Asymptotically free nonabelian Higgs models

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& Luca Zambelli, PRD 92, 025016 (2015) [arXiv:1502.05907], arXiv:1605.XXXXX

Prologue:

& M.M. Scherer, S. Rechenberger, L. Zambelli,
EPJC 73, 2652 (2013)[arXiv:1306.6508], …

DELTA meeting, Heidelberg, April 2016

“If you got a problem, …

…just put a scalar field”

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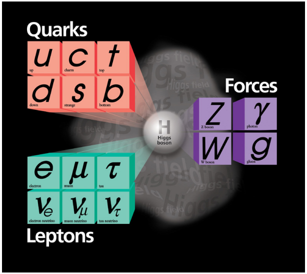
…just put a scalar field"

…, massive gauge bosons, symmetry breaking, CP problem…

…inflation, dark matter, dark energy, …

Standard model Higgs sector

Extremely successful:



Price to be paid:

- naturalness?

$$\delta m^2 \sim \Lambda^2$$

- # of parameters?
- UV completion?

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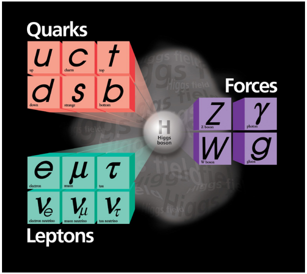

accepted currencies:

- SUSY
- technicolor
- extra-dim's
- ...

Price to be paid:

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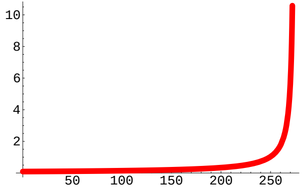
UV completion: triviality problem

▷ $\lambda\phi^4$ theory: perturbation theory predicts its own failure (LANDAU'55)

(GELL-MANN, LOW'54)



$$\frac{1}{\lambda_R} - \frac{1}{\lambda_\Lambda} = b_0 \ \ln \frac{\Lambda}{m_R}, \quad b_0 = \frac{3}{16\pi^2}$$

▷ λ_R and m_R fixed:

$$\Longrightarrow \quad \Lambda_L \quad \simeq \quad m_R \ \exp\left(\frac{1}{b_0 \lambda_R}\right) \quad \gg \quad M_{\text{Planck}}$$

Landau pole singularity

▷ $\lim \ (\Lambda/m_R) \to \infty$: $\Longrightarrow \quad \lambda_R \to 0$

Triviality

▷ lattice evidence:

(LUESCHER,WEISZ'88; HASENFRATZ,JANSEN,LANG,NEUHAUS,YONEYAMA'87; WOLFF'11; BUIVIDOVICH'11; WEISZ,WOLFF'12, …)

Triviality in nonabelian Higgs models?

▷ SU(N) action:

$$S = \int d^4x \left[\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + (D^\mu \phi)^\dagger (D_\mu \phi) + \frac{1}{2} m^2 \phi^\dagger \phi + \frac{\lambda}{8} (\phi^\dagger \phi)^2 \right]$$

$$D^{ab}_\nu = \partial_\nu \delta^{ab} - i\, g\, W^i_\nu (T^i)^{ab}$$

▷ (too) naive argument:

- gauge coupling g is asymptotically free

$\Longrightarrow$ UV theory may reduce to pure scalar sector

$\Longrightarrow$ scalar triviality in λ

Triviality in nonabelian Higgs models?

▷ closer look at perturbation theory: (E.G., GROSS, WILCZEK'73)

$$\begin{aligned} \partial_t g^2 = \beta_{g^2} &= -b_0 g^4 \\ \partial_t \lambda = \beta_\lambda &= A\lambda^2 - B'\lambda g^2 + C g^4 \end{aligned}$$

(mass-independent reg'scheme, deep Euclidean region)

▷ e.g., SU(2):

$$b_0 = \frac{43}{48\pi^2}, \quad A = \frac{3}{4\pi^2}, \quad B' = \frac{9}{16\pi^2}, \quad C = \frac{9}{64\pi^2}$$

▷ analytic solution:

$$\lambda(g^2) = -\frac{g^2}{2A}\left[B + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{2b_0}\ln\frac{g_\Lambda^2}{g^2}\right)\right]$$

$$B = b_0 - B' \quad , \quad \Delta = B^2 - 4AC$$

Triviality in nonabelian Higgs models?

