NONPERTURBATIVE RENORMALIZATION GROUP APPROACH TO FRUSTRATED MAGNETS

with **B. Delamotte, M. Tissier** (Univ. PARIS)

(B. Delamotte, D.M., M. Tissier (2004))

and with **Yu. Holovatch, D. Ivaneyko** (Univ. LVIV)

(B. Delamotte,Yu. Holovatch, D. Ivaneyko, D.M., M. Tissier (2008))

- a class of systems which:

1) have their own physical interest

(studied experimentally since ∼ 30 years)

2) constitute a good laboratory to test the relevance of the perturbative / nonperturbative RG ideas

since:

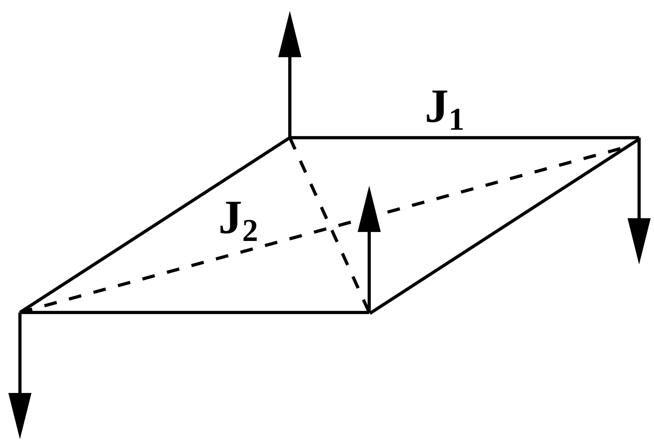
⋆ the associated field theory is the simplest variant of that of the well-controlled $O(N)/O(N-1)$ model

⋆ (almost) nothing works !

- motivation: effects induced by competing interactions on continuous spin systems

- ex: J_1-J_2 antiferro. model

$$H = \sum_{\langle i,j \rangle \; n.n.} J_1 \, \mathrm{S_i.S_j} + \sum_{\langle\langle i,j \rangle\rangle \; n.n.n.} J_2 \, \mathrm{S_i.S_j} \quad \text{with} \quad J_1, J_2 > 0$$



⇝ destabilization of order ? ⇝ "new" physics

- HT_C: 2d Copper oxydes (ex: $La_{2-\delta}Sr_{\delta}CuO_4$)

Hubbard : $$H = -t \sum_{\langle i,j \rangle, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} \, n_{i\downarrow}$$

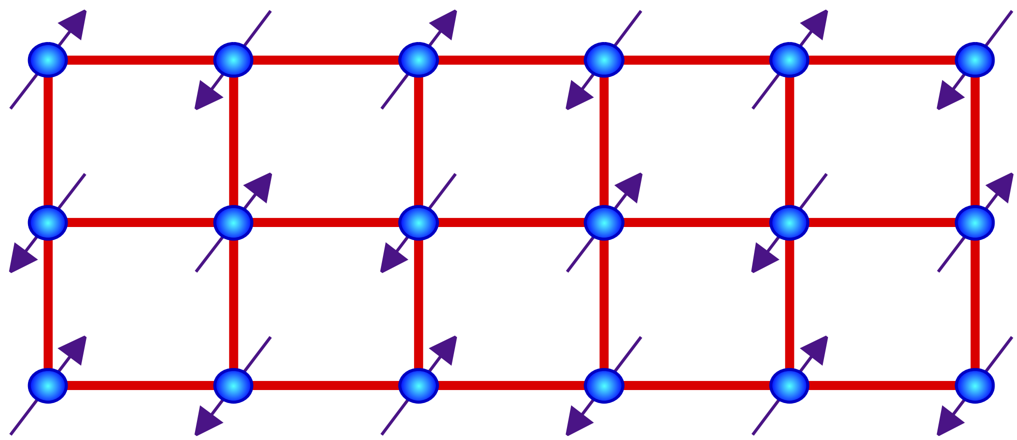
$$\delta \ll 1 \quad \Downarrow \quad \int \textbf{charge}$$

$J_1 - J_2$: $$H = \sum_{\langle i,j \rangle} \mathbf{J_1} \, \mathbf{S}_i . \mathbf{S}_j + \sum_{\langle\langle i,j \rangle\rangle} \mathbf{J_2} \, \mathbf{S}_i . \mathbf{S}_j + \ \ldots$$

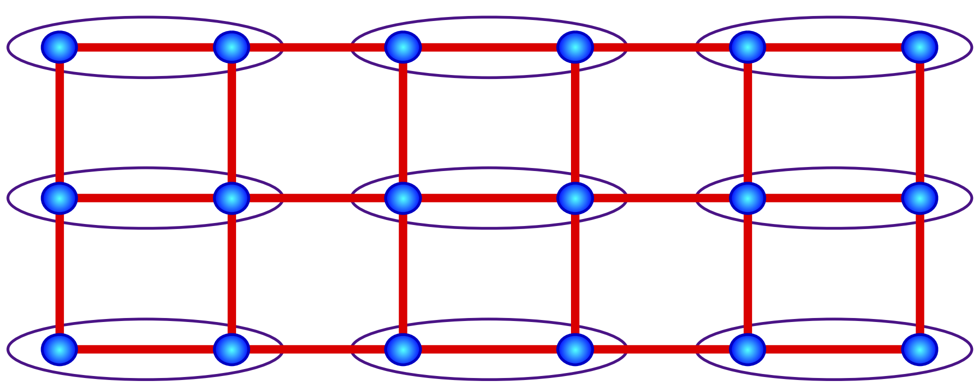
(M. Inui, S. Doniach, et M. Gabay (1988))

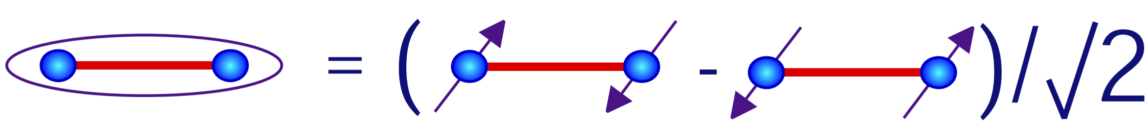
- quantum systems at T=0

$J_2 = 0 \rightsquigarrow$ **Néel (ordered)** phase:



$J_2 \neq 0 \rightsquigarrow$ magnetically **disordered** phase ?:



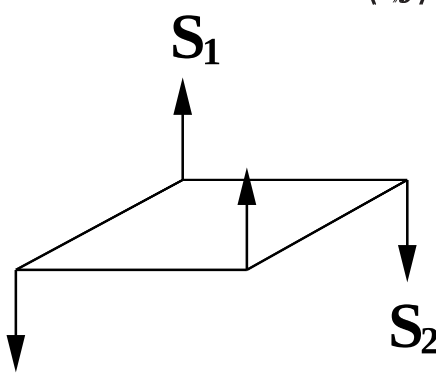


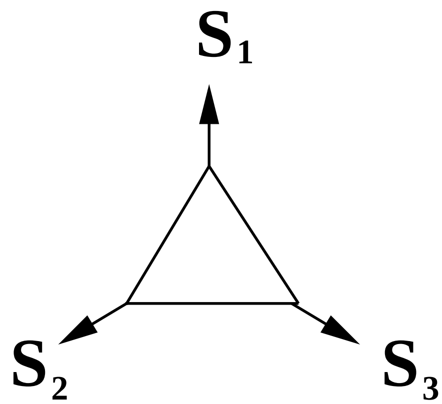
$\rightsquigarrow$ precursor of **HT_c** superconductivity ?

(P.W Anderson (1987))

- prototype: classical Heisenberg "AFT"

$$H = \sum_{\langle i,j \rangle} J \ \mathrm{S_i . S_j} \qquad J > 0$$





geometrical frustration ⇒ noncollinear ground state

$$\Longrightarrow \boxed{\frac{G}{H} \neq \frac{O(N)}{O(N-1)} \text{ (ferromagnet)}}$$

- even at the classical level, situation confused:

  ⋆ nature of the phase transition ambiguous:
  1^{st} or 2^{nd} order ?

