

# Ground state and excitations in BEC of magnons

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Group of NonLinear  
Magnetic Dynamics

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Magnons???

Ground state of a FM:

$$S_z = Ns_z$$

Excited states:

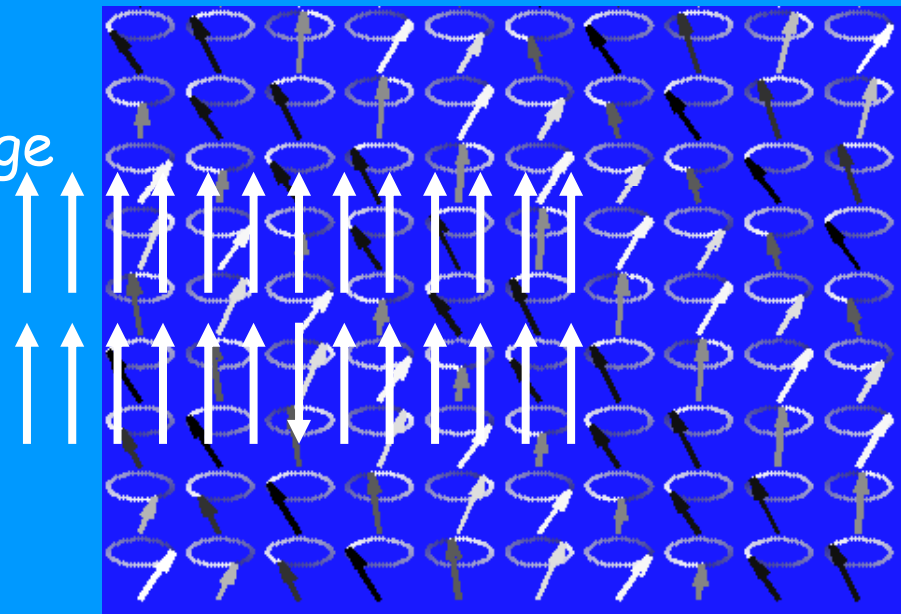
$$S_z = Ns_z - 1, S_z = Ns_z - 2, S_z = Ns_z - 3, \dots$$

1 magnon, 2 magnons, 3 magnons, ...

⇒ magnons are Bose-particles

Carry transverse magnetization

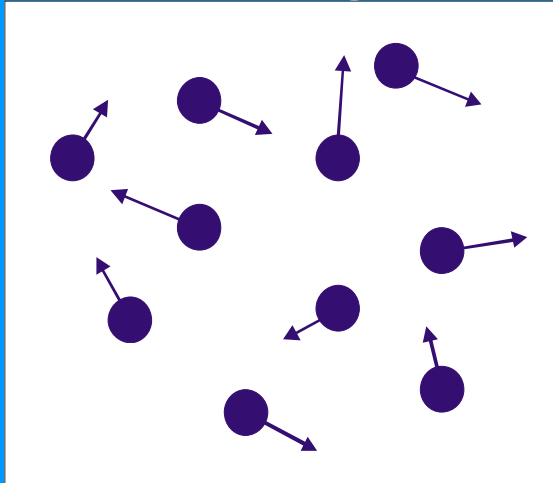
Spin waves ⇒ magnons



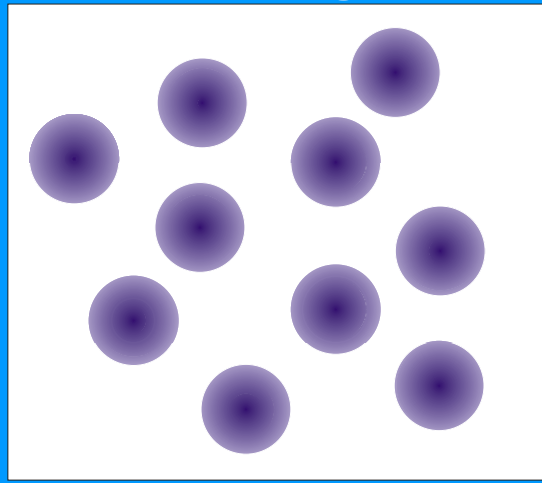
Courtesy: Prof. C. Patton

# Bose-Einstein-Condensation of atoms

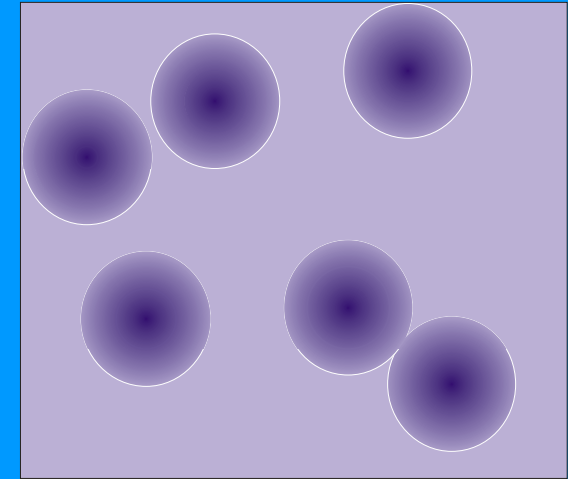
classical gas



quantum gas



BEC



Condition of BEC transition:

$$kT_c = 3.31 \frac{\hbar^2}{m} N^{2/3}$$

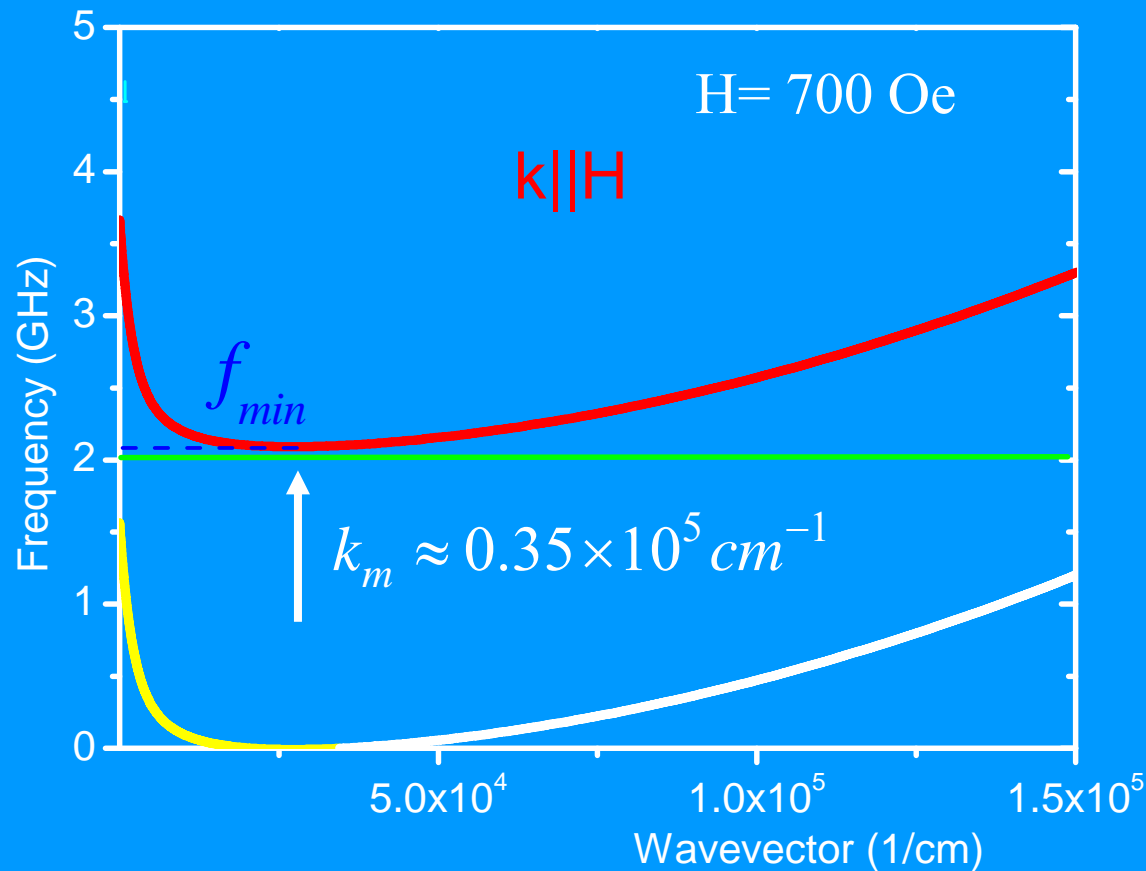
$$N_c^{2/3} = kT \frac{m}{3.31 \hbar^2}$$

Thermodynamics of BEC:

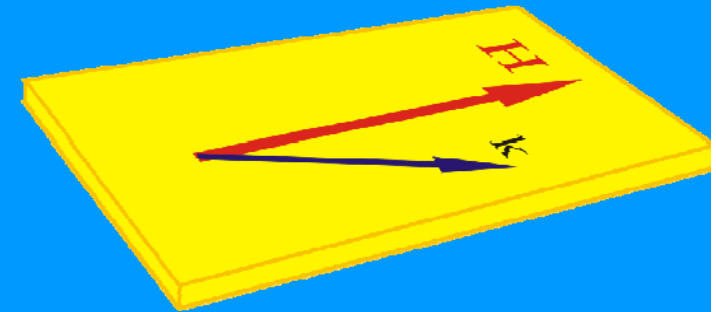
$$n = \frac{1}{\exp\left(\frac{E - \mu}{kT}\right) - 1}$$
$$\mu(T_c, N_c) = E_{\min}$$

# Magnons in ferromagnetic films

YIG (yttrium-iron-garnet)



Transparent ferromagnet  
Films 5-10  $\mu\text{m}$  thick  
No domains

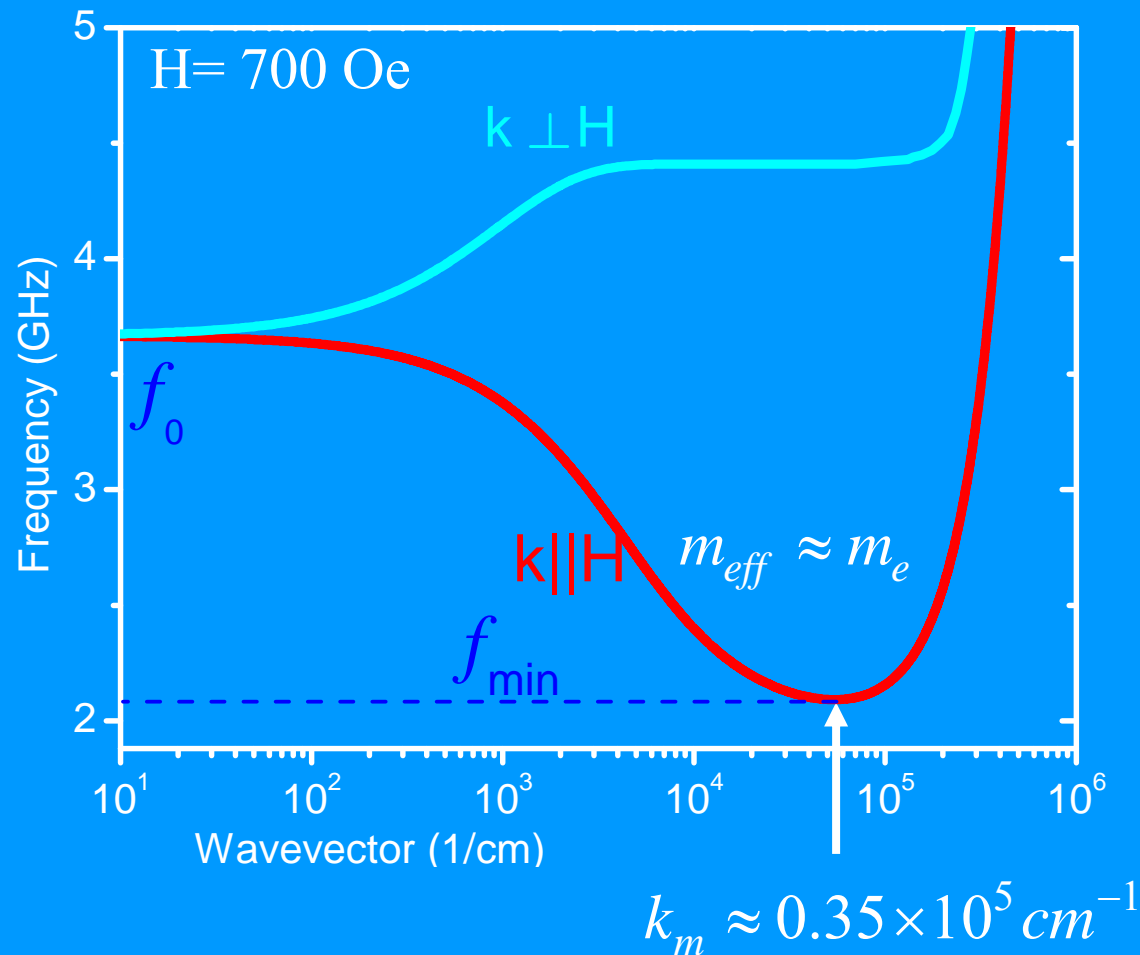


Three contributions to the magnon energy: Zeeman, exchange, and dipole-dipole

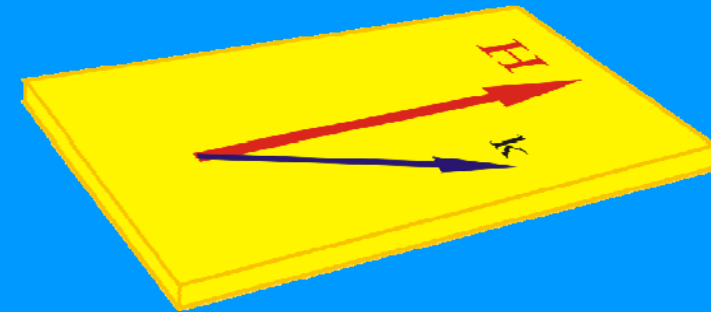
Scattering amplitude depends on wavevector

# Magnons in ferromagnetic films

YIG (yttrium-iron-garnet)



Transparent ferromagnet  
Films 5-10  $\mu\text{m}$  thick  
No domains



$$E_{\text{min}} = h \times 2\text{GHz} =$$

$$= k_B \times 100\text{mK} = 10\mu\text{eV}$$

$$kT_c = 3.31 \frac{\hbar^2}{m} N^{2/3}$$

# (Thermo)dynamic of magnons

In equilibrium:

$$\mu = 0 < E_{\min}$$

at any temperature

Magnons are quasi-particles with variable  $N(T)$ .  
In equilibrium with the lattice ( $F=F_{\min}$ ).

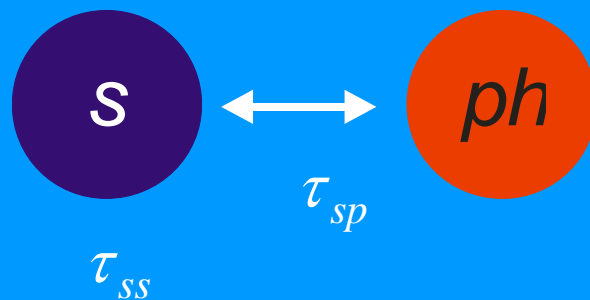
Therefore:

$$\mu = \frac{\partial F}{\partial N} = 0$$

$$E_{\min} > 0.$$

No BEC possible.

