The phase diagram of QCD from low energy models

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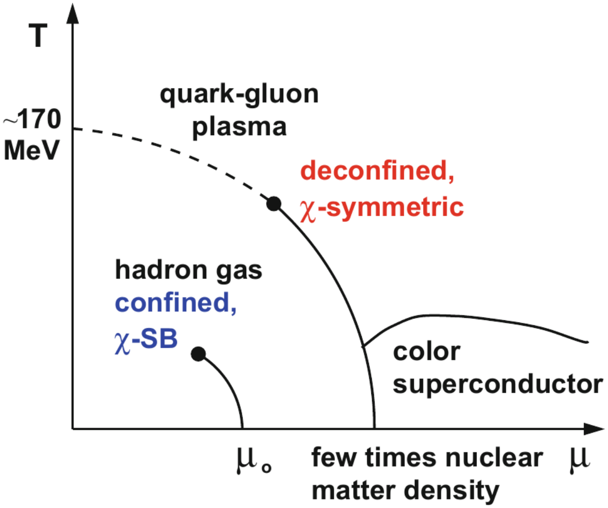
Overview

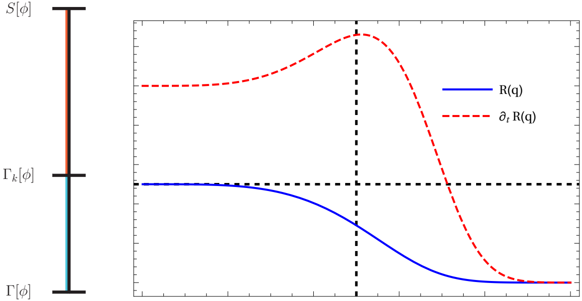
1. The QCD phase diagram - very brief
2. The FRG - very brief
3. The Quark-Meson model
   - Effective scales
4. The Polyakov-Quark-Meson model

The phase diagram of QCD



(Fig. from CBM physics book, Lect. Notes in Physics 814, Springer)

The Functional Renormalization Group

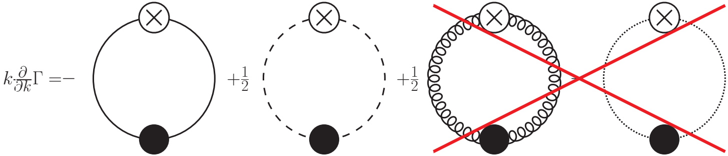


- Wetterich equation:

$$k\frac{\partial}{\partial k}\Gamma_k = \mathrm{STr}\left[k\frac{\partial}{\partial k}R_k\left(\Gamma^{(2)}+R_k\right)^{-1}\right]$$

The Quark-Meson model

The Quark-Meson model in the FRG



- gauge sector decouples at low energies, matter sector drives dynamics

$$\mathcal{L}_{\mathrm{QM}} = \bar{\psi}\left(\partial\!\!\!/ + h\left(\sigma T^0 + i\gamma^5\pi^a T^a\right)\right)\psi$$
$$+ \partial_\mu\pi_i\partial_\mu\pi_i + \partial_\mu\sigma\partial_\mu\sigma + V\left(\pi^2 + \sigma^2\right)$$

- model shows chiral symmetry breaking

- commonly used initialization scale scale: ~ 1 GeV, above chiral symmetry breaking scale

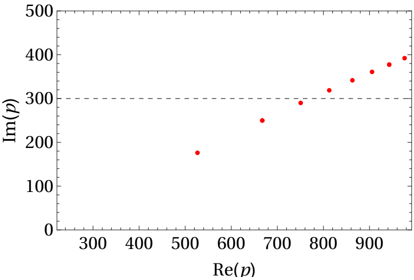
Truncation

- Our Truncation: LPA (no dressing), constant Yukawa coupling
- Yukawa coupling is approximately constant (from full calculation) below ~ 1 GeV (Mitter, Pawlowski, Strodhoff Phys.Rev. D91 (2015) 054035)
- Possible extensions: Field dependent Yukawa coupling and dressing functions change crossover temperature (Pawlowski, Rennecke Phys.Rev. D90 (2014) no.7, 076002 ,Helmboldt, Pawlowski, Strodthoff Phys.Rev. D91 (2015) no.5, 054010)

The phase diagram of the Quark-Meson model in the FRG so far

- Finite chemical potential → complex momenta $p_0 \to p_0 + i\mu$
- common approach: 3d regulators, leave p_0 direction unregularized → can perform trace and get analytical expressions
- problem: why single out one direction?

solution: 4d regulators; best: some analytical smooth cut-off function (fermionic:) (Fister, Pawlowski Phys.Rev. D92 (2015) no.7, 076009, Pawlowski, Strodthoff Phys.Rev. D92 (2015) no.9, 094009)



