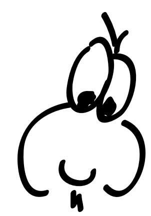
Heavy Quark-Antiquark Free Energy in Medium from AdS/CFT



Carlo Ewerz

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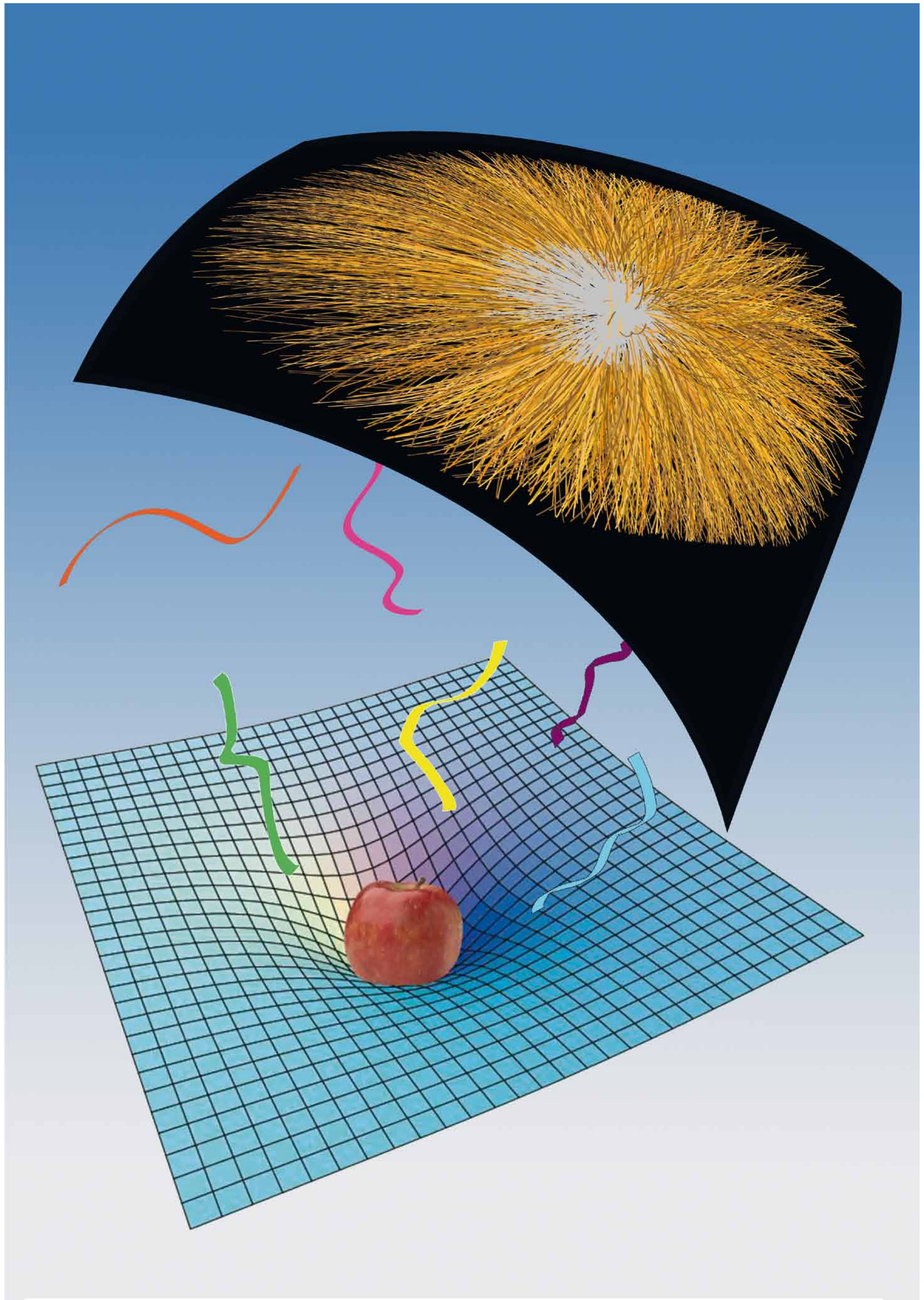
& Universität Heidelberg & FIAS

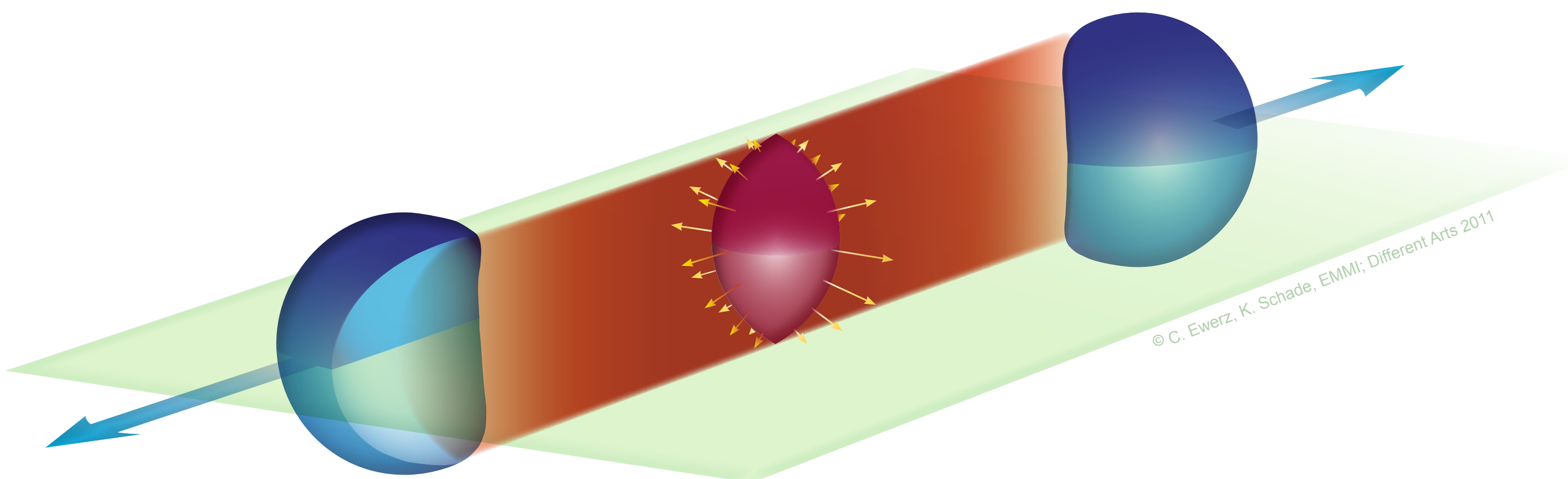
Delta Meeting, Heidelberg, 28 April 2016



in collaboration with

Andreas Samberg
Konrad Schade
Ling Lin
Paul Wittmer
Olaf Kaczmarek

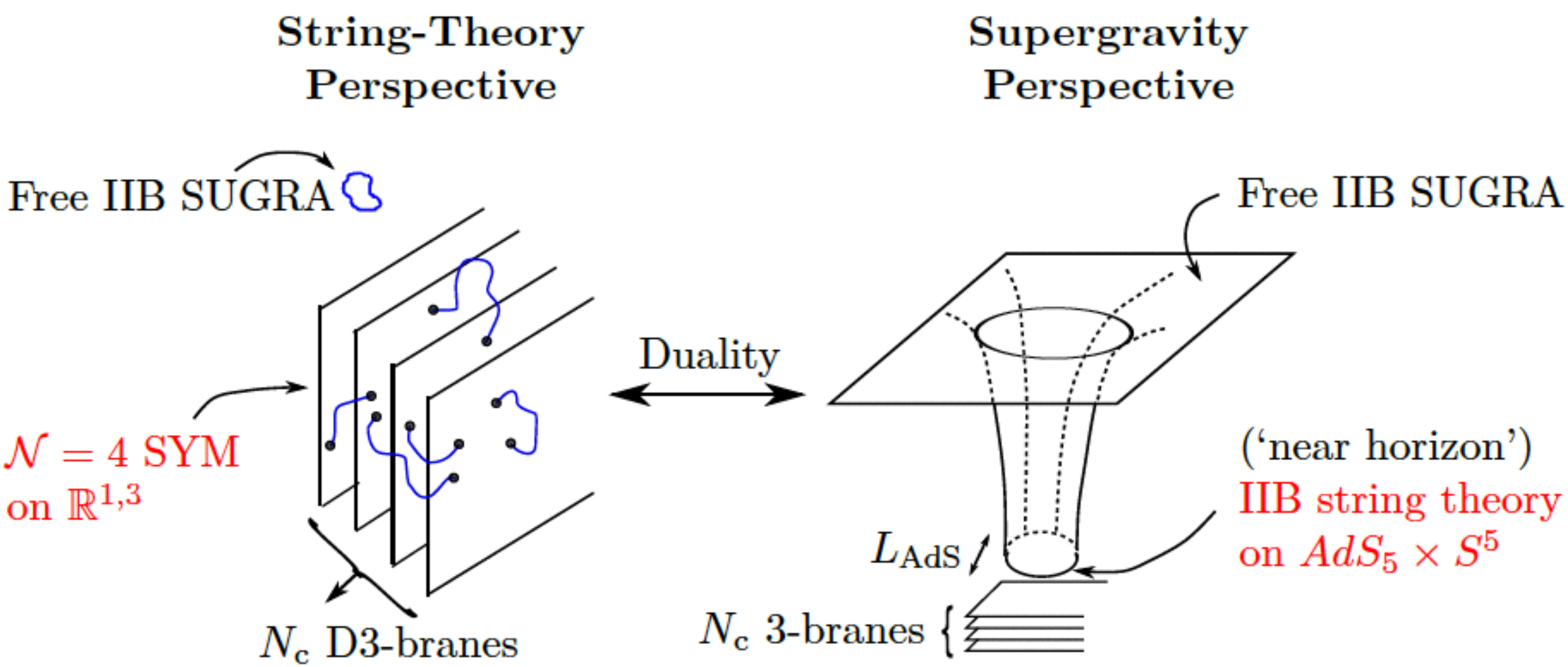
Motivation



data hint at strongly coupled QGP

AdS/CFT is promising method to study strongly coupled theories

Origin of AdS/CFT: two views on a stack of D-branes



Taking limits: gauge / gravity duality

useful (and tractable) limit of correspondence:

$\mathcal{N}$=4 super Yang-Mills SU(N) theory in 3+1 dimensions for

$N \to \infty$ and large $\lambda = g^2 N$

$$\Leftrightarrow$$

(super)gravity on $AdS_5 \times S_5$

strongly coupled QFT $\Leftrightarrow$ classical gravity !

QCD vs $\mathcal{N}$=4 SYM

original AdS/CFT is for $\mathcal{N}$=4 SYM rather than QCD, very different theories:

$\mathcal{N}$=4 SYM:
- max supersymmetric
- conformal
- no confinement, no χSB
- $N_c \to \infty$ required for duality

@ large T less different from QCD:

- above $2T_c$ QCD close to conformal
- no confinement, no χSB in QCD
- T breaks SUSY and conformal invariance

Non-conformal Theories

We can come closer to QCD by including explicit breaking of conformal invariance

→ deformations of AdS_5

but will not find a dual to QCD this way

→ consider large classes of deformations and hope for universality
(example: η/s)

Our Aim

look for universal or robust properties generically emerging in strongly coupled theories

→ classes of holographic models

in general requires choosing suitable observables

aim is not to find a precise model for QCD

Observables

At finite T (and finite μ):

- thermodynamics
- drag force
- heavy meson screening:
  - screening distance
  - free, binding, internal energy
- running coupling
- jet quenching parameter
- energy loss of rotating quarks

includes dynamical observables!

Models

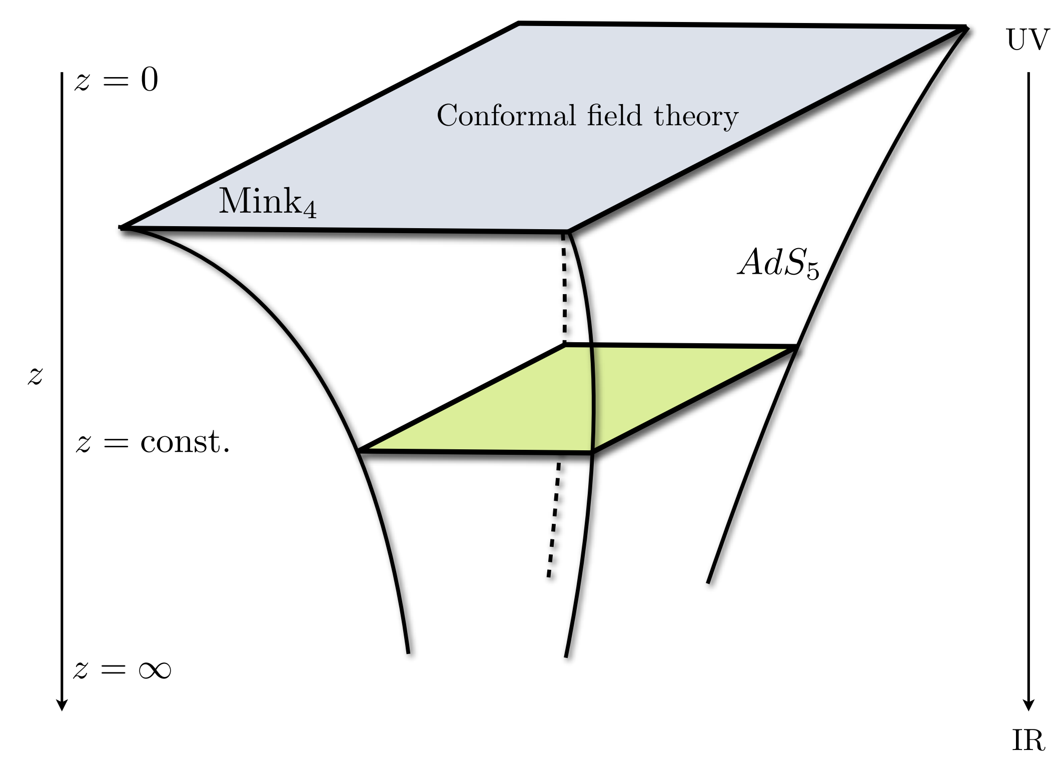
Anti de Sitter Space

- maximally symmetric space with constant negative curvature
- solves vacuum Einstein equations with negative cosmological constant

$$S = \frac{1}{16\pi G_{\rm N}^{(D)}} \int \mathrm{d}^D x \sqrt{-g}\,(\mathcal{R} - 2\Lambda) \qquad \text{(D=5)}$$

$$2\Lambda = -\frac{12}{L^2}$$

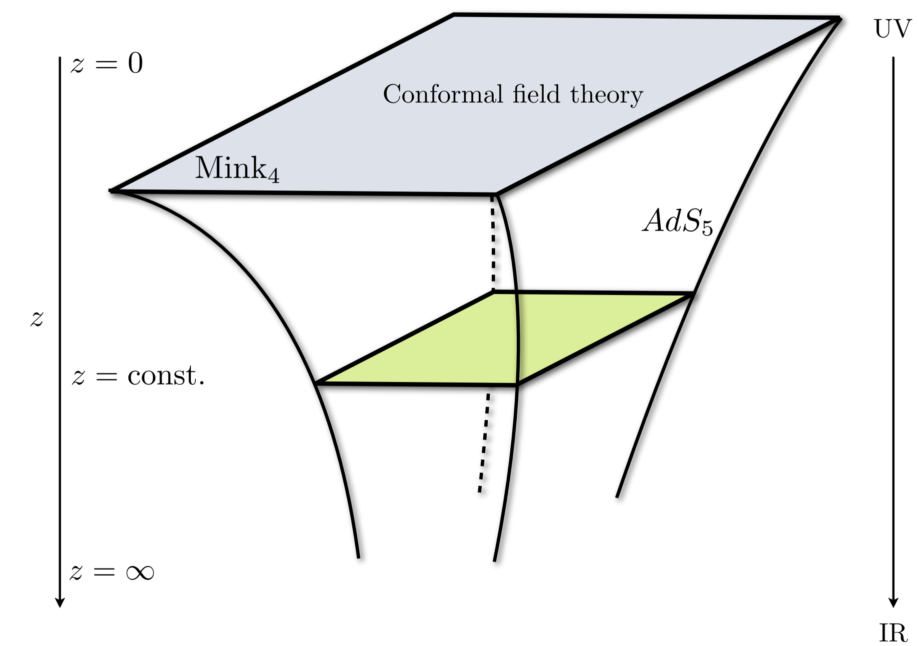
$$\mathrm{d}s^2 = \frac{L^2}{z^2}\Big(-\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \mathrm{d}z^2 \Big)$$



$\mathcal{N}$=4 SYM at Finite Temperature

zero temperature: AdS5

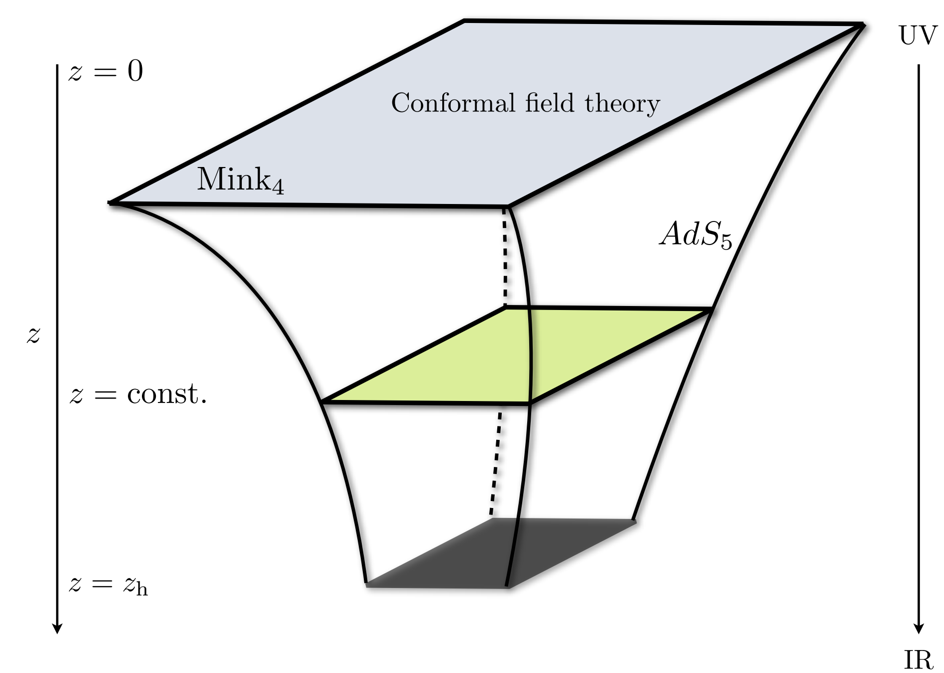
$$ds^2 = \frac{L^2}{z^2}\Big(-\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \mathrm{d}z^2\Big)$$



finite temperature T:
AdS5 with black hole

$$ds^2 = \frac{L^2}{z^2}\Big(-h\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \frac{\mathrm{d}z^2}{h}\Big)$$

with $\quad h = 1 - \dfrac{z^4}{z_h^4}\quad$ and $\quad T = \dfrac{1}{\pi\, z_h}$



Simple Non-conformal Model

- AdS$_5$

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left(-h\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \frac{\mathrm{d}z^2}{h}\right)$$

- A minimal deformation: SW$_T$ model

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\,\mathrm{e}^{cz^2}\left(-h\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \frac{\mathrm{d}z^2}{h}\right)$$

