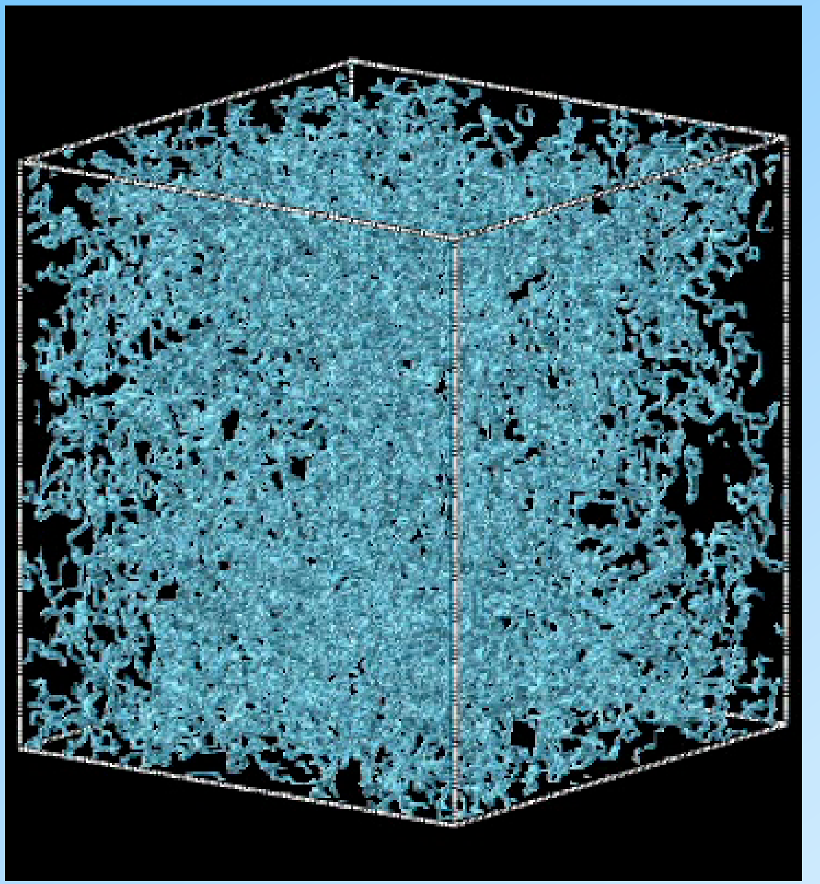
Quantum Hydrodynamic and Turbulence in Bose-Einstein Condensates



Makoto TSUBOTA
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Osaka City University, Japan

Review article

- Progress in Low Temperature Physics Vol.16, eds. W. P. Halperin and M. Tsubota, Elsevier, 2009
- M. Tsubota and K. Kasamatsu, arXiv:1202.1863

Contents

What is “quantum” ?

Element of something

What is “quantum mechanics” ?

Mechanics with element

Energy, momentum and angular momentum *etc*. are quantized.

The element is determined by the Planck’s constant h.

What is “quantum turbulence” ?

Turbulence with some “element”

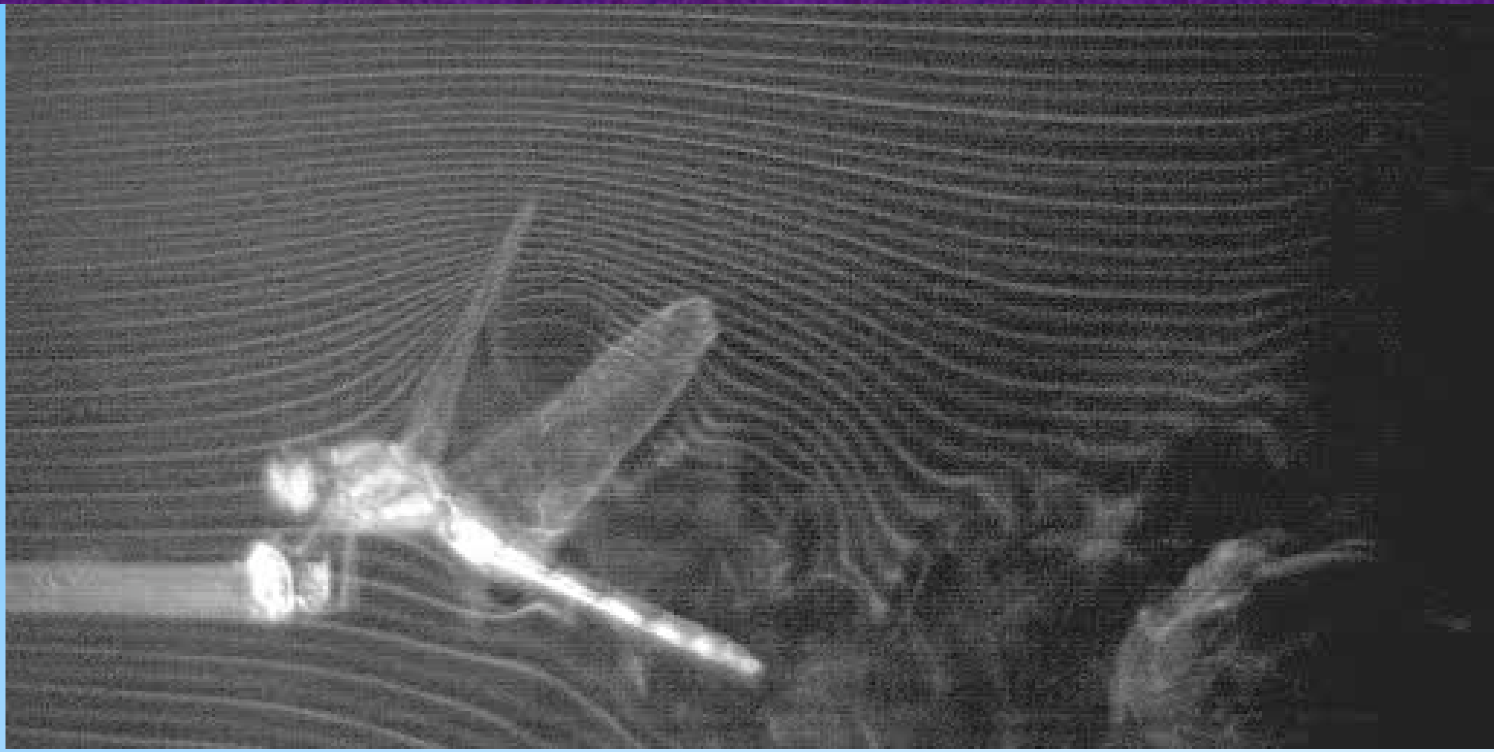
Leonardo Da Vinci
(1452-1519)

Da Vinci observed turbulent flow and found that turbulence consists of many vortices with different scales.

Turbulence is not a simple disordered state but having some structures with vortices.

Certainly turbulence looks to have many vortices.

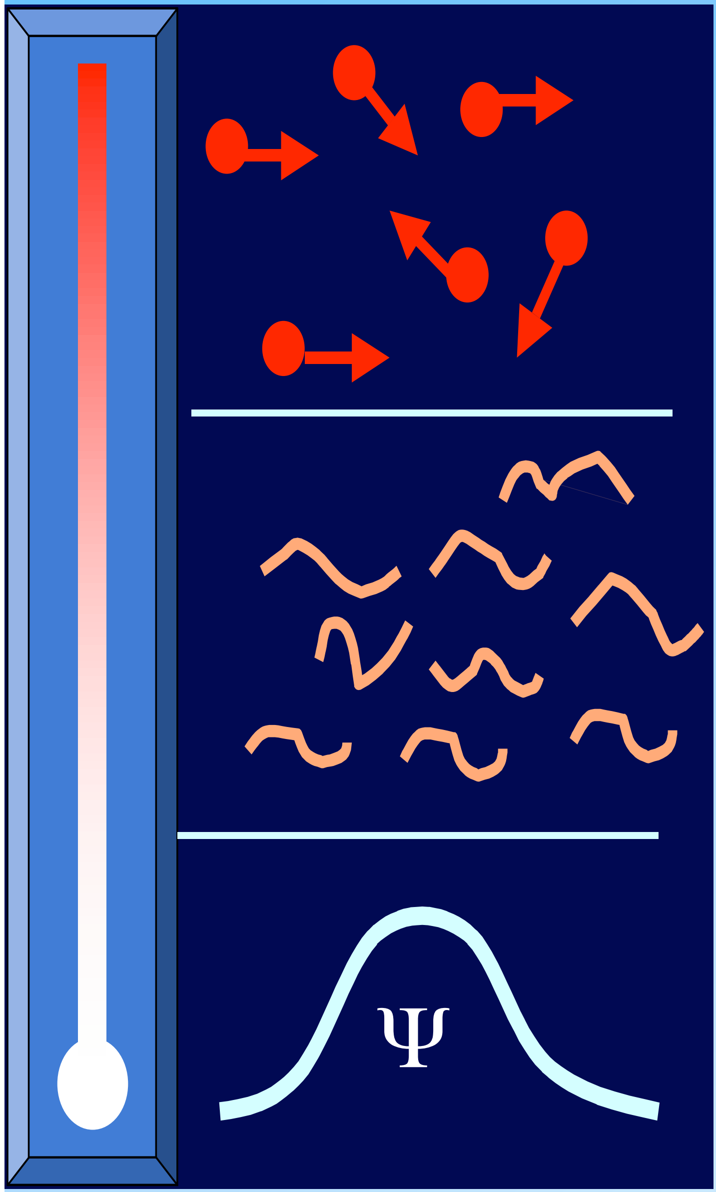
Turbulence is still a great mystery in Nature!



http://www.nagare.or.jp/mm/2004/gallery/iida/dragonfly.html

It is not so straightforward to confirm the Da Vinci message in classical turbulence.

0. Introduction



Quantum mechanics
～ Duality of matter and wave ～

Each atom behaves as a particle at high temperatures.

Thermal de Broglie wave length
～ Distance between particles

Each atom behaves like a wave at low temperatures.

Bose-Einstein condensation (BEC)

Each atom occupies the same single particle ground state. The matter waves become coherent, making a macroscopic wave function Ψ .

Quantum hydrodynamics of the GP model

The wave function Ψ obeys the Gross-Pitaevskii (GP) equation in a weakly interacting Bose system.

$$i\hbar \frac{\partial \Psi}{\partial t} = -\left(\frac{\hbar^2}{2m} \nabla^2 + \mu \right) \Psi + V \Psi + g |\Psi|^2 \Psi$$

V: Potential

g: Interaction parameter

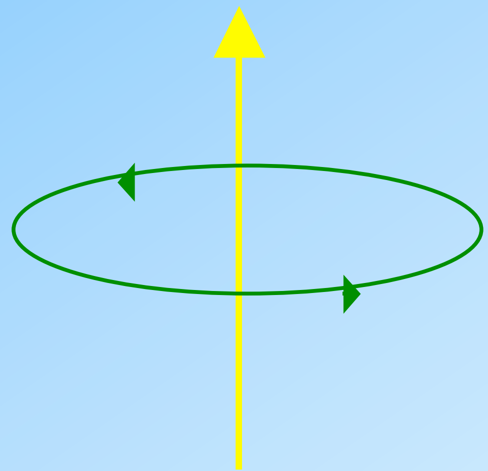
A quantized vortex is a vortex of superflow in a BEC. Any rotational motion in superfluid is sustained by quantized vortices.

(i) The circulation is quantized.

$$\oint \mathbf{v}_s \cdot d\mathbf{s} = \kappa n \qquad (n = 0, 1, 2, \cdots)$$

$$\kappa = h/m$$

A vortex with $n \geqq 2$ is unstable.



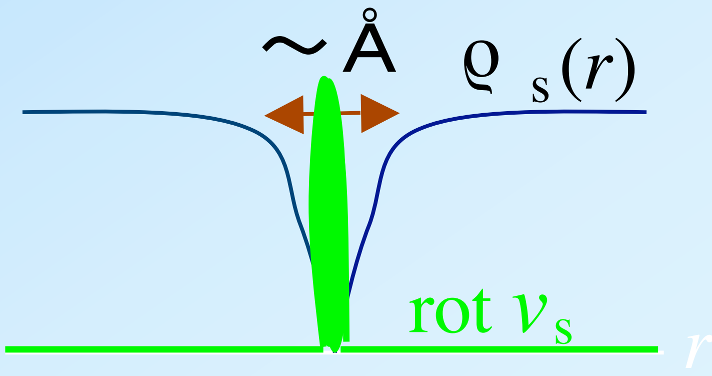
⇨ **Every vortex has the same circulation.**

(ii) Free from the decay mechanism of the viscous diffusion of the vorticity.

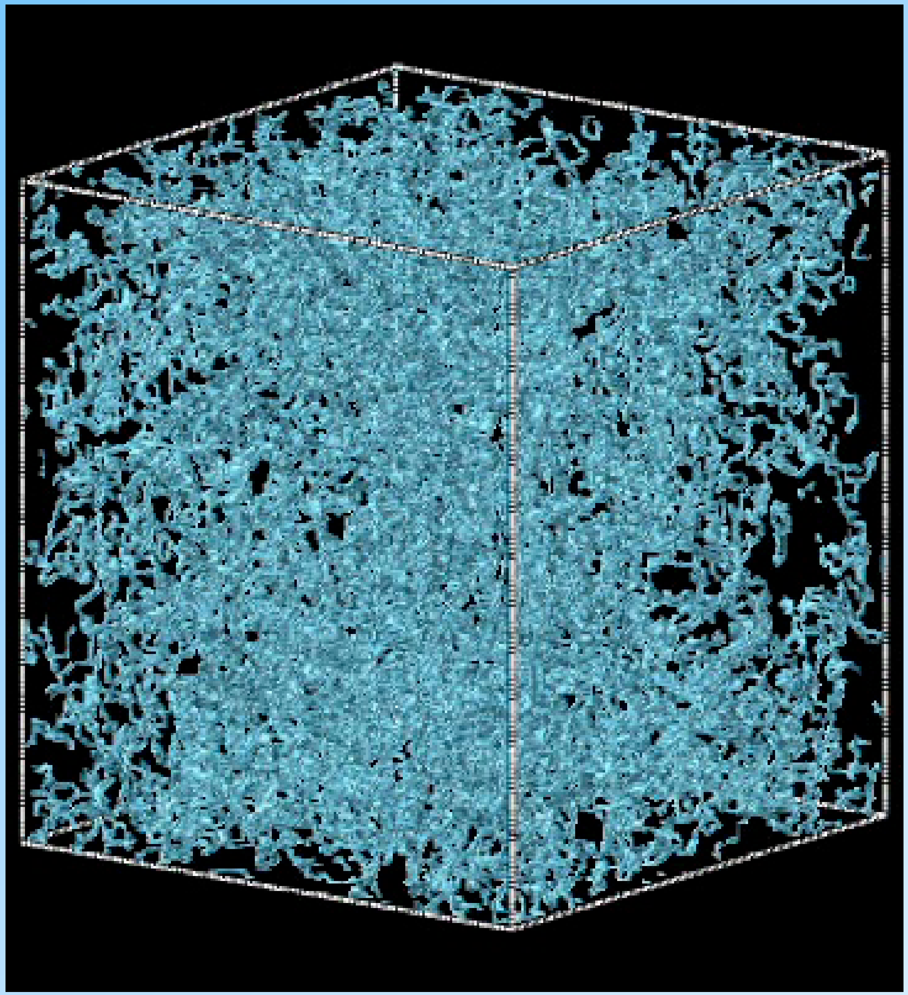
⇨ **The vortex is stable.**

(iii) The core size is very small.

⇨ **The order of the coherence length.**



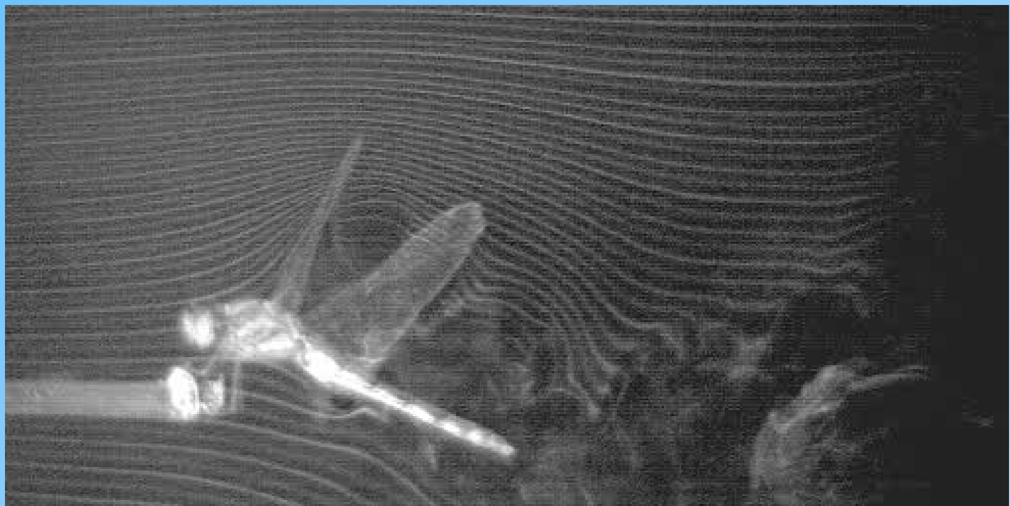
Turbulence consisting of quantized vortices is called ***Quantum Turbulence***.



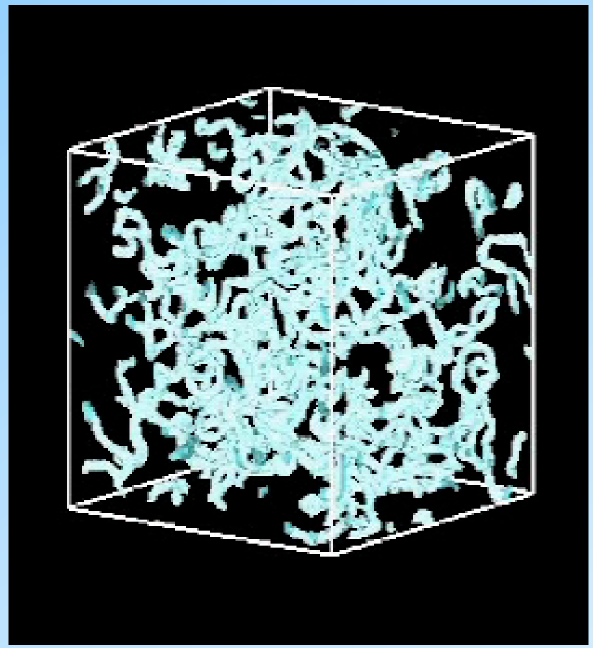
Numerical simulation by the GP model.

Classical Turbulence (CT) vs. Quantum Turbulence (QT)

Classical turbulence



Quantum turbulence



Motion of vortex cores

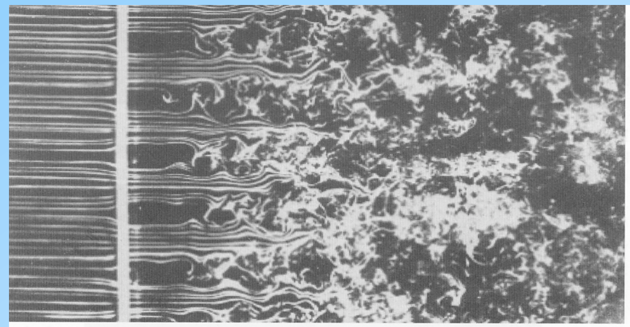
QT can be easier to understand than CT, because each element of turbulence is clearly defined.

・The quantized vortices are stable topological defects.
・Every vortex has exactly the same circulation.
・Circulation is conserved.

Energy spectra of fully developed turbulence

Energy spectrum of the velocity field

$$E = \frac{1}{2}\int \mathbf{v}^2 d\mathbf{r} = \int E(k)dk$$



Energy-containing range

The energy is injected into the system at $k \approx k_0 = 1/\ell_0$ **.**

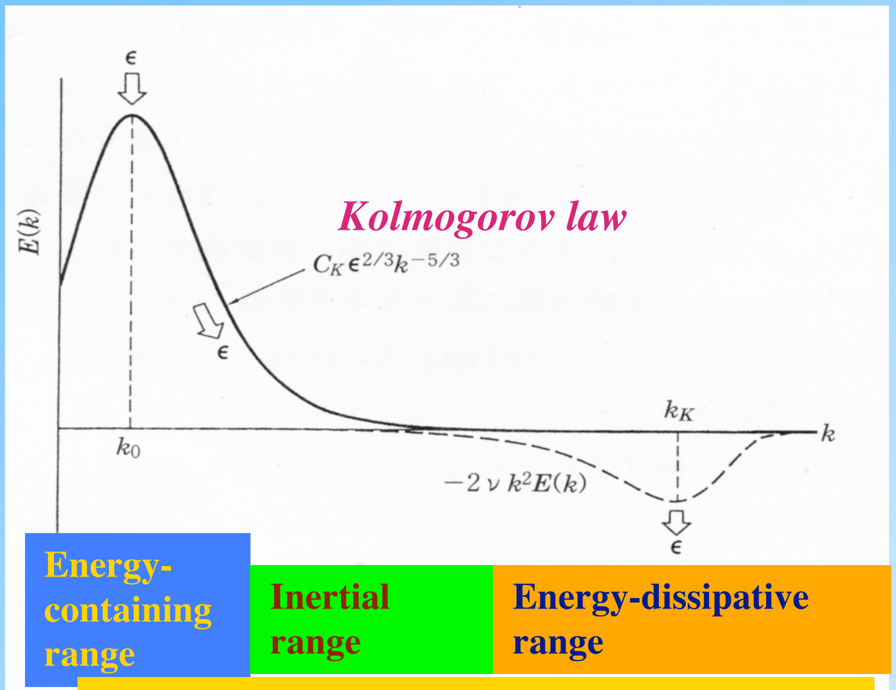
Inertial range

Dissipation does not work. The nonlinear interaction transfers the energy from low *k* region to high *k* region.

