Quantum turbulence and vortex reconnections

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(http://research.ncl.ac.uk/quantum-fluids/)

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Summary

Context: quantum fluids (superfluid helium, atomic condensates)

- Gross-Pitaevskii model
- Vortex filament model
- Classical vortex reconnections
- Quantum vortex reconnections

Gross Pitaevskii Equation

- Macroscopic wavefunction $\Psi = |\Psi| e^{i\phi}$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi + g\Psi|\Psi|^2 - \mu\Psi \qquad \text{(GPE)}$$

- Density $\rho = |\Psi|^2$, Velocity $\mathbf{v} = (\hbar/m)\nabla\phi$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \text{(Continuity)}$$

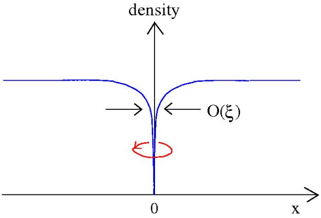
$$\rho \left(\frac{\partial v_j}{\partial t} + v_k \frac{\partial v_j}{\partial x_k} \right) = -\frac{\partial p}{\partial x_j} + \frac{\partial \Sigma_{jk}}{\partial x_k} \qquad (\sim \text{Euler})$$

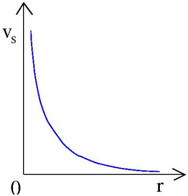
- Pressure $p = \frac{g}{2m^2}\rho^2$, Quantum stress $\Sigma_{jk} = \left(\frac{\hbar}{2m}\right)^2 \rho \frac{\partial^2 \ln \rho}{\partial x_j \partial x_k}$

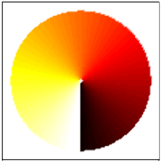
- At length scales $\gg \xi = (\hbar^2/m\mu)^{1/2}$ neglect Σ_{jk} and recover compressible Euler

Vortex solution of the GPE

Vortex: hole of radius $\approx \xi$, around it the phase changes by 2π



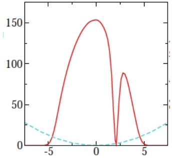


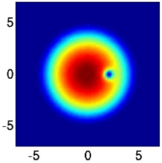


Phase

$$\oint_C \mathbf{v}_s \cdot \mathbf{dr} = \frac{h}{m} = \kappa$$

Quantum of circulation

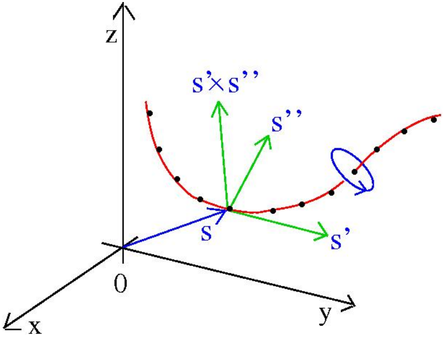




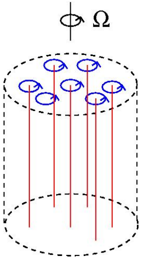
Vortex filament model

- At length scales $\gg \xi \Rightarrow$ GPE becomes compressible Euler
- Away from vortices at speed $\ll c \Rightarrow$ recover incompressible Euler
- Vorticity in thin filaments $\Rightarrow$ Biot-Savart law
- Reconnections performed algorithmically

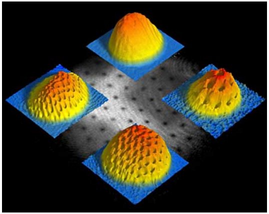
$$\frac{d\mathbf{s}}{dt} = \frac{\kappa}{4\pi} \oint \frac{(\mathbf{z}-\mathbf{s}) \times \mathbf{dz}}{|\mathbf{z}-\mathbf{s}|^3}$$



