

Renormalization and gauge symmetry of 2PI approximation schemes

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CPHT - Ecole Polytechnique

Introduction

- Extra source
- E2PI
- A2PI
- 2PI vertices

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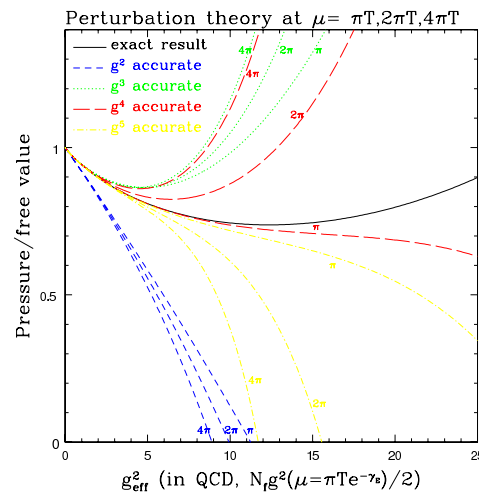
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Applications

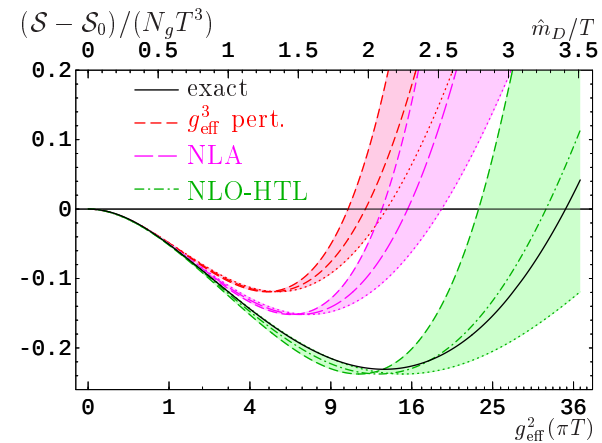
- Allows to go beyond perturbation theory in calculations at finite T .

[Blaizot, Iancu, Rebhan; Ipp, Reinosa]

Perturbations



2PI effective action



Blaizot, Ipp, Rebhan, Reinosa, PRD 72 (2005)

- Allows to study quantum field evolution over long times.

[Aarts, Berges, Borsányi, Serreau; Arrizabalaga, Smit, Tranberg, ...]

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One starts from the generating functional

$$i W[J] = \ln \int \mathcal{D}\varphi \exp \left(i S[\varphi] + i J \cdot \varphi \right)$$

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One starts from the generating functional

$$i W[J, K] = \ln \int \mathcal{D}\varphi \exp \left(i S[\varphi] + i J \cdot \varphi + \frac{i}{2} \varphi \cdot K \cdot \varphi \right)$$

and adds an extra source K – or regulator – which is eventually taken to 0.

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and adds an extra source K – or regulator – which is eventually taken to 0.

One can play with K in many different ways:

- “Variation” w.r.t. $K \Rightarrow$ Functional Renormalization Group.
- Legendre transform $\Rightarrow$ Two-Particle-Irreducible (2PI) effective action.

Exact 2PI effective action

One considers a **double** Legendre transform of $W[J, K]$

$$\Gamma[\phi] \equiv W[J, K] - J \cdot \phi \quad \text{with} \quad \frac{\delta W}{\delta J} =: \phi$$

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Exact 2PI effective action

One considers a **double** Legendre transform of $W[J, K]$

$$\begin{aligned}\Gamma_{2\text{PI}}[\phi, G] \equiv & W[J, K] - J \cdot \phi \quad \text{with} \quad \frac{\delta W}{\delta J} =: \phi \\ & - \frac{1}{2} K \cdot (G + \phi\phi) \quad \text{with} \quad \frac{\delta W}{\delta K} =: \frac{1}{2} (G + \phi\phi)\end{aligned}$$

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$$- \frac{1}{2} K \cdot (G + \phi\phi) \quad \text{with} \quad \frac{\delta W}{\delta K} =: \frac{1}{2} (G + \phi\phi)$$

One then explicitly takes the limit $K \rightarrow 0$ by noticing that

$$K = -2 \frac{\delta \Gamma_{2\text{PI}}}{\delta G}$$

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$$0 = -2 \frac{\delta \Gamma_{2\text{PI}}}{\delta G}$$

This defines the **variational** propagator $\bar{G}[\phi]$.