Kolmogorov law (K41) : $E(k) = C\varepsilon^{2/3}\, k^{-5/3}$

Energy-dissipative range

The energy is dissipated with the rate ε at the Kolmogorov wave number $k_c = (\varepsilon/\nu^3)^{1/4}$**.**



Energy spectrum of turbulence

QT has been studied for a long time in superfluid helium.

Characteristics of atomic BECs

Good news

Theoretically

This is a weakly interacting Bose system. Hence the physics near 0K is quantitatively well described by the GP model.

$$i\hbar\frac{\partial\Psi}{\partial t}=-\left(\frac{\hbar^2}{2m}\nabla^2+\mu\right)\Psi+V\Psi+g|\Psi|^2\Psi$$

Experimentally

1. We can control and visualize directly the condensate and the topological defects such as solitons and quantized vortices.
2. The amplitude and the sign of g is controllable by the Feshbach resonance.
3. It is possible to control the geometry and the dimension of the condensate by changing the trapping potential.

Characteristics of atomic BECs.

Bad news

1. This is essentially metastable with a finite life time.
2. This is a finite-size system usually trapped by a potential. The physics is more or less affected by the finite-size effects.

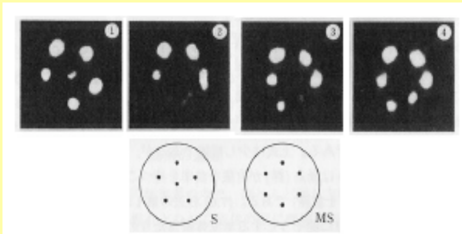
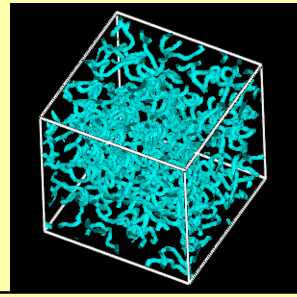
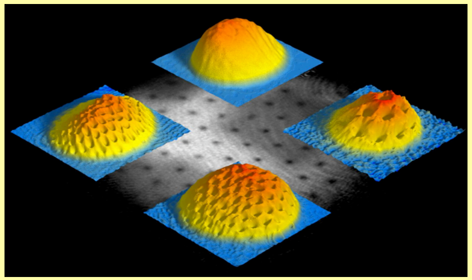
the coherence length $\xi \leqq$ the system size L

cf. In superfluid helium,

the coherence length $\xi \ll$ the system size L

1. QT in a trapped BEC

There are two main cooperative phenomena of quantized vortices; Vortex lattice under rotation and Vortex tangle (Quantum turbulence).

	Vortex lattice	Vortex tangle
Superfluid He		
Atomic BEC		None

Is it possible to make turbulence in a trapped BEC?

(1) We cannot apply some dc flow to the system.

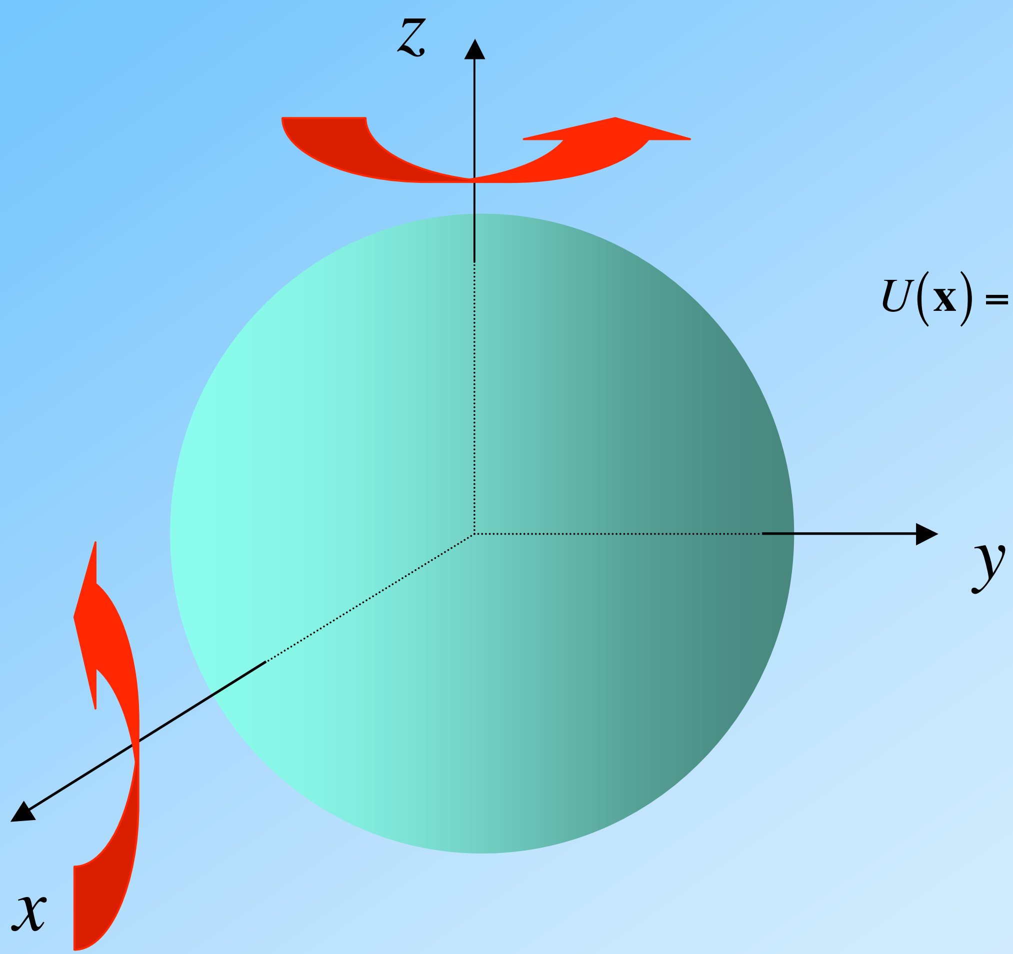
(2) This is a finite-size trapped system. Is this serious?

We use the idea of rotating turbulence.

QT in a trapped BEC

M. Kobayashi and M. Tsubota, Phys. Rev. A76, 045603 (2007)

Making QT by combining two rotations



1. Trap the BEC in a weakly elliptic potential.

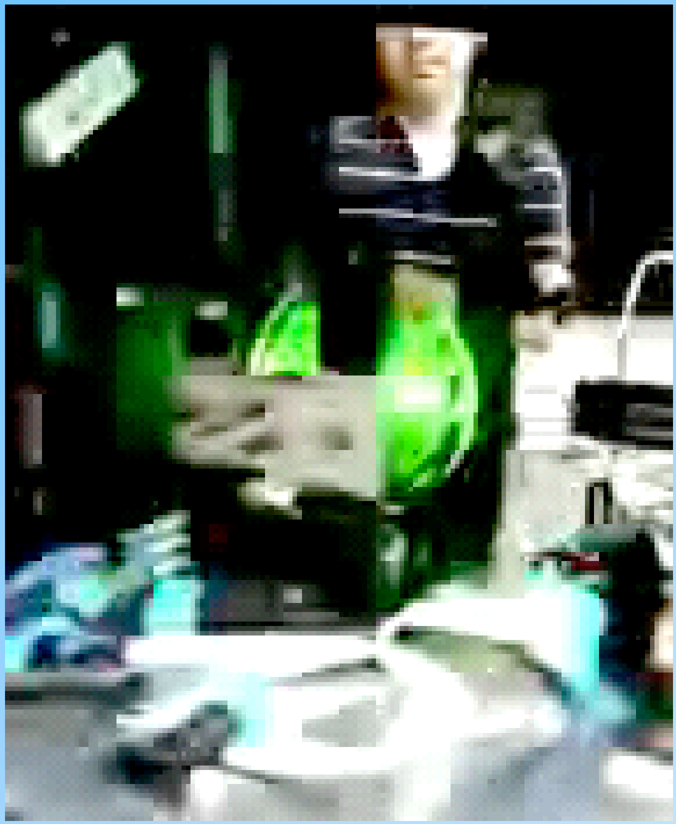
$$U(\mathbf{x}) = \frac{m\omega^2}{2}\left[(1-\varepsilon_1)(1-\varepsilon_2)x^2 + (1+\varepsilon_1)(1-\varepsilon_2)y^2 + (1+\varepsilon_2)z^2\right]$$

2. Rotate the system first around the x-axis, next around the z-axis.

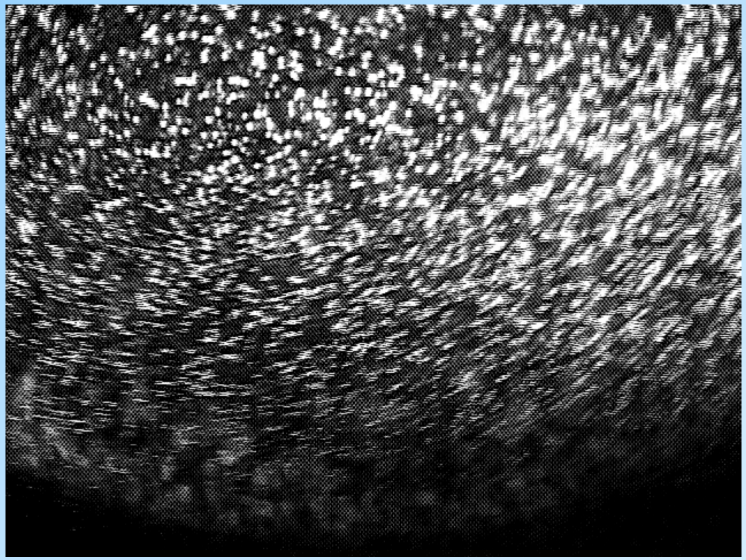
$$\Omega(t) = \left(\Omega_x,\ \Omega_z \sin\Omega_x t, \Omega_z \cos\Omega_x t\right)$$

Actually this idea has been already used in CT.

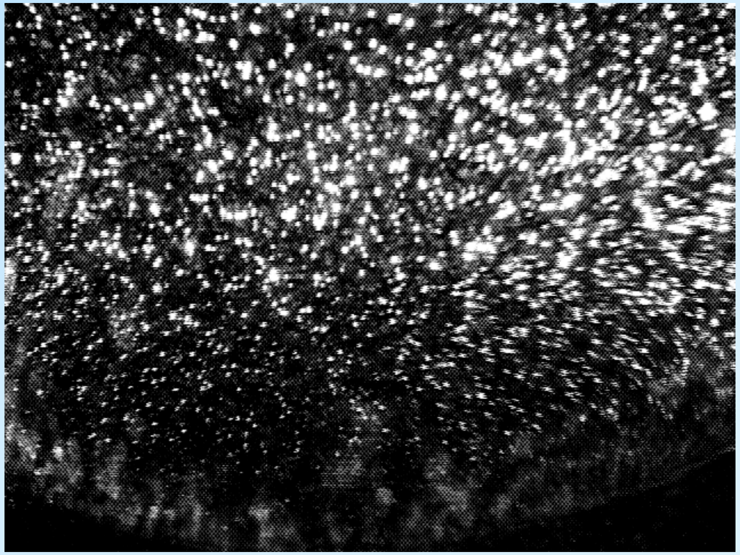
S. Goto, N. Ishii, S. Kida, and M. Nishioka, Phys. Fluids 19, 061705 (2007)



Rotation around one axis



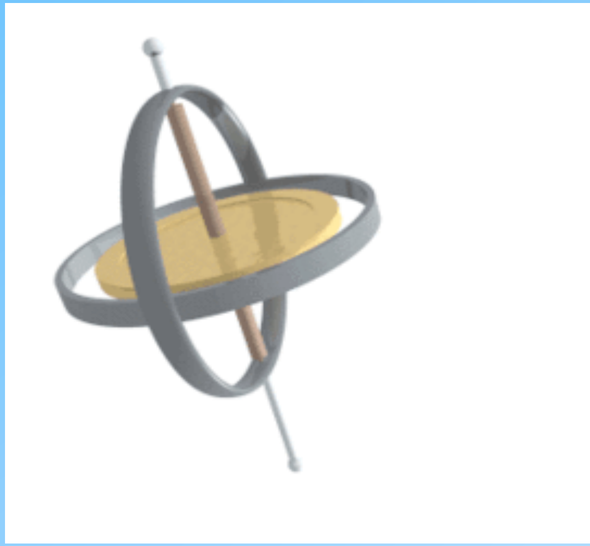
Rotation around two axes



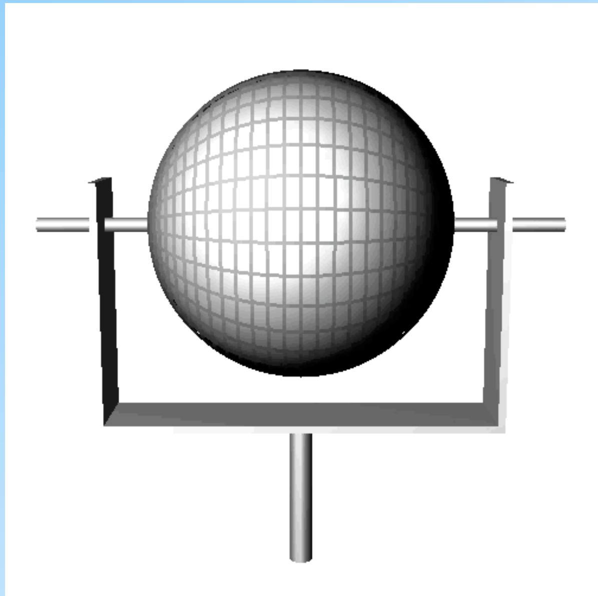
Are these two rotations represented by their sum? No!

Precession

Spin axis itself rotates around another axis.



Precessing motion of a gyroscope



Ω_x

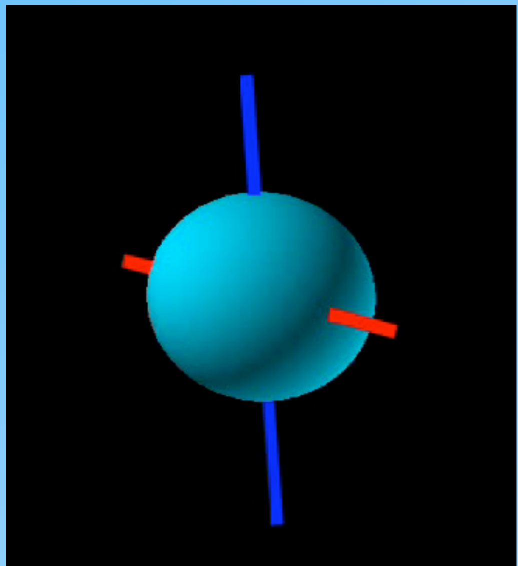
Ω_z

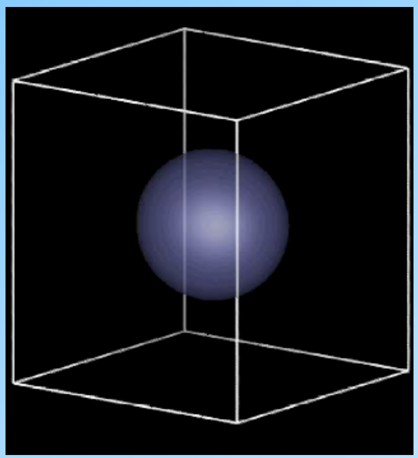
We consider the case where the spinning and precessing rotational axes are perpendicular to each other. Hence, the two rotations do not commute, and thus cannot be represented by their sum.

QT made by two precessions in a trapped BEC

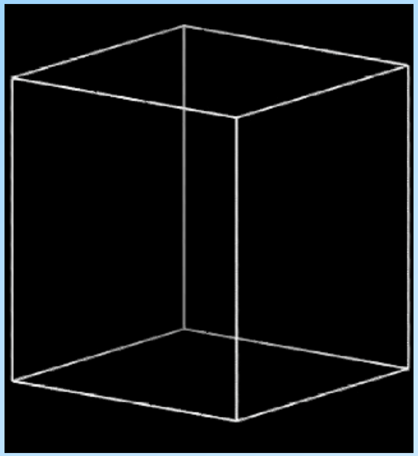
Two precessions ($\omega_x \times \omega_z$)

M. Kobayashi and M. T., Phys. Rev. A76, 045603 (2007)

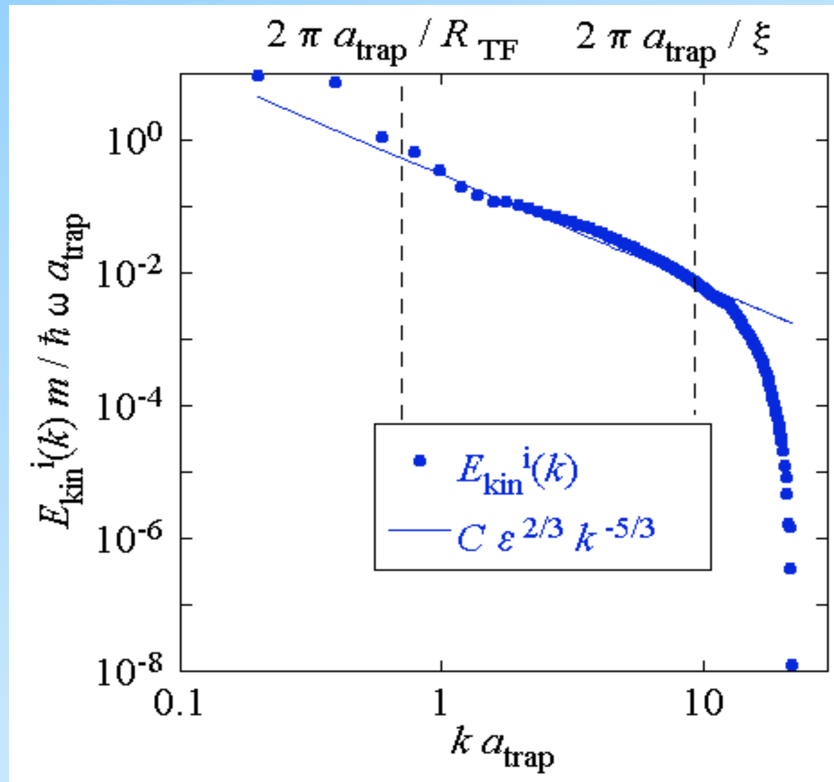




Condensate density



Quantized vortices



$$n \approx 1.78 \pm 0.194$$

We confirmed a scaling law of the energy spectrum similar to the Kolmogorov -5/3 law.

Recently QT has been realized experimentally in atomic BECS too!

1. E.A.L. Henn *et al*., PRL103, 045301(2009)

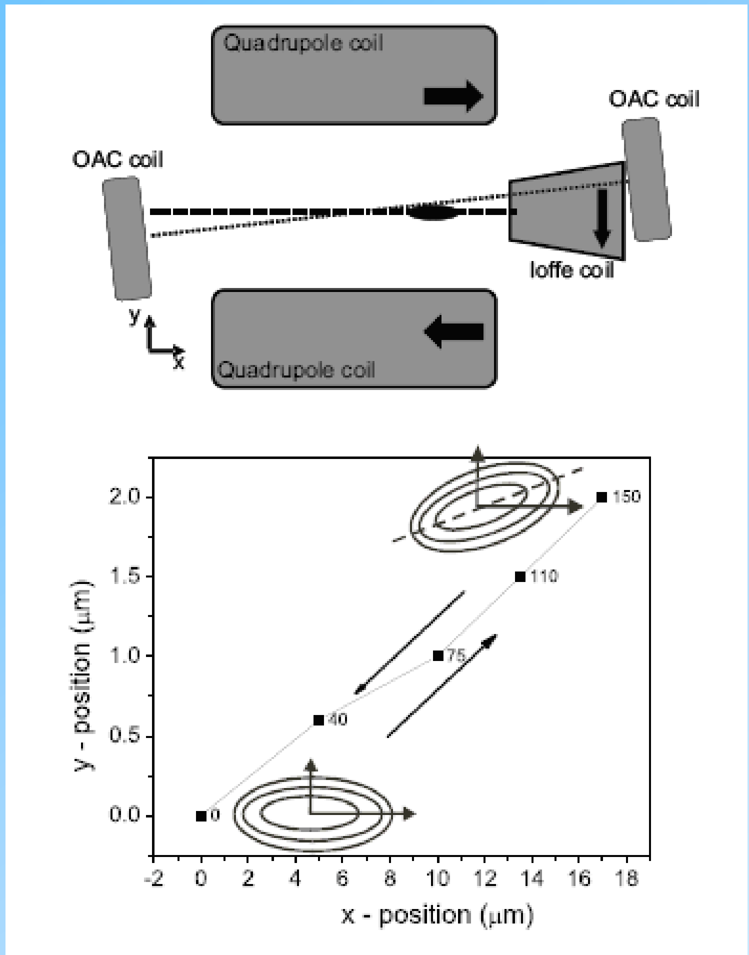
   They made 3D QT by exciting a trapped BEC by some complicated method. Then they observed a power law of the kinetic energy spectrum.

