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Non-Abelian chiral instabilities from the lattice

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Reference:

Y.Akamatsu, A. R., N. Yamamoto: JHEP **1603** (2016) 210

CP violation in heavy-ion collisions?

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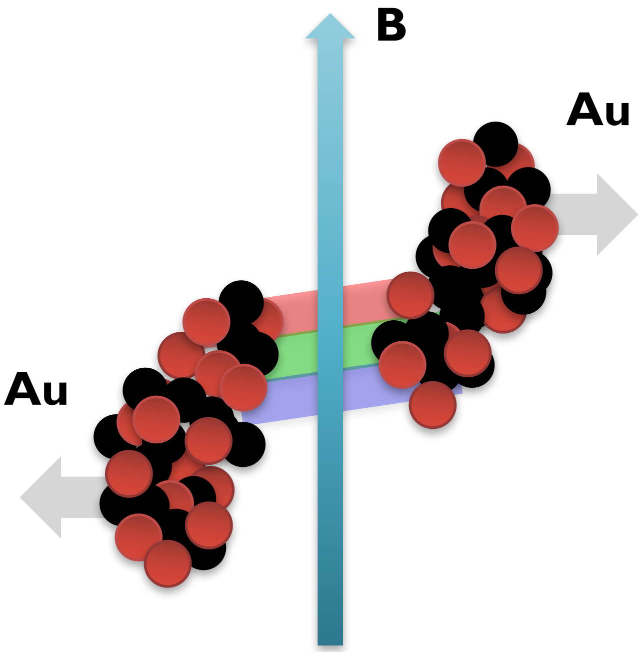
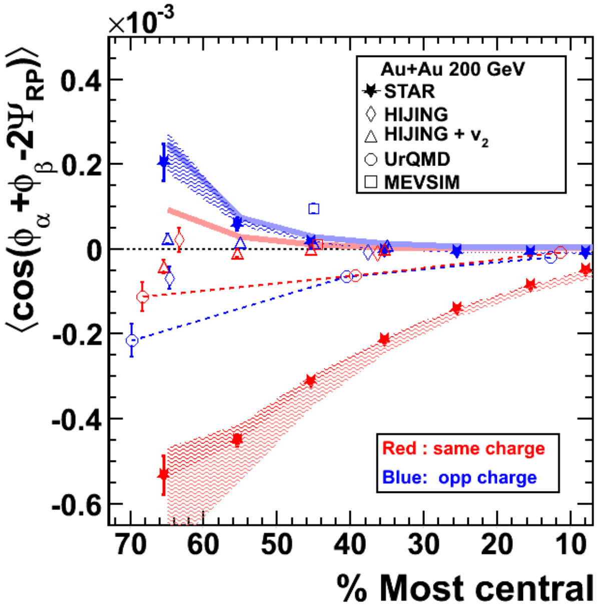


Large U(1) magnetic field B ~10^{16}G ~ m_π^2



STAR Collaboration
PRL 103 (2009) 251601,
PRC 81 (2010) 54908

$$\frac{dN_\pm}{d\phi} \propto 1 + 2v_1\cos[\phi - \Phi_{RP}] + 2v_2\cos[2(\phi - \Phi_{RP})] + \ldots + 2a_\pm\sin[\phi - \Phi_{RP}] + \ldots$$

S. A. Voloshin, Phys. Rev. C 70, 057901 (2004)

- Harmonic analysis of charged particles yields apparently shows CP-odd effects
- Our goal: If present, how do CP-odd domains evolve among dynamical color fields?

Topology and Anomalies in QCD I

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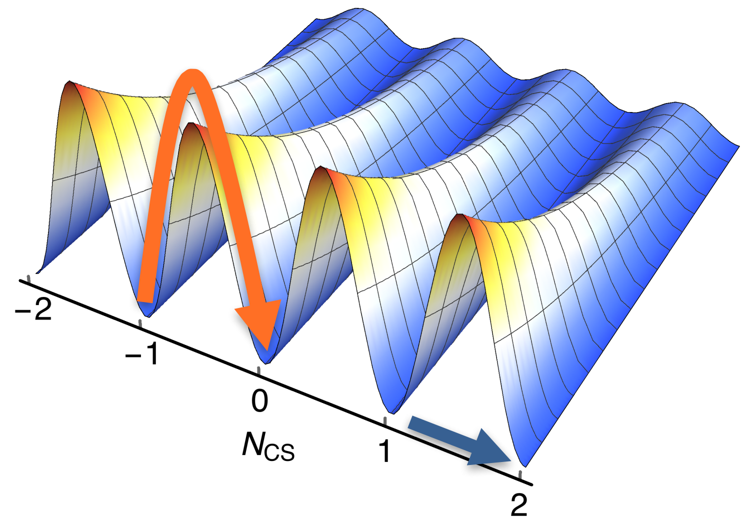
- Infinite number of degenerate vacua characterized via winding number N_{CS}

$$N_{CS} = \frac{g^2}{64\pi^2}\int d^4x F^a_{\mu\nu}(\mathbf{x})\tilde{F}^{\mu\nu}_a(\mathbf{x}) \in \mathbb{Z}$$
$$= -\frac{g^2}{8\pi^2}\int dt \int d^3x\, \mathbf{E}\cdot\mathbf{B}$$

