Far-from-equilibrium universality classes: From heavy-ion collisions to superfluid scalars

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Talk based on:

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J. Berges, KB, S. Schlichting, and R. Venugopalan, ***PRD 92, 096006 (2015); PRL 114, 061601 (2015)***

A. Piñeiro Orioli, KB, and J. Berges, ***PRD 92, 025041 (2015)***

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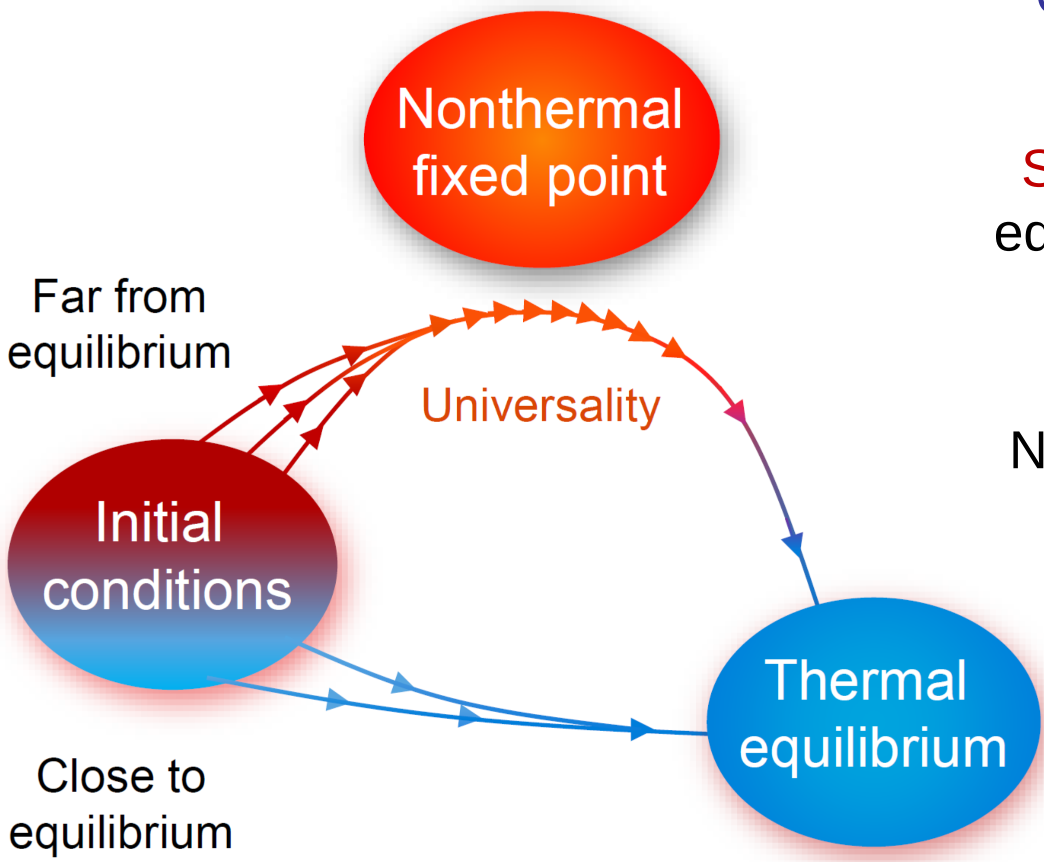
Introduction: Nonthermal fixed points



General picture for weakly coupled field theories

Strong correlations: Far-from-equilibrium initial conditions (IC)

⇓

Nonthermal fixed point (NTFP)

- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

$$\boxed{f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)}$$

Introduction: Nonthermal fixed points

Scaling region (of a nonthermal fixed point)

$$\boxed{f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)}$$

Self-similar evolution of distribution function f

with scaling behavior of typical scales $p_{\text{typ}} \sim t^{-\beta}\ ,\ f(p_{\text{typ}}) \sim t^{\alpha}$

Classification: universality classes far from equilibrium

FP for

$$f = R_b\,[f]$$

Via scaling exponents α, β and the scaling function $f_S(x)$

Analogy: „RG step“ $t \mapsto b\,t$:

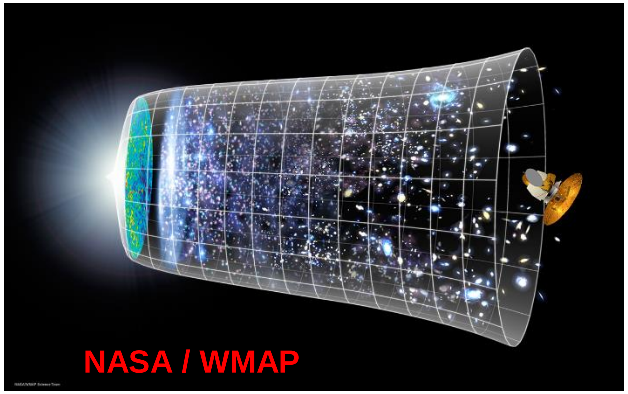
$$f'(p,t) = R_b\,[f(p,t)] = \frac{1}{b^{\alpha}}\, f(b^{-\beta}\,p,\, b\,t)$$

NTFP	Close to 2nd order PT
Time scale τ = t	Temperature scale $\tau = (T-T_c)/T_c$
Self-similar evolution	Critical slowing down, power laws
Scaling exponents & function	Critical exponents & surface

Introduction: Connecting disciplines

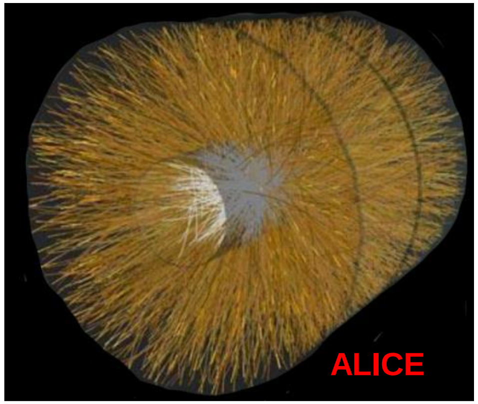
Physics disciplines far from equilibrium

Inflationary cosmology



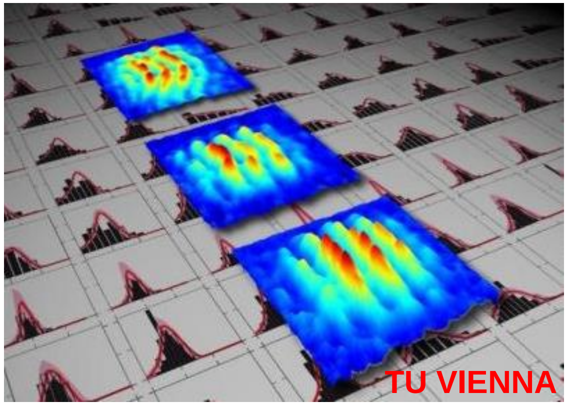
Relativistic O(N) scalars

Heavy-ion collisions at early stages



Longitudinally expanding non-Abelian plasmas

Ultracold atoms



Non-relativistic (Gross-Pitaevski) scalars

Very different field theories and energy scales!