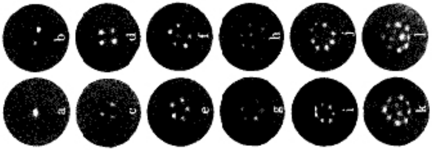
Observations of individual quantum vortices



(Maryland)

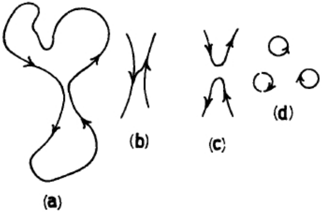


(MIT)



(Berkeley)

Feynman 1955



Consider a large distorted ring vortex (a). If, in a place, two oppositely directed sections of line approach closely, the situation is unstable, and the lines twist about each other in a complicated fashion, eventually coming very close, in places within an atomic spacing. Consider two such lines (b). With a small rearrangement, the lines (which are under tension) may snap together and join connections in a new way to form two loops (c). Energy released this way goes into further twisting and winding of the new loops. This continue until the single loop has become chopped into a very large number of small loops (d)

Quantum turbulence

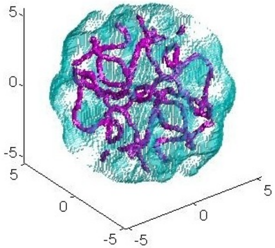
ξ = vortex core, ℓ = average vortex spacing, D = system size

Superfluid ^{4}He and ^{3}He-B:

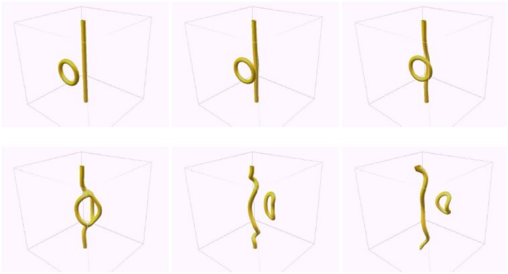
- uniform density,
- $\xi \ll \ell \ll D$
  huge range of length scales
- parameters fixed by nature

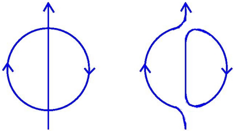
Atomic condensates:

- non-uniform density,
- $\xi < \ell < D$
  restricted length scales
- control geometry, dimensions, strength/type of interaction

