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Invariants of the ERG

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Outline of this Lecture



- Basics
- Technicalities

2 Application to Fixed Points



Outline of this Lecture



- Basics
- Technicalities





Notation

Regularized Propagator:

$$\Delta_m(p, \Lambda) = \frac{c(p^2/\Lambda^2)}{p^2 + m^2(\mu)}$$

Wilsonian Effective Action

$$\mathsf{S}_{\mathsf{n}}[\boldsymbol{\rho}] = \frac{1}{2}\boldsymbol{\rho} \cdot \boldsymbol{\Delta}_{\mathsf{m}}^{-1} \cdot \boldsymbol{\rho} + \mathsf{S}_{\mathsf{n}}[\boldsymbol{\rho}]$$

Flow Equation

• Polchinski Equation

$$-\Lambda \partial_{\Lambda} S^{\mathrm{I}} = \frac{1}{2} \frac{\delta S^{\mathrm{I}}}{\delta \varphi} \cdot \dot{\Delta}_{m} \cdot \frac{\delta S^{\mathrm{I}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta}_{m} \cdot \frac{\delta S^{\mathrm{I}}}{\delta \varphi}$$

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The Dual Action



Properties

The flow of the dual action vanishes

 $-\Lambda \partial_{\Lambda} \mathcal{D}_{m}[\varphi] = 0$

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The Dual Action

Definition

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$$-\mathcal{D}_m[\varphi] = \ln\left[\exp\left(\frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\Delta_m\cdot\frac{\delta}{\delta\varphi}\right)e^{-S^{\mathrm{I}}[\varphi]}\right]$$

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Diagrammatics

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• Wilsonian Effective Action

$$S^{\mathrm{I}}[\varphi] = \frac{1}{2} \left(\begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{S}^{\mathrm{I}} \end{array} \right) \varphi^{2} + \frac{1}{4!} - \begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{S}^{\mathrm{I}} \end{array} + \cdots$$

Diagrammatics

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Dual Action

$$\mathcal{D}_{m}^{(2)} = \left(\begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{I} \end{array} \right) + \frac{1}{2} \left(\begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{S}^{\mathrm{I}} \end{array} \right) - \left(\begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{I} \end{array} \right) - \frac{1}{6} \left(\begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{I} \end{array} \right) + \cdots$$

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Dual Action

 $\mathcal{D}_m^{(2)} = (S^{I}) + \frac{1}{2} (S^{I}) - (S^{I}) - \frac{1}{6} (S^{I}) + \cdots$ • The Dual Action has IR divergences for $m(\mu) \to 0$

IR Divergences I

 Consider constructing n > 2-point connected correlation functions from the bare action

Example

- $G(\rho_1,\ldots,\rho_n) = -\mathcal{D}_m^{(n)}(\rho_1,\ldots,\rho_n)\prod_{i=1}^n \Delta_b(\rho_i), \qquad n>2.$
- $\lim_{m(\mu)\to 0} \mathcal{D}_m^{(n)}(p_1,\ldots,p_n)$ makes sense!
- Any IR divergences have a physical origin



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- Consider constructing the 2-point connected correlation functions from the bare action
- The first contribution is Δ_b
- The full contribution is

$$G(p) = \Delta_b(p) \left[1 - \mathcal{D}_b^{(2)}(p) \Delta_b(p) \right]$$
$$= \Delta_b(p) \left[1 - \mathcal{D}_m^{(2)}(p) \Delta_b(p) \right]$$

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Interpretation

- The dual action is a secondary construction.
- It is not used as the weight in a partition function.
- \otimes It is a convenient way of collecting together the $\mathcal{D}^{(q)}$
- If the $\mathcal{D}^{(q)}$ have IR divergences, so be it!!

Recovering the Wilsonian Effective Action

- Sometimes it is useful to retain the IR regularization.
- Recall:

 $-\mathcal{D}_{m}[\rho] = \ln \left[\exp \left(\frac{1}{2} \frac{\delta}{\delta \rho} + \Delta_{m} + \frac{\delta}{\delta \rho} \right) e^{-2\beta} \ln \theta \right]$

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1PI Vertices

Notation

w Define $\mathcal{D}_{n}^{(2)}$ to be the LPI pieces of $\mathcal{D}_{n}^{(2)}$

Example

1PI Vertices

Notation



Example

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1PI Vertices

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Dressed Effective Propagator

