in 3D for any finite k

Infrared behavior of the propagators
(strongly constrained by symmetries)

NPRG | **Bogoliubov theory**

$$G_{22}(q,\omega)=\frac{-2\mathrm{mc}^2 n_0}{n}\frac{1}{\omega^2+c^2q^2} \qquad -\frac{2\mathrm{mc}^2}{\omega^2+c^2q^2} \quad \text{Goldstone mode}$$

$$G_{12}(q,\omega)=\frac{mc^2}{n}\frac{dn_0}{d\mu}\frac{\omega}{\omega^2+c^2q^2} \qquad \frac{\omega}{\omega^2+c^2q^2}$$

$$G_{11}(q,\omega)=\frac{-1}{2\lambda n_0}\sim\frac{-1}{(\omega^2+c^2q^2)^{(3-d)/2}} \qquad -\frac{q^2/2\mathrm{m}}{\omega^2+c^2q^2} \quad \text{Longitudinal susceptibility}$$

(in agreement with Gavoret-Nozières (1964) for G_22 and G_12, Pistolesi et al. 2003)

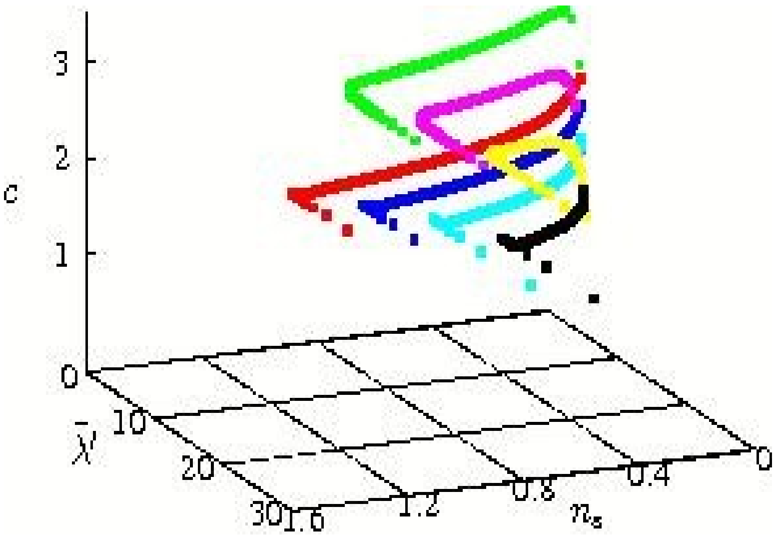
Why are these results exact?

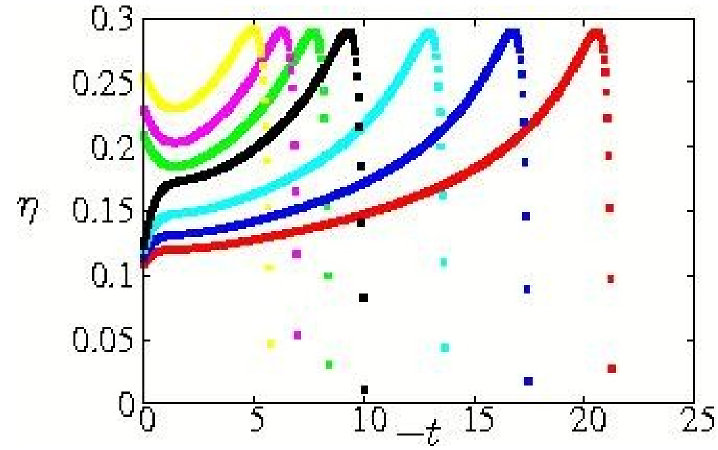
dimensionless coupling $\tilde{\lambda}' \to \tilde{\lambda}'^*$

$$\lambda=\tilde{\lambda}' Z^2 Z_1 \sqrt{Z_2}\, k^{2-d} \to \tilde{\lambda}'^* k^{3-d}$$