Andreev, Zakharov 2006;
Kajantie, Tahkokallio, Yee 2006

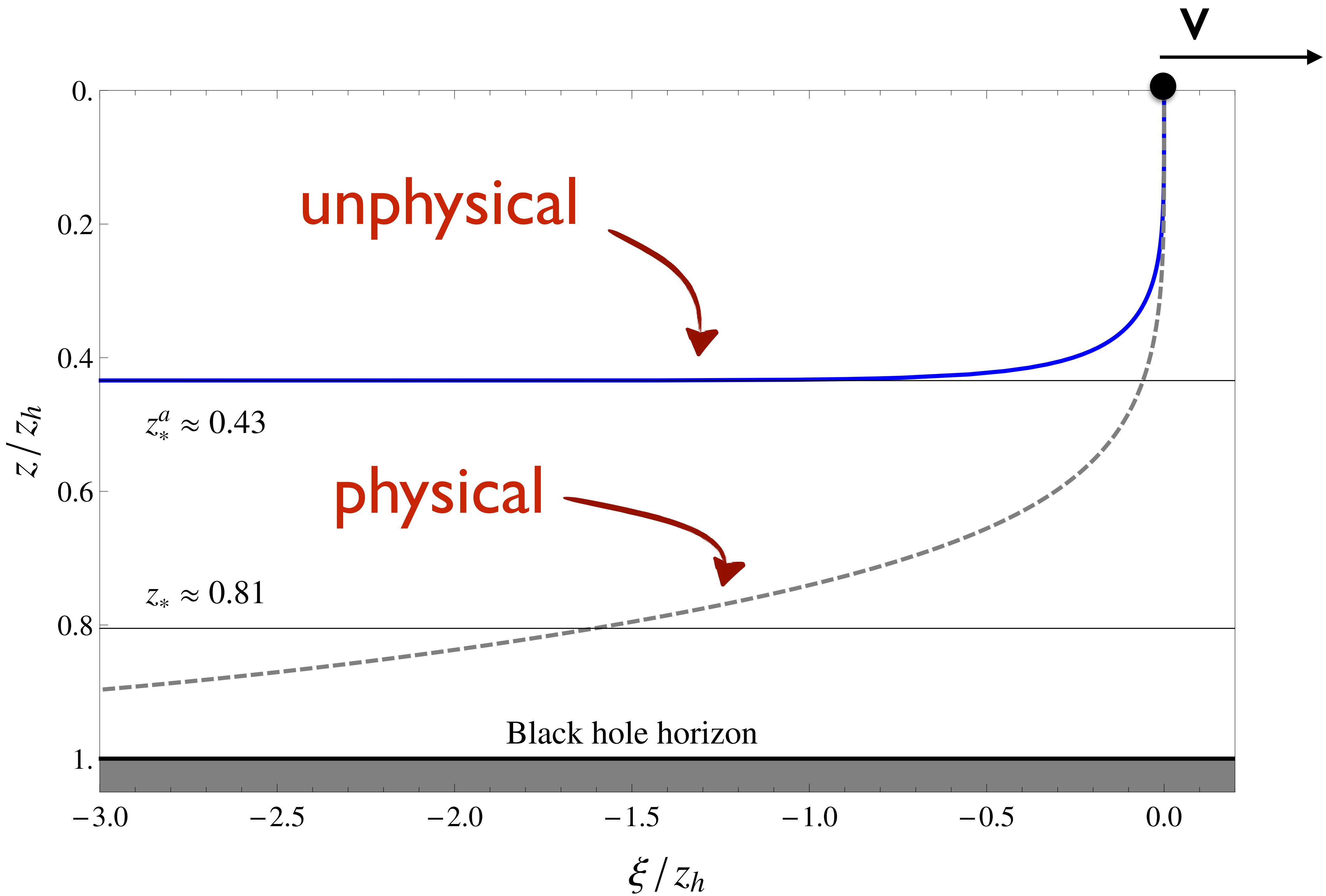
$$h = 1 - \frac{z^4}{z_h^4} \qquad T = \frac{1}{\pi\, z_h}$$

simple non-conformal extension

but: not a solution to any known SUGRA action

likely origin of some problems:
inconsistent thermodynamics, unphysical string configurations

Unphysical Drag Solutions in Ad Hoc Models



L. Lin, A. Samberg, CE

Consistent Non-conformal Model

Start with five dimensional gravity action S_{EHs} :

$$S_{\mathrm{EHs}} = \frac{1}{16\pi\, G_{\mathrm{N}}^{(5)}} \int \mathrm{d}^5x \sqrt{-G} \left(\mathcal{R} - \frac{1}{2} \left(\partial\Phi\right)^2 - V(\Phi) \right)$$

with general ansatz

$$\mathrm{d}s^2 = \mathrm{e}^{2A(z)} \left(-h\, \mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} \right) + \mathrm{e}^{2B(z)} \frac{\mathrm{d}z^2}{h}$$

$$T = \mathrm{e}^{A(z_{\mathrm{h}})-B(z_{\mathrm{h}})} \frac{|h'(z_{\mathrm{h}})|}{4\pi}$$

leads to 3 independent equations of motion but 5 unknown functions $V,\ \Phi,\ A,\ B,\ h$

- 2-parameter model:
  with parameters $\phi,\ c$
  DeWolfe, Rosen; Gubser; 2009

$$\mathrm{e}^{2A(z)} = \mathrm{e}^{c\,z^2} \frac{L^2}{z^2} \quad \text{and} \quad \Phi(z) = \sqrt{\frac{3}{2}}\, \phi\, z^2$$

$$\alpha = \frac{c}{\phi}$$

- 1-parameter model:
  with parameter ϕ
  Schade

$$\mathrm{e}^{2A(z)} = \frac{L^2}{z^2} \quad \text{and} \quad \Phi(z) = \sqrt{\frac{3}{2}}\, \phi\, z^2$$

Consistent Non-conformal Model

Start with five dimensional gravity action S_{EHs} :

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$$T = \mathrm{e}^{A(z_{\mathrm{h}}) - B(z_{\mathrm{h}})} \frac{|h'(z_{\mathrm{h}})|}{4\pi}$$

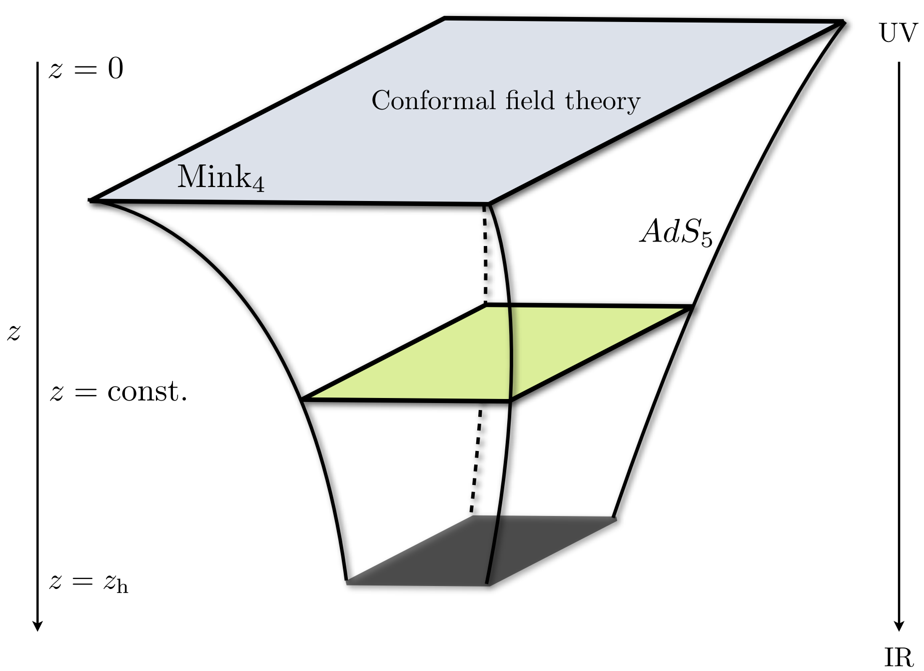
scalar Φ :

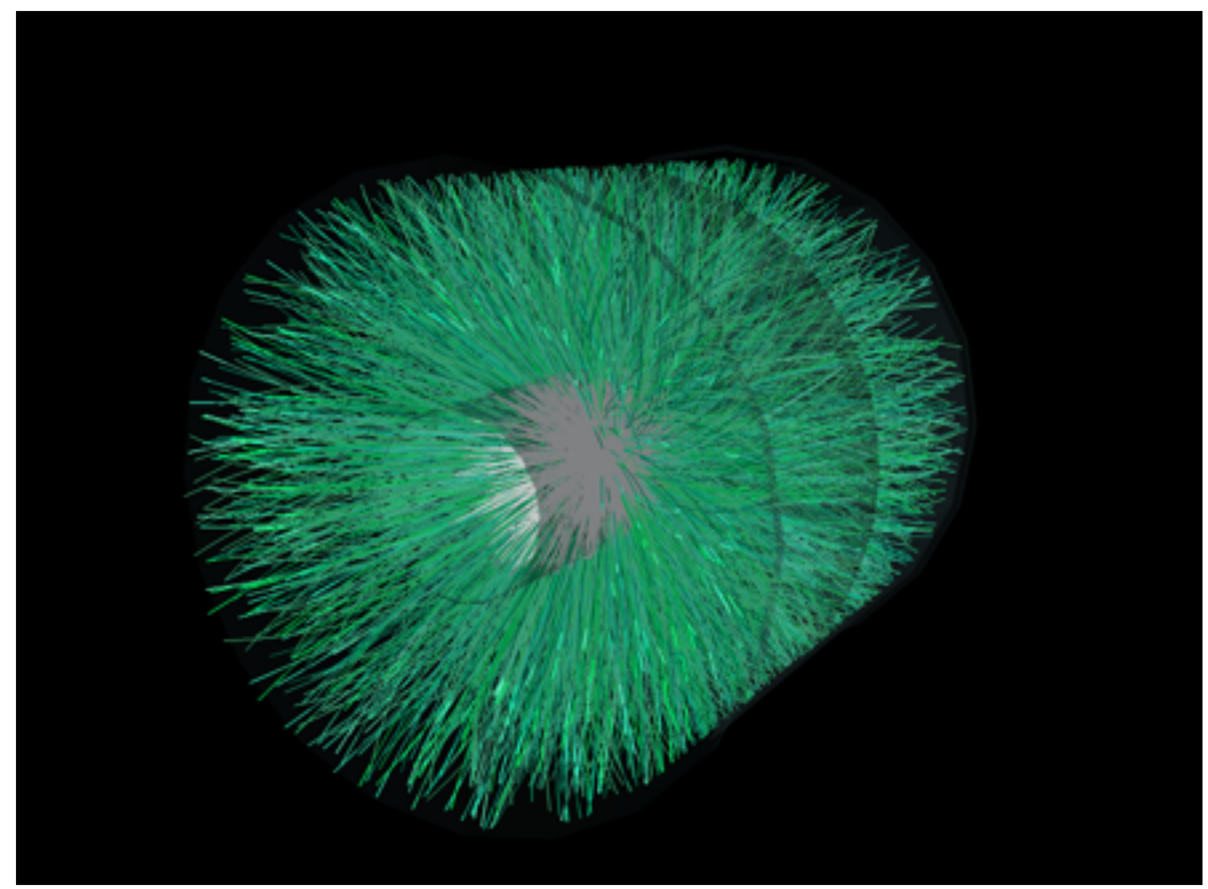
can be dilaton ('string frame model')

or not ('Einstein frame model')

We consider both possibilities as independent models.