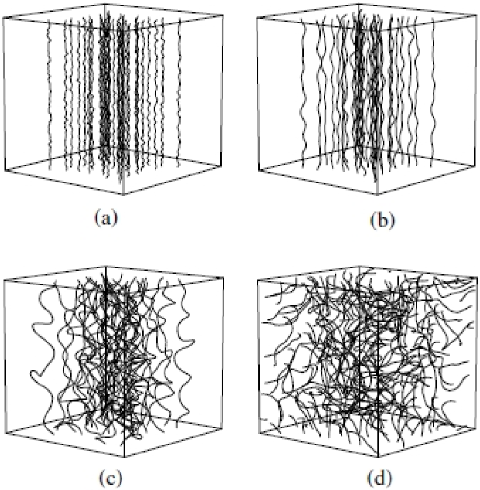


Reconnection of a vortex ring with a vortex line





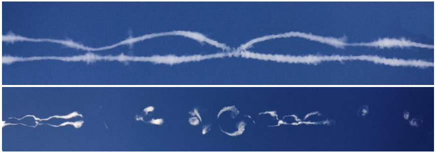
Quantum turbulence

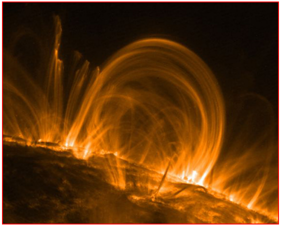


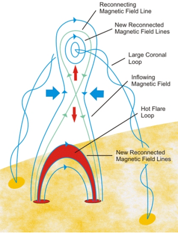
Tsubota, Arachi & Barenghi, PRL 2003

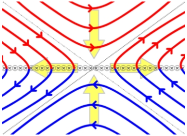
Vortex reconnections in ordinary fluids

Classical reconnection of trailing vortices following the Crow instability





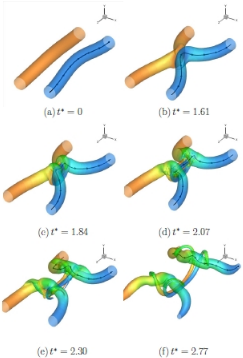




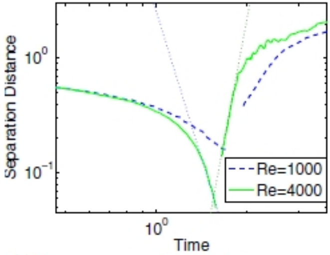
Magnetic reconnection

Vortex reconnections in ordinary fluids

Hussain & Duraisamy 2011



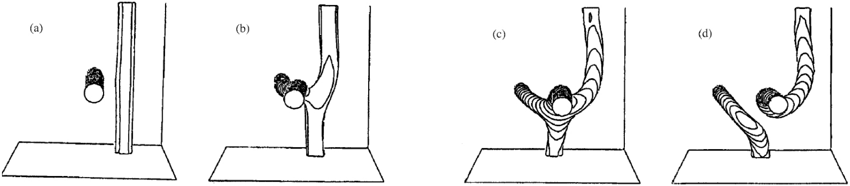
Note the bridges

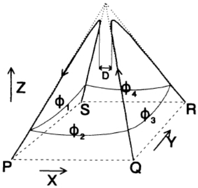


$$\delta(t) \sim (t_0 - t)^{3/4} \text{ before}$$
$$\delta(t) \sim (t - t_0)^2 \text{ after}$$

Quantum vortex reconnections

Koplik & Levine 1993: first GPE reconnection





Aarts & De Waele 1994: cusp is universal

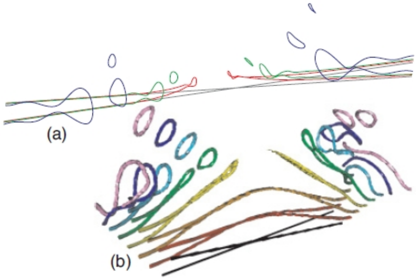
Tebbs, Youd & Barenghi 2011: cusp is not universal

Nazarenko & West 2003: analytic

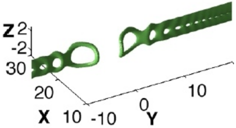
Alamri, Youd & Barenghi: bridges, PRL 2008

Quantum vortex reconnections

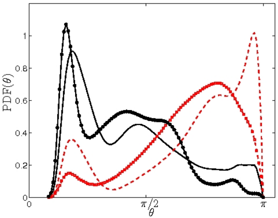
"Cascade of loops" scenario



Kursa, Bajer, & Lipniacki 2011
only if angle $\theta \approx \pi$



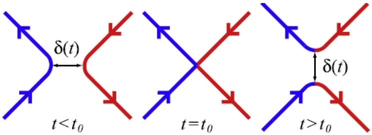
Kerr 2011



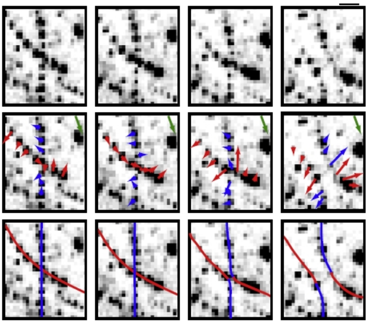
Distribution of θ in turbulence
Sherwin, Baggaley, Barenghi, & Sergeev 2012