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One then explicitly takes the limit $K \rightarrow 0$ by noticing that

$$0 = -2 \frac{\delta \Gamma_{2\text{PI}}}{\delta G}$$

This defines the **variational** propagator $\bar{G}[\phi]$.

The usual effective action is then obtained as the minimum of the 2PI effective action:

$$\Gamma[\phi] \equiv W[J] - J \cdot \phi = \Gamma_{2\text{PI}}[\phi, \bar{G}[\phi]]$$

Approximated 2PI effective action

One usually truncates the 2PI effective action (here scalar theory)

$$\Gamma_{2\text{PI}}[\phi, G] = S[\phi] + \frac{i}{2} \text{Tr} \log G^{-1} + \frac{i}{2} \text{Tr} G_0^{-1} G + \Phi_{2\text{PI}}[\phi, G]$$

$$\Phi_{2\text{PI}}[\phi, G] = \text{diagrams} + \dots$$

The diagrams represent the 2PI effective action terms. The first diagram is a self-energy loop (two circles connected by two lines). The second diagram is a tadpole diagram (a circle with a horizontal line through its center, connected to a vertical line with a cross at the bottom, labeled ϕ and G). The third diagram is a sunset diagram (two circles connected by two lines). The fourth diagram is a bubble diagram (two circles connected by two lines). The fifth diagram is a more complex diagram involving a circle with a horizontal line through its center, connected to a vertical line with a cross at the bottom, labeled ϕ and G .

Hence, one usually minimizes a **truncated** functional.

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$$\Phi_{2\text{PI}}[\phi, G] = \text{diagrams} + \dots$$

The diagrams shown are: a self-energy loop (two circles connected by two lines), a tadpole diagram (a circle with a horizontal line through its center, connected to a vertex labeled ϕ on the left and a vertex labeled G on the right), a sunset diagram (two circles connected by two lines), a vacuum bubble (a circle with two horizontal lines inside), and a more complex diagram with multiple loops and vertices, some highlighted in red.

Hence, one usually minimizes a **truncated** functional.

Does this truncated functional reflects the basic properties of the theory?

symmetry? and renormalization? $\Rightarrow$ vertices

2PI and 2PI-resummed vertices

Two possible definitions of the two-point function:

$$\frac{i \delta^2 \Gamma}{\delta \phi_2 \delta \phi_1} \quad \text{or} \quad \bar{G}_{12}^{-1}$$

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If no approximations, both definitions agree:

$$\frac{i \delta^2 \Gamma}{\delta \phi_2 \delta \phi_1} = \bar{G}_{12}^{-1}$$

2PI and 2PI-resummed vertices

In a given **approximation**, they become **different**:

$$\frac{i \delta^2 \Gamma}{\delta \phi_2 \delta \phi_1} \neq \bar{G}_{12}^{-1}$$

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In a given **approximation**, they become **different**:

$$\frac{i \delta^2 \Gamma}{\delta \phi_2 \delta \phi_1} \neq \bar{G}_{12}^{-1}$$

A similar remark applies for higher vertices

$$\frac{i \delta^n \Gamma}{\delta \phi_n \cdots \delta \phi_1} \neq \frac{\delta^{n-2} \bar{G}_{12}^{-1}}{\delta \phi_n \cdots \delta \phi_3}$$

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$$\boxed{\text{2PI-resummed vertices}} \quad \frac{i \delta^n \Gamma}{\delta \phi_n \cdots \delta \phi_1} \neq \frac{\delta^{n-2} \bar{G}_{12}^{-1}}{\delta \phi_n \cdots \delta \phi_3} \quad \boxed{\text{2PI vertices}}$$

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Approximation **artefact**: at a given order of approximation

$$\frac{i \delta^n \Gamma}{\delta \phi_n \cdots \delta \phi_1} - \frac{\delta^{n-2} \bar{G}_{12}^{-1}}{\delta \phi_n \cdots \delta \phi_3} = \mathcal{O}(\text{higher order contributions})$$