  ⋆ all perturbative approches fail
  although the effective field theory: ϕ^4-like !

some phenomenology

(B. Delamotte, D.M., M. Tissier (2004))

- scaling laws in XY and Heisenberg cases (materials and numerically simulated systems)

but:

- critical exponents for ≠ materials and simulated systems: incompatibles

  ex: XY Tb: $\beta \sim 0.237(4)$ Ho, Dy: $\beta \sim 0.389(7)$

  $\Longrightarrow$ against universality

- violation of scaling relations $\gamma + 2\beta - 3\nu = 0$

  ex: $\gamma + 2\beta - 3\nu = 0.202(92)$ (XY)

- $\eta < 0$ for materials and simulated systems

  ex: $\eta = -0.141(14)$ (XY) $\eta = -0.118(25)$ (H)

  $\Longrightarrow$ against 2^{nd} order

- $\exists$ 1^{st} order : in all recently simulated models with MC and MCRG but one

  (P. Calabrese, P. Parruccini, A. Pelissetto and E. Vicari (2004))

symmetries

N-components order parameter: $O(N) \equiv$ maximal symmetry

if it is not the case:

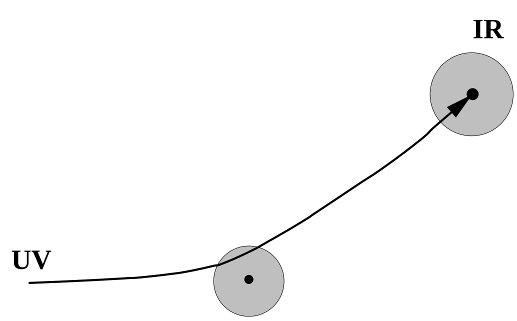
1) structure of the RG flow

(perturbatively in ϵ)

$O(N)$: 1 coupling $\Longrightarrow$ 1 FP in the RG flow

$\neq O(N)$: p coupling $\Longrightarrow$ $2^{p}-1$ FPs

- ⋆ annihilation of FPs
- ⋆ 1^{st} order induced by fluctuations
- ⋆ scattering on FPs



- ⋆ focus FPs, limit cycles, ...

$\rightsquigarrow$ criticality and universality not guaranteed

2) structure and properties of the perturbative series

generically: the renormalized series diverge

$$f(g_r) = \sum_n a_n \, g_r^n \quad \text{with} \quad \boxed{a_n \underset{n\to\infty}{\propto} n! \; n^b (-a)^n}$$

$\Longrightarrow$ you have to resumm the series

⋆ for one coupling constant: it's ok

– within a LGW approach

- in $d = 2, 3$

  (Borel summability rigorously proven)

- ϵ expansion

  (Borel summability expected)

⇝ not within a NLσ model approach

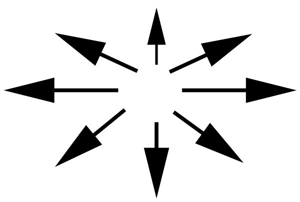
⋆ for several coupling constants: not clear at all

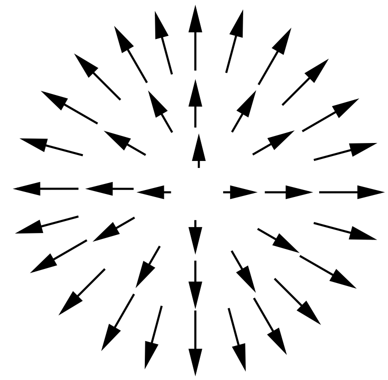
⇝ only recipes that work in the $O(N)$ case

3) nature of the relevant excitations

⋆ spin-waves

⋆ topological excitations: vortices, monopoles in d=2,3





⇝ major role in:

⋆ $O(2)$ case in $d = 2$ (Kosterlitz-Thouless transition)

$\Longrightarrow$ correctly described when adding vortices

$\Longrightarrow$ nonperturbative / T

⋆ $O(2)$ case in $d = 3$: vortex strings necessary
(G. Kohring, R.E. Shrock and P. Wills (1986))

$\Longrightarrow$ correctly described within LGW approach

⋆ not so clear in $O(3)/O(2)$ in d=3
(M. Lau and C. Dasgupta (1989))

$\Longrightarrow$ what about models $\neq O(N)$?

EFFECTIVE FIELD THEORY

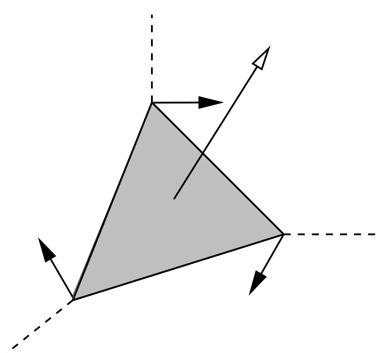
- **Heisenberg** case

unfrustrated spins: ground state specified by **2** angles

$\rightsquigarrow$ symmetry breaking scheme: $O(3) \longrightarrow O(2)$

$\Longrightarrow$ **2** Goldstone modes

frustrated spins: ground state specified by **3** angles



$\rightsquigarrow$ symmetry breaking scheme: $SO(3) \longrightarrow \mathbb{1}$

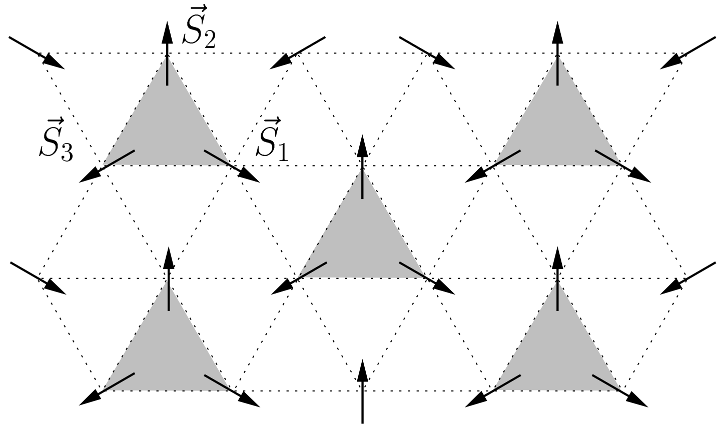
$\Longrightarrow$ order parameter $\equiv$ **matrix**

$\Longrightarrow$ **3** Goldstones modes

- N components: $\boxed{\dfrac{G}{H} \equiv \dfrac{O(N)}{O(N-2)}}$ $\sim$ "Stiefel manifold"

$\Longrightarrow$ $2N-3$ Goldstones modes

• Low energy effective field theory ?



• order parameter ?

⋆ "naïve" magnetization: $\Sigma^I = S_1^I + S_2^I + S_3^I = 0$ at $T = 0$

⋆ "staggered" magnetization:

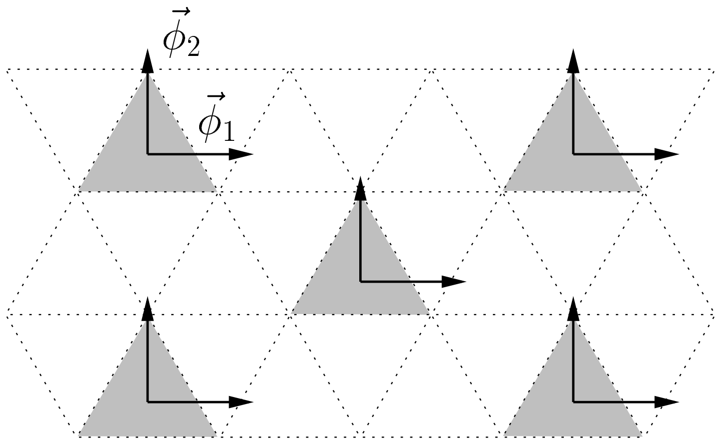
- square antiferro.: $\phi^I = S_1^I - S_2^I$

- triangular antiferro.:

$$\begin{cases} \phi_1^I = \dfrac{1}{\sqrt{3}}\left[S_1^I - S_3^I\right] \\ \phi_2^I = S_2^I \\ \phi_3^I = \phi_1^I \times \phi_2^I \end{cases}$$

$(\phi_1^I, \phi_2^I, \phi_3^I) \equiv$ orthonormal triedral $\equiv$ matrix $R \in SO(3)$

- effective interaction between ϕ_i^I and ϕ_j^J: ferro.



$$H = -J \sum_{\langle I,J \rangle} \left(\phi_1^I . \phi_1^J + \phi_2^I . \phi_2^J \right) \Longrightarrow H = \frac{1}{2} \int d^d\mathbf{r} \left\{ (\partial \phi_1)^2 + (\partial \phi_2)^2 \right\}$$

$$\boxed{\mathcal{Z} = \int \mathcal{D}\phi_{1,2} \prod_{i \leq j} \delta(\phi_i . \phi_j - \delta_{ij}) \exp \left\{ -\frac{1}{2T} \int d^d\mathbf{r} \left\{ (\partial \phi_1)^2 + (\partial \phi_2)^2 \right\} \right\}}$$