Definition

- $= \operatorname{Recall} \left[G(p) \Delta_{b}(p) \right] \left[1 \mathcal{D}_{m}^{(2)}(\rho) \Delta_{b}(\rho) \right]$
- \circ $\hat{\Delta}_m$ is the UV regularized two-point correlation function

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Dressed Effective Propagator

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Dressed Effective Propagator

Definition • $I = I - \bigoplus_{m=1}^{m} + \cdots$ • $\tilde{\Delta}_m = \frac{\Delta_m}{1 + \Delta_m \overline{D}_m^{(2)}}$

Interpretation

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Dressed Effective Propagator

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Dressed Effective Propagator

Definition • $I = I - \bigoplus_{m=1}^{\infty} + \cdots$ • $\tilde{\Delta}_m = \frac{\Delta_m}{1 + \Delta_m \overline{\mathcal{D}}_m^{(2)}} = \frac{1}{\Delta_m^{-1} + \overline{\mathcal{D}}_m^{(2)}} = \Delta_m \Big[1 - \mathcal{D}_m^{(2)} \Delta_m \Big]$

Interpretation

• Recall
$$G(p) = \Delta_b(p) \left[1 - \mathcal{D}_m^{(2)}(p) \Delta_b(p) \right]$$

• $\tilde{\Delta}_m$ is the UV regularized two-point correlation function

We want to investigate fixed pointsIt is convenient to rescale to dimensionless variables

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and to introduce the 'RG-time'

 $t = \ln \mu / \Lambda$

• But scaling out the anomalous dimension produces an annoying change to the Polchinski equation!

 $\left(-\hbar\partial_{h}+\frac{\eta}{2}\varphi-\frac{\delta}{\delta\varphi}\right)S^{1}=\frac{1}{2Z}\frac{\delta S^{1}}{\delta\varphi}-\Delta_{m}\frac{\delta S^{1}}{\delta\varphi}-\frac{1}{2Z}\frac{\delta}{\delta\varphi}-\Delta_{m}\frac{\delta S^{1}}{\delta\varphi}$

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 General ERGs follow from demanding that the partition function is invariant under the flow

$$-\Lambda \partial_{\Lambda} e^{-S[\varphi]} = \int_{x} \frac{\delta}{\delta \varphi(x)} \left(\Psi_{x}[\varphi] e^{-S[\varphi]} \right)$$

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$$-\Lambda \partial_{\Lambda} \mathcal{Z} = -\Lambda \partial_{\Lambda} \int \mathcal{D} \varphi \, \mathrm{e}^{-S[\varphi]} = 0$$

- Parametrizes blocking procedure
- Choose

$$\Psi = \frac{1}{2} \dot{\Delta}^{\text{new}} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- $\Sigma \equiv S 2\hat{S}$
 - The seed action
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- Choose ^{Anew} to eliminate annoying Zs after rescaling

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The New Flow Equation

• After rescaling $\varphi \to \varphi \sqrt{Z}$

$$\left(-\Lambda\partial_{\Lambda}+\frac{\eta}{2}\varphi\cdot\frac{\delta}{\delta\varphi}\right)S=\frac{1}{2}\frac{\delta S}{\delta\varphi}\cdot\dot{\Delta}_{m}\cdot\frac{\delta\Sigma}{\delta\varphi}-\frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}_{m}\cdot\frac{\delta\Sigma}{\delta\varphi}$$

• The dual action is defined as before

But its flow is different

$$-\left(\Lambda\partial_{\Lambda} + \frac{\eta}{2}\varphi \cdot \frac{\delta}{\delta\varphi}\right)\mathcal{D}_{m}[\varphi] = -\frac{\eta}{2}\varphi \cdot \Delta_{m}^{-1} \cdot \varphi$$
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- New two-point term on the right-hand side
- Seed action contribution (surprisingly simple!)
- For the rest of this talk, take $\hat{S}^{\mathrm{I}} = 0$
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Final Rescalings

• Now scale out the canonical dimensions:

$$\varphi \to \varphi \wedge^{(D-2)/2}, \qquad p \to p \wedge, \qquad t \equiv \ln \mu / \Lambda$$

$$\left(\partial_t + \frac{D-2-\eta}{2} \varphi \cdot \frac{\delta}{\delta \varphi} + \Delta_{\partial} - D\right) \mathcal{D}[\varphi] = -\frac{\eta}{2} \varphi \cdot \Delta^{-1} \cdot \varphi,$$
The 'derivative counting constant'