In quasi-equilibrium:



We can change  $N$

Two important time scales:  $\tau_{ss}$   $\tau_{sp}$

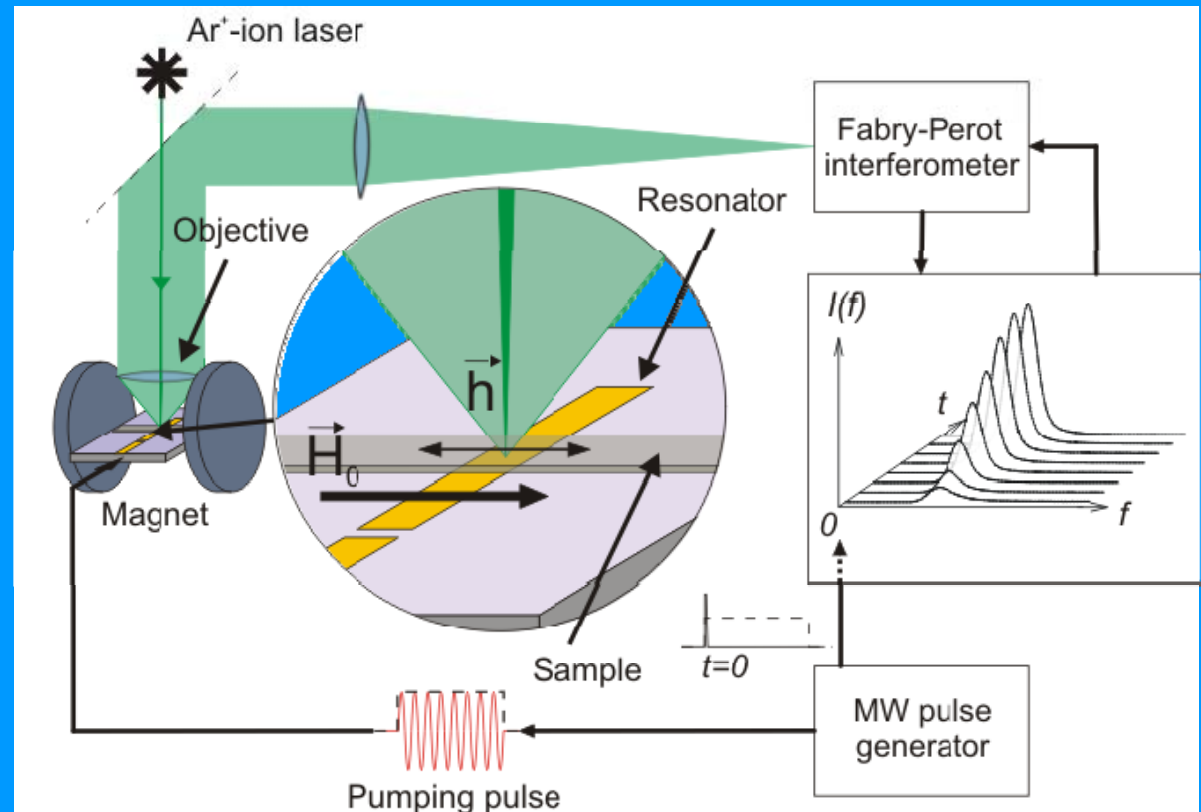
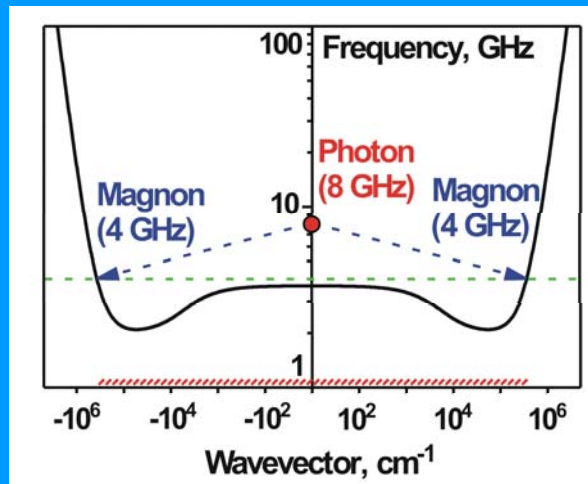
In YIG:

$$\tau_{ss} \approx 10 - 50 ns$$

$$\tau_{sp} \approx 0.2 - 0.5 \mu s$$

# Experimental setup for BEC observation

Magnons created by microwaves and detected by light scattering with time and space resolution

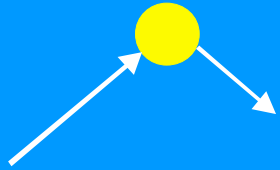


Two thresholds:

#1: pumping itself

#2: BEC

# Mechanisms of magnon thermalization



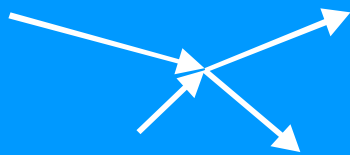
Two-magnon scattering

$$\omega_1 = \omega_2$$

$$k_1 \neq k_2$$

Impurity-scattering, linear effect  
(independent of the magnon density)

Elastic,  $k$ -thermalization



Four-magnon scattering:

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

$$k_1 + k_2 = k_3 + k_4$$

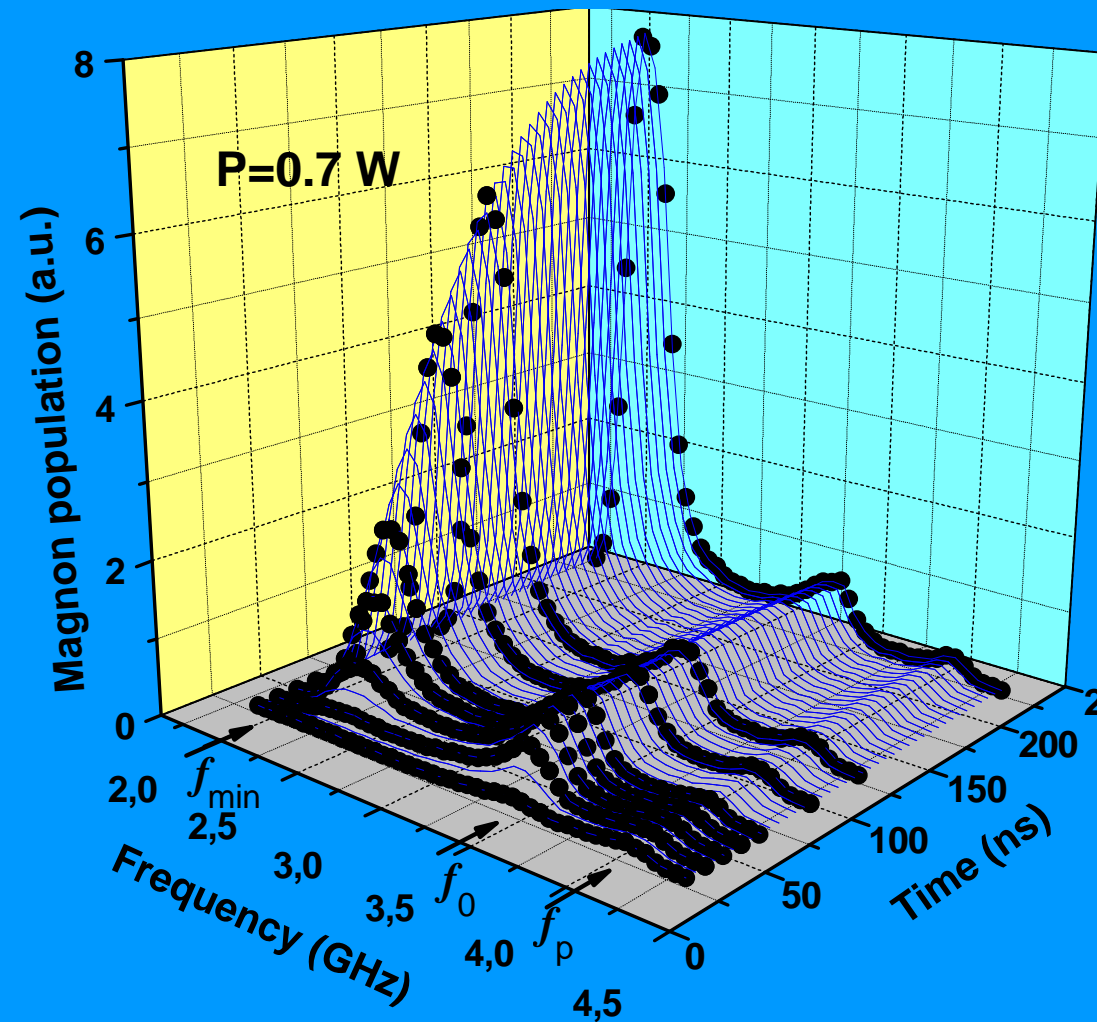
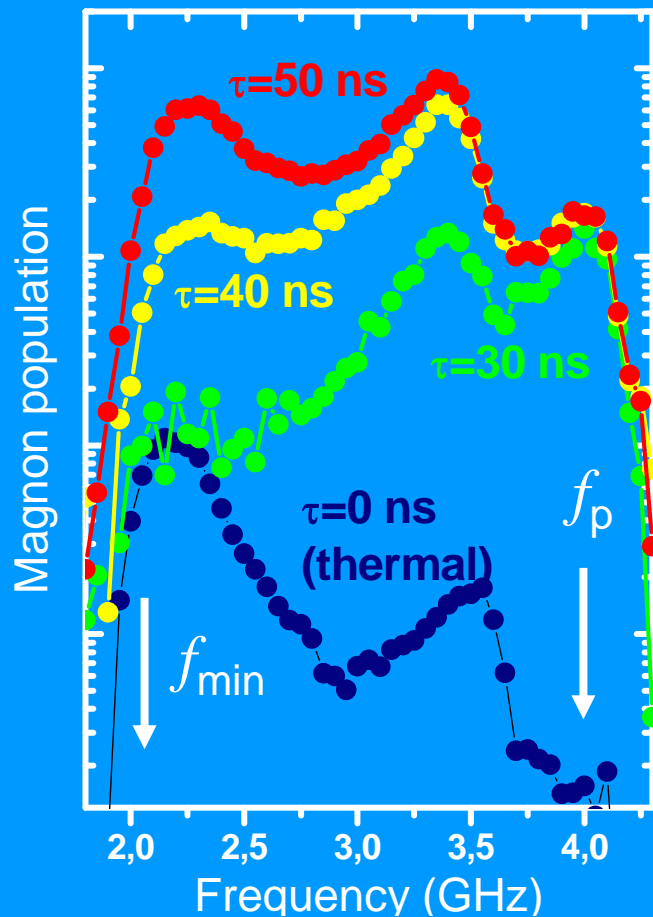
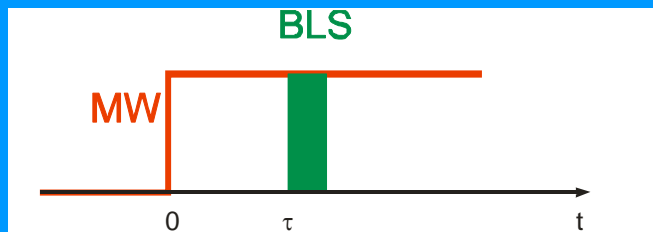
Nonlinear effect  
(increase with increasing density)

Inelastic,  $\omega, k$ -thermalization

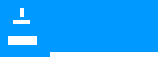
Magnon-magnon scattering keeps the number of particles constant



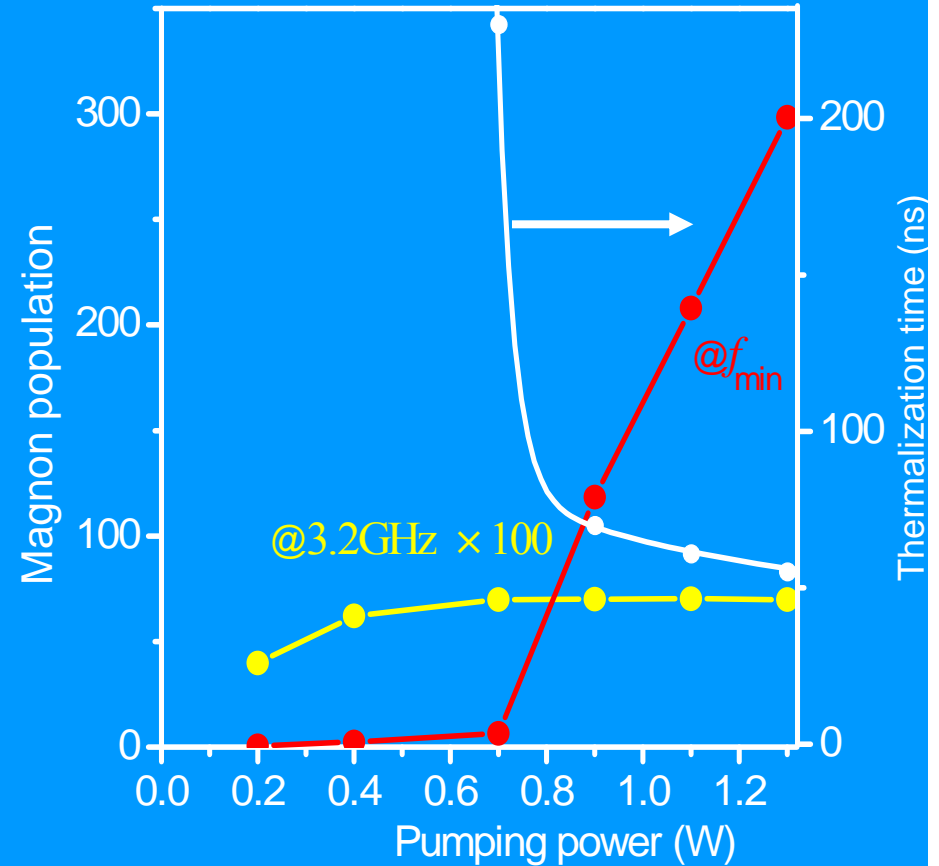
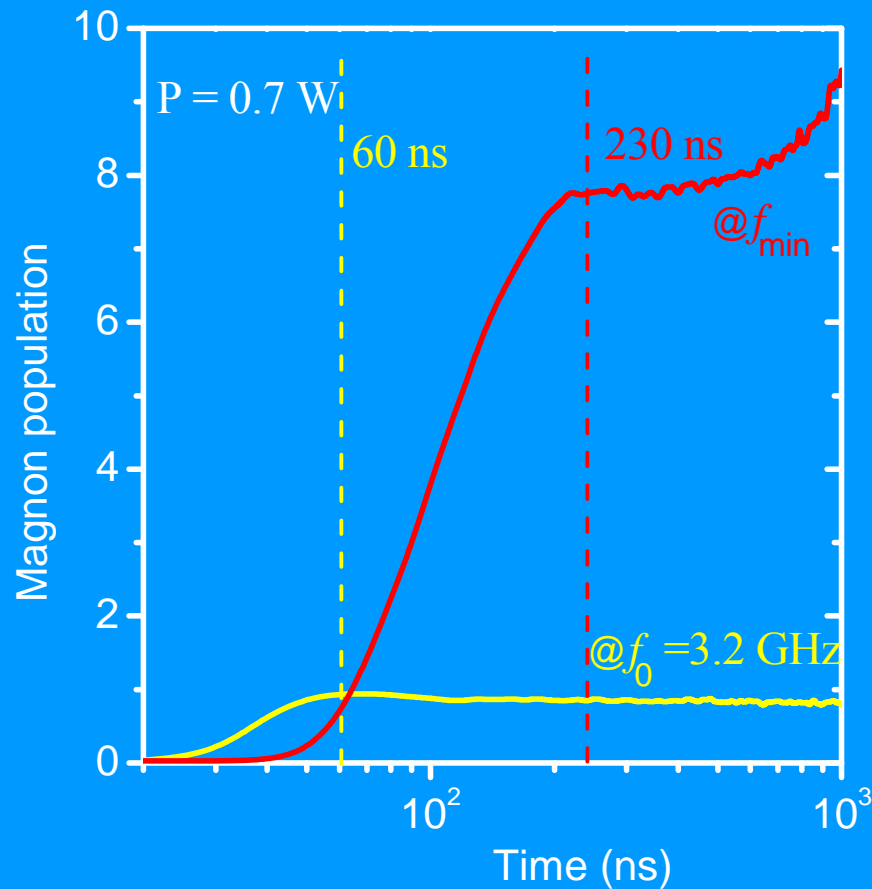
# Magnon thermalization (step-like pumping)



Thermalization happens  
„wave-like“



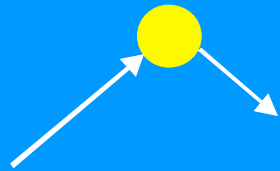
# Thermalization time



Thermalization time depends on the pumping power/magnon density  
At high magnon densities is below 50 ns.