- $R = (\phi_1, \phi_2, \phi_3) \Longleftrightarrow SO(3)$ matrix

$$\boxed{Z = \int \mathcal{D}R \; \delta({}^tR\, R - \mathbb{1}) \exp \left\{ -\frac{1}{2T} \int d^d\mathrm{r} \; \mathrm{Tr} \left(P \, \partial \, {}^tR \, \partial R \right) \right\}}$$

with $P = diag(p_1, p_2 = p_1, 0)$

$$\Longrightarrow \boxed{\frac{G}{H} = \frac{O(N) \times O(2)}{O(N-2) \times O(2)}}$$

• homotopy properties: $\Pi_1[SO(3)] = \mathbf{Z}_2$

⋆ for $R^I \in SO(3)$:

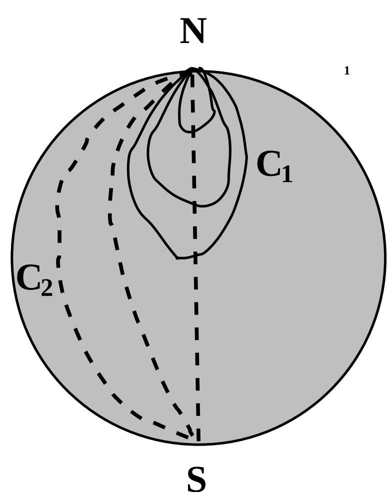
$$R^I_{kl} = 2\left(S^I_k S^I_l - \frac{1}{4}\delta_{kl}\right) + 2\epsilon_{klm} S^I_0 S^I_m + 2\left({S^I_0}^2 - \frac{1}{4}\right)\delta_{kl}$$

with $S^I = (S^I_0, S^I_1, S^I_2, S^I_3) \in S^3 \equiv \frac{SO(4)}{SO(3)}$

$$H = -\frac{1}{2T}\sum_{<I,J>} \mathbf{Tr}\,({}^tR_I.R_J) \propto -\frac{1}{2T}\sum_{<I,J>} (\mathbf{S^I}.\mathbf{S^J})^2$$

$\Longrightarrow$ $\mathbf{Z}_2$ gauge invariance: $S^I \rightarrow -S^I$

$\simeq$ nematic liquid crystal $\in \mathrm{RP}_3 \equiv \frac{S_3}{\mathbf{Z}_2}$

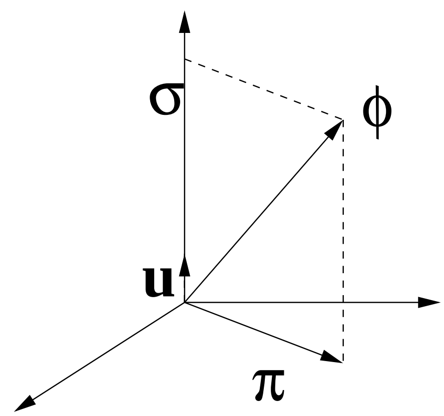


$\Longrightarrow$ spectrum: 3 Goldstone modes + $\mathbf{Z}_2$ vortices

PERTURBATIVE APPROACHES

- low-T expansion in d=2+ϵ: NLσ model

$$\phi(\mathbf{r}) = \sigma(\mathbf{r})\mathbf{u} + \boldsymbol{\pi}(\mathbf{r}) \quad \textbf{with} \quad \sigma^2 + \boldsymbol{\pi}^2 = 1$$



$$\Longrightarrow \boxed{\mathcal{Z} = \int_{|\pi|\leq 1} \mathcal{D}\pi \exp\left\{-\int d^d\mathbf{r}\ g_{ij}[\pi]\ \partial_\mu \pi^i \partial_\mu \pi^j\right\}}$$

with $g_{ij}[\pi]$: metric on the manifold G/H:

$$g_{ij}[\pi] = \delta_{ij} + \frac{T\ \pi_i \pi_j}{1 - T\boldsymbol{\pi}^2}$$

- β functions in d=2+ϵ:

$$\boxed{\beta_{ij}(g) = \frac{\partial g_{ij}}{\partial l} = \epsilon g_{ij} - \frac{1}{2\pi} R_{ij} - \frac{1}{8\pi^2} R_i^{pqr} R_{jpqr}}$$

R_{ij} and R_i^{pqr} $\equiv$ Ricci and Riemann tensors on G/H

(D.H. Friedan (1985))

$\Longrightarrow$ β function geometric, insensitive to the topology

$$\boxed{\frac{O(3)\times O(2)}{O(2)} \text{ model ?}}$$

- RG analysis: a third axe is generated

$\rightsquigarrow$ $\exists$ fixed point in d=2+ϵ with 3 equivalent axes

$$\Longrightarrow \boxed{\text{enlarged } O(3)\times O(3) \text{ symmetry}}$$

$$\Longrightarrow \boxed{\text{symmetry breaking scheme: } \frac{O(3)\times O(3)}{O(3)} \sim \frac{O(4)}{O(3)} \;!}$$

Heisenberg AFT $\Longleftrightarrow$ 4 components collinear spins
(P. Azaria, B. Delamotte, T. Jolicoeur (1990))

$O(4)/O(3)$ behaviour in d=3 ?

- $\nu_{th} = 0.74 \neq \nu_{exp} \in [0.585(9) - 0.62(5)]$

- $\exists$ anomaly in the NLσ model approach in d>2:

  $\rightsquigarrow$ topological content not considered within the low-T expansion

• weak coupling expansion in d=4 − ϵ: LGW model

$$\prod_{i\le j}\delta(\boldsymbol{\phi}_i.\boldsymbol{\phi}_j-\delta_{ij}) \Longrightarrow e^{-V}$$

$$V=\int d^d\mathbf{r}\left\{\frac{r}{2}\left(\phi_1^2+\phi_2^2\right)+\lambda\left(\phi_1^2+\phi_2^2\right)^2-\mu\left(\phi_1\times\phi_2\right)^2\right\}$$

$\Longrightarrow$ enforces the orthonormality of ϕ_1 and ϕ_2 at $T<T_c$

$\rightsquigarrow$ spectrum at $T<T_c$: $2N-3$ Goldstone modes

3 massive modes

• role of $\mathbf{Z}_2$ topological excitations ?

⋆ invalidate the NLσ model approach in $d=2$ at $T=T_V>0$

(M.Caffarel, P.Azaria, B.Delamotte and D.M. (2001))

⋆ role in $d=3$: not established

$\rightsquigarrow$ partially taken into account within the LGW model approach

- $O(N) \times O(2)$ model at one loop: $\exists\ \mathbf{N_c} = 21.8 + O(\epsilon)$

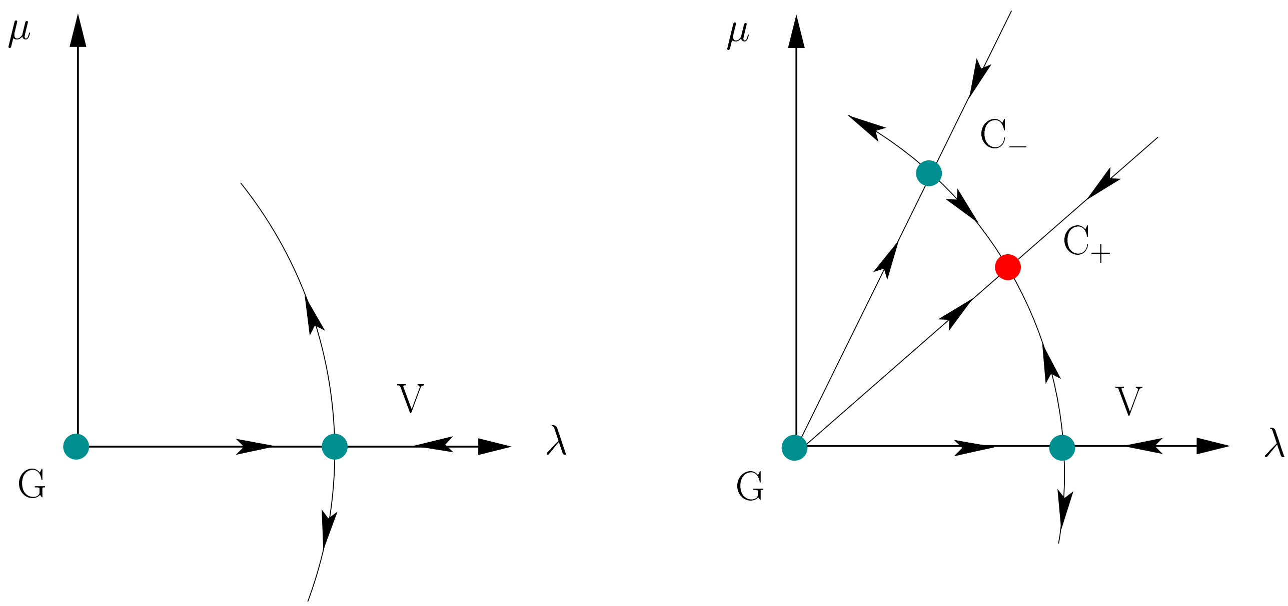
⋆ $N < N_c$: Gaussian and $O(2N)$ **unstables**