AdS/CFT for Hot Plasmas





Screening

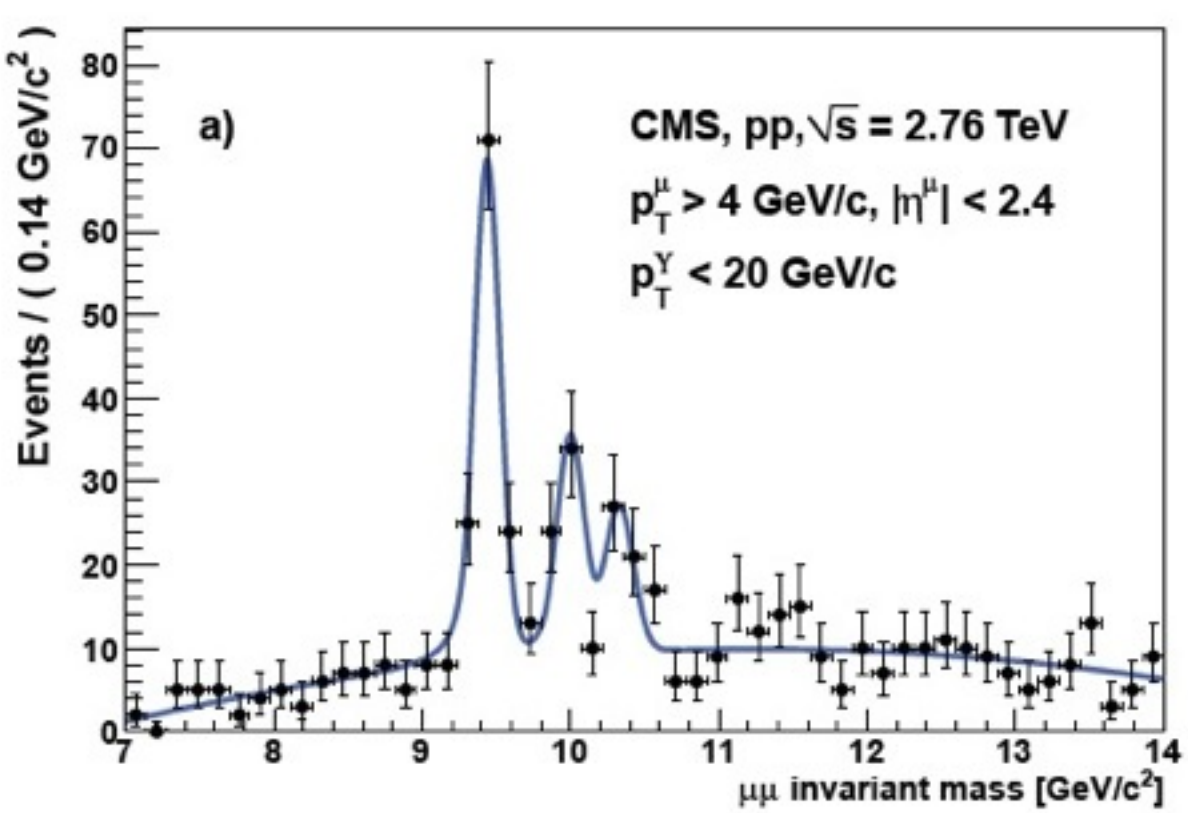
Screening

screening → heavy meson suppression

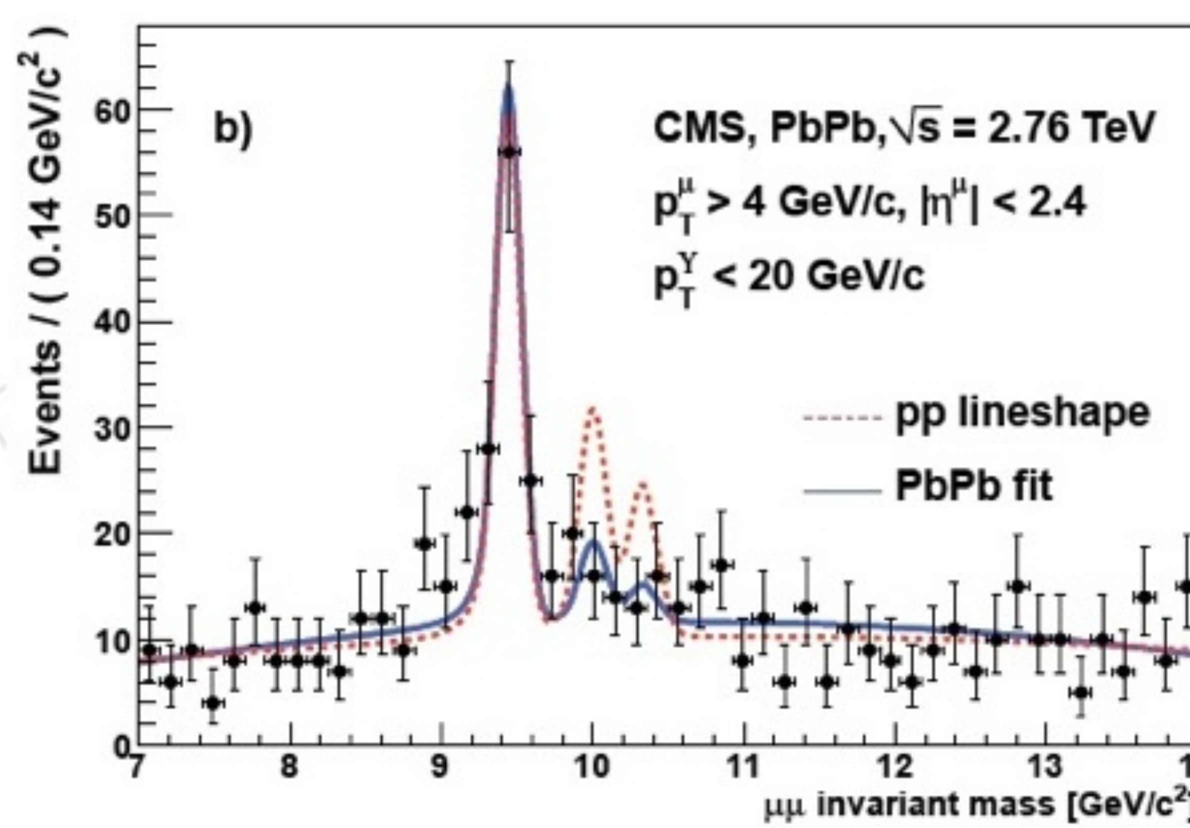
Matsui, Satz

for example: Upsilon suppression at LHC

pp

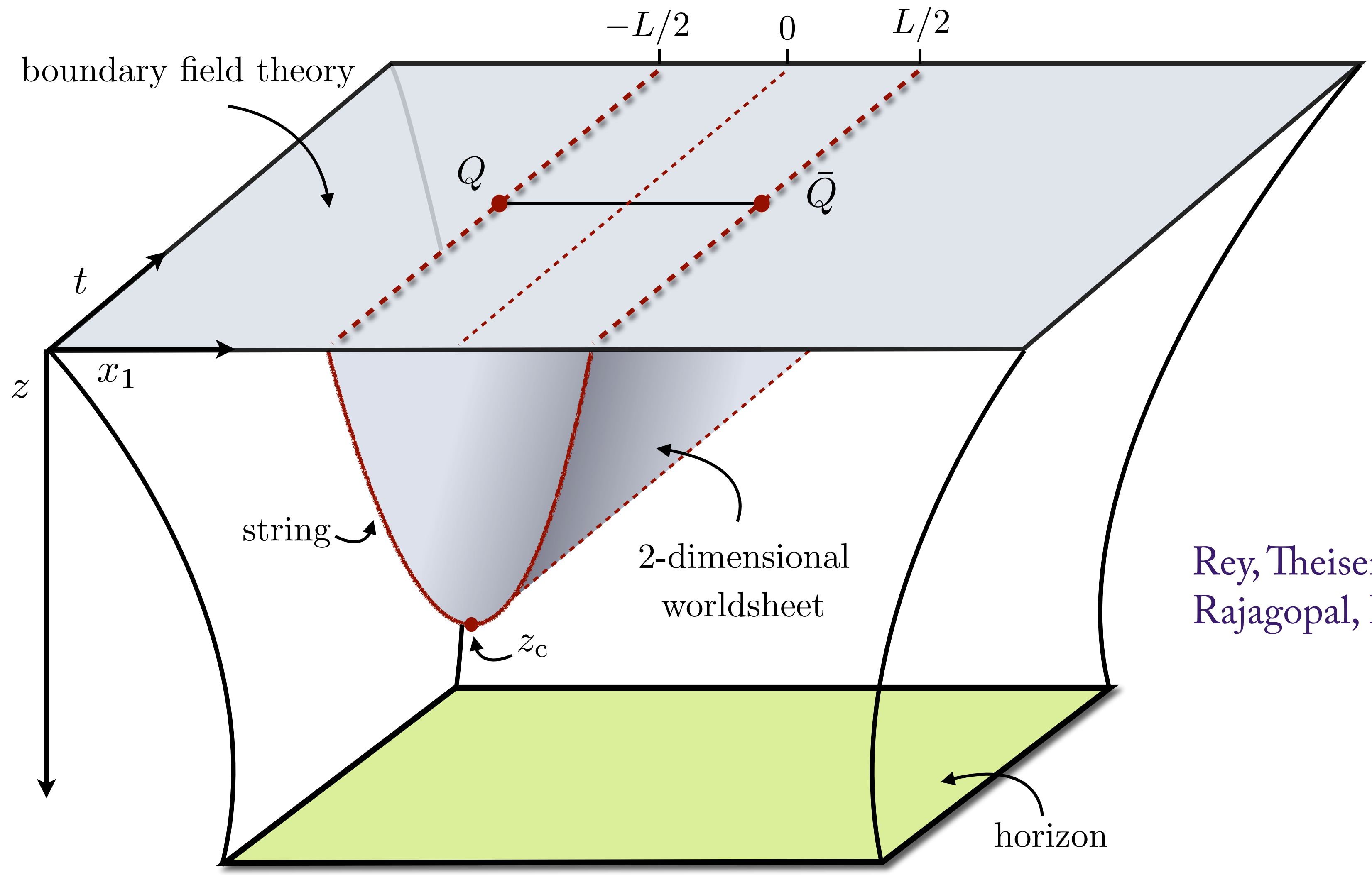


PbPb



Screening Distance

Konrad Schade, CE

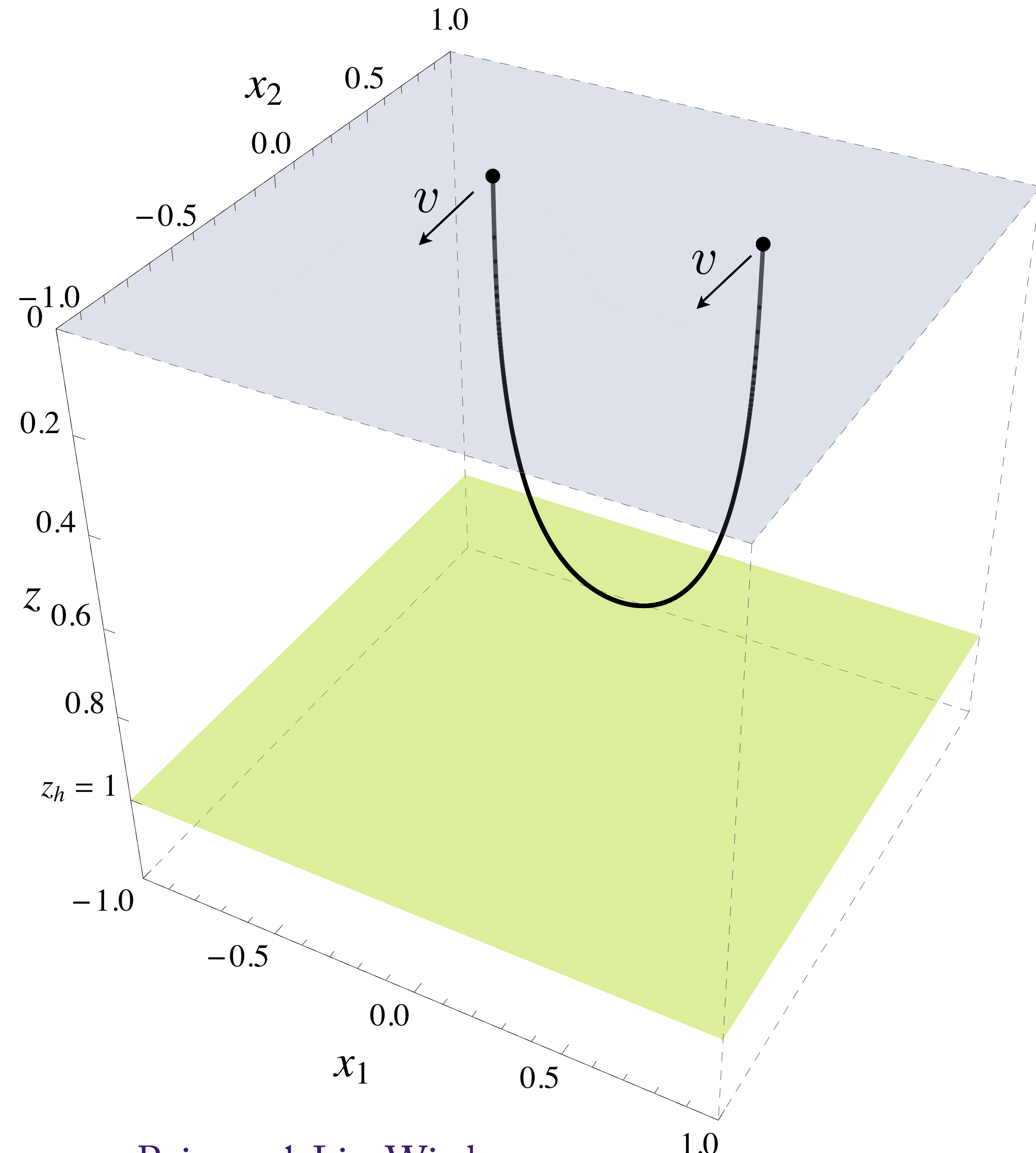


Rey, Theisen, Yee, 1998
Rajagopal, Liu, Wiedemann, 2006

Expectation value of temporal Wegner-Wilson loop in boundary field theory dual to macroscopic string hanging into the bulk

Screening Distance

- Static $Q\bar{Q}$-pair in a hot plasma wind blowing in x_2-direction
- Velocity is given by $v = \tanh \eta$
- Orientation angle θ w.r.t. wind



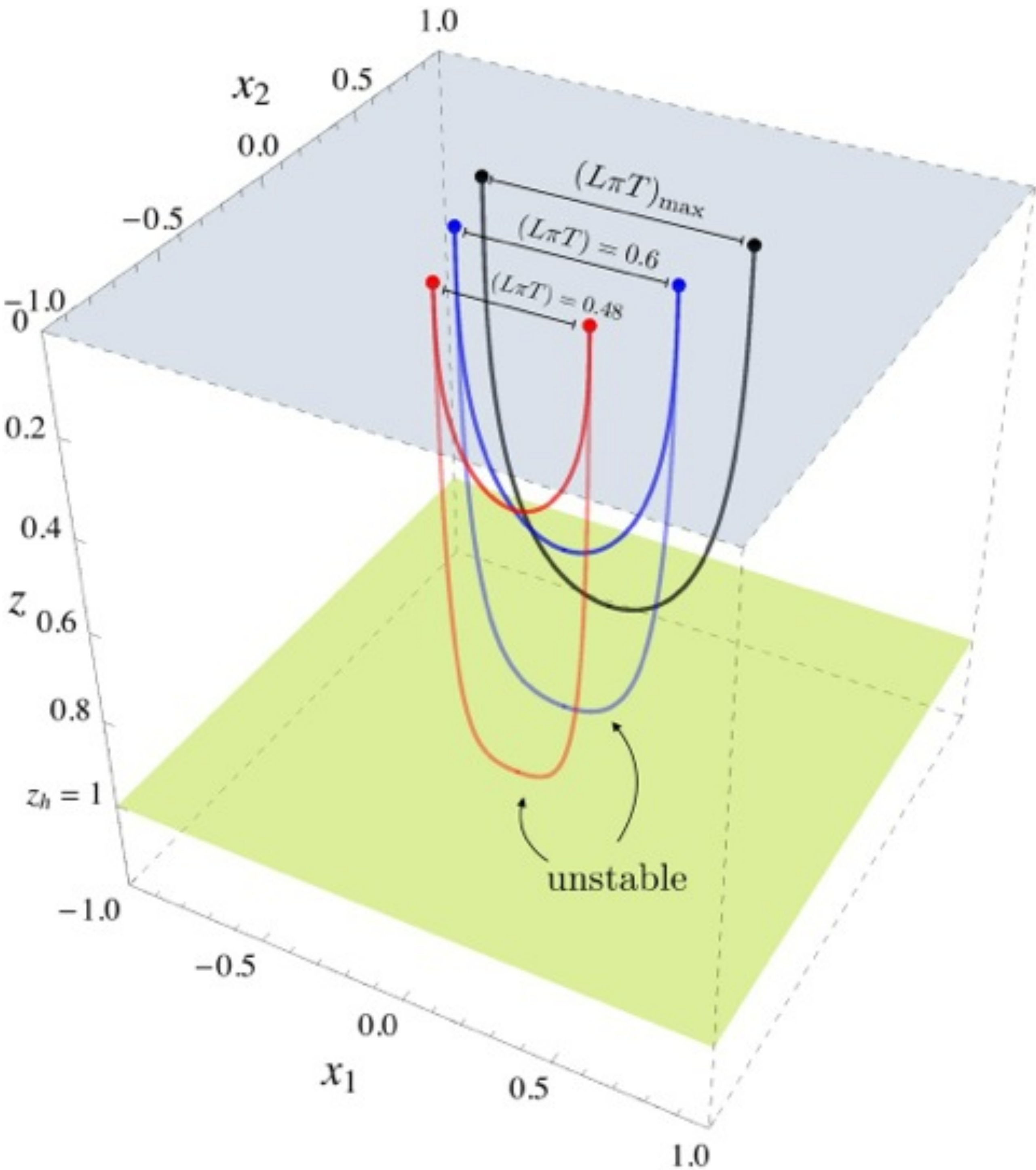
Rajagopal, Liu, Wiedemann

- Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det g_{\alpha\beta}}$$

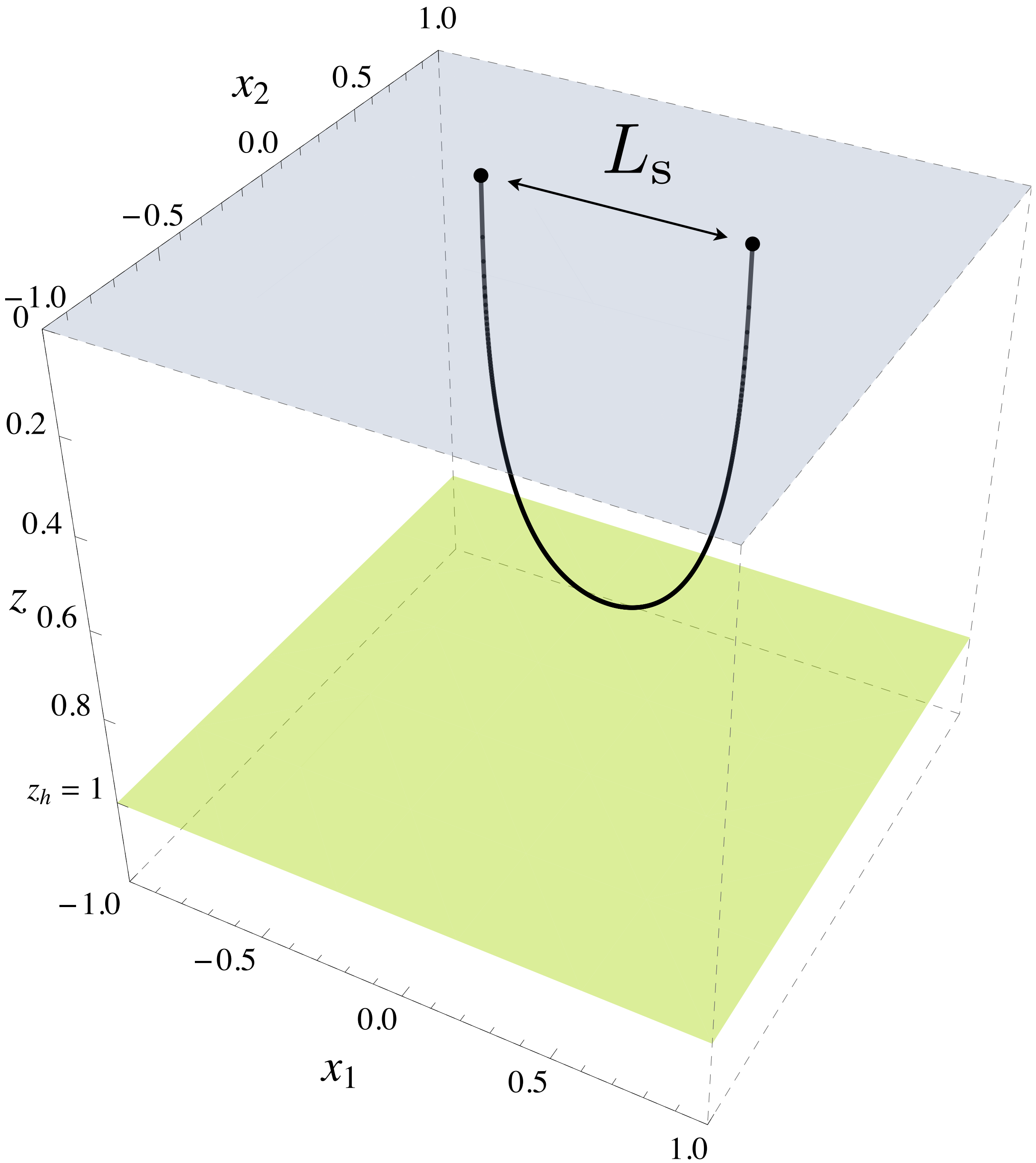
with $g_{\alpha\beta} = G_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu$

Screening Distance



lowest point z_c parametrizes different configurations

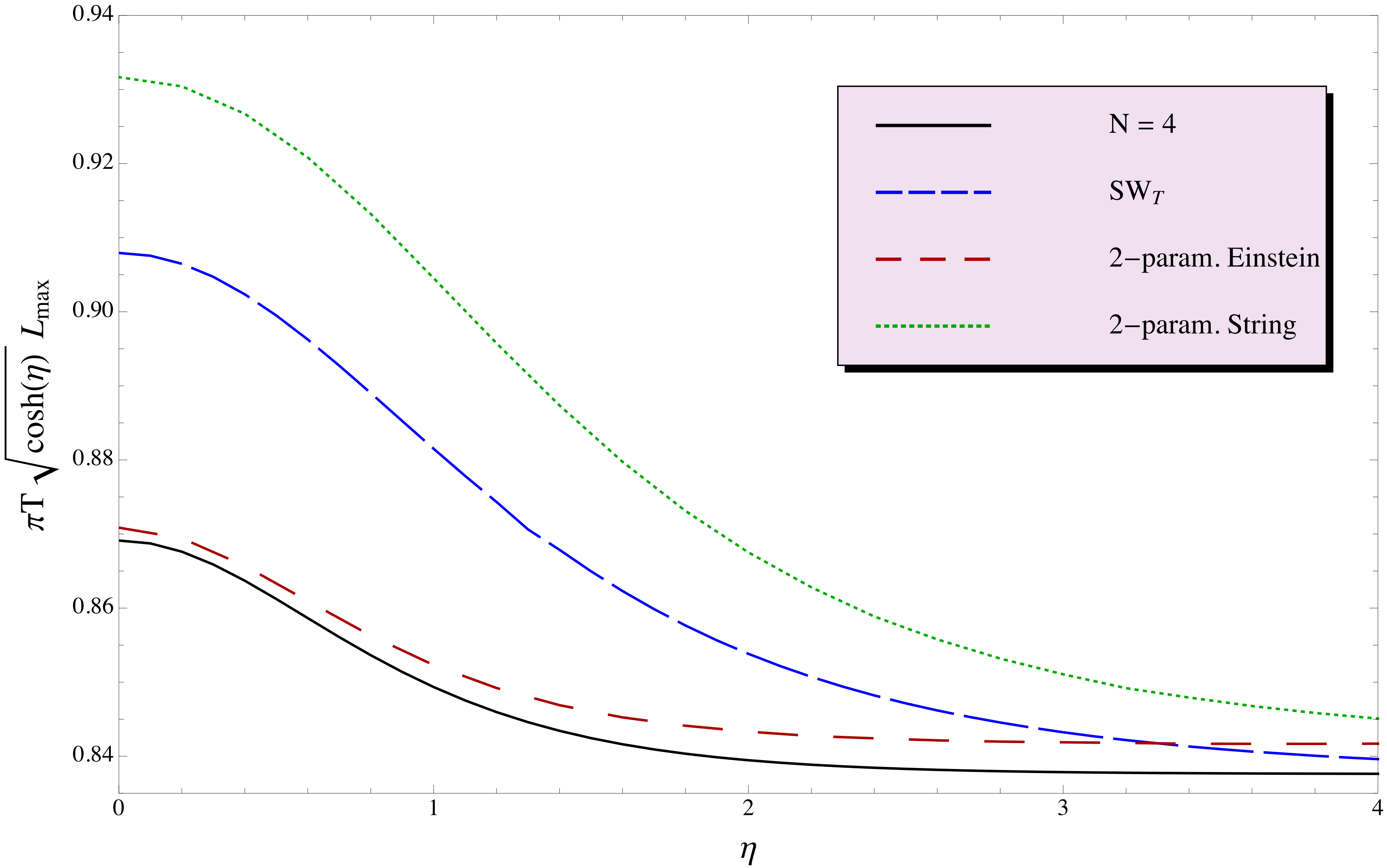
Screening Distance



maximal distance L_s
is screening distance

(different from Debye screening length)

Screening Distance



- velocity lowers screening distance

$$\propto 1/\sqrt{\gamma} \;\; \propto (\text{boosted energy density})^{-1/4}$$

Screening Distance Conjecture

Konrad Schade, CE

Observation:

At given T screening distance in N=4 SYM is smaller than in all consistently deformed models studied.

- holds for all kinematical parameters

Conjecture:

Screening distance in N=4 SYM is lower bound in a large class of (or maybe all?) consistent theories.