S. S. Chern, J. Simons, Annals Math. 99, 48 (1974)

- Tunneling at T=0 via **instantons** OR barrier crossing at T>0 via **sphalerons**

G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976)
N. S. Manton PRD28, 2019 (1983)
F. R. Klinkhamer, N. S. Manton, PRD30, 2212 (1984)



- At high T: diffusion of N_{CS} (sphaleron rate)

$$\Gamma = \lim_{t\to\infty}\frac{\gamma(t)}{t} \propto \lim_{t\to\infty}\frac{\left[\langle (N_{CS}(0)-N_{CS}(t))^2\rangle - \langle N_{CS}(0)-N_{CS}(t)\rangle^2\right]}{N_s^3 t}$$

G. Moore, Phys.Rev. D59 (1999) 014503

Topology and Anomalies in QCD II

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- Quantum anomalies relate topology of fields to chirality of fermions

$$\partial^\mu j^5_\mu(\mathbf{x}) = -\frac{N_f g^2}{32\pi^2} F^a_{\mu\nu}(\mathbf{x}) \tilde{F}^{\mu\nu}_a(\mathbf{x}) \qquad \frac{d(n_R - n_L)}{dt} = \frac{dn_5}{dt} \propto \frac{dN_{CS}}{dt}$$

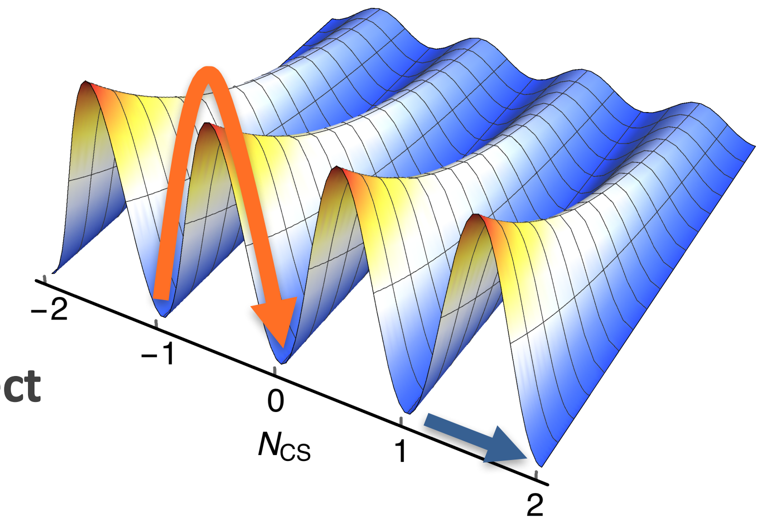
S. L. Adler, Phys. Rev. 177, 2246 (1969)
J. S. Bell, R. Jackiw, Nuovo Cim. A60, 47 (1969)

- Useful technical tool:
  chiral chemical potential $\mu_5 = \mu_5(n_5) = \mu_5(t)$

see e.g. K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRD78 (2008) 074033

- In strong magnetic fields: **Chiral Magnetic Effect**

$$\mathbf{j}^{\mathrm{CME}} = \frac{N_f g^2 \mu_5}{4\pi^2} \mathbf{B} = \kappa \, \mathbf{B}$$



see e.g.: D. E. Kharzeev, L. D. McLerran, H. J. Warringa Nucl.Phys. A803 (2008) 227-253

Chiral Plasma Instabilities (CPI)

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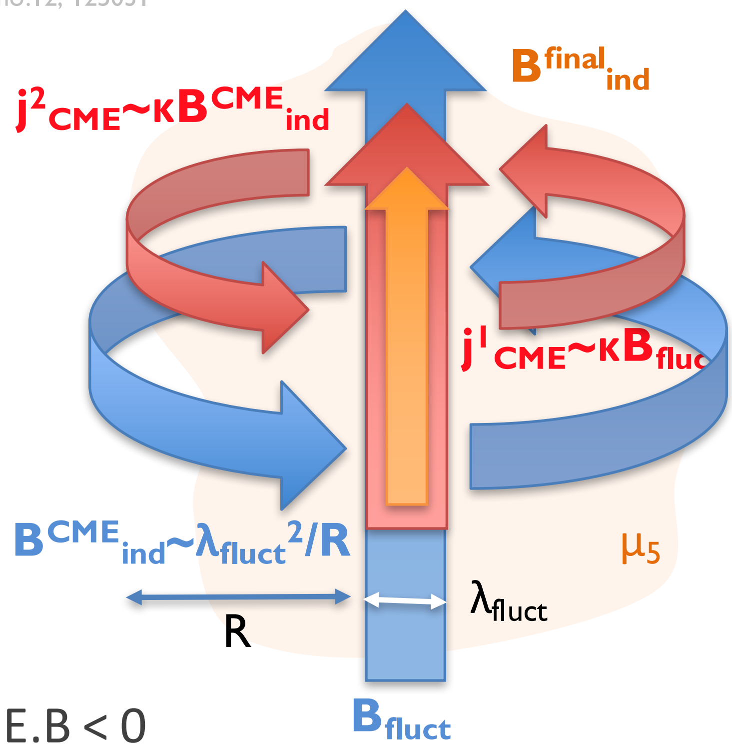
- Origin: Fluctuating gauge fields in the presence of a chiral imbalance ($B_{ext}=0$)