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$\Longrightarrow$ "total" asymptotic freedom

$$\lambda \sim g^2, \quad \text{if } \Delta > 0$$

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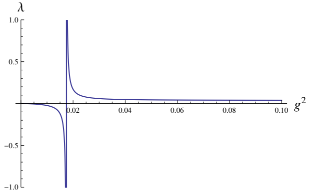
BUT: $\Delta < 0$ for all SU($N \geq 2$):

$\Longrightarrow$ Landau pole



▷ lattice studies: (LANG ET AL.'81; KUHNELT ET AL.'83; JERSAK ET AL. 85; MAAS'13'15)

“Total” asymptotic freedom

$$B = b_0 - B' \quad , \quad \Delta = B^2 - 4AC$$

▷ Δ can be made positive by adjusting N and adding fermions

(Gross,Wilczek'73; Chang'74; Fradkin,Kalashnikov'75; Salam,Strathdee'78)

(Callaway'88; Giudice et al.'14; Holdom,Ren,Zhang'14)

▷ good news:

gauge-Higgs locking $\lambda \sim g^2$ implies $m_{\mathrm{H}}^2 \sim m_{W/Z}^2$

…“reduction of couplings” (Zimmermann'84)

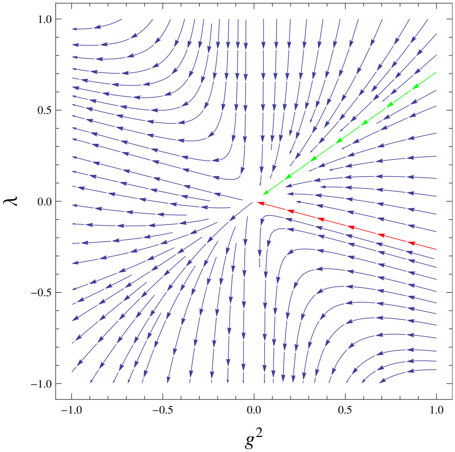
▷ bad news:

- residual symmetry generically too large: $\geq$ SU(2)
- many possibilities but no ordering principle

Asymptotically free Higgs sectors for $\Delta > 0$

▷ gauge-Higgs locking: $\lambda \sim g^2$

▷ phase diagram in (λ, g^2) plane



Asymptotically free Higgs sectors for $\Delta > 0$

▷ flow of gauge-rescaled coupling:

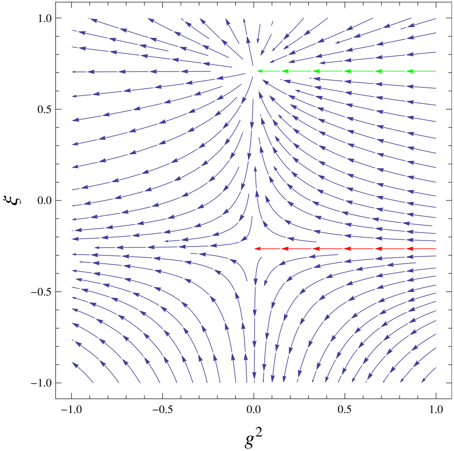
$$\xi := \frac{\lambda}{g^2}, \qquad \partial_t \xi = \beta_\xi = g^2 \left[A\xi^2 + B\xi + C \right]$$

▷ phase diagram in (ξ, g^2) plane

▷ quasi fixed points:

$$\beta_\xi = 0 \quad \text{at} \;\; g^2 > 0$$

$$\Longrightarrow \; \xi_+^*, \xi_-^*$$



Things to ponder

- general relevance/meaning of quasi fixed points

$\beta_\xi = 0$ at finite g^2

- different viewpoint: field rescalings, e.g.

$$\lambda\,\phi^4 = \xi\,\left(\frac{\phi}{\sqrt{g}}\right)^4$$

$\Longrightarrow$ for $\xi \to \xi^*$ and $g \to 0$: large amplitude fluctuations expected

…higher dimensional operators?

- life beyond the deep Euclidean region?

…quasi-fixed point potentials with a minimum?
…quasi-conformal vev?