Proof (for finite T only):

to first order in general perturbation around AdS_5

Energies

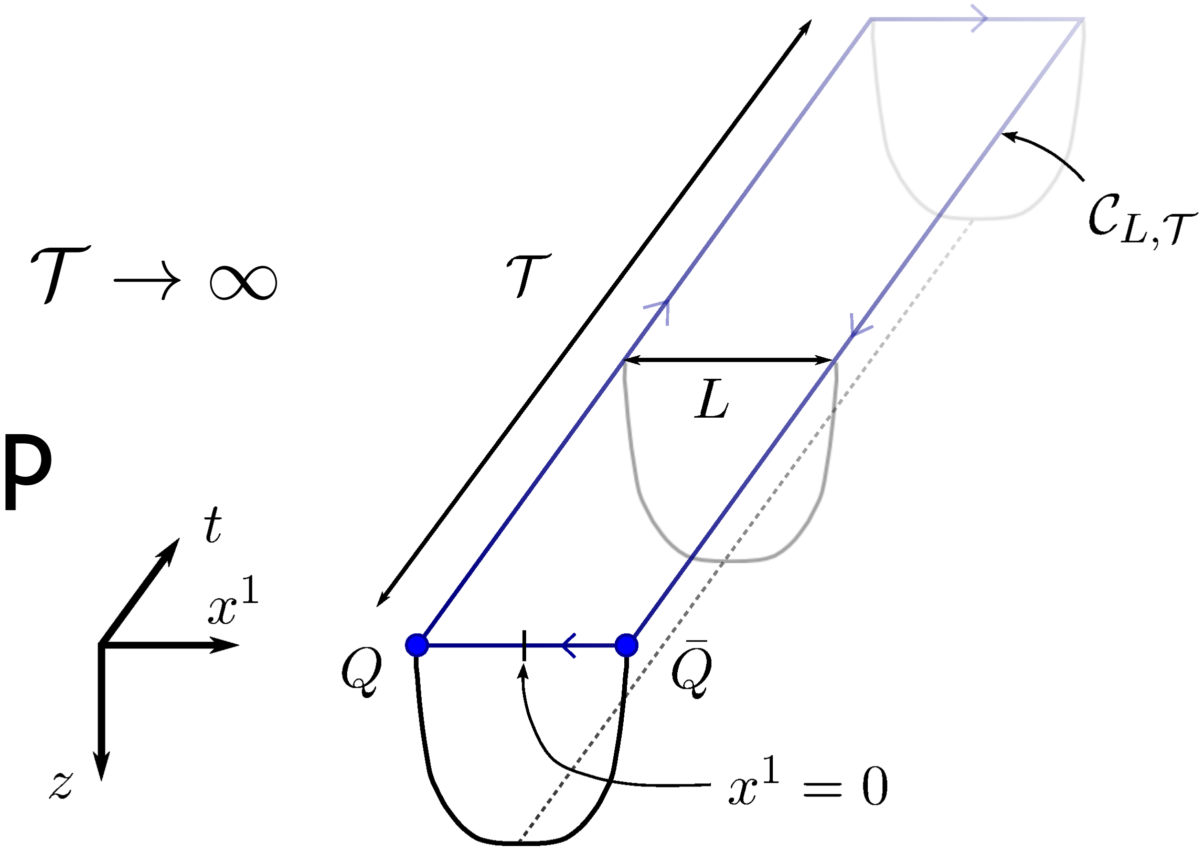
A. Samberg, O. Kaczmarek, CE

Free Energy of Heavy QQbar pair

in field theory:

$$\langle W(\mathcal{C}_{L,\mathcal{T}})\rangle \sim \exp\left(-\mathrm{i}F_{Q\bar{Q}}(L)\mathcal{T}\right)\ , \qquad \mathcal{T}\to\infty$$

for temporal Wilson loop



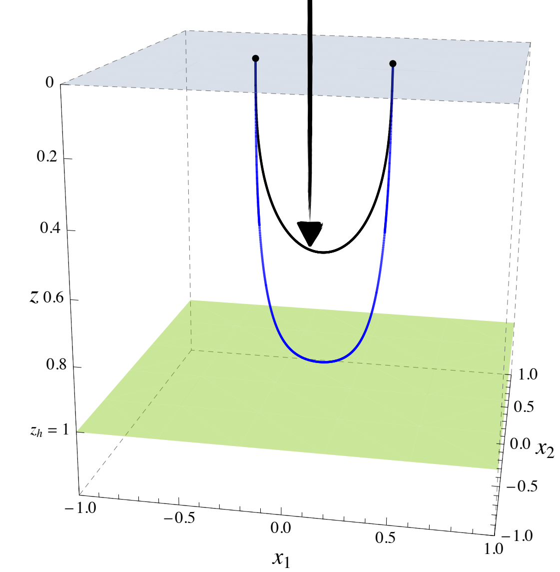
calculation in AdS/CFT:

$$\langle W(\mathcal{C})\rangle \sim \exp\left(\mathrm{i}S_{\mathrm{NG}}[\mathcal{C}]\right)$$

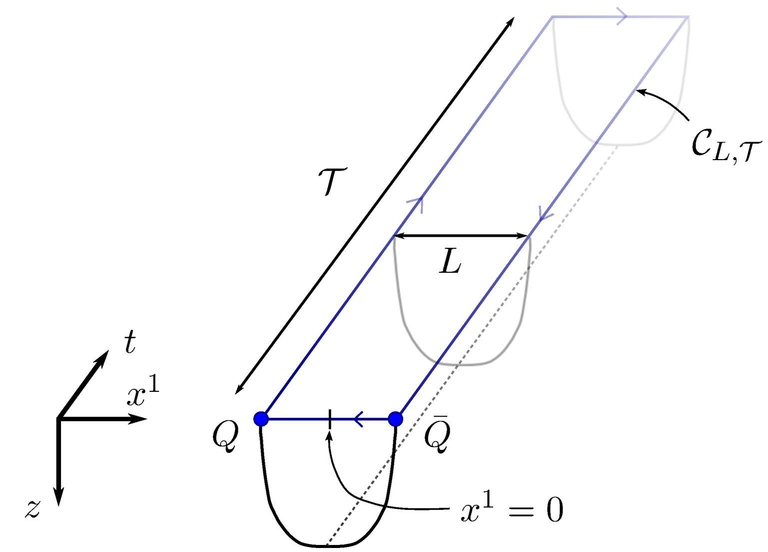
$$S_{\mathrm{NG}} = -\frac{1}{2\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{-\det g_{ab}}\,.$$

$$F_{Q\bar{Q}}(L) \sim -\frac{S_{\mathrm{NG}}[\mathcal{C}_{L,\mathcal{T}}]}{\mathcal{T}}\,, \qquad \mathcal{T}\to\infty$$

energetically favored string



Nambu-Goto action for hanging string



in general metric

$$\mathrm{d}s^2 = \mathrm{e}^{2A(z)}\left(-h(z)\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^2\right) + \frac{\mathrm{e}^{2B(z)}}{h(z)}\,\mathrm{d}z^2$$

we have

$$S_{\mathrm{NG}}[\mathcal{C}_{L,\mathcal{T}}] = -\frac{\mathcal{T}}{\pi\alpha'}\int_0^{z_{\mathrm{t}}} \mathrm{d}z\,\mathrm{e}^{A+B}\sqrt{\frac{\mathrm{e}^{4A}h}{\mathrm{e}^{4A}h - \mathrm{e}^{4A_{\mathrm{t}}}h_{\mathrm{t}}}}$$

z_t: turning point

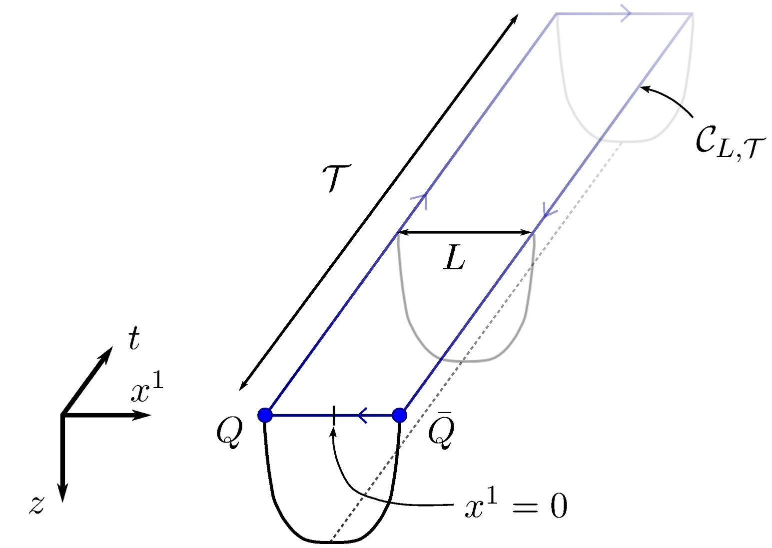
UV divergent:

$$S_{\mathrm{NG}}^{(\mathrm{reg})}[\mathcal{C}_{L,\mathcal{T}}] = -\frac{\mathcal{T}}{\pi\alpha'}\int_\varepsilon^{z_{\mathrm{t}}} \mathrm{d}z\,\mathrm{e}^{A+B}\sqrt{\frac{\mathrm{e}^{4A}h}{\mathrm{e}^{4A}h - \mathrm{e}^{4A_{\mathrm{t}}}h_{\mathrm{t}}}} \sim -\frac{\mathcal{T}L_{\mathrm{AdS}}^2}{\pi\alpha'}\left(\frac{1}{\varepsilon} + \dots\right)$$

Subtraction for Nambu-Goto action

subtraction required:

$$F^{(\mathrm{ren})}_{Q\bar{Q}}(L) = \lim_{\mathcal{T}\to\infty}\left(-\frac{S^{(\mathrm{reg})}_{\mathrm{NG}}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S}{\mathcal{T}}\right)$$



subtractions in the literature:

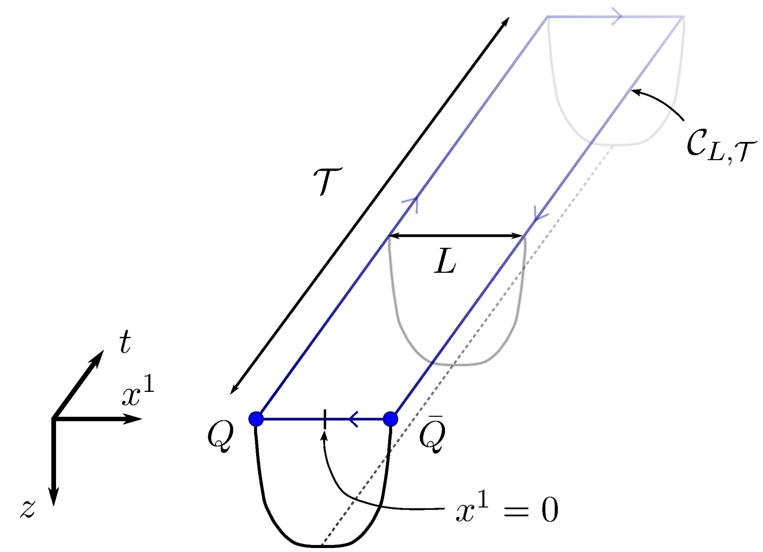
- non-interacting string hanging down into black hole (2x)

$$S^{(\mathrm{reg})}_{\mathrm{NG}}[\text{straight string}] = -\frac{\mathcal{T}}{2\pi\alpha'}\int_{\varepsilon}^{z_\mathrm{h}} \mathrm{d}z\, \mathrm{e}^{A+B} \sim -\frac{\mathcal{T} L^2_{\mathrm{AdS}}}{2\pi\alpha'}\left(\frac{1}{\varepsilon} + \dots\right)$$

- real part of action at L=∞

but: then F is T-dependent for small L - unphysical!

Subtraction for Nambu-Goto action



subtraction required:

$$F^{(\mathrm{ren})}_{Q\bar{Q}}(L) = \lim_{\mathcal{T}\to\infty}\left(-\frac{S^{(\mathrm{reg})}_{\mathrm{NG}}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S}{\mathcal{T}}\right)$$

correct subtraction: only singularity

$$\Delta S_{\min} \equiv -\frac{\mathcal{T} L^2_{\mathrm{AdS}}}{\pi\alpha'}\int_{\varepsilon}^{\infty}\frac{\mathrm{d}z}{z^2} = -\frac{\mathcal{T} L^2_{\mathrm{AdS}}}{\pi\alpha'}\,\frac{1}{\varepsilon}$$

then no unphysical T-dependence!

Binding Energy of Heavy QQbar pair

quantity with hanging-string subtraction is
binding energy

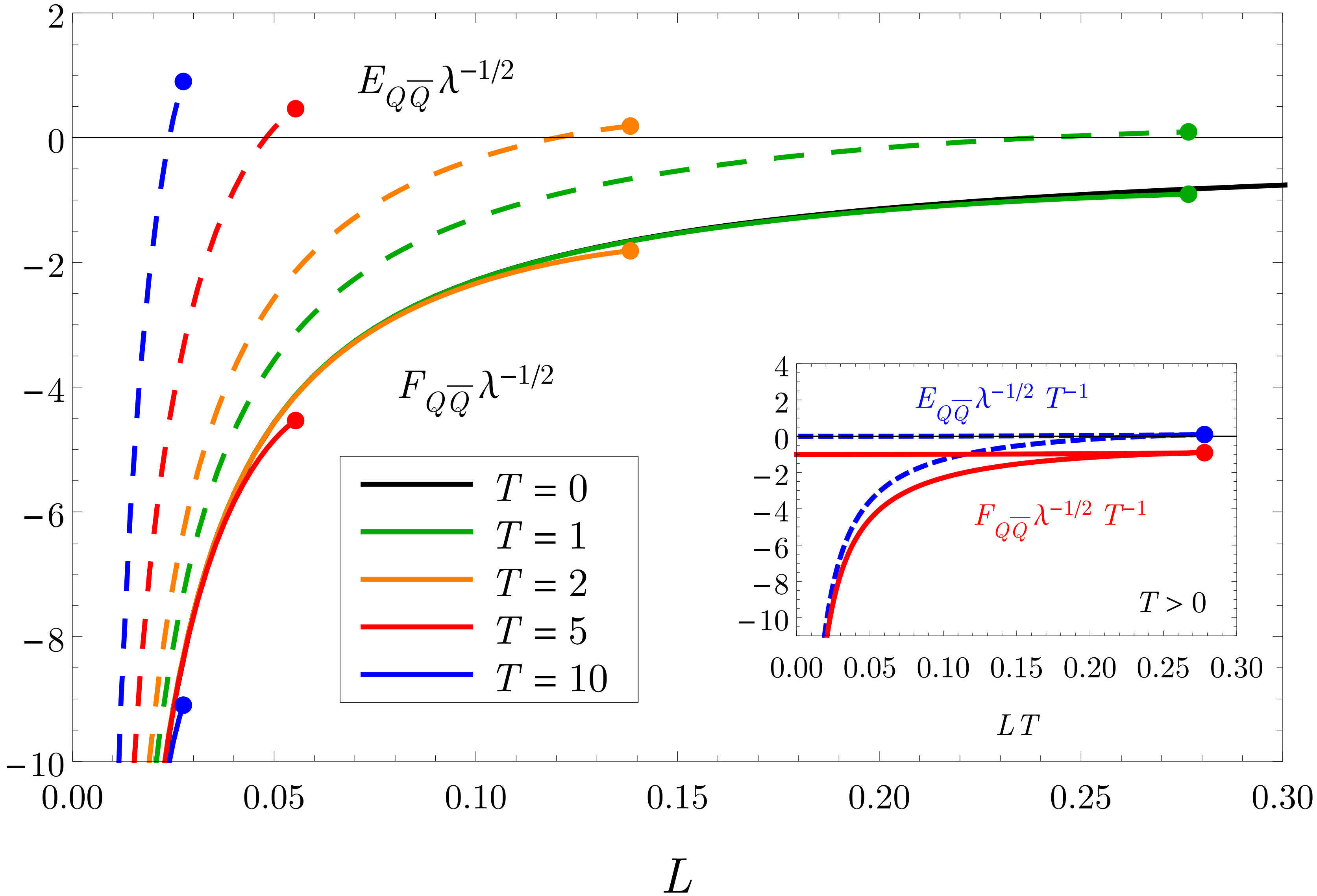
$$E_{Q\bar{Q}}(L) = \lim_{\mathcal{T}\to\infty}\left(-\frac{S_{\rm NG}[\mathcal{C}_{L,\mathcal{T}}] - 2S_{\rm NG}[\text{straight string}]}{\mathcal{T}}\right)$$

in fact difference of free energies:

$$\begin{aligned} E_{Q\bar{Q}}(L) &= \lim_{\mathcal{T}\to\infty}\left[-\frac{\left(S_{\rm NG}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S_{\rm min}\right) - \left(2S_{\rm NG}[\text{straight string}] - \Delta S_{\rm min}\right)}{\mathcal{T}}\right] \\ &= F_{Q\bar{Q}} - F_{Q\,;\,\bar{Q}}\,, \end{aligned}$$

(note: defines single-quark free energy)

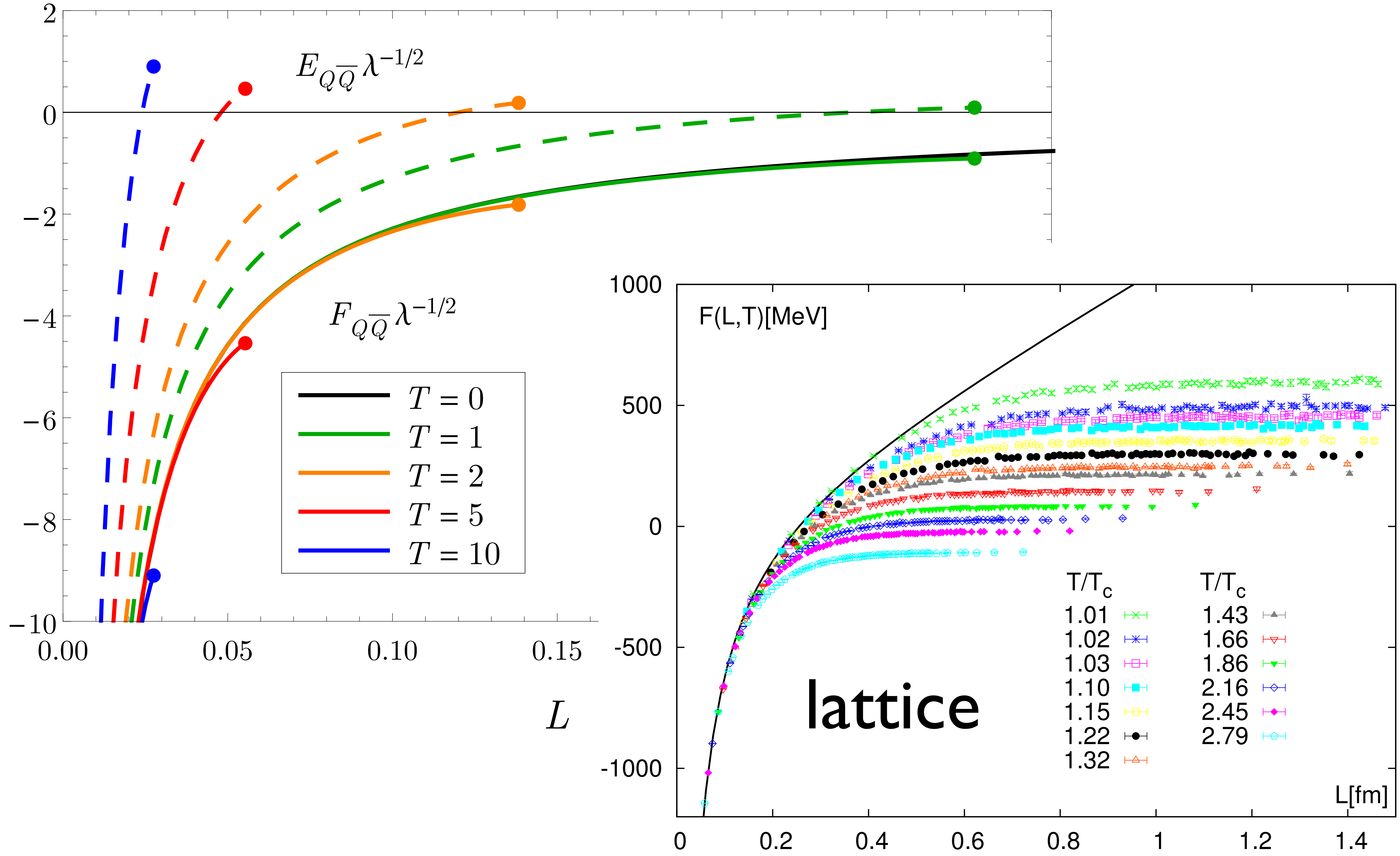
Free vs Binding Energy in $\mathcal{N}$=4