Phys. Rev.Lett. '07.

# Mechanisms of magnon thermalization



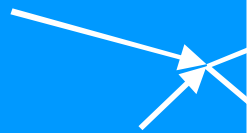
Two-magnon scattering

$$\omega_1 = \omega_2$$

$$k_1 \neq k_2$$

Impurity-scattering, linear effect  
(independent of the magnon density)

Elastic,  $k$ -thermalization



Under external influence magnon gas in YIG first thermalizes itself to a quasi-equilibrium (and then relax as a whole, if pumping is switched off)

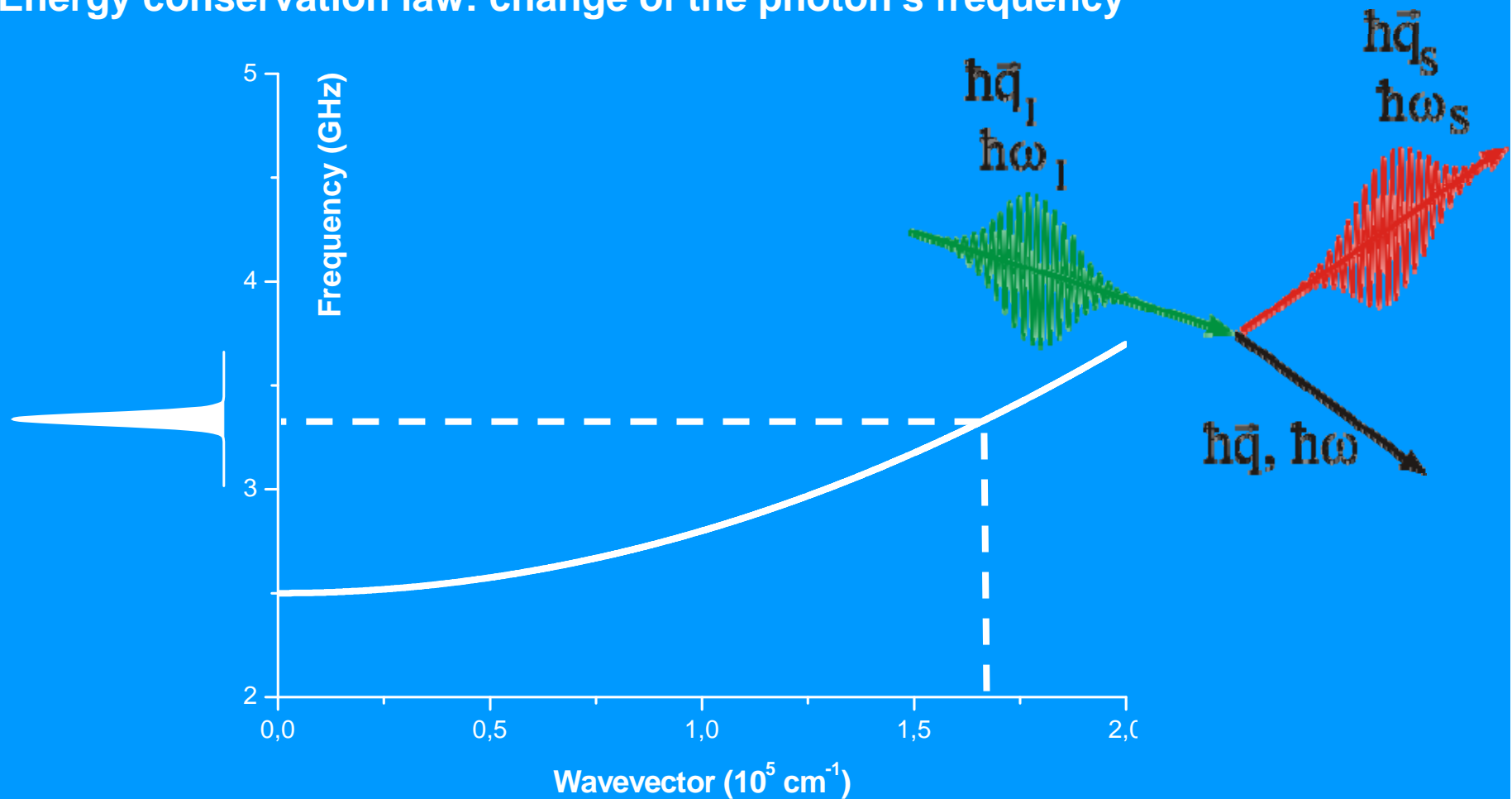
(density)

Magnon-magnon scattering keeps the number of particles constant

Thermalization happens fast if the number of magnons is high enough

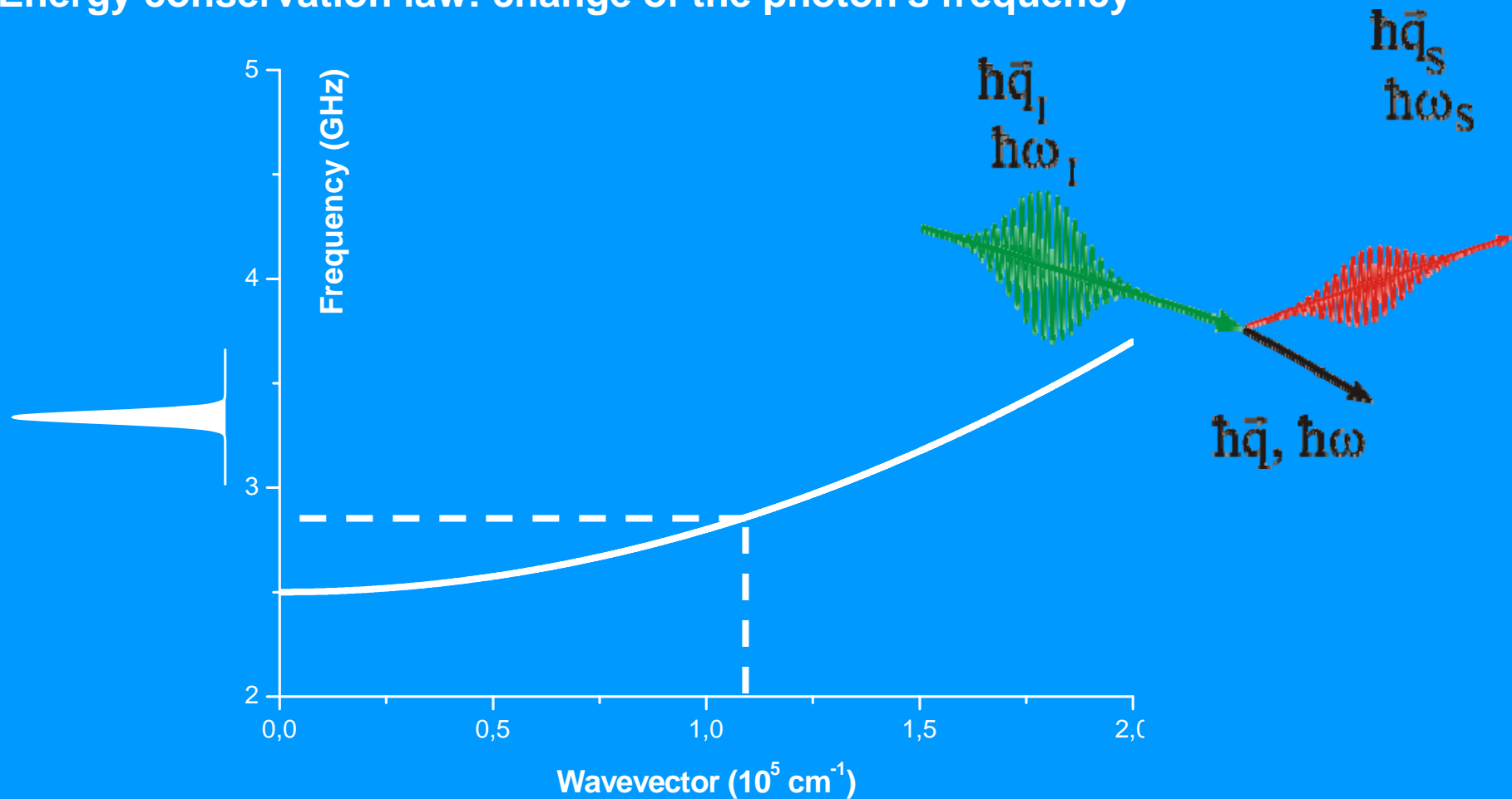
# Brillouin Light Scattering

Momentum conservation law: the geometry defines the spin-wave wavevector  
Energy conservation law: change of the photon's frequency

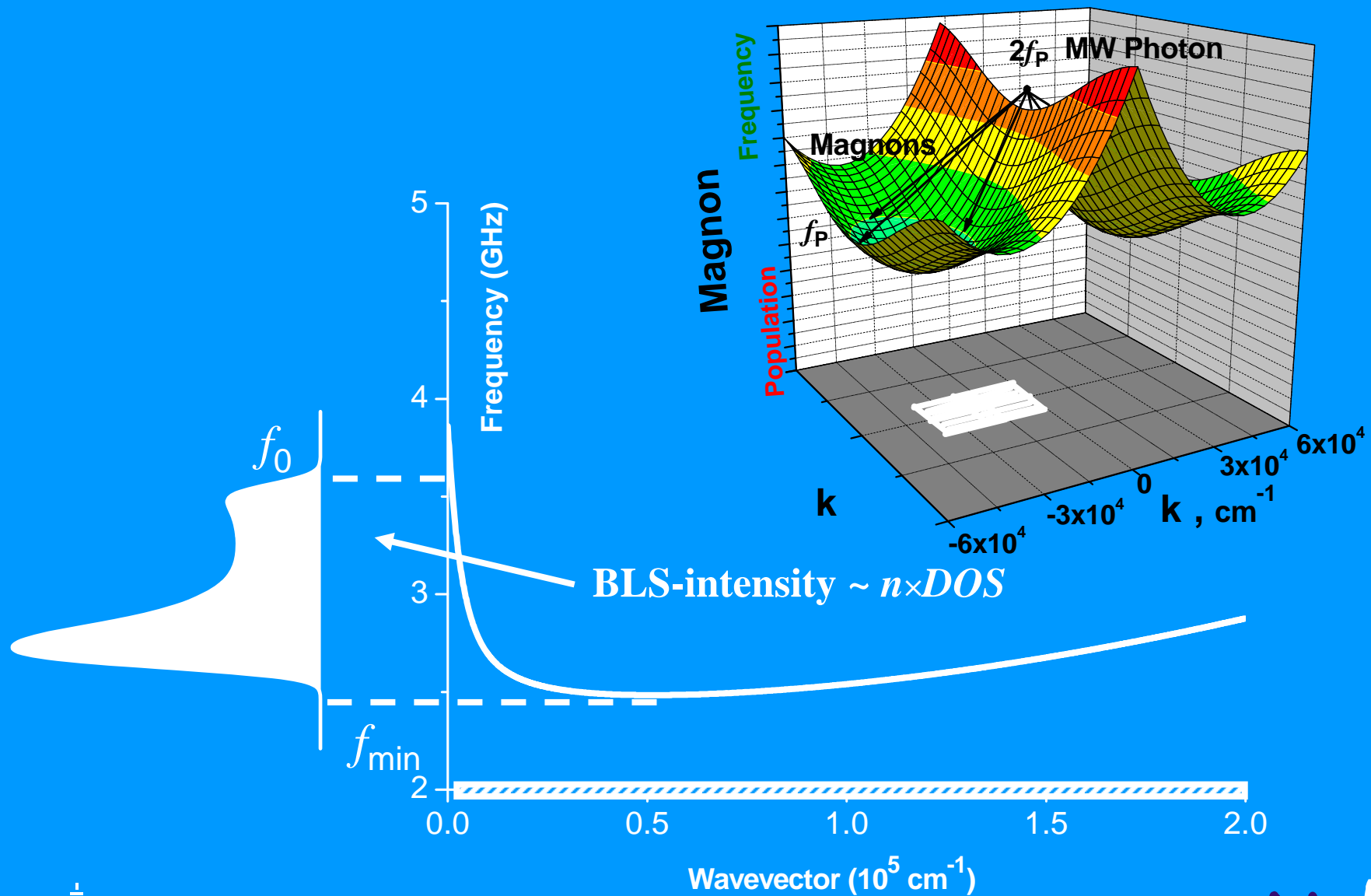


# Brillouin Light Scattering

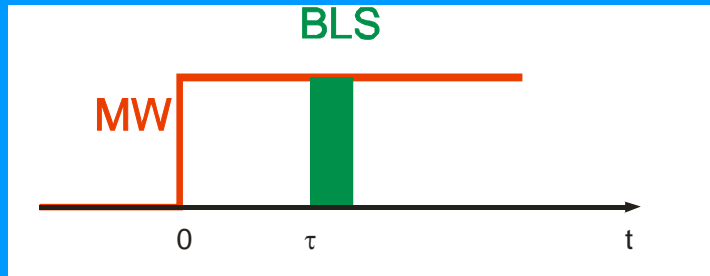
Momentum conservation law: the geometry defines the spin-wave wavevector  
Energy conservation law: change of the photon's frequency



