Renormalization and gauge symmetry of 2PI approximation schemes

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CPHT - Ecole Polytechnique

Introduction

Applications

Introduction
- Extra source
- E2PI
- A2PI
- 2PI vertices

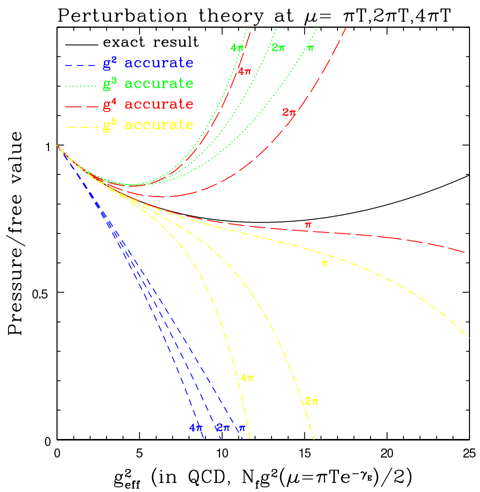
QED

Summary

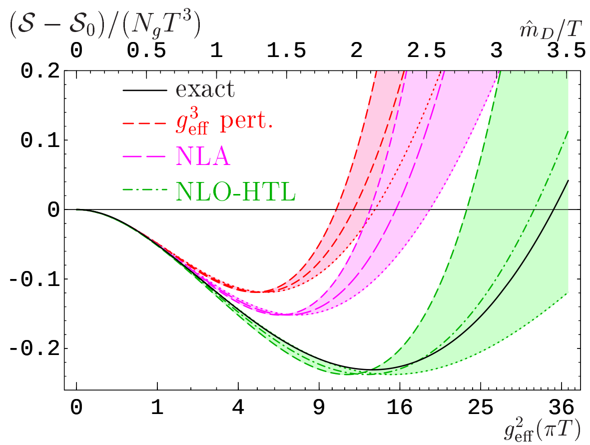
- Allows to go beyond pertubation theory in calculations at finite T.

  [Blaizot, Iancu, Rebhan; Ipp, Reinosa]

Perturbations



2PI effective action



Blaizot, Ipp, Rebhan, Reinosa, PRD 72 (2005)

- Allows to study quantum field evolution over long times.

  [Aarts, Berges, Borsányi, Serreau; Arrizabalaga, Smit, Tranberg, . . .]

Extra source

One starts from the generating functional

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and adds an extra source K – or regulator – which is eventually taken to 0.

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and adds an extra source K – or regulator – which is eventually taken to 0.

One can play with K in many different ways:

- "Variation" w.r.t. K $\Rightarrow$ Functional Renormalization Group.
- Legendre transform $\Rightarrow$ Two-Particle-Irreducible (2PI) effective action.

Exact 2PI effective action

One considers a double Legendre transform of $W[J,K]$

$$\Gamma[\phi] \;\equiv\; W[J,K] - J\cdot\phi \quad \text{with} \quad \frac{\delta W}{\delta J} =: \phi$$

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$$- \frac{1}{2}K\cdot\big(G+\phi\phi\big) \quad \text{with} \quad \frac{\delta W}{\delta K} =: \frac{1}{2}\big(G+\phi\phi\big)$$

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$$K = -2\,\frac{\delta\Gamma_{\rm 2PI}}{\delta G}$$

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The usual effective action is then obtained as the minimum of the 2PI effective action:

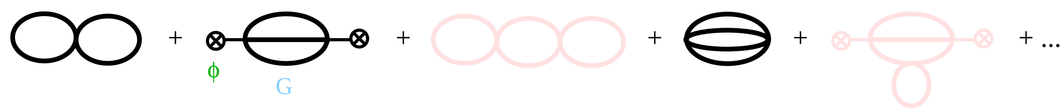
$$\Gamma[\phi]\equiv W[J]-J\cdot\phi=\Gamma_{\rm 2PI}[\phi,\bar{G}[\phi]]$$