$\Longrightarrow$ **1^{st} order**

⋆ $N > N_c$: Gaussian, $O(2N)$, C_-, **unstables**

and C_+ **stable**

$\Longrightarrow$ **2^{nd} order** $\neq O(N)/O(N-1)$



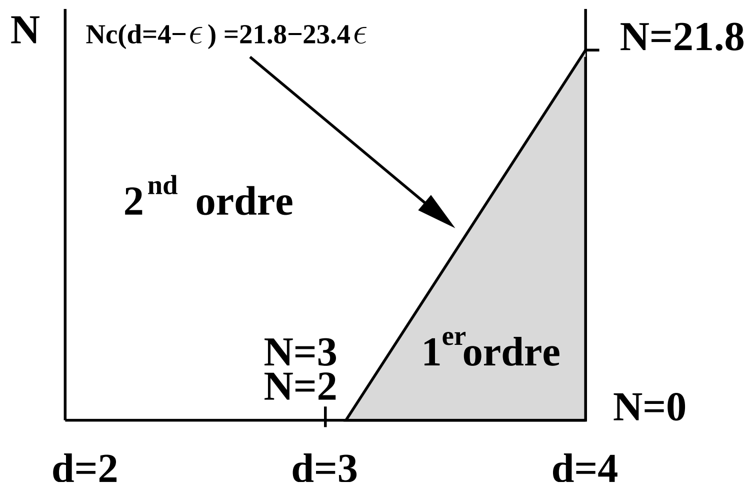
(a) N≤N_c (b) N>N_c

⋆ **N=N$_c$**: **annihilation** of C_+ with C_-

N$_c$(d=3) or line N$_c$(d) ?

ϵ-expansion

- two loops in d=4 - ϵ: $N_c(4-\epsilon)=21.8 - 23.43\,\epsilon + O(\epsilon^2)$

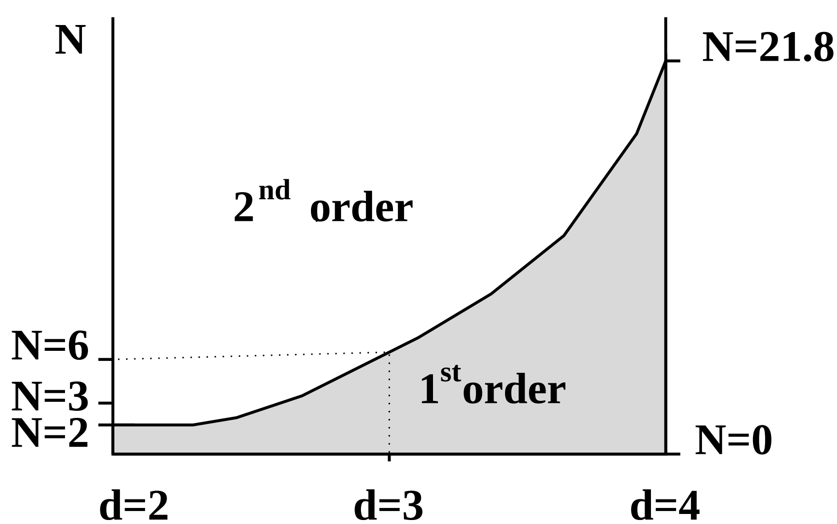


2^{nd} order for N=2, 3 $\Longrightarrow$ new universality class ?

- five loops in d=4 - ϵ: $N_c(4-\epsilon)= 21.8-23.43\,\epsilon+7.09\,\epsilon^2-0.03\,\epsilon^3+4.26\,\epsilon^4$

(P. Calabrese and P. Parruccini (2004))

$\Longrightarrow$ $N_c(d=3) \simeq$ 5-6



1^{st} order for N=2,3 ? $\Longrightarrow$ scaling laws ?

Fixed dimensional perturbative analysis

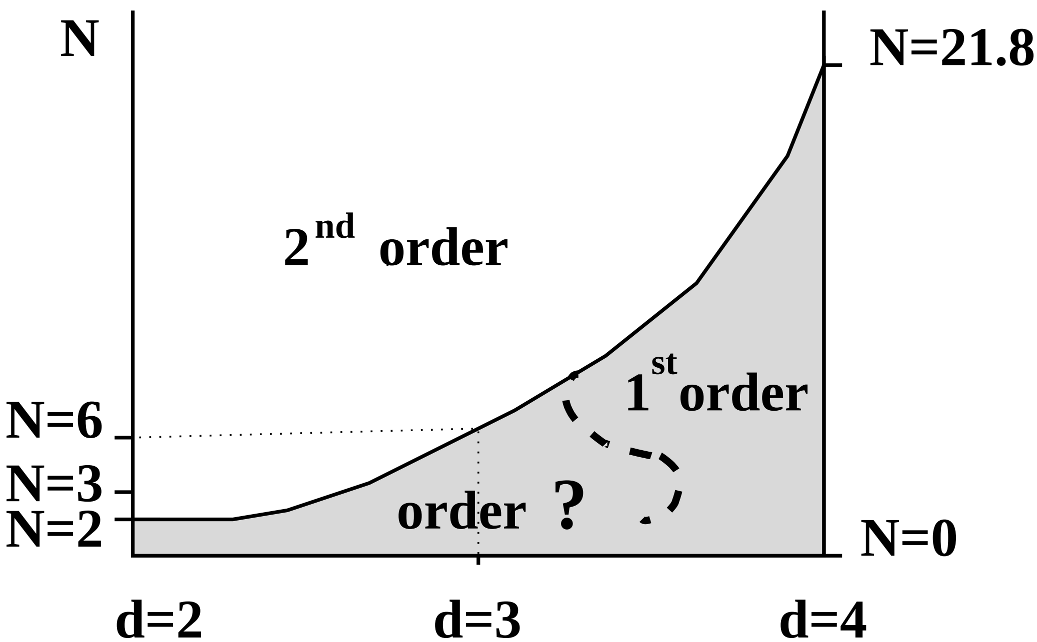
- 6 loops massive scheme in d=3

(A. Pelissetto, P. Rossi and E. Vicari (2001)
P. Calabrese, P. Parruccini and A.I. Sokolov (2003))

- 5 loops in $\overline{\mathrm{MS}}$ without ϵ expansion

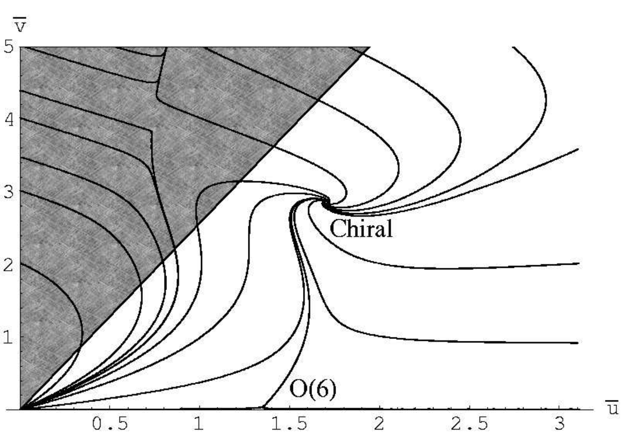
(P. Calabrese, P. Parruccini, A. Pelissetto and E. Vicari (2004))

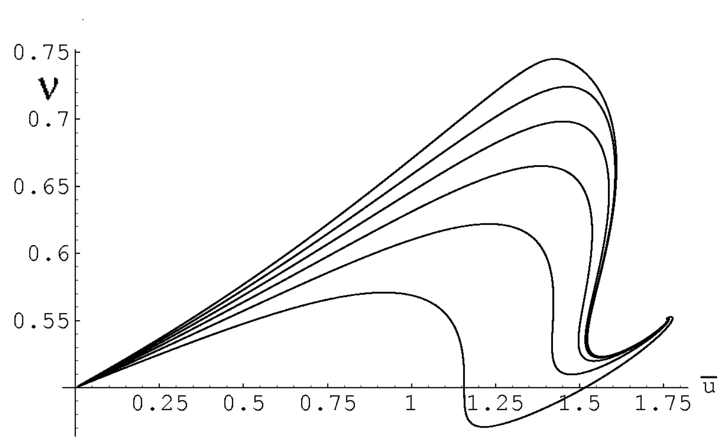
stable fixed point in the XY and Heisenberg cases !



