Invariants of the ERG

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Outline of this Lecture

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Set-up

Notation

- Regularized Propagator:

$$\Delta_m(p,\Lambda) = \frac{c(p^2/\Lambda^2)}{p^2 + m^2(\mu)}$$

- Wilsonian Effective Action

$$S_\Lambda[\varphi] = \frac{1}{2}\varphi \cdot \Delta_m^{-1} \cdot \varphi + S_\Lambda^{\mathrm{I}}[\varphi]$$

Flow Equation

- Polchinski Equation

$$-\Lambda\partial_\Lambda S^{\mathrm{I}} = \frac{1}{2}\frac{\delta S^{\mathrm{I}}}{\delta\varphi} \cdot \dot{\Delta}_m \cdot \frac{\delta S^{\mathrm{I}}}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi} \cdot \dot{\Delta}_m \cdot \frac{\delta S^{\mathrm{I}}}{\delta\varphi}$$

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The Dual Action

Definition

- $-\mathcal{D}_m[\varphi] = \ln \left[\exp\left(\frac{1}{2} \frac{\delta}{\delta\varphi} \cdot \Delta_m \cdot \frac{\delta}{\delta\varphi} \right) e^{-S[\varphi]} \right]$

Properties

- The flow of the dual action vanishes

$$-\Lambda\partial_\Lambda \mathcal{D}_m[\varphi] = 0$$

- Its vertices, $\mathcal{D}_m^{(n)}$, are invariants of the ERG

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Diagrammatics

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- Wilsonian Effective Action

$$S^{\mathrm{I}}[\varphi] = \frac{1}{2}\, S^{\mathrm{I}} \, \varphi^{2} + \frac{1}{4!}\, S^{\mathrm{I}} \, \varphi^{4} + \cdots$$

The Dual Action

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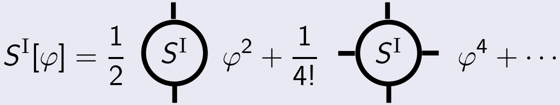

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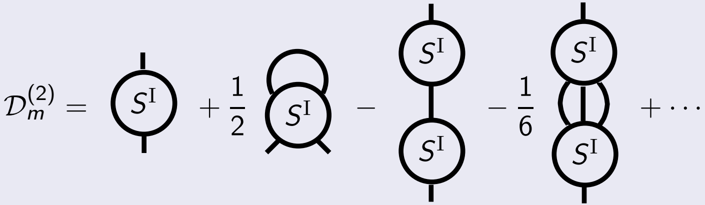

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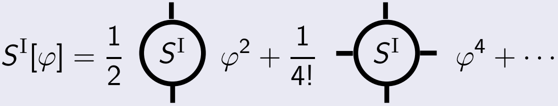

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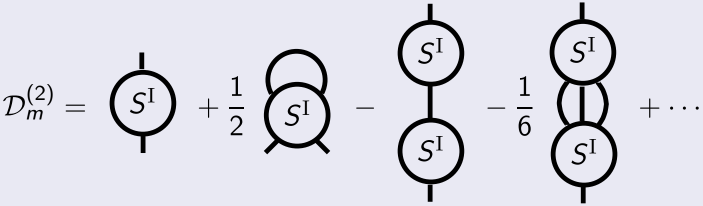

- The Dual Action has IR divergences for $m(\mu) \to 0$

IR Divergences I

- Consider constructing $n > 2$-point connected correlation functions from the bare action

Example

- $G(p_1, \ldots, p_n) = -\mathcal{D}_m^{(n)}(p_1, \ldots, p_n) \prod_{i=1}^{n} \Delta_b(p_i), \qquad n > 2.$
- $\lim_{m(\mu)\to 0} \mathcal{D}_m^{(n)}(p_1, \ldots, p_n)$ makes sense!
- Any IR divergences have a physical origin

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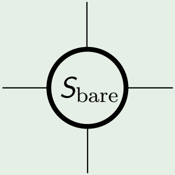
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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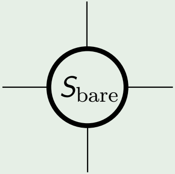
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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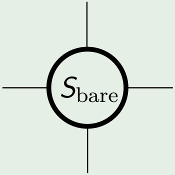
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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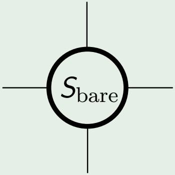
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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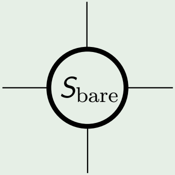
$G(p_1, p_2, p_3, p_4) = -$  $+ \cdots$

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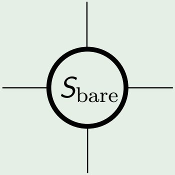
$G(p_1, p_2, p_3, p_4) = -$ 