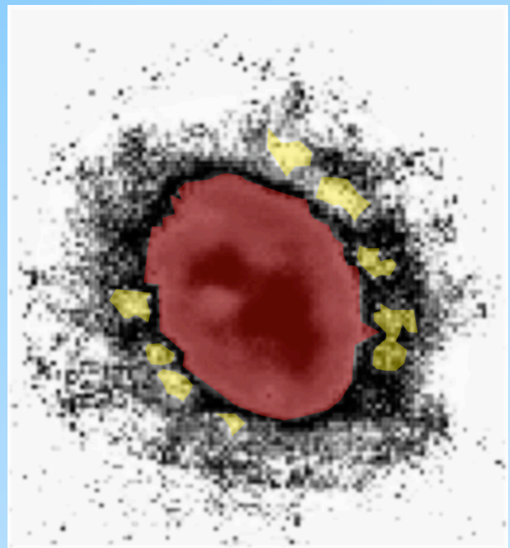
2. T. W. Neely *et al*., arXiv:1204.1102

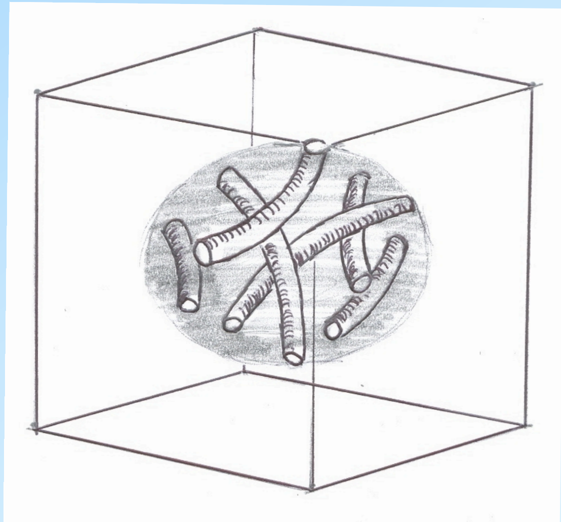
   They made 2D turbulence. The numerical simulation showed the energy spectra compared with the inverse energy cascade.

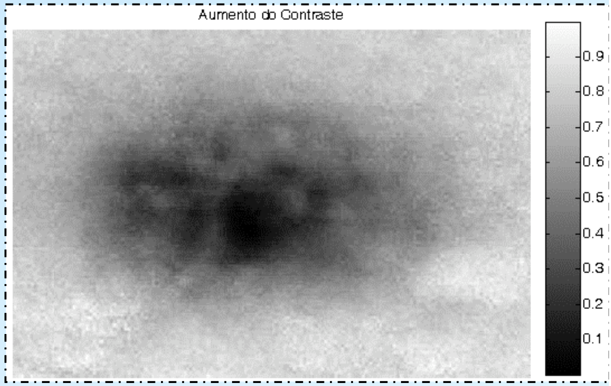
Making 3D QT by exciting a trapped BEC.

E.A.L.Henn *et al.*, PRL103, 045301(2009)





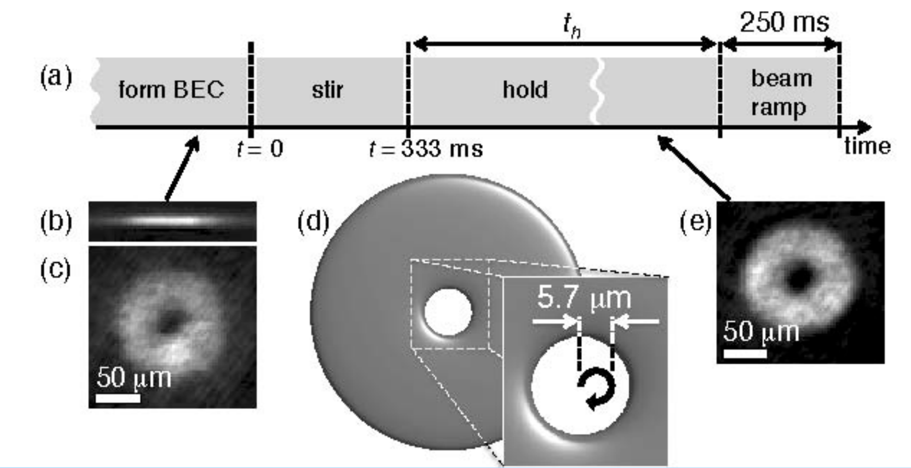


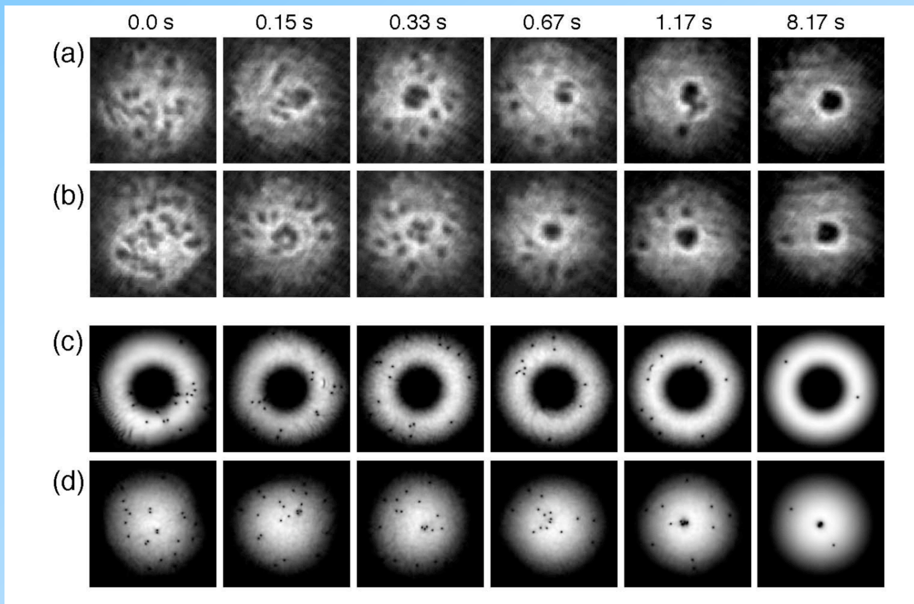


Coupled large amplitude oscillation

Making 2D QT by stirring a trapped BEC

T. W. Neely et al., arXiv:1204.1102





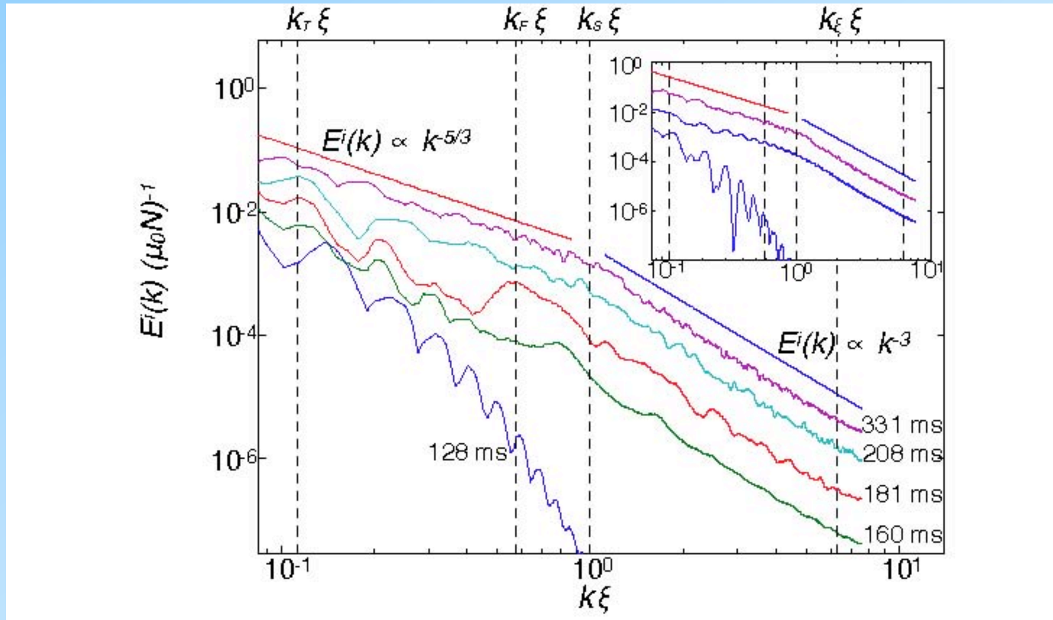


FIG. 3. Log-log plots of $E^i(k)$ (per atom) for times (128, 160, 181, 208, 331) ms over which forcing occurs, plotted

Numerical simulation of the GP model

Contents

0. Introduction ～Why is quantum turbulence interesting?～

1. QT in a trapped BEC

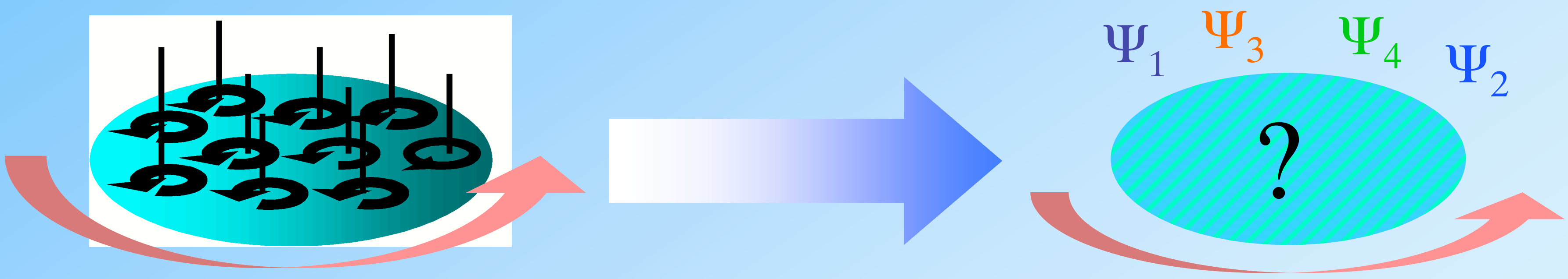
2. Counterflow of two-component BECs: two-component QT

3. Spin turbulence in spin-1 spinor BECs

Vortices and hydrodynamics in multi-component BECs

Depending on the symmetry, multi-component order parameters can yield various kinds of topological defects.

superfluid ^{3}He, superconductivity with non-s-wave symmetry (Sr_2RuO_4, UPt_3), bilayer quantum Hall systen, nonlinear optics, nuclear physics, cosmology(Neutron star), …



Ω Triangular (Abrikosv)lattice

Ω

Topological defects in two-component BECs

Two-component BEC

Two order parameters (macroscopic wave functions) Ψ_1 Ψ_2

Coupled Gross-Pitaevskii(GP) equations

$$i\hbar\partial_t\Psi_1 = \left(-\frac{\hbar^2}{2m_1}\nabla^2 + U_1 + g_{11}|\Psi_1|^2 + g_{12}|\Psi_2|^2\right)\Psi_1$$

$$i\hbar\partial_t\Psi_2 = \left(-\frac{\hbar^2}{2m_2}\nabla^2 + U_2 + g_{12}|\Psi_1|^2 + g_{22}|\Psi_2|^2\right)\Psi_2$$

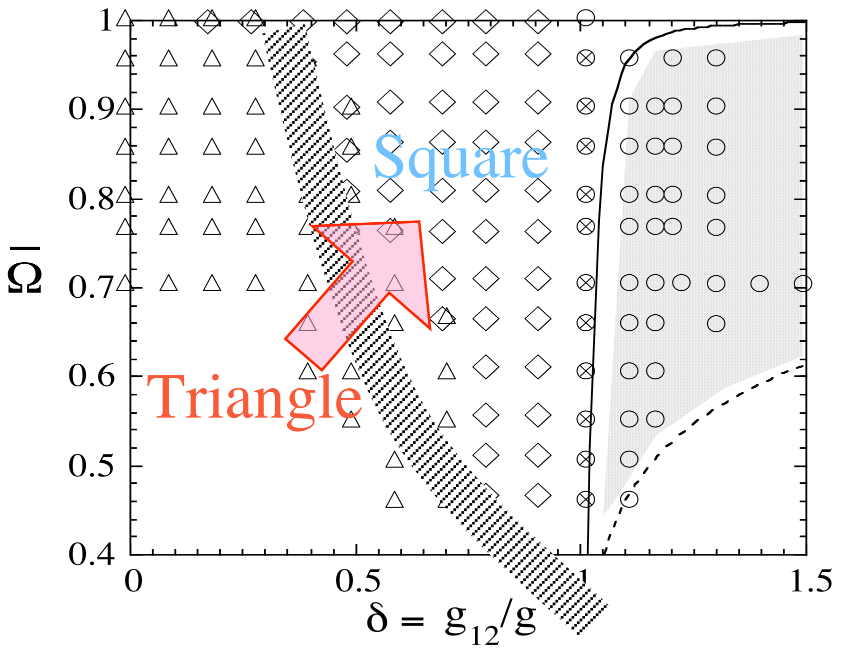
g_{11}, g_{22}: intracomponent interaction

g_{12} : intercomponent interaction

When $g_{11}g_{22} > g_{12}^2$, two BECs are mixed.

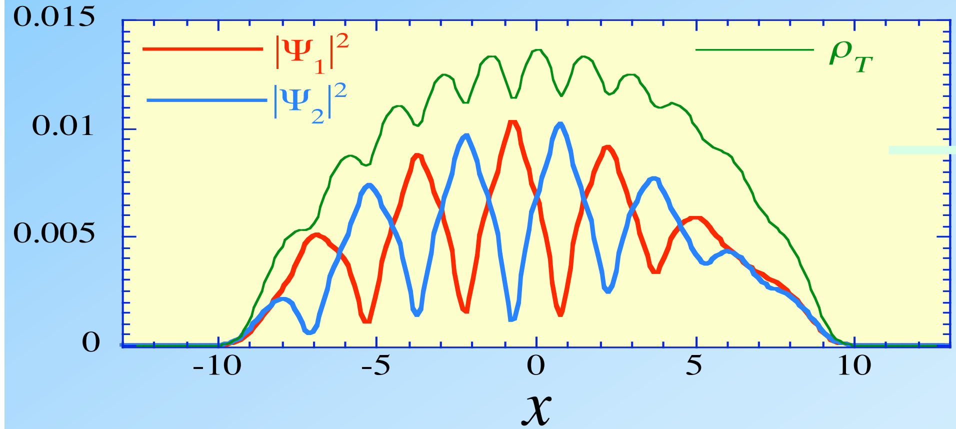
When $g_{11}g_{22} < g_{12}^2$, two BECs are phase-separated.

Vortex lattices in rotating two-component BECs

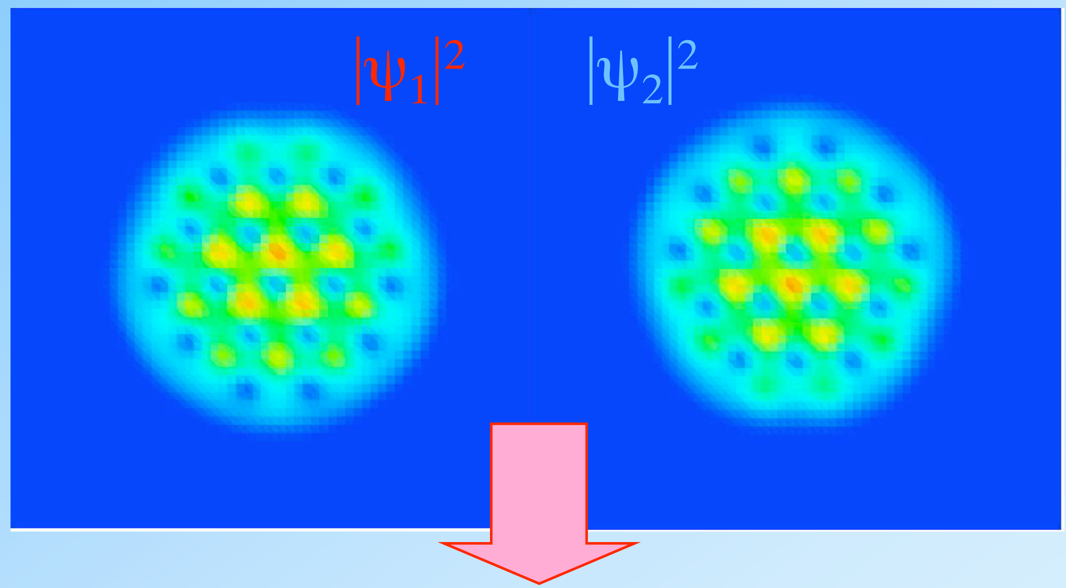


K. Kasamatsu, MT, M. Ueda, PRL91, 150406 (2003)

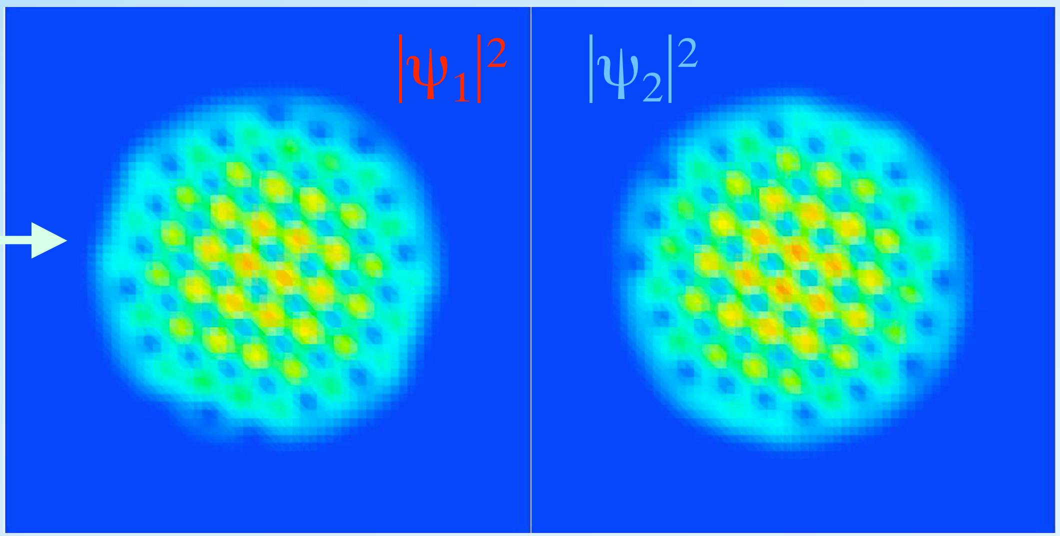
cross section



Triangular lattices



Square lattices



Hydrodynamic instability in two-component BECs

Quantum Kelvin-Helmholtz instability (KHI)

H. Takeuchi, N. Suzuki, K. Kasamatsu, H. Saito, MT, PRB81, 094517 (2010)

Crossover between KHI and counterflow instability

N. Suzuki, H. Takeuchi, K. Kasamatsu, MT, H. Saito, PRA81, 063604 (2010)

Counterflow instability and QT
H. Takeuchi, S. Ishino, MT, PRL105, 205301(2010)
S. Ishino, MT, H. Takeuchi,   PRA83, 063602(2011)
C Hamner, J. J. Chang, P. Engles, M. A. Hoefer, PRL106, 065302(2011)

Rayleigh-Taylor instability

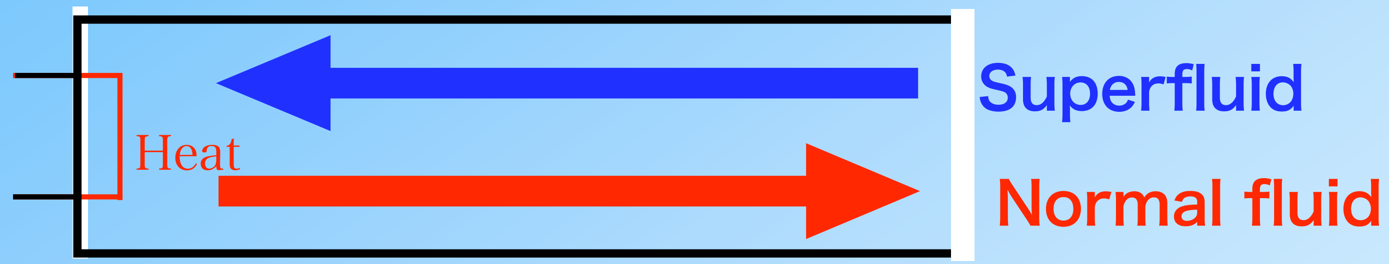
K. Sasaki, N, Suzuki, D. Akamatsu, H. Saito, PRA80, 042704 (2009)

1. Counterflow of two-component BECs: two-component QT

H. Takeuchi, S. Ishino, MT, Phys. Rev. Lett.105, 205301(2010);

S. Ishino, MT, H. Takeuchi, Phys. Rev. A83, 063602(2011)

Thermal counter flow



When the relative velocity exceeds some critical value, the superfluid becomes turbulent.