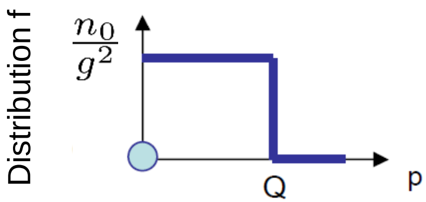
For weak coupling limit: **Universality classes?**

Introduction: Simulation method

Weak couplings but highly correlated system

Typical initial conditions

Over-occupation



$$f(p,t_0) = \frac{n_0}{\lambda}\Theta\left(Q-p\right)$$

Examples:

Micha & Tkachev ; Smit & Tranberg; Nowak, Scholle, Sexty & Gasenzer ; Berges, KB, Schlichting & Venugopalan; Kurkela & Moore; …

Classical-statistical simulations

- Initial fields distributed with $W[\varphi_0, \pi_0]$
- Weak coupling limit $\lambda \to 0$ while $\lambda f = const$
- Solve equations on the lattice, e.g. $\Box\, \varphi_{\mathrm{cl}} + \frac{\lambda}{6}\varphi_{\mathrm{cl}}^3 = 0$
- Observables averaged over (quantum) IC

$$\langle O[\varphi,\pi]\rangle_{\mathrm{cl}} \approx \frac{1}{N_{\mathrm{samp}}} \sum_{(\varphi_0,\pi_0)} O[\varphi^{\mathrm{cl}}(\varphi_0,\pi_0), \pi^{\mathrm{cl}}(\varphi_0,\pi_0)]$$

Classicality condition: $f(p,t) = \sqrt{\langle\varphi\varphi\rangle\langle\partial_t\varphi\,\partial_t\varphi\rangle} \gg 1$

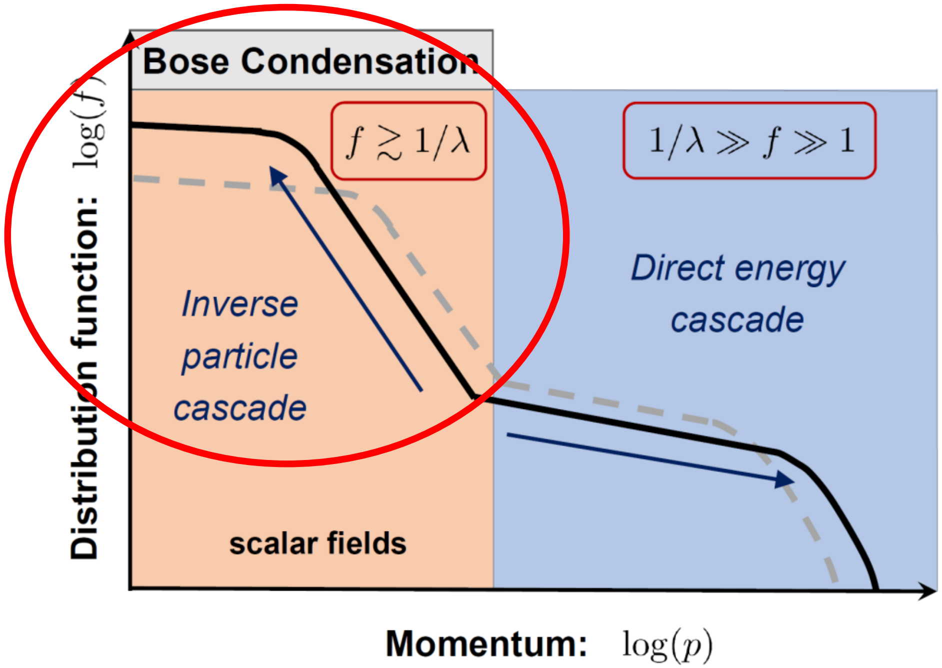
Universality class: scalars in the IR

Massless relativistic scalar

O(N) – symmetric, $\lambda\varphi^4$ interaction

Non-relativistic scalars

(Gross-Pitaevskii)



A. Piñeiro Orioli, KB, and J. Berges,

PRD 92, 025041 (2015)

Universality class: scalars in the IR

Self-similar evolution

$$f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)$$

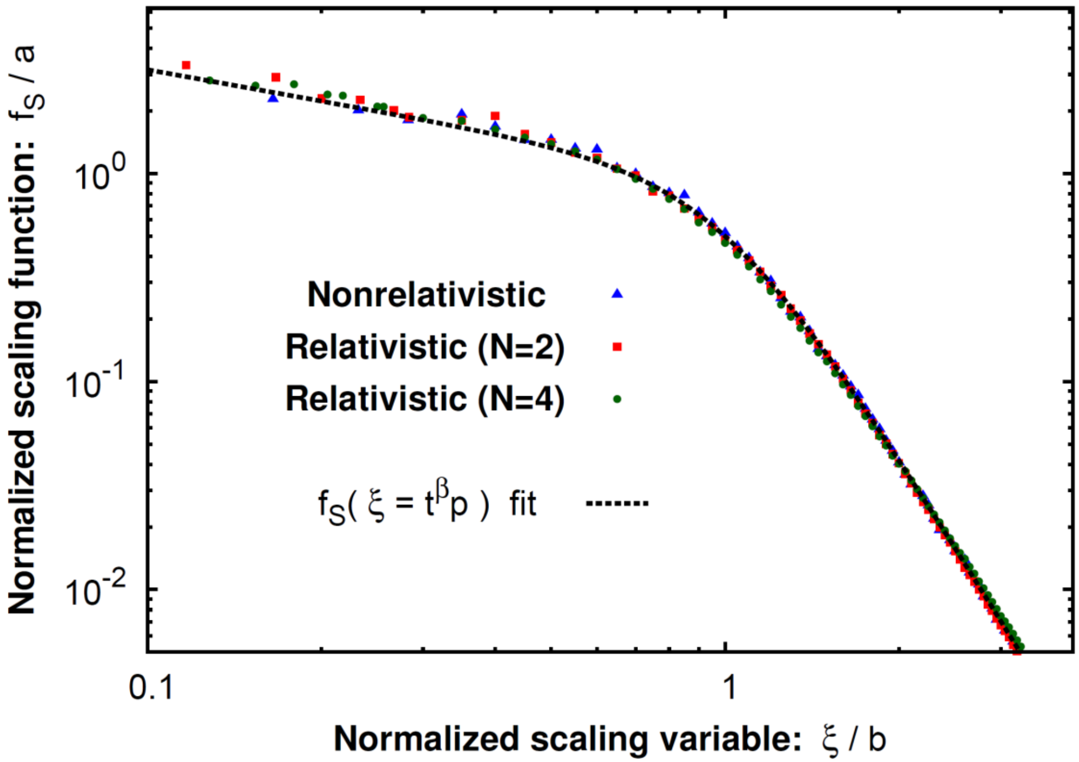
Universal scaling function

Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

Same functional form

$$\lambda f_S = \frac{a}{(|\mathbf{p}|/b)^{\kappa_<} + (|\mathbf{p}|/b)^{\kappa_>}}$$

with $\kappa_< \simeq 0.5\,,\ \kappa_> \simeq 4.5$



Universality class for:

- non-relativistic Bose gases
- O(N) symmetric scalars

Universality class: scalars in the IR

Self-similar evolution

$$f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)$$

Scaling exponents

$$\boxed{\alpha = 3/2\,, \;\; \beta = 1/2}$$

Particle number conservation

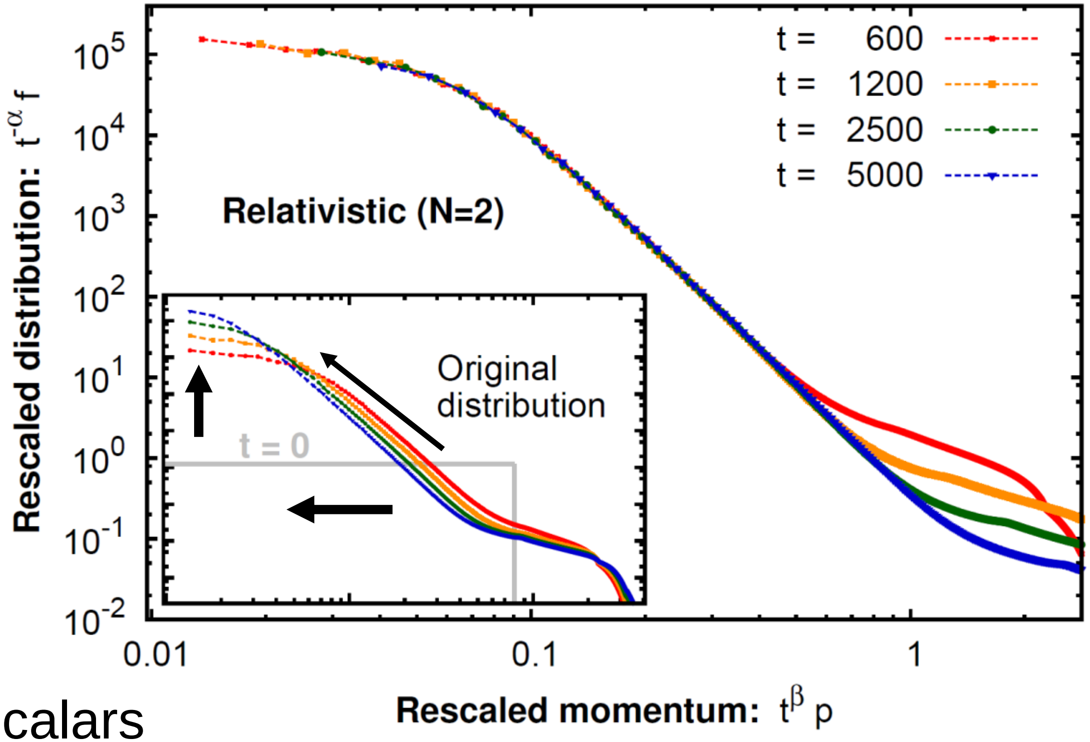
$$n \sim \int d^3p f = const$$

For non-relativistic and Relativistic O(N) symmetric scalars

Self-similar evolution

Pinerio Orioli, KB & Berges, *PRD 92, 025041 (2015)*

Inverse particle cascade to infrared



Universality class: scalars in the IR

Vertex-resummed kinetic theory

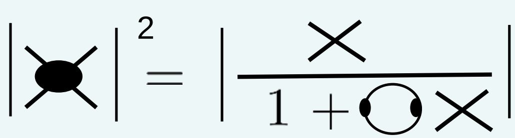
Kinetic theory $\boxed{\dfrac{\partial f(t;\mathbf{p})}{\partial t} = C[f](t;\mathbf{p})}$ often used to explain NTFPs

Micha & Tkachev,
PRD 70, 043538 (2004)

Elastic collision integral:

$$\boxed{C^{2\leftrightarrow 2}(\mathbf{p}) \propto \int \mathrm{d}\Omega^{2\leftrightarrow 2}\, |\lambda^{2\leftrightarrow 2}_{\text{eff}}[f]|^2 \;\; [(1+f_p)(1+f_l)f_q f_r - f_p f_l (1+f_q)(1+f_r)] + \cdots}$$

At low momenta high occupancies → vertex resummations needed

$\lambda^2_{\text{eff}}[f](p) \sim$  $= \dfrac{\lambda^2}{|1+\lambda\Pi^R(p)|^2}$

Berges & Sexty,
PRD 83, 085004 (2011)

Explains scaling exponents correctly!

Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

Universality class: scalars and non-Abelian plasmas

Massless scalar field theory (O(N))

$$S = \int d\tau d^2x_T d\eta \; \tau \left(\frac{g^{\mu\nu}}{2} (\partial_\mu \varphi_a)(\partial_\nu \varphi_a) - \frac{\lambda}{4!N} (\varphi_a \varphi_a)^2 \right)$$

Non-Abelian gauge theory (SU(2))

$$S = \int d\tau d^2x_T d\eta \; \tau \; F^a_{\mu\nu} F^{a,\mu\nu}$$

Bjorken coordinates: $\tau = \sqrt{t^2 - (x^3)^2}\,, \quad \eta = \mathrm{artanh}\left(\frac{x^3}{t}\right)$

Metric in Bjorken coordinates:
longitudinal expansion (in beam direction)

$$g_{\mu\nu}(\tau) = \mathrm{diag}\,(1, -1, -1, -\tau^2)$$

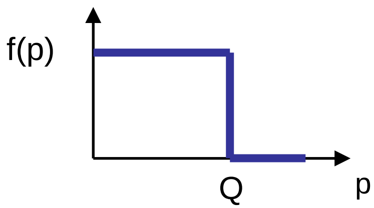
J. Berges, KB, S. Schlichting and R. Venugopalan:
PRL 114, 061601 (2015) ; PRD 92, 096006 (2015)

Universality class: scalars and non-Abelian plasmas

Heavy-ion collisions at early times

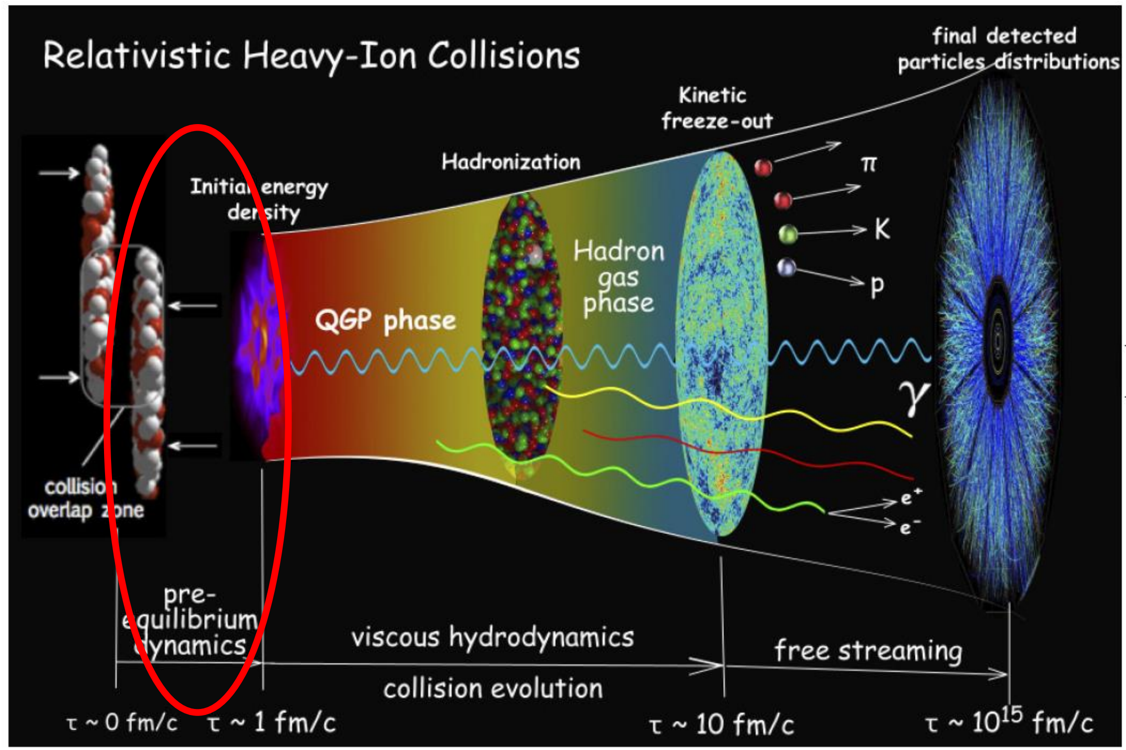
(Ultrarelativistic energies, weak-coupling limit $\alpha_s \ll 1$)