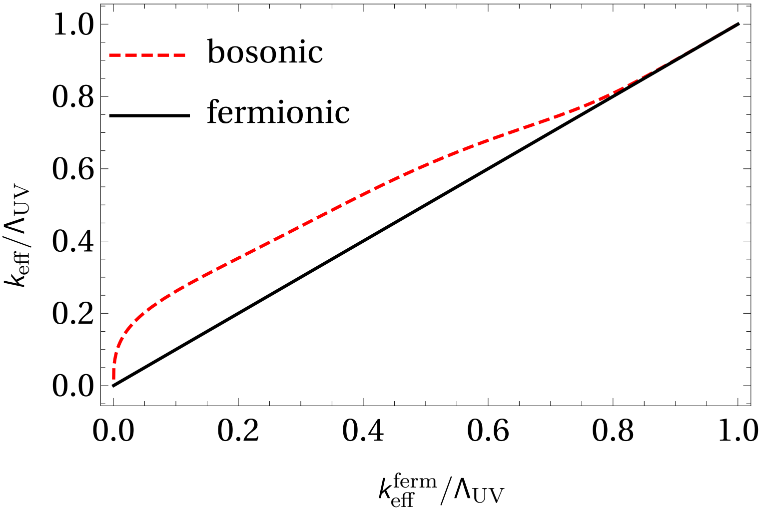
Effective scales in the FRG

- assume a theory with regulators evaluated at scale k. Now assume the same theory always at $c\,k$ $\rightarrow$ FRG only tells us that it is the same at $k = 0$. What happens in between? What if we have a mixed theory with both?
- Likely scenario: completely different regulators for bosons and fermions depending on the choices of Δm
- Solution: physical scales (Pawlowski Annals Phys. 322 (2007) 2831-2915 , Pawlowski, Scherer, Schmidt, Wetzel arXiv:1512.03598)
- map physical scales onto each other (applicable for mixed theories)

$$\frac{1}{k_{\text{eff}}^{d}} = \max_{p} |G(p)| \Big|_{m=0}$$

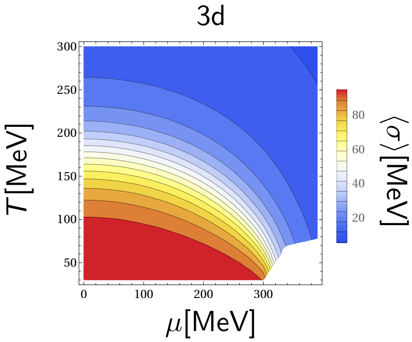
$$k_{\text{eff}}^{\text{bos}}\left(\tilde{k}\right) \overset{!}{=} k_{\text{eff}}^{\text{ferm}}(k)$$

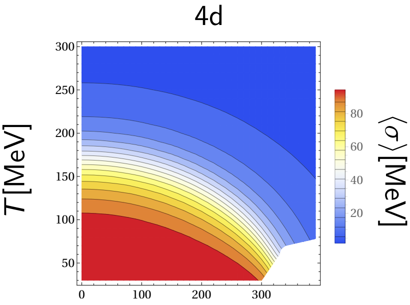
Effective scales

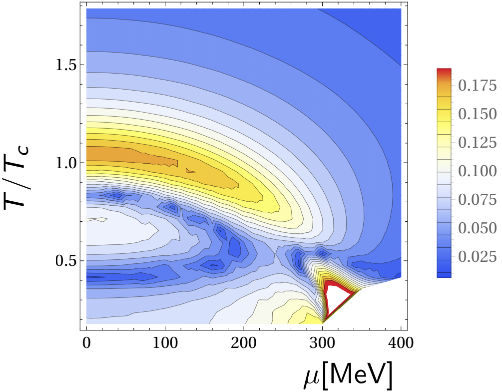


The phase diagram of the QM model 3d vs. 4d

Comparison of LPA (no wavefunction-renormalization factors) results for 3d and 4d







$$\frac{\langle\sigma\rangle_{4d}}{\langle\sigma\rangle_{4d}(0)}\sqrt{\left|\frac{\langle\sigma\rangle_{4d}^2-\langle\sigma\rangle_{3d}^2}{\langle\sigma\rangle_{4d}^2+\langle\sigma\rangle_{3d}^2}\right|}$$

The Polyakov-Quark-Meson model

The Polyakov Loop

- Quark-Meson model does not show confinement, no gauge fields taken into account