Counterflow of two-component BECs

When the relative velocity exceeds some critical value, two BECs are expected to become unstable and turbulent.

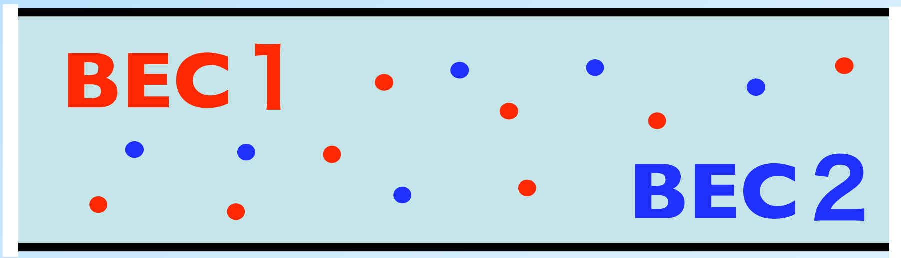
Two-component GP model

$$
\boxed{
\begin{aligned}
i\hbar\frac{\partial}{\partial t}\Psi_1 &= \frac{\mathbf{p}_1^2}{2m_1}\Psi_1 + g_{11}|\Psi_1|^2\Psi_1 + g_{12}|\Psi_2|^2\Psi_1 \\
i\hbar\frac{\partial}{\partial t}\Psi_2 &= \frac{\mathbf{p}_2^2}{2m_2}\Psi_2 + g_{22}|\Psi_2|^2\Psi_2 + g_{12}|\Psi_1|^2\Psi_2
\end{aligned}
}
$$

g_{11}, g_{22} : intracomponent interaction

g_{12} : intercomponent interaction

$g_{11}g_{22} > g_{12}^2$ ➡ The mixture is stable.



Two-component GP model

$$i\hbar\frac{\partial}{\partial t}\Psi_1 = \frac{\mathbf{p}_1^2}{2m_1}\Psi_1 + g_{11}|\Psi_1|^2\Psi_1 + g_{12}|\Psi_2|^2\Psi_1$$

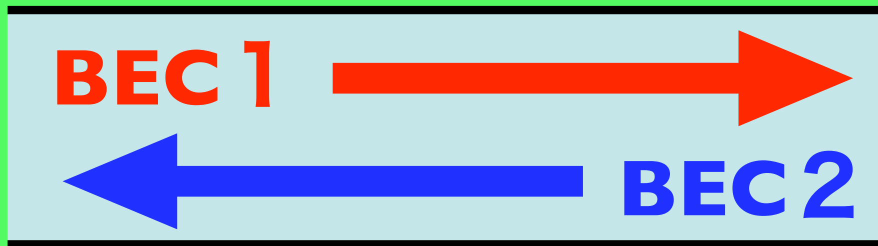
$$i\hbar\frac{\partial}{\partial t}\Psi_2 = \frac{\mathbf{p}_2^2}{2m_2}\Psi_2 + g_{22}|\Psi_2|^2\Psi_2 + g_{12}|\Psi_1|^2\Psi_2$$

g_{11}, g_{22} : intracomponent interaction

g_{12} : intercomponent interaction

$g_{11}g_{22} > g_{12}^2$ ➡ The mixture is stable.

However,



The large relative velocity should make it unstable.

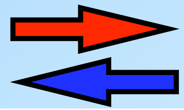
V. I. Yukalov and E. P. Yukalova, Laser Phys. Lett. **1**, 50 (2004).

C. Hamner *et al*., Phys. Rev. Lett. **106**, 065302(2011).

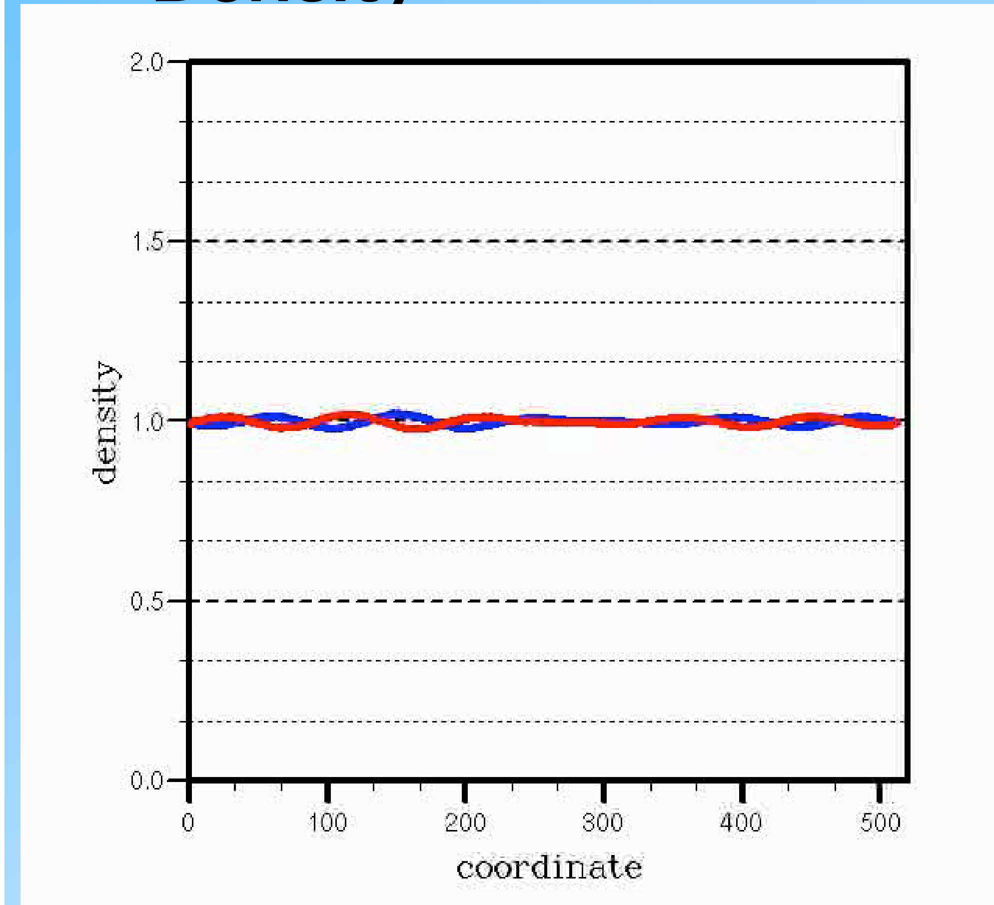
Results (One-dimension)

$$n_1 = n_2 = 1.0\ ,\ \mathbf{v}_1 = -\mathbf{v}_2\ ,\ \gamma = 0.5$$

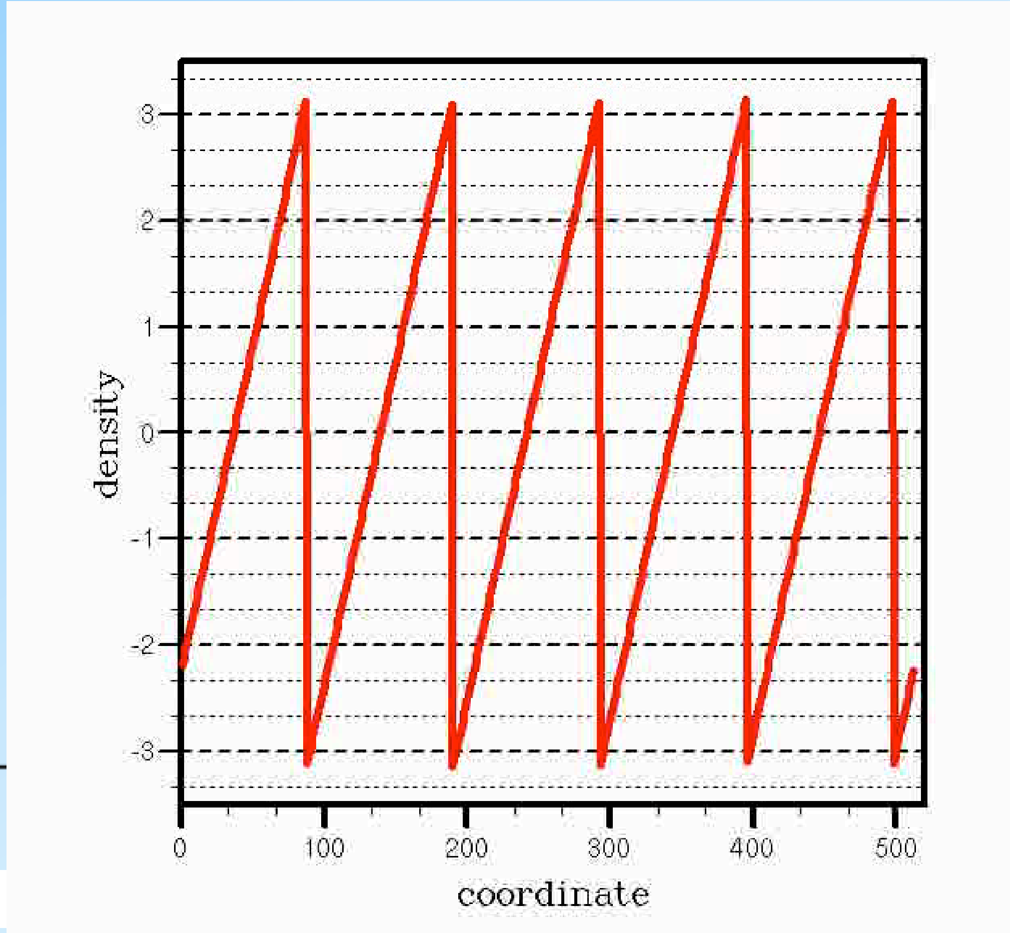
Direction of flow



Density



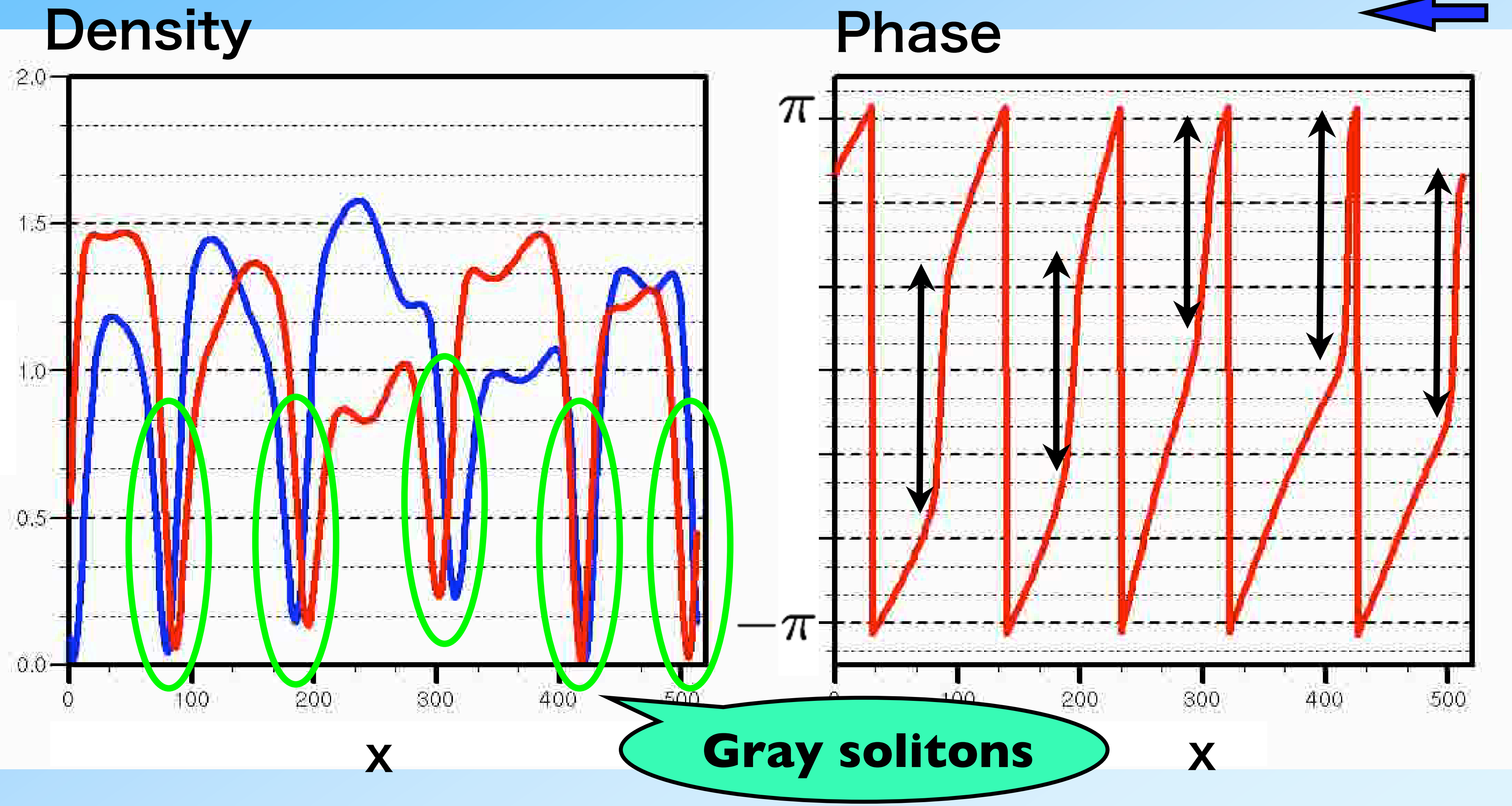
Phase



Results　(One-dimension)

$n_1 = n_2 = 1.0$, $\mathbf{v}_1 = -\mathbf{v}_2$, $\gamma = 0.5$

Flow direction

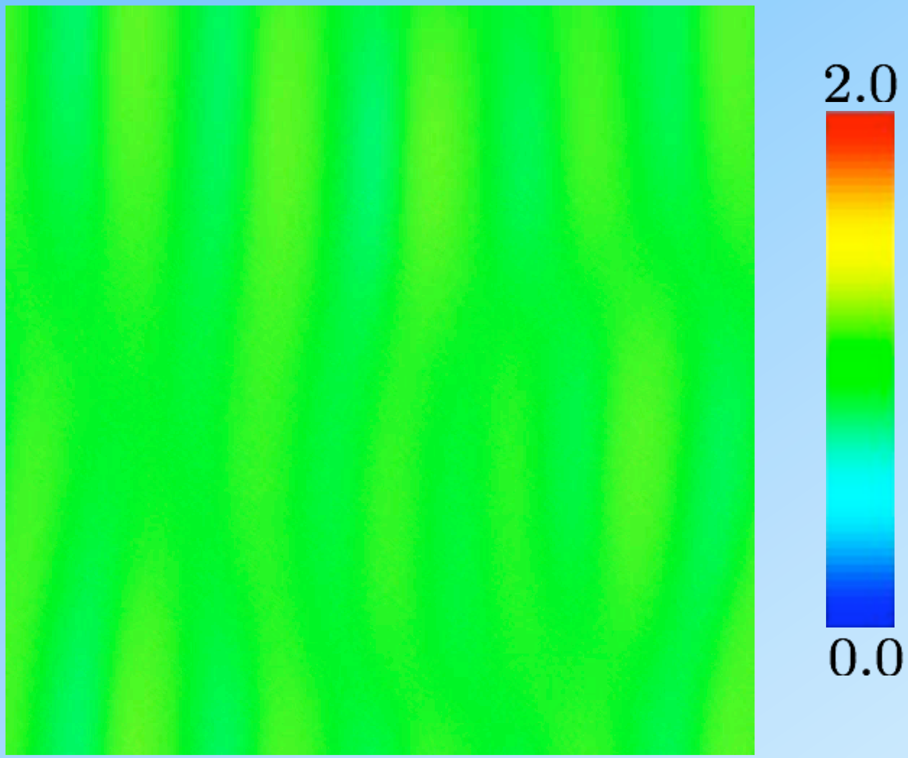


Results (two-dimension)

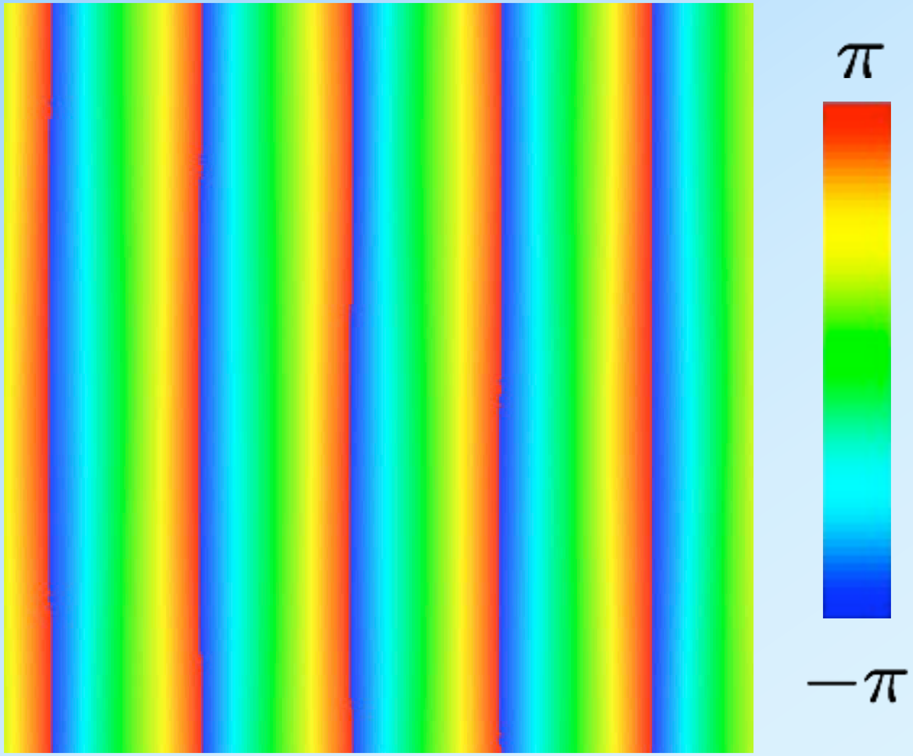
$n_1 = n_2 = 1.0$, $\mathbf{v}_1 = -\mathbf{v}_2$, $\gamma = 0.5$

Flow direction

Density



Phase

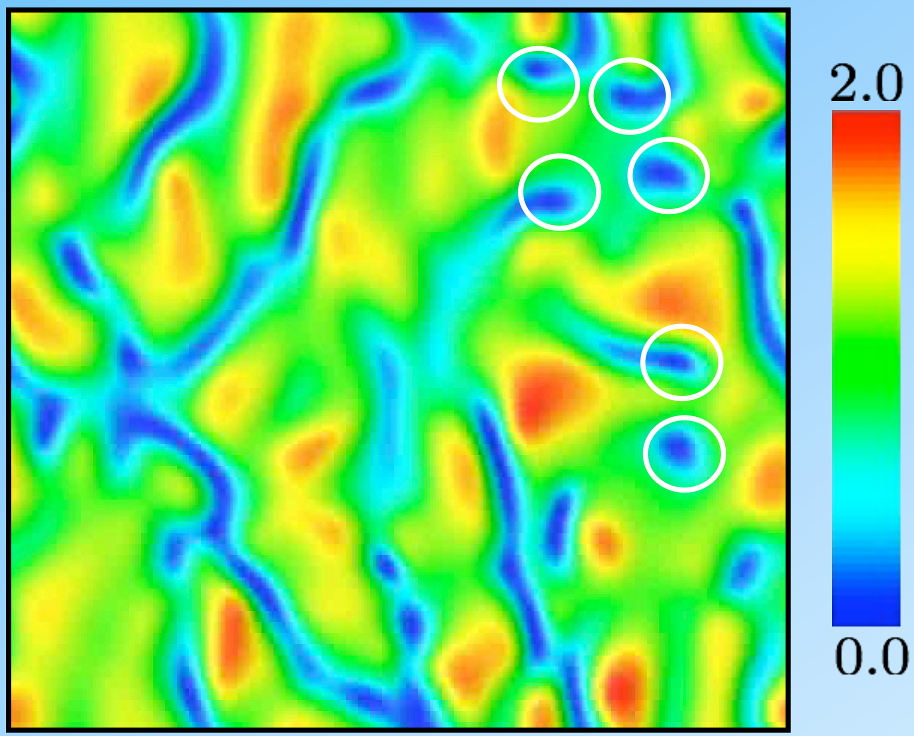


Results　　(two-dimension)