# BLS spectroscopy



# Pumped magnons (step-like pumping)

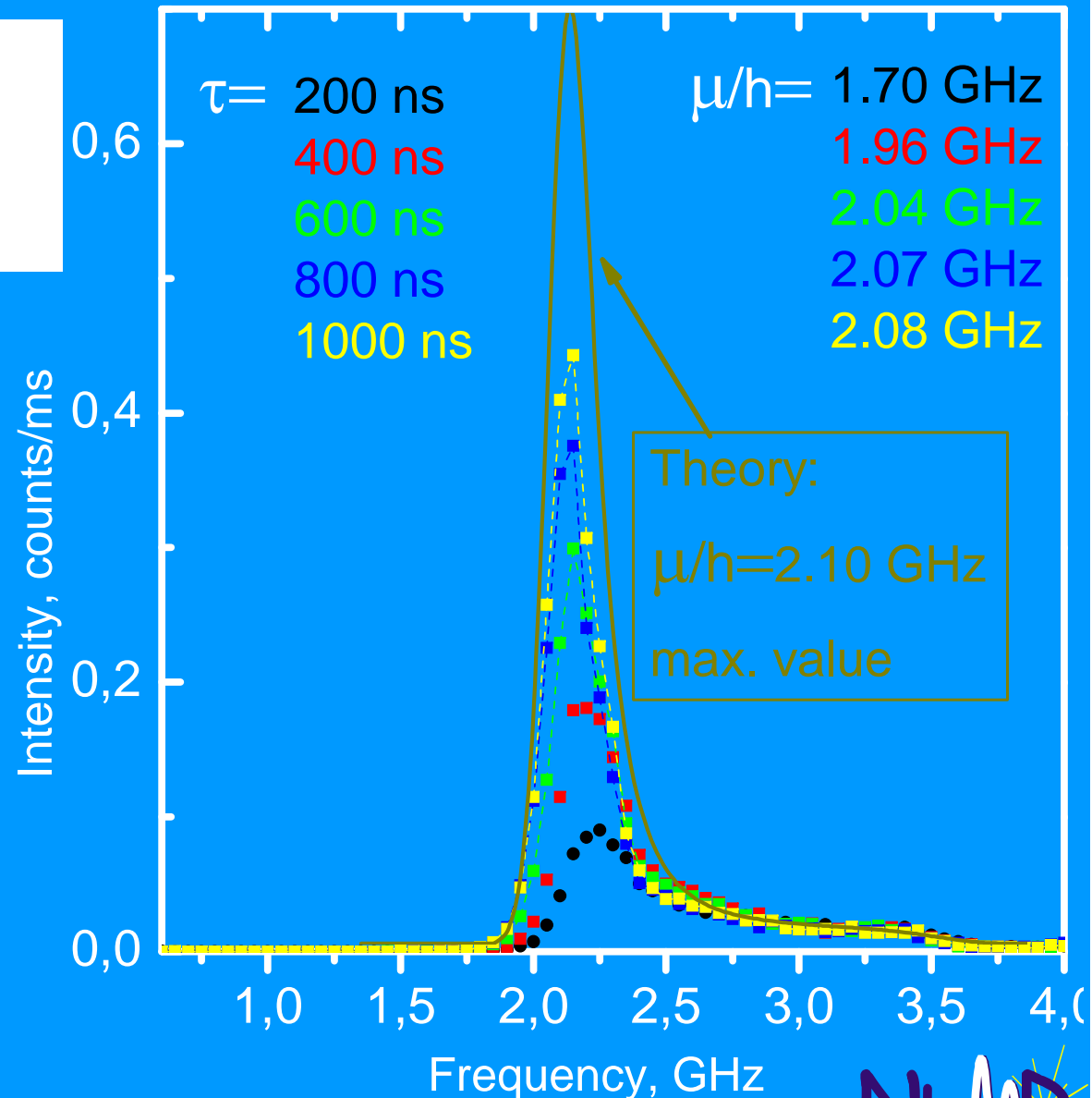


$P = 4 \text{ W}$

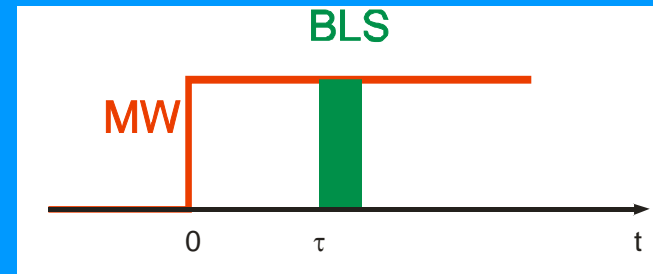
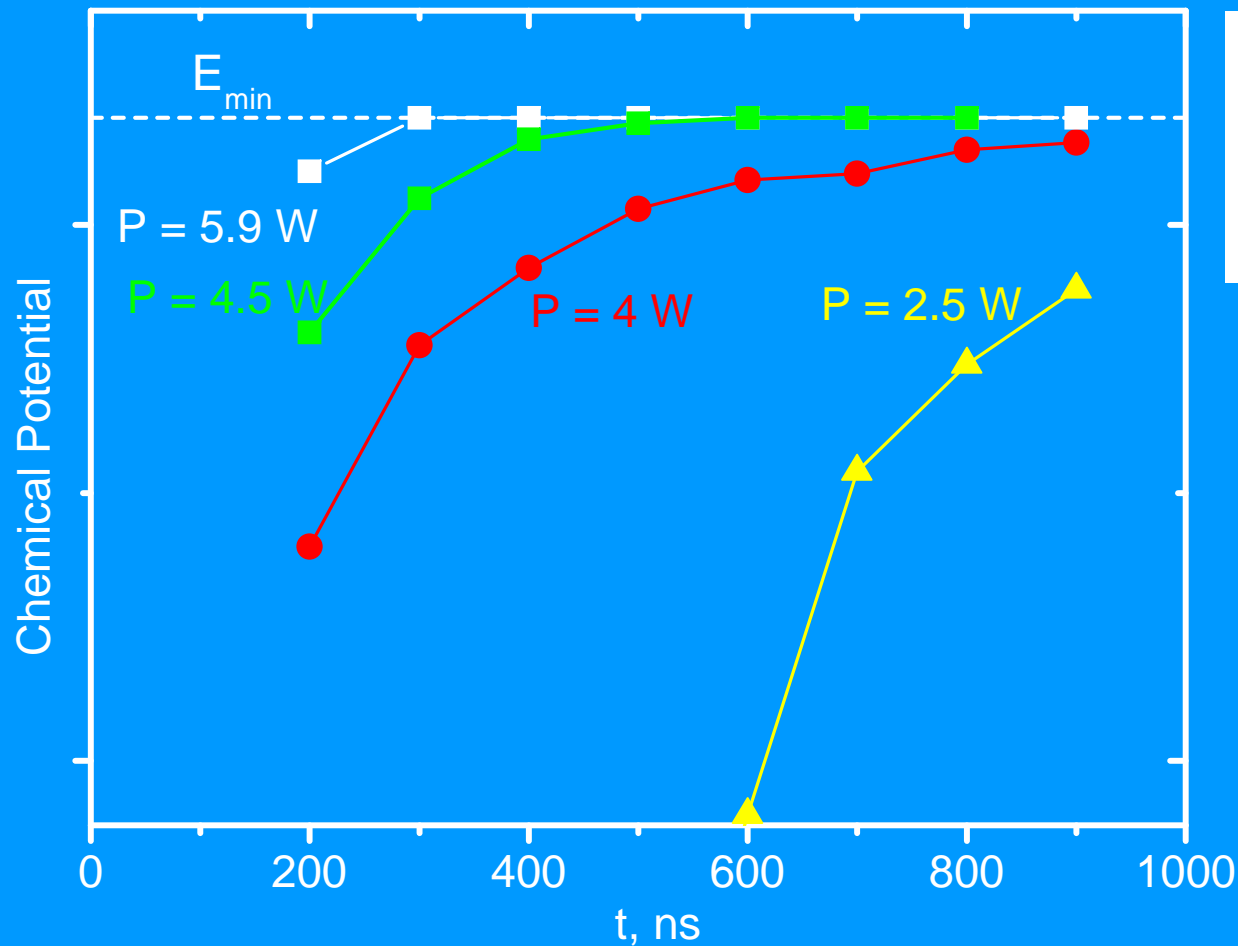
Time development of magnon distribution

Known DOS:  $n(\omega)$  fit with  $\mu$

Bose-statistics with non-zero  $\mu < \mu_{\text{max}}$



# Time dependence of the chemical potential



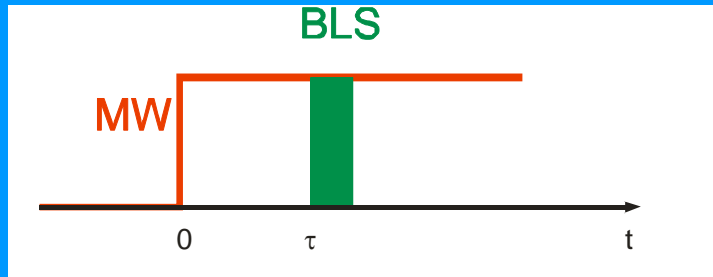
Stationary state  
due to spin-lattice  
relaxation

For high pumping power one can reach the critical  
density of magnons





# Pumped magnons (step-like pumping)



$P = 5.9 \text{ W}$

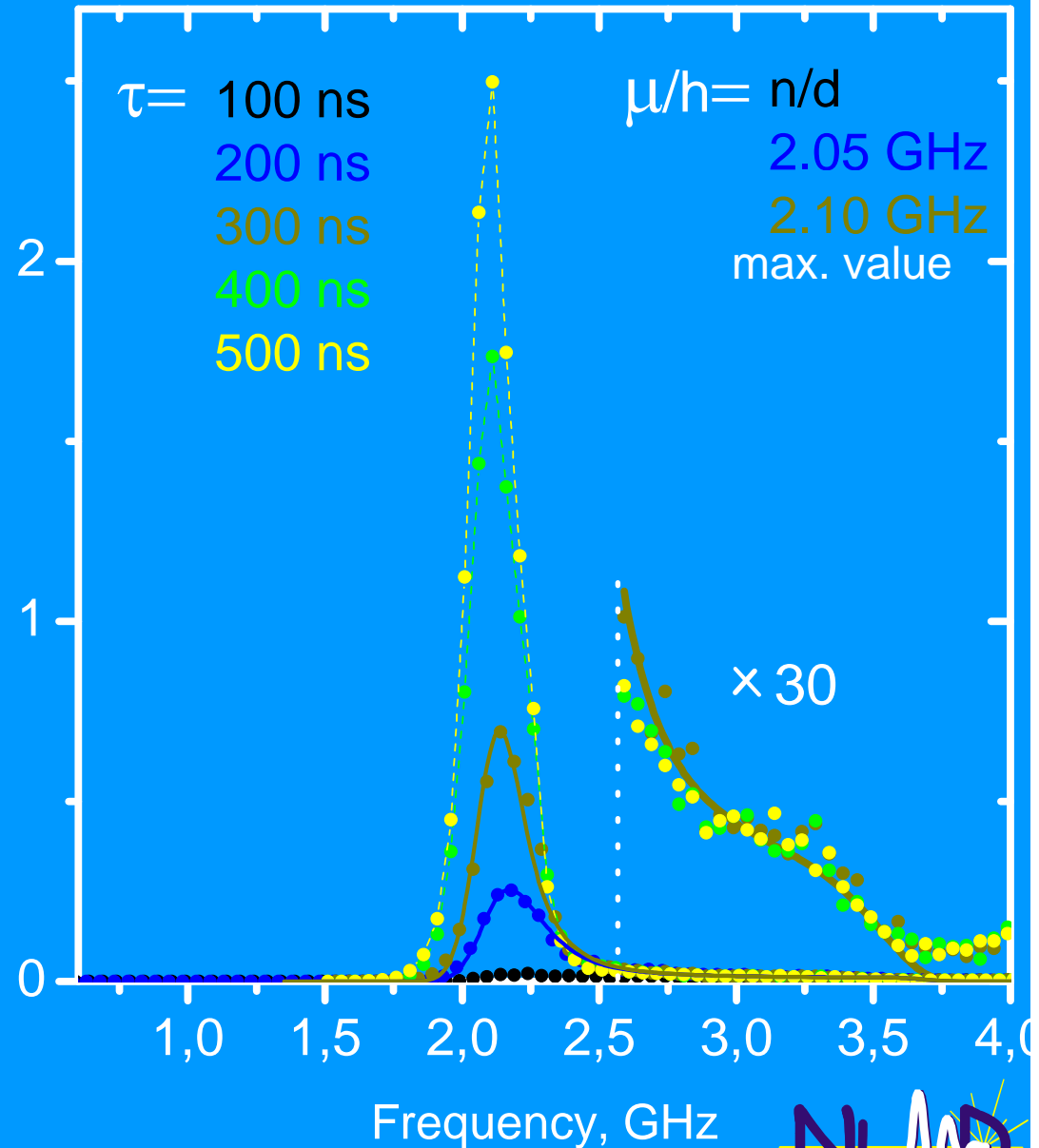
Time development of magnon distribution

Known DOS:  $n(\omega)$  fit with  $\mu$

Bose-statistics.  
At 300 ns critical density:

$$\mu \approx \mu_{\text{max}}$$

Intensity, counts/ms



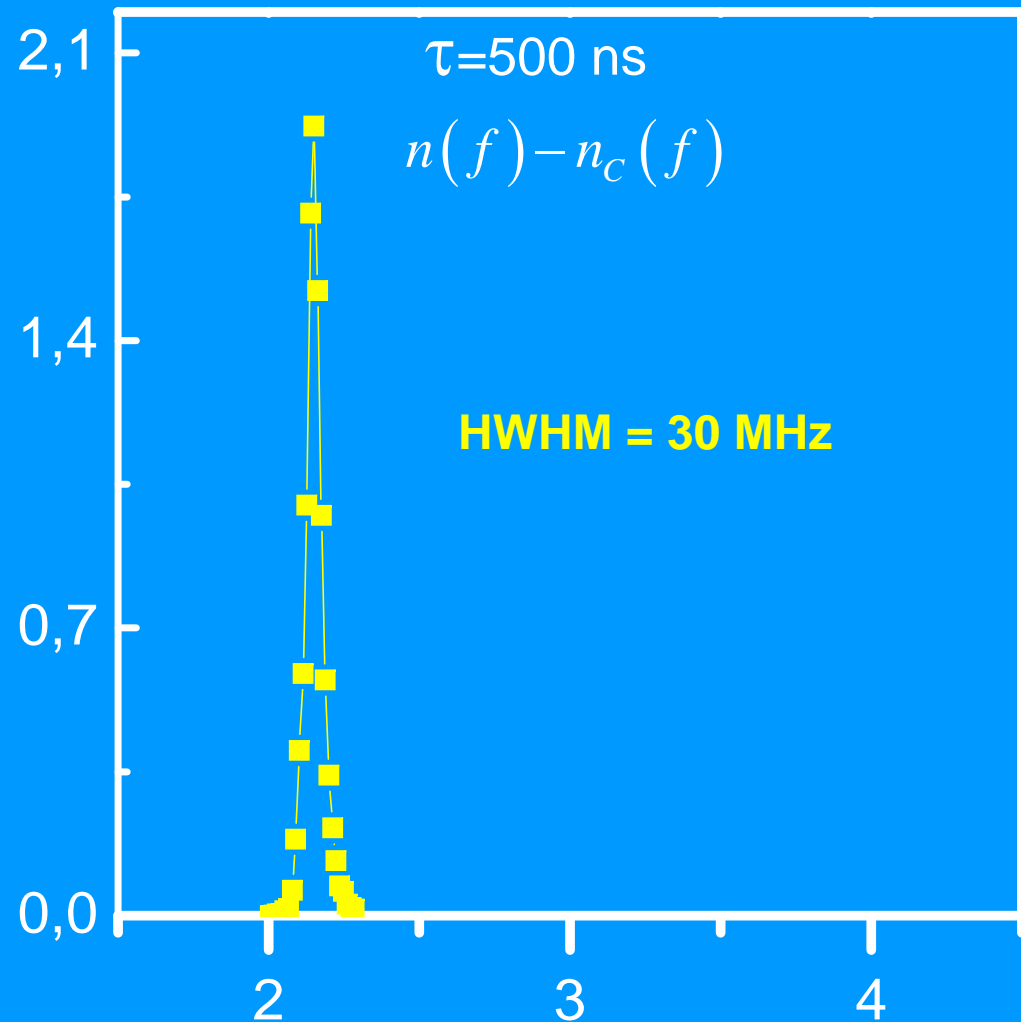
# Experiments with ultimate resolution

The addition to the critical density is of  $\delta$ -type  
(width is  $<1.5$  mK,  
i.e.  $<10^{-5}kT$ ).