Highly correlated plasma



$$f(p \lesssim Q) \sim 1/\alpha_s$$

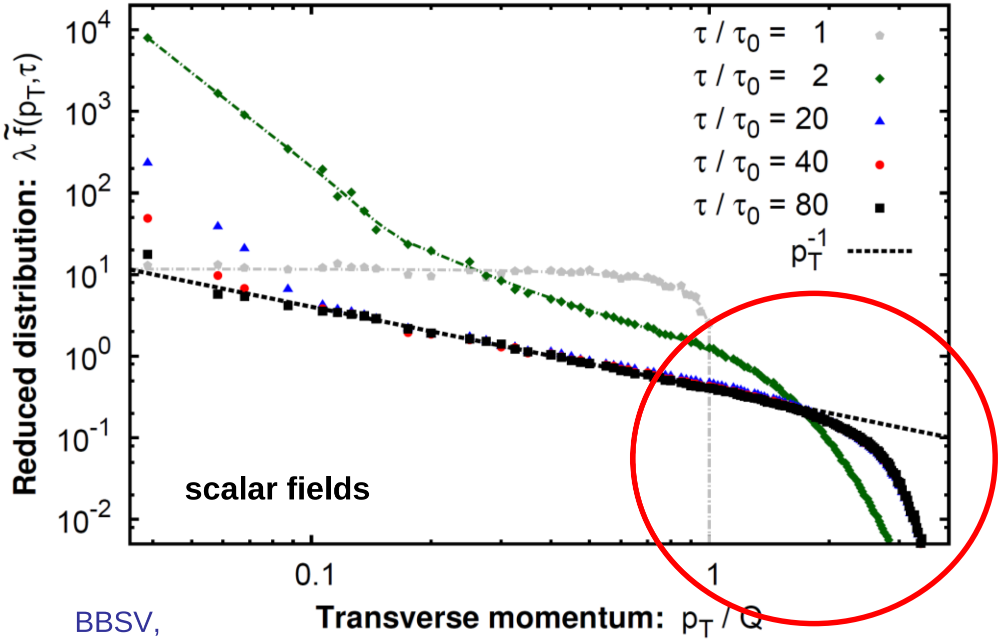
(after initial Glasma fields have decayed)



Little Bang by P. Sorensen and C. Shen

Some literature: *… , Romatschke & Venugopalan (2006); Fukushima & Gelis (2012); Berges & Schlichting (2013)*

Universality class: scalars and non-Abelian plasmas



BBSV,
*PRL **114**, 061601 (2015)*

Reduced distribution

$$\tilde{f}(p_T,\tau) = \frac{\tau}{Q\tau_0}\int \frac{dp_z}{2\pi} f(p_T,p_z,\tau)$$

Becomes *time-independent*

$$\tilde{f}(\tau) \sim \text{const}$$

Self-similar evolution

$$f(p_T,p_z,\tau) = \tau^{\alpha} f_S\left(\tau^{\beta} p_T, \tau^{\gamma} p_z\right)$$

Local conservation of particle number and energy density in transverse momentum

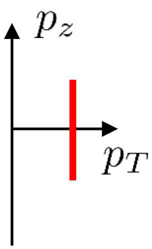
implies $\boxed{\begin{array}{rcr} \alpha-\gamma & \approx & -1 \\ \beta & \approx & 0 \end{array}}$

$$\tau\, dn/dp_T \sim p_T \tilde{f}\;, \quad \tau\, d\epsilon/dp_T \sim p_T^2 \tilde{f} \quad \longleftrightarrow$$

Effectively no flux in p_T!

Universality class: scalars and non-Abelian plasmas

Longitudinal dynamics at $p_T \sim Q/2$



Reminder: *Self-similar evolution*

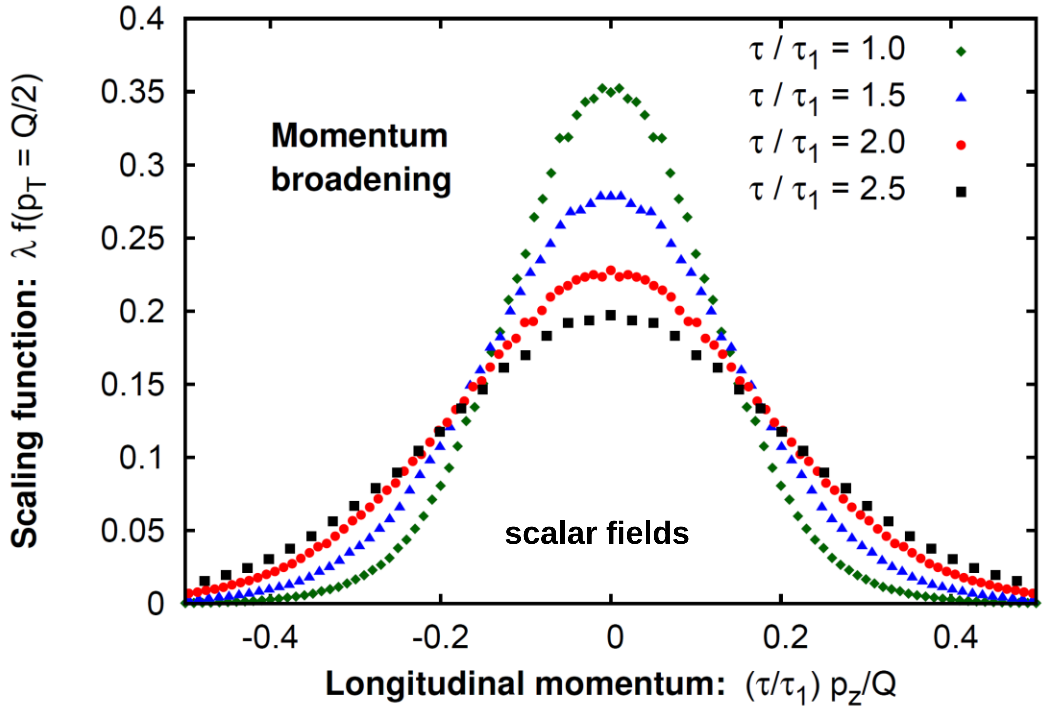
$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Free evolution would imply red shift $p_{z,\mathrm{typ}} \sim \tau^{-1}$

and $\alpha = 0\,, \quad \gamma = 1$

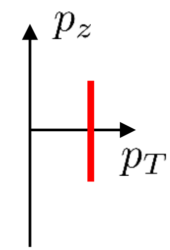
Momentum broadening hints

at non-trivial scattering!



Universality class: scalars and non-Abelian plasmas

Longitudinal dynamics at $p_T \sim Q/2$



Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Dynamical exponents

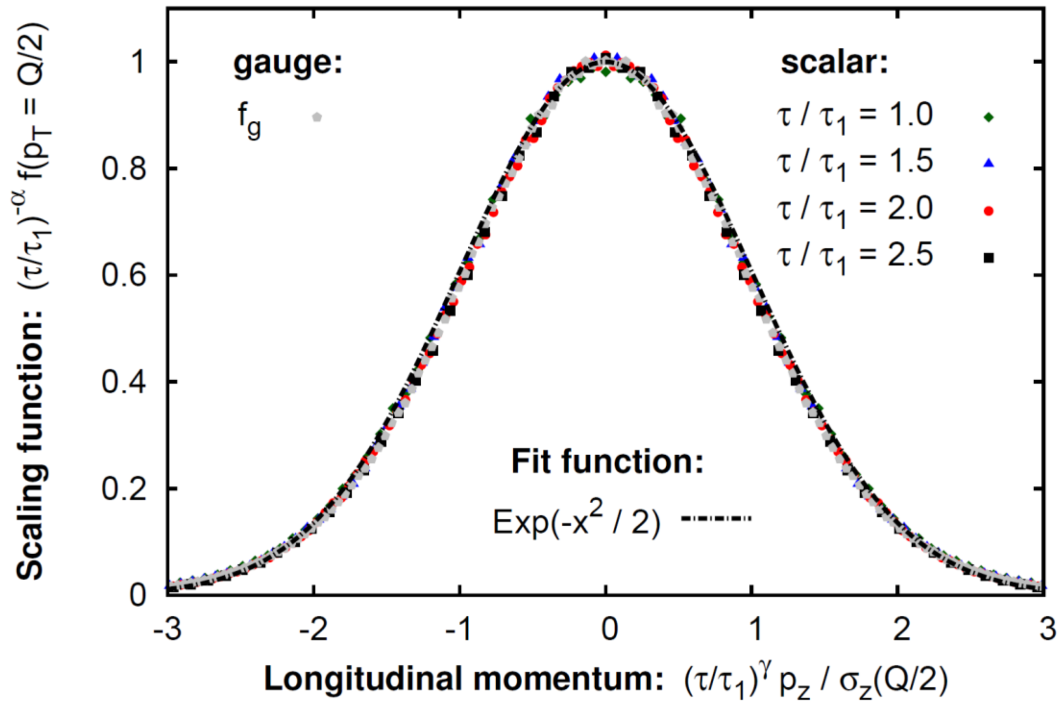
$$\boxed{\begin{aligned} \alpha &= -2/3 \\ \gamma &= 1/3 \end{aligned}}$$

Common functional form

$$f_S(p_z) \sim e^{-p_z^2/2\sigma_z^2}$$

Same exponents and function as in gauge theory!