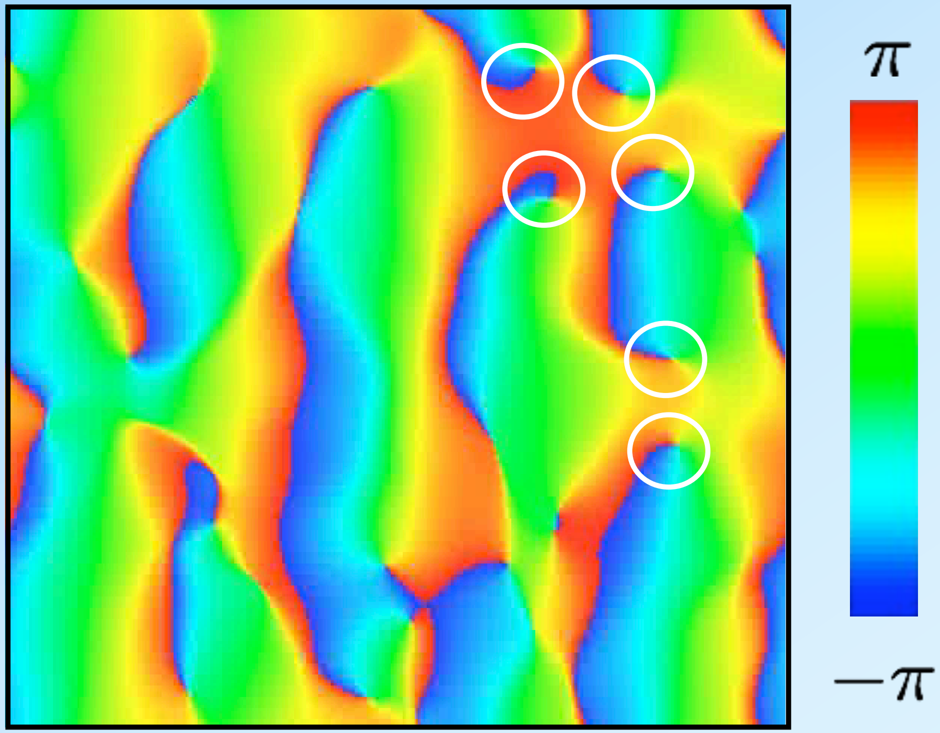
$n_1 = n_2 = 1.0$, $\mathbf{v}_1 = -\mathbf{v}_2$, $\gamma = 0.5$

Flow direction

Density



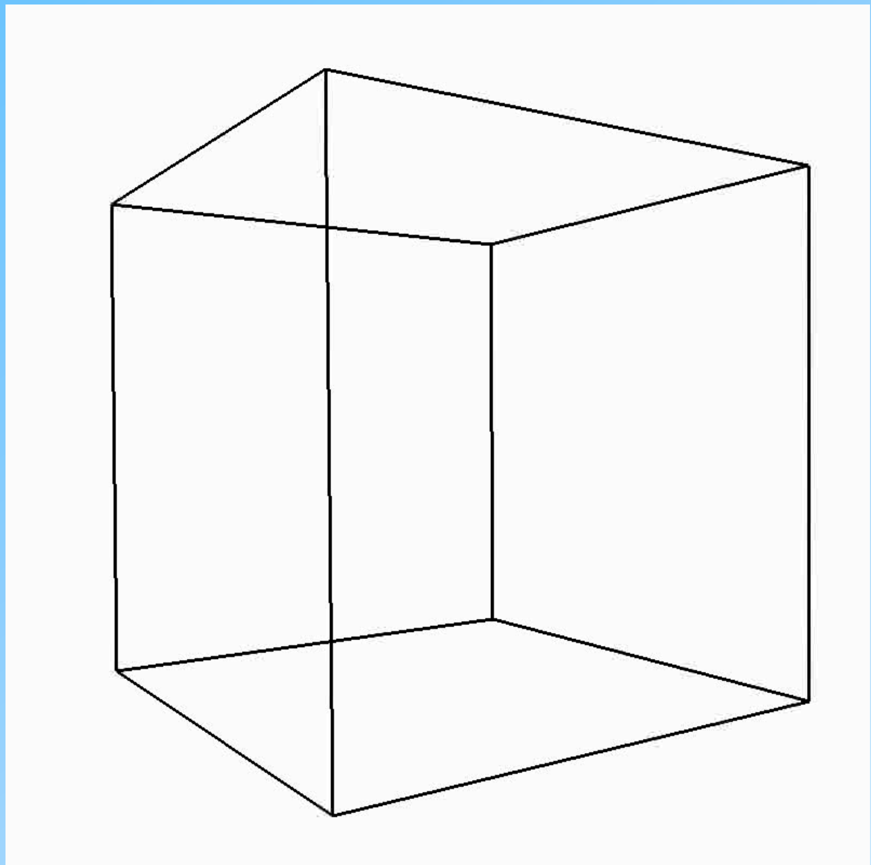
Phase

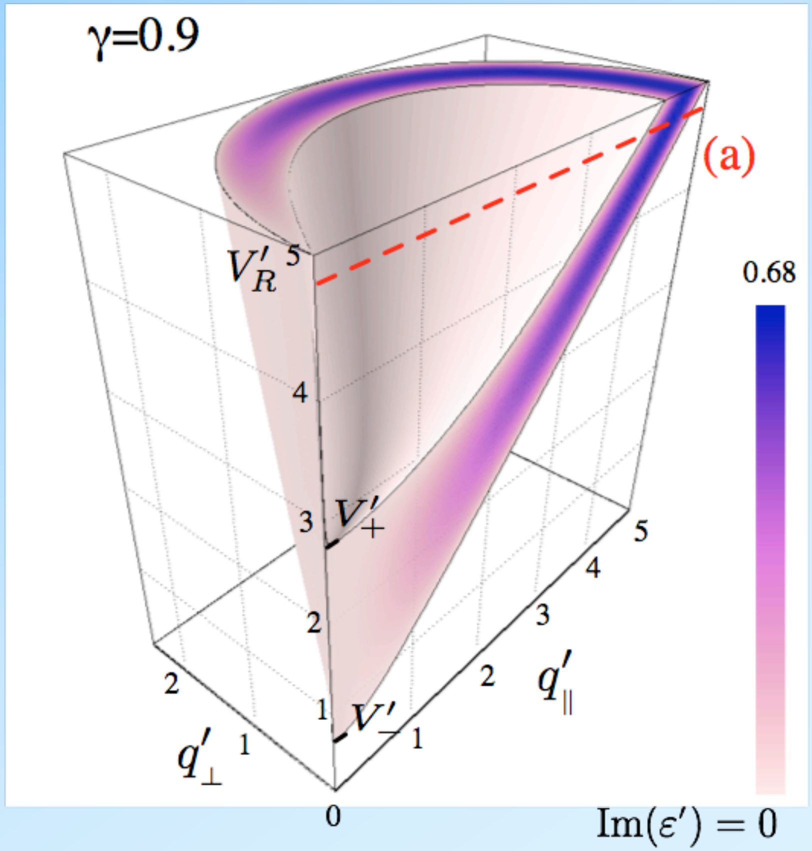


The solitons decay to vortex pairs through snake instability.

3D 2-component QT

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \quad V'_R = 4.71, \quad \gamma = 0.9$$



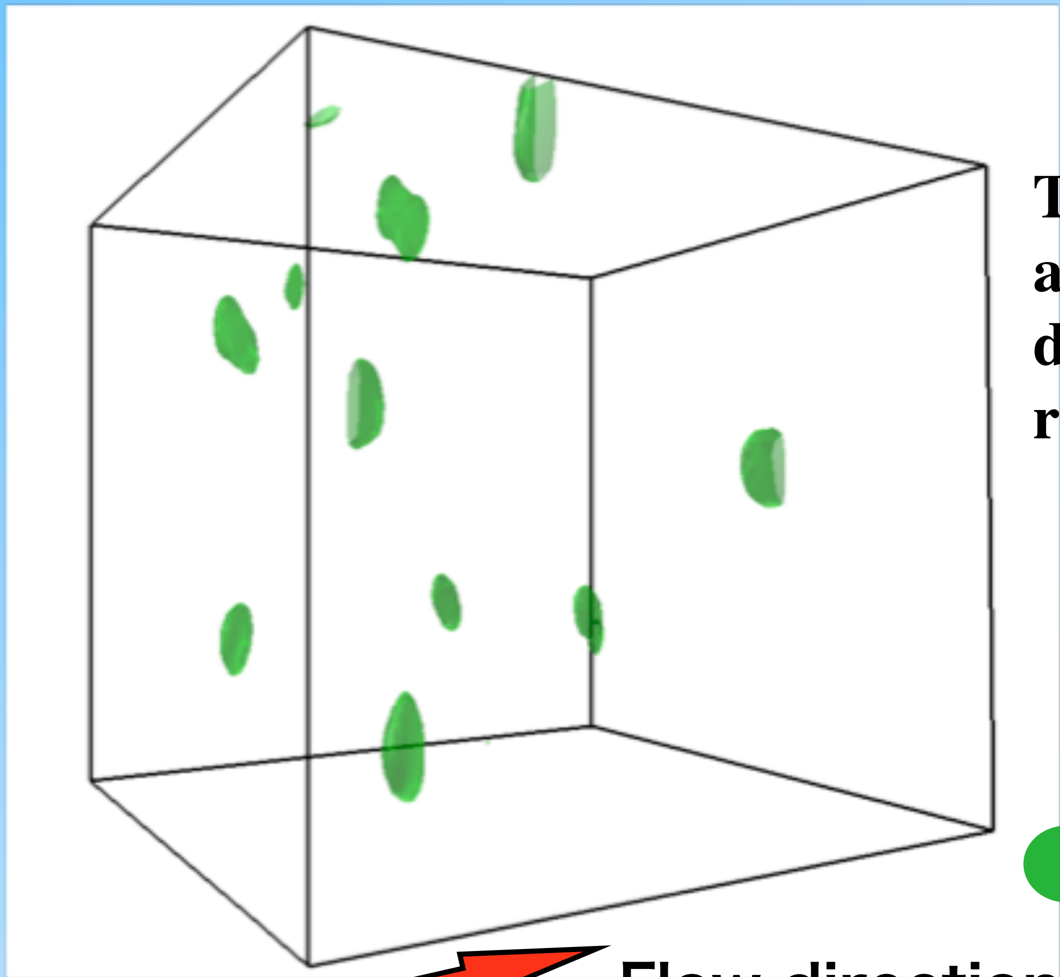


Flow direction

Solitons → Vortex loops → QT

Scenario to turbulence (1)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1,\ \ V_R' = 4.71,\ \ \gamma = 0.9$$



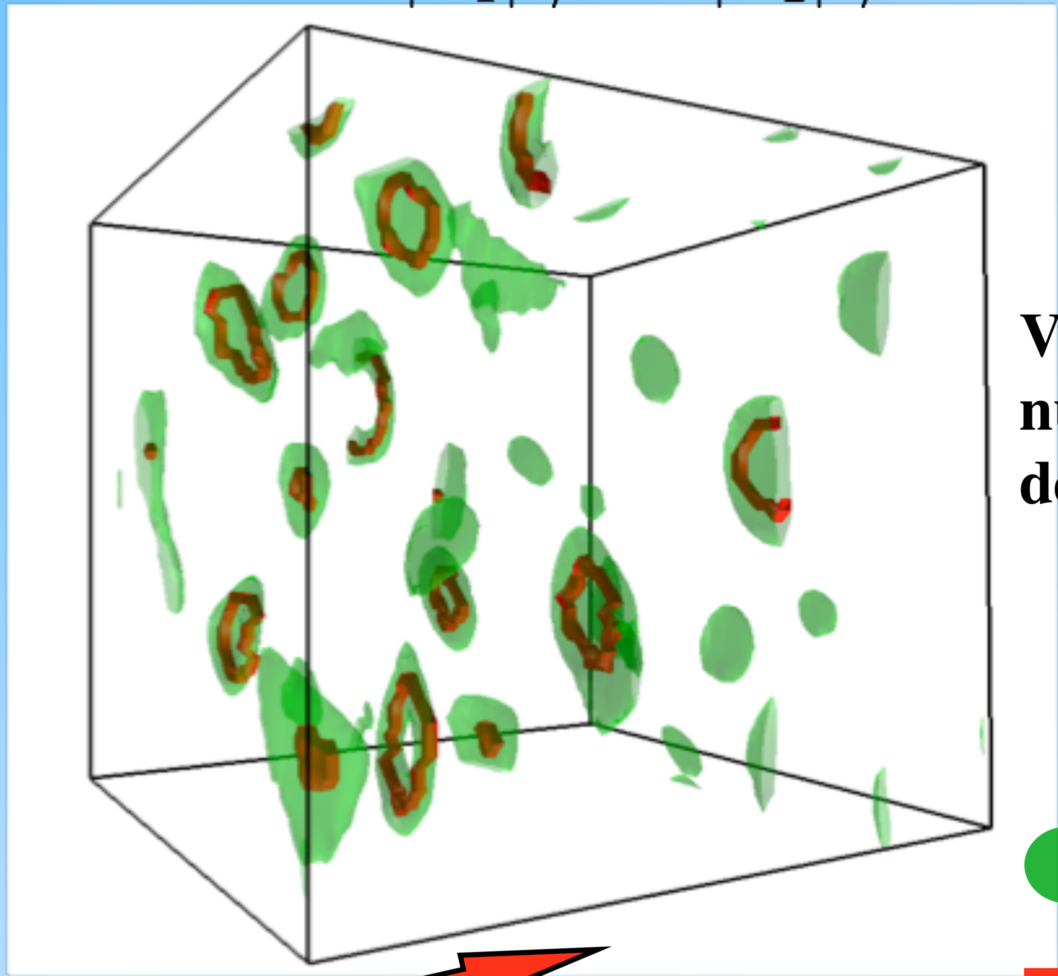
The unstable mode is amplified to lead to the disk-shaped low density regions.

Isosurface of $|\Psi_1|^2/n = 0.1$

Flow direction

Scenario to turbulence (2)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \;\; V_R' = 4.71, \;\; \gamma = 0.9$$



Vortex rings are nucleated inside the low density regions.

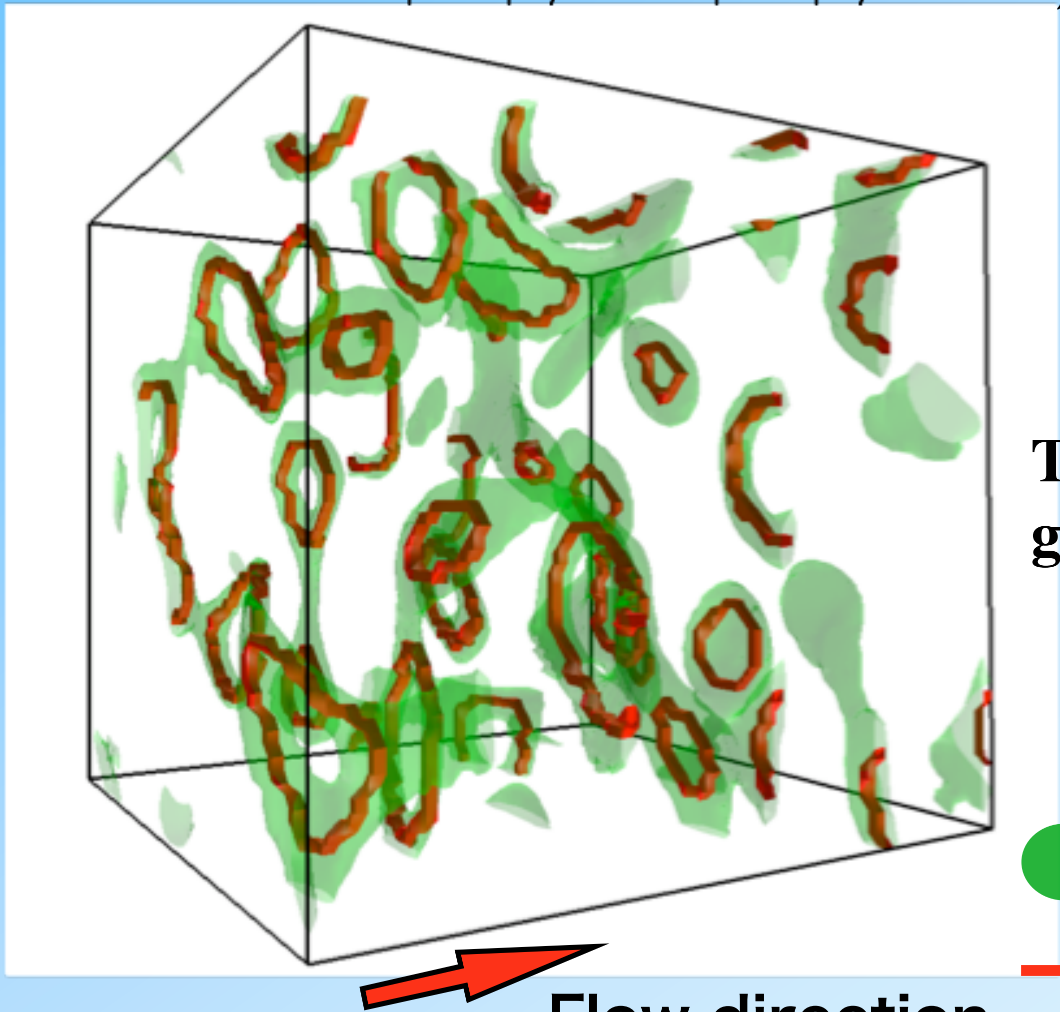
Flow direction

Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Scenario to turbulence (3)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \;\; V'_R = 4.71, \;\; \gamma = 0.9$$



The vortices expand and grow.