- Order Parameter for confinement: expectation value of Polyakov loop

$$L\left[A_0\right] = \frac{1}{N}\mathrm{Tr}_f\left[\mathcal{P}e^{ig\int_0^\beta dx_0 A_0(x_0,\vec{x})}\right]$$

$$\langle L[A_0]\rangle \begin{cases} = 0 & \text{confined} \\ > 0 & \text{deconfined} \end{cases}$$

The Polyakov Loop part II

- Different order parameter $L\left[\langle A_0\rangle\right]$: Go to Polyakov gauge (A_0 depends on $\vec{x}$ only and is rotated into Cartan)

$$L\left[A_0\right] = \frac{1}{N}\mathrm{Tr}_f\, e^{g\beta A_0} = \frac{1}{N}\mathrm{Tr}_f\, e^{2\pi i\varphi}$$

- Single out expectation value of A_0 from minimum of effective potential $V\left(A_0\right)$

- Jensen inequality:

$$\langle L\left[A_0\right]\rangle \leq L\left[\langle A_0\rangle\right]$$

- Which order parameter should we use? (Herbst, Luecker, Pawlowski arXiv:1510.03830)

Including the Polyakov loop into the model

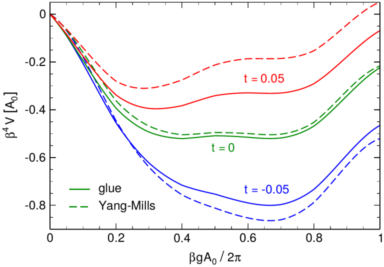
- How to include confinement into the QM model? Use background potential. (Schaefer, Pawlowski, Wambach Phys.Rev. D76 (2007) 074023, Herbst, Pawlowski, Schaefer Phys.Lett. B696 (2011) 58-67)
- explicit appearance of A_0 via covariant derivative in our equations, use $L\left[\langle A_0 \rangle\right]$
- Perturbative potential known (Weiss Phys.Rev. D24 (1981) 475, Gross, Pisarski, Yaffe Rev.Mod.Phys. 53 (1981) 43)
- Non-perturbative potential from fit, $\varphi = \beta g A_0 / 2\pi$ (Herbst, Luecker, Pawlowski arXiv:1510.03830, Fister, Pawlowski Phys.Rev. D88 (2013) 045010)

$$V_{SU(2)}(\varphi) = a(T) V_W(\varphi) + b(T) V_W^2(\varphi)$$
$$V_{SU(N)} = \sum_{\text{adj.EV}} V_{SU(2)}(\varphi) \tag{1}$$

Backreaction

- Backreaction of quarks on the gauge sector
- Rescaling of reduced temperatures mimics backreaction (Haas, Stiele, Braun, Pawlowski, Schaffner-Bielich Phys.Rev. D87 (2013) no.7, 076004 , Herbst, Mitter, Pawlowski, Schaefer, Stiele Phys.Lett. B731 (2014) 248-256)

$$t = \frac{T - T_{\rm crit}}{T_{\rm crit}}$$
$$t_{\rm YM}\left(t_{\rm glue}\right) \approx 0.57\, t_{\rm glue}$$

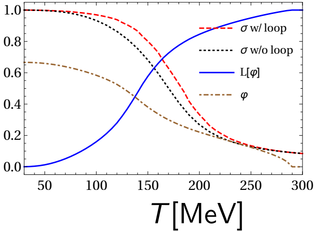


- TODO: fix scales between background potential and our computations, e.g. via T_c in the chiral limit, deconfinement and chiral critical temperatures should coincide (Braun, Haas, Marhauser, Pawlowski Phys.Rev.Lett. 106 (2011) 022002)

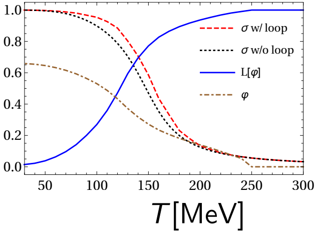
Chiral and deconfinement crossover at vanishing density

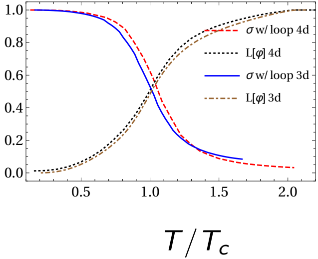
Preliminary & w/o scale fixing

3d



4d





critical temperatures

	T_c[MeV]
3d, conf	141
4d, conf	123
3d, χ	180
4d, χ	154

Summary and outlook

1. necessity of effective scales for mixed theories
2. phase diagram of QM model with 4d reg. $\rightarrow$ necessary for quantitative full QCD calculations
3. Background potential of the gauge field instead of the Polyakov loop variable should be used

Outlook: PQM at finite μ in progress

Thank you for your attention.