1D superfluids: superfluidity without BEC

Flow trajectories $(n_s, \tilde{\lambda}, c)$ for various initial conditions





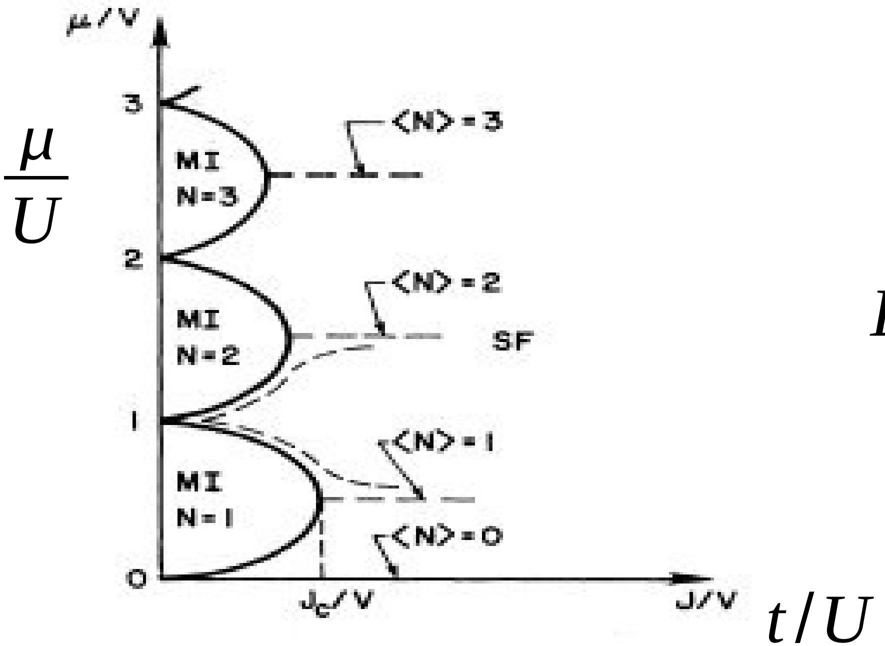
For a sufficiently small interaction, the trajectories rapidly hit a plane of quasi fixed points defined by $\tilde{\lambda} \sim 15$ where the running of n_s and c is very slow.

To be compared with exact solution of the Lieb-Liniger model:

$$\eta_{\max} = 1/2 \quad \text{for} \quad g \to \infty \quad \text{(hard core bosons)}$$

Towards an NPRG description of the superfluid-Mott transition in the Bose-Hubbard model?

$$H=-t\sum_{\langle i,j\rangle}\left(\psi_i^+\psi_j+\text{h.c.}\right)-\mu\sum_i n_i+\frac{U}{2}\sum_i n_i(n_i-1)$$



Fisher et al. (1989)

$$L_{\text{eff}}=r_0|\psi|^2+Z\,t\,|\nabla\psi|^2+\lambda|\psi|^4$$
$$+Z_1\psi^*\partial_\tau\psi+V\left|\partial_\tau\psi\right|^2$$

At the tip of the Mott lob: $Z_1=0 \;\Rightarrow\; z=1$ (XY model in d+1)

Away from the tip: $Z_1\neq 0 \;\Rightarrow\; z=2$

(NPRG for lattice models: ND & K. Sengupta, arXiv:0806.4257)

Conclusion

- NPRG solves the problem of the IR divergences in interacting boson systems. It provides an efficient way to go beyond Bogoliubov theory while satisfying Hugenholtz-Pines (Goldstone) theorem.
- Bogoliubov theory corresponds to the true fixed point only in dimension $d>3$.
- For $d<=3$: emergence of a new fixed point characterized by a space-time $SO(d+1)$ symmetry.
- 1D superfluidity (Luttinger liquid) well described for weak interaction.
- Non-trivial infrared behavior in the superfluid phase linked to the superfluid-Mott transition of lattice bosons (Bose-Hubbard model).