Common universality class

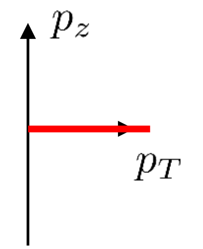


Gauge attractor from elastic scatterings

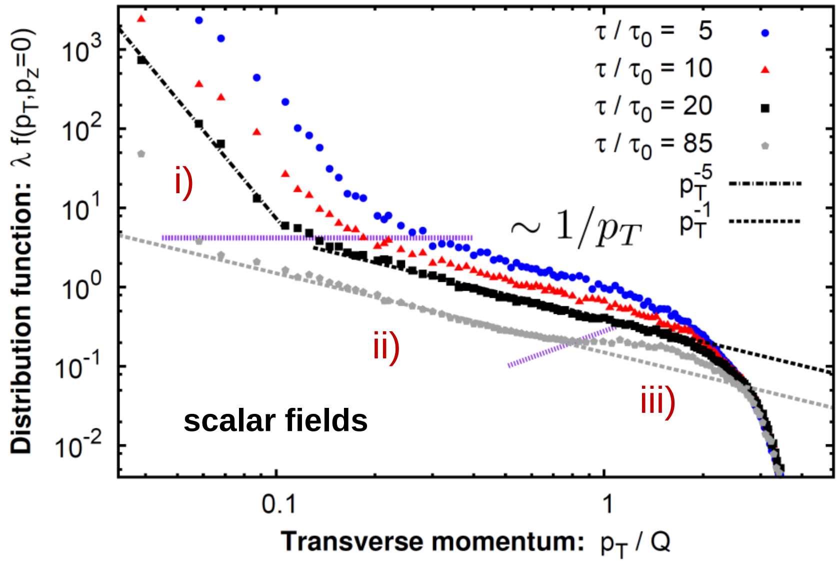
Baier, Mueller, Schiff & Son, PLB 502, 51 (2001)

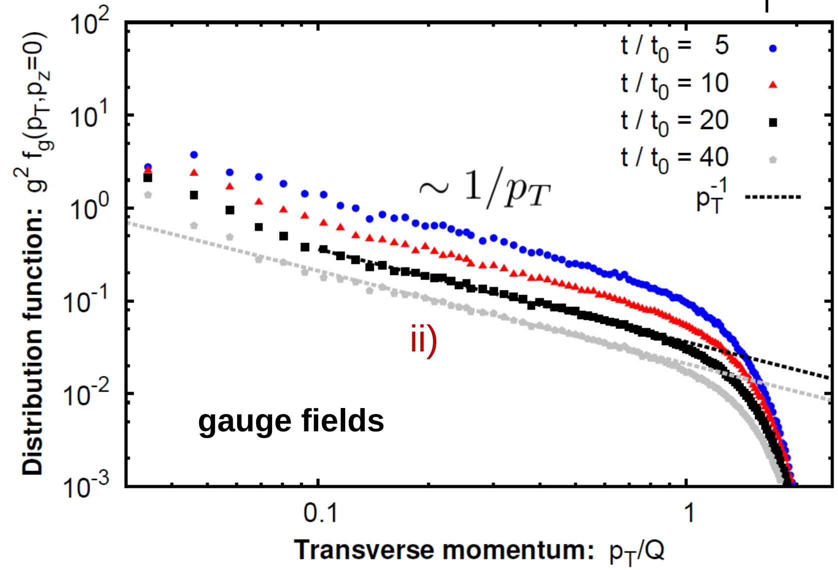
Berges, KB, Schlichting, Venugopalan, *PRD 89, 074011 + 114007 (2014)*

Universality class: scalars and non-Abelian plasmas



Where is common scaling region?



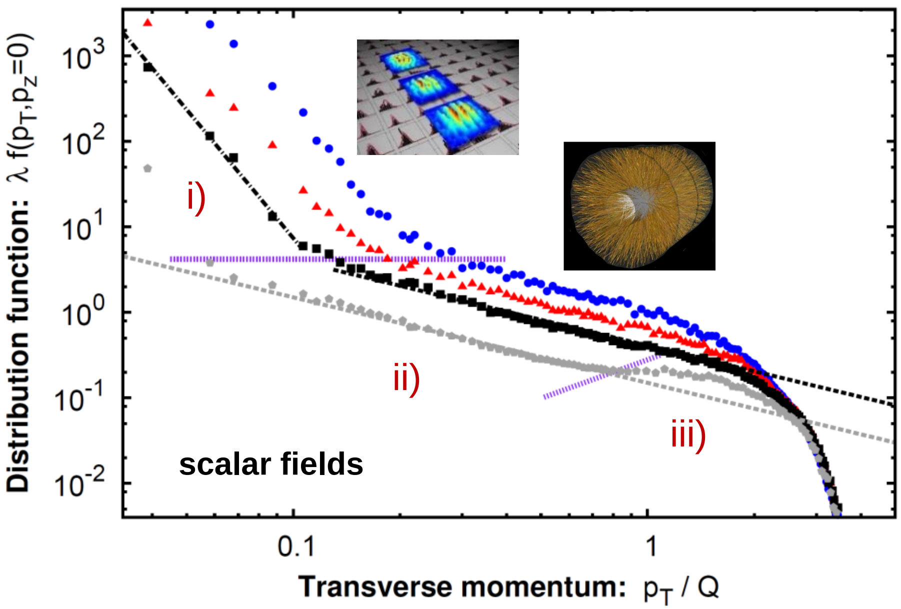


- Scaling range ii) given by $\lambda f \sim \dfrac{\tau^{-2/3}}{p_T} e^{-p_z^2/2\sigma_z^2}$ with $\sigma_z^2 = \dfrac{\int dp_z \, p_z^2 f}{\int dp_z f} \sim \tau^{-2/3}$
- Exponents and form insensitive to initial conditions (memory loss), see

J. Berges, KB, S. Schlichting, and R. Venugopalan: *PRD 89, 074011 + 114007 (2014) ; PRD 92, 096006 (2015)*

Universality class: scalars and non-Abelian plasmas

The entire attractor in longitudinally expanding scalars



Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Berges, KB, Schlichting & Venugopalan, *PRD 92, 096006 (2015)*

Scaling regions	α	β	γ	λf_S
i)	1	$2/3$	$2/3$	$\left((p/b)^{-1/2} + (p/b)^{-5}\right)^{-1}$
ii)	$-2/3$	0	$1/3$	$p_T^{-1}\ e^{-p_z^2/2\sigma_z^2}$
iii)	$-1/2$	0	$1/2$	$\mathrm{sech}(p_z/\sigma_z)$

Universality class: scalars and non-Abelian plasmas

Puzzles

BBSV, *PRD 92, 096006 (2015)*

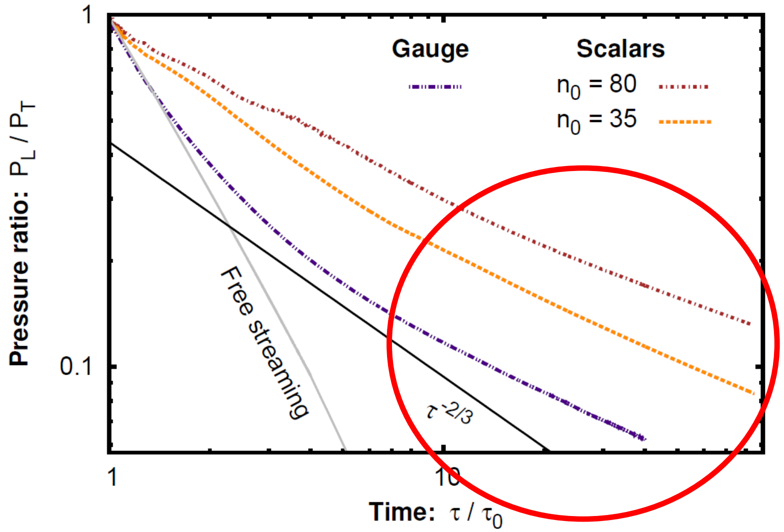
Pressure ratio:

Parametrically:

$$\frac{P_L}{P_T} \overset{\text{kinetic theory}}{\sim} \frac{\int d^3p\, p_z^2/\omega f}{\int d^3p\, p_T^2/\omega f} \overset{\text{late times}}{\sim} (Q\tau)^{-2/3}$$

Discrepancies because of IR?