A condensate is created!

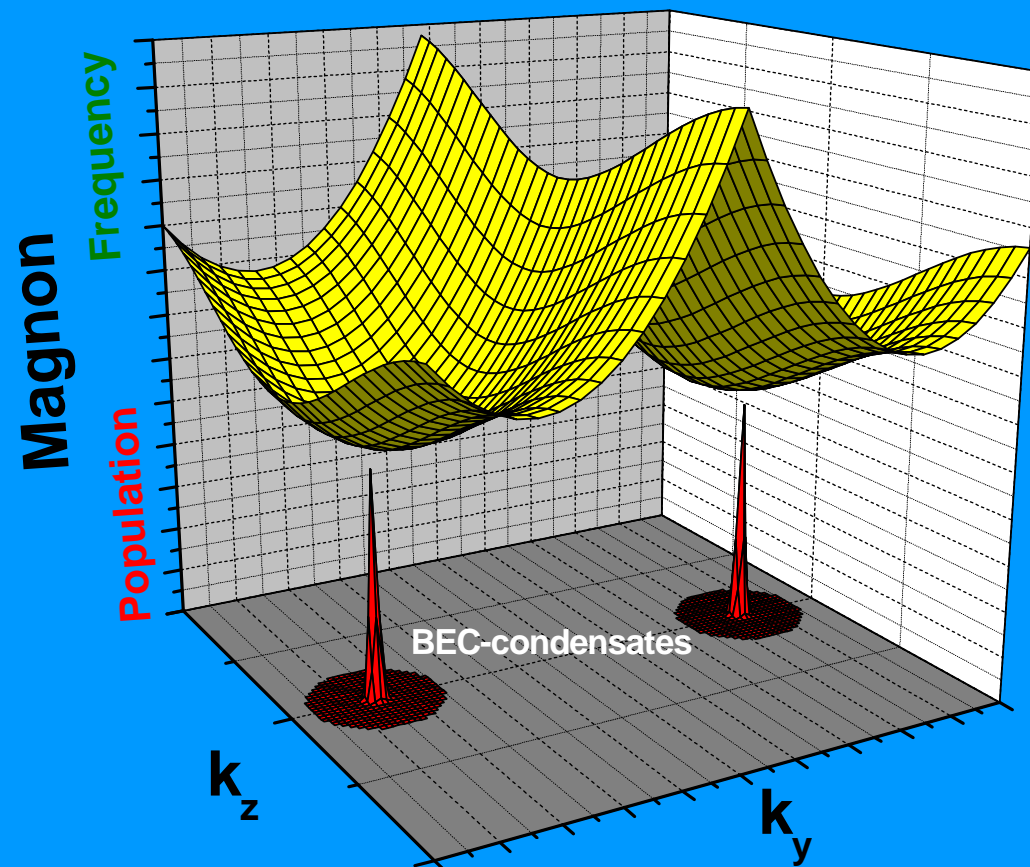
Condensate: a lot of spins  
precess in phase.

*Nature* 443 430 '06



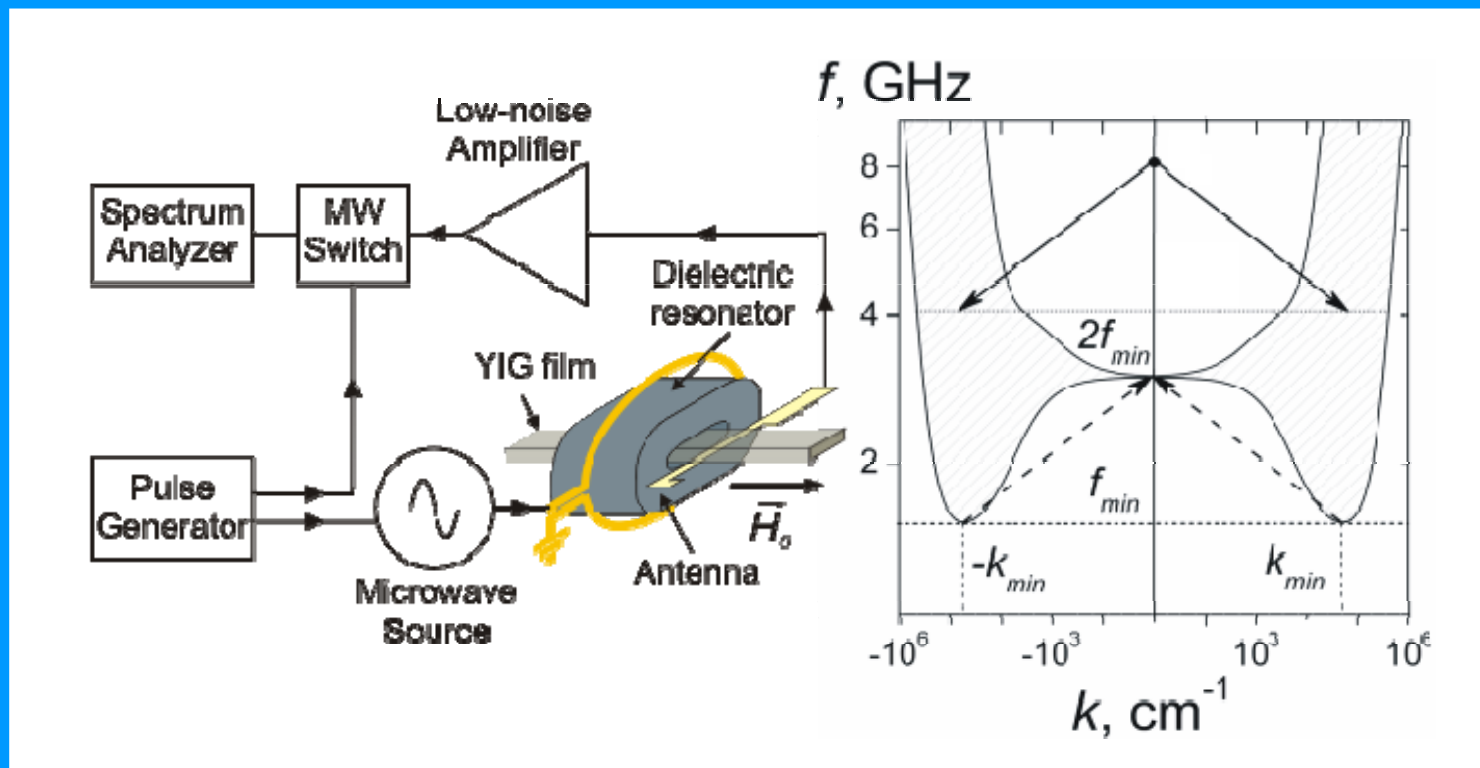
# The condensate is doubly degenerate

$$\psi(y, z, t) = \left( \psi_+(y, z, t) e^{ik_0 z} + \psi_-(y, z, t) e^{-ik_0 z} \right) e^{-\frac{i\mu_c t}{\hbar}}$$



# Detection of the coherent magnetic precession

Condensate: a lot of spins precess in phase.  
The precessing spins should radiate at  $f_{min}$



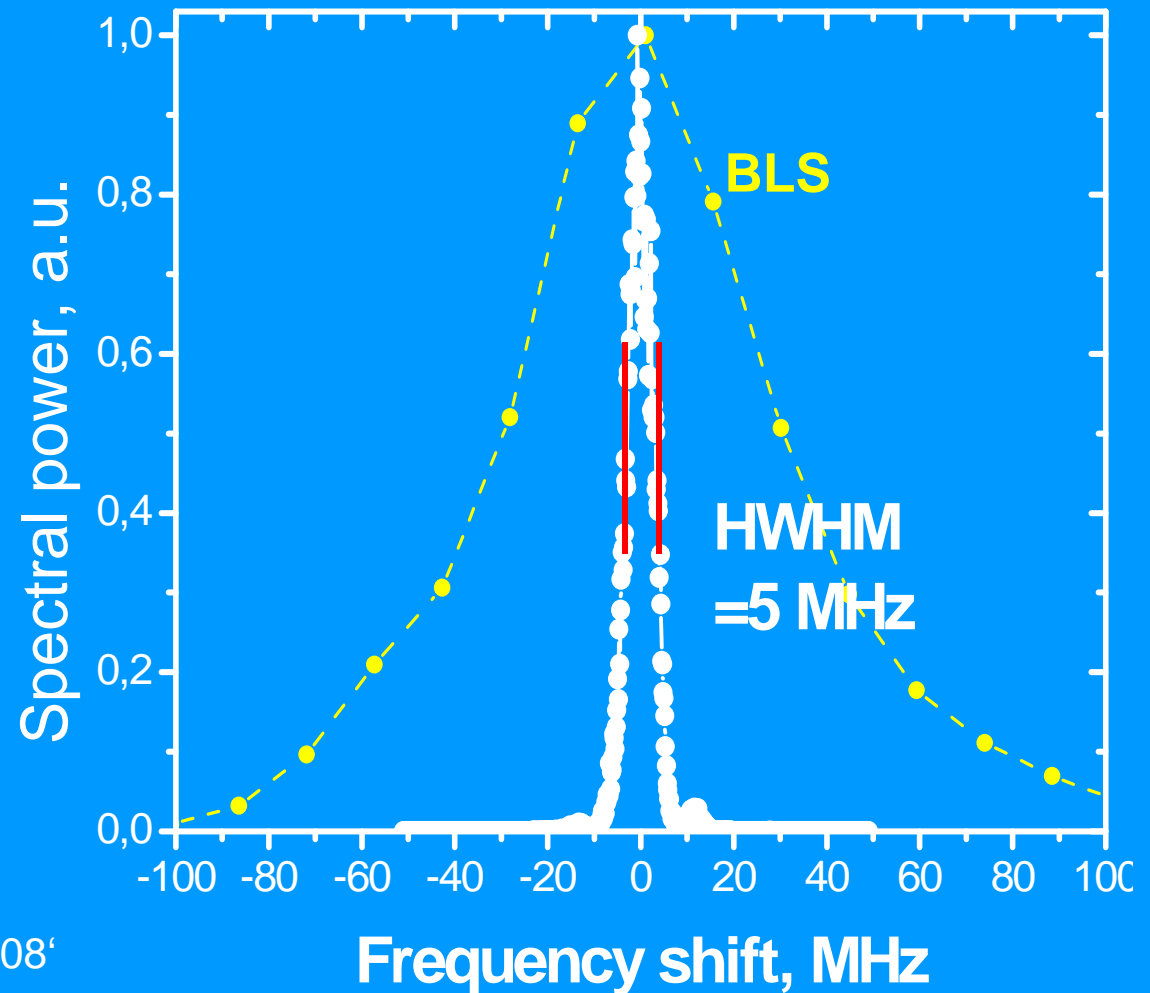
Pump magnons.

Analyze the ringing of the sample using MW spectrum-analyzer.

# Spectrum of magnetic precession

The measured width corresponds to 0.3 mK, i.e.  $2 \cdot 10^{-6} \text{ kT}$

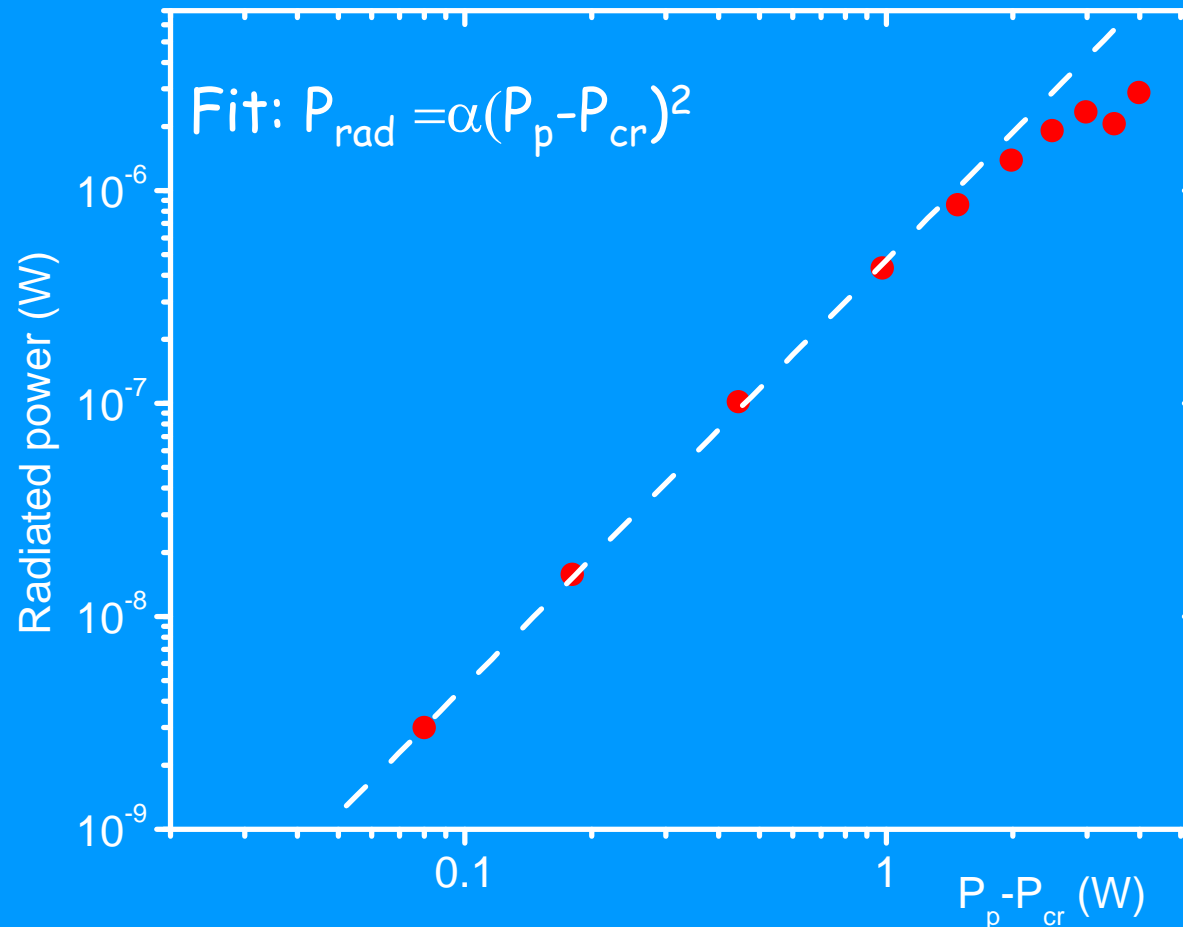
Very high temporal coherence of the condensate



*Appl. Phys. Rev. Lett.* **92** 162510 08'