Approximated 2PI effective action

One usually truncates the 2PI effective action (here scalar theory)

$$\Gamma_{2\mathrm{PI}}[\phi, G] = S[\phi] + \frac{i}{2}\mathrm{Tr}\,\log G^{-1} + \frac{i}{2}\mathrm{Tr}\, G_0^{-1}\, G + \Phi_{2\mathrm{PI}}[\phi, G]$$

$$\Phi_{2\mathrm{PI}}[\phi, G] =$$



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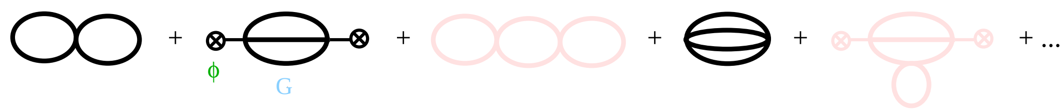

Hence, one usually minimizes a truncated functional.

Does this truncated functional reflects the basic properties of the theory?

symmetry? and renormalization? $\Rightarrow$ vertices

2PI and 2PI-resummed vertices

Two possible definitions of the two-point function:

$$\frac{i\,\delta^2\Gamma}{\delta\phi_2\,\delta\phi_1} \quad \text{or} \quad \bar{G}^{-1}_{12}$$

2PI and 2PI-resummed vertices

If no approximations, both definitions agree:

$$\frac{i\,\delta^2\Gamma}{\delta\phi_2\,\delta\phi_1} = \bar{G}_{12}^{-1}$$

2PI and 2PI-resummed vertices

In a given approximation, they become different:

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A similar remark applies for higher vertices

$$\frac{i\,\delta^n\Gamma}{\delta\phi_n\cdots\delta\phi_1} \neq \frac{\delta^{n-2}\bar{G}_{12}^{-1}}{\delta\phi_n\cdots\delta\phi_3}$$

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Approximation artefact: at a given order of approximation

$$\frac{i\,\delta^n\Gamma}{\delta\phi_n\cdots\delta\phi_1} - \frac{\delta^{n-2}\bar{G}_{12}^{-1}}{\delta\phi_n\cdots\delta\phi_3} = \mathcal{O}(\text{higher order contributions})$$

QED

Symmetry

QED in the covariant gauge with gauge fixing parameter ξ

$$S_{\rm qed} = \int d^d x \left\{ \bar{\psi} \left[i\partial\!\!\!/ - eA\!\!\!/ - m \right] \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\} - \frac{1}{\xi} \int d^d x \, (\partial^\mu A_\mu)^2$$

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At any order of approximation, the 2PI effective action is gauge invariant

$$\delta_\alpha \Big(\Gamma_{\rm 2PI}[\psi, A, D, G] - S_{\rm gf}[A] \Big) = 0$$

under a gauge transformation of the fields:

$$\delta_\alpha \psi(x) = i\alpha(x)\, \psi(x) \quad \text{and} \quad \delta_\alpha A^\mu(x) = -(1/e)\partial^\mu_x \alpha(x)$$

and a gauge transformation of the propagators:

$$\delta_\alpha D(x) = i\alpha(x)\, D(x,y) - iD(x,y)\, \alpha(y) \quad \text{and} \quad \boxed{\delta_\alpha G(x,y) = 0}$$

Ward-Takahashi identities (1/2)

2PI-resummed vertices fulfill usual WT identities [UR & J. Serreau, JHEP 0711:097 (2007)]

$$0 = \partial^\mu \frac{i\,\delta^4\Gamma}{\delta A^\mu \delta A^\nu \delta A^\rho \delta A^\sigma}$$

2PI vertices fulfill partial WT identities [UR & J. Serreau, JHEP 0711:097 (2007)]

$$\partial^\rho \frac{\delta^2 \bar{G}^{-1}_{\mu\nu}}{\delta A^\rho \delta A^\sigma} = 0 \quad \text{but} \quad \partial^\mu \frac{\delta^2 \bar{G}^{-1}_{\mu\nu}}{\delta A^\rho \delta A^\sigma} \neq 0$$

Ward-Takahashi identities (2/2)

2PI-resummed vertices fulfill usual WT identities [UR & J. Serreau, JHEP 0711:097 (2007)]

$$0 = \partial^\mu \left[\frac{i\,\delta^2\Gamma}{\delta A^\mu \delta A^\nu} - G^{-1}_{0,\mu\nu} \right]$$