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Symmetry

QED in the covariant gauge with gauge fixing parameter ξ

$$S_{\text{qed}} = \int d^d x \left\{ \bar{\psi} [i\partial - eA - m] \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\} - \frac{1}{\xi} \int d^d x (\partial^\mu A_\mu)^2$$

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QED in the covariant gauge with gauge fixing parameter ξ

$$S_{\text{qed}} = \int d^d x \left\{ \bar{\psi} [i\partial - eA - m] \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\} - \frac{1}{\xi} \int d^d x (\partial^\mu A_\mu)^2$$

At any order of approximation, the 2PI effective action is gauge invariant

$$\delta_\alpha \left(\Gamma_{2\text{PI}}[\psi, A, D, G] - S_{\text{gf}}[A] \right) = 0$$

under a gauge transformation of the fields:

$$\delta_\alpha \psi(x) = i\alpha(x) \psi(x) \quad \text{and} \quad \delta_\alpha A^\mu(x) = -(1/e) \partial_x^\mu \alpha(x)$$

and a gauge transformation of the propagators:

$$\delta_\alpha D(x) = i\alpha(x) D(x, y) - iD(x, y) \alpha(y) \quad \text{and} \quad \boxed{\delta_\alpha G(x, y) = 0}$$

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Ward-Takahashi identities (1/2)

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2PI-resummed vertices fulfill **usual** WT identities [UR & J. Serreau, JHEP 0711:097 (2007)]

$$0 = \partial^\mu \frac{i \delta^4 \Gamma}{\delta A^\mu \delta A^\nu \delta A^\rho \delta A^\sigma}$$

2PI vertices fulfill **partial** WT identities [UR & J. Serreau, JHEP 0711:097 (2007)]

$$\partial^\rho \frac{\delta^2 \bar{G}_{\mu\nu}^{-1}}{\delta A^\rho \delta A^\sigma} = 0 \quad \text{but} \quad \partial^\mu \frac{\delta^2 \bar{G}_{\mu\nu}^{-1}}{\delta A^\rho \delta A^\sigma} \neq 0$$

Ward-Takahashi identities (2/2)

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2PI-resummed vertices fulfill **usual** WT identities [UR & J. Serreau, JHEP 0711:097 (2007)]

$$0 = \partial^\mu \left[\frac{i \delta^2 \Gamma}{\delta A^\mu \delta A^\nu} - G_{0,\mu\nu}^{-1} \right]$$

2PI vertices fulfill **partial** WT identities [UR & J. Serreau, JHEP 0711:097 (2007)]

$$\partial^\mu \left[\bar{G}_{\mu\nu}^{-1} - G_{0,\mu\nu}^{-1} \right] = \mathcal{O}(\text{higher order contributions})$$

Exact theory

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QED is renormalizable: one can redefine the fields

$$A_b^\mu \equiv Z_3^{1/2}(d) A^\mu \quad \psi_b \equiv Z_2^{1/2}(d) \psi \quad \bar{\psi}_b \equiv Z_2^{1/2}(d) \bar{\psi}$$

as well as the parameters of the theory

$$Z_2(d) m_b \equiv Z_0(d) m \quad Z_2(d) Z_3^{1/2}(d) e_b \equiv Z_1(d) e \quad \frac{Z_3(d)}{\xi_b} = \frac{Z_4(d)}{\xi}$$

such that the vertices are convergent as $d \rightarrow 4$.

Exact theory

In terms of renormalized fields, the 2PI effective action gets the additional contribution

$$\begin{aligned}\delta\Gamma_{2\text{PI}} = & \frac{\delta Z_3}{2} \int_x A_\mu(x) \left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) A_\nu(x) \\ & + \frac{\delta Z_3}{2} \int_x \left[\left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) G_{\mu\nu}(x, y) \right]_{x=y} + \dots\end{aligned}$$

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The 2PI-resummed photon propagator
gets the additional contribution

$$\begin{aligned}\frac{i \delta^2 \Gamma}{\delta A^\mu \delta A^\nu} = & i \delta Z_3 (g_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) \\ & + \dots\end{aligned}$$

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$$\begin{aligned}\bar{G}_{\mu\nu}^{-1} = & i \delta Z_3 (g_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) \\ & + \dots\end{aligned}$$

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$$\frac{i \delta^2 \Gamma}{\delta A^\mu \delta A^\nu} = \bar{G}_{\mu\nu}^{-1}$$