The 'derivative counting operator'

$$\Delta_{\partial} \equiv D + \int \! rac{d^D p}{(2\pi)^D} \, arphi(p)
ho \cdot rac{\partial}{\partial p} rac{\delta}{\delta arphi(p)}.$$

• The massless effective propagator is independent of t

$$\Delta(\rho) = \frac{c(\rho^2)}{\rho^2}$$

• At a fixed point

$$\partial_t S_{\star} = 0, \qquad \Rightarrow \qquad \partial_t \mathcal{D}_{\star} = 0$$

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$$\Delta_{\partial} \equiv D + \int\!\! rac{d^D p}{(2\pi)^D} \, arphi(p) p \cdot rac{\partial}{\partial p} rac{\delta}{\delta arphi(p)}.$$

• The massless effective propagator is independent of t

$$\Delta(p) = \frac{c(p^2)}{p^2}$$

$$\partial_t S_{\star} = 0, \qquad \Rightarrow \qquad \partial_t \mathcal{D}_{\star} = 0$$

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Final Rescalings

• Now scale out the canonical dimensions:

$$\varphi \to \varphi \Lambda^{(D-2)/2}, \qquad p \to p \Lambda, \qquad t \equiv \ln \mu / \Lambda$$

•
$$\left(\partial_t + \frac{D-2-\eta}{2}\varphi \cdot \frac{\delta}{\delta\varphi} + \Delta_\partial - D\right)\mathcal{D}[\varphi] = -\frac{\eta}{2}\varphi \cdot \Delta^{-1} \cdot \varphi,$$

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- Basics
- Technicalities





The 2-point Vertex

• Define
$$x \equiv p^2$$

• Set $\partial_t \mathcal{D}^{(2)}_*(p) = 0$
• $\Rightarrow -\frac{2 + \eta_*}{2} \mathcal{D}^{(2)}_*(x) + x \frac{\partial \mathcal{D}^{(2)}_*(x)}{\partial x} = -\frac{\eta_*}{2} \Delta^{-1}(x)$

• The solution is:

$$\mathcal{D}_{\star}^{(2)}(x) = x^{1+\eta_{\star}/2} \left[b - \frac{\eta_{\star}}{2} \int dx \frac{c^{-1}(x)}{x^{1+\eta_{\star}/2}} \right]$$

• Taylor Expanding the cutoff function:

$$\mathcal{D}_{\star}^{(2)}(x) = \begin{cases} bx^{1+\eta_{\star}/2} + (x + \text{subleading}) & \eta_{\star} \neq 0\\ bx & \eta_{\star} = 0 \end{cases}$$

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Application to Fixed Points

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Sanity Check

• Recall:
$$G(p) = \Delta_b(p) \left[1 - \mathcal{D}_m^{(2)}(p) \Delta_b(p) \right]$$

• Using

$$\mathcal{D}_{\star}^{(2)}(p) = \begin{cases} bp^{2(1+\eta_{\star}/2)} + (p^2 + \text{subleading}) & \eta_{\star} \neq 0\\ bp^2 & \eta_{\star} = 0 \end{cases}$$

• Gives the expected result at a critical fixed point

$$G(p)\sim rac{1}{p^{2(1-\eta_{\star}/2)}}$$

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IR Finiteness

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IR Finiteness

• We can resum classes of diagrams contributing to $\overline{\mathcal{D}}_{\star}^{(2)}$:

$$\overline{\mathcal{D}}_{\star}^{(2)} = \left(\begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{F} \end{array} \right) + \frac{1}{2} \left(\begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{S}^{\mathrm{I}} \end{array} \right) - \frac{1}{6} \left(\begin{array}{c} \mathsf{S}^{\mathrm{I}} \\ \mathsf{S}^{\mathrm{I}} \end{array} \right) + \cdots$$

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IR Finiteness

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• We can resum classes of diagrams contributing to $\overline{\mathcal{D}}_{\star}^{(2)}$:

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Recall: $\widetilde{\Delta}_{\star}(p) \sim \frac{1}{p^{2(1-\eta_{\star}/2)}}, \qquad \overline{\mathcal{D}}_{\star}^{(2)} \sim p^{2(1-\eta_{\star}/2)}$

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• Recall: $\tilde{\Delta}_{\star}(p) \sim \frac{1}{p^{2(1-\eta_{\star}/2)}}, \qquad \overline{\mathcal{D}}_{\star}^{(2)} \sim p^{2(1-\eta_{\star}/2)}$ • So, for D > 4 and $\eta_{\star} \ge 0$ or D = 4 and $\eta_{\star} > 0$