Y. Akamatsu, N. Yamamoto, PRL 111 (2013) 052002, PRD90 (2014) no.12, 125031

System remains stable if

$$\mathbf{B}^{\mathrm{final}}_{\mathrm{ind}} \leq \mathbf{B}_{\mathrm{fluct}}$$

$$\lambda_{\mathrm{fluct}} \leq \lambda_c \sim 1/\kappa$$



- Instability weakens itself due to $dn_5/dt \propto E.B < 0$

- Q I : Can we find numerical evidence of chiral instabilities for $n_5>0$?
- Q II: How does $n_5>0$ influence topology changing processes (sphaleron rate)?

An effective theory for CPI's

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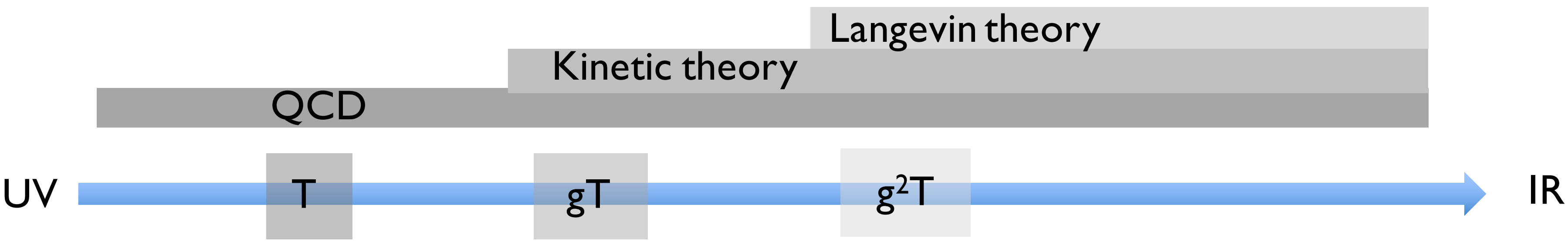
- At high temperature (g<<1) a clear separation of scales is present



- CPI operates at the non-perturbative magnetic scale ($\lambda_c \sim (g^2T)^{-1}$)

- Anomalous effective theory for soft classical fields ($k \sim g^2T$) & hard modes ($k \sim T$)

Y. Akamatsu, N. Yamamoto, PRD90 (2014) no.12, 125031

$$\mathbf{D} \times \mathbf{B} = D_t \mathbf{E} + \mathbf{j}_{\mathrm{hard}} \qquad \mathbf{j}_{\mathrm{hard}} = \sigma_c \mathbf{E} + \zeta + \mu_5 \frac{N_f g^2}{4\pi^2} \mathbf{B}$$

$$\langle \zeta_i^a(x) \zeta_j^b(y) \rangle = 2\sigma_c T \delta^{ab} \delta_{ij} \delta^{(4)}(x-y) \qquad \frac{dn_5}{dt} = -\frac{1}{V} \frac{dN_{CS}}{dt}$$

Lattice implementation

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- Naïve Wilson Hamiltonian for classical soft gauge fields (A^0=0 gauge, SU(2))

$$H[E,U] = \frac{2N_c}{g^2 a_s} \sum_{\mathbf{x}} \Big[\sum_i a_s^2 E_i^a E_i^a - \frac{1}{N_c} \sum_{i<j} \Big(\mathrm{ReTr}[\square_x] - N_c \Big) \Big]$$

$$\dot{U}_k(\mathbf{x},t) = \left(i2\sqrt{N_c} E_k^a(\mathbf{x},t) T^a \right) U_k$$

YM Hamiltonian dynamics

$$\partial E_k^b(\mathbf{x},t) = -\frac{1}{\sqrt{N_c a_s^2}} \mathrm{ImTr}\Big[T^b U_k(\mathbf{x},t) \sum_{j\neq k} (S_j^{\sqsubset} + S_j^{\sqsupset}) \Big]$$
$$- \sigma_c E_k^b(\mathbf{x},t) - \frac{g a_s}{2\sqrt{N_C}} \sigma_a(n_5) B_k^a(\mathbf{x},t) + \frac{g a_s}{2\sqrt{N_C}} \xi_k^b(\mathbf{x},t)$$

$$\beta_L = \frac{2N_c}{g^2 a T}$$

Dissipative Ohmic terms

Non-diss. Anomalous term

$$\partial_t n_5(t) = -\frac{2N_f}{a_s^3 N_s^3} \frac{dN_{CS}^{lat}}{dt}$$

Anomaly relation for n_5

Y.Akamatsu, A. R., N. Yamamoto JHEP 1603 (2016) 210

- Classical thermal equilibrium initial conditions: $P[U,E] \sim \exp\Big[-\frac{1}{T} H[U,E] \Big] \prod_{\mathbf{x}} \delta\big[G(\mathbf{x}) \big]$