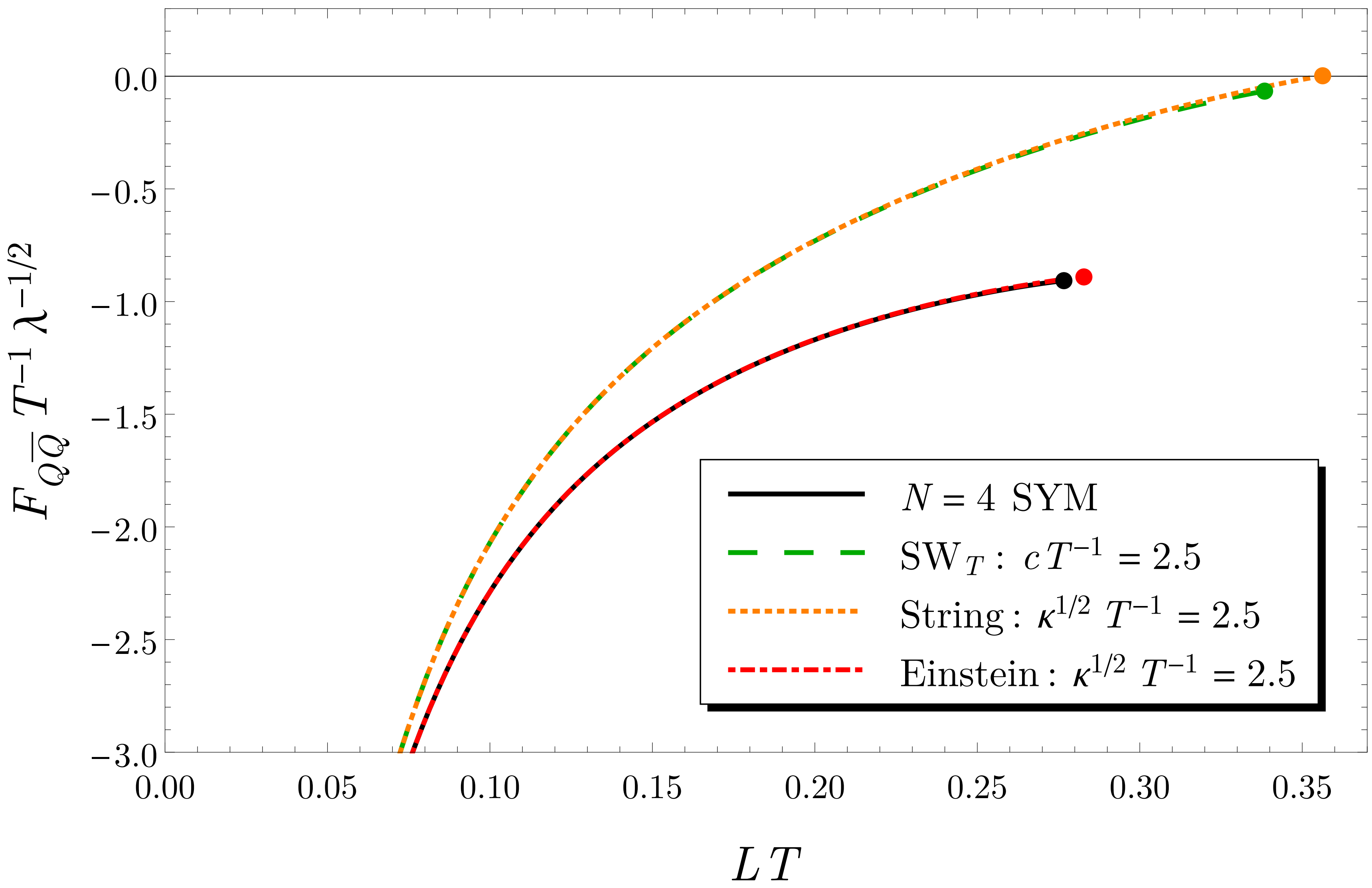
black: T=0 potential

$$V_{Q\bar{Q}}(L) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4\left(\frac{1}{4}\right)L}$$

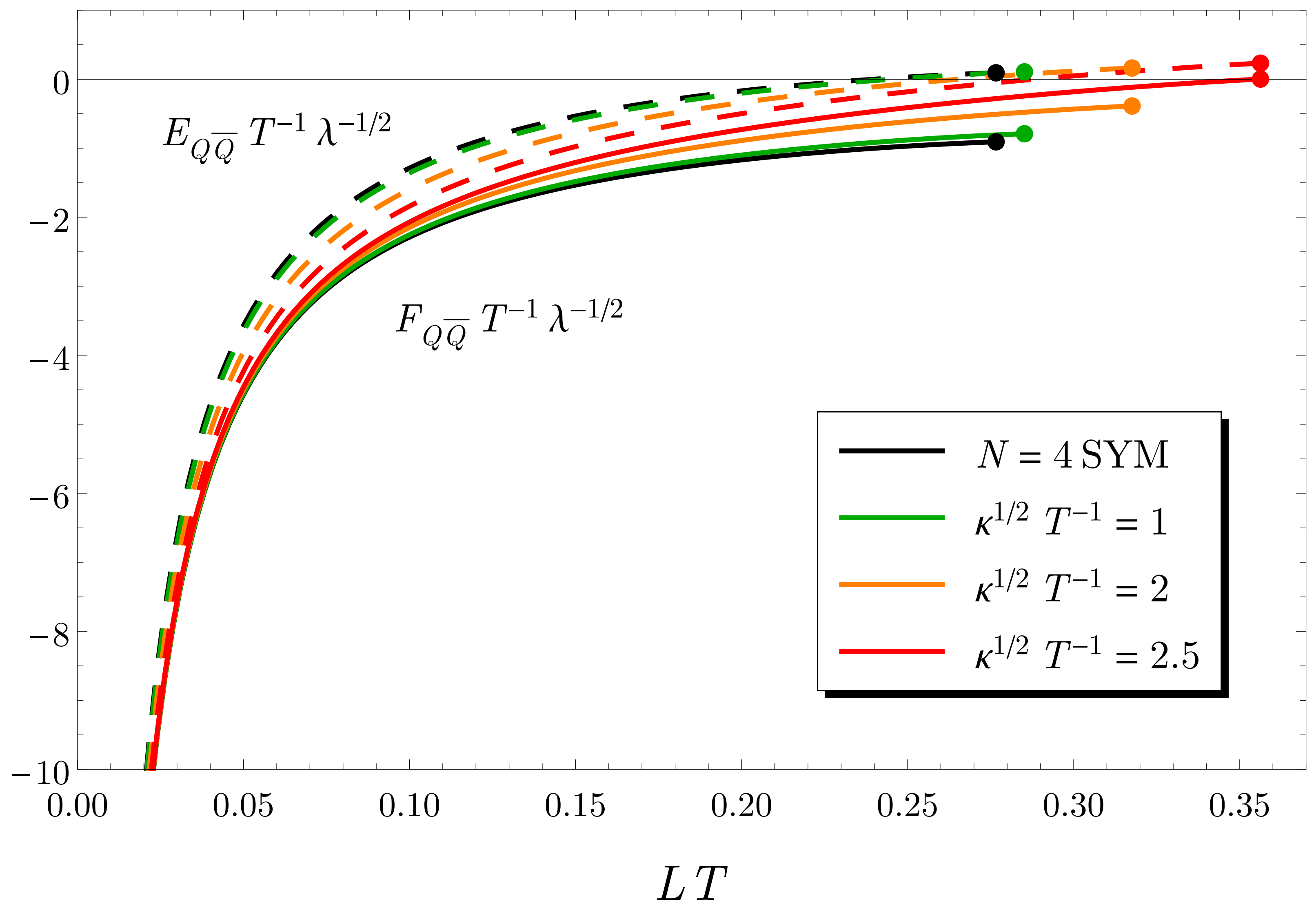
Free vs Binding Energy in $\mathcal{N}$=4



Free Energy - different models



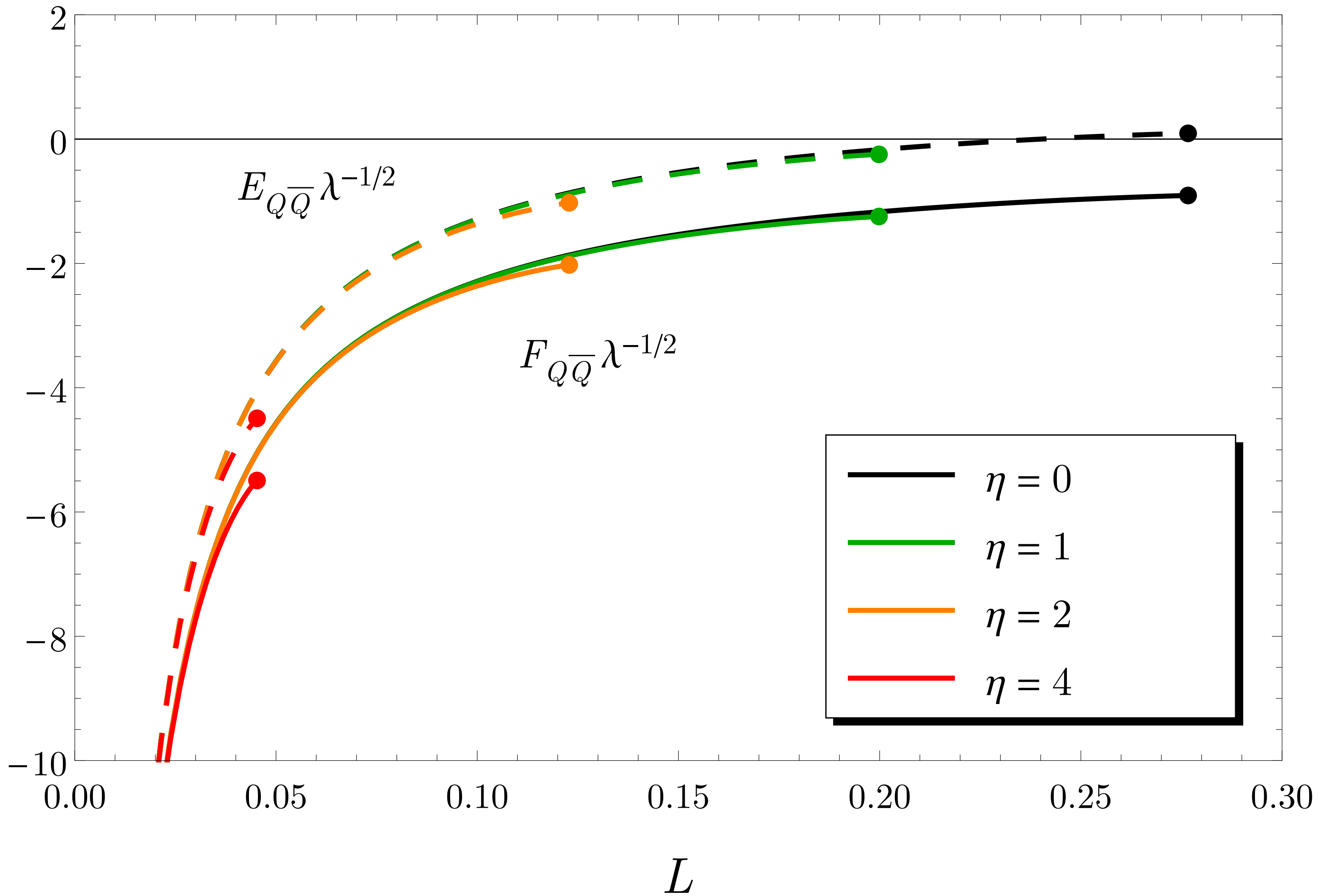
Free and Binding Energy - different non-conformalities



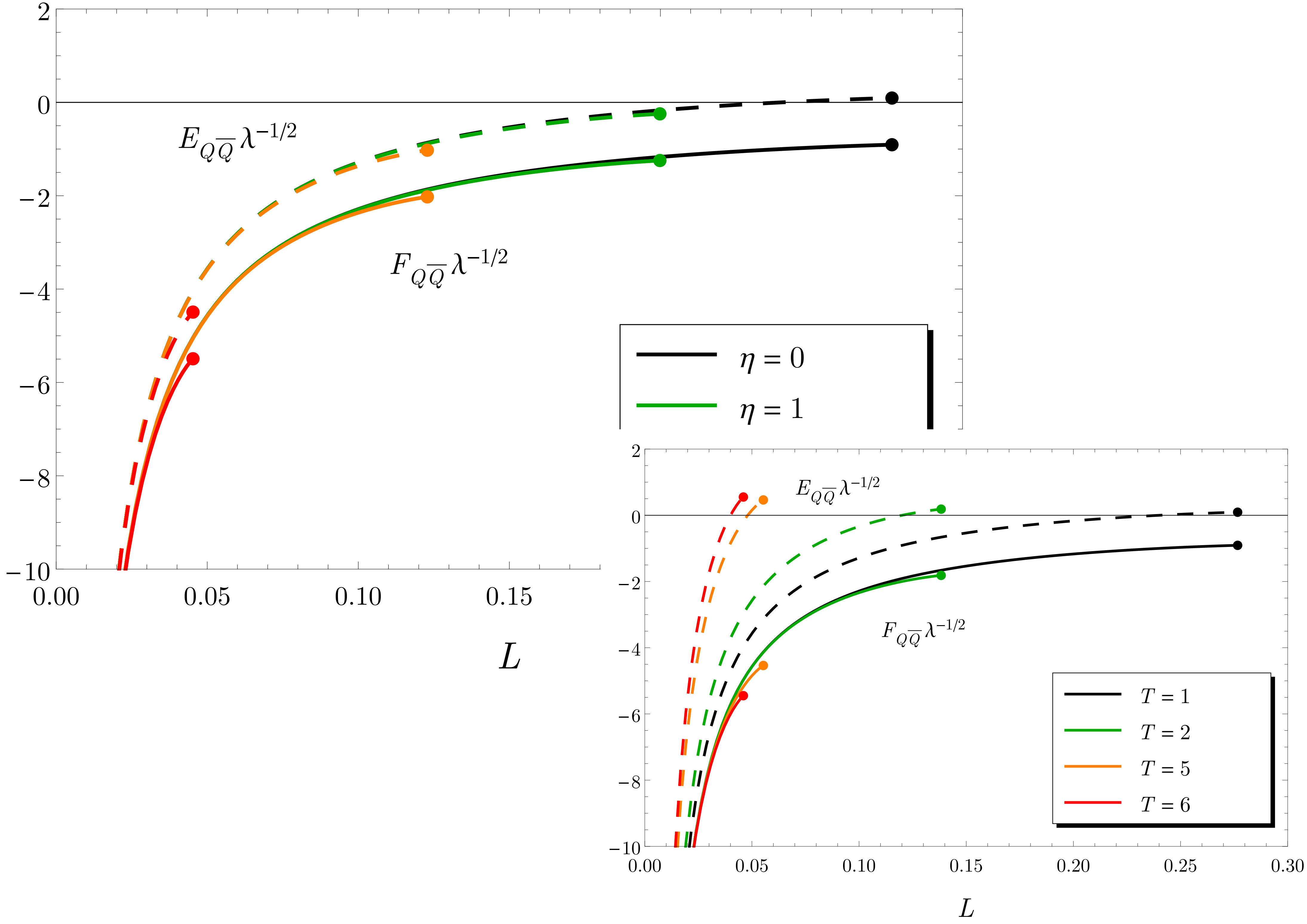
1-parameter string frame

Free and Binding Energy - quarks in motion

P.Wittmer, CE



Free and Binding Energy - quarks in motion



Entropy and Internal Energy of QQbar pair

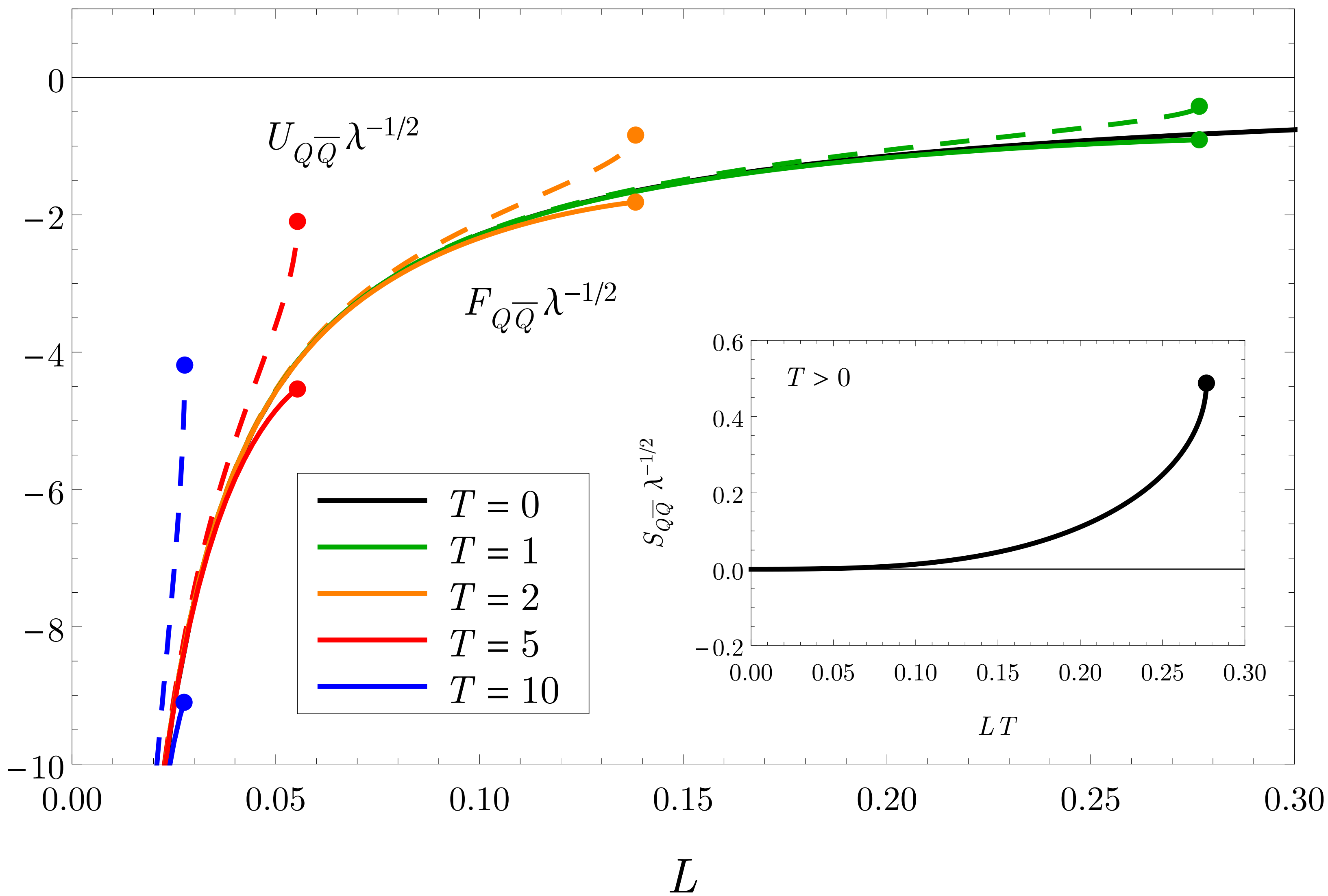
with (correct!) free energy obtain entropy