Effective field theory

▷ inclusion of higher-dimensional operators, e.g. potential:

$$U(\phi) = \frac{1}{2} m^2 (\phi^\dagger \phi) + \frac{1}{8} \lambda (\phi^\dagger \phi)^2 + \frac{1}{48} \frac{\lambda_3}{\Lambda_{\text{eff}}^2} (\phi^\dagger \phi)^3 + \ldots$$

▷ Λ_{eff}: "UV" scale of effective field theory

Effective field theory

- inclusion of higher-dimensional operators, e.g. potential:

$$U(\phi) = \frac{1}{2}m^2(\phi^\dagger\phi) + \frac{1}{8}\lambda(\phi^\dagger\phi)^2 + \frac{1}{48}\frac{\lambda_3}{k^2}(\phi^\dagger\phi)^3 + \dots$$

- k: sliding scale, UV behavior visible for $k \to \infty$

Effective field theory

- inclusion of higher-dimensional operators, e.g. potential:

$$U(\phi) = \sum_{n=1}^{N_\mathrm{p}} \frac{\lambda_{2n}}{n!\,k^{2(n-2)}} \left(\frac{\phi^\dagger \phi}{2}\right)^n$$

- with $N_\mathrm{p} \to \infty$

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▷ with $N_\mathrm{p} \to \infty$

▷ conventional studies: λ_1 and v can be ignored in deep Euclidean region

"asymptotic symmetry"

(Lee,Weisberger'74)

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▷ structure of the RG flow:

$$\partial_t \lambda_n = \beta_{\lambda_n}(g^2, \lambda_1, \lambda_2, \ldots, \lambda_{n+1})$$

$\Longrightarrow$ infinite tower of coupled ODE's

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▷ infinite tower of coupled ODE's

$$\begin{aligned} \partial_t \lambda_2 &= \beta_{\lambda_2}(g^2, \lambda_1, \lambda_2, \lambda_3) \\ \partial_t \lambda_3 &= \beta_{\lambda_2}(g^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) \\ \vdots & \qquad\qquad \vdots \end{aligned}$$

Effective field theory

- structure of the RG flow:

$$\partial_t \lambda_n = \beta_{\lambda_n}(g^2, \lambda_1, \lambda_2, \ldots, \lambda_{n+1})$$

- e.g., truncating at $N_\mathrm{p} = 2$ (in deep Euclidean region):

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BUT: since $\partial_t \lambda_3 \neq 0$, setting $\lambda_3 = 0$ is as good/bad as any other choice:

$$\text{e.g.,} \qquad \lambda_3 = \text{const.} \qquad \text{or} \quad \lambda_3 = f(g^2)$$

Effective field theory

▷ simplest "agnostic" approximation $N_\mathrm{p} = 2$ (deep E):

$$\partial_t \lambda_2 = A\lambda_2^2 - B'\lambda_2 g^2 + Cg^4 - D\lambda_3, \qquad D_{SU(2)} = \frac{1}{4\pi^2}$$

Effective field theory

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$$\partial_t \lambda_2 = A\lambda_2^2 - B'\lambda_2 g^2 + Cg^4 - D\lambda_3, \qquad D_{SU(2)} = \frac{1}{4\pi^2}$$

$\Longrightarrow$ asymptotically free trajectory, if

$$\lambda_3 = \chi\, g^4, \qquad \chi = const.$$

▷ integrated flow:

$$\lambda_2(g^2) = -\frac{g^2}{2A}\left[B + \sqrt{\Delta'}\tanh\left(\frac{\sqrt{\Delta'}}{2b_0}\ln\frac{g_\Lambda^2}{g^2}\right)\right]$$

$$\Delta' = B^2 - 4AC', \qquad C' = C - D\chi$$

$\Longrightarrow$ one-parameter family of asymptotically free trajectories

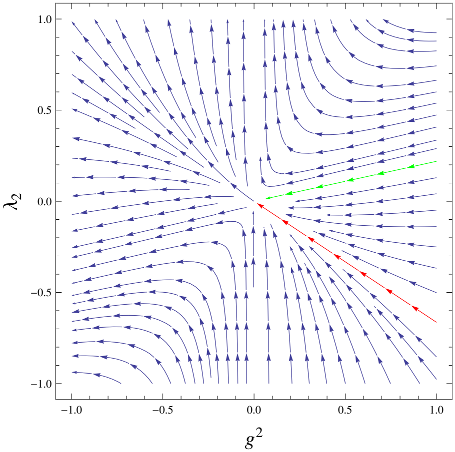
Asymptotically free trajectory: simple approximation