See eg.: G. D. Moore, Phys.Rev. D59 (1999) 014503

Measuring topology on the lattice

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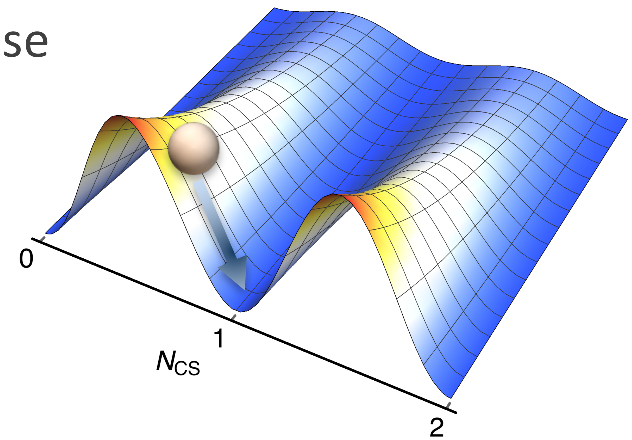
- Naïve discretization of dN_{CS}/dt susceptible to UV noise

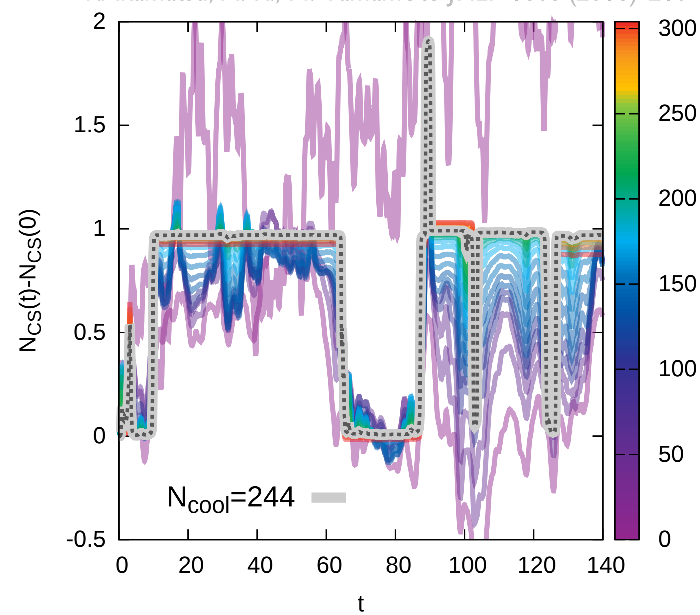
$$\frac{dN_{\rm CS}}{dt} = \frac{g^2}{64\pi^2}\int d^3x\, F^a_{\mu\nu}(\boldsymbol{x})\tilde{F}^{\mu\nu}_a(\boldsymbol{x})$$

$$F^{\mu\nu} = -\frac{1}{4}\frac{i}{g a_\mu a_\nu}\sum_{\square}\log\left(\boxplus_{\mu\nu}\right)$$

See e.g. discussion in G. D. Moore, Nucl.Phys. B480 (1996) 657-688



Y.Akamatsu, A. R., N. Yamamoto JHEP 1603 (2016) 210



- Dissipative cooling drives the system to the vacuum of its current topological sector

$$\frac{\partial U}{\partial \tau} = -\frac{\delta H_{cool}}{\delta A^a_j} i g a_s T^a U_k$$

See e.g.: P. de Forcrand, M. G. Perez, I.-O. Stamatescu, Nucl.Phys. B499 (1997) 409-449
M. Lüscher JHEP 1008 (2010) 071

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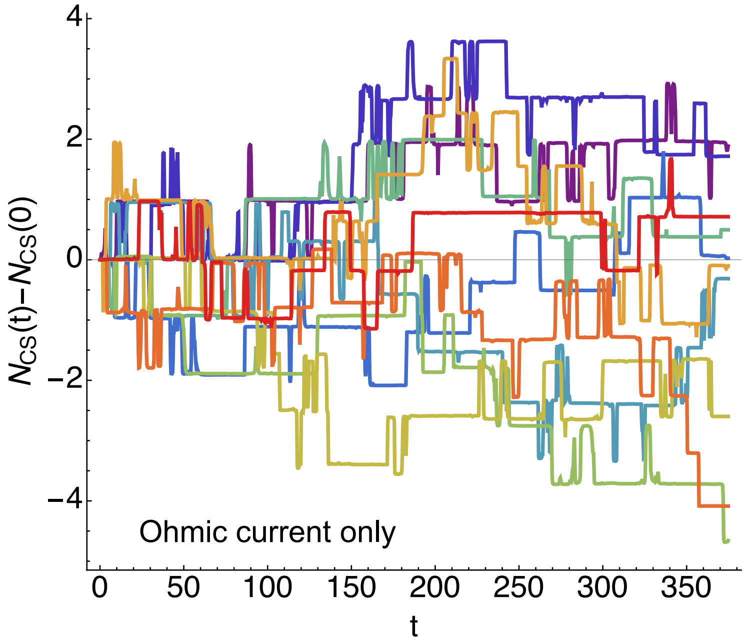
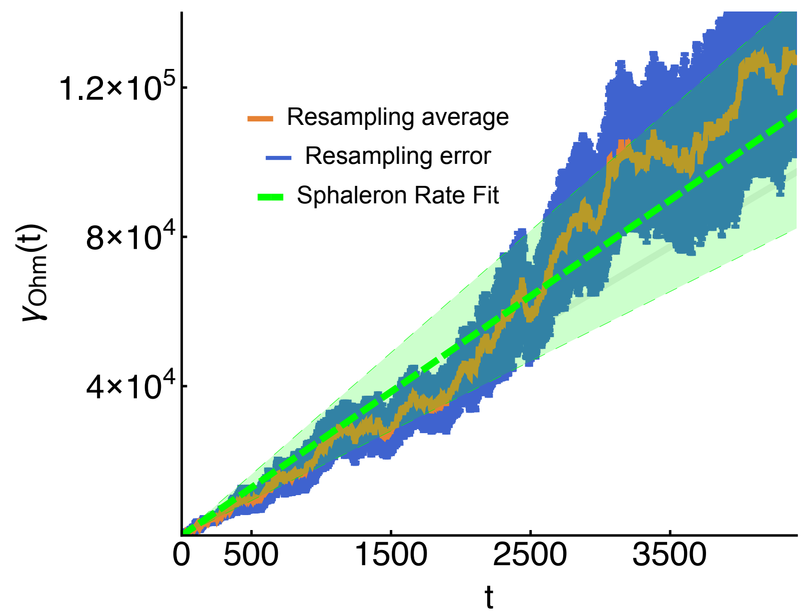
Langevin evolution w/o anomaly