Quantum vortex reconnections

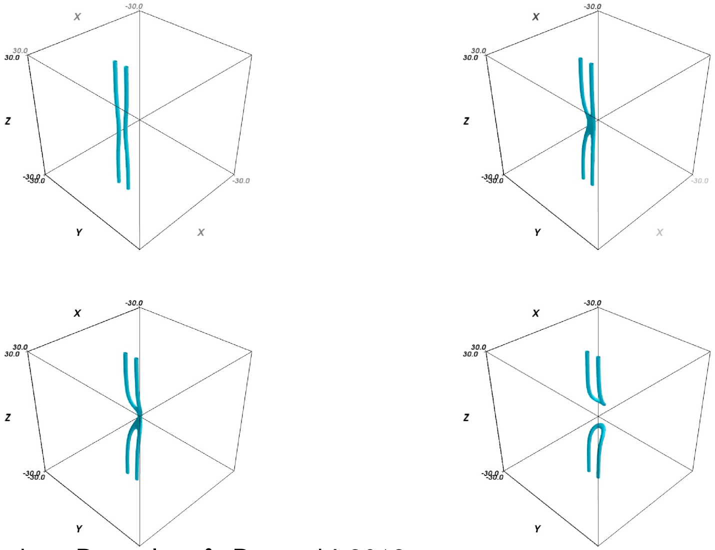
Direct observation of quantum vortex reconnections:
lines visualised by micron-size trapped solid hydrogen particles
Bewley, Paoletti, Sreenivasan, & Lathrop 2008



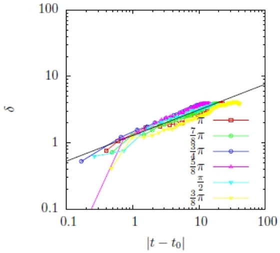
$$\delta(t) \sim (t_0 - t)^{1/2} \text{ before}$$
$$\delta(t) \sim (t - t_0)^{1/2} \text{ after}$$

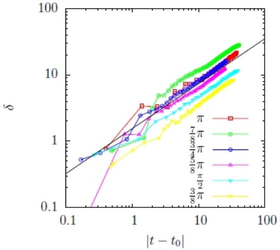


Quantum vortex reconnections



Zuccher, Baggaley, & Barenghi 2012





GPE reconnections:
$\delta(t) \sim (t_0 - t)^{0.39}$ before
$\delta(t) \sim (t - t_0)^{0.68}$ after

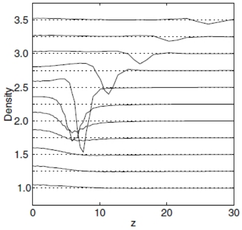
Biot-Savart reconnections:
$\delta(t) \sim |t_0 - t|^{1/2}$ before and after

Why the difference between GPE and Biot-Savart reconnections ?
Why the difference between GPE and experiments ?

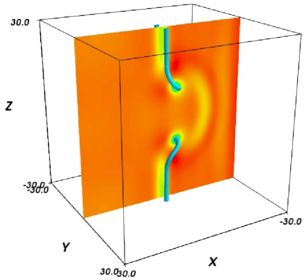
Zuccher, Baggaley, & Barenghi 2012

Quantum vortex reconnections

Sound wave emitted at reconnection event



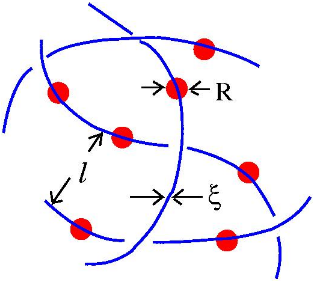
Leabeater, Adams, Samuels, & Barenghi 2001



Zuccher, Baggaley, & Barenghi 2012

Conclusions

- Vortex reconnections are essential for turbulence
- Analogies between classical and quantum vortex reconnections:
  bridges, time asymmetry
- Visualization of individual vortex reconnections
- Cascade of vortex loops scenario ?
- Time asymmetry probably related to acoustic emission
- GPE, Biot-Savart and experiments probe different length scales:



vortex core $\xi \approx 10^{-8}$ cm
tracer particle $R \approx 10^{-4}$ cm
intervortex distance $\ell \approx 10^{-2}$ cm