$$S_{Q\bar{Q}}(L,T) = -\frac{\partial F_{Q\bar{Q}}(L,T)}{\partial T}$$

and internal energy

$$U_{Q\bar{Q}}(L,T) = F_{Q\bar{Q}}(L,T) + T S_{Q\bar{Q}}(L,T)$$

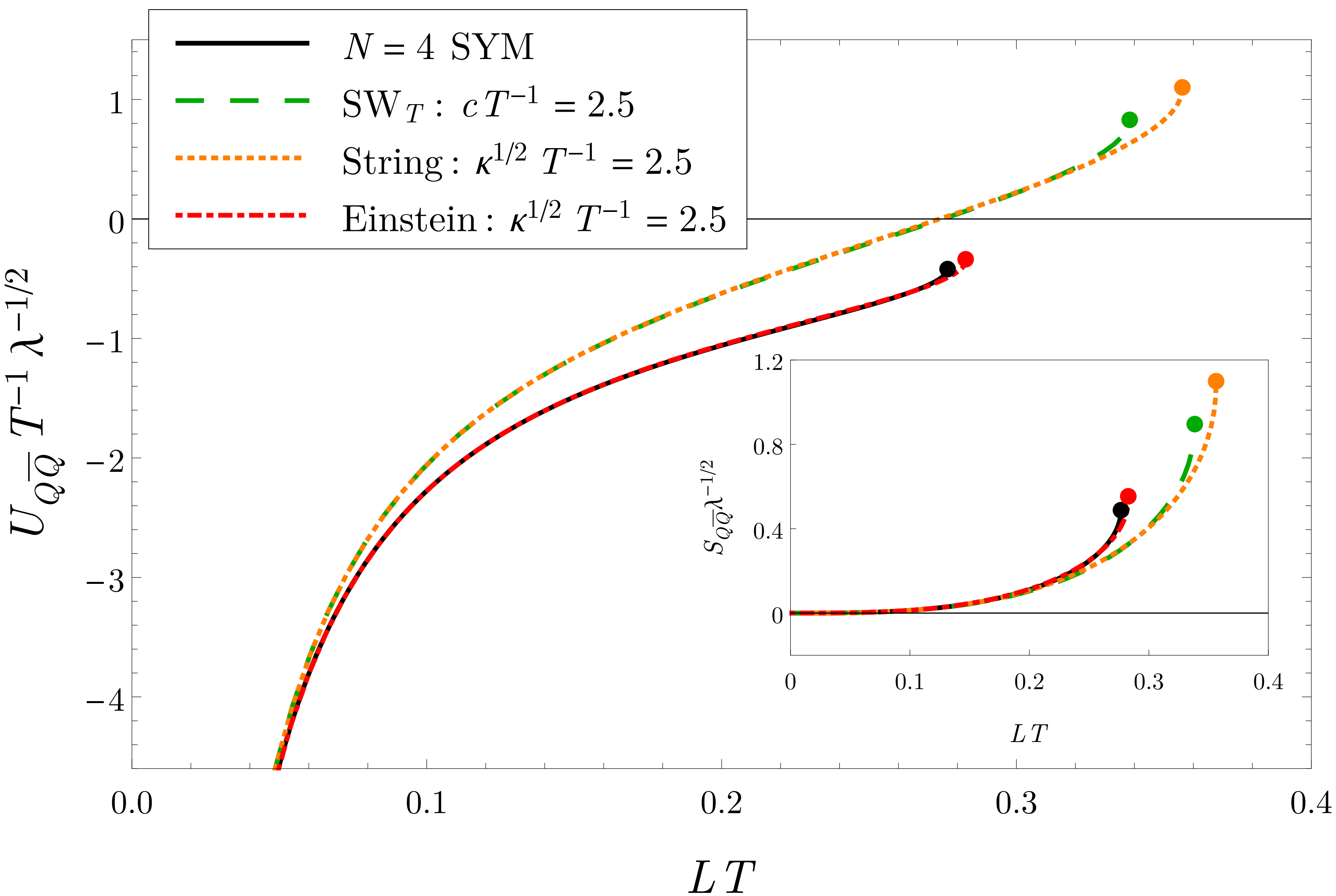
Entropy and Internal Energy in $\mathcal{N}$=4



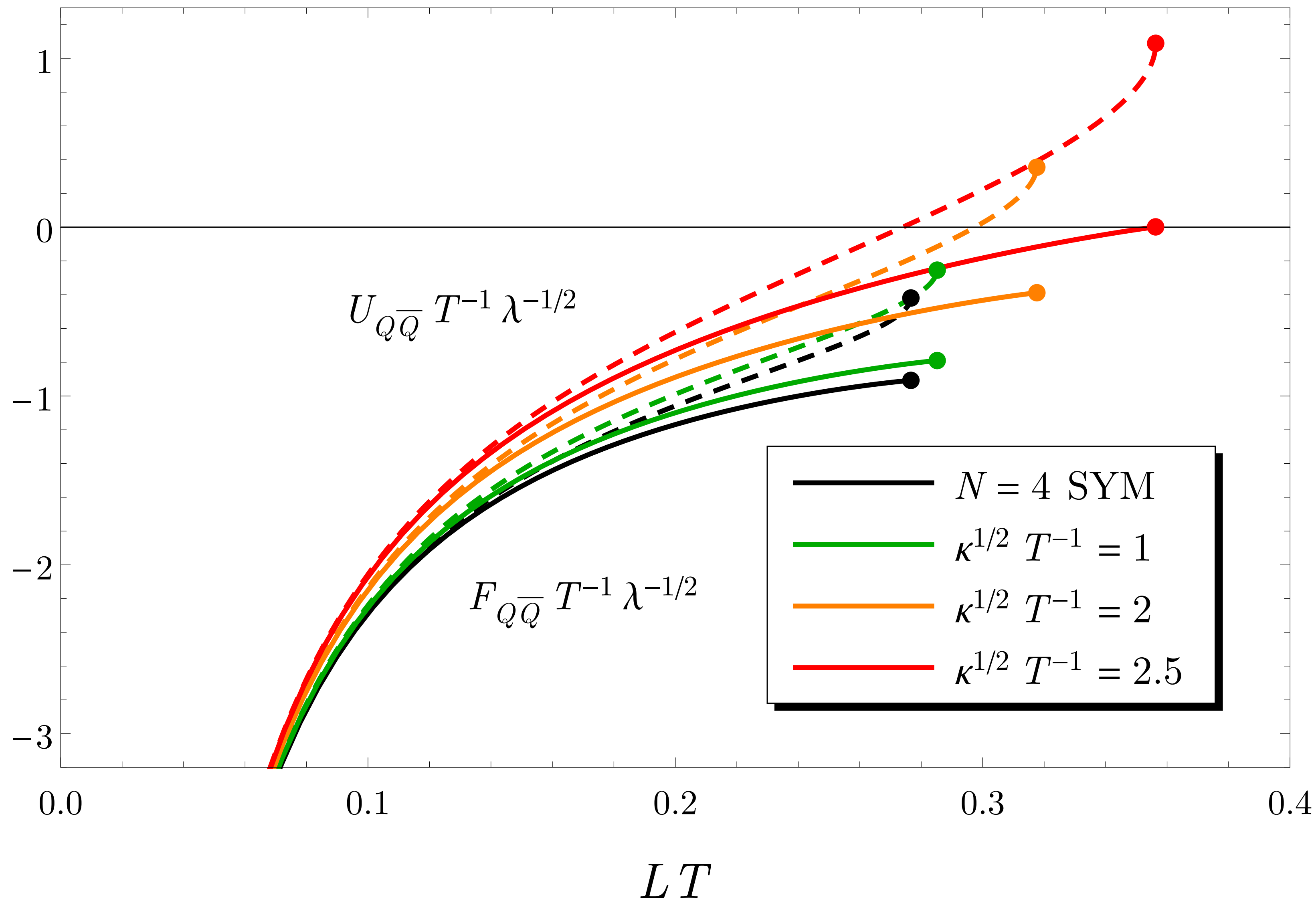
black: T=0 potential

$$V_{Q\bar{Q}}(L) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4\left(\frac{1}{4}\right) L}$$

Internal Energy - different models



Internal Energy - different non-conformalities



1-parameter string frame

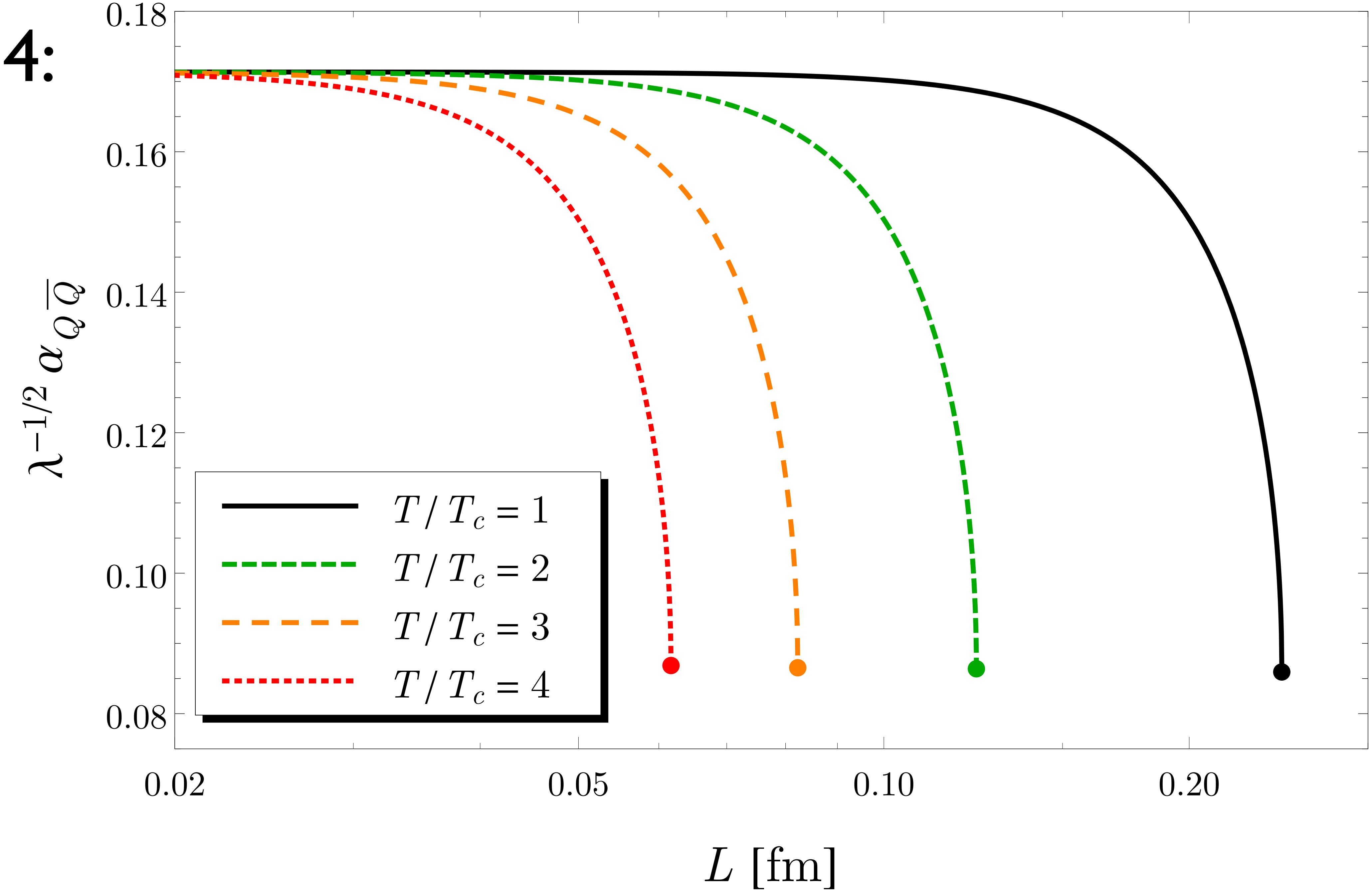
Running Coupling

Running Coupling α_{qq}

$$\alpha_{Q\bar{Q}} = \frac{3L^2}{4}\frac{dF(L,T)}{dL}$$

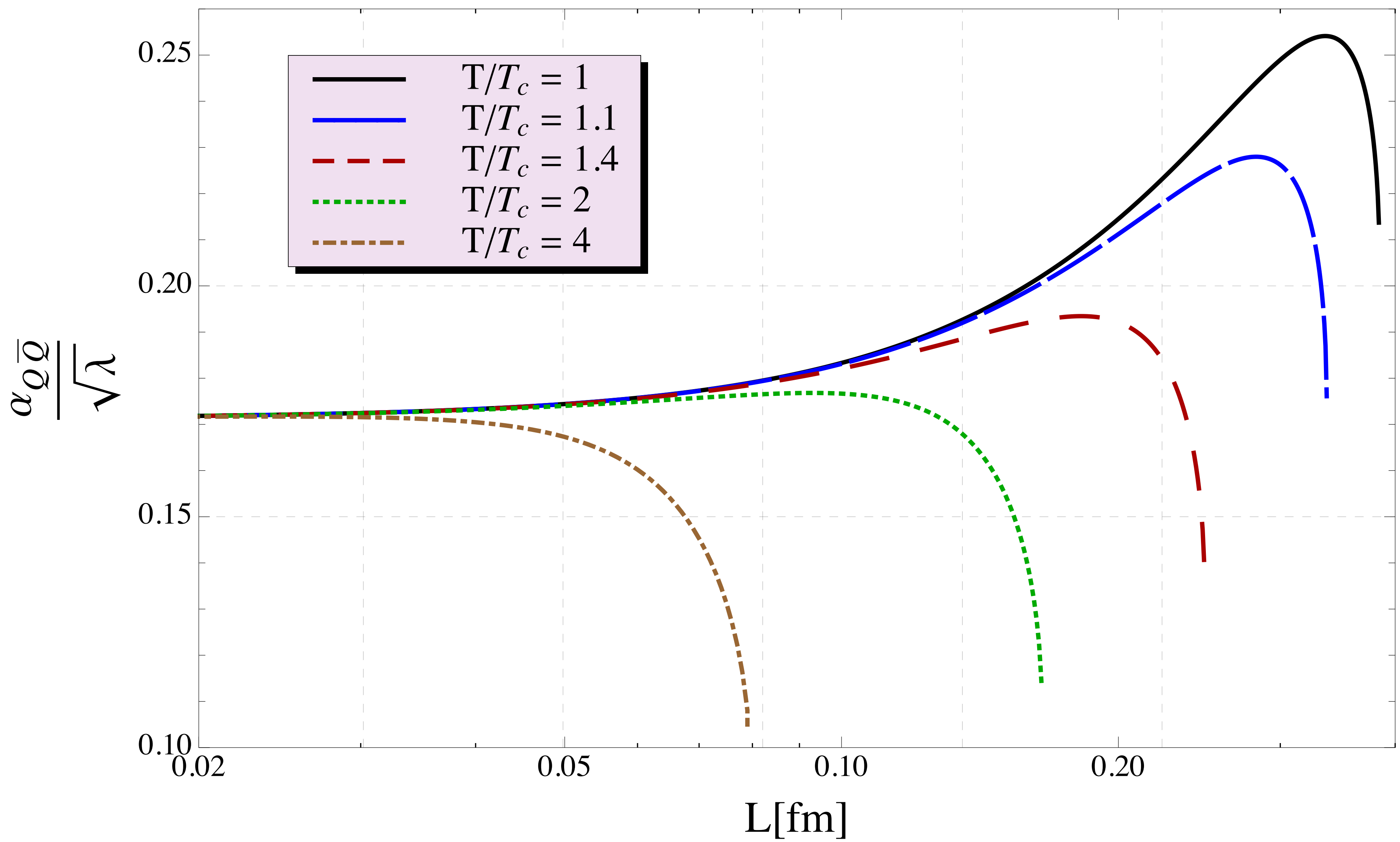
Konrad Schade, CE

$\mathcal{N}$=4:



Running Coupling α_{qq}

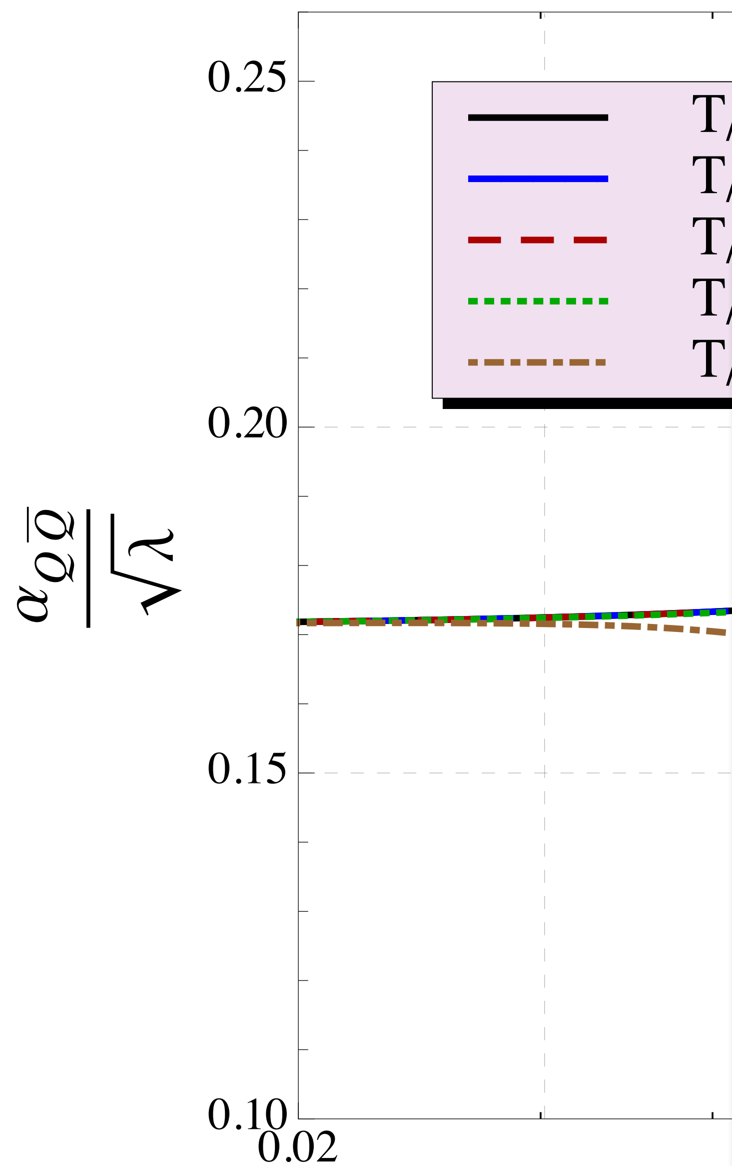
with deformation:

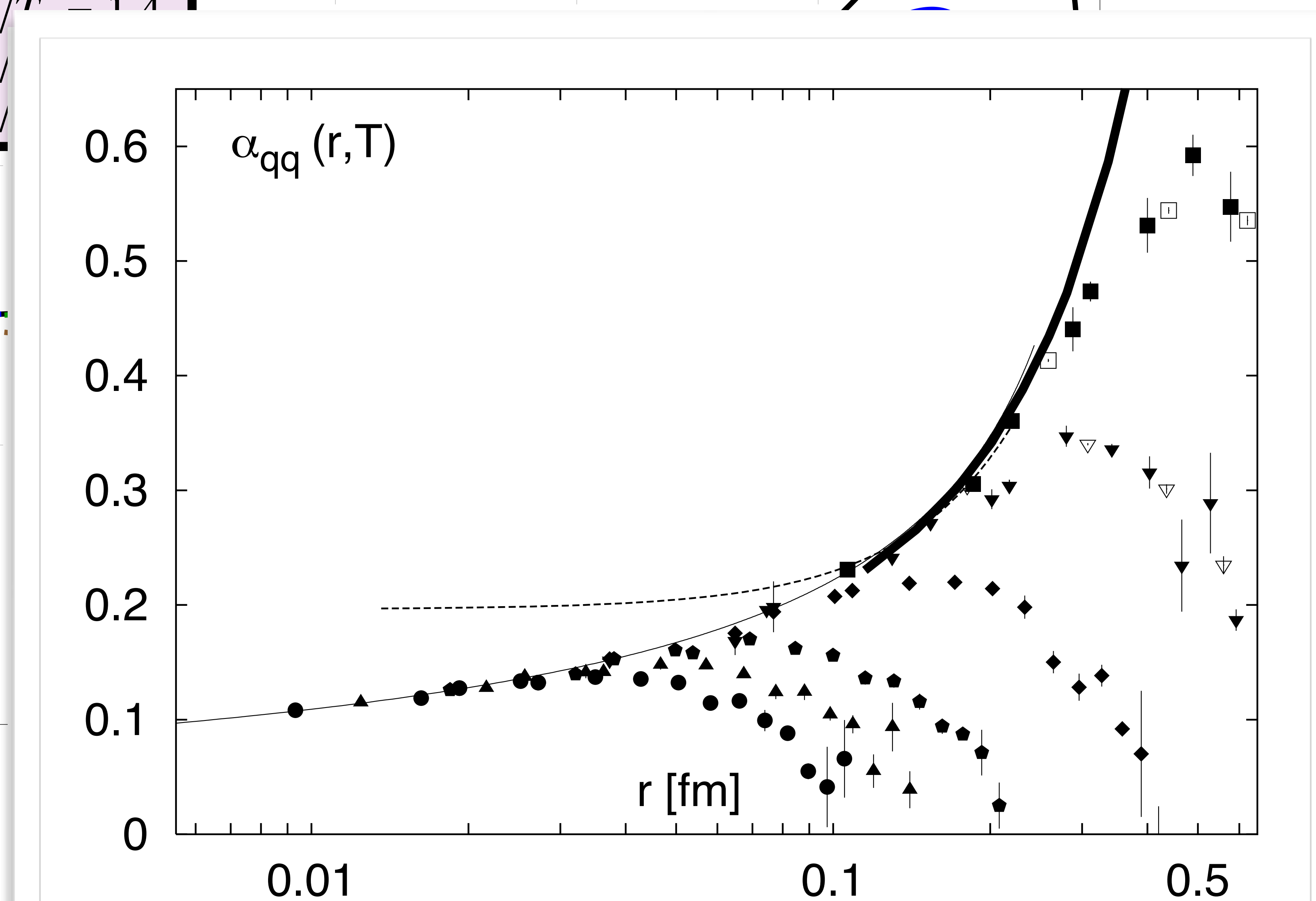


- universal rise above conformal value

Running Coupling α_{qq}

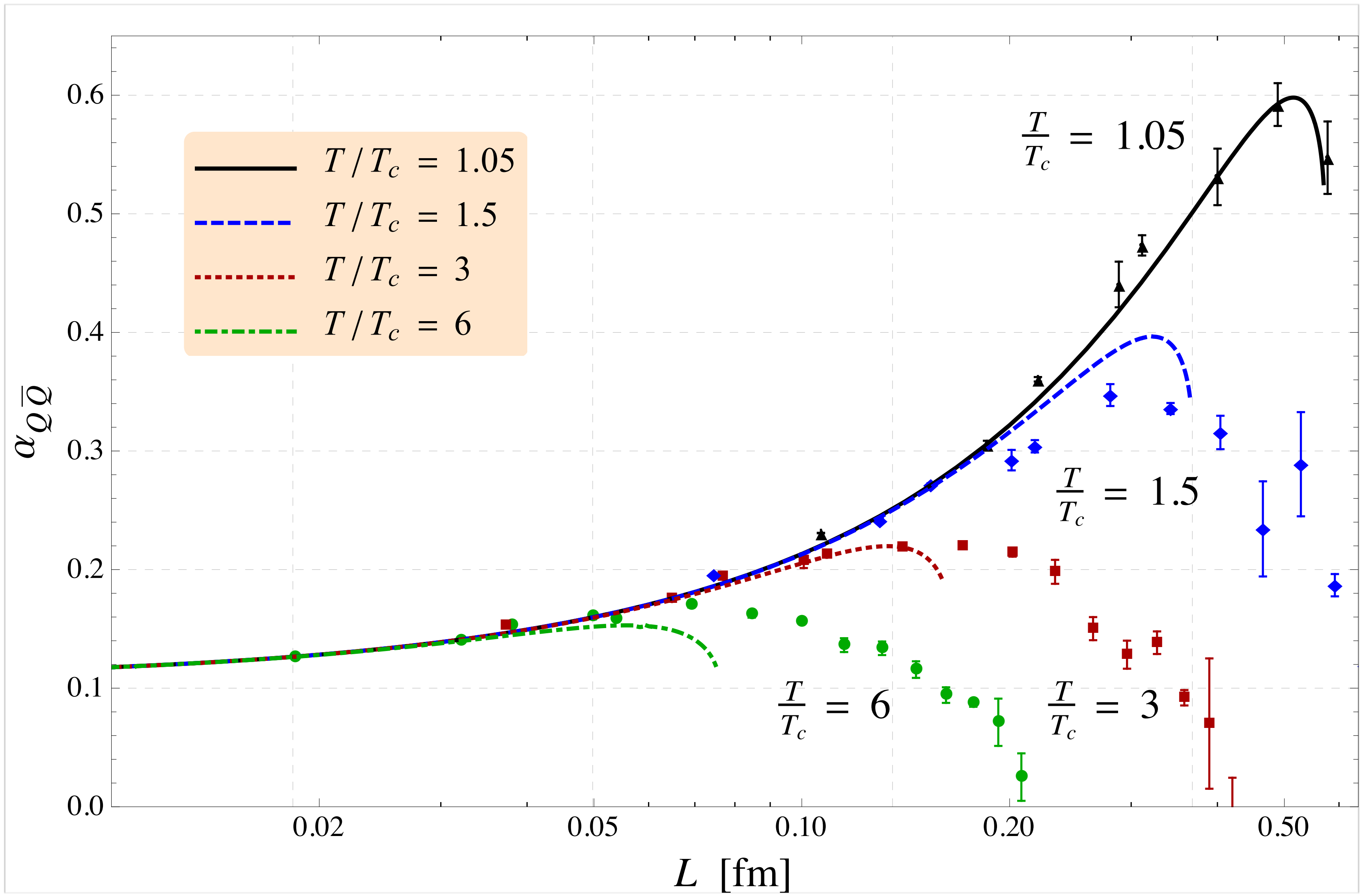
with deformation:





Kaczmarek, Karsch, Petreczky and Zantow, 2004

Running Coupling α_{qq}



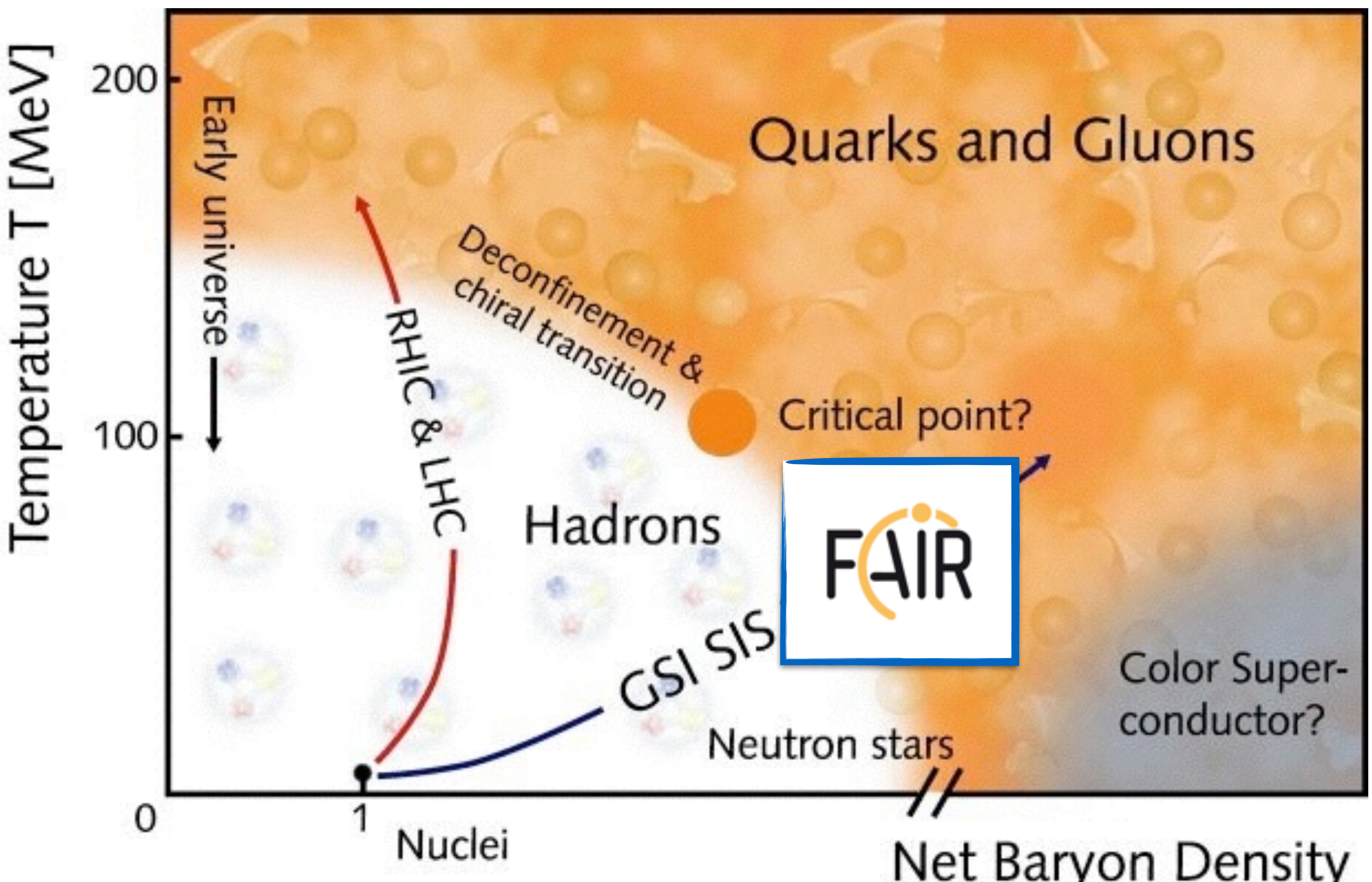
Schade, CE

- Coming close to QCD data if free parameters properly adjusted
- Parameters fixed from thermodynamics; only λ by hand

Finite Chemical Potential

Finite Chemical Potential

Extending these calculations to finite μ

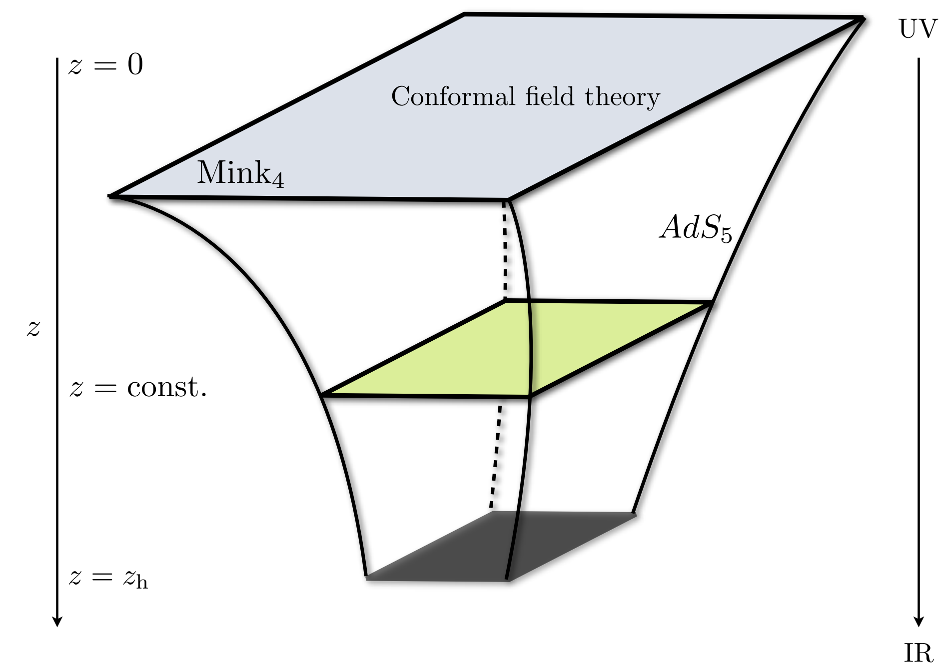


$\mathcal{N}$=4 SYM at Finite T and μ

- finite T: AdS-BH

$$ds^2 = \frac{L^2}{z^2}\left(-h\,dt^2 + d\vec{x}^{\,2} + \frac{dz^2}{h}\right)$$

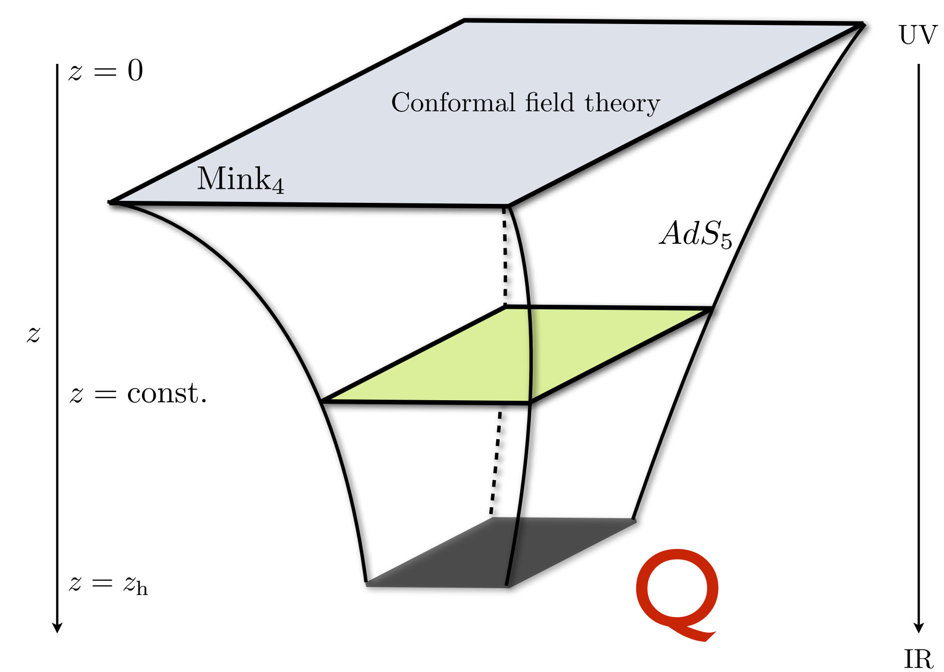
$$h = 1 - \frac{z^4}{z_h^4}$$



- finite T and μ: charged BH, AdS Reissner-Nordström BH

$$ds^2 = \frac{L^2}{z^2}\left(-h\,dt^2 + d\vec{x}^{\,2} + \frac{dz^2}{h}\right)$$

$$h(z) = 1 - \left(1 + \frac{\mu^2 z_h^2}{3}\right)\frac{z^4}{z_h^4} + \frac{\mu^2 z_h^2}{3}\frac{z^6}{z_h^6}$$



Finite Chemical Potential

simple models:

- conformal: AdS-RN ↔ N=4 SYM

$$ds^2 = \frac{L^2}{z^2}\left(-h\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \frac{\mathrm{d}z^2}{h}\right)$$

$$h(z) = 1 - \left(1 + \frac{\mu^2 z_h^2}{3}\right)\frac{z^4}{z_h^4} + \frac{\mu^2 z_h^2}{3}\frac{z^6}{z_h^6}$$

- non-conformal: $\mathrm{SW}_{T,\,\mu}$ model — Colangelo, Giannuzzi, Nicotri 2011

$$ds^2 = \underline{\mathrm{e}^{c^2 z^2}}\,\frac{L^2}{z^2}\left(-h\,dt^2 + d\vec{x}^2 + \frac{1}{h}dz^2\right)$$

- ad hoc deformation of soft-wall type
- some shortcomings at small T, μ

Finite Chemical Potential

consistent model:

Maxwell U(1) field

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^5x \sqrt{-g} \left(\mathcal{R} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{f(\phi)}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

(action as in DeWolfe, Gubser, Rosen)

solve with ansatz

$$\mathrm{d}s^2 = \mathrm{e}^{2A(z)} \left(-h\, \mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} \right) + \mathrm{e}^{2B(z)} \frac{\mathrm{d}z^2}{h}$$

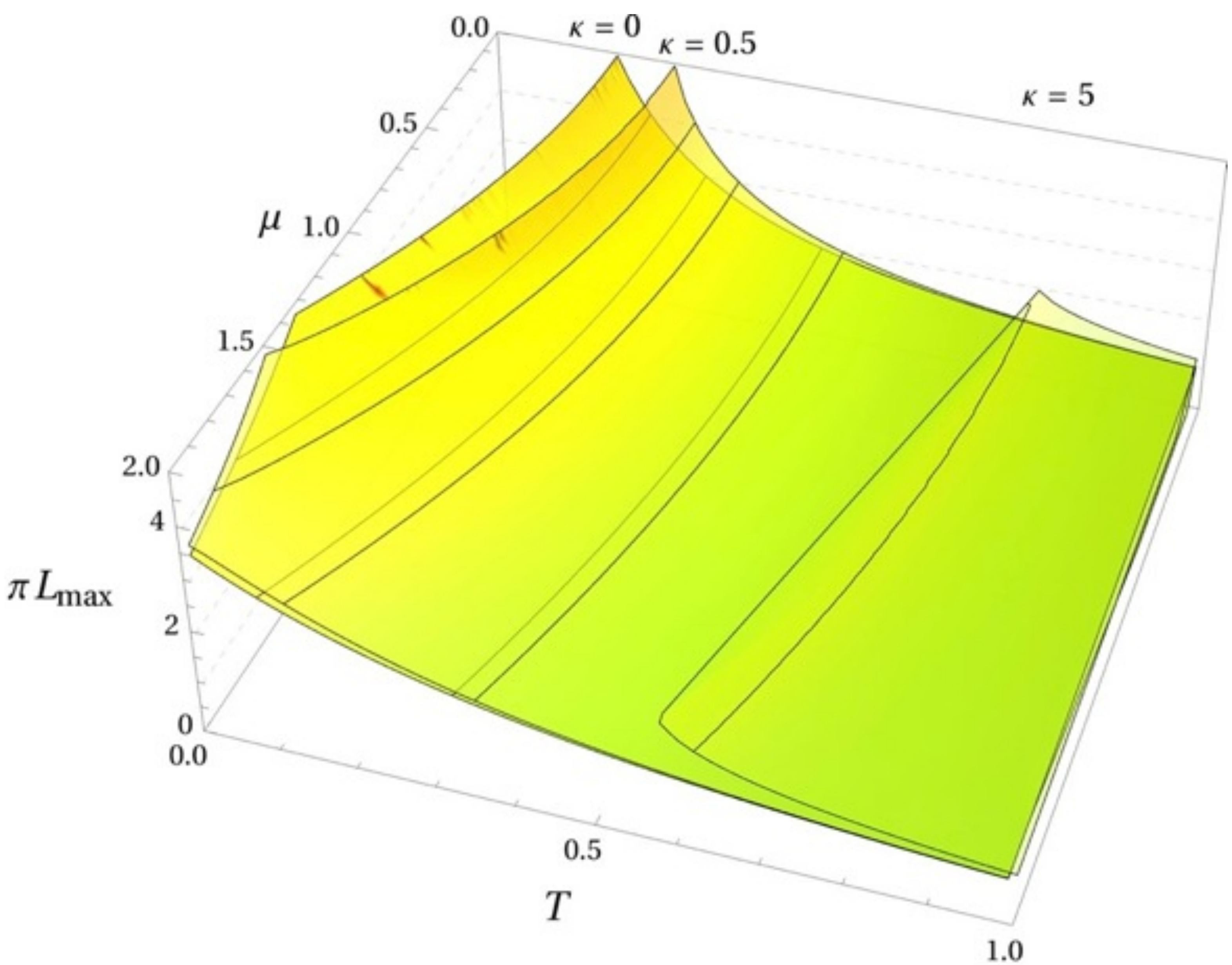
A. Samberg, CE

$$A(z) = \log\left(\frac{L}{z}\right) \qquad \phi(z) = \sqrt{\frac{3}{2}}\, \kappa z^2$$

and choice $f(\phi) = \cosh(12/5)/\cosh(6(\phi-2)/5)$

- solves 5d gravity action, consistent deformation with scale κ
- evades problems at small T, μ