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In terms of renormalized fields, the 2PI effective action gets the additional contribution

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$$\frac{i \delta^2 \Gamma}{\delta A^\mu \delta A^\nu} \neq \bar{G}_{\mu\nu}^{-1}$$

The 2PI photon propagator
gets the additional contribution

$$\begin{aligned}\bar{G}_{\mu\nu}^{-1} = & i \delta Z_3 (g_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) \\ & + \dots\end{aligned}$$

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In terms of renormalized fields, the 2PI effective action gets the additional contribution

$$\begin{aligned}\delta\Gamma_{2\text{PI}} = & \frac{\delta Z_3}{2} \int_x A_\mu(x) \left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) A_\nu(x) \\ & + \frac{\delta \bar{Z}_3}{2} \int_x \left[\left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) G_{\mu\nu}(x, y) \right]_{x=y} + \dots\end{aligned}$$

The 2PI-resummed photon propagator
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$$\begin{aligned}\frac{i \delta^2 \Gamma}{\delta A^\mu \delta A^\nu} = & i \delta Z_3 (g_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) \\ & + \dots\end{aligned}$$

$$\frac{i \delta^2 \Gamma}{\delta A^\mu \delta A^\nu} \neq \bar{G}_{\mu\nu}^{-1}$$

$$\delta Z_3 \neq \delta \bar{Z}_3$$

The 2PI photon propagator
gets the additional contribution

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In terms of renormalized fields, the 2PI effective action gets the additional contribution

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The 2PI-resummed photon propagator gets the additional contribution

$$\begin{aligned}\frac{i \delta^2 \Gamma}{\delta A^\mu \delta A^\nu} = & i \delta Z_3 (g_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) \\ & + \dots\end{aligned}$$

$$\frac{i \delta^2 \Gamma}{\delta A^\mu \delta A^\nu} \neq \bar{G}_{\mu\nu}^{-1}$$

$$\delta Z_3 \neq \delta \bar{Z}_3, \delta \bar{Z}_L, \delta \bar{M}^2$$

The 2PI photon propagator gets the additional contribution

$$\begin{aligned}\bar{G}_{\mu\nu}^{-1} = & i \delta \bar{Z}_3 (g_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) \\ & + i \delta \bar{Z}_L \partial_\mu \partial_\nu + i \delta \bar{M}^2 g_{\mu\nu} + \dots\end{aligned}$$

Approximations

In terms of renormalized fields, the 2PI effective action gets the additional contribution

$$\begin{aligned}\delta\Gamma_{2\text{PI}} = & \frac{\delta Z_3}{2} \int_x A_\mu(x) \left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) A_\nu(x) \\ & + \frac{\delta \bar{Z}_3}{2} \int_x \left[\left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) G_{\mu\nu}(x, y) \right]_{x=y} + \dots \\ & + \frac{\delta \bar{Z}_L}{2} \int_x \left[\partial_x^\mu \partial_x^\nu G_{\mu\nu}(x, y) \right]_{x=y} + \frac{\delta \bar{M}^2}{2} \int_x G_\mu^\mu(x, x)\end{aligned}$$

All these new contributions do not affect the gauge invariance of the 2PI effective action

$$\delta_\alpha G = 0 \Rightarrow \delta_\alpha \left(\Gamma_{2\text{PI}} + \delta\Gamma_{2\text{PI}} - S_{\text{gf}}[A] \right) = 0$$

$\Rightarrow$ new counterterms allowed by symmetry [UR & J. Serreau, in preparation]

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In terms of renormalized fields, the 2PI effective action gets the additional contribution

$$\begin{aligned}
 \delta\Gamma_{2\text{PI}} = & \frac{\delta Z_3}{2} \int_x A_\mu(x) \left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) A_\nu(x) \\
 & + \frac{\delta \bar{Z}_3}{2} \int_x \left[\left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) G_{\mu\nu}(x, y) \right]_{x=y} + \dots \\
 & + \frac{\delta \bar{Z}_L}{2} \int_x \left[\partial_x^\mu \partial_x^\nu G_{\mu\nu}(x, y) \right]_{x=y} + \frac{\delta \bar{M}^2}{2} \int_x G_\mu^\mu(x, x) \\
 & + \frac{\delta \bar{g}_1}{8} \int_x G_\mu^\mu(x, x) G_\nu^\nu(x, x) + \frac{\delta \bar{g}_2}{4} \int_x G^{\mu\nu}(x, x) G_{\mu\nu}(x, x) \\
 \boxed{\text{not wanted!}} : & \int_x A_\mu(x) G^{\mu\nu}(x, y) A_\nu(x), \quad \int_x A_\mu(x) A^\mu(x) G_\nu^\nu(x, x)
 \end{aligned}$$

