Critical phenomena in random field models: a nonperturbative functional renormalization group approach II

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Introduction

- Systems with disorder impurities, dislocations, etc.: generic situation!
- Influence of disorder on physical properties? in particular critical properties
- In case of Random field, new properties:
    - Dimensional reduction (relate disordered system to pure system in 2 spacial dimensions less)
    - hidden supersymmetry, spontaneously broken in some cases
    - quasi long-range order (?)
    - Critical hysteresis properties and avalanches.
- Need for alternative nonperturbative approaches.

Exact Equations

Description of the model: need to introduce arbitrary number of copies (replicas) $\vec{\phi}_a$ $\rightarrow$ enables to compute all cumulants. Partition function:

$$\mathcal{Z}_k[\{\vec{J}_a\}] = \int \mathcal{D}\vec{\phi}_a \exp\left\{ -\frac{1}{T}\sum_a \left(S[\vec{\phi}_a] + \int \left(\frac{1}{2}\vec{\phi}_a(q)\vec{\phi}_a(-q)\hat{R}_k(q) \right) \right) \right.$$

$$\left. + \frac{1}{2T^2}\sum_{a,b}\left(\int (\tilde{R}_k(q) + \Delta)\ \vec{\phi}_a(q)\vec{\phi}_b(-q) \right) + \sum_a \int \vec{J}_a(q)\vec{\phi}_a(-q) \right\}$$

Symmetries:

- Simultaneous rotation of copies ($O(N)$);
- Permutation of copies;
- Supersymmetry (somehow hidden in this formalism).

Flow Equations

- Legendre transform of $\log \mathcal{Z}[\{J_a\}]$: $\Gamma_k[\{\Phi_a\}]$
- Write Γ_k in an expansion in number of different replicas:

$$\Gamma_k[\{\Phi_a\}] = \sum_a \Gamma_{1,k}[\vec{\Phi}_a] - \frac{1}{2}\sum_{ab} \Gamma_{2,k}[\vec{\Phi}_a, \vec{\Phi}_b] + \dots$$

- Infinite hierarchy of coupled flow equations: Flow for $\Gamma_{n,k}$ depends on $\Gamma_{1,k} \dots \Gamma_{n+1,k}$
- Linear dependence on the effective temperature T_k.

$$\partial_t \Gamma_{n,k} = \mathcal{F}_n[\Gamma_{1,k}, \dots, \Gamma_{n+1,k}] + T_k\, \mathcal{G}_n[\Gamma_{1,k}, \dots, \Gamma_{n,k}]$$

  However, T_k flows to 0, so fixed point obtained for $T_k = 0$.
- Possible to study phase transition at $T = 0$.

Role of temperature

- Around fixed point, $T_k \sim k^{\theta}$: dangerously irrelevant term, like ϕ^4 coupling constant of Wilson-Fisher fixed point above $d = 4$.
  Modification of hyperscaling relation:

  $$\alpha = 2 - \nu(d - \theta)$$

  In mean field, $\theta = 2$, $\alpha = 0$, $\nu = 1/2$, upper critical dimension 6.
- Two 2-point correlation function. At criticality:
  $\overline{\langle\phi(q)\phi(-q)\rangle} - \overline{\langle\phi(q)\rangle\,\langle\phi(-q)\rangle} \sim q^{-2+\eta}$ and
  $\overline{\langle\phi(q)\rangle\langle\phi(-q)\rangle} - \overline{\langle\phi(q)\rangle}\;\overline{\langle\phi(-q)\rangle} \sim q^{-4+\bar{\eta}}$
  θ, η and $\bar{\eta}$ are not independent: $\theta = 2 + \eta - \bar{\eta}$
- Theory at finite temperature gives access to rare events as opposed to typical results (droplet picture).