Y.Akamatsu, A. R., N. Yamamoto JHEP 1603 (2016) 210

- Reproduces the diffusive behavior reported in the literature

$$\Gamma^{\mathrm{Ohm}} = 26 \pm 7, \quad \Gamma^{\mathrm{Ohm}}_{\mathrm{lit}} = 22.1 \pm 0.62$$

G. D. Moore, M. Tassler JHEP 1102 (2011) 105

$\beta_L=20$ $N_s=20$
$a_t=0.0375$ $a=1$
$\sigma_c=1$ $N_f=2$

- System on average stays in the same vacuum sector $\langle N_{CS}\rangle=0$

Langevin evolution with anomaly

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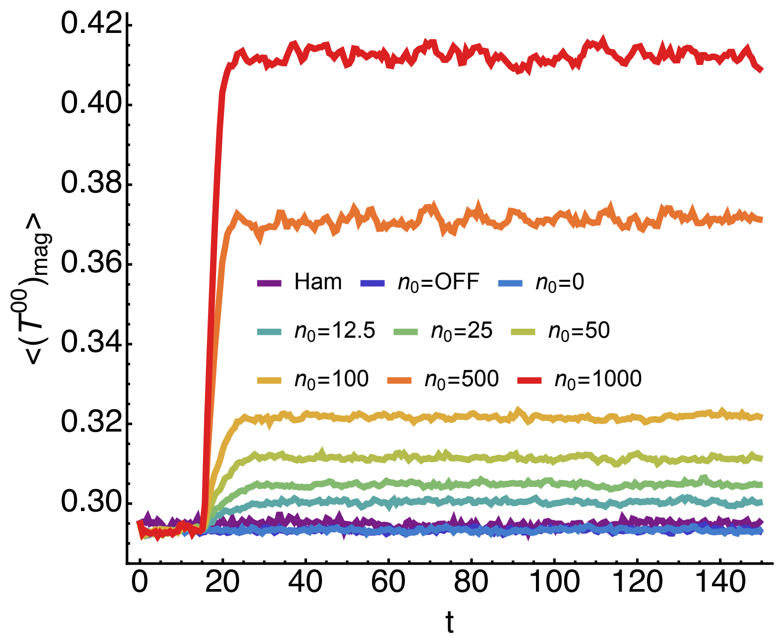
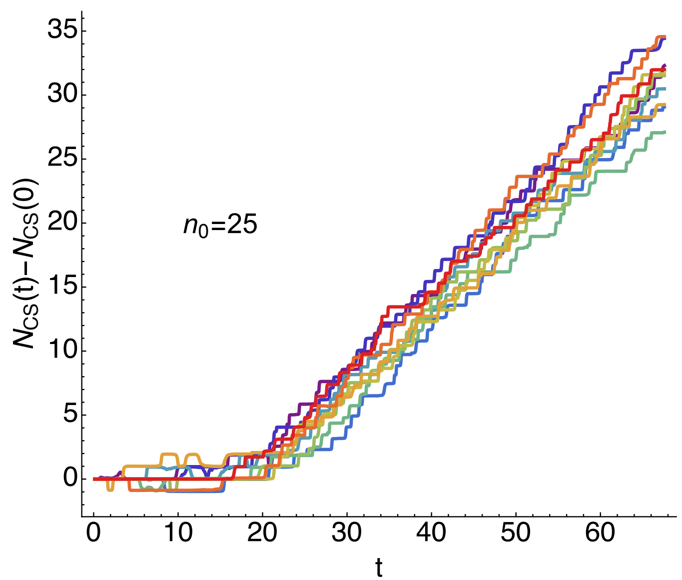
- Provide initial chiral imbalance $n(t=15)=n_0$