All these new contributions do not affect the gauge invariance of the 2PI effective action

$$\delta_\alpha G = 0 \Rightarrow \delta_\alpha \left(\Gamma_{2\text{PI}} + \delta\Gamma_{2\text{PI}} - S_{\text{gf}}[A] \right) = 0$$

$\Rightarrow$ new counterterms allowed by symmetry [UR & J. Serreau, in preparation]

Four photon leg subgraphs

The 2PI photon propagator $\bar{G}_{\mu\nu}^{-1}$ contains subgraphs involving four photons legs

$$\bar{G}_{\mu\nu}^{-1} = \cdots + \text{diagram} + \cdots$$

The diagram shows a central horizontal loop with two green arrows pointing in opposite directions. This loop is connected to four external wavy lines (blue). The top and bottom connections are labeled G_0 . The left and right connections are labeled μ and ν respectively.

$$\frac{\delta}{\delta G_0^{\rho\sigma}} \left(\bar{G}_{\mu\nu}^{-1} \right) = \cdots + \text{diagram} + \cdots \propto \bar{V}_{\mu\nu,\rho\sigma}$$

The diagram is similar to the one above, but the top and bottom wavy lines are red and labeled ρ and σ . A red label $\frac{\delta}{\delta G_0}$ is placed to the right of the diagram.

The same applies for the 2PI-resummed photon propagator $\delta^2 \Gamma / \delta A^\mu \delta A^\nu$

$$\frac{\delta}{\delta G_0^{\rho\sigma}} \left(\frac{\delta^2 \Gamma_{2\text{PI}}}{\delta A^\mu \delta A^\nu} \right) \propto \frac{\delta^2 \bar{G}_{\rho\sigma}^{-1}}{\delta A^\mu \delta A^\nu}$$

Renormalized 2PI effective action

A similar analysis on other 2PI and 2PI-resummed vertices leads to

$$\begin{aligned}
 \delta\Gamma_{2\text{PI}} = & \frac{\delta Z_3}{2} \int_x A_\mu(x) \left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) A_\nu(x) \\
 & + \delta Z_2 \int_x \bar{\psi}(x) i \not{\partial} \psi(x) - m \delta Z_0 \int_x \bar{\psi}(x) \psi(x) \\
 & - e \delta Z_1 \int_x \bar{\psi}(x) A(x) \psi(x) \\
 & + \frac{\delta \bar{Z}_3}{2} \int_x \left[\left(g^{\mu\nu} \partial_x^2 - \partial_x^\mu \partial_x^\nu \right) G_{\mu\nu}(x, y) \right]_{x=y} \\
 & + \frac{\delta \bar{Z}_L}{2} \int_x \left[\partial_x^\mu \partial_x^\nu G_{\mu\nu}(x, y) \right]_{x=y} + \frac{\delta \bar{M}^2}{2} \int_x G_\mu^\mu(x, x) \\
 & + \frac{\delta \bar{g}_1}{8} \int_x G_\mu^\mu(x, x) G_\nu^\nu(x, x) + \frac{\delta \bar{g}_2}{4} \int_x G^{\mu\nu}(x, x) G_{\mu\nu}(x, x) \\
 & - \delta \bar{Z}_2 \int_x \text{tr} [i \not{\partial}_x D(x, y)]_{x=y} + \delta \bar{m} \int_x \text{tr} D(x, x) \\
 & + e \delta \bar{Z}_1 \int_x \text{tr} [A(x) D(x, x)]
 \end{aligned}$$

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Renormalized 2PI effective action

A similar analysis on other 2PI and 2PI-resummed vertices leads to

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- Approximations of the 2PI effective action for QED are **gauge invariant**:

$$\delta_{\alpha} \left(\Gamma_{2\text{PI}} - S_{\text{gf}}[A] \right) = 0$$

- The corresponding – 2PI and 2PI-resummed – vertices are **renormalizable**:

★ Doubled counterterms: $(\delta Z_3, \delta \bar{Z}_3)$, $(\delta Z_2, \delta \bar{Z}_2)$, $(\delta Z_1, \delta \bar{Z}_1)$, $(\delta Z_0, \delta \bar{Z}_0)$.