2PI vertices fulfill partial WT identities [UR & J. Serreau, JHEP 0711:097 (2007)]

$$\partial^\mu \left[\bar{G}^{-1}_{\mu\nu} - G^{-1}_{0,\mu\nu} \right] = \mathcal{O}(\text{higher order contributions})$$

Exact theory

QED is renormalizable: one can redefine the fields

$$A_b^\mu \equiv Z_3^{1/2}(d)A^\mu \qquad \psi_b \equiv Z_2^{1/2}(d)\psi \qquad \bar{\psi}_b \equiv Z_2^{1/2}(d)\bar{\psi}$$

as well as the parameters of the theory

$$Z_2(d)m_b \equiv Z_0(d)m \qquad Z_2(d)Z_3^{1/2}(d)e_b \equiv Z_1(d)e \qquad \frac{Z_3(d)}{\xi_b} = \frac{Z_4(d)}{\xi}$$

such that the vertices are convergent as $d \to 4$.

Exact theory

In terms of renormalized fields, the 2PI effective action gets the additional contribution

$$\delta\Gamma_{\rm 2PI} = \frac{\delta Z_3}{2}\int_x A_\mu(x)\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)A_\nu(x) + \frac{\delta Z_3}{2}\int_x \Big[\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)G_{\mu\nu}(x,y)\Big]_{x=y} + \cdots$$

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The 2PI-resummed photon propagator gets the additional contribution

$$\frac{i\,\delta^2\Gamma}{\delta A^\mu \delta A^\nu} = i\,\delta Z_3\big(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\big) + \cdots$$

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$$\frac{i\,\delta^2\Gamma}{\delta A^\mu\delta A^\nu}\neq\bar G^{-1}_{\mu\nu}$$

Approximations

Introduction

QED

- Symmetry
- Renormalization

Summary

In terms of renormalized fields, the 2PI effective action gets the additional contribution

$$\delta\Gamma_{\rm 2PI} = \frac{\delta Z_3}{2}\int_x A_\mu(x)\Big(g^{\mu\nu}\partial_x^2 - \partial_x^\mu\partial_x^\nu\Big)A_\nu(x) + \frac{\delta \bar{Z}_3}{2}\int_x \Big[\Big(g^{\mu\nu}\partial_x^2 - \partial_x^\mu\partial_x^\nu\Big)G_{\mu\nu}(x,y)\Big]_{x=y} + \cdots$$

The 2PI-resummed photon propagator gets the additional contribution

$$\frac{i\,\delta^2\Gamma}{\delta A^\mu \delta A^\nu} = i\,\delta Z_3\big(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\big) + \cdots$$

$$\delta Z_3 \neq \delta\bar{Z}_3$$

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Approximations

In terms of renormalized fields, the 2PI effective action gets the additional contribution

$$\delta\Gamma_{2\mathrm{PI}} = \frac{\delta Z_3}{2}\int_x A_\mu(x)\Big(g^{\mu\nu}\partial_x^2 - \partial_x^\mu\partial_x^\nu\Big)A_\nu(x)$$

$$+ \frac{\delta\bar Z_3}{2}\int_x \Big[\Big(g^{\mu\nu}\partial_x^2 - \partial_x^\mu\partial_x^\nu\Big)G_{\mu\nu}(x,y)\Big]_{x=y} + \cdots$$

$$+ \frac{\delta\bar Z_L}{2}\int_x \Big[\partial_x^\mu\partial_x^\nu G_{\mu\nu}(x,y)\Big]_{x=y} + \frac{\delta\bar M^2}{2}\int_x G^\mu_\mu(x,x)$$

The 2PI-resummed photon propagator gets the additional contribution

$$\frac{i\,\delta^2\Gamma}{\delta A^\mu\delta A^\nu} = i\,\delta Z_3\big(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\big) + \cdots$$

$$\delta Z_3 \neq \delta\bar Z_3,\ \delta\bar Z_L,\ \delta\bar M^2$$

The 2PI photon propagator gets the additional contribution

$$\bar G^{-1}_{\mu\nu} = i\,\delta\bar Z_3\big(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\big) + i\,\delta\bar Z_L\,\partial_\mu\partial_\nu + i\,\delta\bar M^2 g_{\mu\nu} + \cdots$$