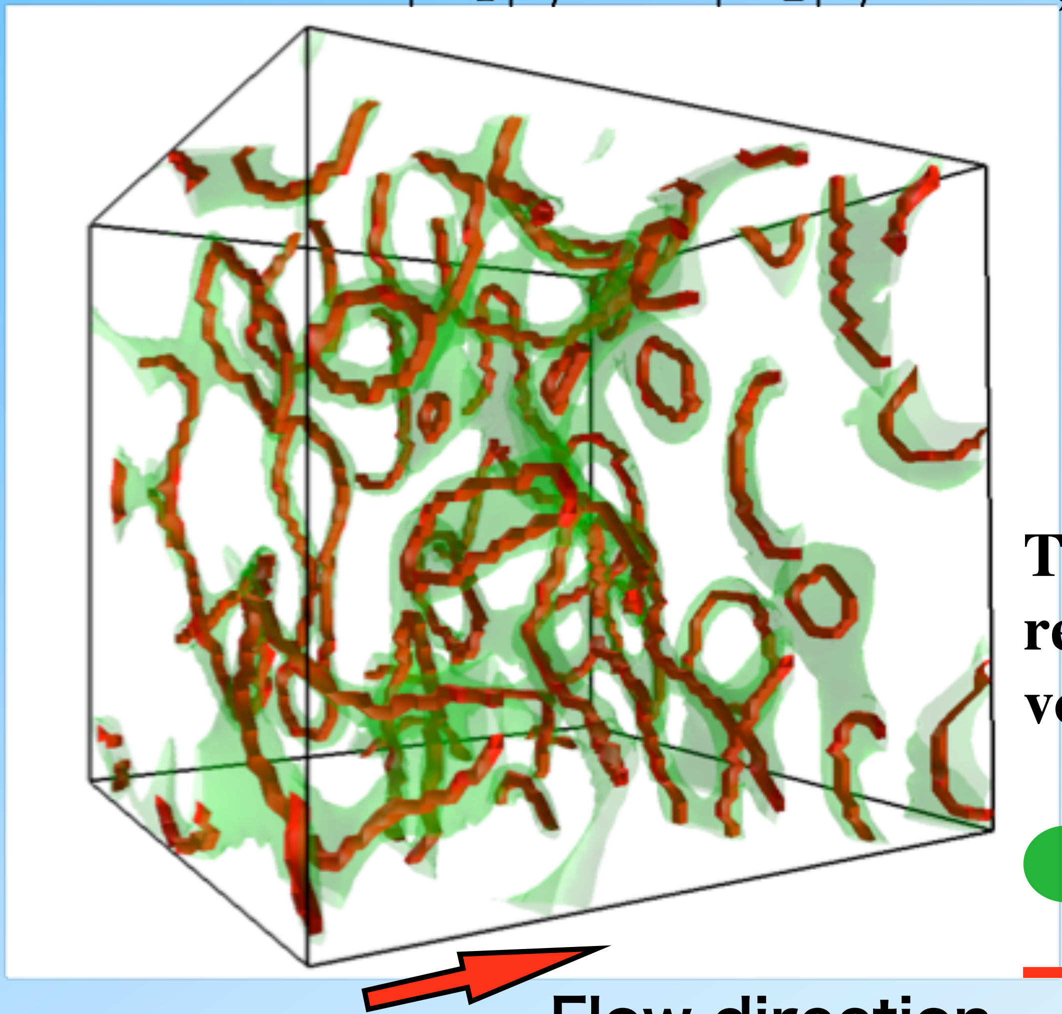
Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Flow direction

Scenario to turbulence (4)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \;\; V_R' = 4.71, \;\; \gamma = 0.9$$



Flow direction

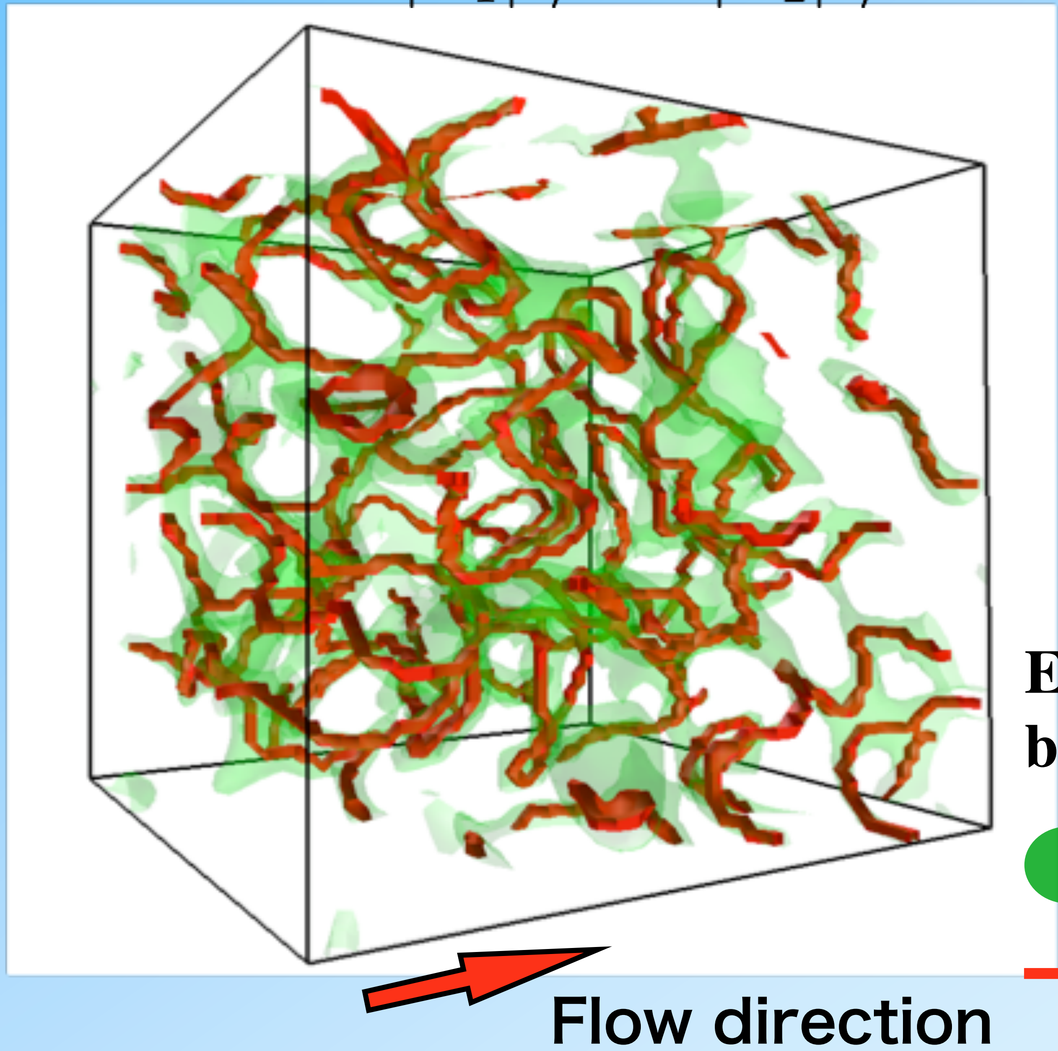
The vortices expand to reconnect with other vortices.

Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Scenario to turbulence (5)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \ \ V_R' = 4.71, \ \ \gamma = 0.9$$

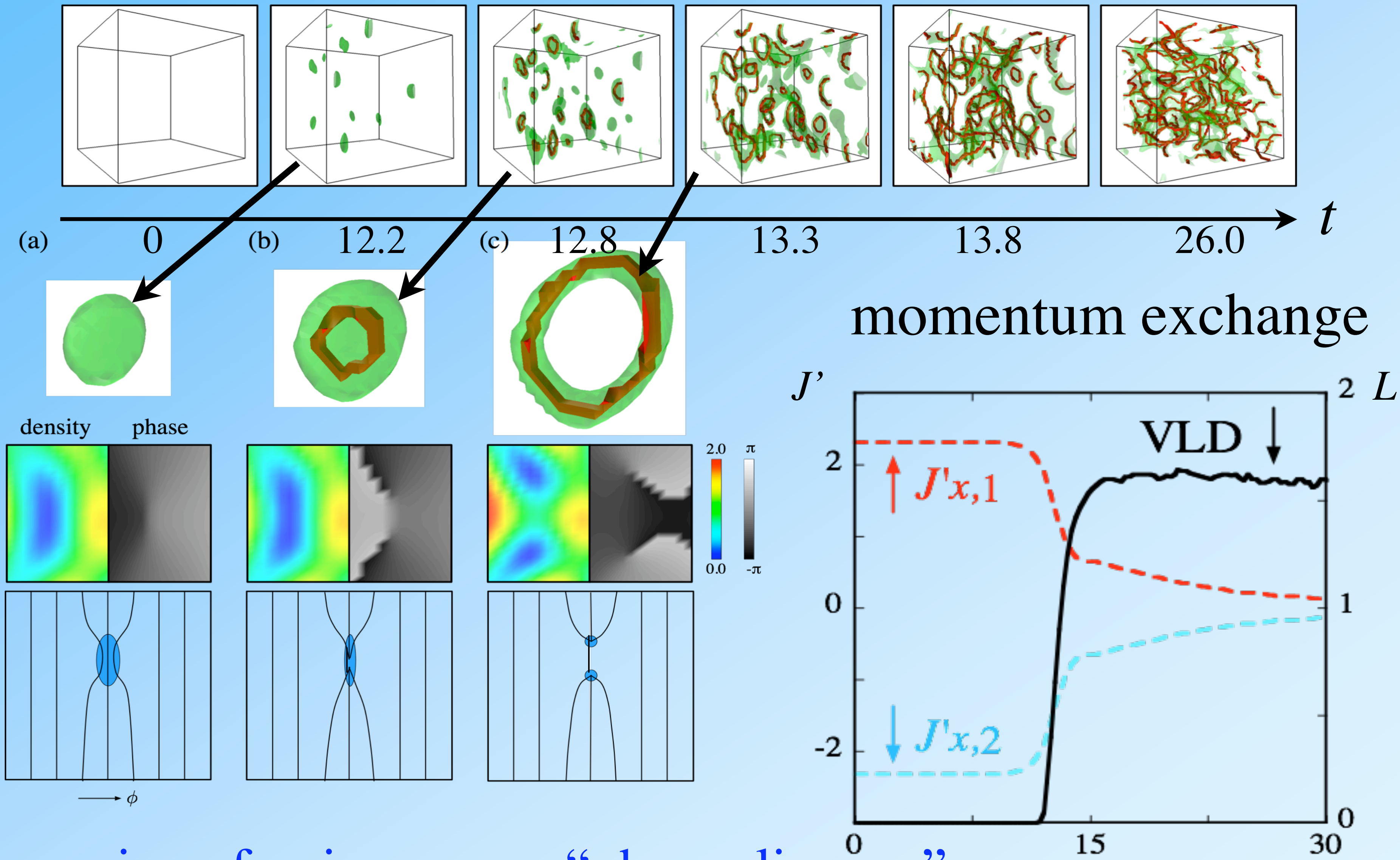


Eventually the vortices become tangled.

Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

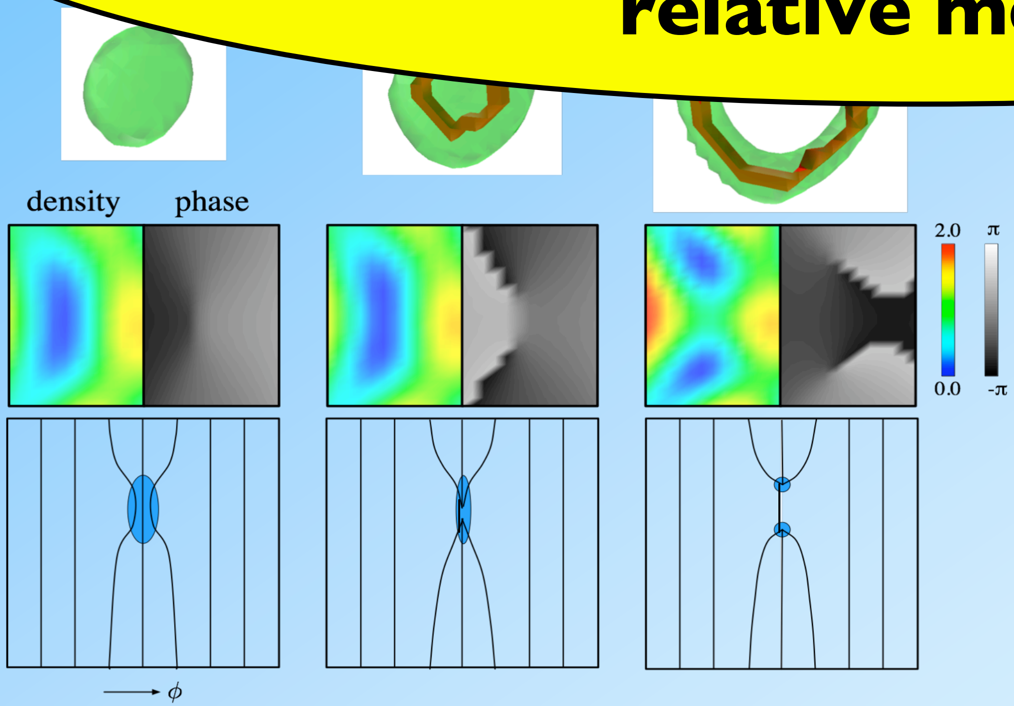
Scenario to turbulence

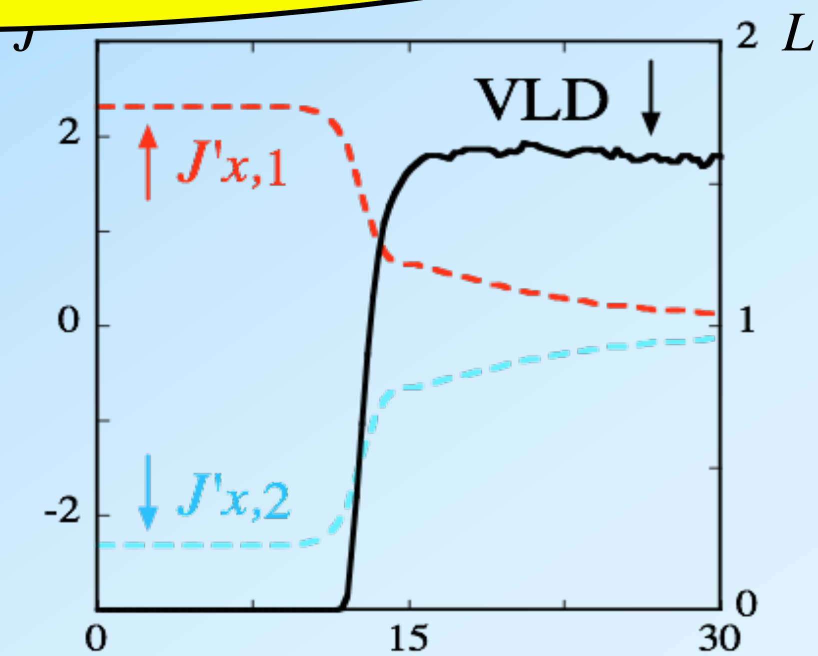


Expansion of a ring means “phase slippage”.

Scenario to turbulence

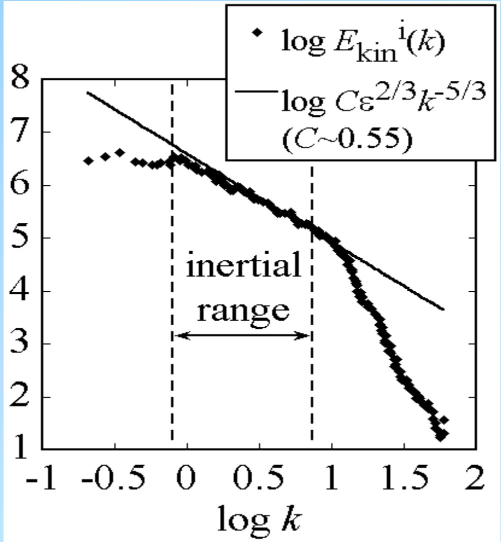
The "mutual friction" between two condensates exchange momentum between them to reduce their relative motion.





Why is binary QT interesting?

We know one-component QT obeys the Kolmogorov law(K41).



M. Kobayashi, MT, J. Phys. Soc. Jpn. 74, 3248 (2005)

What happens to two-component QT?

- By changing g_{12}, we can control their coupling.
- By considering the unsymmetric case $g_{11} \neq g_{22}$, we can consider the coupling of different QTs. *etc.*

Currently several groups try to perform the experiments!

3. Spin turbulence in a spin-1 spinor BEC

K. Fujimoto and MT, Phys. Rev. A83, 053609(2011): Phys. Rev. A85, 033642(2012)

Spin-1 spinor Bose-Einstein condensate

^{23}Na, ^{87}Rb, etc.

S = 1

$S_z = 1$

$S_z = 0$

$S_z = -1$

Now we confine ourselves to the ferromagnetic case.

Spin-1 spinor Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi_m = -\frac{\hbar^2}{2M}\nabla^2\psi_m + c_0 n\psi_m + c_1\sum_{m=-1}^{1}\mathbf{s}\cdot\mathbf{S}_{mn}\psi_n \qquad (m = 1, 0, -1)$$

Spin density vector is $\quad s_i = \sum_{m,n=-1}^{1}\psi_m^*(S_i)_{mn}\psi_n$

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix},\quad S_y = \frac{i}{\sqrt{2}}\begin{pmatrix}0 & -1 & 0\\ 1 & 0 & -1\\ 0 & 1 & 0\end{pmatrix},\quad S_z = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

Total energy

$$E = \int\sum_{m=-1}^{1}\left[\psi_m^*\left(-\frac{\hbar^2}{2M}\nabla^2\right)\psi_m\right]d\mathbf{r} + \frac{c_0}{2}\int n^2 d\mathbf{r} + \underline{\frac{c_1}{2}\int\mathbf{s}^2 d\mathbf{r}}$$

Spin-dependent energy

Why is spinor BEC interesting from the view points of quantum hydrodynamics?

It has BOTH degrees of freedom of superflow and spins.

Superflow has quantized vortices and can be turbulent.

Spins can have their own vortices and show another turbulence.

$$i\hbar\frac{\partial}{\partial t}\psi_m = -\frac{\hbar^2}{2M}\nabla^2\psi_m + c_0 n\psi_m + c_1\sum_{m=-1}^{1}\mathbf{s}\cdot\mathbf{S}_{mn}\psi_n \quad (m=1,0,-1)$$

Our theoretical and numerical works

K. Fujimoto and MT, Phys. Rev. A83, 053609(2011)

By using the counterflow instability in a uniform system, we made spin turbulence and found the spectrum of the spin dependent energy $E_s \propto k^{-7/3}$.

$$E = \int \sum_{m=-1}^{1} \left[\psi_m^* \left(-\frac{\hbar^2}{2M} \nabla^2 \right) \psi_m \right] d\mathbf{r} + \frac{c_0}{2} \int n^2 d\mathbf{r} + \underline{\frac{c_1}{2} \int \mathbf{s}^2 d\mathbf{r}}$$

Spin-dependent energy

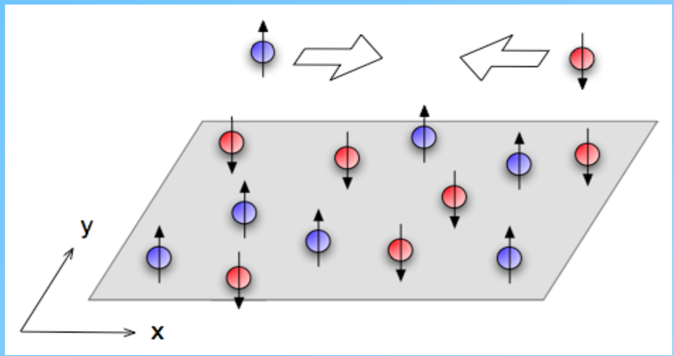
K. Fujimoto and MT, Phys. Rev. A85, 033642(2012)

Starting from a helical structure in a trapped system, we also obtained spin turbulence with the spectrum $E_s \propto k^{-7/3}$ and characteristic behavior of spins.

These states should be accessible experimentally.