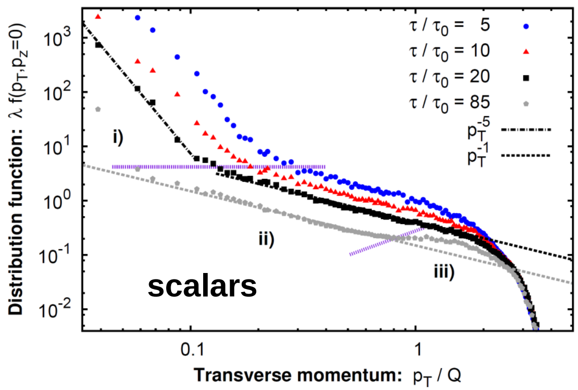
In non-Abelian plasmas nontrivial IR dynamics similar to scalars?

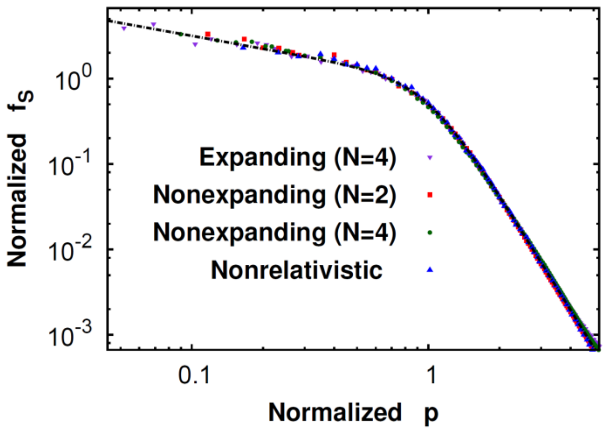


Scalar theory:

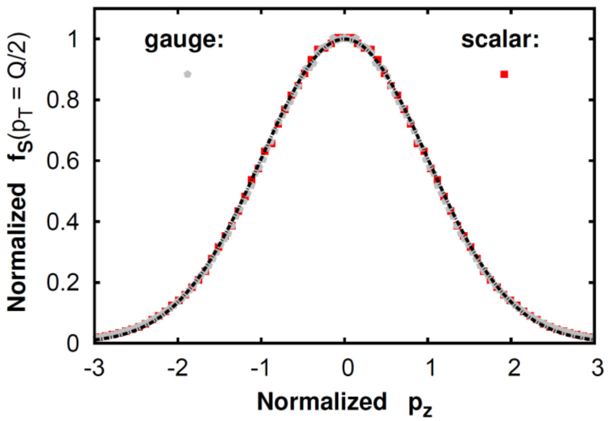
How can regions ii) and iii) be microscopically understood?

How important is soft region for it?





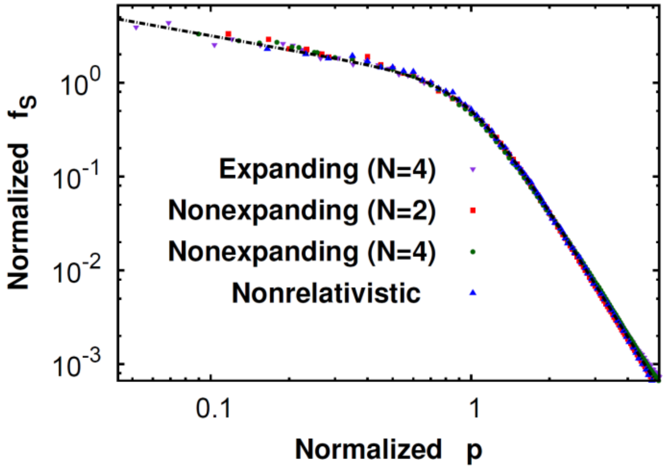
Conclusion

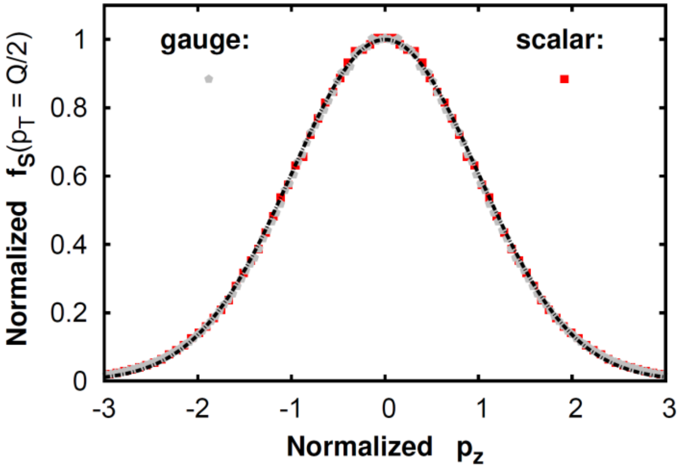


- Highly correlated quantum systems may approach non-thermal fixed-points
- Scalars lie in the same infrared universality class
- Expanding scalar and gauge systems have mutual universality class

Outlook:

- Infrared region should be better understood in gauge theories
- How comes that exp. scalars show same scaling region as gauge theory?
- What pattern classifies far-from-equilibrium universality classes?





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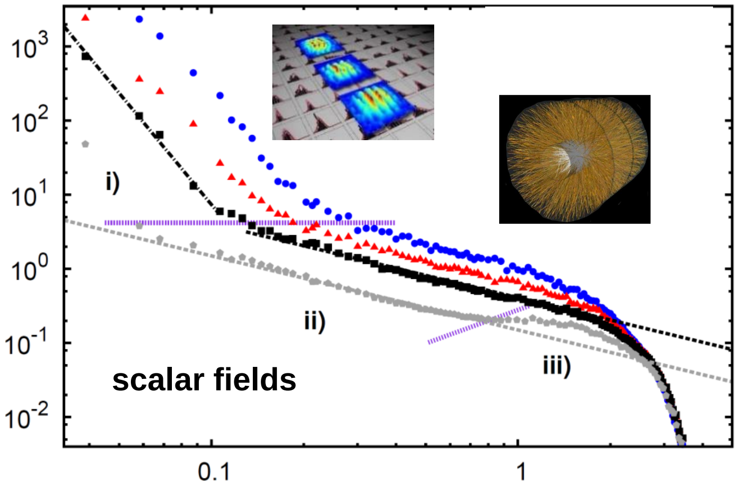