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Approximations

In terms of renormalized fields, the 2PI effective action gets the additional contribution

$$\begin{aligned}\delta\Gamma_{\rm 2PI} \;=\; & \frac{\delta Z_3}{2}\int_x A_\mu(x)\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)A_\nu(x)\\ & +\frac{\delta \bar{Z}_3}{2}\int_x\Big[\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)G_{\mu\nu}(x,y)\Big]_{x=y}+\cdots\\ & +\frac{\delta \bar{Z}_L}{2}\int_x\Big[\partial_x^\mu\partial_x^\nu G_{\mu\nu}(x,y)\Big]_{x=y}+\frac{\delta \bar{M}^2}{2}\int_x G_\mu^\mu(x,x)\end{aligned}$$

All these new contributions do not affect the gauge invariance of the 2PI effective action

$$\delta_\alpha G=0 \;\Rightarrow\; \delta_\alpha\Big(\Gamma_{\rm 2PI}+\delta\Gamma_{\rm 2PI}-S_{\rm gf}[A]\Big)=0$$

$\Rightarrow$ new counterterms allowed by symmetry [UR & J. Serreau, in preparation]

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not wanted! : $\int_x A_\mu(x)G^{\mu\nu}(x,y)A_\nu(x)\,,\quad\int_x A_\mu(x)A^\mu(x)G_\nu^\nu(x,x)$

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Four photon leg subgraphs

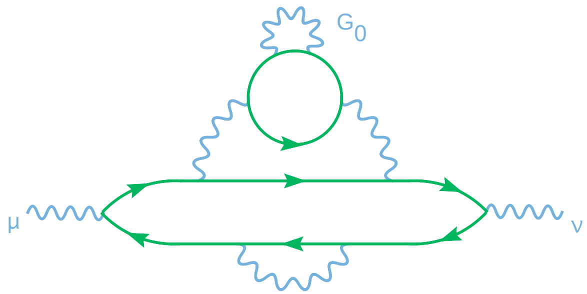
Introduction

QED

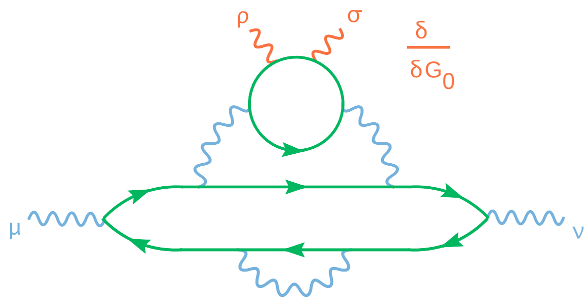
- Symmetry
- Renormalization

Summary

The 2PI photon propagator $\bar{G}^{-1}_{\mu\nu}$ contains subgraphs involving four photons legs

$$\bar{G}^{-1}_{\mu\nu} = \cdots +$$ 

$$+ \cdots$$

$$\frac{\delta}{\delta G_0^{\rho\sigma}}\left(\bar{G}^{-1}_{\mu\nu}\right) = \cdots +$$ 

$$+ \cdots \propto \bar{V}_{\mu\nu,\rho\sigma}$$

The same applies for the 2PI-resummed photon propagator $\delta^2\Gamma/\delta A^\mu \delta A^\nu$

$$\frac{\delta}{\delta G_0^{\rho\sigma}}\left(\frac{\delta^2\Gamma_{2\text{PI}}}{\delta A^\mu \delta A^\nu}\right) \propto \frac{\delta^2 \bar{G}^{-1}_{\rho\sigma}}{\delta A^\mu \delta A^\nu}$$