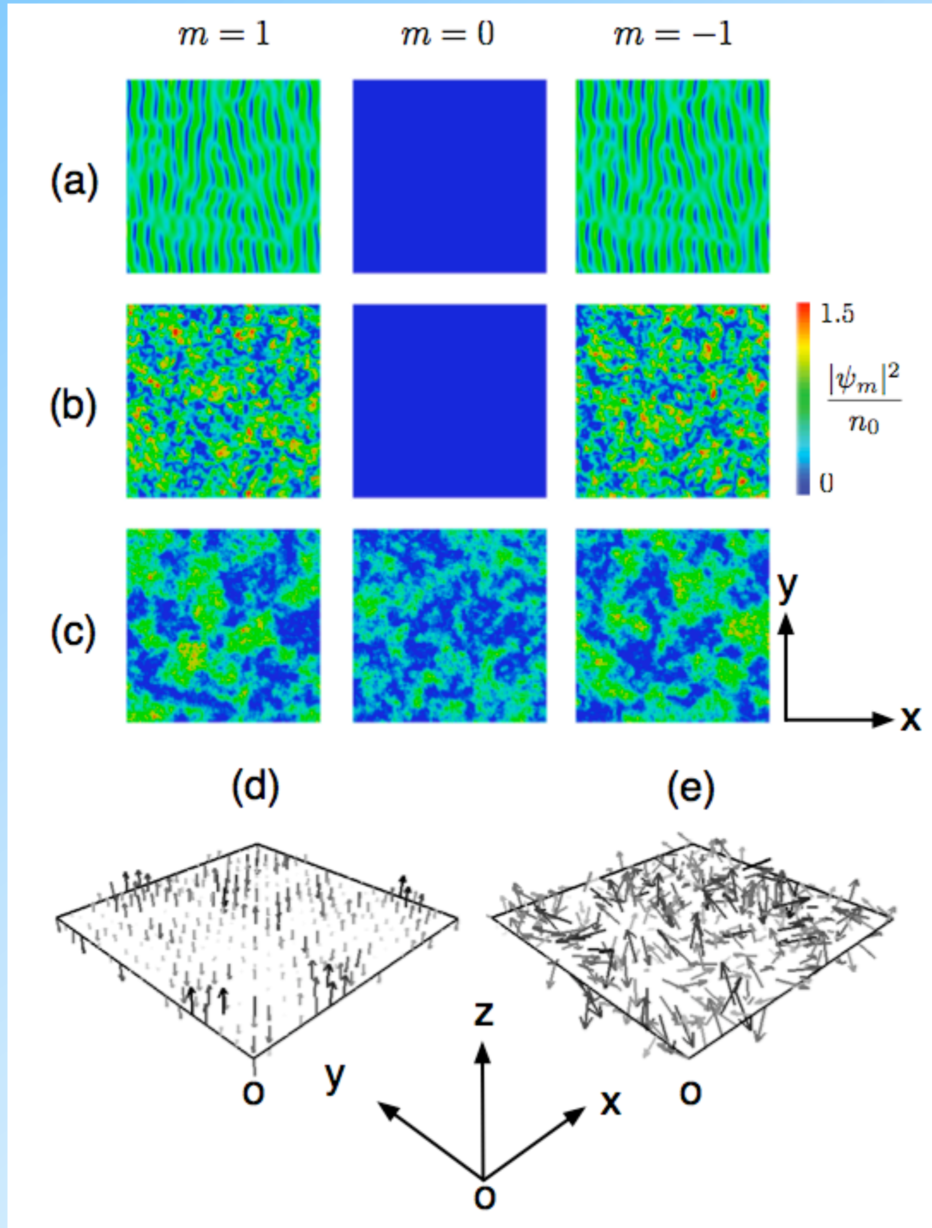
K. Fujimoto and MT, Phys. Rev. A83, 053609(2011)

S_z=1 S_z= -1



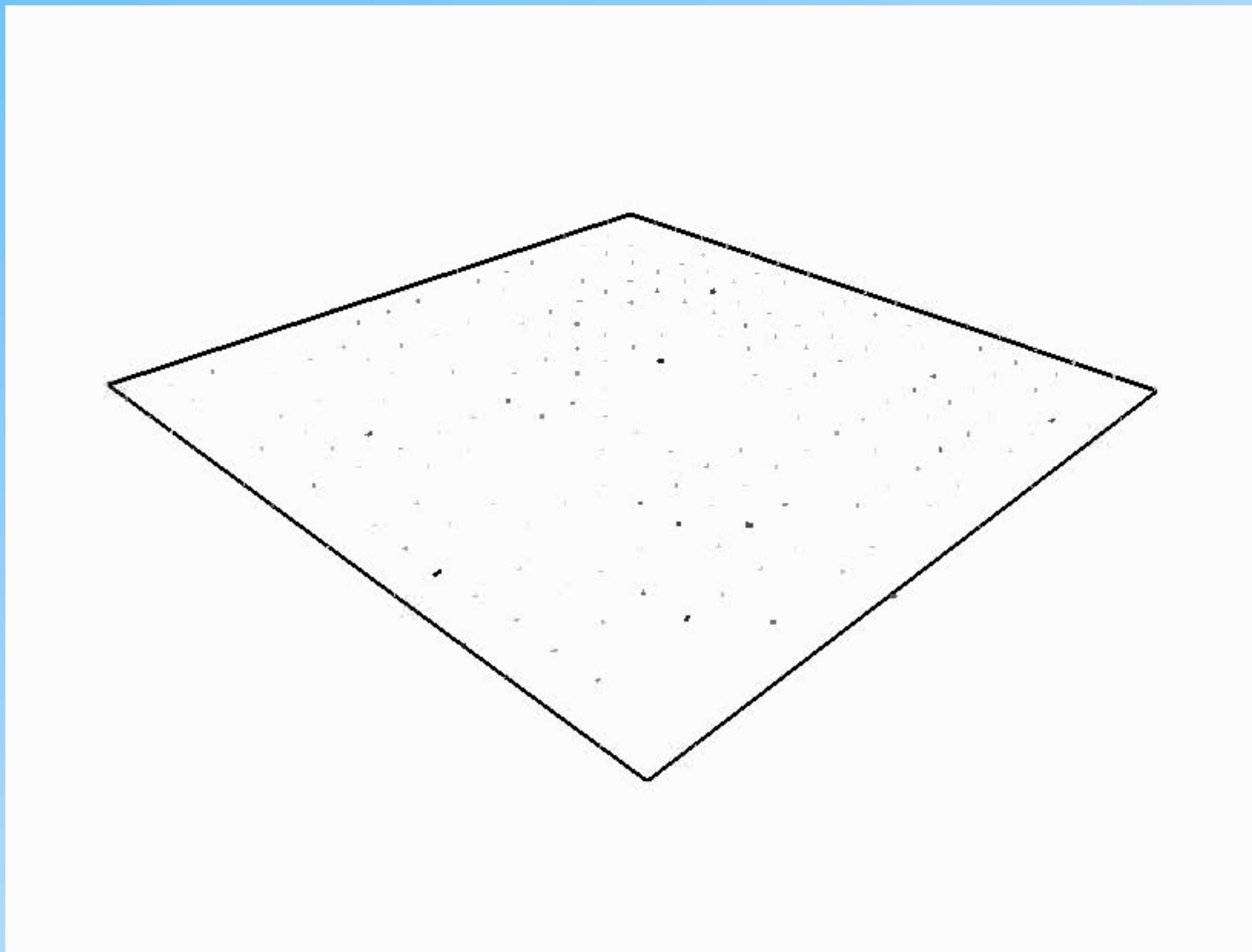
Counterflow between S_z=±1 components

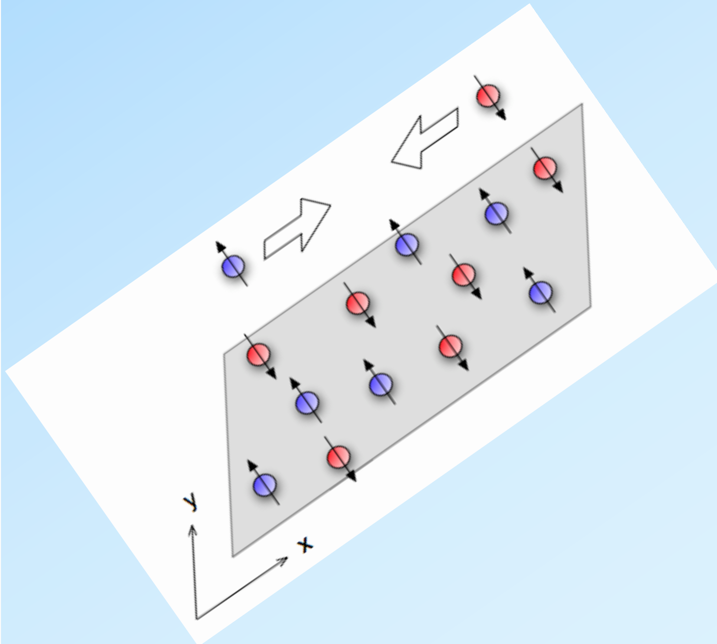
The counterflow makes the system unstable towards spin turbulence.



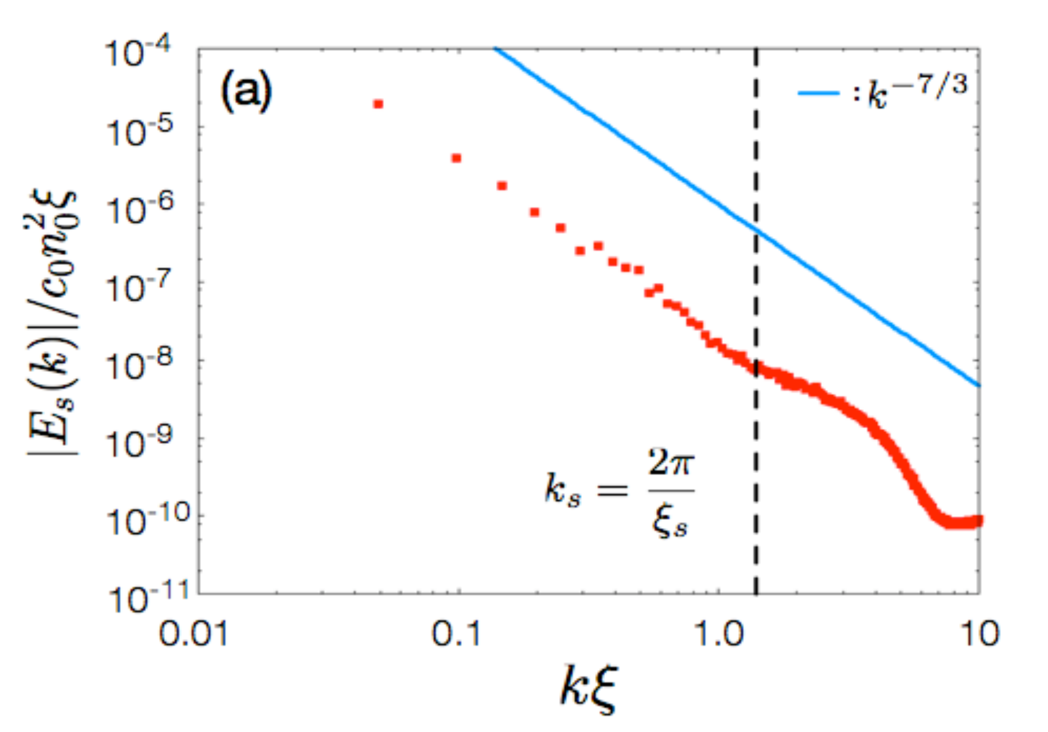
Time-development of the spin density vector **s**

z





Energy spectrum of the spin energy in the turbulent state



$$E_s = \frac{c_1}{2A}\int \mathbf{s}(\mathbf{r})^2 d\mathbf{r}, \quad \mathbf{s}(\mathbf{r}) = \sum_{\mathbf{k}} \tilde{\mathbf{s}}(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{r})$$

$$E_s = \frac{c_1}{2}\sum_{\mathbf{k}} \left|\tilde{\mathbf{s}}(\mathbf{k})\right|^2, \quad E_s(k) = \frac{c_1}{2\,\Delta k}\sum_{k<|\mathbf{k}_1|<k+\Delta k} \left|\tilde{\mathbf{s}}(\mathbf{k}_1)\right|^2$$

$$E_s \propto k^{-7/3}$$

This characteristic scaling law is understood through the scaling of the equation of the motion of **s**.

$$\frac{\partial}{\partial t}\hat{\mathbf{s}} + (\mathbf{v}\cdot\nabla)\hat{\mathbf{s}} = \frac{\hbar}{2M}\hat{\mathbf{s}} \times \left[\nabla^2\hat{\mathbf{s}} + \left(\frac{\nabla n}{n}\cdot\nabla\right)\hat{\mathbf{s}}\right], \quad \hat{\mathbf{s}} \equiv \frac{\mathbf{s}}{n}$$

Stamper-Kurn asked me whether the spin turbulence can occur in a trapped system too.

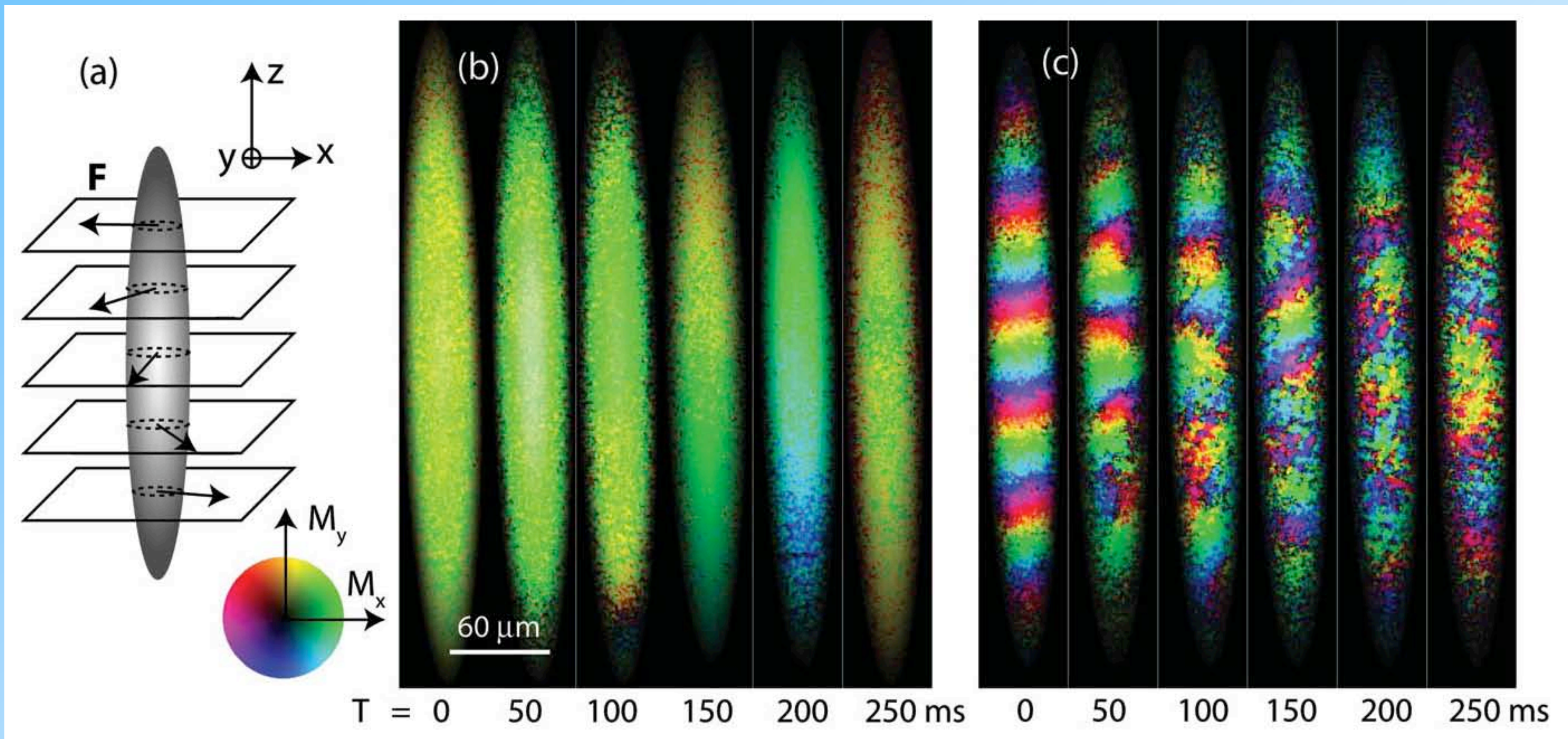
PR… week ending … MAY 2008

Spontaneously Modulated Spin Textures in a Dipolar Spinor Bose-Einstein Condensate

M. Vengalattore,[1] S. R. Leslie,[1] J. Guzman,[1] and D. M. Stamper-Kurn[1,2]

[1]*Department of Physics, University of California, Berkeley, California 94720, USA*

[2]*Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*



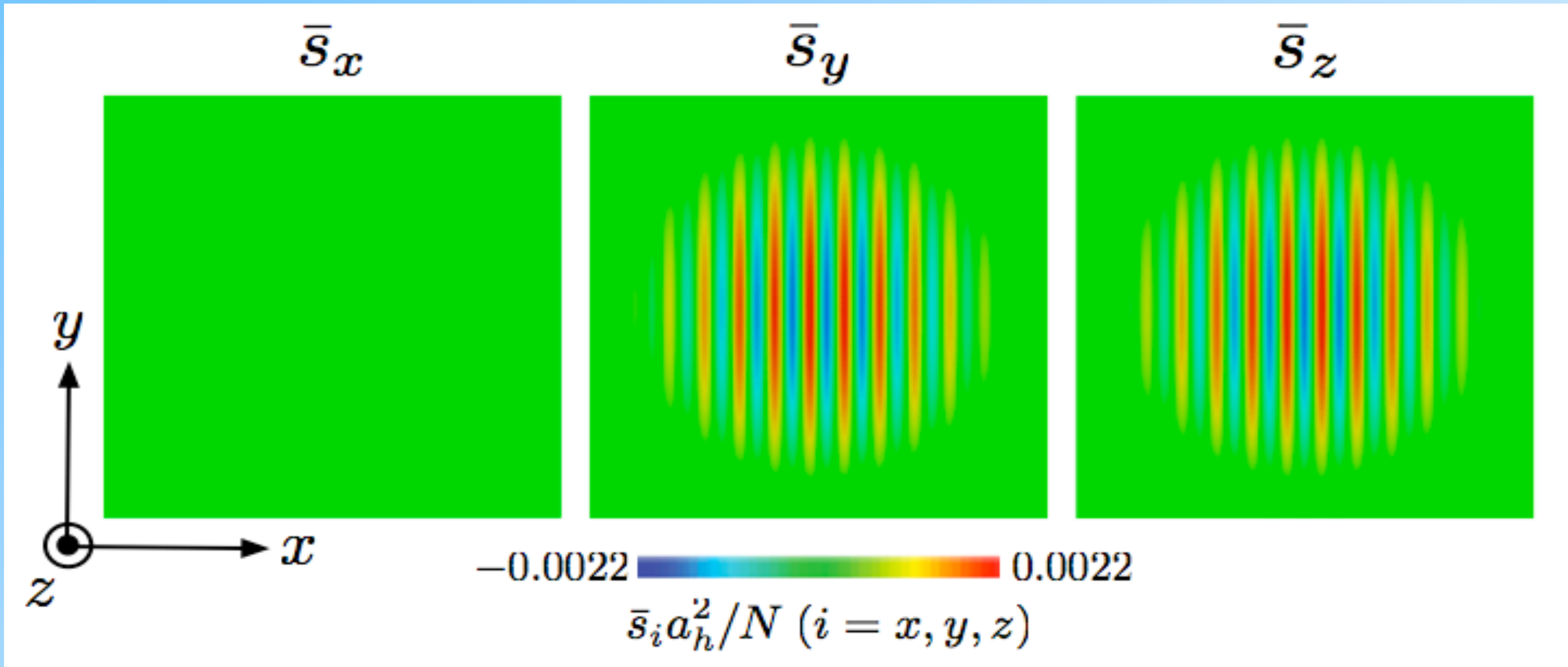
Starting with a helical structure

Possible to observe the spin vector field.

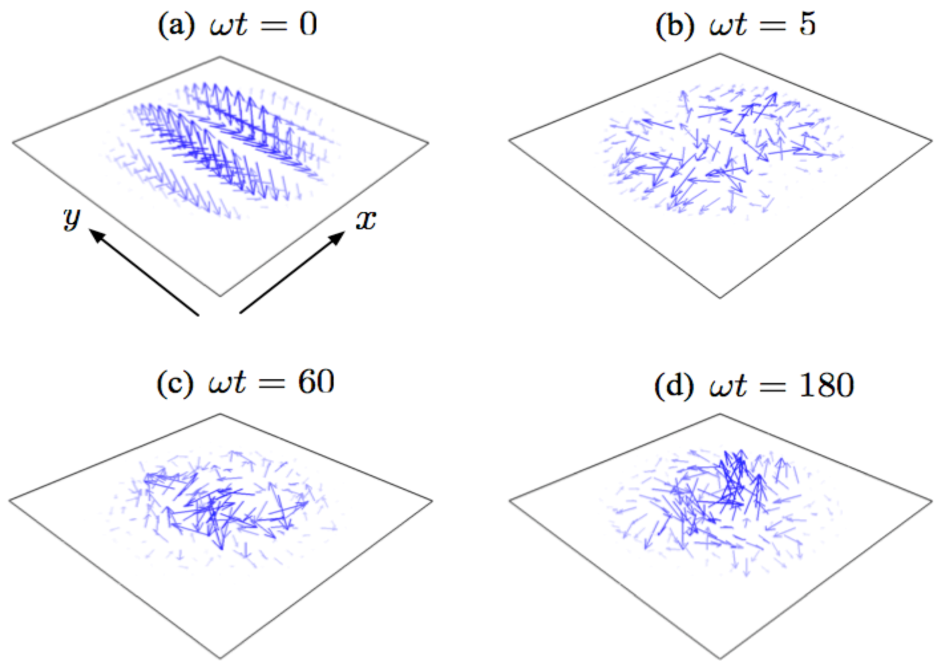
PHYSICAL REVIEW A **85**, 053641 (2012)

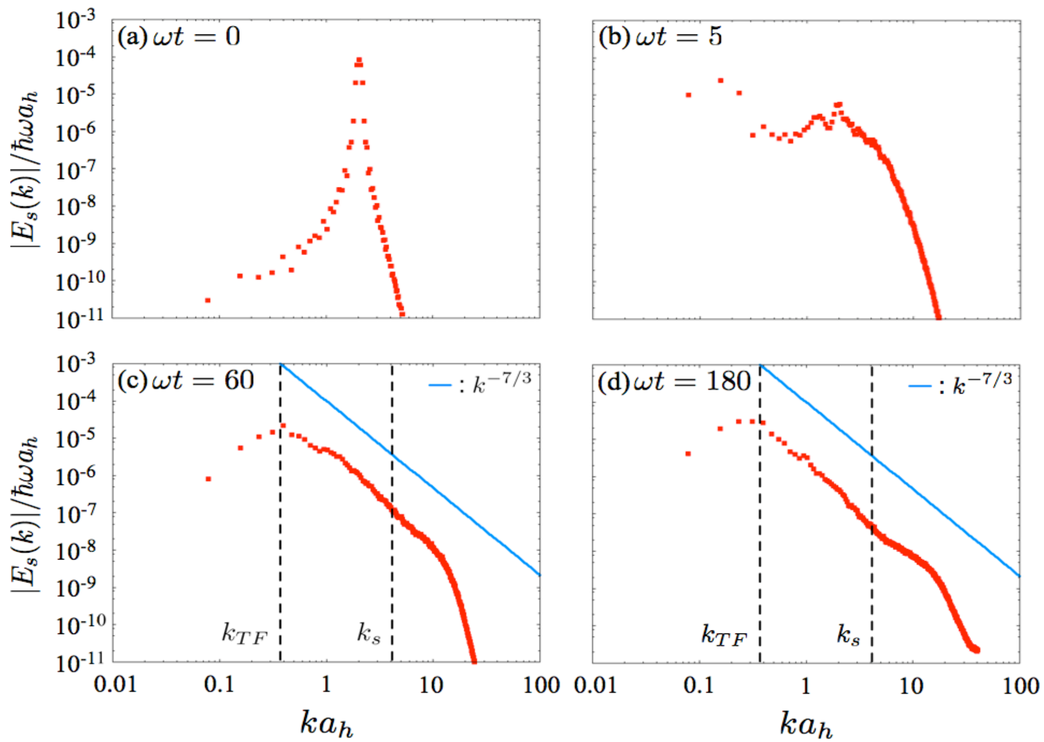
Spin turbulence in a trapped spin-1 spinor Bose-Einstein condensate