- simplest "agnostic" approximation $N_\mathrm{p} = 2$ (deep E):

$$\partial_t \lambda_2 = A\lambda_2^2 - B'\lambda_2 g^2 + Cg^4 - D\lambda_3, \qquad D_{SU(2)} = \frac{1}{4\pi^2}$$

- phase diagram in (λ_2, g^2) plane

e.g., $\chi = 1$



Asymptotically free trajectory: simple approximation

▷ gauge-rescaled coupling:

$$\xi_2 := \frac{\lambda_2}{g^2}, \qquad \partial_t \xi_2 = \beta_{\xi_2} = g^2\ \left[A\xi_2^2 + B\xi_2 + C - D\chi\right]$$

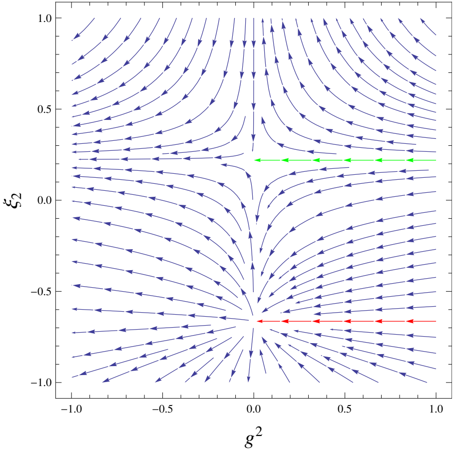
▷ phase diagram in (ξ_2, g^2) plane

▷ quasi fixed points:

$$\beta_\xi = 0 \quad \text{at} \ \ g^2 > 0$$

$\Longrightarrow$ perturbation about ξ_+^* is RG irrelevant

$\Longrightarrow$ trade ξ_2 for χ



Asymptotically free trajectory: higher-orders?

▷ same conclusion for any finite truncation

$$
\begin{aligned}
\partial_t \lambda_2 &= \beta_{\lambda_2}(g^2, \lambda_2, \lambda_3) \\
\partial_t \lambda_3 &= \beta_{\lambda_2}(g^2, \lambda_2, \lambda_3, \lambda_4) \\
\vdots \quad & \qquad\qquad \vdots \\
\partial_t \lambda_n &= \beta_{\lambda_n}(g^2, \lambda_2, \lambda_3, \ldots, \lambda_{n+1})
\end{aligned}
$$

▷ if

$$\lambda_{n+1} = \text{const.} \times g^{n-1}$$

$\Longrightarrow$ one free parameter χ.

Existence of asymptotically free trajectories?

Crucial questions:

- Does the series expansion $\sim \sum \lambda_n (\phi^\dagger \phi)^n$ sum up to a

  global fixed point potential : $U^*(\phi)$?

  $\Longrightarrow$ χ parameter $\rightarrow$ boundary condition for $U^*(\phi)$

- Are perturbations about $U^*(\phi)$ self-similar? (Morris'98)

  ...quantization of RG directions, predictivity

- Classification of perturbations?

  ...# of physical parameters, ...

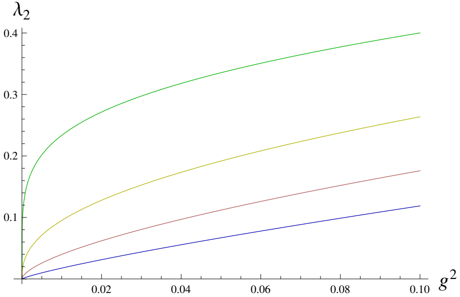
P-scaling solutions

▷ …yet another parameter:

$$\text{general field rescalings} \quad \phi \rightarrow \frac{\phi}{g^P} \quad \rightarrow \xi_n = g^{-2Pn}\lambda_n$$

$\Longrightarrow$ quasi fixed points
for all values of $P > 0$

▷ e.g., $P \in \{0.1, 0.2, 0.3, 0.4\}$,
$\chi = 1$



$\Longrightarrow$ χ and P parametrize boundary conditions for correlation functions