Renormalized 2PI effective action

A similar analysis on other 2PI and 2PI-resummed vertices leads to

$$\begin{aligned}
\delta\Gamma_{2\mathrm{PI}} = & \frac{\delta Z_3}{2}\int_x A_\mu(x)\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)A_\nu(x) \\
& + \delta Z_2\int_x \bar\psi(x) i\partial\!\!\!/\,\psi(x) - m\,\delta Z_0\int_x \bar\psi(x)\psi(x) \\
& - e\,\delta Z_1\int_x \bar\psi(x) A\!\!\!/(x)\psi(x) \\
& + \frac{\delta\bar Z_3}{2}\int_x \Big[\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)G_{\mu\nu}(x,y)\Big]_{x=y} \\
& + \frac{\delta\bar Z_L}{2}\int_x \Big[\partial_x^\mu\partial_x^\nu G_{\mu\nu}(x,y)\Big]_{x=y} + \frac{\delta\bar M^2}{2}\int_x G^\mu_\mu(x,x) \\
& + \frac{\delta\bar g_1}{8}\int_x G^\mu_\mu(x,x)G^\nu_\nu(x,x) + \frac{\delta\bar g_2}{4}\int_x G^{\mu\nu}(x,x)\,G_{\mu\nu}(x,x) \\
& - \delta\bar Z_2\int_x \mathrm{tr}\,\big[i\partial\!\!\!/_x D(x,y)\big]_{x=y} + \delta\bar m\int_x \mathrm{tr} D(x,x) \\
& + e\,\delta\bar Z_1\int_x \mathrm{tr}\,\big[A\!\!\!/(x)D(x,x)\big]
\end{aligned}$$

Renormalized 2PI effective action

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$$\begin{aligned}
\delta\Gamma_{\rm 2PI} = {} & \frac{\delta Z_3}{2}\int_x A_\mu(x)\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)A_\nu(x) \\
& + \delta Z_2\int_x \bar\psi(x) i\partial\!\!\!/\psi(x) - m\,\delta Z_0\int_x \bar\psi(x)\psi(x) \\
& - e\,\delta Z_1\int_x \bar\psi(x) A\!\!\!/(x)\psi(x) \\
& + \frac{\delta\bar Z_3}{2}\int_x \Big[\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)G_{\mu\nu}(x,y)\Big]_{x=y} \\
& + \frac{\delta\bar Z_L}{2}\int_x \Big[\partial_x^\mu\partial_x^\nu G_{\mu\nu}(x,y)\Big]_{x=y} + \frac{\delta\bar M^2}{2}\int_x G_\mu^\mu(x,x) \\
& + \frac{\delta\bar g_1}{8}\int_x G_\mu^\mu(x,x)G_\nu^\nu(x,x) + \frac{\delta\bar g_2}{4}\int_x G^{\mu\nu}(x,x)\,G_{\mu\nu}(x,x) \\
& - \delta\bar Z_2\int_x \mathrm{tr}\,\big[i\partial\!\!\!/_x D(x,y)\big]_{x=y} + m\,\delta\bar Z_0\int_x \mathrm{tr} D(x,x) \\
& + e\,\delta\bar Z_1\int_x \mathrm{tr}\,\big[A\!\!\!/(x)D(x,x)\big]
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\begin{aligned}
\delta\Gamma_{\rm 2PI} = \; & \frac{\delta Z_3}{2}\int_x A_\mu(x)\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)A_\nu(x)\\
& + \delta Z_2\int_x \bar\psi(x) i\partial\!\!\!/\psi(x) - m\,\delta Z_0\int_x \bar\psi(x)\psi(x)\\
& - e\,\delta Z_1\int_x \bar\psi(x) A\!\!\!/(x)\psi(x)\\
& + \frac{\delta \bar Z_3}{2}\int_x \Big[\Big(g^{\mu\nu}\partial_x^2-\partial_x^\mu\partial_x^\nu\Big)G_{\mu\nu}(x,y)\Big]_{x=y}\\
& + \frac{\delta \bar Z_L}{2}\int_x \Big[\partial_x^\mu\partial_x^\nu G_{\mu\nu}(x,y)\Big]_{x=y} + \frac{\delta \bar M^2}{2}\int_x G^\mu_\mu(x,x)\\
& + \frac{\delta \bar g_1}{8}\int_x G^\mu_\mu(x,x)G^\nu_\nu(x,x) + \frac{\delta \bar g_2}{4}\int_x G^{\mu\nu}(x,x)\,G_{\mu\nu}(x,x)\\
& - \delta \bar Z_2\int_x {\rm tr}\,\big[i\partial\!\!\!/_x D(x,y)\big]_{x=y} + m\,\delta\bar Z_0\int_x {\rm tr}\, D(x,x)\\
& + e\,\delta \bar Z_1\int_x {\rm tr}\,\big[A\!\!\!/(x) D(x,x)\big]
\end{aligned}
$$