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- Consider constructing the 2-point connected correlation functions from the bare action
- The first contribution is Δ_b
- The full contribution is

$$\begin{aligned} G(p) &= \Delta_b(p)\left[1 - \mathcal{D}_b^{(2)}(p)\Delta_b(p)\right] \\ &= \Delta_b(p)\left[1 - \mathcal{D}_m^{(2)}(p)\Delta_b(p)\right] \end{aligned}$$

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More about the Dual Action

Interpretation

- The dual action is a secondary construction
- It is not used as the weight in a partition function
- It is a convenient way of collecting together the $\mathcal{D}^{(n)}$
- If the $\mathcal{D}^{(n)}$ have IR divergences, so be it

Recovering the Wilsonian Effective Action

- Sometimes it is useful to retain the IR regularization
- Recall:

$$-\mathcal{D}_m[\varphi] = \ln\left[\exp\left(-\frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\Delta_m\cdot\frac{\delta}{\delta\varphi}\right)e^{-S^{\Lambda}[\varphi]}\right]$$

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1PI Vertices

Notation

- Define $\overline{\mathcal{D}}^{(n)}_m$ to be the 1PI piece of $\mathcal{D}^{(n)}_m$

Example

1PI Vertices

Notation

- Define $\overline{\mathcal{D}}_m^{(n)}$ to be the 1PI pieces of $\mathcal{D}_m^{(n)}$

Example

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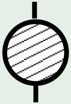
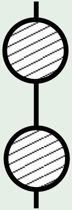
- $\mathcal{D}_m^{(2)} =$  $-$  $+ \cdots$

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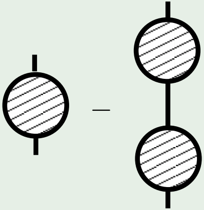
- $\mathcal{D}_m^{(2)} =$  $+ \cdots$

- $$\mathcal{D}_m^{(2)}(p) = \frac{\overline{\mathcal{D}}_m^{(2)}(p)}{1 + \Delta_m(p)\overline{\mathcal{D}}_m^{(2)}(p)}$$

Dressed Effective Propagator

Definition

Interpretation

- Recall $G(p) = \Delta_b(p)\left[1 - \mathcal{D}_m^{(2)}(p)\Delta_b(p)\right]$
- $\tilde{\Delta}_m$ is the UV regularized two-point correlation function

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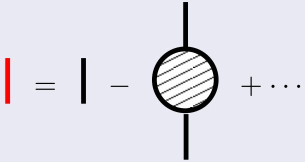
- 

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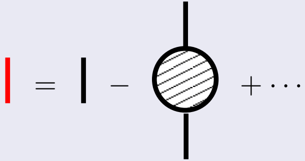
- 

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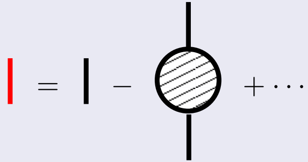
- 
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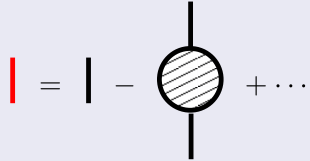
- 
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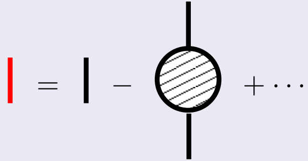
- 
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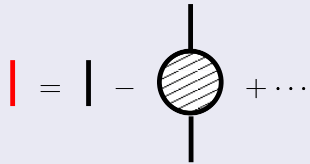
- 
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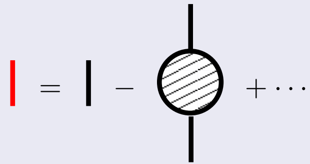
- 
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Rescalings

- We want to investigate fixed points
- It is convenient to rescale to dimensionless variables

$$\varphi \rightarrow \varphi\sqrt{Z}\Lambda^{(D-2)/2}$$
$$p \rightarrow p\Lambda$$

and to introduce the 'RG-time'

$$t = \ln \mu/\Lambda$$

- But scaling out the anomalous dimension produces an annoying change to the Polchinski equation!

$$\left(-\Lambda\partial_\Lambda + \frac{\eta}{2}\varphi\cdot\frac{\delta}{\delta\varphi}\right)S^I = \frac{1}{2Z}\frac{\delta S^I}{\delta\varphi}\cdot\Delta_\Lambda\cdot\frac{\delta S^I}{\delta\varphi} - \frac{1}{2Z}\frac{\delta}{\delta\varphi}\cdot\Delta_\Lambda\cdot\frac{\delta S^I}{\delta\varphi}$$