Functional RG analysis

▷ RG flow of the dimensionless scalar potential, $u = k^{-d}U(\phi)$

$$\partial_t u = -du+(d-2+\eta_\phi)\tilde{\rho}u'+2v_d\left\{(d-1)\sum_{i=1}^{N^2-1} l_0^{(\mathrm{G})d}\left(\mu^2_{W,i}(\tilde{\rho})\right)+(2N-1)l_0^{(\mathrm{B})d}\left(u'\right)+l_0^{(\mathrm{B})d}\left(u'+2\tilde{\rho}u''\right)\right\}$$

$$\eta_\phi = \frac{8v_d}{d}\left\{\tilde{\rho}(3u''+2\tilde{\rho}u''')^2 m_{2,2}^{(\mathrm{B})d}(u'+2\tilde{\rho}u'',u'+2\tilde{\rho}u'')+(2N-1)\tilde{\rho}u''^2 m_{2,2}^{(\mathrm{B})d}(u',u')\right.$$
$$\left.\left. -2g^2(d-1)\sum_{a=1}^{N}\sum_{i=1}^{N^2-1} T^i_{\hat{n}a}T^i_{a\hat{n}}\, l_{1,1}^{(\mathrm{BG})d}\left(u',\mu^2_{W,i}\right)+(d-1)\sum_{i=1}^{N^2-1}\frac{\mu^4_{W,i}}{\tilde{\rho}}\left[2a_1^d\left(\mu^2_{W,i}\right)+m_2^{(\mathrm{G})d}\left(\mu^2_{W,i}\right)\right]\right\}\right|_{\tilde{\rho}=\tilde{\rho}_{\min}}$$

▷ 2nd order PDE in k and ϕ (Gies,Scherer,Rechenberger,Zambelli'13)

Functional RG analysis

▷ RG flow of the dimensionless scalar potential, $u = k^{-d}U(\phi)$

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\partial_t u &= -du+(d-2+\eta_\phi)\tilde{\rho}u'+2v_d\Big\{(d-1)\textstyle\sum_{i=1}^{N^2-1} l_0^{(\mathrm{G})d}\big(\mu_{W,i}^2(\tilde{\rho})\big)+(2N-1)l_0^{(\mathrm{B})d}\big(u'\big)+l_0^{(\mathrm{B})d}\big(u'+2\tilde{\rho}u''\big)\Big\} \\
\eta_\phi &= \tfrac{8v_d}{d}\Big\{\tilde{\rho}(3u''+2\tilde{\rho}u''')^2 m_{2,2}^{(\mathrm{B})d}(u'+2\tilde{\rho}u'',u'+2\tilde{\rho}u'')+(2N-1)\tilde{\rho}u''^2 m_{2,2}^{(\mathrm{B})d}(u',u') \\
&\quad -2g^2(d-1)\textstyle\sum_{a=1}^{N}\sum_{i=1}^{N^2-1} T^i_{\hat{n}a}T^i_{a\hat{n}}\, l_{1,1}^{(\mathrm{BG})d}\big(u',\mu_{W,i}^2\big)+(d-1)\sum_{i=1}^{N^2-1}\frac{\mu_{W,i}^4}{\tilde{\rho}}\Big[2a_1^d\big(\mu_{W,i}^2\big)+m_2^{(\mathrm{G})d}\big(\mu_{W,i}^2\big)\Big]\Big\}\Big|_{\tilde{\rho}=\tilde{\rho}_{\min}}
\end{aligned}
$$

▷ 2nd order PDE in k and ϕ (Gies,Scherer,Rechenberger,Zambelli'13)

▷ gauge-rescaled variables: (Gies,Zambelli'15)

$$
x = g^{2P}\,\frac{Z_\phi|\phi|^2}{k^2}, \qquad f(x) = u
$$

$$
\begin{aligned}
\partial_t f &= \beta_f \equiv -4f+(2+\eta_\phi-P\eta_W)xf' \\
&\quad +\tfrac{1}{16\pi^2}\Big\{3\textstyle\sum_{i=1}^{N^2-1} l_{0\mathrm{T}}^{(\mathrm{G})4}\big(g^{2(1-P)}\omega_{W,i}^2(x)\big)+(2N-1)l_0^{(\mathrm{B})4}\big(g^{2P}f'\big)+l_0^{(\mathrm{B})4}\big(g^{2P}(f'+2xf'')\big)\Big\}
\end{aligned}
$$