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IR Finiteness

• We can resum classes of diagrams contributing to $\overline{\mathcal{D}}^{(2)}_{\star}$:

$$\overline{\mathcal{D}}_{\star}^{(2)} = \overbrace{S^{\mathrm{I}}}^{\mathbf{I}} + \frac{1}{2} \overbrace{S^{\mathrm{I}}}^{\mathbf{S}^{\mathrm{I}}} - \frac{1}{6} \overbrace{S^{\mathrm{I}}}^{\mathbf{S}^{\mathrm{I}}} + \cdots$$

• Recall: $\tilde{\Delta}_{\star}(p) \sim \frac{1}{p^{2(1-\eta_{\star}/2)}}, \qquad \overline{\mathcal{D}}_{\star}^{(2)} \sim p^{2(1-\eta_{\star}/2)}$
• So, for $D > 4$ and $\eta_{\star} \ge 0$ or $D = 4$ and $\eta_{\star} > 0$
• $\lim_{p \to 0} \overline{\mathcal{D}}^{(2)}(p) = \text{const}$

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IR Finiteness

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• We can resum classes of diagrams contributing to $\overline{\mathcal{D}}_{\star}^{(2)}$:

$$\overline{\mathcal{D}}_{\star}^{(2)} = \left(\overbrace{S^{1}}^{I} + \frac{1}{2} \right) \left(\overbrace{S^{1}}^{I} - \frac{1}{6} \right) \left(\overbrace{S^{1}}^{I} + \cdots \right)$$
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$$\eta_{\star} = 0$$
• Re-express $\mathcal{D}^{(4)}$ in terms of 1PI pieces:

$$\mathcal{D}^{(4)}(p_1, p_2, p_3, p_4) = rac{\overline{\mathcal{D}}^{(4)}(p_1, p_2, p_3, p_4)}{\prod_{i=1}^4 \left[1 + \Delta(p_i)\overline{\mathcal{D}}^{(2)}(p_i)
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• For $\eta_{\star}=0$:

$$D - 4 + r = 0$$

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- Two conditions: r < 0 and $r \ge 0$, $\Rightarrow D_s^{(0)} = 0$
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$$\left(4\frac{D-2-\eta_{\star}}{2}+\sum_{i=1}^{4}p_{i}\cdot\partial_{p_{i}}-D\right)\mathcal{D}_{\star}^{(4)}(p_{1},p_{2},p_{3},p_{4})=0$$

• For $\eta_{\star} = 0$:

$$D-4+r=0$$

- Two conditions: r < 0 and $r \ge 0$, $\Rightarrow \mathcal{D}_{*}^{(4)} = 0$
- For D = 4, r = 0 is a solution with $\mathcal{D}^{(4)}_{\star} \sim \text{const} \Rightarrow \mathcal{D}^{(4)}_{\star} = 0$

• Re-express $\mathcal{D}^{(4)}$ in terms of 1PI pieces:

$$\mathcal{D}^{(4)}(p_1, p_2, p_3, p_4) = rac{\overline{\mathcal{D}}^{(4)}(p_1, p_2, p_3, p_4)}{\prod_{i=1}^4 \left[1 + \Delta(p_i)\overline{\mathcal{D}}^{(2)}(p_i)
ight]}$$

•
$$\mathcal{D}^{(4)}_{\star}$$
 satisfies

$$\left(4\frac{D-2-\eta_{\star}}{2}+\sum_{i=1}^{4}p_{i}\cdot\partial_{p_{i}}-D\right)\mathcal{D}_{\star}^{(4)}(p_{1},p_{2},p_{3},p_{4})=0$$

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The 6-pt Vertex

• Re-express $\mathcal{D}^{(6)}$ in terms of 1PI pieces



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The 6-pt Vertex

• Re-express $\mathcal{D}^{(6)}$ in terms of 1PI pieces



• Re-express $\mathcal{D}^{(6)}$ in terms of 1PI pieces

$$\mathcal{D}^{(6)} = \begin{array}{c} \overline{\mathcal{D}} \\ \overline{\mathcal{D}} \\ \overline{\mathcal{D}} \end{array} - \begin{array}{c} \overline{\mathcal{D}} \\ \overline{\mathcal{D}} \\ \overline{\mathcal{D}} \end{array}$$