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- Sanity check at $n_0=0$: stable with same energy as purely ohmic case (n_0=OFF)
- Fast increase of magnetic field energy after switching on $n_0 > 0$
- Topological drift of the Chern-Simons number N_{CS} ensues

Langevin evolution with anomaly II

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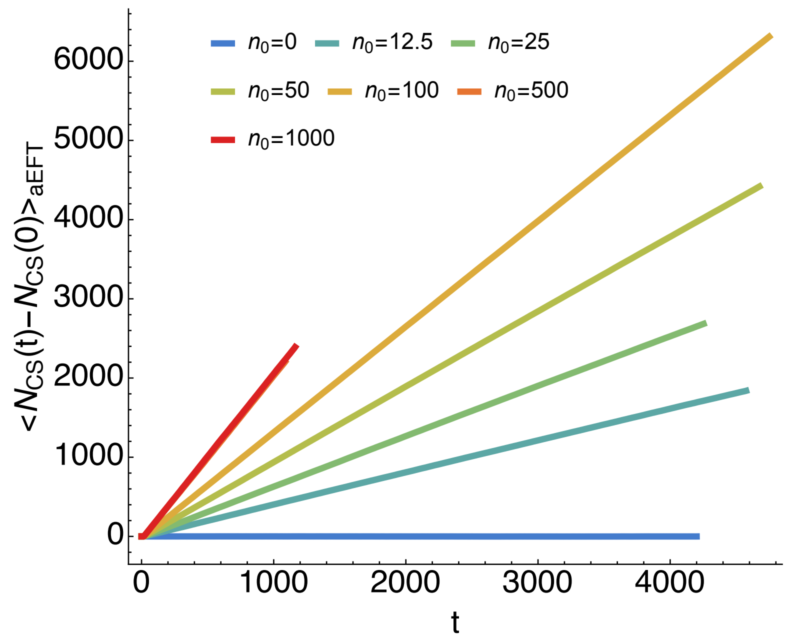
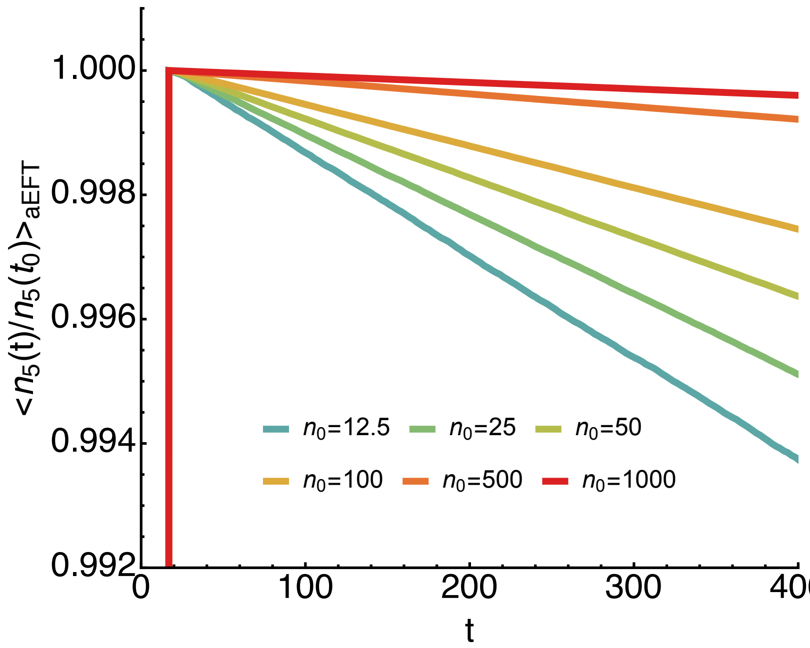
- In the absence of external B field: helicity conservation balances N_{CS} and n_5