# Critical index

Sweeping pumping power just above the BEC threshold



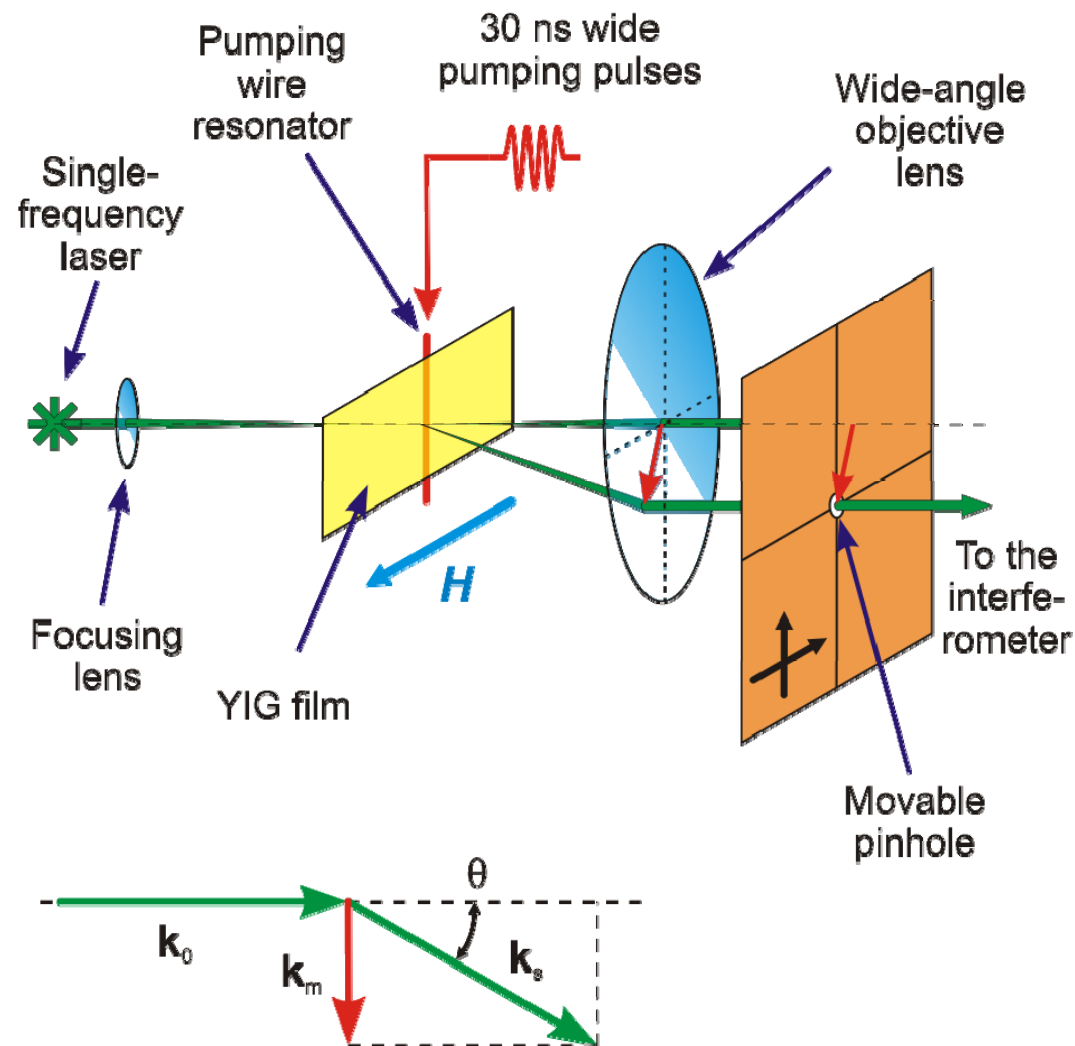
Kalafati & Safonov  
predicted (1993)

$$P_{\text{rad}} \propto (P_p - P_{\text{cr}})^2$$

for BEC of magnons  
due to double  
degeneracy of the  
spectrum and  
phase-locking  
between to  
components of the  
condensate



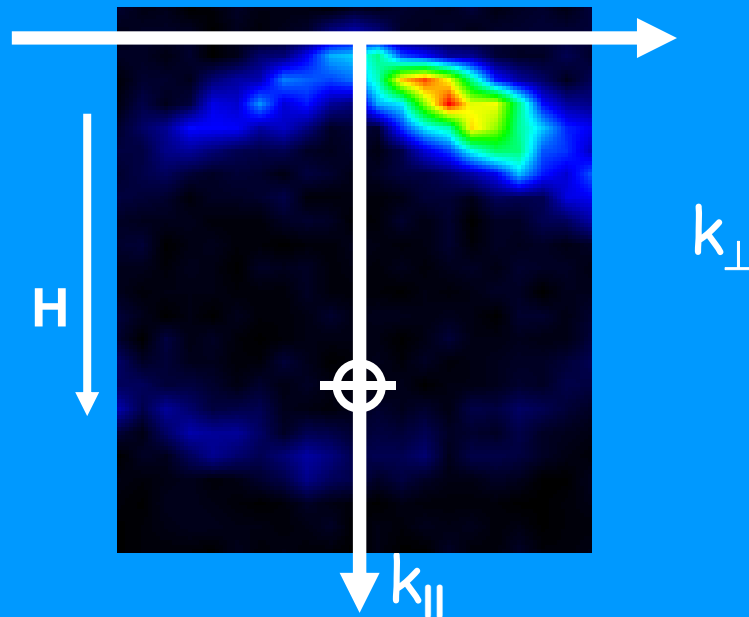
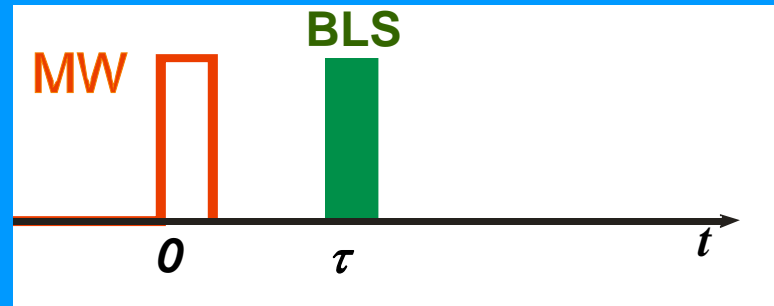
# Study with k-resolution



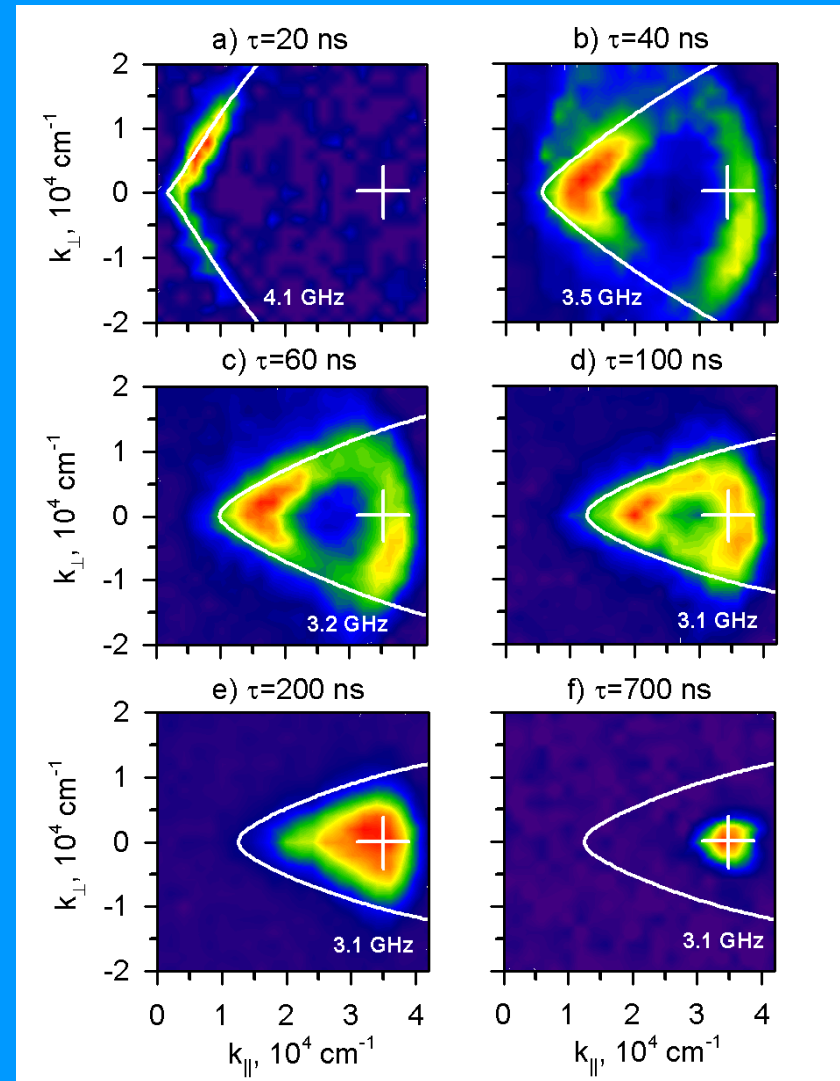
Instead integrating the signal over  $(k_{||}, k_{\perp})$  k-resolved measurements are performed.

Goal: investigation of magnon kinetics during the formation of the condensate and spatial coherence properties of the condensate.

# Magnon kinetics in the phase space

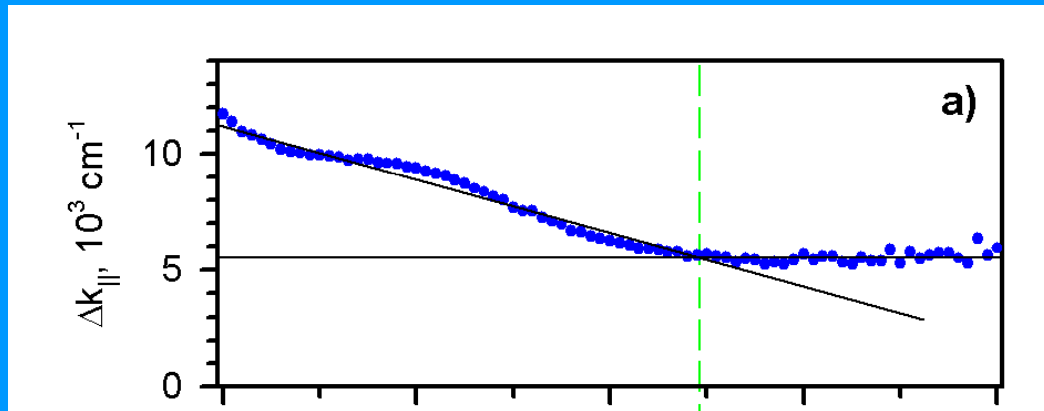


Magnons are gathered at the point in the phase space corresponding to the minimum frequency.





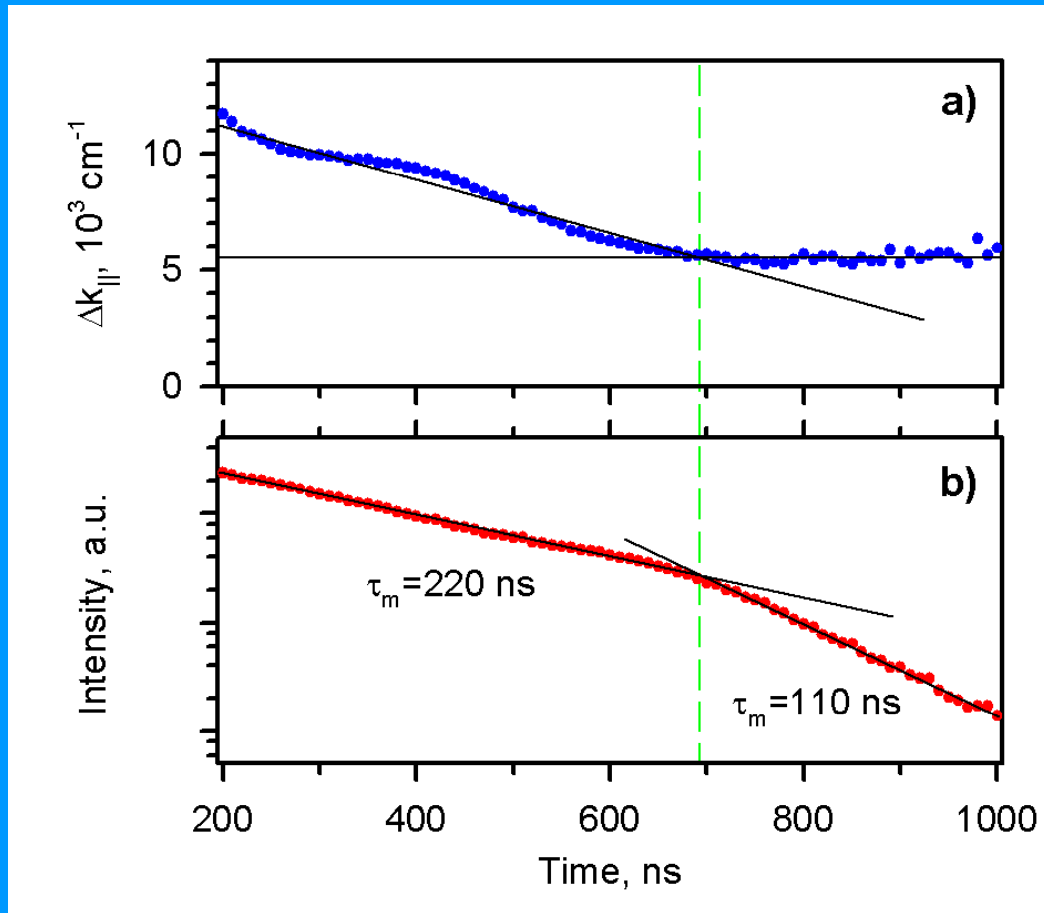
# Spatial coherence of the condensate



The width of the magnon cloud in the  $k$ -space first decreases and then saturates.

The corresponding coherence length  $\xi = \pi/\Delta k$  can be determined:

# Spatial coherence of the condensate



The width of the magnon cloud in the k-space first decreases and then saturates.