Summary

Summary

- Approximations of the 2PI effective action for QED are gauge invariant:

$$\delta_\alpha\Big(\Gamma_{\rm 2PI} - S_{\rm gf}[A]\Big) = 0$$

- The corresponding – 2PI and 2PI-resummed – vertices are renormalizable:
  - ⋆ Doubled counterterms: $(\delta Z_3, \delta\bar{Z}_3)$, $(\delta Z_2, \delta\bar{Z}_2)$, $(\delta Z_1, \delta\bar{Z}_1)$, $(\delta Z_0, \delta\bar{Z}_0)$.
  - ⋆ Additional counterterms: $\delta\bar{g}_1$, $\delta\bar{g}_2$, $\delta\bar{Z}_L$ and $\delta\bar{M}^2$.

- The renormalization procedure is consistent:
  - ⋆ All these new features are allowed by the symmetry:

$$\delta_\alpha\Big(\Gamma_{\rm 2PI} + \delta\Gamma_{\rm 2PI} - S_{\rm gf}[A]\Big) = 0$$

  - ⋆ The number of renormalization conditions is the same as usual.

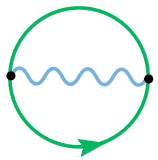
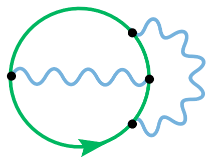
⇒ **Systematic application of 2PI techniques to abelian gauge theories** ⇐

Backup slides

Gauge-fixing dependence

Example: Consider the pressure of the system

$$\mathcal{P}_{\rm 2PI} = -\mathrm{Tr}\left[\ln \bar{D}^{-1}(\xi) + D_0^{-1}\,\bar{D}(\xi)\right] + \frac{1}{2}\mathrm{Tr}\left[\ln \bar{G}^{-1}(\xi) + G_0^{-1}(\xi)\,\bar{G}(\xi)\right]$$

$+$  $+$  $+ \dots$

At a given order of approximation, there is a residual gauge-fixing dependence

At order e^2, $\quad \dfrac{d}{d\xi}\mathcal{P}_{\rm 2PI} = \mathcal{O}(e^4)$

At order e^4, $\quad \dfrac{d}{d\xi}\mathcal{P}_{\rm 2PI} = \mathcal{O}(e^6)$, and so on

Two-loop result (1/2)

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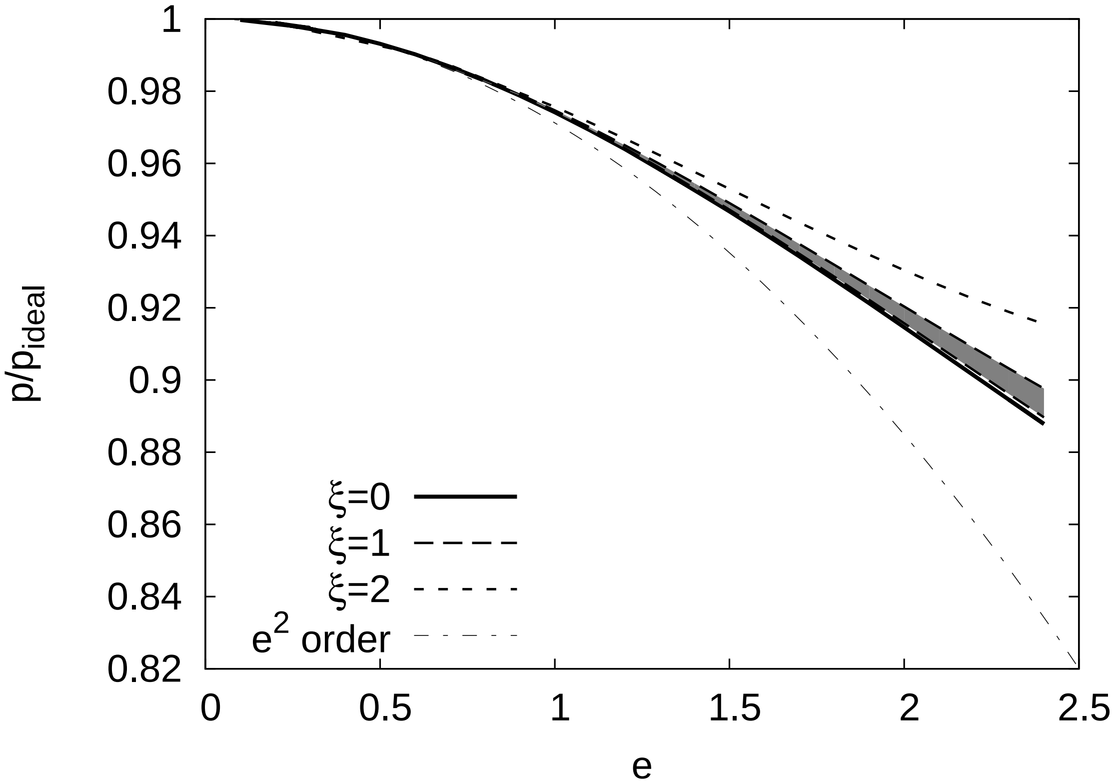
Summary