Nonlinear σ model I

Around lower critical dimension (4 here, not 2, because of dimensional reduction), Radial excitations are frozen. field constrained in norm ($\phi^2 = 1$). Marginal operators:

- Kinetic term for 1-replica part:

$$\int d^d x \frac{1}{2g} (\partial \vec{\phi})^2$$

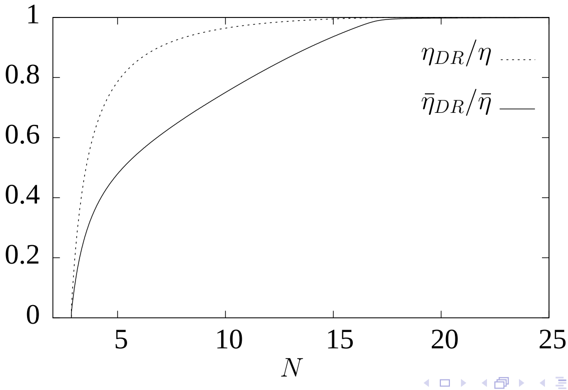
- Potential part for 2-replica part

$$\int d^d x \ V(\vec{\phi}_a, \vec{\phi}_b) = \int d^d x \ V(z = \cos\theta)$$

with θ the angle between 2 vectors. Full function to renormalize: functional renormalization group.

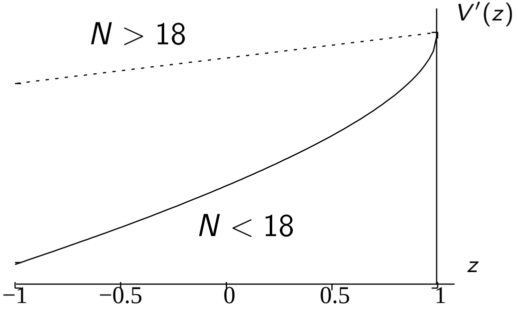
Integration of the flow equations displays 2 regimes:

- for $N > 18$, V is regular along the flow, and critical exponents satisfy dimensional reduction ($\eta = \frac{d-4}{N-2}$, $\nu = \frac{1}{d-4}$, etc.)
- for $N < 18$, nonanalyticity shows up at finite RG time (Larkin length): $V(z) \sim (1-z)^{3/2}$. Dimensional reduction is not satisfied.



Nonlinear σ model III

- Fixed point value for large and small N:

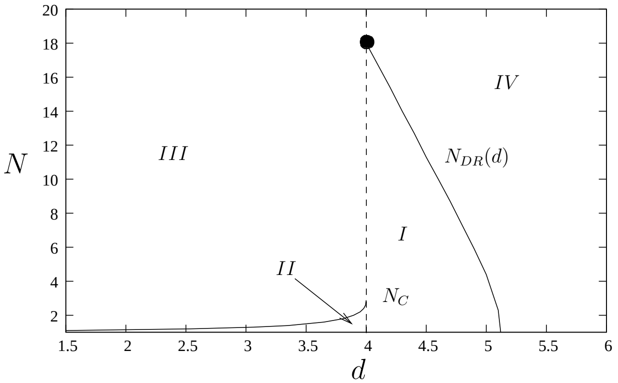


- for small N if we expand $V(z)$ around $z = 1$
  - At leading order ($V(z) = a + b(1-z)$), we retrieve dimensional reduction (wrong!)
  - At next to leading order ($V(z) = a + b(1-z) + c(1-z)^2$), see the Larkin length, but unable to go beyond.
  - Truncation with *a priori* knowledge of the form of the cusp: ($V(z) = a + b(1-z) + d(1-z)^{3/2} + c(1-z)^2$)

Truncations

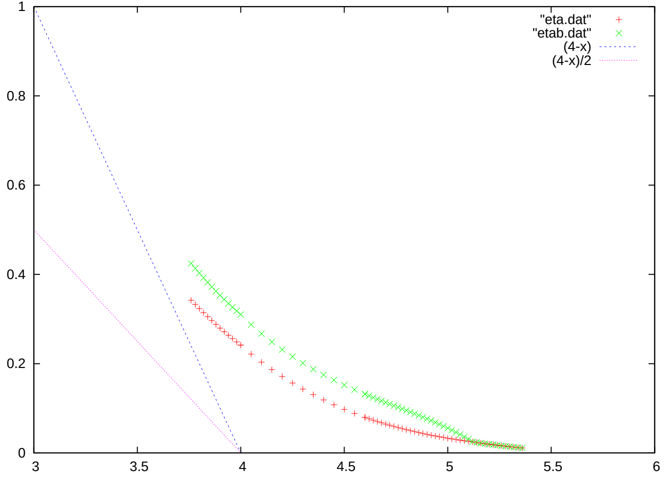
- Expansion in number of free replicas, keep only first 2 terms $\Gamma_{k,1}[\vec{\Phi}_a]$ and $\Gamma_{k,2}[\vec{\Phi}_a, \vec{\Phi}_b]$
- Derivative expansion: keep only Potential + Kinetic term in $\Gamma_{1,k}$ and only Potential for $\Gamma_{2,k}$
- Remains difficult to treat numerically because the 2-replica potential depends on 3 invariants for RFO(N), 2 for RFIM. Need extra truncations.
- Supersymmetry relates 1-replica kinetic term and 2-replica potential. Use truncation that do not violate this relation.
    - Expand in directions in which there is no cusp (RFO(N))
    - With knowledge on singularity, expand in the direction in which there is a cusp (RFIM).