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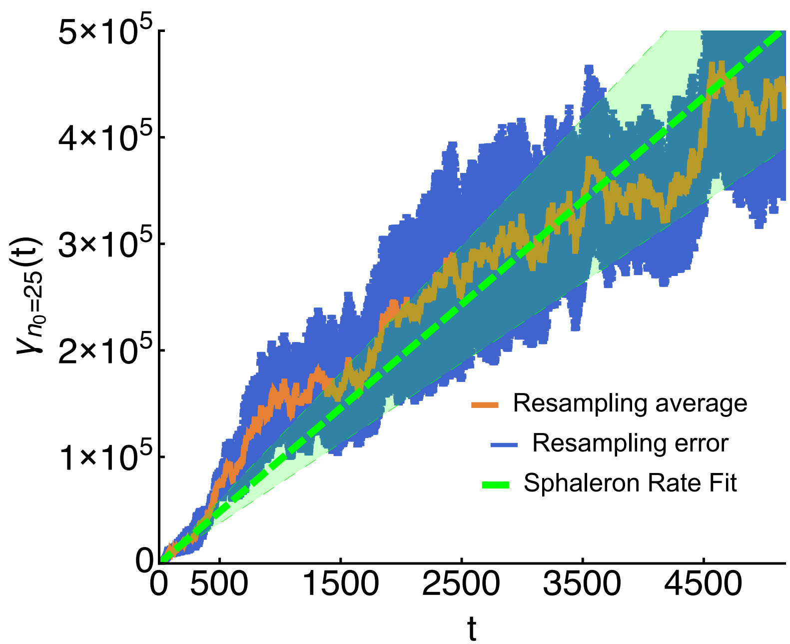
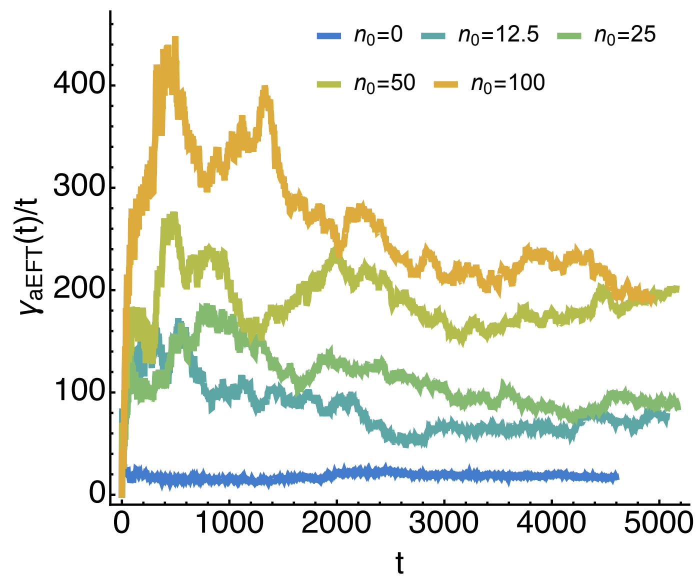
- Eventually drift in N_{CS} abates and the system will diffuse around a new vacuum

Sphaleron rate with anomaly

- Is the diffusion of N_{CS} around the drift influenced by n_5?





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- We find a clear ordering of the sphaleron rate with $n_5(t=15)=n_0$

	$n_0 = 0$	$n_0 = 12.5$	$n_0 = 25$	$n_0 = 50$	$n_0 = 100$
Γ	20 ± 4	65 ± 12	104 ± 22	182 ± 40	225 ± 50

Anomalous effects w/o initial n_5

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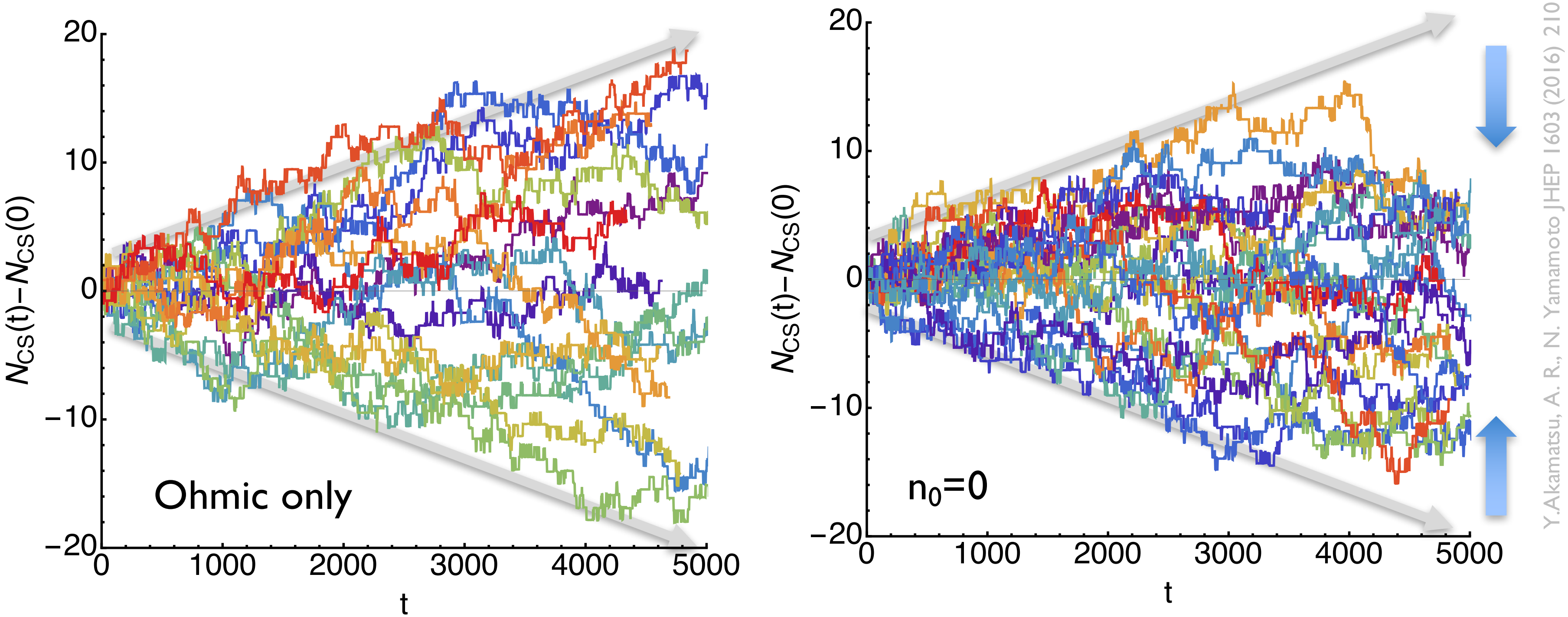
- Diffusion of N_{CS} leads to excursions towards large values: buildup of n_5



- Reduction of the conventional thermal sphaleron rate due to the anomaly?