Kazuya Fujimoto[1] and Makoto Tsubota[1,2]



Starting with a helical structure

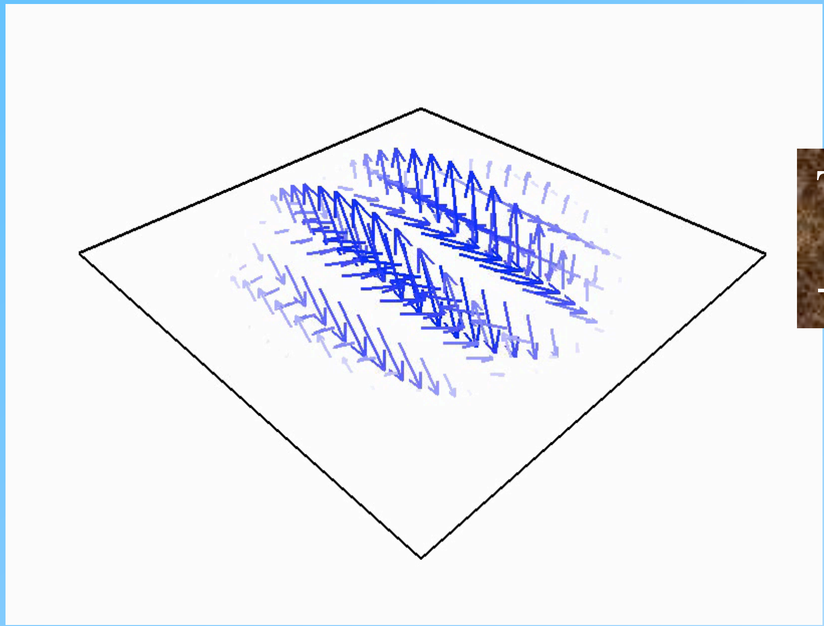




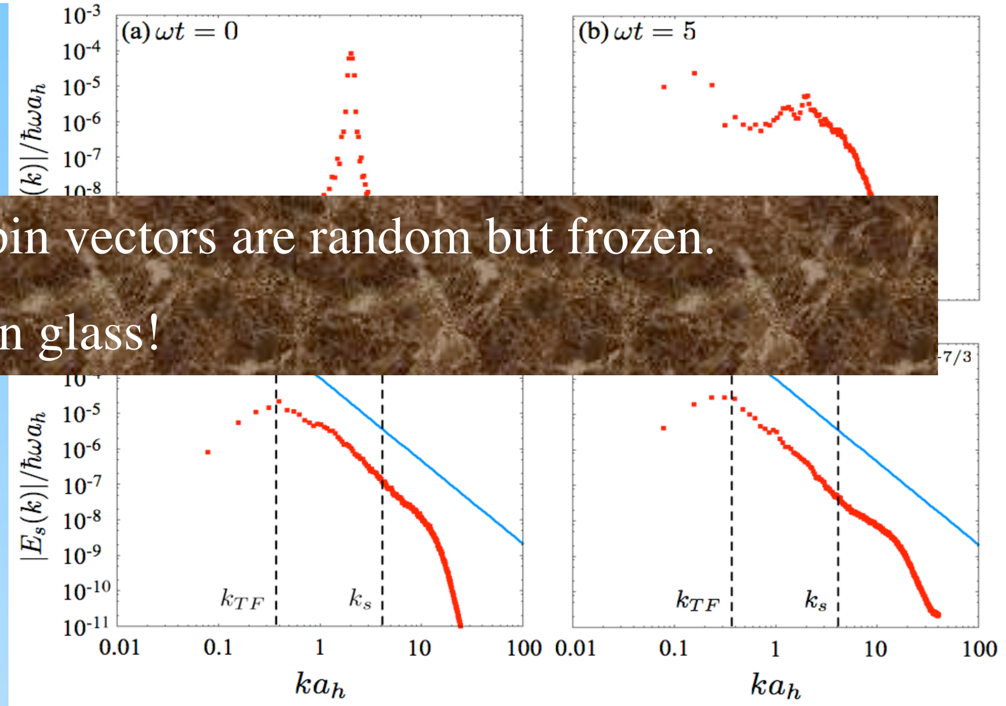
Time development of the spin density vector

Energy spectra

The -7/3 power law seems to be sustained in the trapped system too. What does this mean?

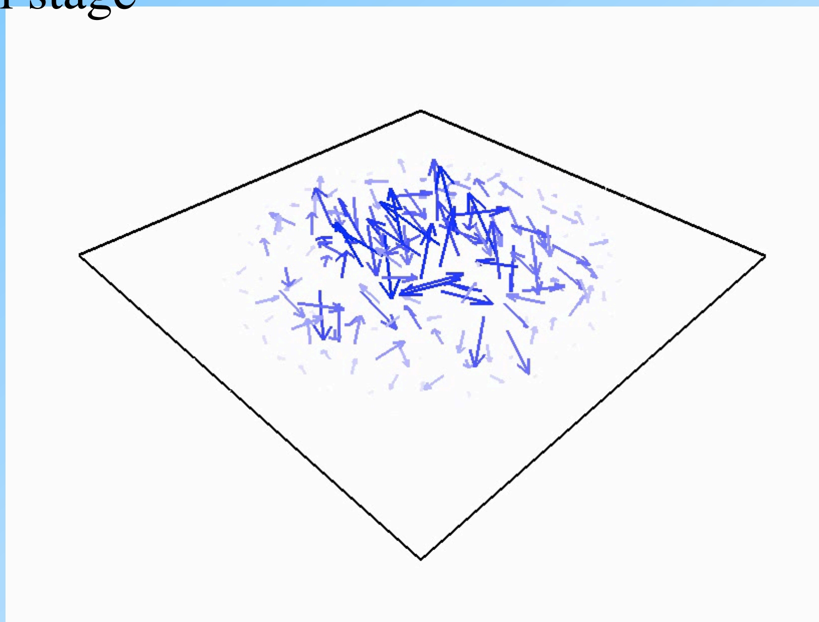


Initial stage

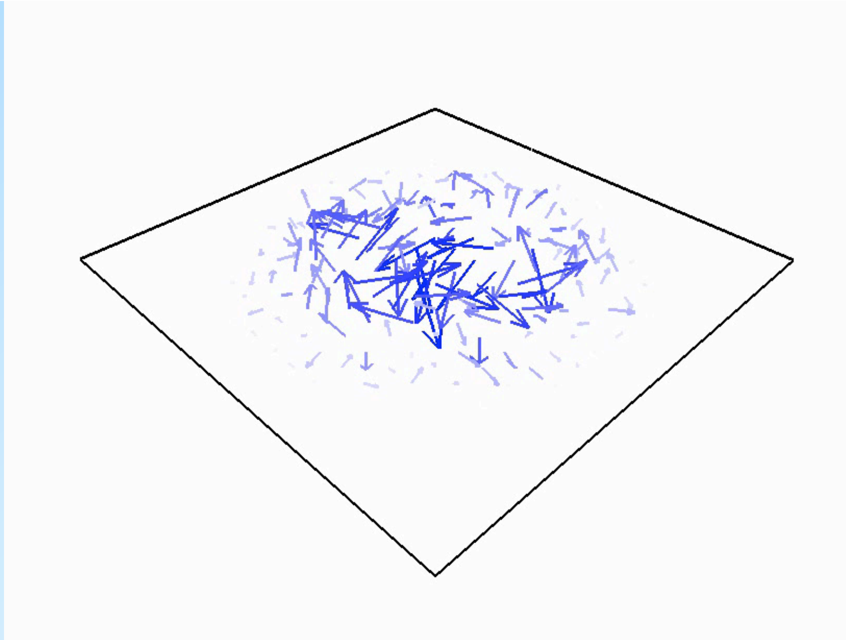


The spin vectors are random but frozen.

-> Spin glass!



From the onset of the instability to the powerlaw



The -7/3 power stands up.

How to characterize the dynamics?

Depicting the orbits in the phase space

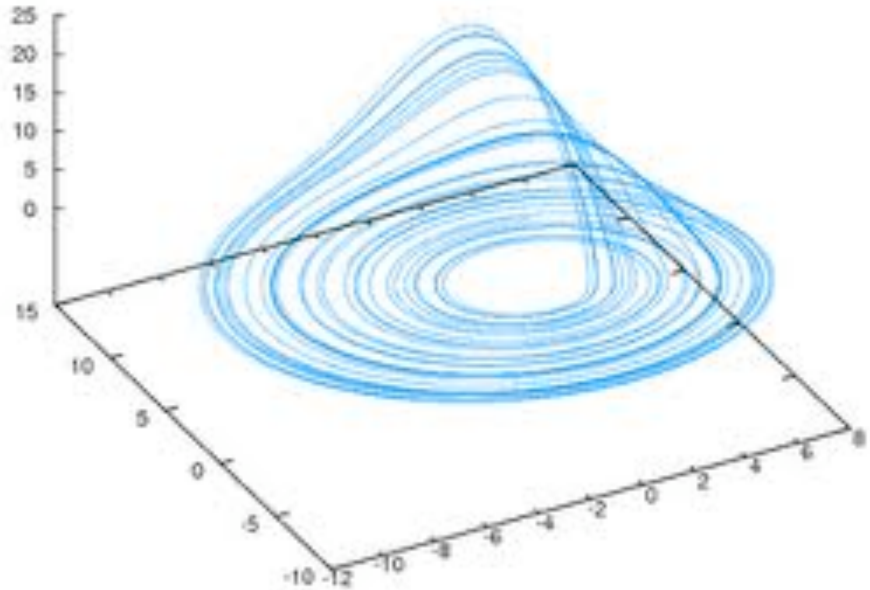
For example Rossler model

$$\dot{x} = -(y+z)$$

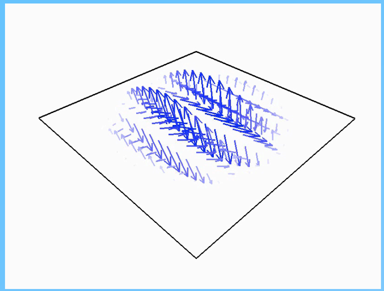
$$\dot{y} = x + \frac{1}{5}y$$

$$\dot{z} = \frac{1}{5} + z(x-\mu)$$

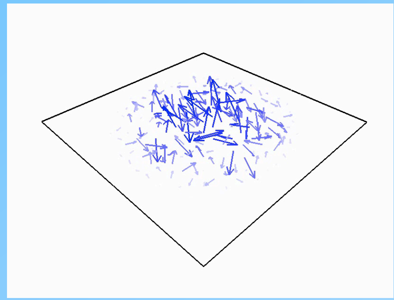
μ : parameter



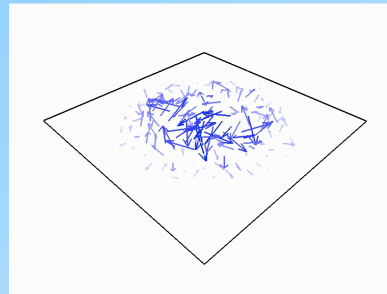
Strange attractor



Early stage

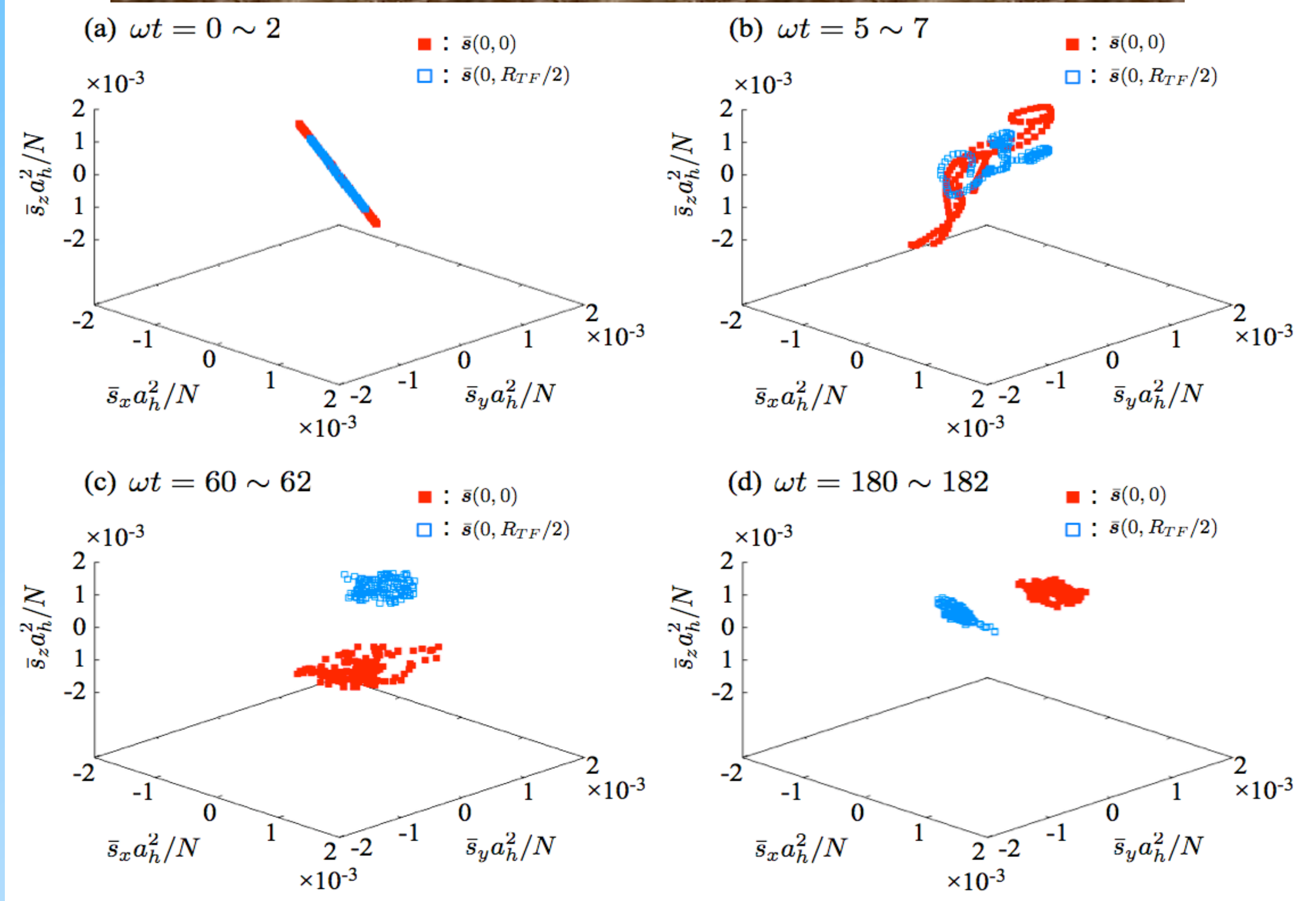


Middle stage

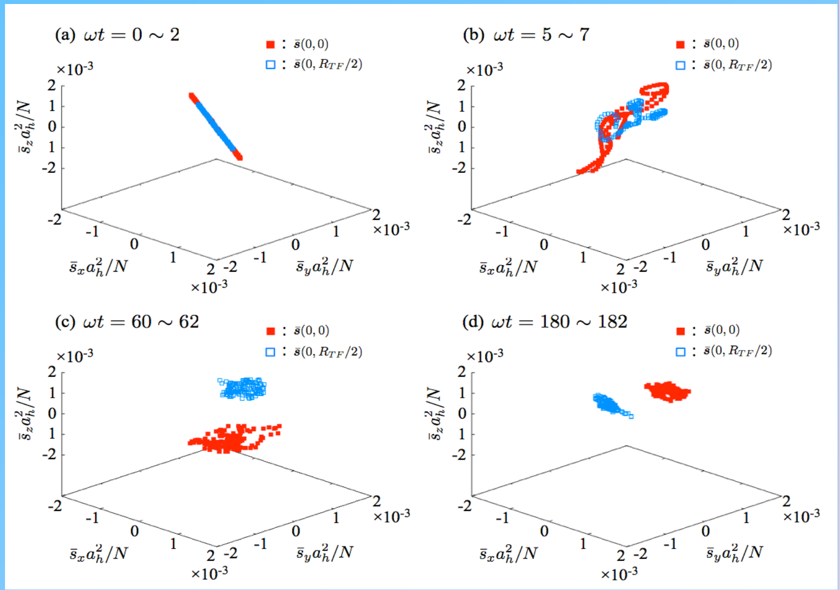


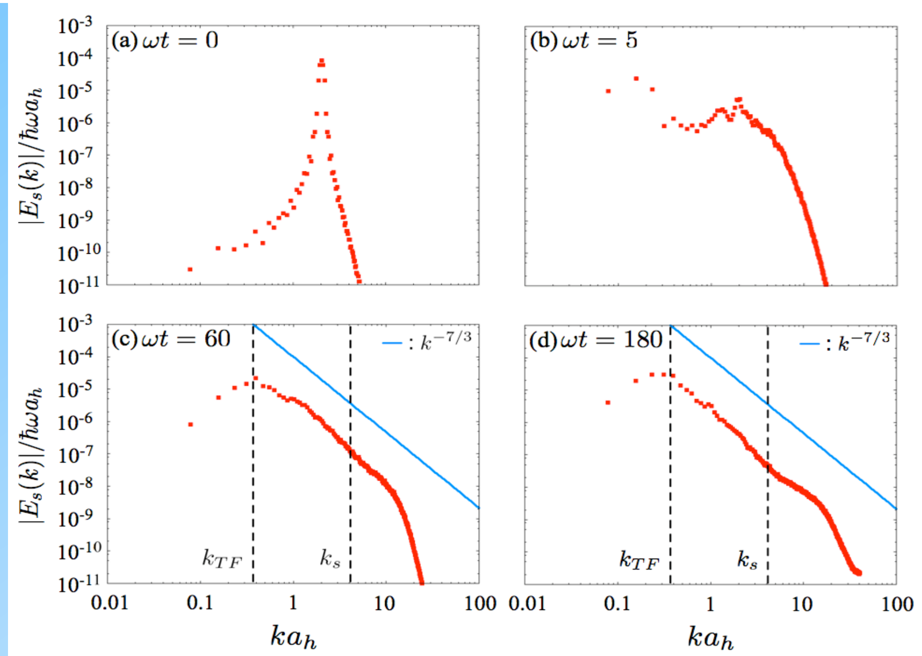
Late stage

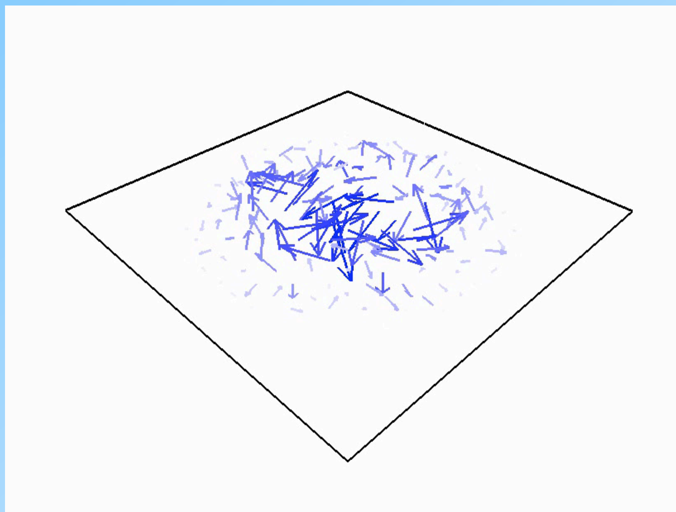
Orbits in the spin space (S_x, S_y, S_z)



- In the (S_x,S_y,S_z) space the orbits shrink.
- The spin orbits shrink to different direction depending on the position.







The -7/3 power law means that the spin system is frozen to some kind of attractor.

The ordered state may be described by the order parameter of spin glass.

Summary

0. Introduction ～Why is quantum turbulence interesting?～
1. QT in a trapped BEC
2. Counterflow of two-component BECs: two-component QT
3. Spin turbulence in spin-1 spinor BECs

How to characterize such turbulent states and the transition?