BACKUP SLIDES

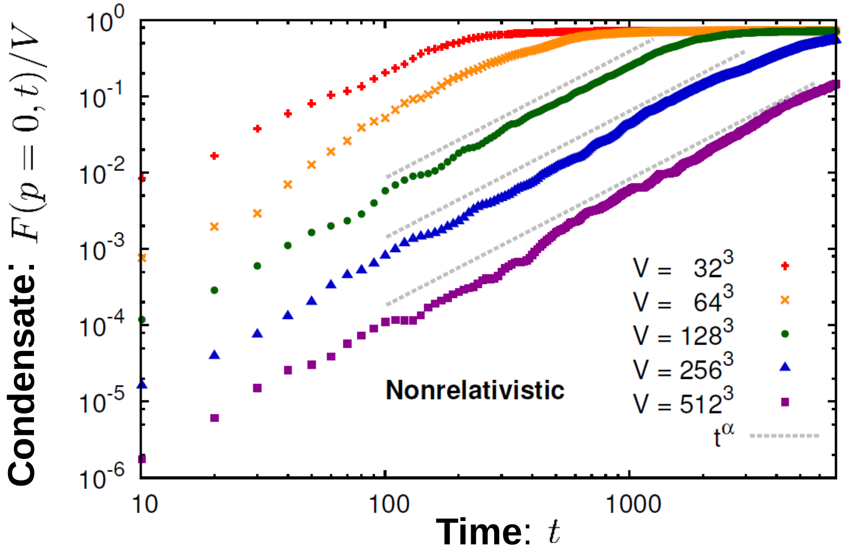
Universality class: scalars in the IR

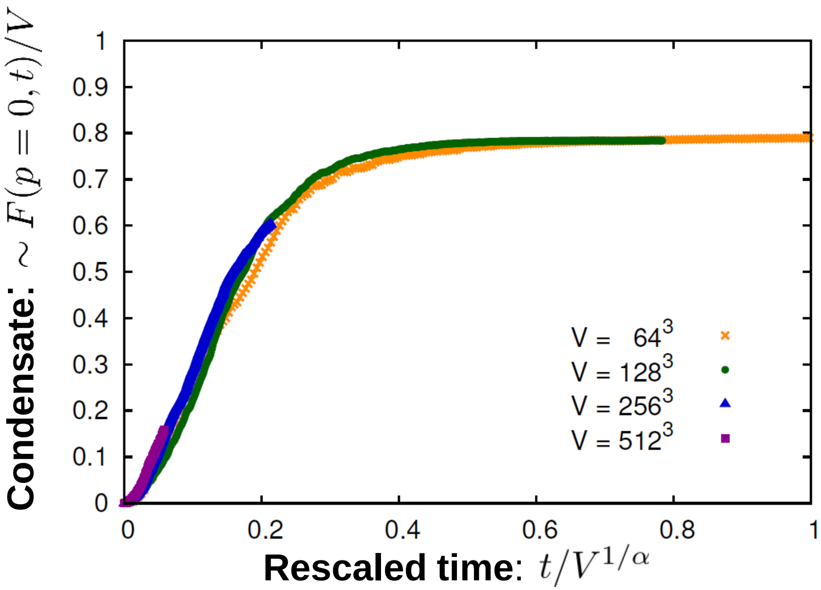
Self-similar evolution

$$f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)$$

Bose-Einstein condensation

Pinerio Orioli, KB & Berges, *PRD 92, 025041 (2015)*





Correlation function: $F(p,t) = \langle \varphi(\mathbf{p},t)\varphi(-\mathbf{p},t)\rangle$

Condensation: $F(p=0,t) \sim f(p=0,t) \sim t^{\alpha}$

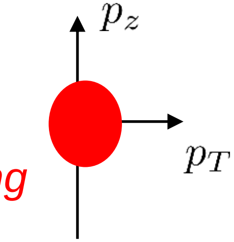
Stops when: $F(p=0,t_c) \sim (2\pi)^3 \delta^{(3)}(\boldsymbol{p})\, \phi^2 \sim V$

Condensation time: $t_c \sim V^{1/\alpha}$

In contrast: perturbative kinetic theory provides finite t_c

Semikoz & Tkachev (1994)

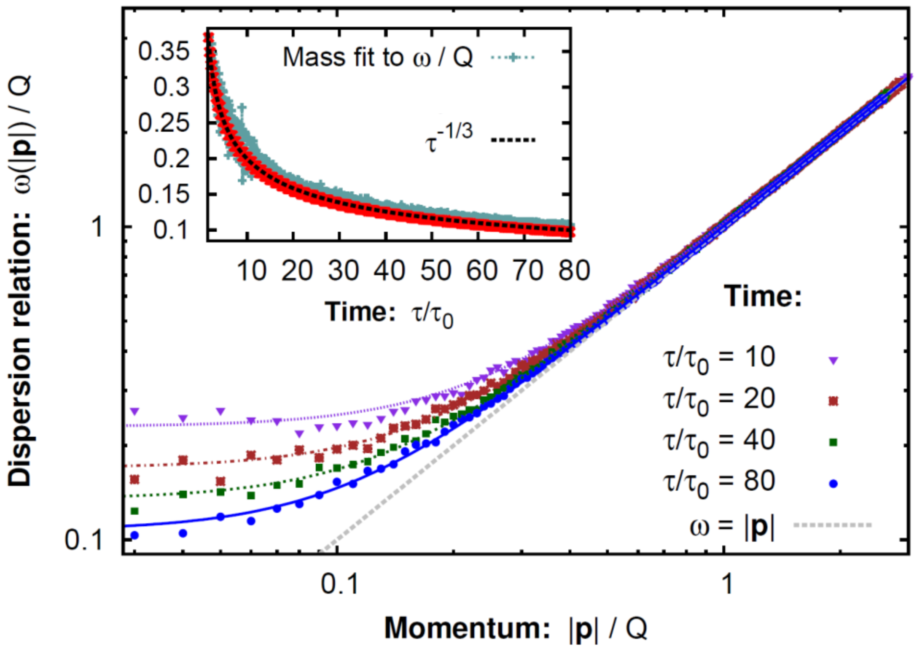
Universality classes and remaining puzzles



Scalar fields infrared scaling region: i)