- This factor of Z also appears in the dual action

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General ERGs

- General ERGs follow from demanding that the partition function is invariant under the flow

$$-\Lambda\partial_\Lambda \, e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] \, e^{-S[\varphi]} \right)$$

- $-\Lambda\partial_\Lambda \mathcal{Z} = -\Lambda\partial_\Lambda \int \mathcal{D}\varphi \, e^{-S[\varphi]} = 0$
- Parametrizes blocking procedure
- Choose

$$\Psi = \frac{1}{2} \dot{\Delta}^{\text{new}} \cdot \frac{\delta\Sigma}{\delta\varphi}$$

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The New Flow Equation

- After rescaling $\varphi \to \varphi\sqrt{Z}$

$$\left(-\Lambda\partial_\Lambda + \frac{\eta}{2}\varphi\cdot\frac{\delta}{\delta\varphi}\right) S = \frac{1}{2}\frac{\delta S}{\delta\varphi}\cdot\dot{\Delta}_m\cdot\frac{\delta\Sigma}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}_m\cdot\frac{\delta\Sigma}{\delta\varphi}$$

- The dual action is defined as before
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- New two-point term on the right-hand side
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- After rescaling $\varphi \to \varphi\sqrt{Z}$

$$\left(-\Lambda\partial_\Lambda + \frac{\eta}{2}\varphi \cdot \frac{\delta}{\delta\varphi} \right) S = \frac{1}{2}\frac{\delta S}{\delta\varphi} \cdot \dot{\Delta}_m \cdot \frac{\delta\Sigma}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi} \cdot \dot{\Delta}_m \cdot \frac{\delta\Sigma}{\delta\varphi}$$

- The dual action is defined as before
- But its flow is different

$$-\left(\Lambda\partial_\Lambda + \frac{\eta}{2}\varphi \cdot \frac{\delta}{\delta\varphi} \right) \mathcal{D}_m[\varphi] = -\frac{\eta}{2}\varphi \cdot \Delta_m^{-1} \cdot \varphi$$

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- New two-point term on the right-hand side
- Seed action contribution (surprisingly simple!)
- For the rest of this talk, take $\hat{S}^{\mathrm{I}} = 0$
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Final Rescalings

- Now scale out the canonical dimensions:

$$\varphi \longrightarrow \varphi \Lambda^{(D-2)/2}, \qquad p \longrightarrow p\Lambda, \qquad t \equiv \ln \mu/\Lambda$$

- $\left(\partial_t + \frac{D-2-\eta}{2}\varphi \cdot \frac{\delta}{\delta\varphi} + \Delta_\partial - D\right)\mathcal{D}[\varphi] = -\frac{\eta}{2}\varphi \cdot \Delta^{-1} \cdot \varphi,$
- The 'derivative counting operator'

$$\Delta_\partial \equiv D + \int \frac{d^D p}{(2\pi)^D} \varphi(p) p \cdot \frac{\partial}{\partial p} \frac{\delta}{\delta\varphi(p)}.$$

- The massless effective propagator is independent of t

$$\Delta(p) = \frac{c(p^2)}{p^2}$$

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- Define $x = p^2$
- Set $\partial_t \mathcal{D}_\star^{(2)}(p) = 0$
- $\Rightarrow -\frac{2+\eta_\star}{2}\mathcal{D}_\star^{(2)}(x) + x\frac{\partial \mathcal{D}_\star^{(2)}(x)}{\partial x} = -\frac{\eta_\star}{2}\Delta^{-1}(x)$
- The solution is:

$$\mathcal{D}_\star^{(2)}(x) = x^{1+\eta_\star/2}\left[b - \frac{\eta_\star}{2}\int dx \frac{c^{-1}(x)}{x^{1+\eta_\star/2}}\right]$$

- Taylor Expanding the cutoff function:

$$\mathcal{D}_\star^{(2)}(x) = \begin{cases} bx^{1+\eta_\star/2} + (x + \text{subleading}) & \eta_\star \neq 0 \\ bx & \eta_\star = 0. \end{cases}$$

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- Recall: $G(p) = \Delta_b(p)\left[1 - \mathcal{D}^{(2)}_m(p)\Delta_b(p)\right]$
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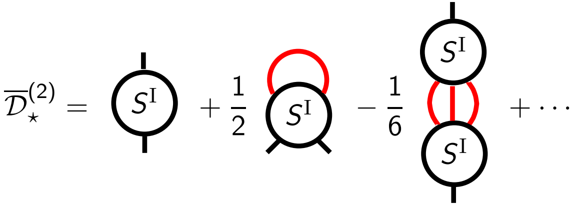