• $\mathcal{D}^{(6)}_{\star}$ satisfies (with $\eta_{\star} = 0$)

$$\left(6\frac{D-2}{2}+\sum_{i=1}^{4}p_i\cdot\partial_{p_i}-D\right)\mathcal{D}_{\star}^{(6)}(p_1,\ldots,p_6)=0$$

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• Re-express $\mathcal{D}^{(6)}$ in terms of 1PI pieces

$$\mathcal{D}^{(6)} = \overline{\mathcal{D}} - \overline{\mathcal{D}}$$

• $\mathcal{D}_{\star}^{(6)}$ satisfies (with $\eta_{\star} = 0$)

$$\left(6\frac{D-2}{2}+\sum_{i=1}^4p_i\cdot\partial_{p_i}-D\right)\mathcal{D}^{(6)}_\star(p_1,\ldots,p_6)=0$$

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D > 4: D⁽⁶⁾_⋆ ~ const + subleading ⇒ D⁽⁶⁾_⋆ = 0
D = 4: require D⁽⁶⁾_⋆ ~ mom⁻²

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• D > 4: $\mathcal{D}_{\star}^{(6)} \sim \text{const} + \text{subleading} \Rightarrow \mathcal{D}_{\star}^{(6)} = 0$ • D = 4: require $\mathcal{D}_{\star}^{(6)} \sim \text{mom}^{-2} \Rightarrow \mathcal{D}_{\star}^{(6)} = 0$

- For $D \ge 4$ and $\eta_{\star} \ge 0$: • $\mathcal{D}_{\star}^{(2)}$ tells us that $\eta_{\star} = 0$ • This forces $\mathcal{D}_{\star}^{(2)} = 0$ • This forces $\mathcal{D}_{\star}^{(2)} = 0$ • By induction, $\mathcal{D}_{\star}^{(n+2)} = 0$
- Therefore, the only critical fixed point in $D\geq 4$ with $\eta_{\star}\geq 0$ is the Gaussian one

The n-pt Vertex

• For $D \ge 4$ and $\eta_{\star} \ge 0$:

- $\mathcal{D}^{(2)}_{\star}$ tells us that $\eta_{\star} = 0$
- This forces $\mathcal{D}_{\star_{c1}}^{(4)} = 0$
- This forces $\mathcal{D}_{\star}^{(6)} = 0$
- By induction, $\mathcal{D}_{\star}^{(n>2)} = 0$
- Therefore, the only critical fixed point in $D\geq 4$ with $\eta_{\star}\geq 0$ is the Gaussian one

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- For D > 4 and $\eta_{\star} > 0$:
 - $\mathcal{D}^{(2)}_{\star}$ tells us that $\eta_{\star} = 0$
 - This forces $\mathcal{D}_{\star}^{(4)} = 0$ This forces $\mathcal{D}_{\star}^{(6)} = 0$

 - By induction. $\mathcal{D}^{(n>2)}_{+} = 0$
- Therefore, the only critical fixed point in $D \ge 4$ with $\eta_{\star} \ge 0$ is

- For D > 4 and $\eta_{\star} > 0$:
 - $\mathcal{D}^{(2)}_{\star}$ tells us that $\eta_{\star} = 0$
 - This forces $\mathcal{D}_{\star}^{(4)} = 0$ This forces $\mathcal{D}_{\star}^{(6)} = 0$

 - By induction, $\hat{\mathcal{D}}_{\star}^{(n>\tilde{2})} = 0$
- Therefore, the only critical fixed point in $D \ge 4$ with $\eta_{\star} \ge 0$ is

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- For D > 4 and $\eta_{\star} > 0$:
 - $\mathcal{D}_{\star}^{(2)}$ tells us that $\eta_{\star} = 0$
 - This forces $\mathcal{D}_{\star}^{(4)} = 0$ This forces $\mathcal{D}_{\star}^{(6)} = 0$

 - By induction, $\mathcal{D}_{+}^{(n>2)} = 0$
- Therefore, the only critical fixed point in $D \ge 4$ with $\eta_{\star} \ge 0$ is the Gaussian one

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Conclusion

- To complete the proof of the triviality of scalar field theory in $D \ge 4$, negative anomalous dimensions need to be considered
- Exactly the same method can be used to prove the triviality of
 - > Non-compact, pure U(1) gauge theory in any D
 - Theories of a chiral superfield in D = 4 (Wess-Zumino model)
- The million dollar question:
 - Can this methodology be usefully applied to non-trivial theories??

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Thank you for listening