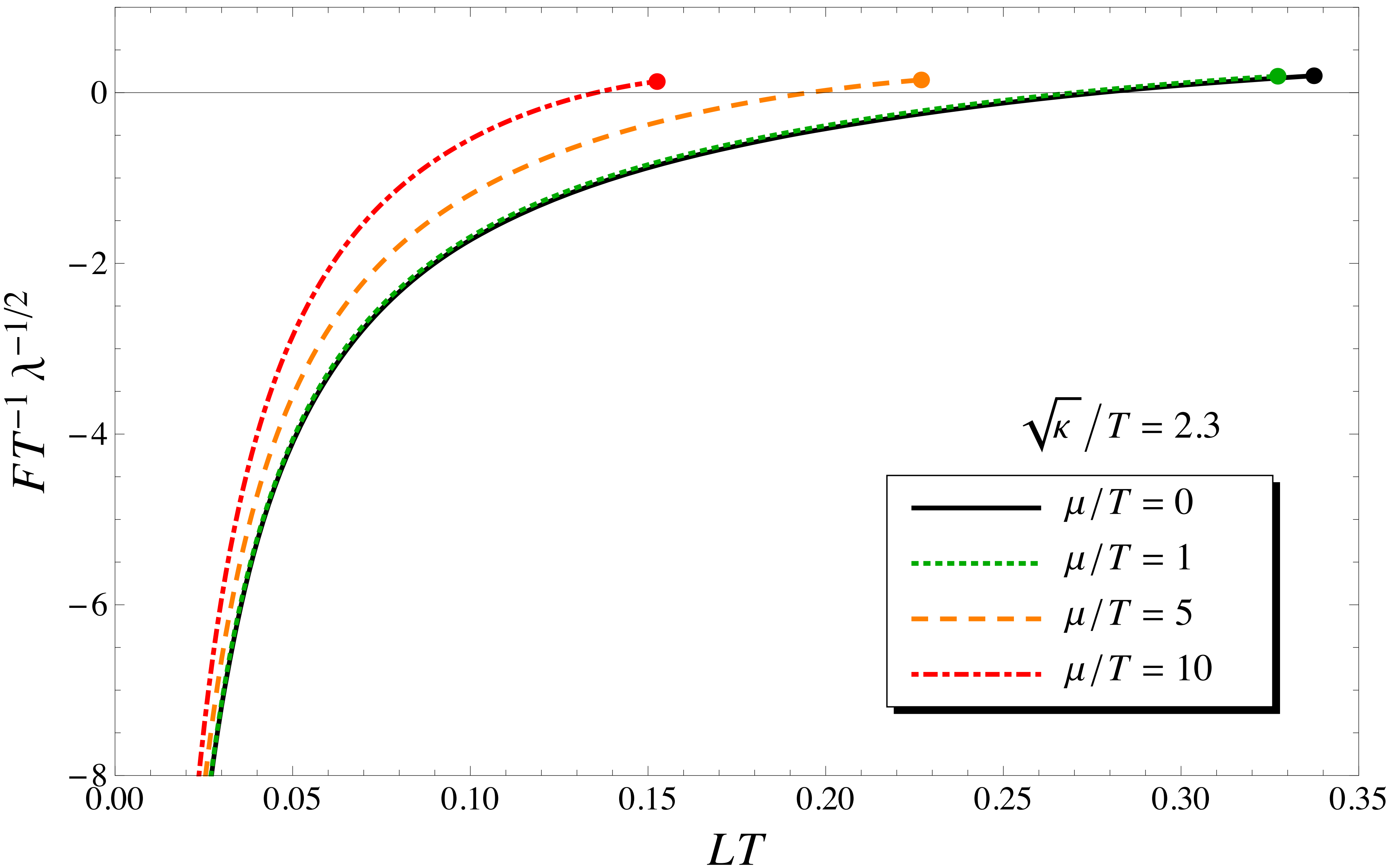
Screening Distance at Finite T, μ



Andreas Samberg, CE

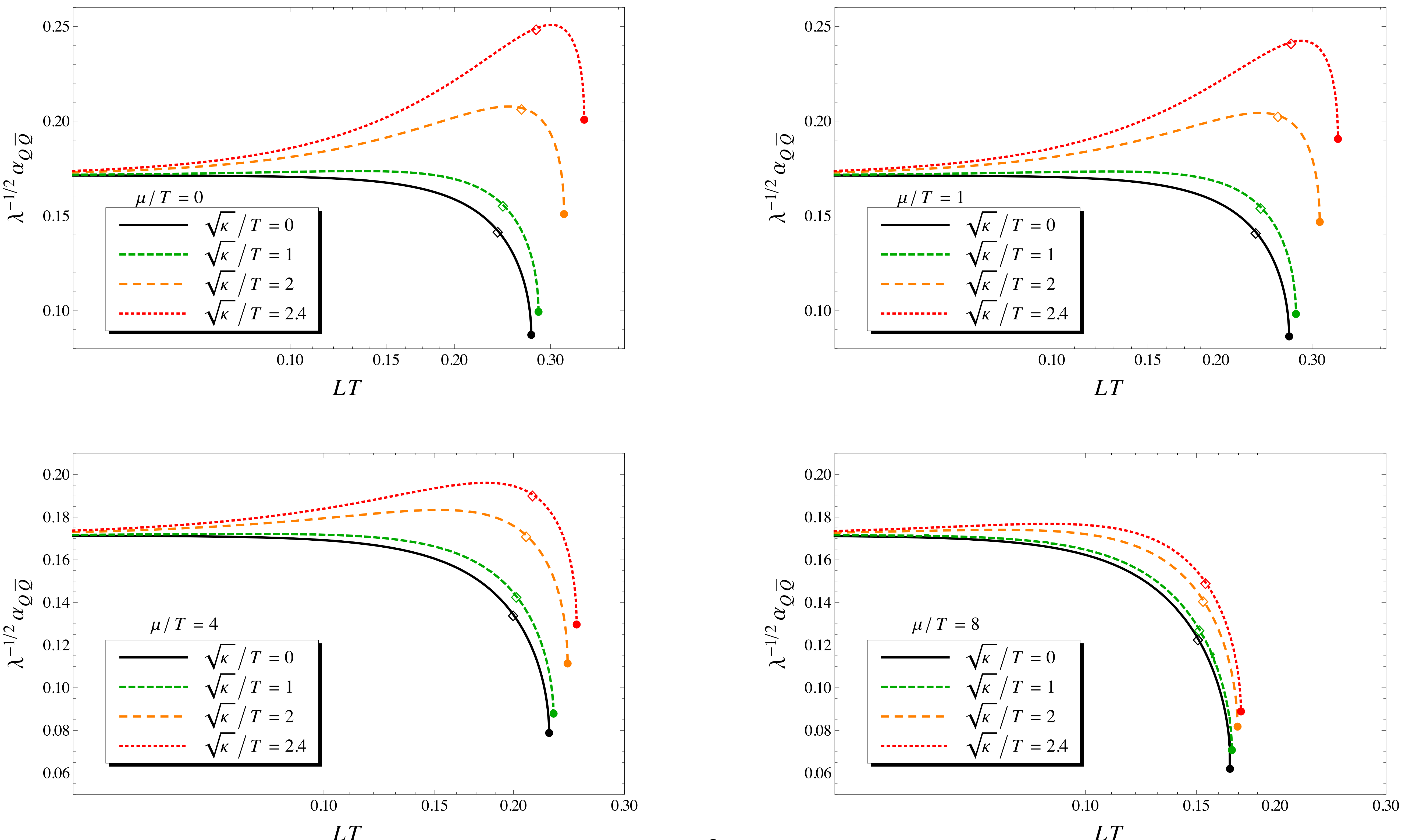
- at finite μ and velocity: screening distance in N=4 no longer lower bound
- but deviations due to finite μ small

Binding Energy at Finite T, μ



- increasing μ decreases binding strength

Running Coupling α_{qq} at Finite T, μ



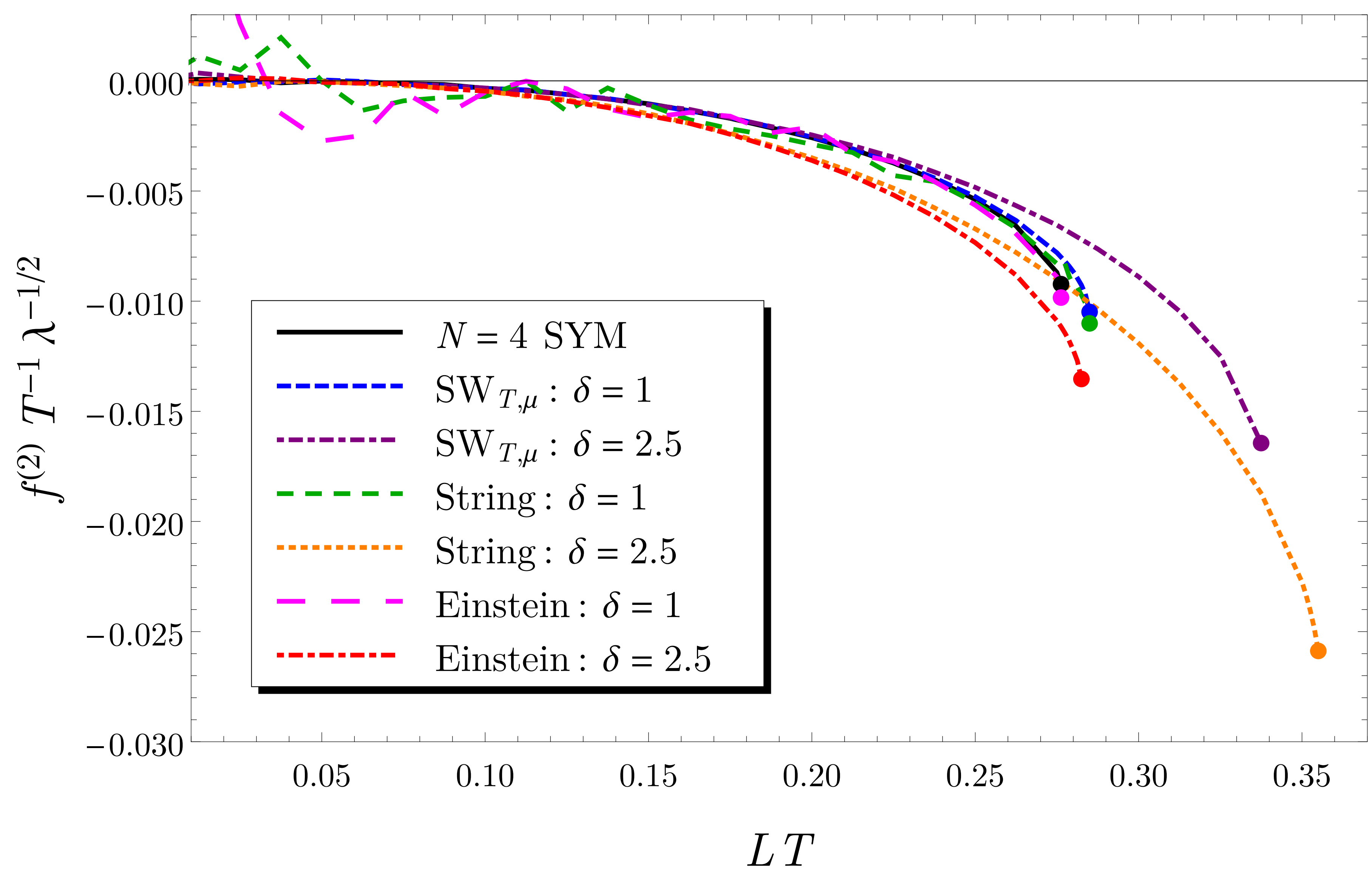
$$\alpha_{Q\bar{Q}} = \frac{3L^2}{4}\frac{dF(L,T,\mu)}{dL}$$

A. Samberg, CE

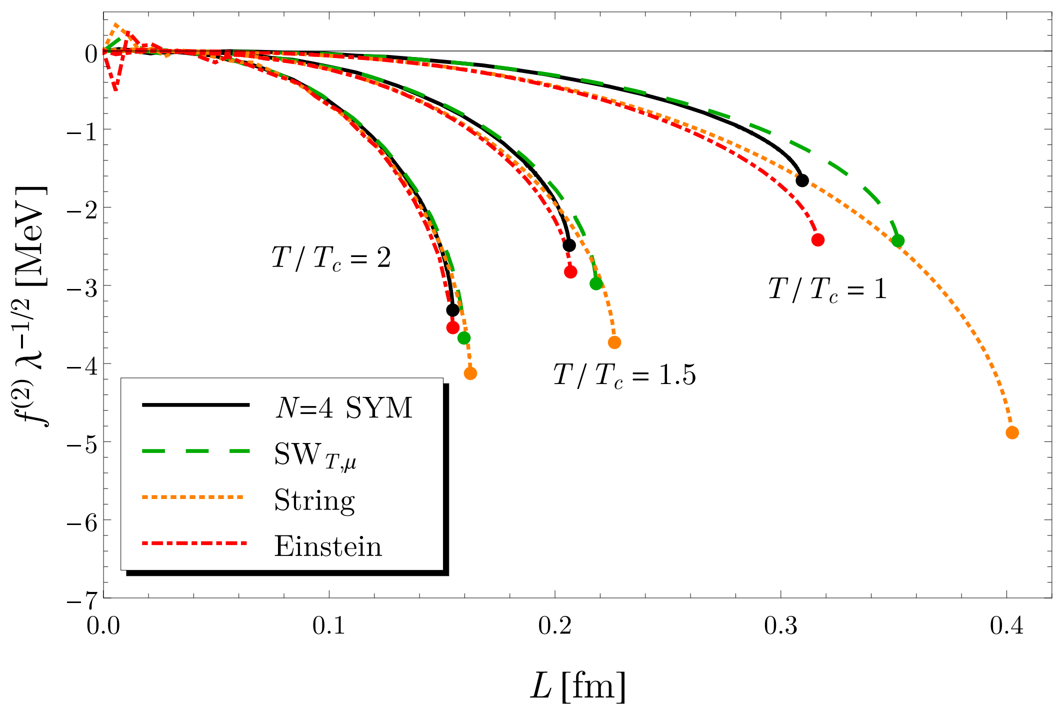
Taylor Coefficients for α_{qq} in Expansion in μ

$$F_{Q\bar{Q}}(L;T,\mu) = \sum_{n=0}^{\infty} f^{(n)}(L;T)\left(\frac{\mu}{T}\right)^n$$

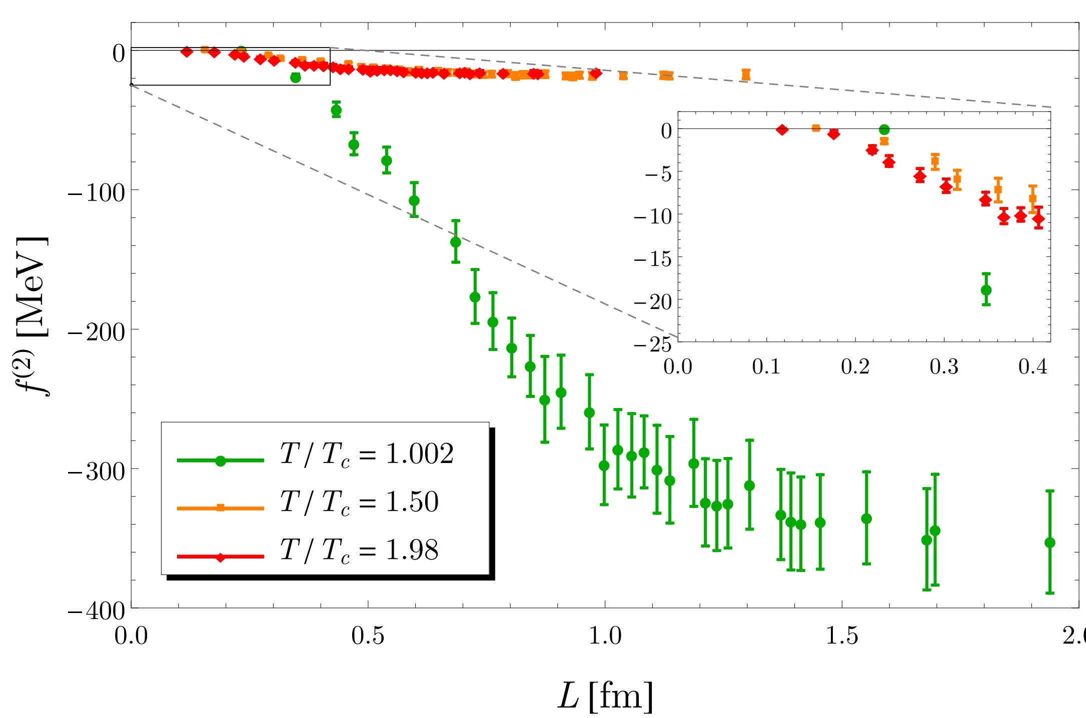
coefficient of order $(\mu/T)^2$:



Taylor Coefficients for α_{qq} in Expansion in μ



(a) $f^{(2)}/\sqrt{\lambda}$ in $\mathcal{N} = 4$ SYM and our three non-conformal models. See the text for an explanation of the scale T_c used here. The noisy behavior for small L is a numerical artifact. Note the smaller range in L as compared to the lattice data (lower panel).



(b) $f^{(2)}$ in 2-flavor lattice QCD [226]. We have chosen from the data of [226] the three temperatures closest to the ones used in the holographic models (upper panel), and converted the data to physical units by using $\sqrt{\sigma} = 420\,\mathrm{MeV}$ for the string tension.

Summary

- Study of various dynamical quantities in hot plasmas via non-conformal deformations of AdS_5 solving Einstein-Hilbert-scalar action
- Screening distance conjecture:
  L_s is bounded from below by its value in N=4 SYM
- first actual calculation of free and internal energy of QQbar pair
- Several observables studied at finite chemical potential with consistent metrics
  - typically T has stronger effect than μ
  - screening distance conjecture violated at finite μ

Thanks for your interest!