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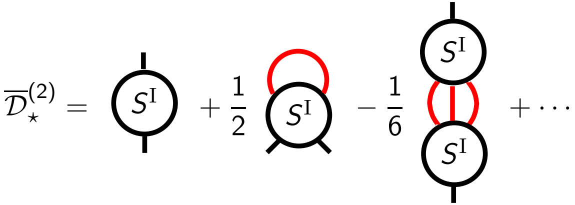

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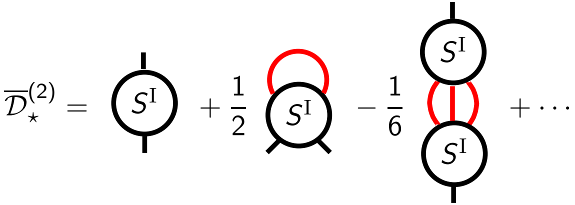

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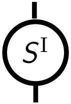
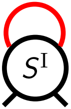
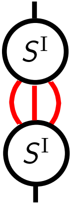
$\overline{\mathcal{D}}^{(2)}_\star =$ 

$+\frac{1}{2}$ 

$-\frac{1}{6}$ 

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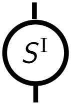
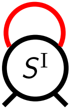
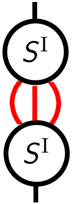
$\overline{\mathcal{D}}^{(2)}_\star =$ 

$+\frac{1}{2}$ 

$-\frac{1}{6}$ 

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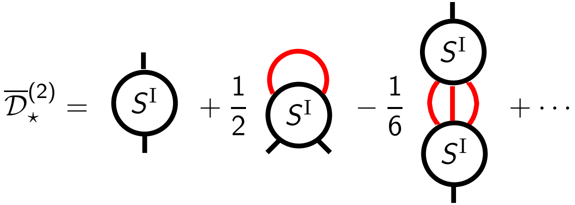

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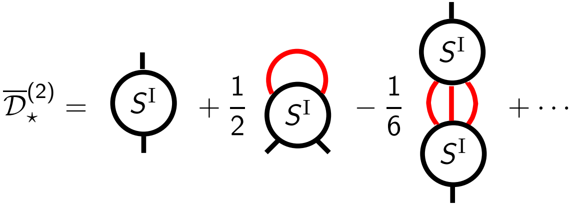

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The 4-pt Vertex

- Re-express $\mathcal{D}^{(4)}$ in terms of 1PI pieces:

$$\mathcal{D}^{(4)}(p_1, p_2, p_3, p_4) = \frac{\overline{\mathcal{D}}^{(4)}(p_1, p_2, p_3, p_4)}{\prod_{i=1}^{4}\left[1 + \Delta(p_i)\overline{\mathcal{D}}^{(2)}(p_i)\right]}$$

- $\mathcal{D}_\star^{(4)}$ satisfies

$$\left(4\frac{D-2-\eta_\star}{2} + \sum_{i=1}^{4} p_i \cdot \partial_{p_i} - D\right)\mathcal{D}_\star^{(4)}(p_1, p_2, p_3, p_4) = 0$$

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$$D - 4 + r = 0$$

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- Two conditions: $r < 0$ and $r > 0$, $\Rightarrow \mathcal{D}_\star^{(4)} = 0$
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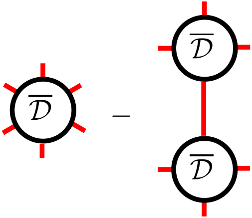

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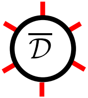

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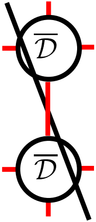

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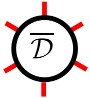
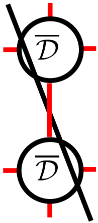
$\mathcal{D}^{(6)} =$ 

$-$ 

- $\mathcal{D}_\star^{(6)}$ satisfies (with $\eta_\star = 0$)

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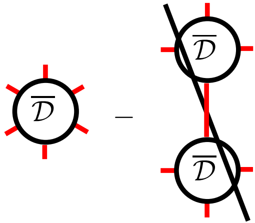

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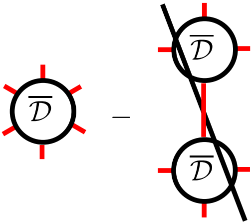

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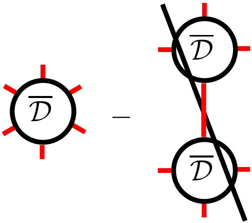

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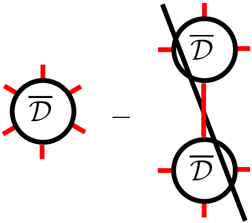

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- For $D \geq 4$ and $\eta_\star \geq 0$:
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- To complete the proof of the triviality of scalar field theory in $D \geq 4$, negative anomalous dimensions need to be considered
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Can this methodology be usefully applied to non-trivial theories??

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Thank you for listening