Determination of the region where supersymmetry is spontaneously broken:



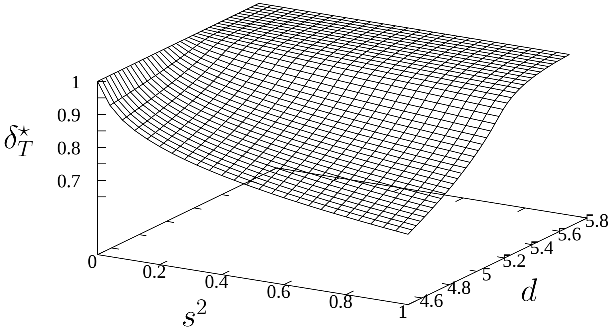
Critical properties II

Determination of the critical exponents (for RFIM), as function of dimension.



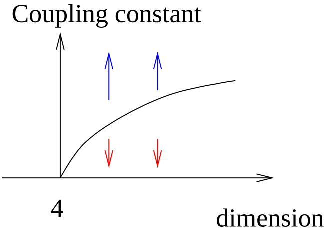
Critical properties III

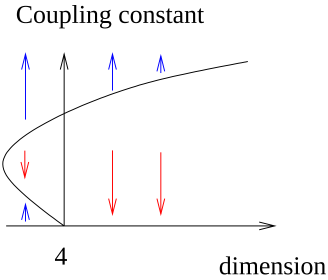
Appearance of a cusp at the fixed point.



Quasi Long-Range Order I

2-loop calculation and nonperturbative approach give the following behavior for N small:





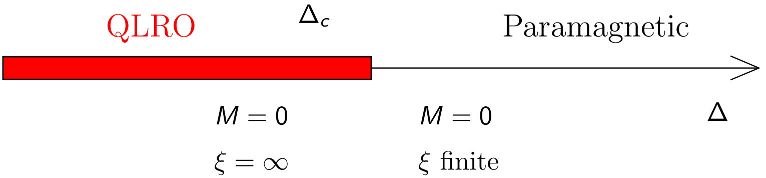
$N > 2.83\ldots$ $\qquad$ $2 < N < 2.83\ldots$

For $d < 4$, N small, there exists a fully stable fixed point.

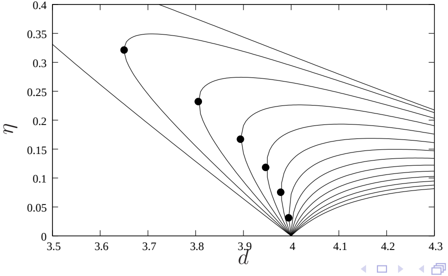
Criticality without fine-tuning! Similar to Self-Organized Criticality

Quasi Long-Range Order II

At fully stable fixed point, minimum of dimensionless potential is finite: magnetization is 0.



2 sets of critical exponents: in low disorder phase, and at transition.



Quasi Long-Range Order III

- Proposed to explain physics of Bragg glass [Le Doussal, Giamarchi] experimental results on vortex lattices in type II superconductor, pinned by disorder.
- However QLRO exist only for $d > 3.8$
- Actual hamiltonian has same spin-wave modes but different massive modes

Conclusions

- Framework which enables to study the theory whether or not supersymmetry is spontaneously broken.
- Numerical work to obtain critical exponents.
- Supersymmetry breaking associated with non-zero order parameter (analogy with Gross-Neveu model)? [Wschebor, Tarjus, MT]
- 0-d model? [Wschebor, Tarjus, MT]
- Spin-glass phases in RFIM [Mouhanna, Tarjus]
- Spontaneous supersymmetry breaking in other theories (Critical dynamics)? [Delamotte, Kzakala, Tarjus, MT]