	Ohmic	$n_0 = 0$
Γ	26 ± 7	20 ± 4

later times and more statistics necessary!

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Conclusions

- The chiral magnetic effect (CME) allows for a new class of instabilities:

  Chiral Plasma Instabilities (CPI)

- Numerical evidence for the existence of CPI in non-Abelian SU(2) EFT
- In the presence of CPI: drift of N_{CS} and significantly increased $\Gamma^{sphaleron}$
- Helicity conservation (B_{ext}=0) leads to diminished imbalance, abating instability
- Even if $n_5(0)=0$: sphaleron rate might be reduced due to intermediate n_5 buildup

- Need to significantly increase statistics to pin down anomalous effects at $n_0=0$
- Connect to HIC phenomenology: Attach physical units to the EFT simulations

Thank you for your attention

The Chiral Magnetic Effect (CME)

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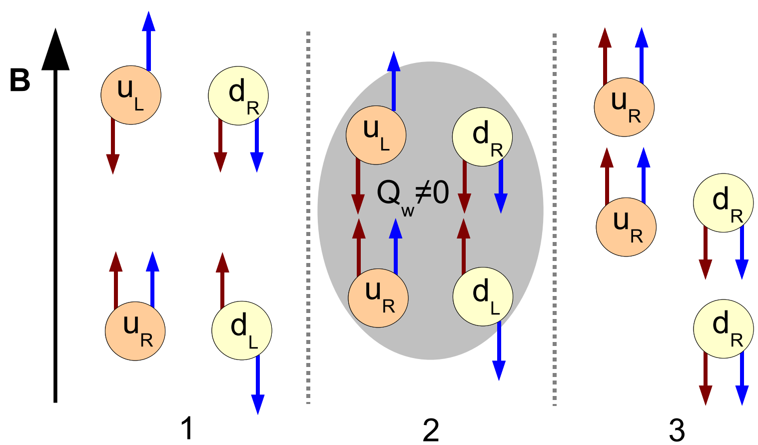
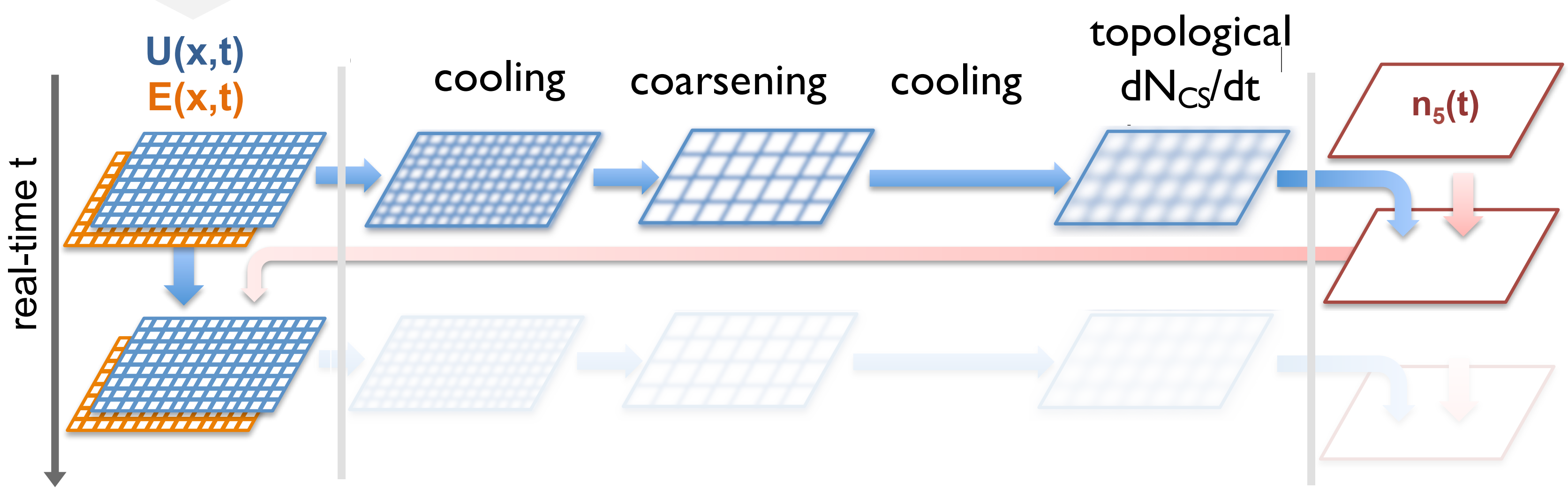
- In strong magnetic fields: Anomaly leads to novel transport phenomena



$$\mathbf{j}^{\mathrm{CME}} = \frac{N_f g^2 \mu_5}{4\pi^2}\mathbf{B} = \kappa \ \mathbf{B}$$

Summary: lattice evolution

Starting from
thermal
initial conditions



β_L=20 N_s=20 a_t=0.0375 a=1 σ_c=1 N_f=2