- nonuniversal scaling laws ⟹ focus fixed point

(P. Calabrese, P. Parruccini and A.I. Sokolov(2003))





⟹ effective varying critical exponents

NONPERTURBATIVE APPROACH

(B. Delamotte, D. M., M. Tissier (2000, 2003, 2004))

- running effective action Γ_k

(C. Wetterich (1993))

- motivation: - to reconcile NLσ and LGW approaches
  - to reproduce the physics in d=3

- need for Γ_k ?

ex: minimal truncation O(N) model:

$$\Gamma_k[\phi] = \int d^d\mathbf{r} \left\{ \frac{Z}{2}(\partial\boldsymbol{\phi})^2 + \frac{1}{2}\lambda(\rho - \rho_0)^2 \right\} \quad \text{with} \quad \rho \sim \phi^2$$

$$\begin{cases} \partial_t \rho_0 = -(d-2)\,\rho_0 + 2v_d\,(N-1)\, l_1^d(0) + 6v_d\; l_1^d(2\lambda\rho_0) \\ \partial_t \lambda = (d-4)\,\lambda + 2v_d\,(N-1)\,\lambda^2\, l_2^d(0) + 18v_d\,\lambda^2\, l_2^d(2\lambda\rho_0) \end{cases}$$

- $l_n^d(w)$: – nonperturbative in λ, T (NLσ coupling)
  – arguments: masses $m_1 = 0$ and $m_2 = 2\lambda\rho_0$

$$\boxed{l_n^d(w) \propto w^{-n-1}} \quad \text{for} \quad w \gg 1 \quad \text{(decoupling)}$$

$\Longrightarrow$ interpolation between LGW (d=4) and NLσ (d=2)

$\Longrightarrow$ correct description of the physics in d=3

- $O(N) \times O(2)$ at order ∂^2:

$$\Gamma_k = \int d^d\mathbf{r} \Big\{ U_k(\rho,\tau) + Z_k(\rho,\tau)\Big((\partial\boldsymbol{\phi}_1)^2 + (\partial\boldsymbol{\phi}_2)^2\Big) +$$
$$Y_k^{(1)}(\rho,\tau)\Big(\boldsymbol{\phi}_1\cdot\partial\boldsymbol{\phi}_2 - \boldsymbol{\phi}_2\cdot\partial\boldsymbol{\phi}_1\Big)^2 + Y_k^{(2)}(\rho,\tau)\Big(\boldsymbol{\phi}_1\cdot\partial\boldsymbol{\phi}_1 + \boldsymbol{\phi}_2\cdot\partial\boldsymbol{\phi}_2\Big)^2 +$$
$$Y_k^{(3)}(\rho,\tau)\Big[\Big(\boldsymbol{\phi}_1\cdot\partial\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2\cdot\partial\boldsymbol{\phi}_2\Big)^2 + \Big(\boldsymbol{\phi}_1\cdot\partial\boldsymbol{\phi}_2 + \boldsymbol{\phi}_2\cdot\partial\boldsymbol{\phi}_1\Big)^2\Big]\Big\}$$

with

$$\begin{cases} \rho = \mathbf{Tr}({}^t\Phi.\Phi) \\ \tau = \frac{1}{2}\,\mathbf{Tr}\left({}^t\Phi.\Phi - \frac{\mathbb{1}}{2}\rho\right)^2 \\ \Phi = (\boldsymbol{\phi}_1, \boldsymbol{\phi}_2) \end{cases}$$

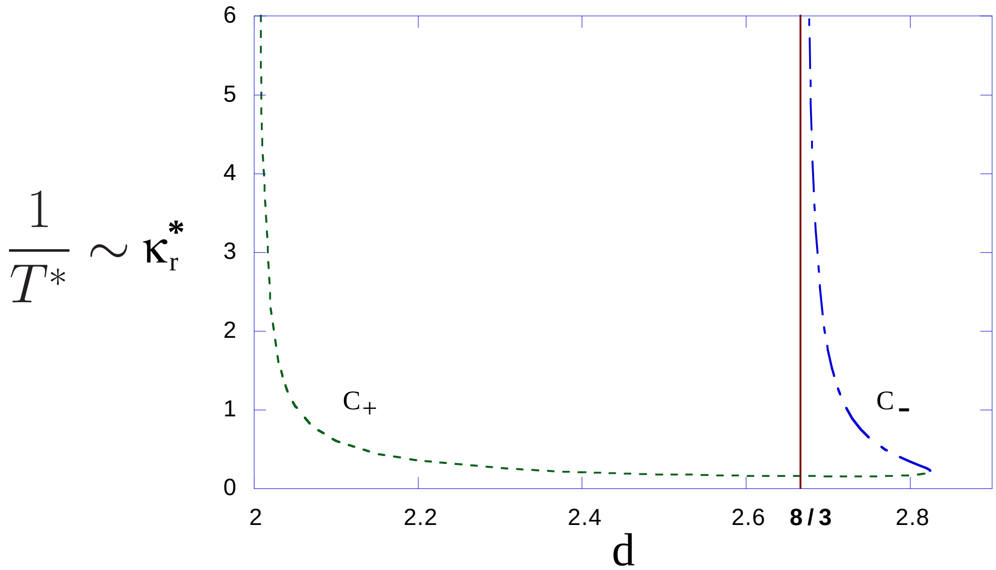
- all terms compatible with the symmetries up to order ∂^2 and ϕ^{10}

(M. Tissier, B. Delamotte et D.M. (2000-2004))

- d=4 - $\epsilon \Longrightarrow$ N_c(d=4 - ϵ) of LGW model

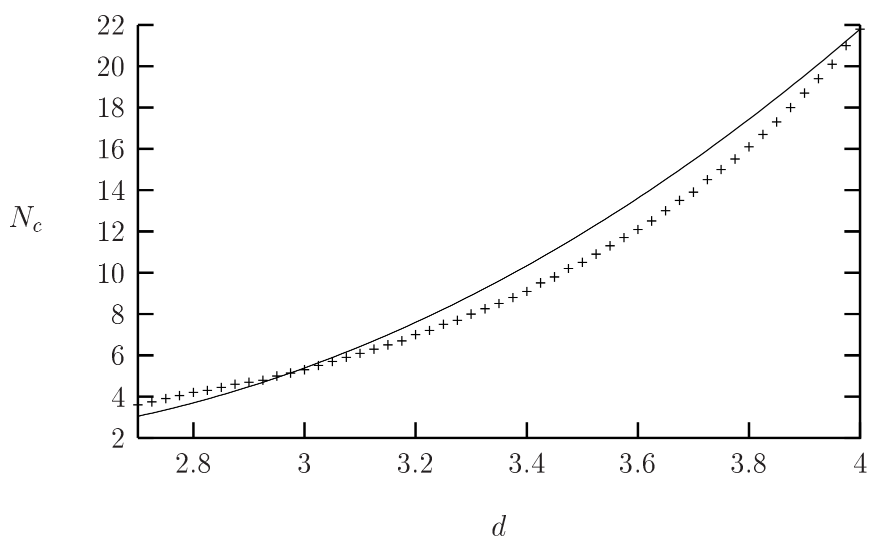
- d=2 + $\epsilon \Longrightarrow$ fixed point $O(4)/O(3)$ of NLσ model !

- 2<d<4 ?



$\Longrightarrow$ annihilation of C_+ with C_-

- $N_c(d)$?

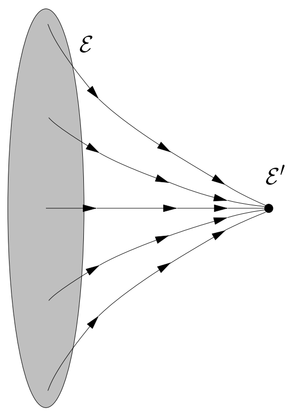


$\Longrightarrow N_c(3) \simeq 5.3$ +++ : NPRG —— : $4 - \epsilon$

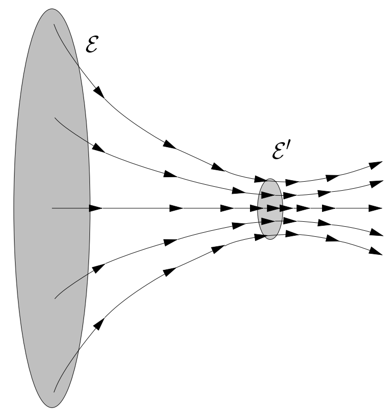
• no fixed point in d=3 for XY and Heisenberg spins

$\Longrightarrow$ how to explain the scaling behaviour ?