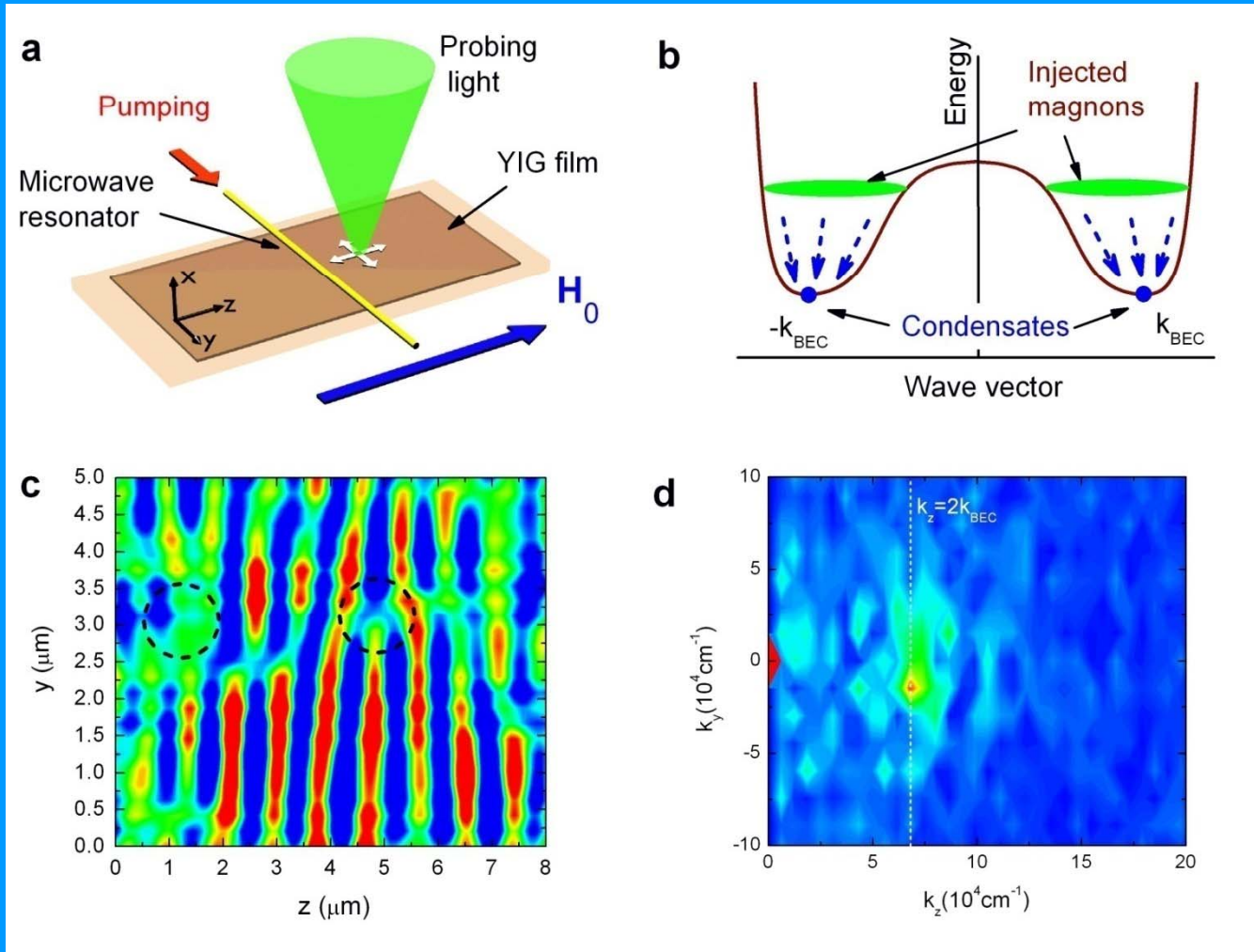
The corresponding coherence length  $\xi = \pi/\Delta k$  can be determined:

$$\xi_{||} = 6 \mu\text{m} \quad \xi_{\perp} > 10 \mu\text{m}$$

The coherence length is anisotropic, reflecting the anisotropy of the magnon spectrum.

# Phase-locking between the $k$ and $-k$ components

## CW measurements

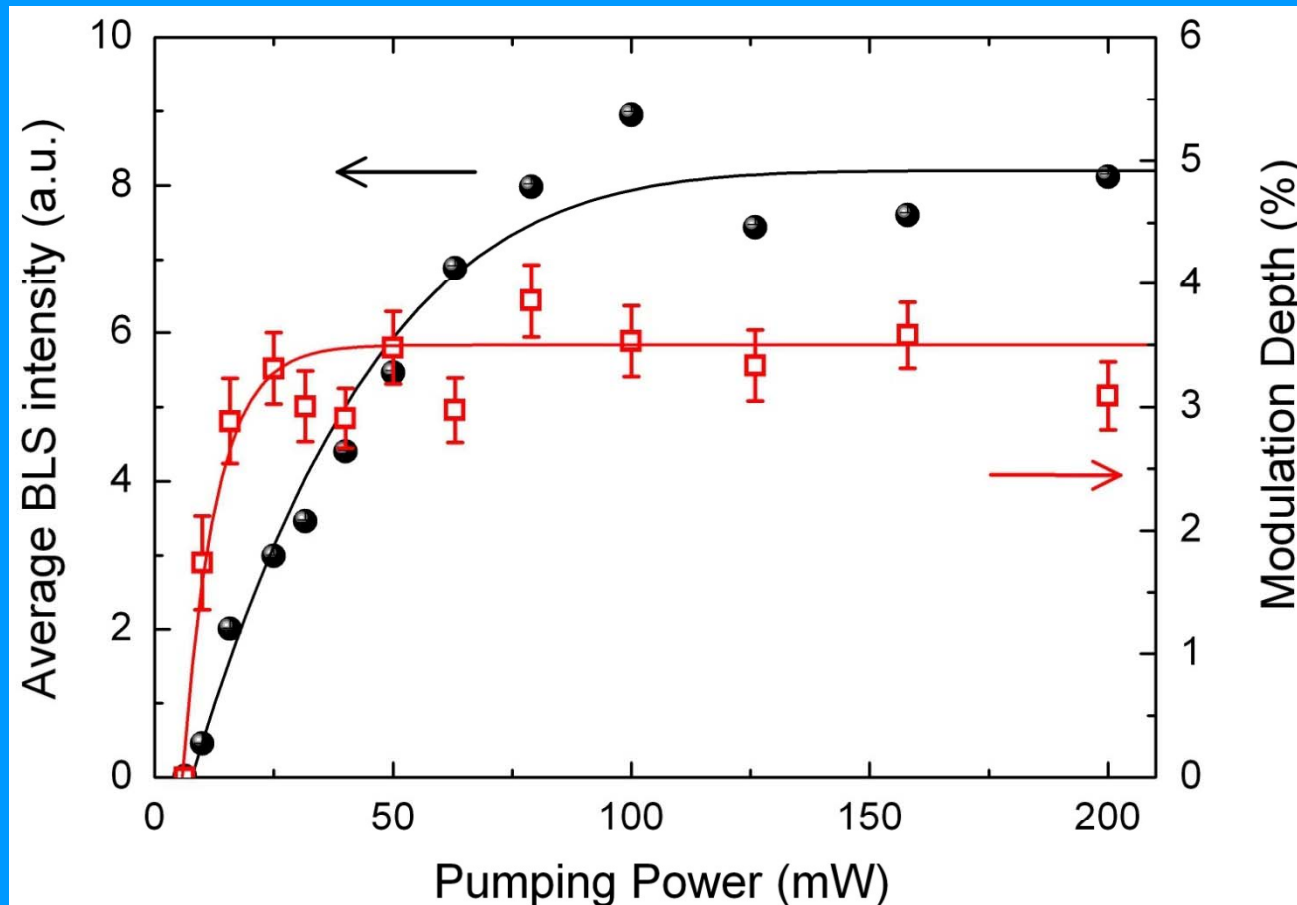


Two components of the condensate are phase locked  
The phase-locking is probably due to the defect-mediated coupling

In addition to regular periodic structure stationary vortices are observed

# Correlation of the $k$ and $-k$ components

## CW measurements

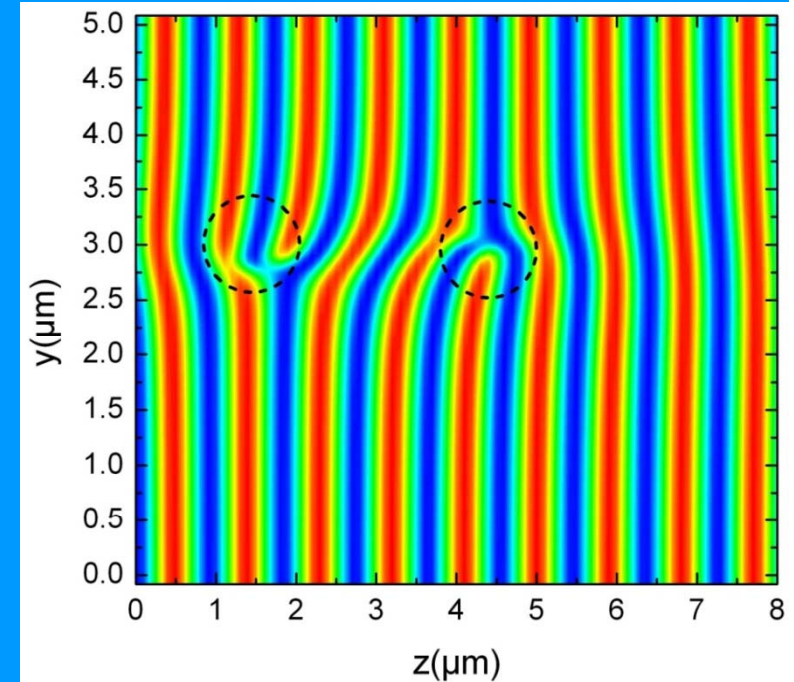
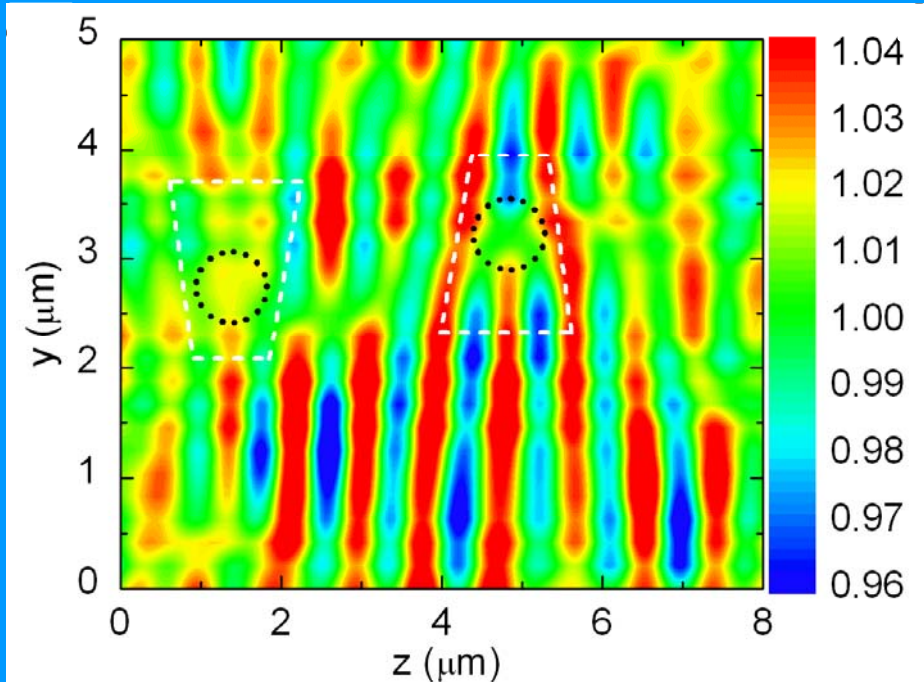


The amplitude of the modulation grows faster than the total density.



The phase-locking is due to a nonlinear interaction between the components of the condensate

# Vortices in the condensate



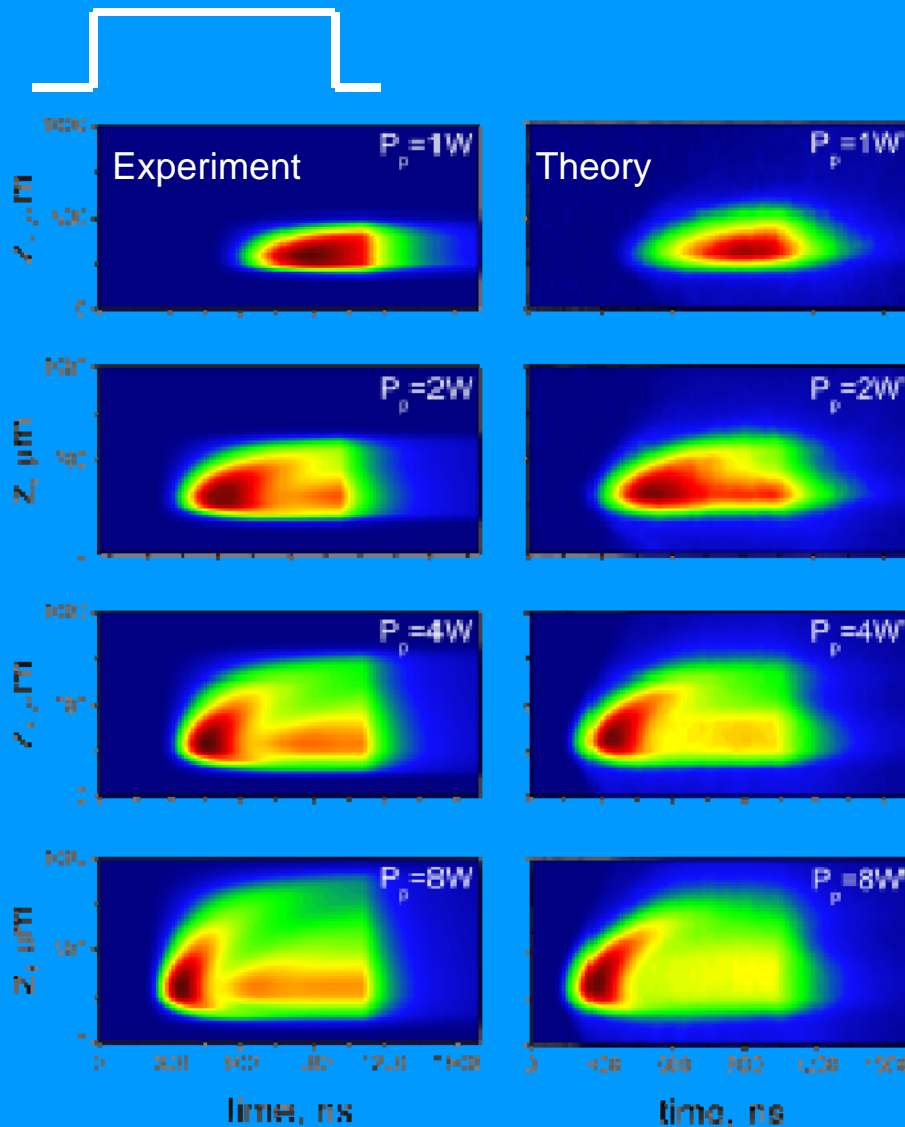
$$\psi(y, z, t) = \left( \psi_+(y, z, t) e^{ik_0 z} + \psi_-(y, z, t) e^{-ik_0 z} \right) e^{-\frac{i\mu_c t}{\hbar}}$$

$$i\hbar\partial_t \psi_{\pm} = \left[ -\frac{\hbar^2 \partial_{zz}}{2m_{\parallel}} - \frac{\hbar^2 \partial_{yy}}{2m_{\perp}} - \mu_c + U_s |\psi_{\pm}|^2 + iP(\psi_{\pm}) \right] \psi_{\pm} + J\psi_{\mp}^*$$

$$P(\psi_{\pm}) = \gamma_{\text{eff}} - i\hbar\eta\partial_t - \Gamma_s |\psi_{\pm}|^2 \quad \Gamma_s(\vec{r})$$

