Beyond the static approximation in the fermionic fRG

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Outline

Goal: Functional differential equation for effective action Γ.

Multiplicative cut-off: $G_{0,k}^{-1} = G_0^{-1} M_k$

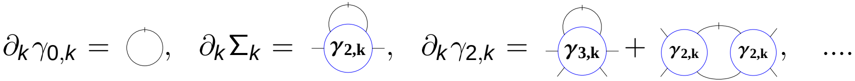
$$\frac{\partial}{\partial k}\Gamma_k[\phi^*,\phi] = -\frac{1}{2}\mathrm{Tr}\left\{\left([\Gamma_k^{(2)}[\phi^*,\phi] + G_{0,k}^{-1}]^{-1} - G_{0,k}\right)\frac{\partial}{\partial k}G_{0,k}^{-1}\right\}$$

Additive cut-off R_k:

$$\frac{\partial}{\partial k}\Gamma_k[\phi^*,\phi] = -\frac{1}{2}\mathrm{Tr}\left\{[\Gamma_k^{(2)}[\phi^*,\phi] + R_k]^{-1}\frac{\partial}{\partial k}R_k\right\}$$

Truncation schemes

- Expand flow equation in terms of vertex functions



Static approximation: neglect flow of $\gamma_{2,k}$,
energy independent flow of $\gamma_{2,k}$.

- Derivative expansion of effective action

$$\Gamma_k[\phi^*, \phi] = \int_0^\beta \mathrm{d}\tau \sum_{\alpha=1}^{N} \phi_\alpha^*(\tau) \frac{\partial}{\partial \tau} \phi_\alpha(\tau) + U_k\left(\phi^*(\tau), \phi(\tau)\right)$$

- Both methods yield the same flow equations for Σ_k, $\gamma_{2,k}$ in static approximation
(MW, D. Sibold, PRB 77, 125309 (2008))

Expand U according to its Grassmann structure

$$U(\phi^*,\phi) = \sum_{j=1}^{N} a_{jj}\phi_j^*\phi_j + a_{j,j+1}\phi_j^*\phi_{j+1} + a_{j,j-1}\phi_j^*\phi_{j-1} + \mathcal{U}\sum_{j=1}^{N}\phi_j^*\phi_j\phi_{j+1}^*\phi_{j+1}$$

Flow equations for running couplings a:

$$a'_{jj} = \frac{\mathcal{U}}{2\pi}\sum_{\lambda=\pm k}\left(g_{j+1,j+1}(i\lambda) + g_{j-1,j-1}(i\lambda)\right)$$

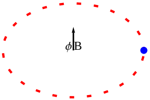
$$a'_{j,j\pm 1} = -\frac{\mathcal{U}}{2\pi}\sum_{\lambda=\pm k} g_{j,j\pm 1}(i\lambda), \qquad g_{ij}^{-1} = a_{ij} + i\lambda\delta_{ij}$$

Beyond the static approximation: Wavefunction renormalization Z_k

$$\Gamma_k[\phi^*, \phi] = \int_0^\infty \mathrm{d}\tau \; Z_k(\phi^*, \phi) \; \phi^*(\tau) \frac{\partial}{\partial \tau} \phi(\tau) + U_k(\phi^*, \phi)$$

Expand U and Z acoording to its Grassmann structure.

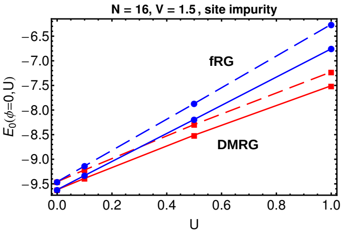
Mesoscopic rings with a single impurity

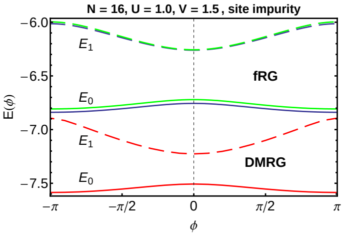


1D Hubbard model:

$$H = -t \sum_{j=1}^{N} \left(c_j^\dagger c_{j+1} \exp^{i\phi/N} + \text{h.c.} \right) + U \sum_{j=1}^{N} n_j n_{j+1} + V n_1$$

periodic boundary conditions, half filling, $t = 1$

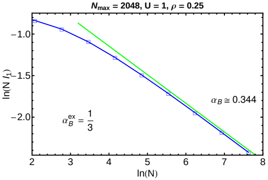




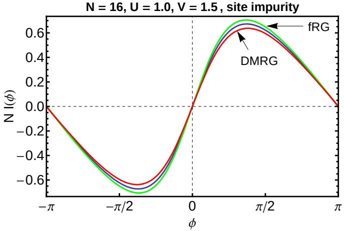
γ_2 renormalization included in a static approximation.