• qualitative RG flow around N_c:



(c) $N > N_c$



(d) $N \lessapprox N_c$

flow stops at $\mathcal{E}'$

$\Downarrow$

$\mathcal{E}'$: fixed point

$\Downarrow$

2^{nd} order

$\Downarrow$

scaling + universality

slow RG flow within $\mathcal{E}'$

$\Downarrow$

$\mathcal{E}'$: pseudo-fixed region

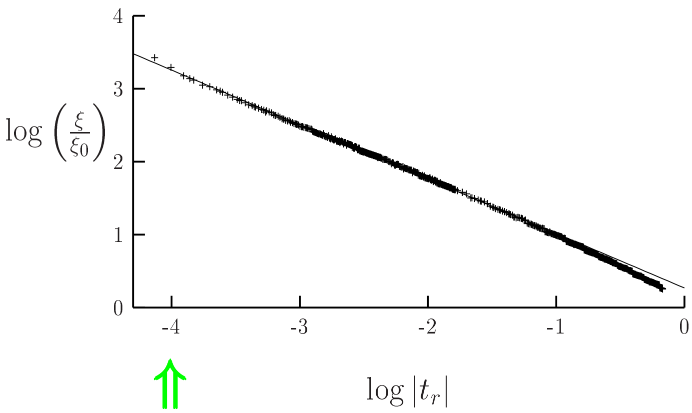
$\Downarrow$

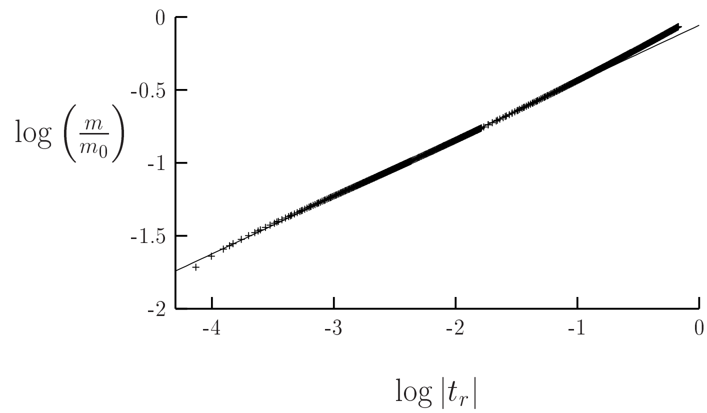
generic weak 1^{st} order

$\Downarrow$

pseudo-scaling
without
universality

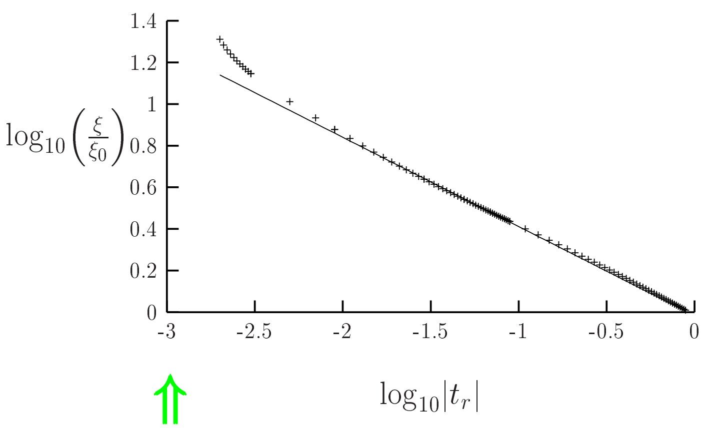
• **Heisenberg**

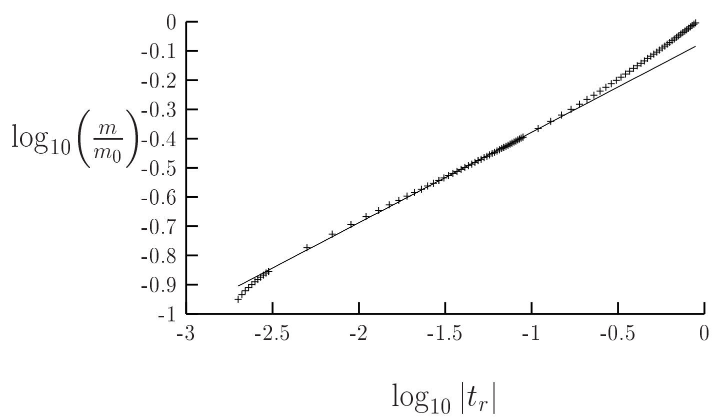




$$\beta \in [0.27 - 0.42] \text{ and } \nu \in [0.56, 0.71]$$

• **XY**





$$\beta \in [0.25 - 0.38] \text{ and } \nu \in [0.47, 0.59]$$

⋆ **agreement theory − experiment ?**

$$\begin{cases} \textbf{RG} : \beta = 0.38, \nu = 0.58, \gamma = 1.13 \\ \textbf{Ho} - \textbf{Dy} : \beta = 0.389(7), \nu = 0.558(25), \gamma = 1.10(5) \end{cases}$$

Nonperturbative vs fixed-d perturbative approach

discrepancy between:

- NPRG (and weak-coupling with ϵ-expansion)

$\Downarrow$

weak first order with (pseudo-)scaling

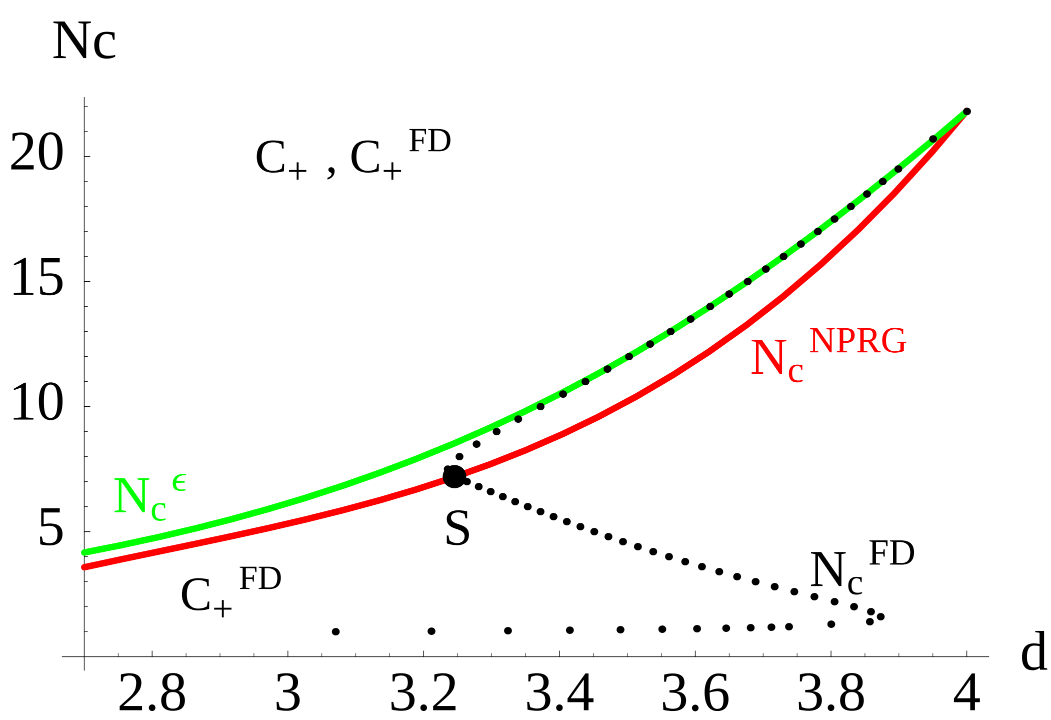
AND

- perturbative "FD approach"

  - $\overline{\text{MS}}$ scheme without ϵ-expansion
  - massive scheme in d=3

$\Downarrow$

second order behaviour without universality



$\hookrightarrow$ FD perturbative approach

(B. Delamotte, Yu. Holovatch, D. Ivaneyko, D.M., M. Tissier (2008))

ex: $\overline{\text{MS}}$ scheme

$$O(N) \text{ model: } \beta(u) = -u\left(\epsilon - u + \frac{3(3n+14)}{(n+8)^2}\, u^2 + \ldots\right)$$