In the range of converge $\xi \in [0, 2]$, the ξ-dependence is not dramatically big: comparable to the μ-dependence in the range $\mu \in [\pi T, 4\pi T]$



UR & Sz. Borsányi, PLB 661 (2008)

Two-loop result (2/2)

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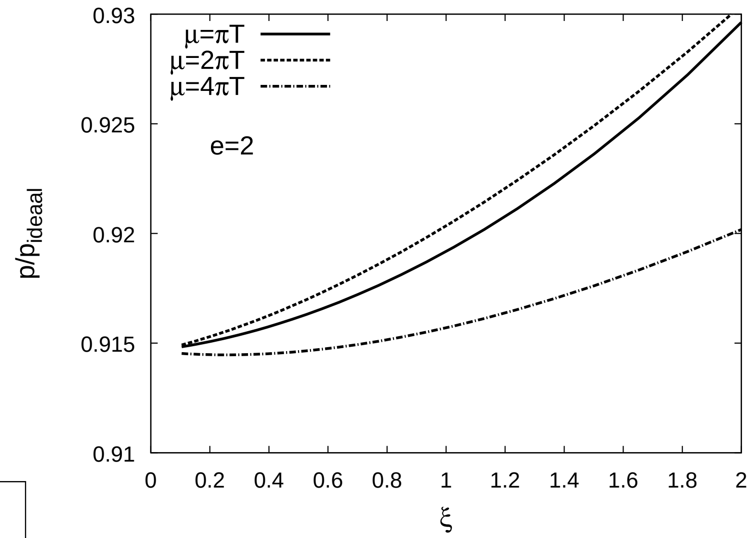
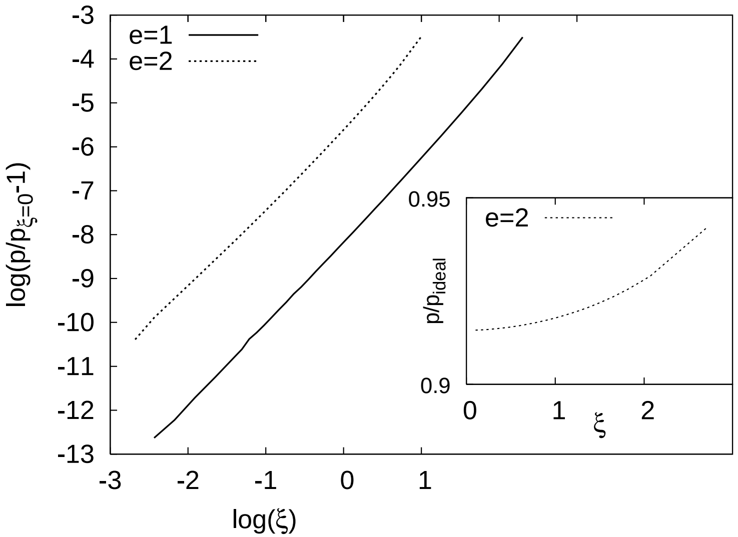
Minimum sensitivity obtained for $\xi = 0$ (Landau gauge):



μ dependence minimal for $\xi = 0$



$$\mathcal{P}_{\rm 2PI}(\xi) - \mathcal{P}_{\rm 2PI}(0) \sim \alpha(e,\mu)\,\xi^2$$