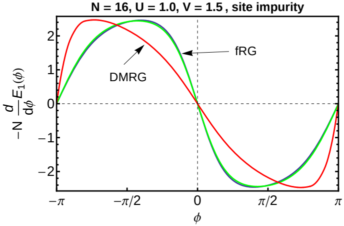
DMRG calculations by Andrej Gendiar (Bratislava)

Mesoscopic rings with a single impurity



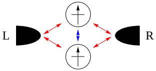
Persistent currents:





Result: For larger interactions fRG and DMRG agree qualitatively but not quantitatively.

Coupled quantum dots



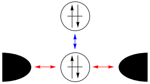
$$H = H_{\rm D} + H_{\rm E} + H_{\rm DE}$$

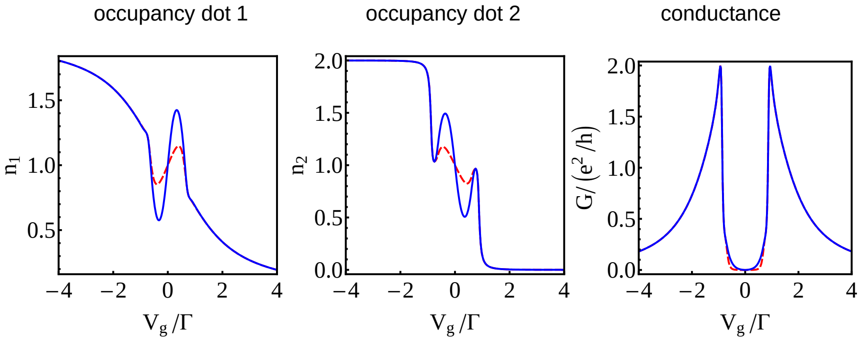
$$H_{\rm D} = \sum_{j\sigma} (\epsilon_{j\sigma} + V_g) c^\dagger_{j\sigma} c_{j\sigma} - \sum_{j>j',\sigma} \left(t_{jj'} c^\dagger_{j\sigma} c_{j'\sigma} + h.c. \right) + \frac{1}{2} \sum_{jj',\sigma\sigma'} U^{\sigma\sigma'}_{jj'} n_{j\sigma} n_{j'\sigma'}$$

$$H_{\rm E} = -\tau_h \sum_{j\sigma l} d^\dagger_{j\sigma l} d_{(j+1)\sigma l} + h.c.$$

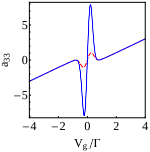
$$H_{\rm DE} = -\sum_{j\sigma l} t^l_j d^\dagger_{0,\sigma,l} c_{j,\sigma} + h.c. \qquad (l = L,\ R)$$

Side coupled double QD with small interdot hopping:

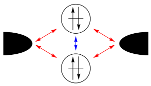


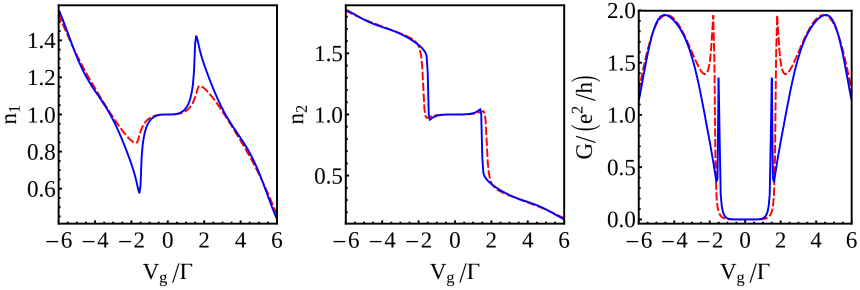


$t_{12}/\Gamma = .2$, $U/\Gamma = 2$, $\Gamma_1^L = \Gamma_1^R = \Gamma/2$



Parallel coupled double QD:





$t_{12}/\Gamma = 0.$, $U/\Gamma = 4$, $\Gamma_1^L/\Gamma = .5$, $\Gamma_1^R/\Gamma = .25$, $\Gamma_2^L = 0.07$, $\Gamma_2^R = 0.18$

Summary

- In static approximation expansion in terms of vertex functions and derivative expansion yield the same set of ODE.
- This set develops unphysical singularities for large interactions. To fix this requires e.g. wave function renormalizations.
- Hubbard model for 1D chains. Comparison with DMRG
- Hubbard-like model for coupled quantum dots