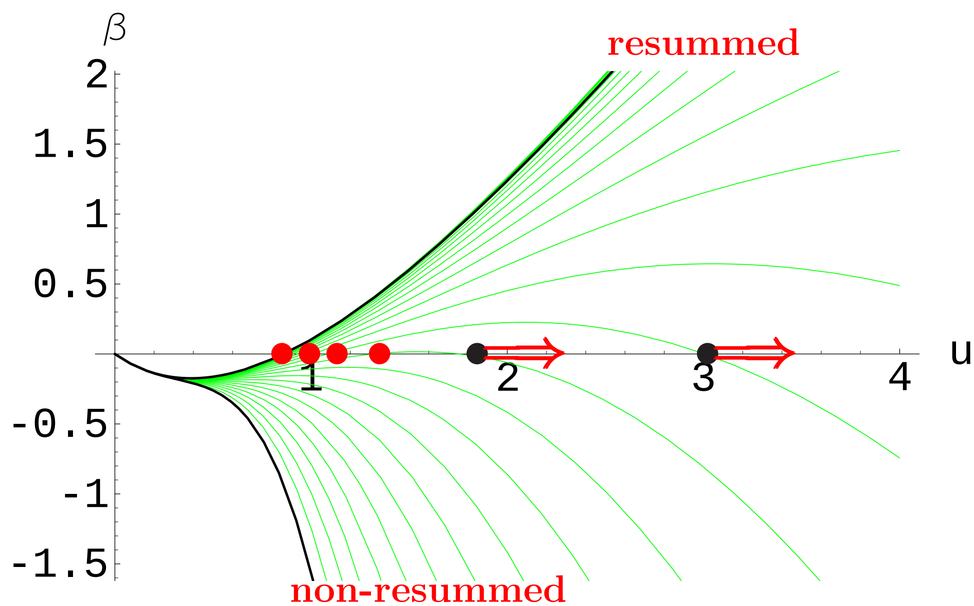
- within ϵ-expansion: two roots: $u^* = 0$ and $u^* \sim \epsilon$
- without ϵ-expansion: at order L $\Longrightarrow$ L roots $\in \mathbb{C}$!

$\Longrightarrow$ several solutions or no solution at all

How to cure this problem ?

$\Longrightarrow$ you have to resum the divergent β-functions

ex: $O(3)$ model in d=3 at 4 loops



•: Wilson-Fisher FP   •: "spurious" FP

$\hookrightarrow$ possibly more involved with several couplings

$\Longrightarrow$ spurious FPs even after resummation ?

<u>in practice</u>: series $f(u_1, u_2) \Longrightarrow$ series in u_1:

$$f\,(u_1, z = u_2/u_1) = \sum_n f_n(z)\; u_1^n$$

with: $f_n(z) \underset{n\to+\infty}{\sim} n!\;(-a(z))^n\; n^b$

Borel-Leroy sum: $\quad B(u_1, z) = \sum_n \frac{f_n(z)}{\Gamma[n+b+1]}\; u_1^n$

$\Longrightarrow$ resummed expression:

$$f_R(u_1, z) = \int_0^\infty dt\; e^{-t}\, t^b\; B^{AN}(u_1\, t, z)$$

with $B^{AN}(u_1, z) \equiv$ analytic c. of $B(u_1, z)$ for $u_1 > 1/a$

$\hookrightarrow$ conformal mapping: $\omega(u; z) = \frac{\sqrt{1+a(z)\,u}-1}{\sqrt{1+a(z)\,u}+1}$

$$f_R(u_1, z) = \sum_n d_n(\alpha, a(z), b; z) \int_0^\infty dt\; \frac{e^{-t}\, t^b\, [\omega(u_1 t; z)]^n}{[1-\omega(u_1 t; z)]^\alpha}$$

with α given by strong coupling: $f(u_1, z) \underset{u_1\to+\infty}{\sim} u_1^{\alpha/2}$

(J.C. Le Guillou, J. Zinn-Justin (70's, 80's))

$\exists\ d^\circ$ of freedom: $a(z)$ $\sim$ known; b, α: free parameters

$\Longrightarrow$ best apparent convergence

(J.C. Le Guillou, J. Zinn-Justin (70's, 80's))

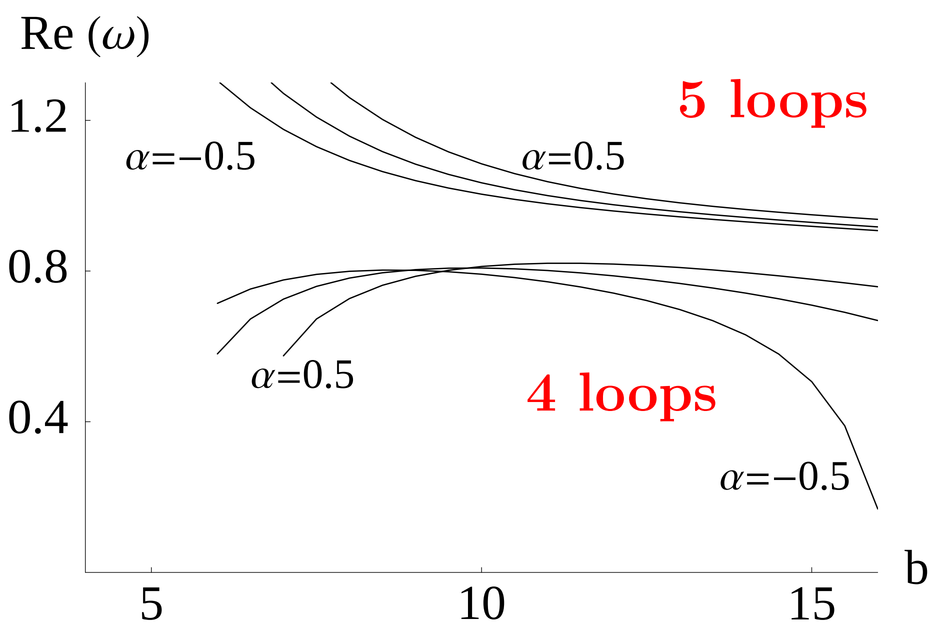
$\Longrightarrow + \sim$ principle of minimal sensitivity

(A.I. Mudrov and K.B. Varnashev (1998))

- α that minimizes $|Q(\alpha,b,L+1) - Q(\alpha,b,L)|$
- b such that $Q(\alpha,b,L)$ stationary

frustrated magnets in the $\overline{\mathrm{MS}}$ scheme without ϵ-exp.

(B. Delamotte, Yu. Holovatch, D. Ivaneyko, D.M., M. Tissier (2008))



$\Longrightarrow \sim$ apparent convergence but with strong error:

100 times larger than for Ising !

$\Longrightarrow$ only a problem of convergence ?

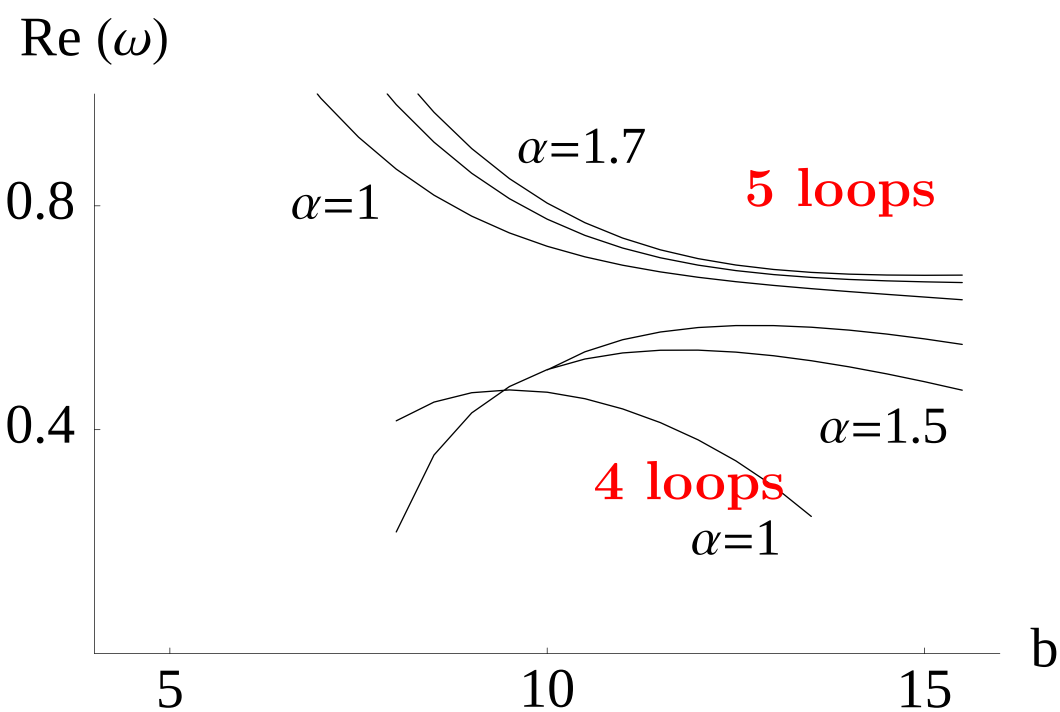
$O(N)$ model + cubic anisotropy: (well-known physics)

$$H = \int \mathrm{d}^d x \left\{ \frac{1}{2} \left[(\partial\phi)^2 + m^2\phi^2 \right] + \frac{u}{4!} \left[\phi^2\right]^2 + \frac{v}{4!} \sum_{i=1}^{N} \phi_i^4 \right\}$$

$\Longrightarrow$ **new FP** "P" within the FD analysis !