★ Additional counterterms: $\delta \bar{g}_1$, $\delta \bar{g}_2$, $\delta \bar{Z}_L$ and $\delta \bar{M}^2$.

- The renormalization procedure is **consistent**:

★ All these new features are **allowed by the symmetry**:

$$\delta_{\alpha} \left(\Gamma_{2\text{PI}} + \delta \Gamma_{2\text{PI}} - S_{\text{gf}}[A] \right) = 0$$

★ The number of **renormalization conditions** is the same as usual.

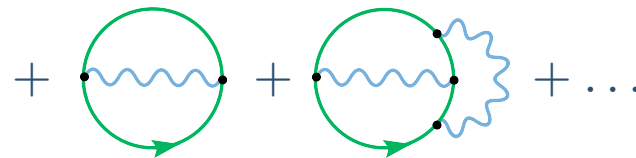
⇒ Systematic application of 2PI techniques to **abelian gauge theories** ⇐

Backup slides

Gauge-fixing dependence

Example: Consider the pressure of the system

$$\mathcal{P}_{2\text{PI}} = -\text{Tr} \left[\ln \bar{D}^{-1}(\xi) + D_0^{-1} \bar{D}(\xi) \right] + \frac{1}{2} \text{Tr} \left[\ln \bar{G}^{-1}(\xi) + G_0^{-1}(\xi) \bar{G}(\xi) \right]$$



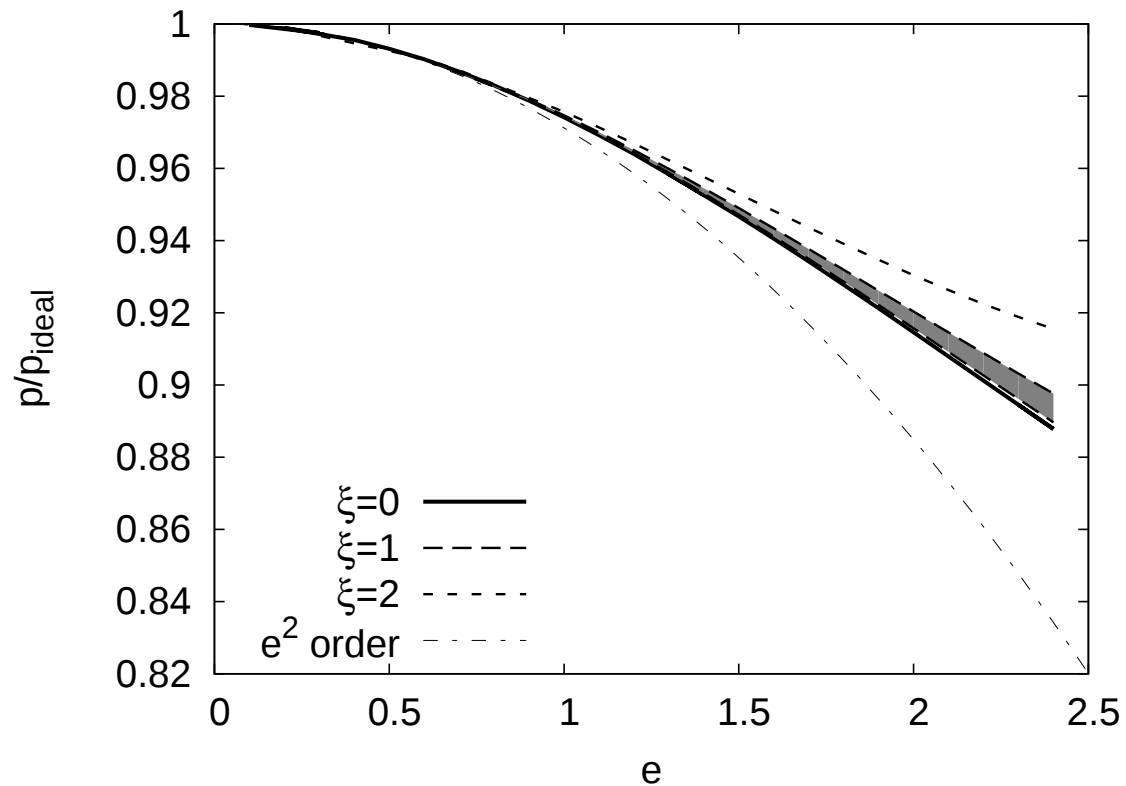
At a given order of approximation, there is a **residual** gauge-fixing dependence