Expanding

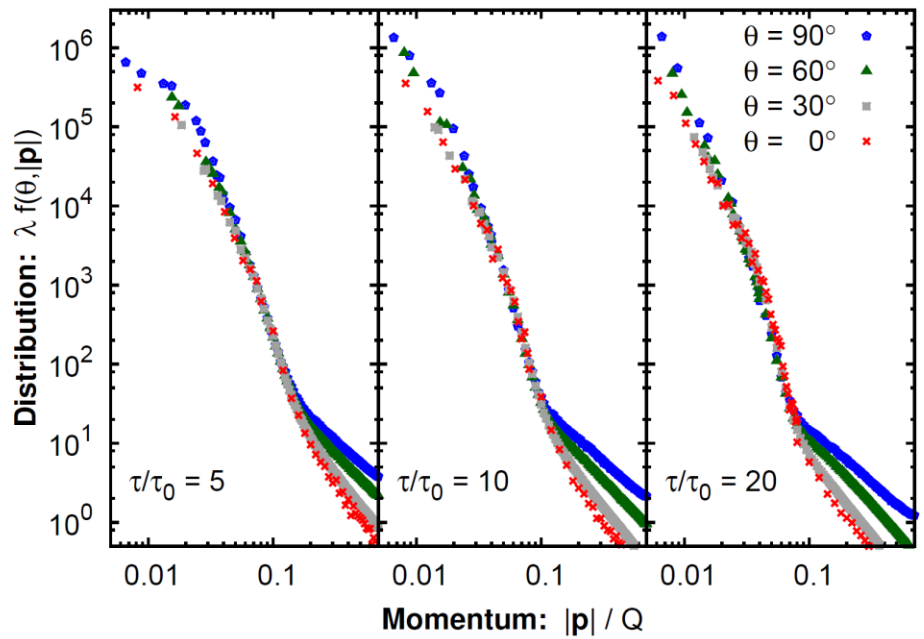
Dynamically generated *mass*



Dispersion relation fit: $\sqrt{m^2 + p^2}$

$$m(\tau) \sim \tau^{-1/3}$$

(approx.) *isotropic* distribution

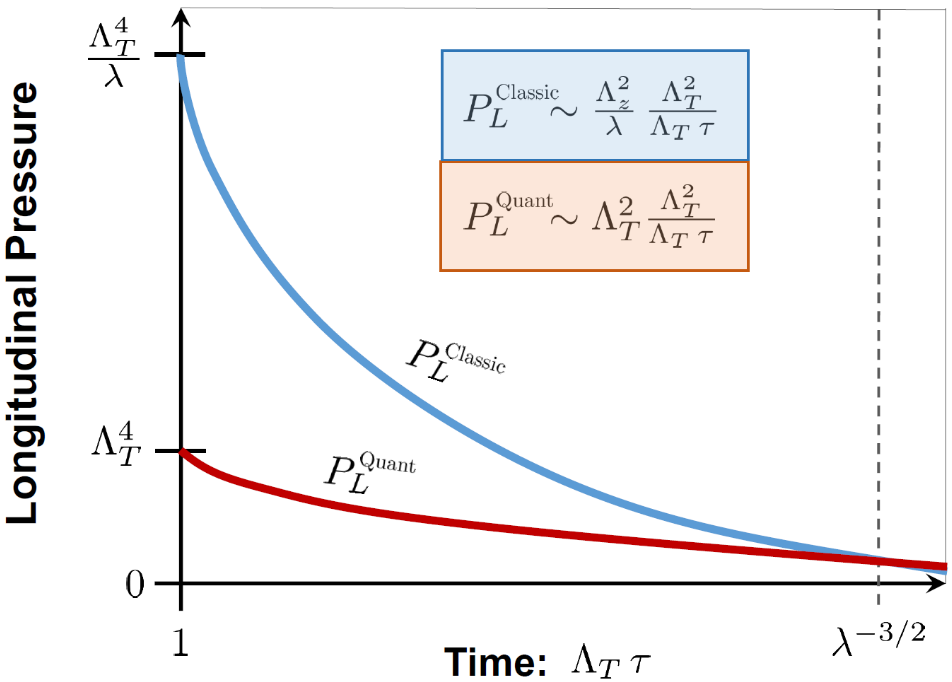


All momenta in i) below mass! $p \lesssim m$

Effectively nonrelativistic infrared region

Universality classes and remaining puzzles

Break-down of classical dynamics in scalar theory



Classical pressure (intermediate p_T)

$$P_L^{\rm Classic} \sim \int d^3p\, \frac{p_z^2}{p_T} f(p_T, p_z)$$

Quantum part: $\sigma_{\rm Large-angle} \sim \lambda^2/\Lambda_T^2$

$$\frac{dN^{\rm Coll}_{\rm Large-angle}}{d\tau} \sim \sigma_{\rm Large-angle} N_{\rm hard}(\tau)$$

$$N_{\rm Large-angle}(\tau) \sim N_{\rm hard}(\tau) \frac{dN^{\rm Coll}_{\rm Large-angle}}{d\tau}\tau$$

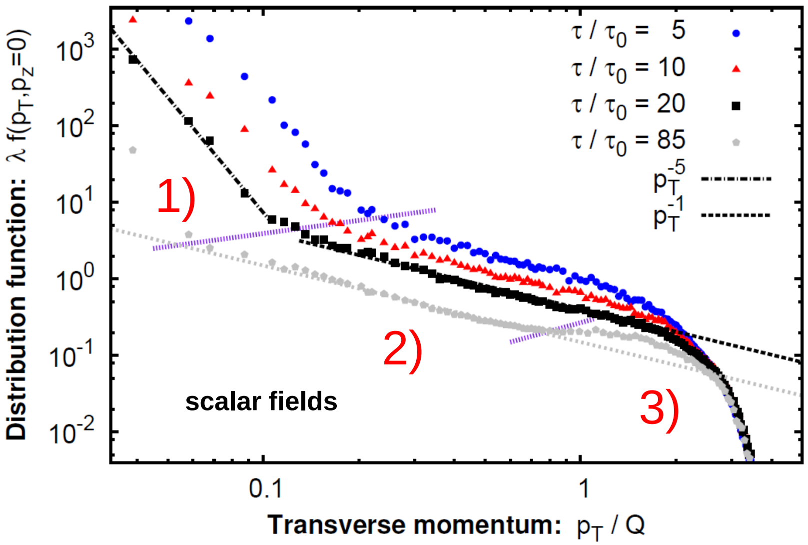
$$P_L^{\rm Quant}(\tau) \sim N_{\rm Large-angle}(\tau)\Lambda_T$$

Break-down at same time when classical approximation breaks down: $f \sim 1$

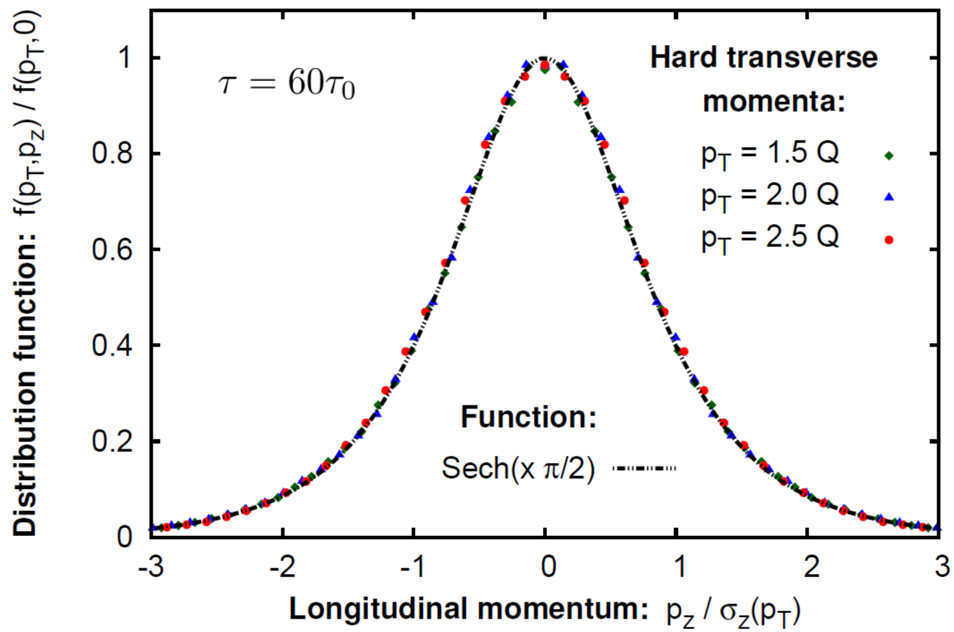
No new constraint!

Longitudinally expanding systems

Scalars fields hard-momentum fixed-point: Inertial range 3)



Longitudinal distribution at hard p_T



At late times a non-thermal fixed-point emerges at *large momenta*.

It has a hyperbolic secant shape, which has a broader tail than the Gaussian function.

Sign for large angle scatterings? $(2 \leftrightarrow 2)$

Longitudinally expanding systems

Scalars fields hard-momentum fixed-point: Inertial range 3)

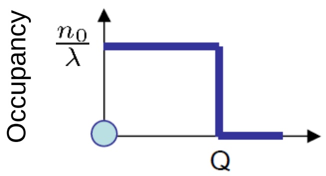
Self-similar evolution at large p_T

$$f(p_T, p_z, \tau) = \tau^{\alpha'} f'_S(p_T, \tau^{\gamma'} p_z)$$

Longitudinal hard scale

$$\Lambda_L^2(\tau) \approx \frac{\int d^2p_T \int dp_z\ p_z^2\, \omega(\boldsymbol{p})\ f(p_T, p_z, \tau)}{\int d^2p_T \int dp_z\ \omega(\boldsymbol{p})\ f(p_T, p_z, t)}$$
$$\sim \tau^{-2\gamma'}$$

Independent of initial conditions!



$$f(p_T, p_z, \tau_0) = \frac{n_0}{\lambda}\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

Logarithmic slope of hard scale