$\Longrightarrow$ P is a **spurious focus** FP

- same convergence analysis for P:



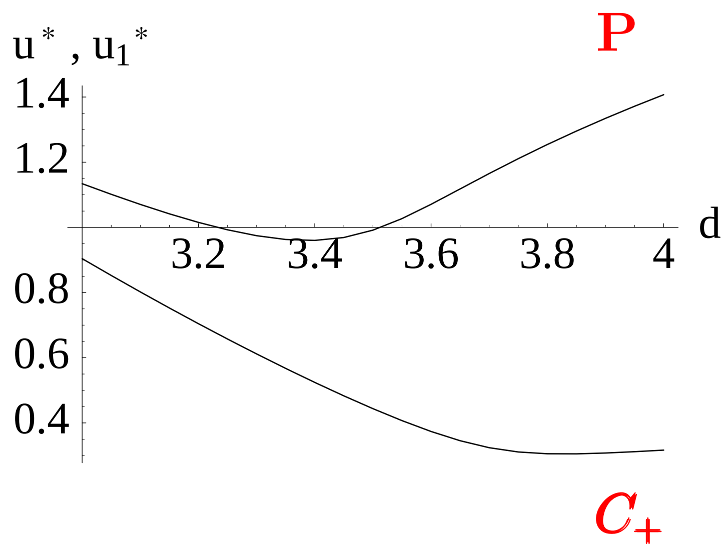
$\Longrightarrow$ $\sim$ same convergence properties than for C^+ !

$\Longrightarrow$ C^+ is **doubtful**

- calls for another criterion to conclude

- criterion:

  ϕ^4-like theories are very likely trivial in d=4

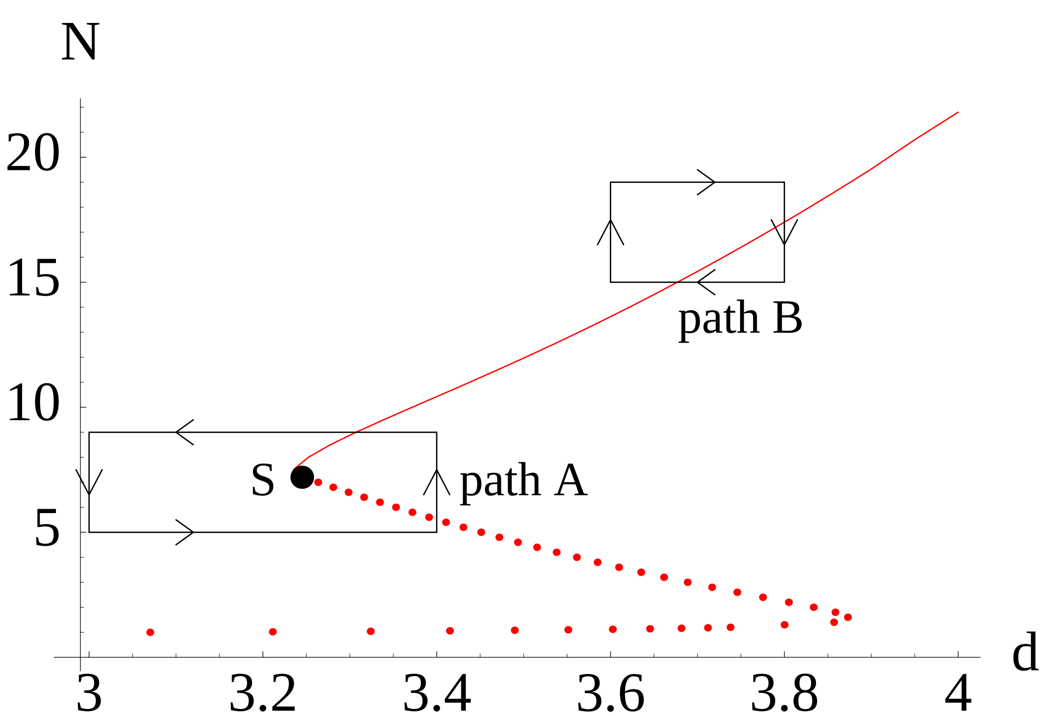


$\Longrightarrow$ P and C^+ not Gaussian in d=4 !

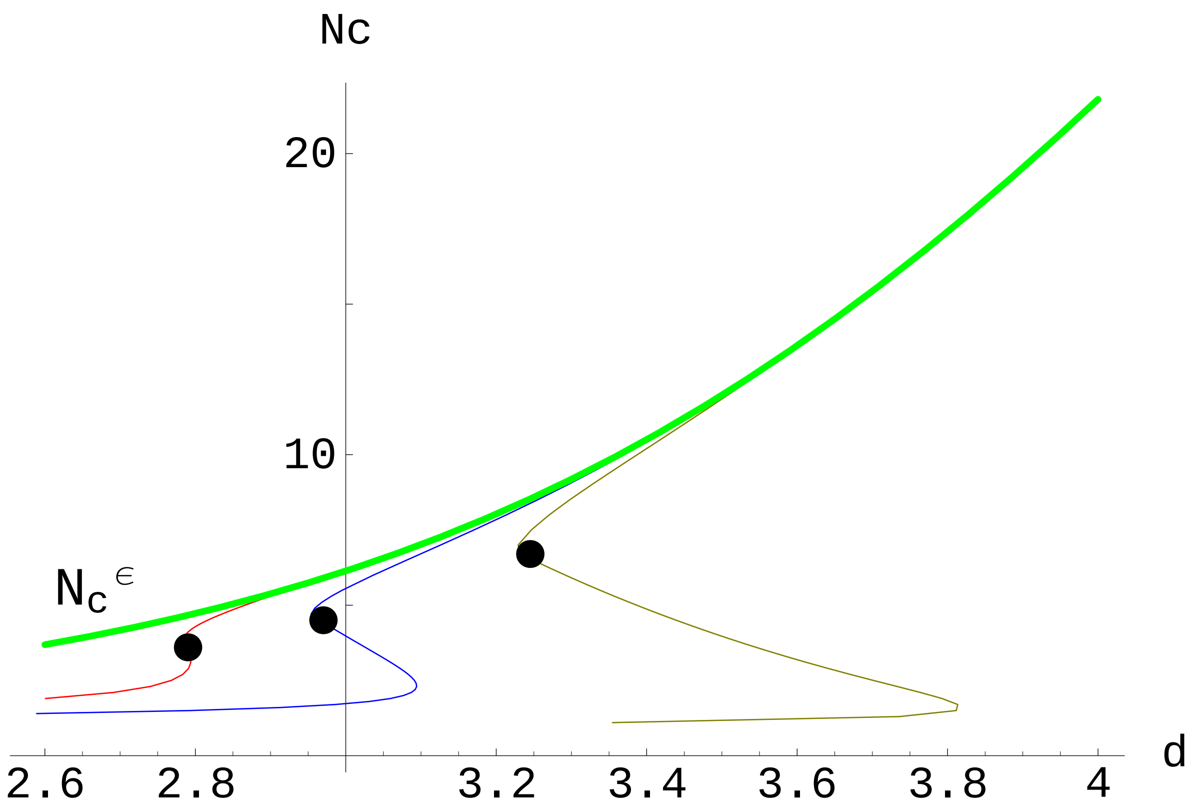
$\Longrightarrow$ they are both spurious FPs

- a check: point S is a topological singularity in the mapping:

$$(d, N) \to (u_1^*(d, N), u_2^*(d, N))$$



- lack of convergence ? $\Longrightarrow$ vary $a(z)$



$N^{FD}(a = 1.5)$ $N^{FD}(a = 1.3)$ $N^{FD}(a = 0.5)$

$\Longrightarrow$ all approaches coincide !

CONCLUSION

Frustrated magnets:

- **breakdown of - almost - all perturbative approches**

  - NLσ model $\Longrightarrow$ $\mathbb{Z}_2$ vortices

  - FD perturbative approaches $\Longrightarrow$ to be elucidated

    $\star$ lack of convergence ?

    $\star$ more fondamental origin ?

- **nonperturbative approach provides a solution**

  $\Longrightarrow$ extension to other models where:

  - $\exists$ N_c

  - $\exists$ discrepancy between NLσ and LGW

    ex: scalar QED

  - $\exists$ suspicion of spurious FP

- **question of the relevant excitations:**

$\star$ classically: spin-waves, massive modes, vortices

$\star$ quantum mechanically: anyons, bound states, ...