Global fixed point solution

▷ e.g., for $P = 1$ and SU(2): (Gies,Zambelli'15)

$$f^*(x) = \xi x^2 - \left(\frac{3}{16\pi}\right)^2 \left[2x + x^2 \ln\left(\frac{x}{2+x}\right)\right]$$

fixed-point solution $\sim$ Coleman-Weinberg type, one-parameter family ξ

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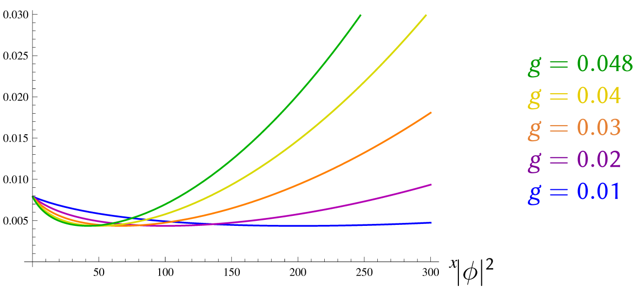
fixed-point solution ∼ Coleman-Weinberg type, one-parameter family ξ

▷ leading-order scaling solution for different P:

$$f(x) = \begin{cases} \xi x^2 - \xi \frac{3}{16\pi^2} g^{2P} x & \text{for } P \in (0, 1/2) \\ \xi x^2 - \frac{3(3+8\xi)}{128\pi^2} g\, x & \text{for } P = 1/2 \\ \xi x^2 - \frac{9}{128\pi^2} g^{2(1-P)} x & \text{for } P \in (1/2, 1) \end{cases} .$$

Asymptotically free UV gauge scaling solutions

▷ gauge scaling towards flatness, $P = 1$ (Rechenberger,Scherer,HG,Zambelli'13; HG,Zambelli'15)



▷ approach to UV $k \to \infty$:

$$g^2 \to 0, \quad |\phi_{\text{min}}|^2 \sim \frac{1}{g^2} \to \infty, \quad \underline{\underline{\lambda \sim g^4 \to 0}}, \quad \frac{m_W^2}{k^2} \to \text{const.}$$

$\Longrightarrow$ deep Euclidean region is sidestepped

…no "asymptotic symmetry"

Asymptotically free perturbations

▷ classification of (ir-)relevant perturbations for given ξ and P:

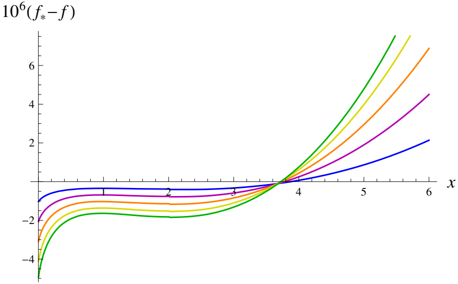
δm^2	relevant	"as usual" → naturalness?
$\delta g^2 \oplus \delta f(x)$	marginally relevant	"natural"

▷ marginal-relevant direction:

$\Longrightarrow$ self-similar
& polynomially bounded

as should be

(Morris'98)

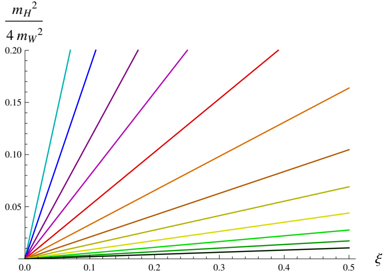


Estimates of IR Observables

▷ Higgs to W boson mass ratio:

$$\frac{m_{\mathrm{H}}^2}{m_W^2} \sim c_P\ \xi$$

▷ UV-IR mapping
of physical parameters:



$$\left.\begin{array}{c} v \\ m_{\mathrm{H}} \\ m_W \end{array}\right\} \quad \Longleftrightarrow \quad \left\{\begin{array}{cl} \delta m^2 & \text{relevant} \\ \delta g^2 \oplus \delta f & \text{marginal-relevant} \\ \xi, P & \text{“exactly marginal”} \end{array}\right.$$

$\Longrightarrow$ pheno-relevant parameter regime is accessible

Conclusions

- Interplay: asymptotic freedom $\longleftrightarrow$ boundary conditions

  b.c.'s for correlation functions

- Non-abelian Higgs models can be asymptotically free and UV complete

  if our choice of b.c.'s is legitimate

- scaling solutions satisfy necessary criteria

  self-similarity, boundedness, predictivity