Scientific Reports, in press

# Spatio-temporal evolution of the condensate



$$\Psi(z,t) = \Psi_+(z,t)e^{ik_0 z} + \Psi_-(z,t)e^{-ik_0 z}$$

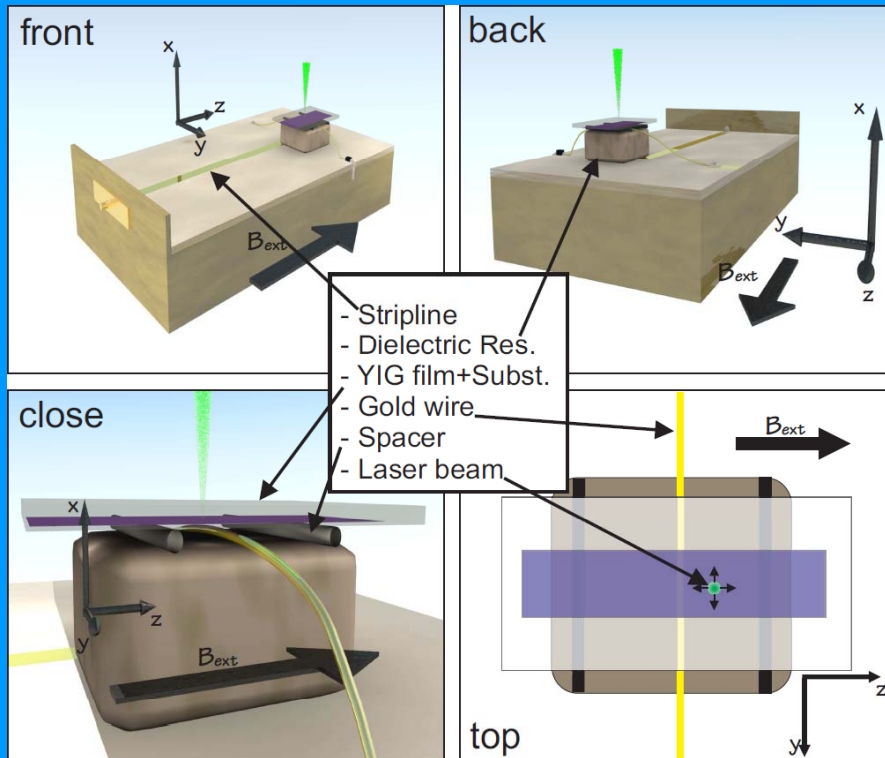
$$i(\Psi_+)_t + \mu_0 \Psi_+ + \frac{1}{2}(\Psi_+)_{zz} + i\eta \Psi_+ + (|\Psi_+|^2 + \sigma_1 |\Psi_-|^2) \Psi_+ + i\tau (|\Psi_+|^2 + \sigma_2 |\Psi_-|^2) \Psi_+ = -f \delta(z)$$

$$i(\Psi_-)_t + \mu_0 \Psi_- + \frac{1}{2}(\Psi_-)_{zz} + i\eta \Psi_- + (|\Psi_-|^2 + \sigma_1 |\Psi_+|^2) \Psi_- + i\tau (|\Psi_-|^2 + \sigma_2 |\Psi_+|^2) \Psi_- = -f \delta(z)$$

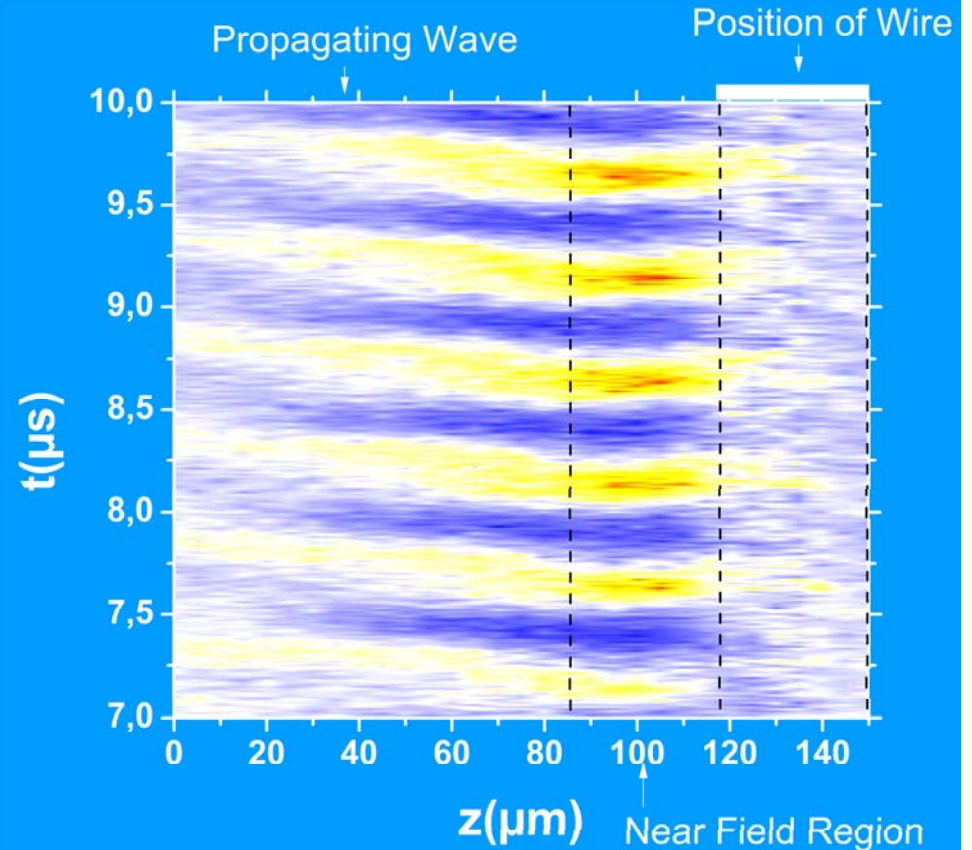
The nonlinearly coupled equations, written for amplitudes of the right- ( $\vec{k}_0$ ) and left-traveling ( $-\vec{k}_0$ ) waves, combining basic features of the Gross-Pitaevskii and complex Ginzburg-Landau models.

B.Malomed et al., *Phys. Rev. B* 81 024418 '10

# Sound in the condensate

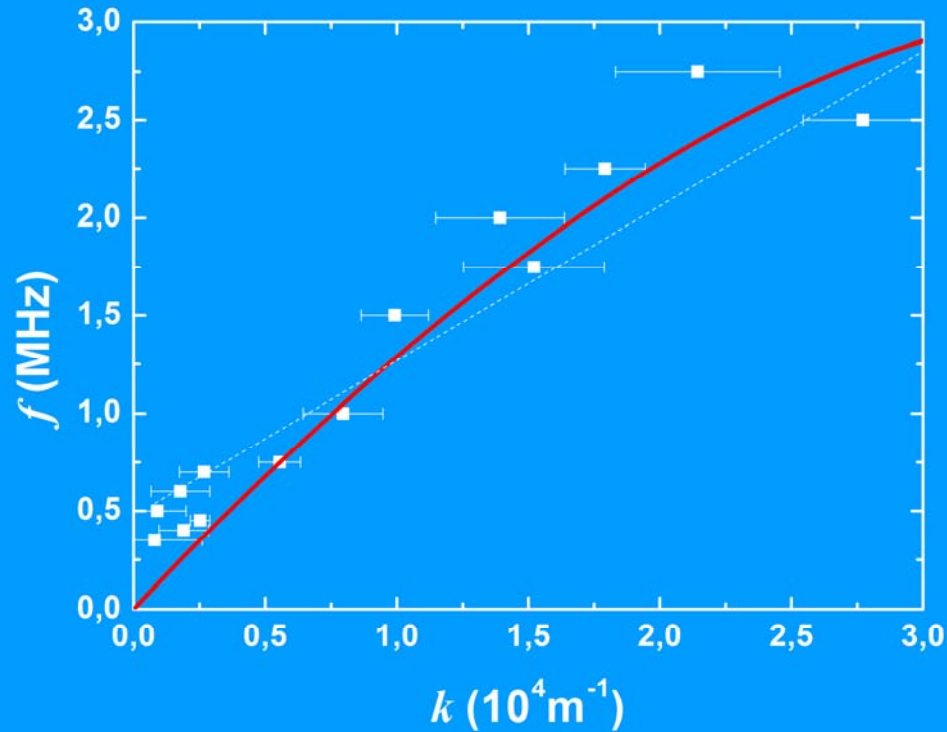


Condensate is created by the microwave pumping via dielectric resonator  
 It is disturbed by radio-field using a narrow wire

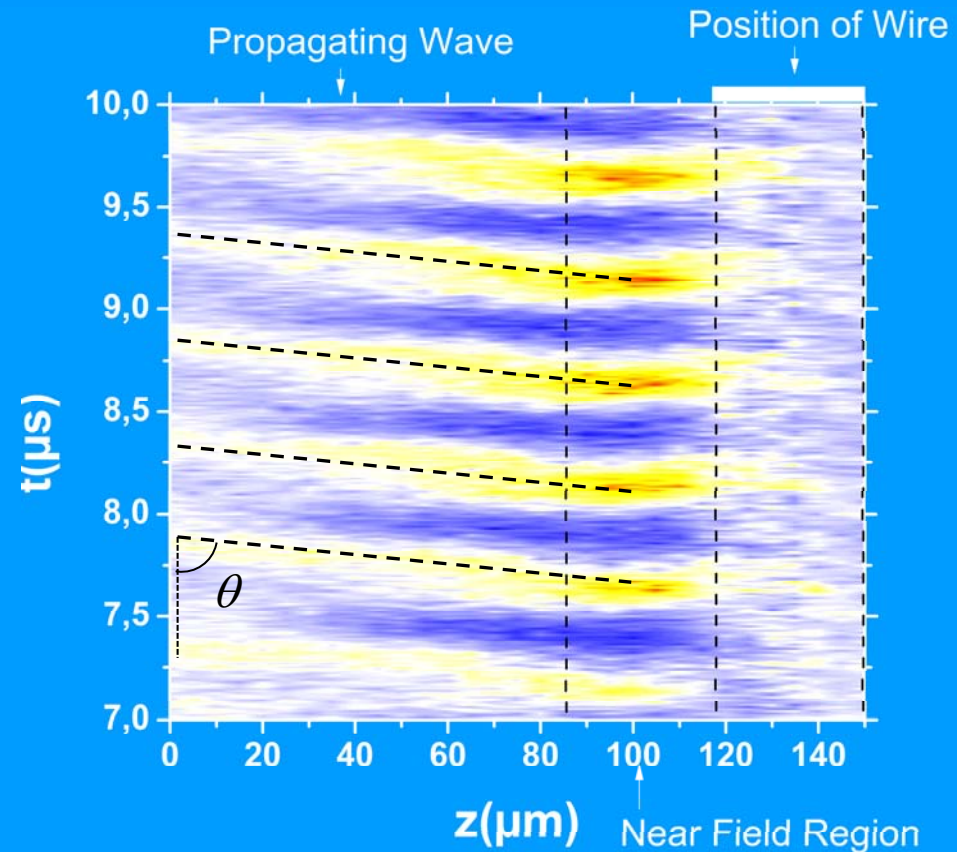


The wire excites waves propagating in the condensate

# Sound in the condensate



Theory based on the GPE and the known spectrum of magnons.

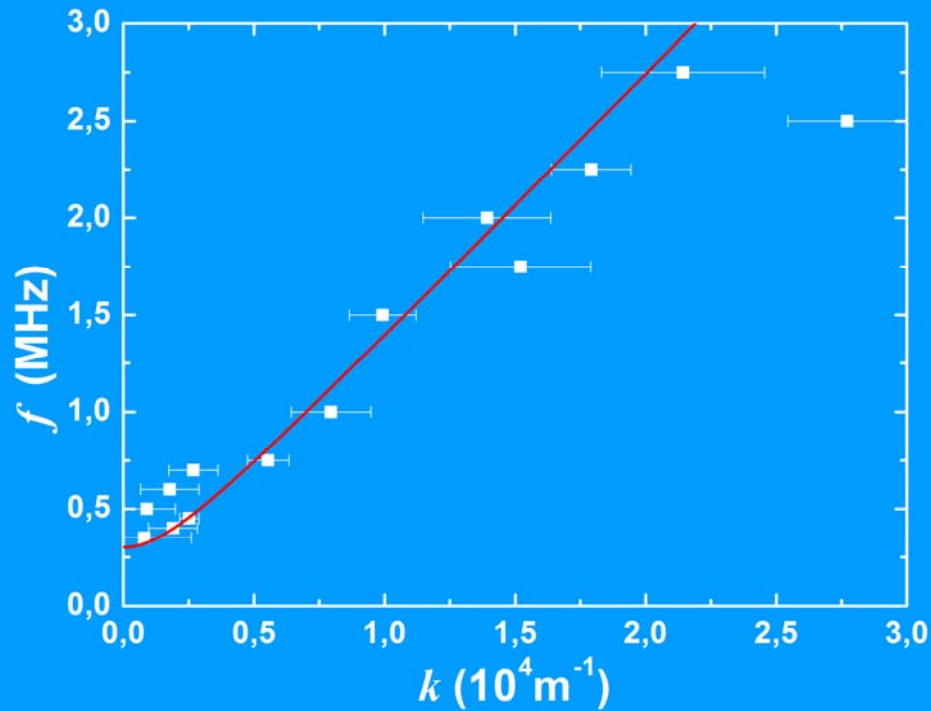


$$v_{ph} = \frac{\omega}{k} = \tan \theta$$

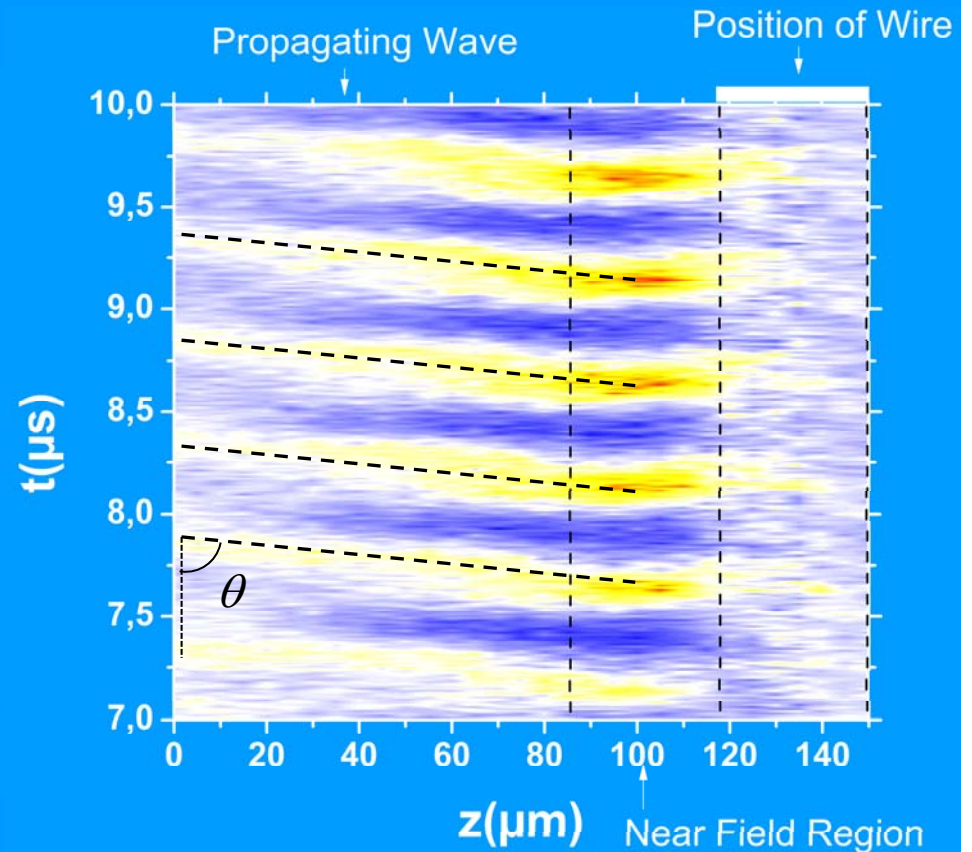
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# Sound in the condensate



Theory based on the GPE and the known spectrum of magnons.



$$v_{ph} = \frac{\omega}{k} = \tan \theta$$

The wire excites waves propagating in the condensate

# Summary

- Doubly degenerated Bose-Einstein condensate of magnons is created at room temperature
- Coherence properties of the condensate as well its spatio-temporal dynamics are studied

<http://www.uni-muenster.de/Physik/AP/Demokritov/>