$$\text{At order } e^2, \quad \frac{d}{d\xi} \mathcal{P}_{2\text{PI}} = \mathcal{O}(e^4)$$

$$\text{At order } e^4, \quad \frac{d}{d\xi} \mathcal{P}_{2\text{PI}} = \mathcal{O}(e^6), \quad \text{and so on}$$

Two-loop result (1/2)

In the range of converge $\xi \in [0, 2]$, the ξ -dependence is not dramatically big:
comparable to the μ -dependence in the range $\mu \in [\pi T, 4\pi T]$

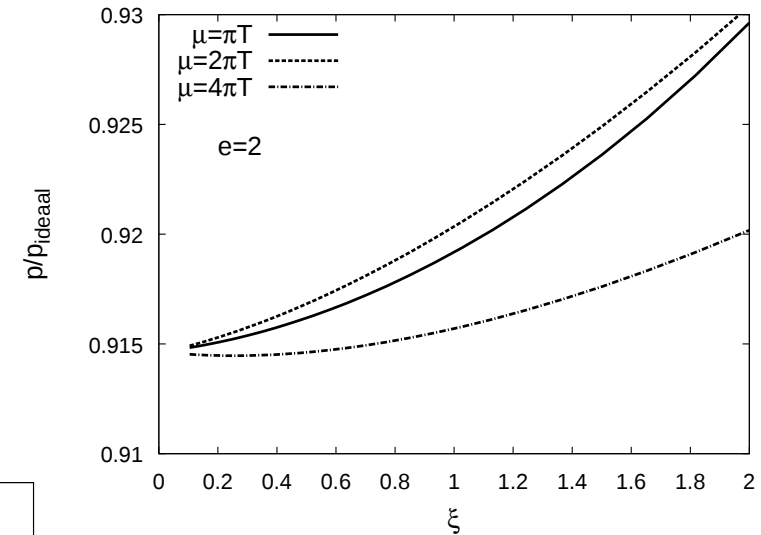
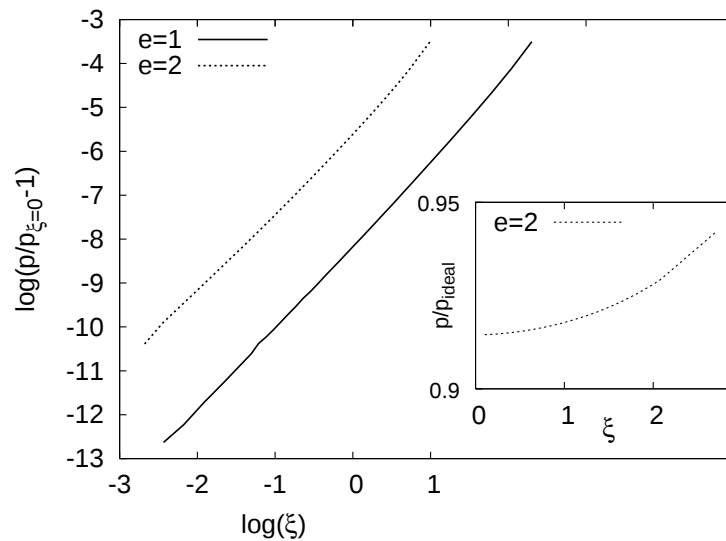


UR & Sz. Borsányi, PLB 661 (2008)

Two-loop result (2/2)

Minimum sensitivity obtained for $\xi = 0$ (Landau gauge):

μ dependence minimal for $\xi = 0$



$$\mathcal{P}_{2PI}(\xi) - \mathcal{P}_{2PI}(0) \sim \alpha(e, \mu) \xi^2$$